Hydrodynamic Long-time Tails in Large-N Systems

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Gauge/Gravity Duality

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1. Outline

- Fluid mechanics
- Hydrodynamics
- Dynamical fluctuations
- Feedback on hydrodynamics
- Large N systems
- SK-EFT
- Prediction for non-causal diffusion→ experiment
- Prediction for causal diffusion → experiment
- Why important?

2. Fluid mechanics!

Let us consider a fluid system:

fluid element

"The smallest component in fluid"

- Dynamics:

$$\boxed{\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \mathbf{grad})\mathbf{v} = -\frac{1}{\rho}\,\mathbf{grad}\,p + \frac{\eta}{\rho} \triangle\,\mathbf{v}.}$$

[Euler, 1755]

[Navier, Stokes, 1822-1850]

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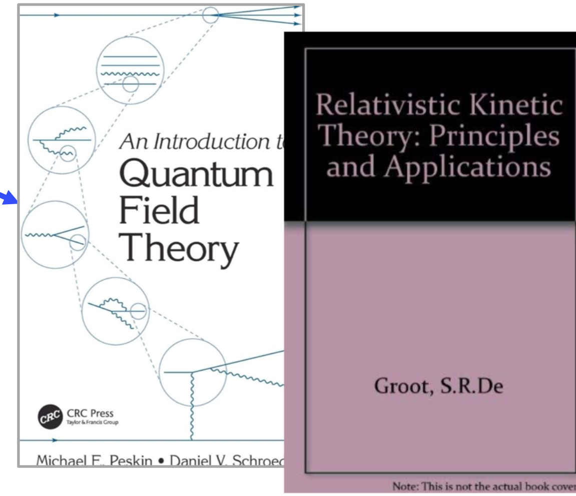

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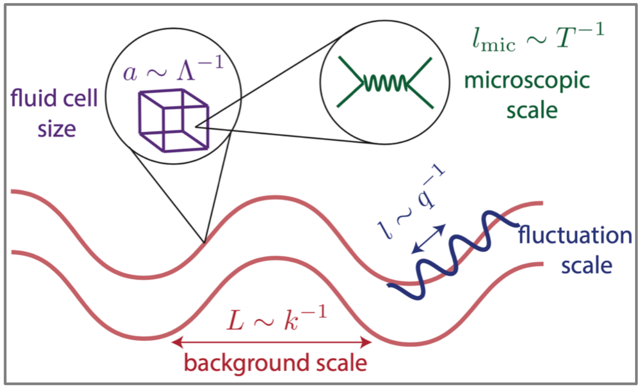
- $\ell_{mic} \ll a$

3. Hydrodynamics

Extend to more general systems:



[Xin An, Holotube]

In the limit: $\ell_{mic} \ll L \quad \rightarrow \quad \ell_{mic} \ll a \ll L$

- QFT inside the box of size a is integrated out: "fluid cell"
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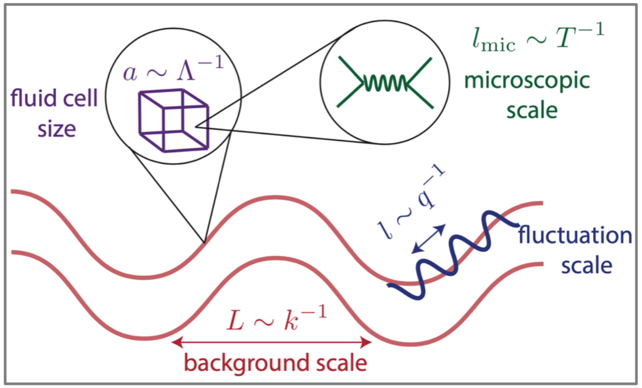

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4. Structure of $T^{\mu\nu}(x)$ in hydrodynamics

- From a microscopic viewpoint: (in scalar QFT)

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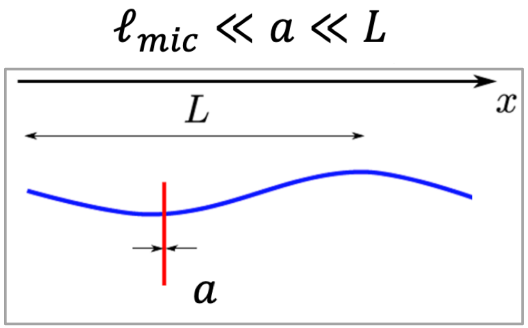

- In the hydrodynamic limit:
  - local equilibrium: $\rho_{eq}(x) = \frac{1}{Z} e^{\beta(x) P^\mu u_\mu(x)}$
  - Dof $T^{\mu 0}(x) = \langle \hat{T}^{\mu 0} \rangle$
  - Stat-FT techniques: $T^{\mu\nu}(x) = \mathrm{Tr}\left(\rho_{eq}(x)\hat{T}^{\mu\nu}\right)$

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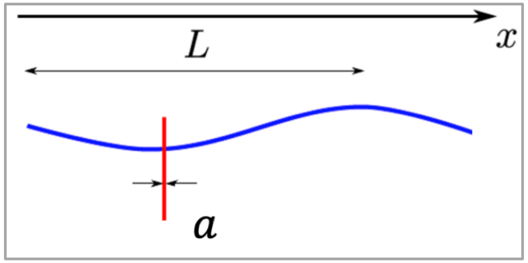

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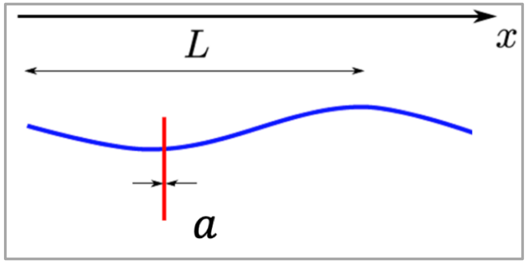

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$$\downarrow$$

scalar field theory : $\epsilon(x) = 3\,p(x) = \frac{\pi^2 T(x)^4}{30}$

5. Deviation from local equilibrium

- Local equilibrium ≡ **leading term in the expansion over** $\varepsilon = \frac{\ell_{mic}}{L}$

- Out of Local equilibrium: **derivative expansion**: $\varepsilon = \frac{\ell_{mic}}{L} \sim \ell_{mic}\partial$

$$\boxed{\mathrm{Tr}\Big[\big(\rho_{eq}(x) + \delta\rho(x)\big)\hat{T}^{\mu\nu}\Big] = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + \mathcal{O}(\partial^2)}$$

ideal fluid

viscous

$$T^{\mu\nu}_{(1)} = -\eta\sigma^{\mu\nu}(x) - \xi\Delta^{\mu\nu}\partial_\mu u^\mu(x)$$

- Exp: In Scalar QFT:

$$\boxed{\hat{T}^{00}(x) = \frac{1}{2}\big(\dot{\hat{\varphi}}(x)\big)^2 + \frac{1}{2}\big(\nabla\hat{\varphi}(x)\big)^2 + \frac{1}{4!}\lambda\,\hat{\varphi}(x)^4}$$

$$\epsilon(x) = 3\,p(x) = \frac{\pi^2 T(x)^4}{30} - \frac{\pi^2\lambda}{16}$$

$$\eta(x) = a\frac{T(x)^3}{\lambda^2}, \qquad \xi = 0$$

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- So far dissipation; what about fluctuations?

6. Diffusion scale

- In the static background:

$$\langle H^2 \rangle - \langle H \rangle^2 = k_B T^2 \, C_V$$

$$\left\langle \delta\epsilon(\vec{x}_1) \, \delta\epsilon(\vec{x}_2) \right\rangle = k_B T^2 c_V \, \delta^3(\vec{x}_1 - \vec{x}_2)$$

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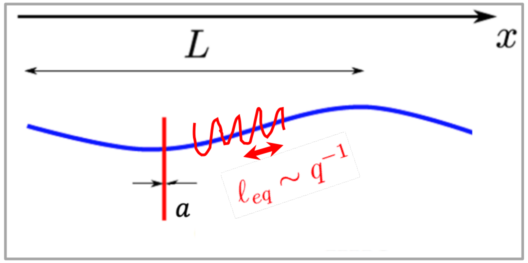
- In the hydro limit:
  - non-zero correlations

$$\boxed{\tau_{ev} = \frac{L}{c_s}} \xrightarrow{x\sim\sqrt{Dt}} \boxed{\ell_{eq}\sim\sqrt{\frac{D\,L}{c_s}} \ll L}$$



[An, Basar, Stephanov, Yee PRC (2019)]

  - Correlations find a finite **width**

$$G(\vec{x}_1,\,\vec{x}_2) \quad\to\quad G(t,\vec{x}_1,\,\vec{x}_2) \equiv G(t,\vec{x},\vec{y})$$

  - The separation of scales suggests to work with:

$$\xrightarrow{\ell_{mic}\ll a \ll |\vec{y}| \ll L} \boxed{G(x,\boldsymbol{y}) = \int\frac{d^3q}{(2\pi)^3}G(x,\boldsymbol{q})e^{i\boldsymbol{q}\cdot\boldsymbol{y}}}$$

$$k \ll q \ll \Lambda \ll \ell_{mic}^{-1}$$

7. Fluctuations dynamics

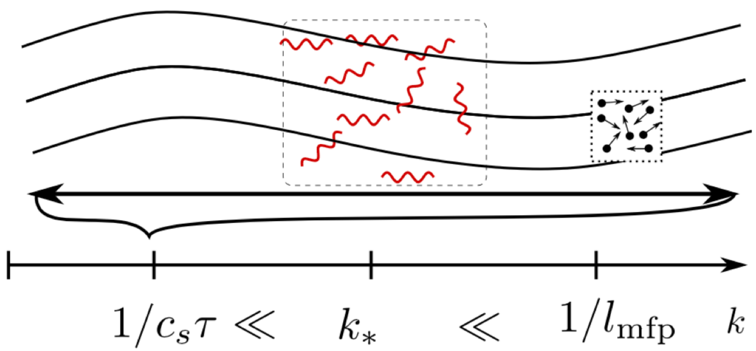
Equal-time correlators:

$$\left\langle \delta\phi_{a_1}(x)\delta\phi_{a_2}(x)\right\rangle = \int \frac{d^3k}{(2\pi)^3} G_{a_1 a_2}(x, \boldsymbol{k})$$

hydrodynamic 2pt function

(hydrodynamic fluctuation)

- How equilibrate?
- Feedback on the flow?
- How large?
- How to calculate?



[Akamatsu, Mazeliauskas, Teaney PRC (2017)]

8. Stochastic hydro

- Landau-Lifshitz idea:
  - $\partial_\mu T^{\mu\nu}(x) = 0$ are indeed $\partial_\mu \langle \breve{T}^{\mu\nu}(x) \rangle = 0$
  - In general, stochastic: $\partial_\mu \breve{T}^{\mu\nu}(x) = 0$

$$\boxed{\underbrace{\breve{T}^{\mu\nu}(x)}_{\text{Stochastic EM tensor}} = \mathrm{Tr}\Big(\rho(x)\,\hat{T}^{\mu\nu}\Big) = \underbrace{\breve{T}^{\mu\nu}_c(x)}_{\text{EM tensor with stochastic fields}} + \underbrace{S^{\mu\nu}(x)}_{\text{random noise}}}$$

Stochastic EM tensor

random noise

EM tensor with stochastic fields

$$\breve{T}^{\mu\nu}_c = \underbrace{(\breve{\epsilon} + \breve{p})\breve{u}^\mu \breve{u}^\mu + \eta^{\mu\nu}\breve{p}}_{\mathcal{O}(\partial^0)} - \underbrace{\eta\breve{\sigma}^{\mu\nu} - \zeta\breve{\Delta}^{\mu\nu}\partial_\mu \breve{u}^\mu}_{\mathcal{O}(\partial)} + \mathcal{O}(\partial^2)$$

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- Landau-Lifshitz focuses on linear order.

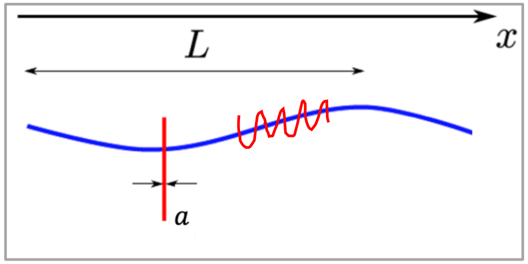
9. Kinetic-hydro

- Kinetic hydro for simple diffusion: $\partial_t \breve{n} - D\nabla^2 \breve{n} = \theta$

$$\xrightarrow{G_{nn}(t,\mathbf{k}) = \left\langle \delta n(t,\mathbf{k})\delta n(t,-\mathbf{k})\right\rangle} \quad \partial_t G_{nn}(t,\mathbf{k}) = -2D\mathbf{k}^2\big(G_{nn} - \bar{G}_{nn}\big)$$

FDT

- Back reaction on **J**:
  - Stochastic current $\breve{\mathbf{J}} = -\left(D_0 + D_1\breve{n} + \cdots\right)\boldsymbol{\nabla}\breve{n}$
  - Averaged current: contains $\int \frac{d^3k}{(2\pi)^3} G_{nn}(t,\mathbf{k})$



$$\boxed{\left\langle \breve{\mathbf{J}} \right\rangle = \Big(D_0\, n + \mathcal{O}(\Lambda^3 T) + D_1\chi\,\big(\frac{\omega}{D_0}\big)^{3/2} T\Big)\mathbf{k}}$$

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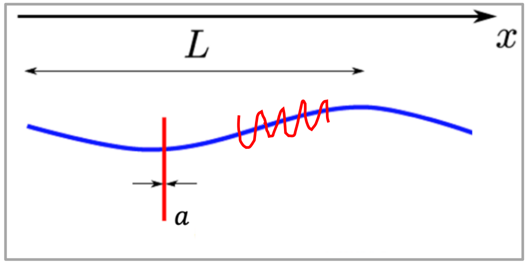

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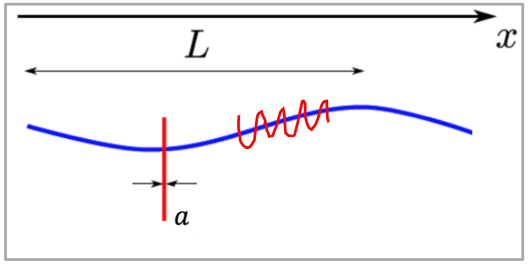

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renormalization

Longtime tail

$$\mathbf{J} \sim \frac{1}{t^{5/2}}\mathbf{k}$$

10. Largeness of fluctuations

- From our calculations

$$\frac{dD}{dn}$$

$$\left\langle \breve{\mathbf{J}} \right\rangle = \Big(\underbrace{D_0\, n}_{\mathcal{O}(\chi)} + \mathcal{O}(\Lambda^3 T) + \underbrace{D_1 \chi \left(\frac{\omega}{D_0}\right)^{3/2} T}_{\mathcal{O}(1)} \Big)\mathbf{k}$$

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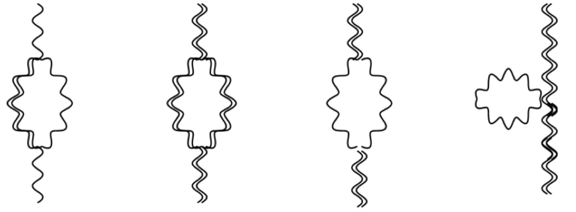
- The effect of fluctuations should be small in
    - Weakly coupled systems
    - Strongly coupled systems at large N

11. Large N systems

- In holography, usually linear hydro is sufficient!
- Nonlinear effects are suppressed by $\frac{1}{N^{\#}}$

[Saremi Caron-Huot JHEP 2009]

$$ {}^{(1)}G_{\mathrm{R}}^{xx}(\omega, k{=}0) \simeq \frac{T\Xi}{2w}(d-2)\Gamma(\frac{1-d}{2})\left(\frac{-i\omega}{4\pi(D+\gamma_\eta)}\right)^{\frac{d-1}{2}} $$



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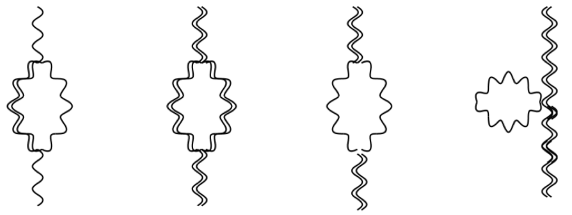

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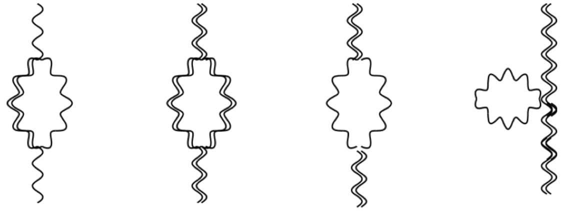

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interacting EFT ~ nonlinear effects in hydro

loop effect in EFT ~ longtime tail

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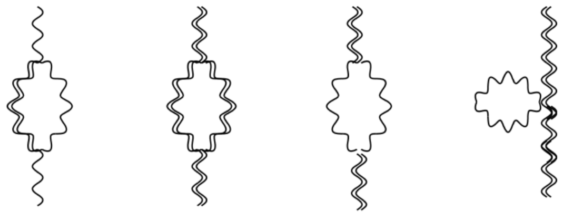

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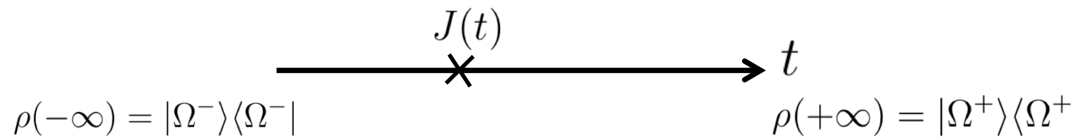
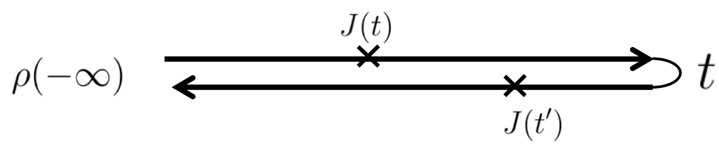
- Towards Schwinger Keldysh EFT

12. SK-EFT

- QFT at $T = 0$:

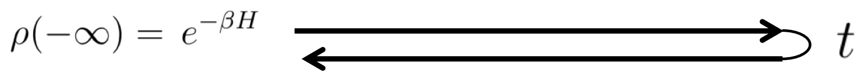


- QFT at $T > 0$:



CTP contour

- SK-EFT:
  - QFT on a CTP



  - Integrate out the fast modes
  - Express the long wavelength limit by "conserved densities"
  - In the simple case of diffusion: $S_{EFT} \equiv S[n(x), \phi_a(x)]$
  - $\langle n(x_1) n(x_2) \rangle$ and then $G_{JJ}(x_1, x_2)$

[Crossley, Glorioso, Liu]
[Jensen, Pinzani-Fokeeva, Yarom]
[Heahl, Loganayagam, Rangamani]

- How to construct SK-Lagrangian:
  - Classical hydro equation = EoM
  - Symmetries of $\rho(-\infty)$

13. Simple diffusion

- Theory of diffusive fluctuations

[Chen-Lin, Delacretaz, Hartnoll, PRL (2019)]

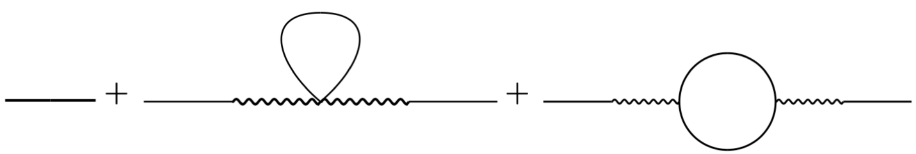
$$\mathcal{L} = iT^2\kappa(\nabla\varphi_a)^2 - \varphi_a\left(\dot{n} - D\nabla^2 n\right) + \nabla^2\varphi_a\left[\frac{1}{2}\lambda n^2 + \frac{1}{3}\lambda' n^3\right] + icT^2(\nabla\varphi_a)^2\left[\tilde{\lambda} n + \tilde{\lambda}' n^2\right] + \cdots$$

  - EoM at leading order: $\partial_t n - D\nabla^2 n = 0$

$$\langle n\varphi_a\rangle_p = \frac{1}{\omega + iDk^2}$$

$$\langle nn\rangle_p = \frac{2T\chi Dk^2}{\omega^2 + (Dk^2)^2}$$

  - EoM at sub-leading order: $\partial_t n - D\nabla^2 n - \frac{1}{2}\lambda\boldsymbol{\nabla}^2 n^2 = 0$



$$\delta D(\omega, \mathbf{k}) = \frac{\lambda^2 T\chi}{4D^{3/2}}(-i\omega)\left(D\mathbf{k}^2 - 2i\omega\right)^{-1/2}$$

[NA, Kaminski, Tavakol PRL 2024]

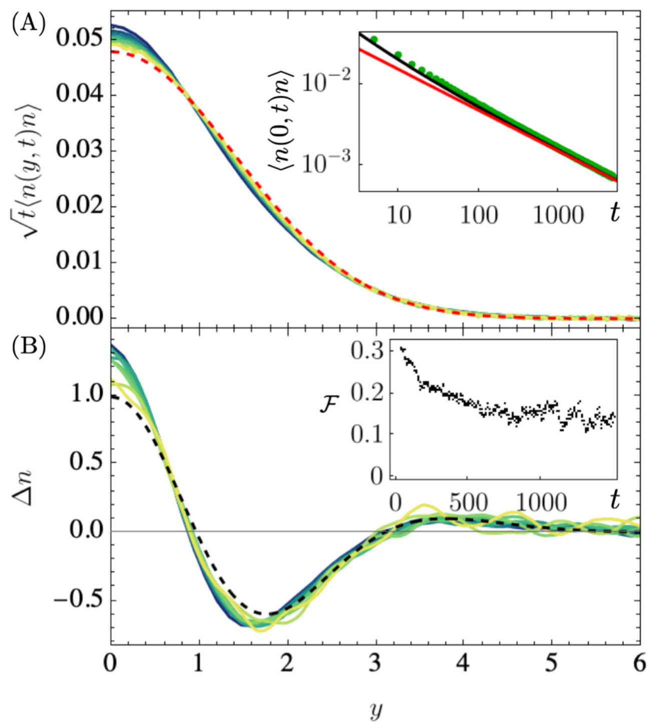
14. Precision test of SK-EFT

- Real time

$$\langle n(x,t)n\rangle = \frac{\chi}{\sqrt{4\pi D t}}\left[F_{0,0}(y) + \frac{1}{\sqrt{t}}F_{1,0}(y) + O\left(\frac{\log t}{t}\right)\right]$$

$$F_{0,0}(y) = e^{-y^2/4}$$
$$y \equiv x/\sqrt{Dt}$$

- Simulation results



$$\Delta n \equiv \sqrt{t}\left[\langle n(x,t)n\rangle - \frac{\chi}{\sqrt{4\pi D t}}F_{0,0}(y)\right]$$

[Michailidis, Abanin, Delacretaz PRX (2024)]

15. Causal diffusion

- A closed look

$$\langle n(x,t)n\rangle = \frac{\chi}{\sqrt{4\pi D t}}\left[F_{0,0}(y) + \frac{1}{\sqrt{t}}F_{1,0}(y) + O\left(\frac{\log t}{t}\right)\right]$$

$$F_{0,0}(y) = e^{-y^2/4}$$
$$y \equiv x/\sqrt{Dt}.$$

[Michailidis, Abanin, Delacretaz PRX (2024)]

The spread of fluctuations is non-causal!

- Towards a causal theory of diffusion:

$$\mathbf{J} = -D\,\boldsymbol{\nabla} n \xrightarrow{\partial_t n + \boldsymbol{\nabla}\cdot\mathbf{J} = 0} \partial_t n - D\boldsymbol{\nabla}^2 n = 0$$

[Cattaneo, Atti Semin.Mat.Fis.Univ.Modena 3 (1948)]
[Kadanoff, Martin, Annals of Physics (1963)]

15. Causal diffusion

- A closed look

$$\langle n(x,t)n\rangle = \frac{\chi}{\sqrt{4\pi D t}}\left[F_{0,0}(y) + \frac{1}{\sqrt{t}}F_{1,0}(y) + O\left(\frac{\log t}{t}\right)\right]$$

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$$y \equiv x/\sqrt{Dt}.$$

[Michailidis, Abanin, Delacretaz PRX (2024)]

The spread of fluctuations is non-causal!

- Towards a causal theory of diffusion:

$$\tau\,\partial_t \mathbf{J} + \mathbf{J} = -D\,\nabla n \xrightarrow{\partial_t n + \nabla\cdot\mathbf{J} = 0} \tau\partial_t^2 n + \partial_t n - D\nabla^2 n = 0$$

[Cattaneo, Atti Semin.Mat.Fis.Univ.Modena 3 (1948)]
[Kadanoff, Martin, Annals of Physics (1963)]

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$$\langle n(x,t)n\rangle = \frac{\chi}{\sqrt{4\pi D t}}\left[F_{0,0}(y) + \frac{1}{\sqrt{t}}F_{1,0}(y) + O\left(\frac{\log t}{t}\right)\right]$$
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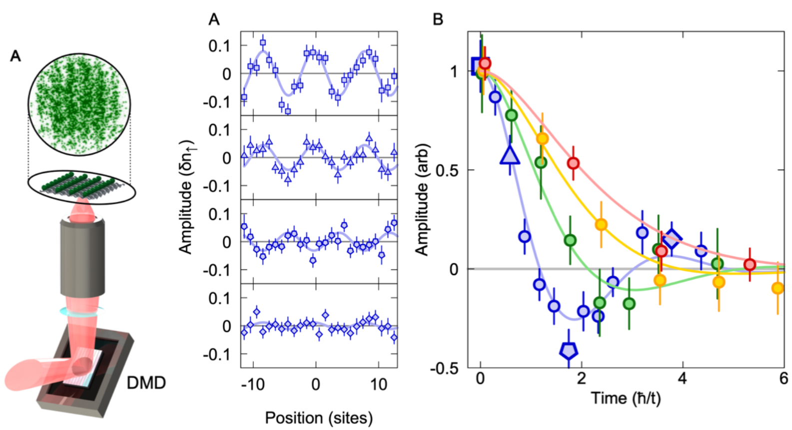
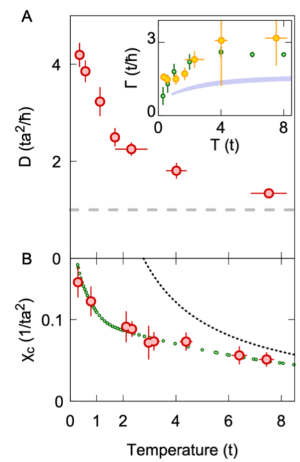
[Cattaneo, Atti Semin.Mat.Fis.Univ.Modena 3 (1948)]
[Kadanoff, Martin, Annals of Physics (1963)]

- Experimental realization:





[Brown et al, Science, (2019) 1802.09456]

16. Fluctuations in Causal theory

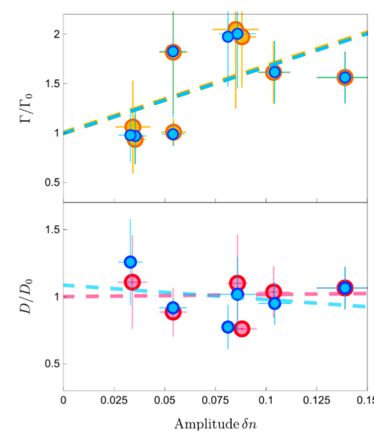
- SK-EFT for causal nonlinear diffusion

$$\mathcal{L} = iT\sigma(\nabla n_a)^2 - n_a\left(\tau\partial_t^2 n + \partial_t n - D\nabla^2 n\right) + iT\chi\lambda_\sigma\, n(\nabla n_a)^2 + \frac{\lambda_D}{2}\,\nabla^2 n_a\, n^2 + \frac{1}{2} iT\chi\lambda'_\sigma\, n^2(\nabla n_a)^2 + \frac{\lambda'_D}{6}\,\nabla^2 n_a\, n^3$$

- Loop corrections

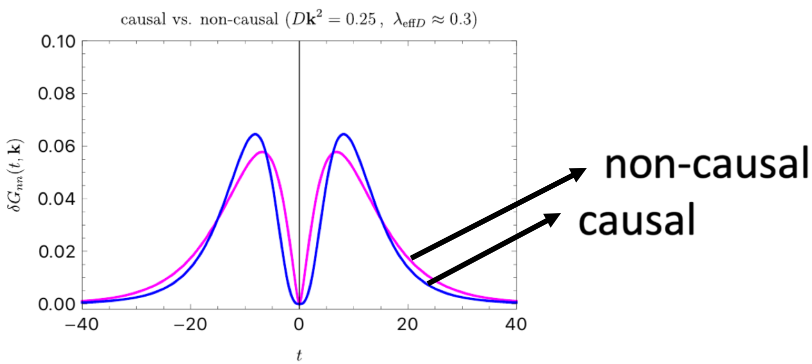
$$G^R_{nn}(\omega,\mathbf{k}) = \frac{i\,\sigma\,\mathbf{k}^2}{-i(\tau+\delta\tau(\omega,\mathbf{k}))\omega^2 + \omega + i(D+\delta D(\omega,\mathbf{k}))\mathbf{k}^2}$$

  - $\delta\tau(\omega,\mathbf{k}) = 0$: consistent with fitting results



[NA, Kaminski, Tavakol PRL 2024]

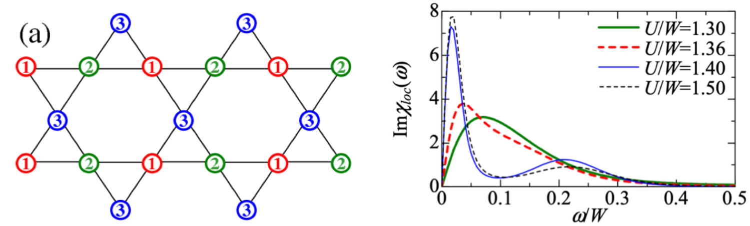
  - Correction to the correlation function



17. Condensed matter systems with large N

- Hubbard model → Mott transitions …

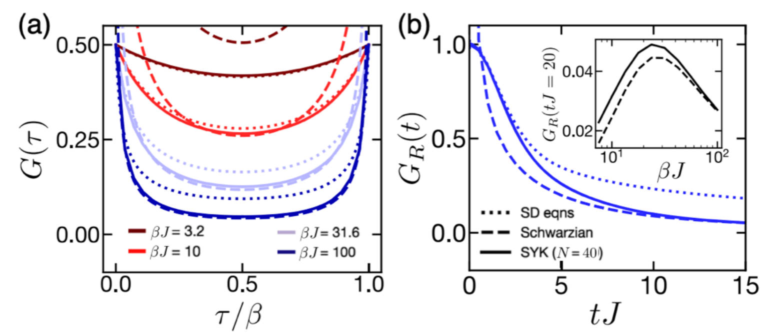
$$H = -t \sum_{\langle i,j\rangle,\sigma} (c^\dagger_{i\sigma} c_{j\sigma} + \mathrm{h.c.}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



[Ohashi, Kawakami, Tsunetsugu PRL (2006)]

- SYK model at large N → quantum spin fluid

$$H = \frac{6}{N^3} \sum_{i<j<k<l} J_{ijkl} \chi_i \chi_j \chi_k \chi_l .$$



[Kobrin, Yang, Kahanamoku-Meyer, Olund, Moore, Stanford, Yao PRL (2021)]

- …

18. Discussion

- We expect it works for a wider range time scales in large-N systems.

  [NA, Kaminski, Tavakol PRL 2024]

- We expect the prediction of causal EFT to be applicable to a wider range of times scales.

  [NA, Kaminski, Tavakol PRL 2024]

- An interesting model to explore the “corrections”: Holography.

  [Liu, Sun, Wu 2411.16306]
  [Baggioli, Bu, Ziogas JHEP (2023)]
  [Bu, Fujita, Lin JHEP (2021)]

Thank you for your attention!

- SK-EFT universally ties:
    1. Nonlinear response
    2. Corrections to linear response
    3. Dependence of D on experimental tuning parameter n: $D_1 = \frac{dD}{dn}$

[Michailidis, Abanin, Delacretaz PRX (2024)]

Thank you for your attention!

$$n \quad \to \quad n\left(1+\frac{\langle \delta n^2\rangle}{n^2}\right) \sim n + \frac{\left(\frac{T\chi}{V}\right)}{n} \sim n + \frac{T\chi \int k^2 dk}{n}$$

$$\boxed{k^* \sim \left(\frac{\omega}{D_0}\right)^{1/2}}$$

$$\boxed{\frac{1}{n} \sim \frac{D_1}{D_0}}$$