中国科学院大学
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Holographic description of QCD

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Gauge Gravity Duality 2024, Sanya, Nov.30-Dec.4,2024

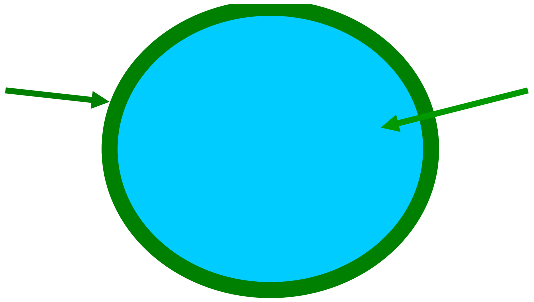
From UV to IR: RG flow and holography

AdS/CFT : Original discovery of duality

Juan Martin Maldacena. Adv. Theor. Math. Phys., 2:231–252, 1998
Edward Witten. Adv. Theor. Math. Phys., 2:253–291, 1998.

Holographic Duality: (d+1)-Gravity/ (d)-QFT

$$Z_{\text{gauge}} = Z_{\text{AdS}} \; ,$$



AdS/CFT as an RG flow

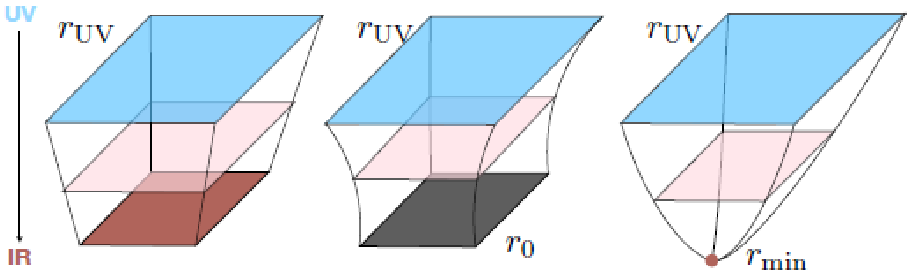


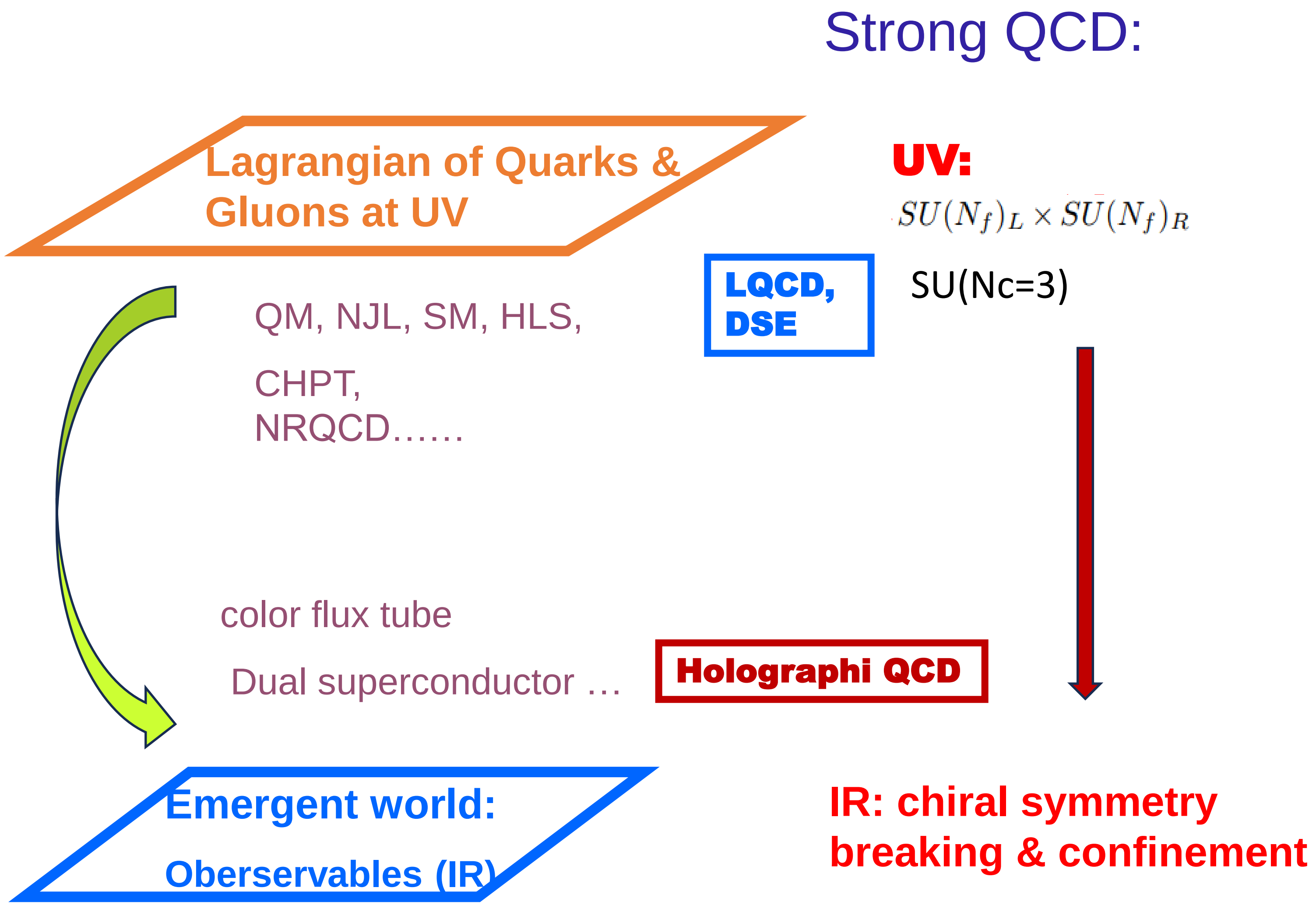
FIG. 1. AdS/CFT as an RG flow. The left panel represents an RG fixed point, so that the entire geometry is scale-invariant (empty AdS). The middle panel shows a thermal state, where the IR geometry is instead a black hole with horizon at r_0. The third panel represents an RG flow where the UV fixed point flows to gapped theory in the IR, ending smoothly at a minimum radius $r_{\rm min}$. Only the first geometry is fully scale invariant.

Hong Liu,Julian Sonner, 1810.02367,
Rept.Prog.Phys. 83 (2019) 1, 016001

Symmetry and symmetry breaking

Weinberg: if you get the symmetries right, then the theory is the right theory

Anderson: More is different! Broken symmetry and the nature(emergence) of the hierarchical structure of science



Effective Field Theory

Joseph Polchinski, TASI lecture 1992, hep-th/9210046

A characteristic energy scale E_0

Choose a cutoff Λ at or slightly below E_0

$$\phi = \phi_{\mathrm{H}} + \phi_{\mathrm{L}}$$

$$\phi_{\mathrm{H}}: \quad \omega > \Lambda \qquad \phi_{\mathrm{L}}: \quad \omega < \Lambda.$$

High frequency low frequency

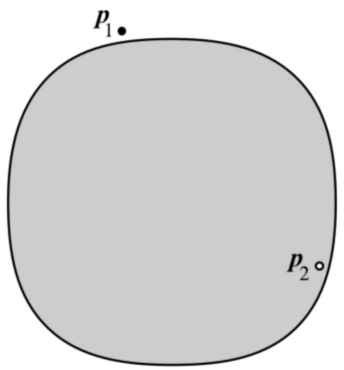
low energy or Wilsonian effective action:

$$\int \mathcal{D}\phi_{\mathrm{L}}\, \mathcal{D}\phi_{\mathrm{H}}\, e^{iS(\phi_{\mathrm{L}},\phi_{\mathrm{H}})} = \int \mathcal{D}\phi_{\mathrm{L}}\, e^{iS_\Lambda(\phi_{\mathrm{L}})},$$

$$S_\Lambda = \int d^D x \sum_i g_i \mathcal{O}_i.$$

$$e^{iS_\Lambda(\phi_{\mathrm{L}})} = \int \mathcal{D}\phi_{\mathrm{H}}\, e^{iS(\phi_{\mathrm{L}},\phi_{\mathrm{H}})}$$

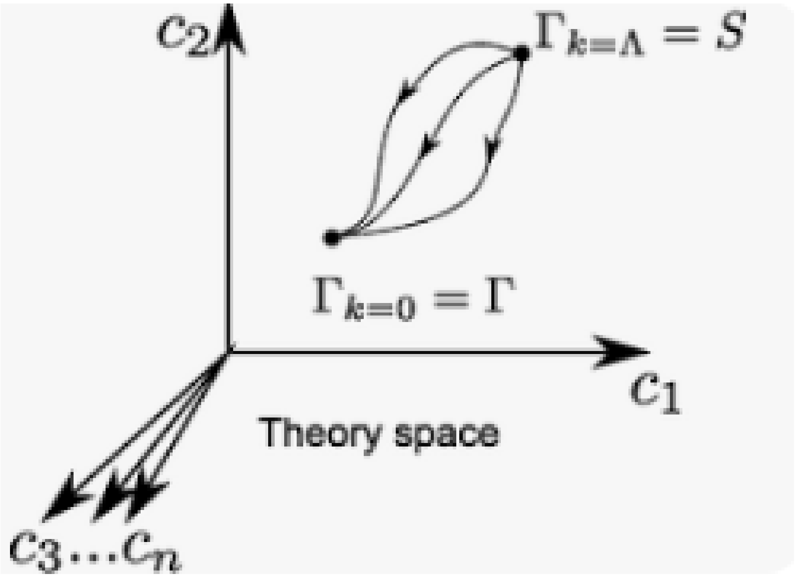
only a finite number of relevant and marginal terms, relevant or marginal operators can be regarded as effective DOF at E_0



e.g., Cooper pairing for SC (BCS),
pion for low-energy effective QCD theory (χPT),
Chiral condensate for ciral symmetry breaking(NJL),
…….

RG flow: from UV to IR

FRG

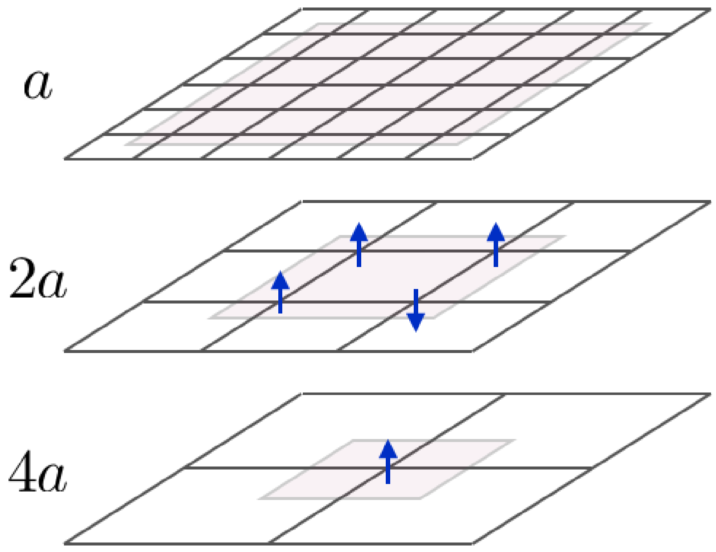


Christof WetterichJan, J. Pawlowski, Yuxin Liu, Weijie Fu, Fei Gao……

Coarse graining spins on a lattice: Kadanoff and Wilson

$$H = \sum_{x,i} J_i(x) \mathcal{O}^i(x)$$

J(x): coupling constant or source for the operator



$$H = \sum_i J_i(x, a)\, \mathcal{O}^i(x)$$

$$H = \sum_i J_i(x, 2a)\, \mathcal{O}^i(x)$$

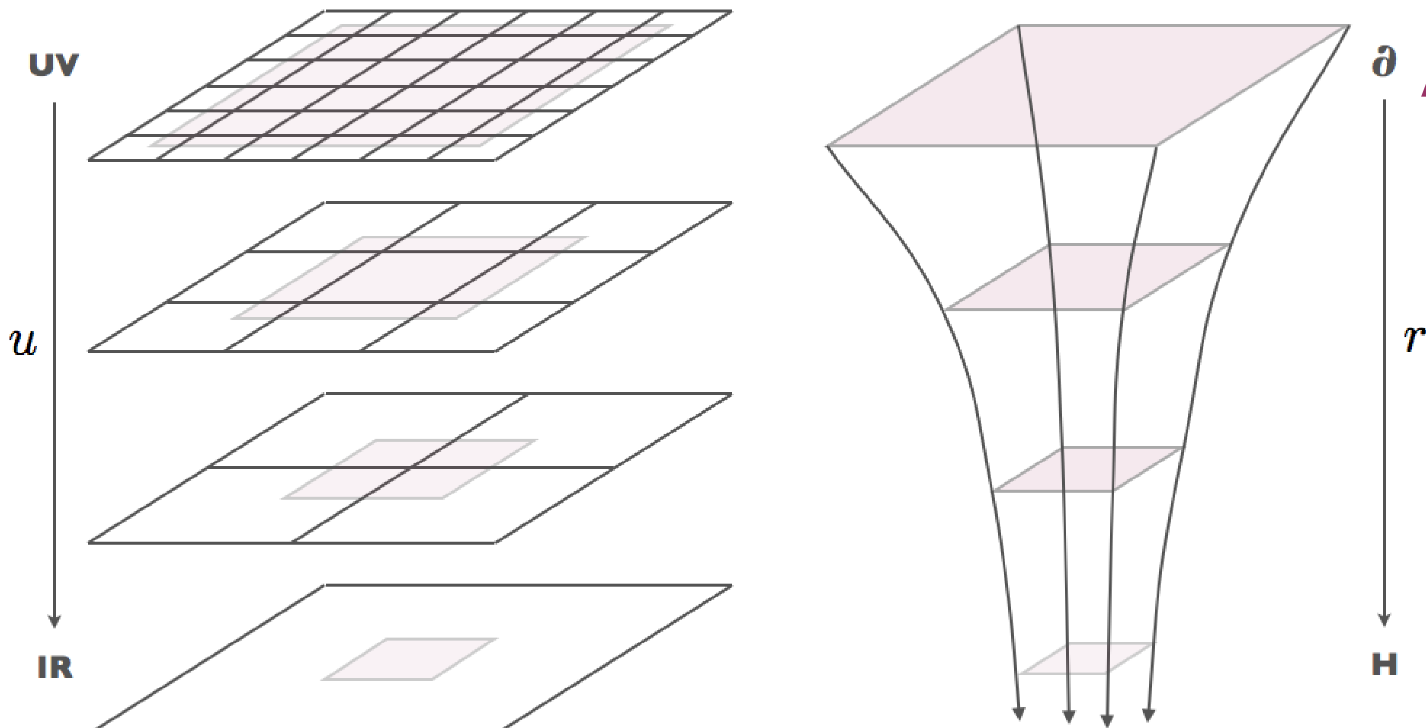
$$H = \sum_i J_i(x, 4a)\, \mathcal{O}^i(x)$$

$$u \frac{\partial}{\partial u} J_i(x, u) = \beta_i(J_j(x, u), u)$$

Holography Framework: Graviton-dilaton-scalar system

$$J_i|_{UV} = \Phi_i|_\partial$$

5D field theory
or bulk field theory

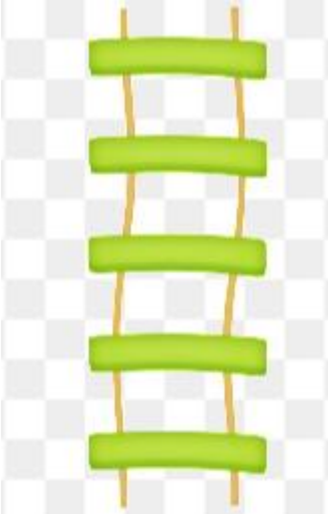


A AdS_5

$\Phi(z)$

$\chi(z)$

From UV to IR



$\mathrm{Tr}\langle G^2\rangle \ \langle g^2A^2\rangle \ \ \langle \bar{q}q\rangle$

IR boundary

Dynamical holographic QCD !

Deformation of AdS_5

Principle of holographic Duality

Bulk field theory or
5D field theory

Boundary QFT **Bulk Gravity**

$$\Delta(d-\Delta) = m^2 L^2$$

Local operator $\mathcal{O}_i(x)$ **Bulk field** $\Phi_i(x, r)$

- Operator/Field correspondence:

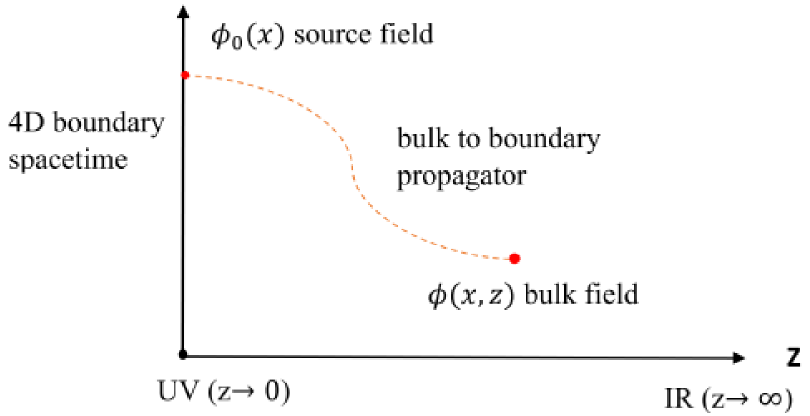
4D boundary operator $\mathcal{O}(x)$
local, gauge invariant, scaling dim. Δ $\Longleftrightarrow$ 5D bulk field
$\phi(x, z \to 0) \to z^{4-\Delta}\phi_0(x) + z^{\Delta} < \mathcal{O}(x) >$

$$\left\langle e^{i\int d^4x \phi_0(x)\mathcal{O}(x)} \right\rangle_{CFT} = e^{iS_{5D}[\phi(x,z)]}|_{\phi(x,z\to 0)\to\phi_0(x)}$$

$$\boxed{Z_{\rm QFT}[J_i] \ = \ Z_{\rm QG}[\Phi[J_i]]}$$

$$Z_{\rm QFT}[J] \ \simeq \ e^{-I_{\rm GR}[\Phi[J]]}$$

$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = \frac{\delta^n I_{\rm GR}[\Phi[J_i]]}{\delta J_1(x_1)\ldots\delta J_n(x_n)}\Big|_{J_i=0}$$



Using bulk field to calculate observables: two-point correlation gives mass spectra, three-point correlation gives form factor, and so on.

Operator/field correspondence

Quark/meson:

TABLE I: Operators/fields of the model

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

$$S = \int d^5x\,\sqrt{g}\;\mathrm{Tr}\Big\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\Big\}$$

$$\left[\partial_z\left(\frac{1}{z}\partial_z A^a_\mu\right) + \frac{q^2}{z}A^a_\mu - \frac{g_5^2 v^2}{z^3}A^a_\mu\right]_\perp = 0;$$

$$\partial_z\left(\frac{1}{z}\partial_z\varphi^a\right) + \frac{g_5^2 v^2}{z^3}(\pi^a - \varphi^a) = 0;$$

$$-q^2\partial_z\varphi^a + \frac{g_5^2 v^2}{z^2}\partial_z\pi^a = 0.$$

Phys.Rev.Lett. 95 (2005) 261602

Observable	Measured (MeV)	Model A (MeV)	Model B (MeV)
m_π	139.6±0.0004 [8]	139.6*	141
m_ρ	775.8±0.5 [8]	775.8*	832
m_{a_1}	1230±40 [8]	1363	1220
f_π	92.4±0.35 [8]	92.4*	84.0
$F_\rho^{1/2}$	345±8 [15]	329	353
$F_{a_1}^{1/2}$	433±13 [6, 16]	486	440
$g_{\rho\pi\pi}$	6.03±0.07 [8]	4.48	5.29

Pure gluon sector

$$\mathcal{L}_G = -\frac{1}{4} G^a_{\mu\nu}(x) G^{\mu\nu,a}(x),$$

Danning Li, M.H., JHEP2013, arXiv:1303.6929

IR: Gluon condensate D=4 $\mathrm{Tr}\langle G^2 \rangle$

Effective gluon mass D=2 $\langle g^2 A^2 \rangle$ →

$$\Phi(z) = \mu^4_{G^2} z^4,$$

$$\Phi = \pm \mu^2_G z^2$$

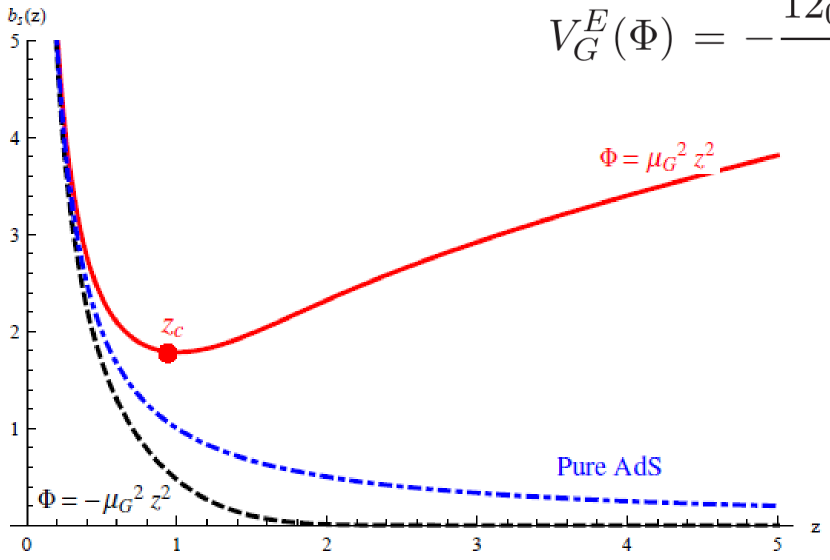
$$\Phi(z) = \mu^2_G z^2 \tanh(\mu^4_{G^2} z^2 / \mu^2_G).$$

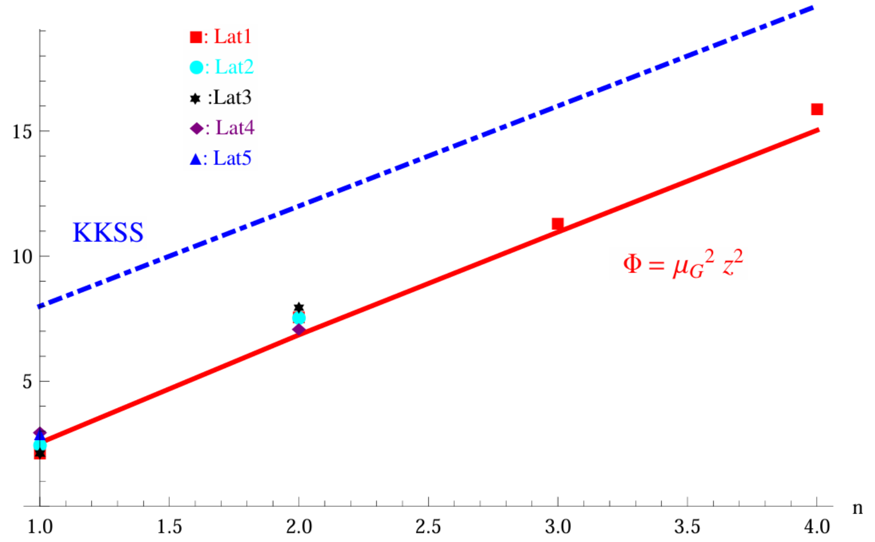
String tension, linear confinement

Dimension-2 dilaton field

$$A_E(z) = \log(\frac{L}{z}) - \log({}_0F_1(5/4, \frac{\Phi^2}{9})),$$

$$V^E_G(\Phi) = -\frac{12\,{}_0F_1(1/4, \frac{\Phi^2}{9})^2}{L^2} + \frac{16\,{}_0F_1(5/4, \frac{\Phi^2}{9})^2 \Phi^2}{3L^2},$$





Graviton-Dilaton-Scalar system

Danning Li, M.H., JHEP2013, arXiv:1303.6929

Total action:

$$S = S_G + \frac{N_f}{N_c} S_{KKSS},$$

Gluonic background+matter part

Action for pure gluon system: Graviton-dilaton coupling, linear confinement

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - V_G(\Phi)\right)$$

Gluonic background

Action for light hadrons: KKSS model (promote dilaton field to a dynamical field)

Chiral symmetry breaking

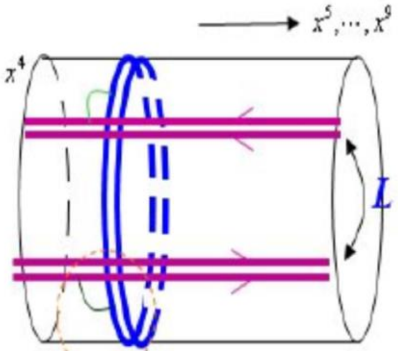
$$S_{KKSS} = -\int d^5x\sqrt{g_s}e^{-\Phi}Tr(|DX|^2 + V_X(X^+X,\Phi) + \frac{1}{4g_5^2}(F_L^2 + F_R^2)).$$

Matter part

Promote the dilaton field in the soft wall model to a dynamical field representing gluodynamics

Dynamical hQCD model(DhQCD)

	Gluodynamics	Quark dynamics
DhQCD	Dilaton Background	Flavor probe
SS:D4-D8 D3-D7	Dp brane: D4, D3	Dq brane: D8, D7
PNJL	Polyakov-loop potential	NJL model



Gluon background + matter field

DhQCD model:
(simplest version)
Can add any probe action

$$S = S_G + \frac{N_f}{N_c} S_{KKSS},$$

Gluonic background + matter field
(Confinement & chiral symmetry breaking)

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - V_G(\Phi)\right),$$

$$S_{\mathrm{KKSS}} = -\int d^5x e^{-\Phi(z)}\sqrt{g_s}Tr\left(|DX|^2 + V_X(|X|,\Phi) + \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right).$$

V-QCD model:

U. Gursoy, E. Kiritsis, F. Nitti

$$S_{\mathrm{V-QCD}} = S_g + S_f$$

$$S_g = M^3N_c^2\int d^5x\,\sqrt{-g}\left(R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)\right)$$

$$S_f = -M^3N_fN_c\int d^5x\,V_f(\lambda,\tau)\sqrt{-\det\left(g_{\mu\nu} + \kappa(\lambda)\partial_\mu\tau\partial_\nu\tau + w(\lambda)F_{\mu\nu}\right)}$$

$$V_g(\lambda) = 12\left[1 + V_1\lambda + \frac{V_2\lambda^2}{1+\lambda/\lambda_0} + V_{\mathrm{IR}}e^{-\lambda_0/\lambda}(\lambda/\lambda_0)^{4/3}\sqrt{\log(1+\lambda/\lambda_0)}\right].$$

DeWolfe-Gubser-Rosen model:

$$V_\phi(\phi) = -12\cosh(c_1\phi) + (6c_1^2 - \frac{3}{2})\phi^2 + c_2\phi^6,$$

Gluon background + matter field

DhQCD model:
(simplest version)
Can add any probe action

$$S = S_G + \frac{N_f}{N_c} S_{KKSS},$$

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - V_G(\Phi)\right),$$

$$S_{\rm KKSS} = -\int d^5x e^{-\Phi(z)}\sqrt{g_s}Tr\left(|DX|^2 + V_X(|X|,\Phi) + \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right).$$

Confinement &
chiral symmetry breaking

5D(Bulk) field theory, has operator/bulk field correspondence,

flexible to extend to pure gluon sector, 2-flavor, 3-flavor, 4-flavor

II. Hadron spectra: glueball, light flavor, heavy flavor

Glueballs

J^{PC}	4-dimensional operator: $\mathcal{O}(x)$	Δ	p	M_5^2
0^{++}	$Tr(G^2) = \vec{E}^a \cdot \vec{E}^a - \vec{B}^a \cdot \vec{B}^a$	4	0	0
0^{-+}	$Tr(G\tilde{G}) = \vec{E}^a \cdot \vec{B}^a$	4	0	0
0^{+-}	$\mathrm{Tr}\left(\{(D_\tau G_{\mu\nu}), (D_\tau G_{\rho\nu})\}(D_\mu G_{\rho\alpha})\right)$	9	0	45
0^{--}	$\mathrm{Tr}\left(\{(D_\tau G_{\mu\nu}), (D_\tau G_{\rho\nu})\}\left(D_\mu \tilde{G}_{\rho\alpha}\right)\right)$	9	0	45
1^{-+}	$f^{abc}\partial_\mu\left[G^a_{\mu\nu}\right]\left[G^b_{\upsilon\rho}\right]\left[G^c_{\rho\alpha}\right]$, $f^{abc}\partial_\mu\left[G^a_{\mu\nu}\right]\left[\tilde{G}^b_{\upsilon\rho}\right]\left[\tilde{G}^c_{\rho\alpha}\right]$, $f^{abc}\partial_\mu\left[\tilde{G}^a_{\mu\nu}\right]\left[G^b_{\upsilon\rho}\right]\left[\tilde{G}^c_{\rho\alpha}\right]$, $f^{abc}\partial_\mu\left[\tilde{G}^a_{\mu\nu}\right]\left[\tilde{G}^b_{\upsilon\rho}\right]\left[G^c_{\rho\alpha}\right]$	7	1	24
1^{+-}	$d^{abc}\left(\vec{E}_a \cdot \vec{E}_b\right)\vec{B}_c$	6	1	15
1^{--}	$d^{abc}\left(\vec{E}_a \cdot \vec{E}_b\right)\vec{E}_c$	6	1	15
2^{++}	$E^a_i E^a_j - B^a_i B^a_j - trace$	4	2	4
2^{-+}	$E^a_i B^a_j + B^a_i E^a_j - trace$	4	2	4
2^{+-}	$d^{abc}\mathcal{S}\left[E^i_a\left(\vec{E}_b \times \vec{B}_c\right)^j\right]$	6	2	16
2^{--}	$d^{abc}\mathcal{S}\left[B^i_a\left(\vec{E}_b \times \vec{B}_c\right)^j\right]$	6	2	16
3^{+-}	$d^{abc}\mathcal{S}\left[B^i_a B^j_b B^k_c\right]$	6	3	15
3^{--}	$d^{abc}\mathcal{S}\left[E^i_a E^j_b E^k_c\right]$	6	3	15

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Glueballs

$$S_{\mathscr{G}} = -\frac{1}{2}\int d^5x\sqrt{g_s}e^{-p\Phi}(\partial_M\mathscr{G}\partial^M\mathscr{G} + M^2_{\mathscr{G},5}(z)\mathscr{G}^2)$$

$$S_V = -\frac{1}{2}\int d^5x\sqrt{g_s}e^{-p\Phi}(\frac{1}{2}F^{MN}F_{MN} + M^2_{\mathscr{V},5}(z)\mathscr{V}^2),$$

$$S_T = -\frac{1}{2}\int d^5x\sqrt{g_s}e^{-p\Phi}(\nabla_L h_{MN}\nabla^L h^{MN} - 2\nabla_L h^{LM}\nabla^N h_{NM} + 2\nabla_M h^{MN}\nabla_N h - \nabla_M h\nabla^M h + M^2_{h,5}(z)(h^{MN}h_{MN} - h^2)),$$

$$-\mathscr{G}_n'' + V_{\mathscr{G}}\mathscr{G}_n = m^2_{\mathscr{G},n}\mathscr{G}_n,$$

$$V_{\mathscr{G}} = \frac{3A_s'' + \frac{3}{z^2} - p\Phi''}{2} + \frac{\left[3A_s' - \frac{3}{z} - p\Phi'\right]^2}{4} + \frac{1}{z^2}e^{2A_s}e^{-c_{r.m.}\Phi}M^2_{\mathscr{G},5}.$$

$$-\mathscr{V}_n'' + V_{\mathscr{V}}\mathscr{V}_n = m^2_{\mathscr{V},n}\mathscr{V}_n,$$

$$V_{\mathscr{V}} = \frac{A_s'' + \frac{1}{z^2} - p\Phi''}{2} + \frac{\left[A_s' - \frac{1}{z} - p\Phi'\right]^2}{4} + \frac{1}{z^2}e^{2A_s}e^{-c_{r.m.}\Phi}M^2_{\mathscr{V},5}.$$

$$-\mathscr{T}_n'' + V_{\mathscr{T}}\mathscr{T}_n = m^2_{\mathscr{T},n}\mathscr{T}_n,$$

$$V_{\mathscr{T}} = \frac{3A_s'' + \frac{3}{z^2} - p\Phi''}{2} + \frac{\left[3A_s' - \frac{3}{z} - p\Phi'\right]^2}{4} + \frac{1}{z^2}e^{2A_s}e^{-c_{r.m.}\Phi}M^2_{\mathscr{T},5}.$$

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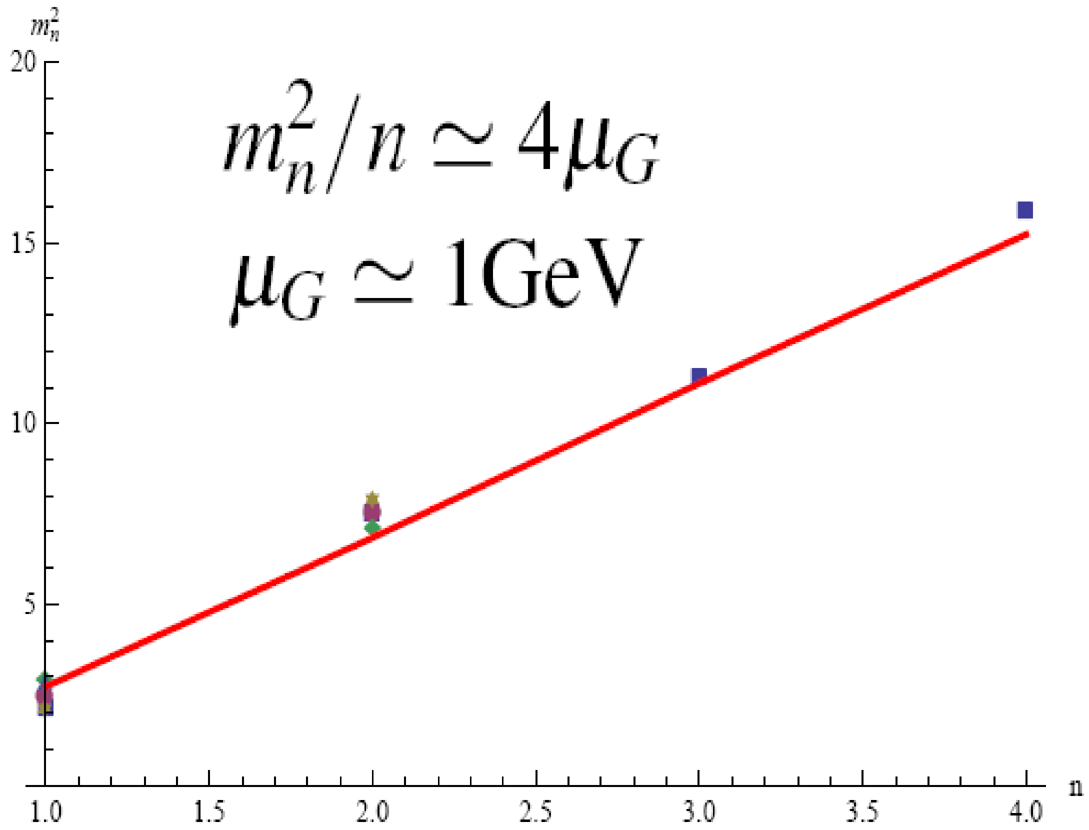
Scalar glueball

Danning Li, M.H., JHEP2013, arXiv:1303.6929



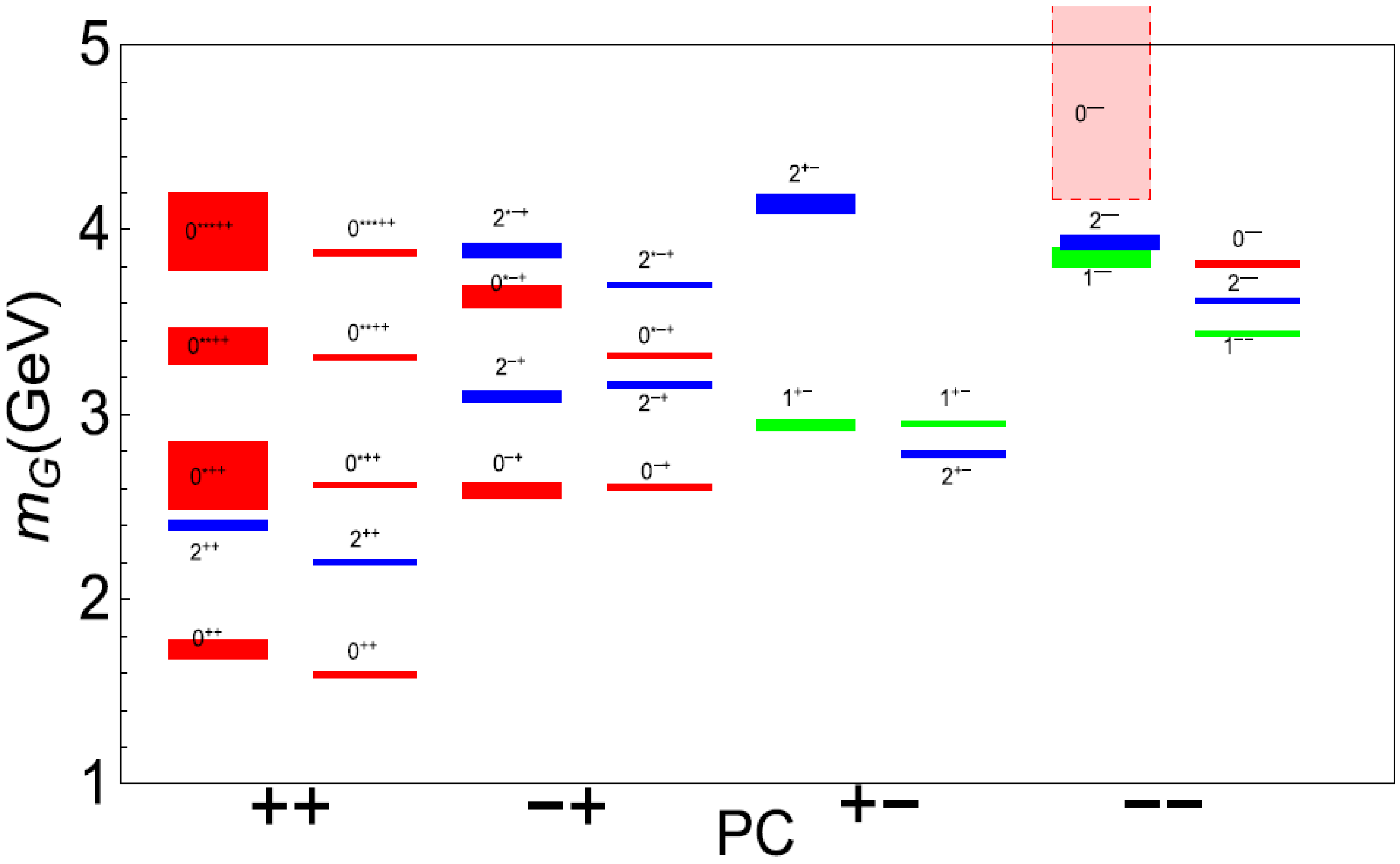
hep-lat/0508002.

[hep-lat/0510074].

[hep-lat/0103027].

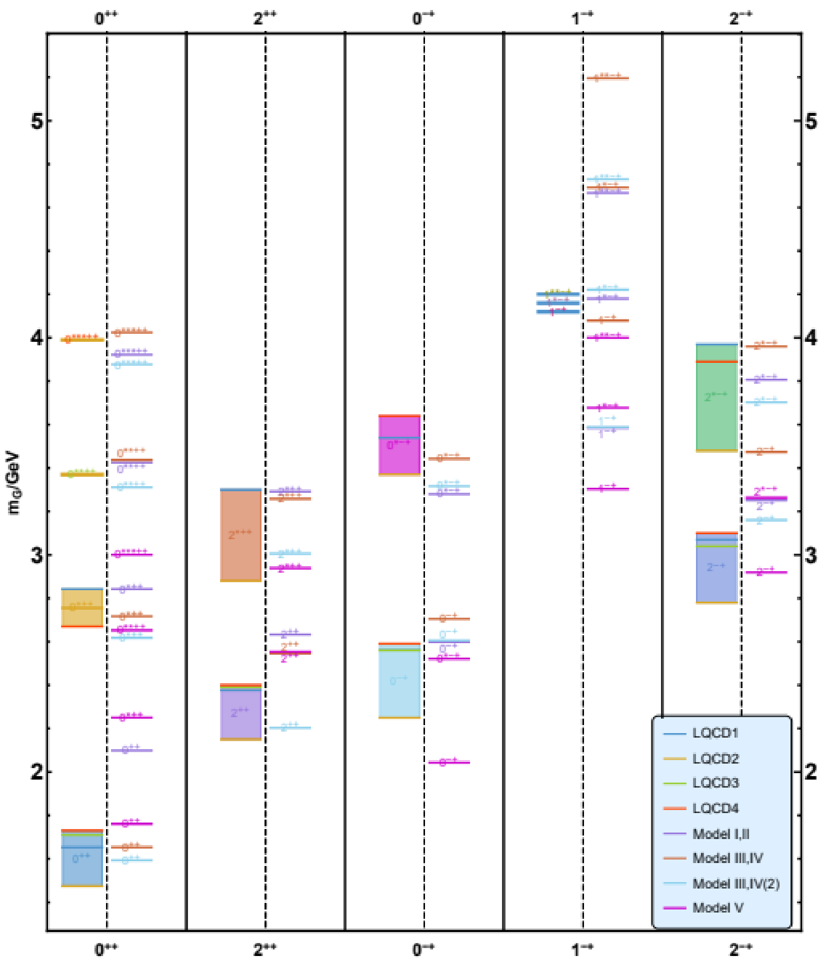
[hep-lat/9901004]

Glueball spectra: **Yidian Chen, M.H., 1511.07018**

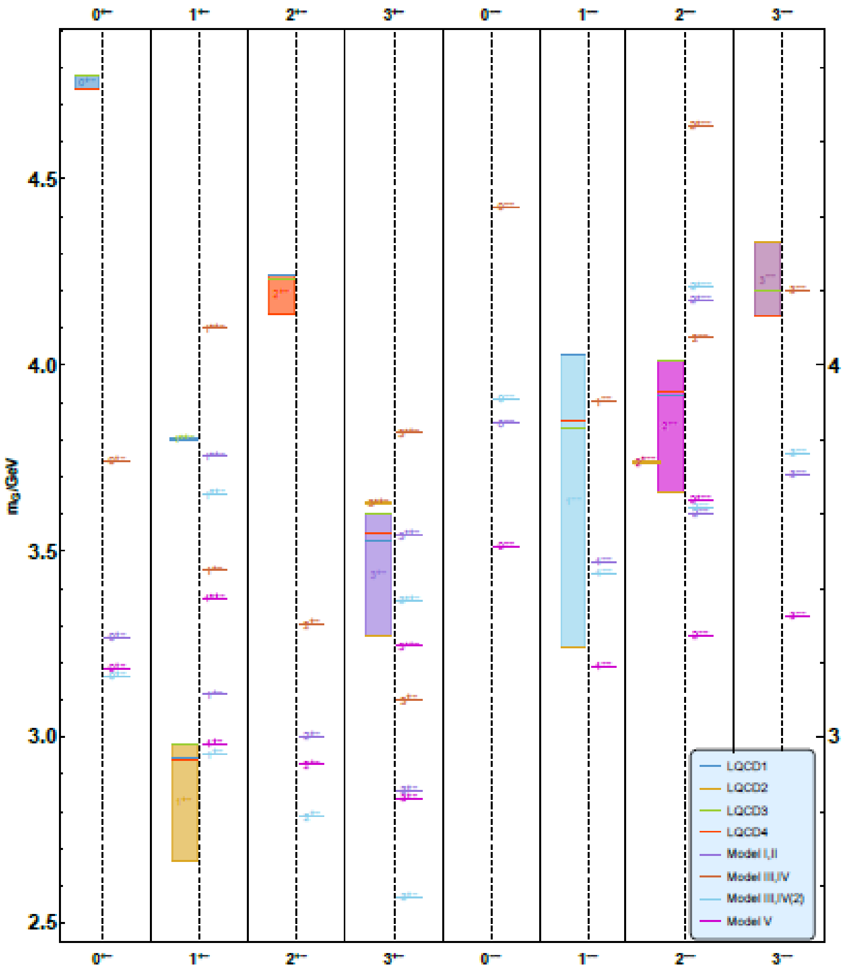


Agree well with lattice result except 0^{--} and 2^{+-} but …

Glueball/Oddball spectra:



Glueball



Odd ball

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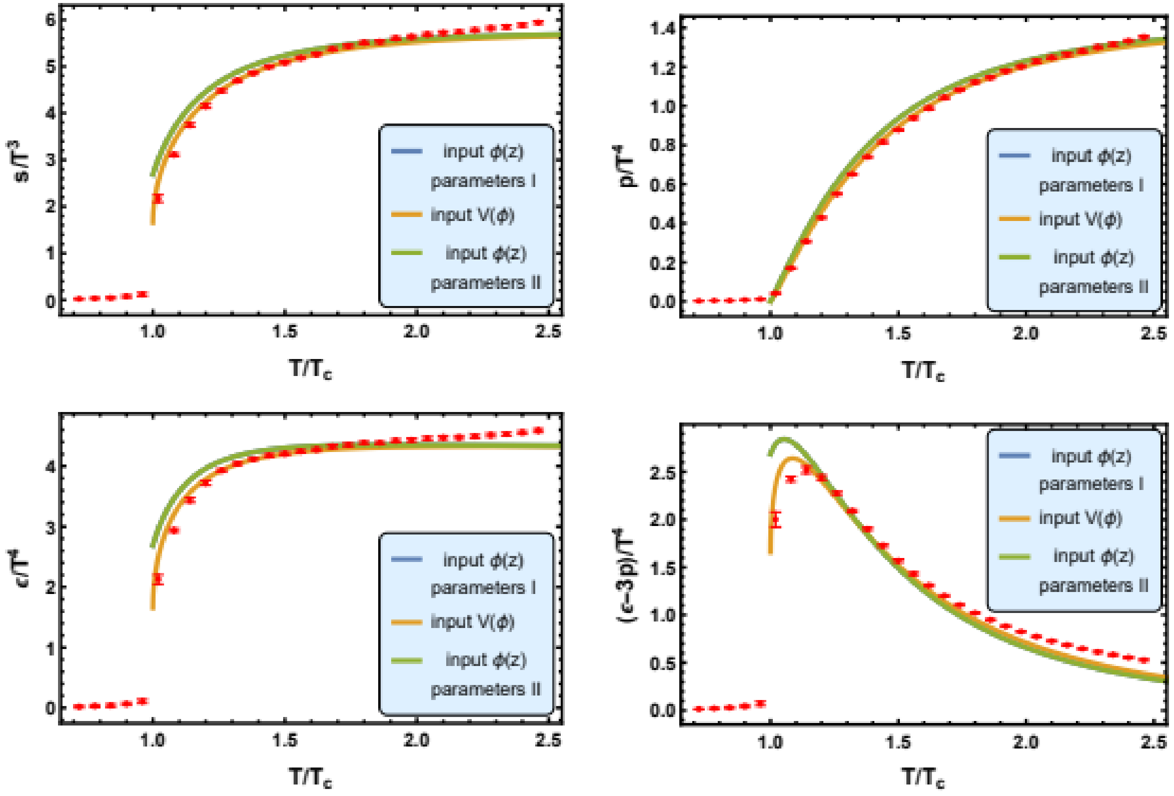
gluon background

$$\phi(z) = c_1 z^2,$$

Agree well with scaled lattice results on EOS for pure gluon system



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Quardratic dilaton field describes pure gluon system reasonably well.

2-flavor system

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

Action for pure gluon system: Graviton-dilaton coupling

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - V_G(\Phi)\right)$$

Gluonic background

Action for light hadrons: KKSS model (promote dilaton field to a dynamical field)

$$S_{KKSS} = -\int d^5x\sqrt{g_s}e^{-\Phi}Tr(|DX|^2 + V_X(X^+X,\Phi) + \frac{1}{4g_5^2}(F_L^2 + F_R^2)).$$

5D linear sigma model

Total action: $$S = S_G + \frac{N_f}{N_c}S_{KKSS}.$$

Two-point correlator gives the spectra:

$$
\begin{aligned}
-s_n'' + V_s(z)s_n &= m_n^2 s_n, \\
-\pi_n'' + V_{\pi,\varphi}\pi_n &= m_n^2(\pi_n - e^{A_s}\chi\varphi_n), \\
-\varphi_n'' + V_\varphi\varphi_n &= g_5^2 e^{A_s}\chi(\pi_n - e^{A_s}\chi\varphi_n), \\
-v_n'' + V_v(z)v_n &= m_{n,v}^2 v_n, \\
-a_n'' + V_a a_n &= m_n^2 a_n,
\end{aligned}
$$

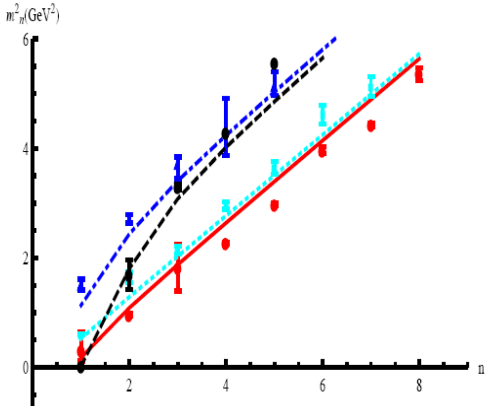
$$
\begin{aligned}
V_s &= \frac{3A_s'' - \phi''}{2} + \frac{(3A_s' - \phi')^2}{4} + e^{2A_s}V_{C,\chi\chi}, \\
V_{\pi,\varphi} &= \frac{3A_s'' - \phi'' + 2\chi''/\chi - 2\chi'^2/\chi^2}{2} \\
&\quad + \frac{(3A_s' - \phi' + 2\chi'/\chi)^2}{4}, \\
V_\varphi &= \frac{A_s'' - \phi''}{2} + \frac{(A_s' - \phi')^2}{4}, \\
V_v &= \frac{A_s'' - \phi''}{2} + \frac{(A_s' - \phi')^2}{4}, \\
V_a &= \frac{A_s' - \phi'}{2} + \frac{(A_s' - \phi')^2}{4} + g_5^2 e^{2A_s}\chi^2.
\end{aligned}
$$

Three-point correlator gives form factor:

$$
f_\pi^2 F_\pi(Q^2) = \frac{N_f}{g_5^2 N_c}\int dz e^{A_s - \Phi} V(q^2, z)\{(\partial_z\varphi)^2 + g_5^2\chi^2 e^{2A_s}(\pi - \varphi)^2\},
$$

Produced hadron spectra and pion form factor comparing with data

D.N. Li, M.H., JHEP2013, arXiv:1303.6929



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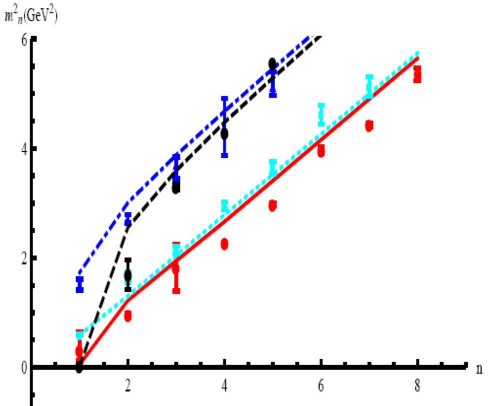
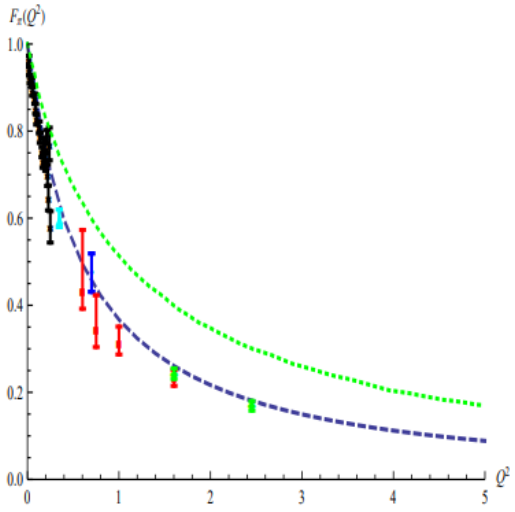
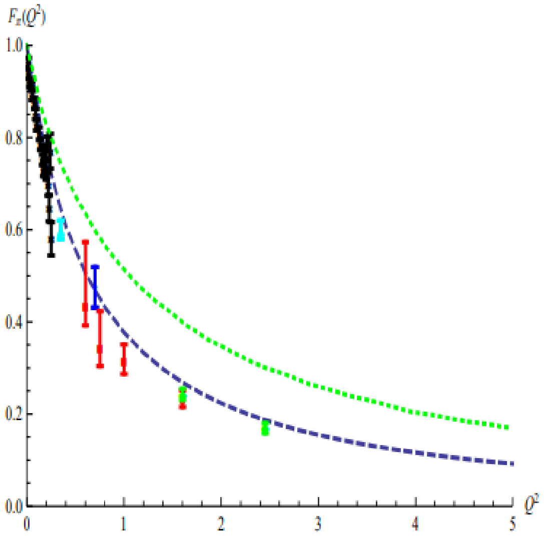


(Mod IB)

$m_n^2/n \simeq 4\mu_G$ $\mu_G \simeq 0.43\text{GeV}$



(Mod IA)

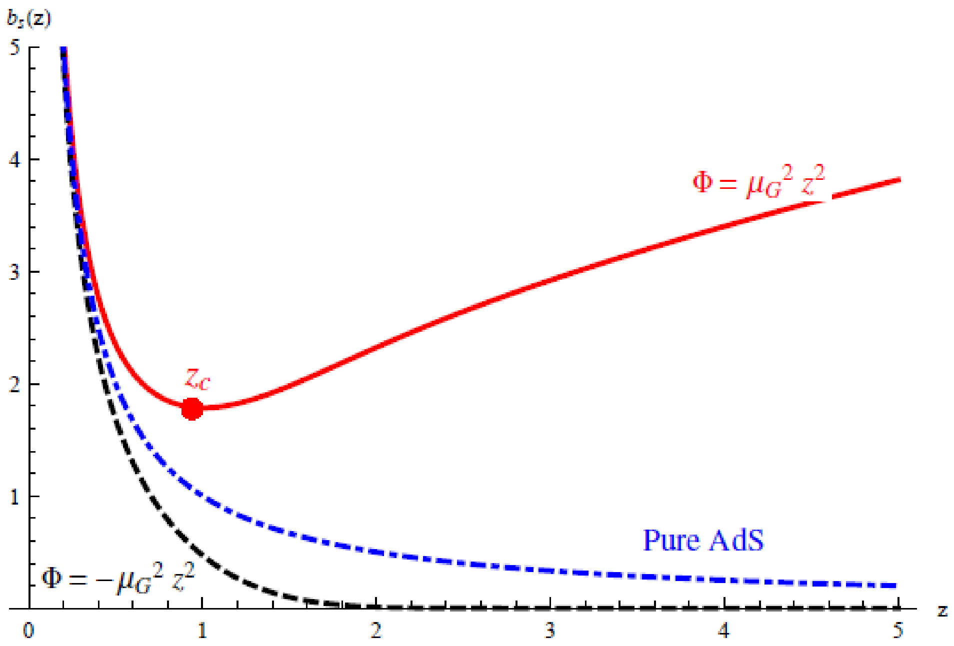


(Mod IIA)

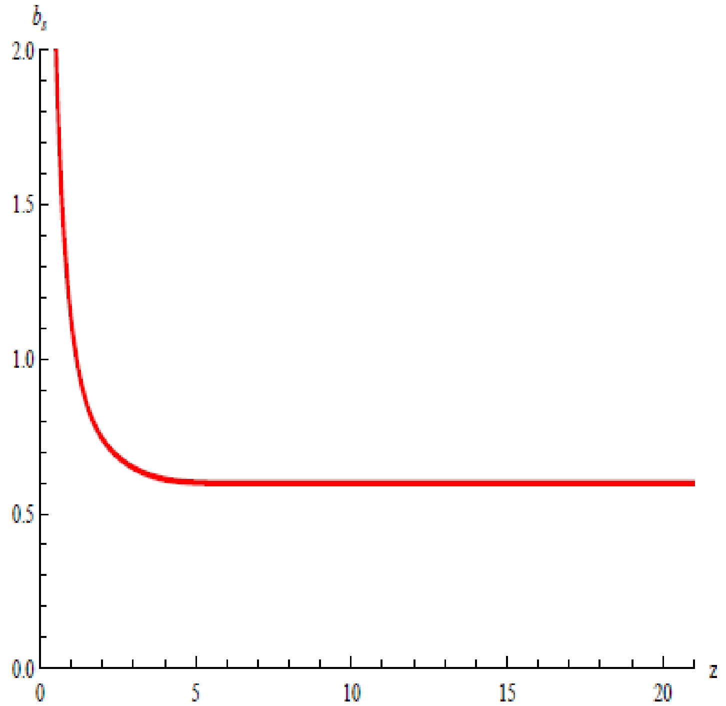
Ground states: chiral symmetry breaking
Excitation states: linear confinement

Spectra and pion form factor cannot be simultaneously produced!

Quenched background



Unquenched background



$$-A_s'' + A_s'^2 + \frac{2}{3}\Phi'' - \frac{4}{3}A_s'\Phi' - \frac{\lambda}{6}e^{\Phi}\chi'^2 = 0,$$

$$\Phi'' + (3A_s' - 2\Phi')\Phi' - \frac{3\lambda}{16}e^{\Phi}\chi'^2 - \frac{3}{8}e^{2A_s - \frac{4}{3}\Phi}\partial_\Phi\left(V_G(\Phi) + \lambda e^{\frac{7}{3}\Phi}V_C(\chi,\Phi)\right) = 0,$$

$$\chi'' + (3A_s' - \Phi')\chi' - e^{2A_s}V_{C,\chi}(\chi,\Phi) = 0.$$

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

4-flavor system

$$S_M = -\int_\epsilon^{z_m} d^5x \sqrt{-g} e^{-\phi} \,\mathrm{Tr}\Big\{ \left(D^M X\right)^\dagger \left(D_M X\right) + M_5^2 |X|^2 + \frac{1}{4g_5^2}\left(L^{MN} L_{MN} + R^{MN} R_{MN}\right) + \left(D^M H\right)^\dagger \left(D_M H\right) + M_5^2 |H|^2 \Big\},$$

$$V = V^a t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} + \frac{\psi}{\sqrt{12}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} + \frac{\psi}{\sqrt{12}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega' + \frac{\psi}{\sqrt{12}} & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & -\frac{3}{\sqrt{12}}\psi \end{pmatrix},$$

$$A = A^a t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{a_1^0}{\sqrt{2}} + \frac{f_1}{\sqrt{6}} + \frac{\chi_{c1}}{\sqrt{12}} & a_1^+ & K_1^+ & \bar{D}_1^0 \\ a_1^- & -\frac{a_1^0}{\sqrt{2}} + \frac{f_1}{\sqrt{6}} + \frac{\chi_{c1}}{\sqrt{12}} & K_1^0 & D_1^- \\ K_1^- & \bar{K}_1^0 & -\sqrt{\frac{2}{3}}(f_1) + \frac{\chi_{c1}}{\sqrt{12}} & D_{s1}^- \\ D_1^0 & D_1^+ & D_{s1}^+ & -\frac{3}{\sqrt{12}}\chi_{c1} \end{pmatrix},$$

$$\pi = \pi^a t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta + \frac{\eta_c}{\sqrt{12}} & D_s^- \\ D^0 & D^+ & D_s^+ & -\frac{3}{\sqrt{12}}\eta_c \end{pmatrix}.$$

Solving background with chiral condensate:

$$S^{(0)} = -\frac{1}{4}\int_\epsilon^{z_m} d^5x \Big\{ \frac{e^{-\phi(z)}}{z^3}\left(2v_l'(z)v_l'(z) + v_s'(z)v_s'(z) + v_c'(z)v_c'(z)\right) - \frac{e^{-\phi(z)}}{z^5}\left(3\left(2v_l(z)^2 + v_s(z)^2 + v_c(z)^2\right) - \frac{\kappa}{4}\left(2v_l(z)^4 + v_s(z)^4 + v_c(z)^4\right)\right) + \frac{e^{-\phi(z)}}{z^3}\left(h_c'(z)h_c'(z)\right) - \frac{3e^{-\phi(z)}}{z^5}h_c(z)^2 \Big\}$$

YiDian Chen, M.H.arXiv: 2110.08215,
Phys.Rev.D 105 (2022) 2, 026021
Hiwa Ameld, Y.D. Chen, M.H.arXiv:2308.14975,
Phys.Rev.D 108 (2023) 8, 086034) arXiv:2309.06156

Two-point correlation function gives mass

$$S^{(2)} = -\int d^5x \left\{ \eta^{MN} \frac{e^{-\phi(z)}}{z^3} \left((\partial_M \pi^a - A_M^a)(\partial_N \pi^b - A_N^b) M_A^{ab} - V_M^a V_N^b M_V^{ab} + V_{HM}^a V_{HN}^b M_{V_H}^{ab} \right) + \frac{e^{-\phi(z)}}{4 g_5^2 z} \eta^{MP} \eta^{NQ} \left(V_{MN}^a V_{PQ}^b + A_{MN}^a A_{PQ}^b \right) \right\}$$

$$\langle J_\mu^{V,a} J_\nu^{V,b} \rangle \simeq \delta^{ab} (q_\mu q_\nu - q^2 g_{\mu\nu}) \times \frac{F_\rho^2}{q^2 - m_\rho^2},$$

Vector field

$$V = V^a t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} + \frac{\psi}{\sqrt{12}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} + \frac{\psi}{\sqrt{12}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega' + \frac{\psi}{\sqrt{12}} & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & -\frac{3}{\sqrt{12}}\psi \end{pmatrix},$$

Axial vector field

$$A = A^a t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{a_1^0}{\sqrt{2}} + \frac{f_1}{\sqrt{6}} + \frac{\chi_{c1}}{\sqrt{12}} & a_1^+ & K_1^+ & \bar{D}_1^0 \\ a_1^- & -\frac{a_1^0}{\sqrt{2}} + \frac{f_1}{\sqrt{6}} + \frac{\chi_{c1}}{\sqrt{12}} & K_1^0 & D_1^- \\ K_1^- & \bar{K}_1^0 & -\sqrt{\frac{2}{3}} f_1 + \frac{\chi_{c1}}{\sqrt{12}} & D_{s1}^- \\ D_1^0 & D_1^+ & D_{s1}^+ & -\frac{3}{\sqrt{12}}\chi_{c1} \end{pmatrix},$$

$$a = 1, 2, \ldots, 15$$

Pseudoscalar field

$$\pi = \pi^a t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta + \frac{\eta_c}{\sqrt{12}} & D_s^- \\ D^0 & D^+ & D_s^+ & -\frac{3}{\sqrt{12}}\eta_c \end{pmatrix}.$$

EOM for hadrons: eigenvalues give hadron spectra

$$
\begin{aligned}
&-s_n'' + V_s(z)s_n = m_n^2 s_n,\\
&-\pi_n'' + V_{\pi,\varphi}\pi_n = m_n^2(\pi_n - e^{A_s}\chi\varphi_n),\\
&-\varphi_n'' + V_\varphi\varphi_n = g_5^2 e^{A_s}\chi(\pi_n - e^{A_s}\chi\varphi_n),\\
&-v_n'' + V_v(z)v_n = m_{n,v}^2 v_n,\\
&-a_n'' + V_a a_n = m_n^2 a_n,
\end{aligned}
$$

$$\langle J_\mu^{V,a} J_\nu^{V,b}\rangle \simeq \delta^{ab}(q_\mu q_\nu - q^2 g_{\mu\nu}) \times \frac{F_\rho^2}{q^2 - m_\rho^2},$$

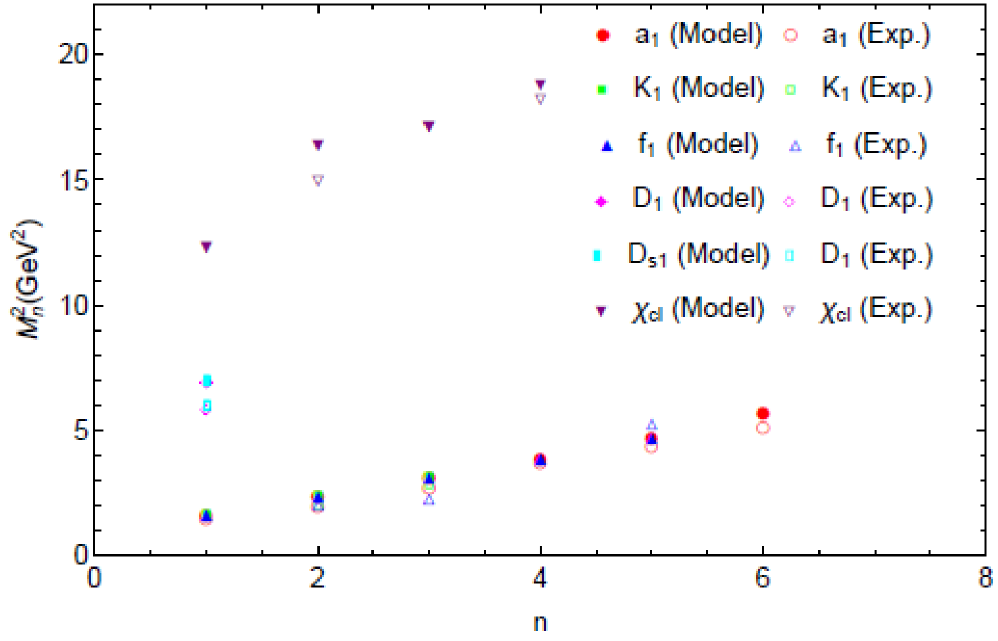
$$
\begin{aligned}
V_s &= \frac{3A_s'' - \Phi''}{2} + \frac{(3A_s' - \Phi')^2}{4} + e^{2A_s}V_{C,\chi\chi},\\
V_{\pi,\varphi} &= \frac{3A_s'' - \Phi'' + 2\chi''/\chi - 2\chi'^2/\chi^2}{2} + \frac{(3A_s' - \Phi' + 2\chi'/\chi)^2}{4},\\
V_\varphi &= \frac{A_s'' - \Phi''}{2} + \frac{(A_s' - \Phi')^2}{4},\\
V_v &= \frac{A_s'' - \Phi''}{2} + \frac{(A_s' - \Phi')^2}{4},\\
V_a &= \frac{A_s' - \Phi'}{2} + \frac{(A_s' - \Phi')^2}{4} + g_5^2 e^{2A_s}\chi^2.
\end{aligned}
$$

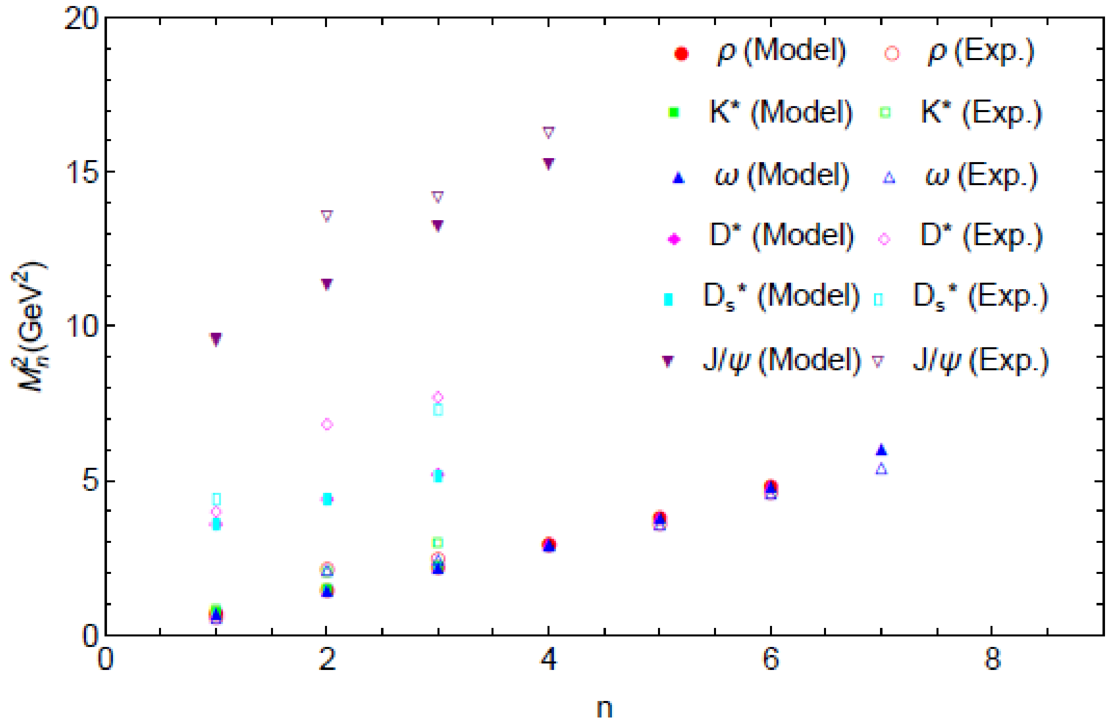
4-flavor hadron spectra: ground state and excitation states

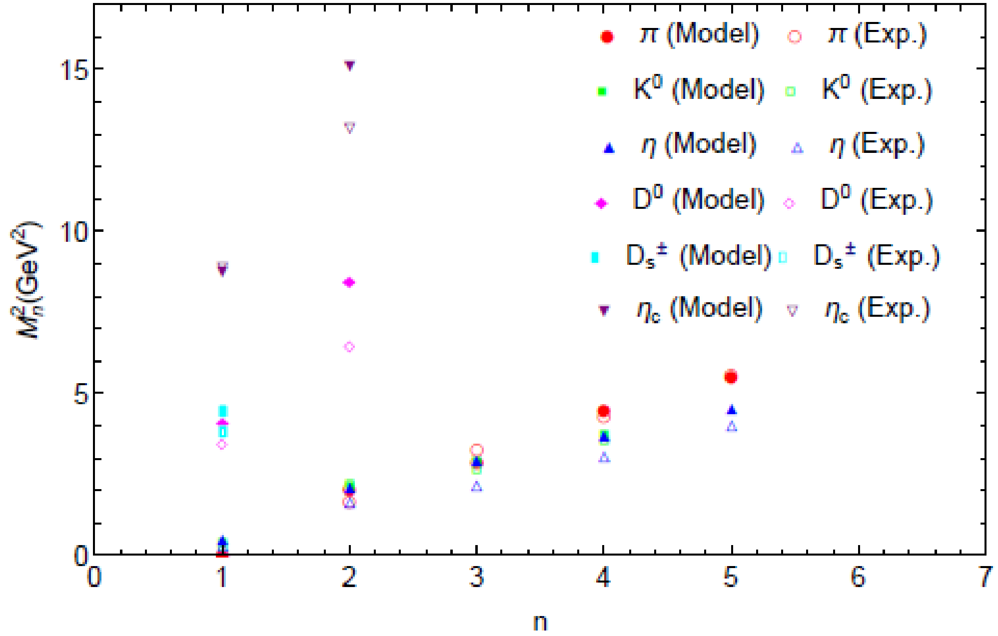
H.Ameld, Y.D. Chen, M.H.arXiv:2308.14975 *Phys.Rev.D 108 (2023) 8, 086034)*, arXiv:2309.06156

$\mu = 0.43$	$\sigma_u = (0.2962)^3$
$m_u = 0.0032$	$\sigma_s = (0.2598)^3$
$m_s = 0.1423$	$\sigma_c = (0.302)^3$
$m_c = 1.5971$	$z_m = 10$
$m_{hc} = 1.985$	$\kappa = 30$

The values of the free parameters with the unit of GeV







Three-point correlation function gives EM and semi-leptonic form factors

$$S^{(3)} = -\int d^5x \left\{\eta^{MN}\frac{e^{-\phi(z)}}{z^3}\left(2\left(A_M^a - \partial_M\pi^a\right)V_N^b\pi^c g^{abc} + V_M^a\left(\partial_N\left(\pi^b\pi^c\right) - 2A_M^b\pi^c\right)h^{abc}\right.\right.$$
$$\left.\left. - V_M^a V_N^b \pi^c k^{abc}\right) + \frac{e^{-\phi(z)}}{2g_5^2 z}\eta^{MP}\eta^{NQ}\left(V_{MN}^a V_P^b V_Q^c + V_{MN}^a A_P^b A_Q^c + A_{MN}^a V_P^b A_Q^c + A_{MN}^a A_P^b V_Q^c\right)f^{bca}\right\}$$

$$\left\langle 0\left|\mathcal{T}\left\{J_{A\parallel}^{\alpha a}(x)J_{V\perp}^{\mu b}(y)J_{A\parallel}^{\beta c}(w)\right\}\right|0\right\rangle = -\frac{\delta^3\, \mathrm{S}(V\pi\pi)}{\delta A_{\parallel\alpha}^{0a}(x)\delta V_{\perp\mu}^{0b}(y)\delta A_{\parallel\beta}^{0c}(w)},$$

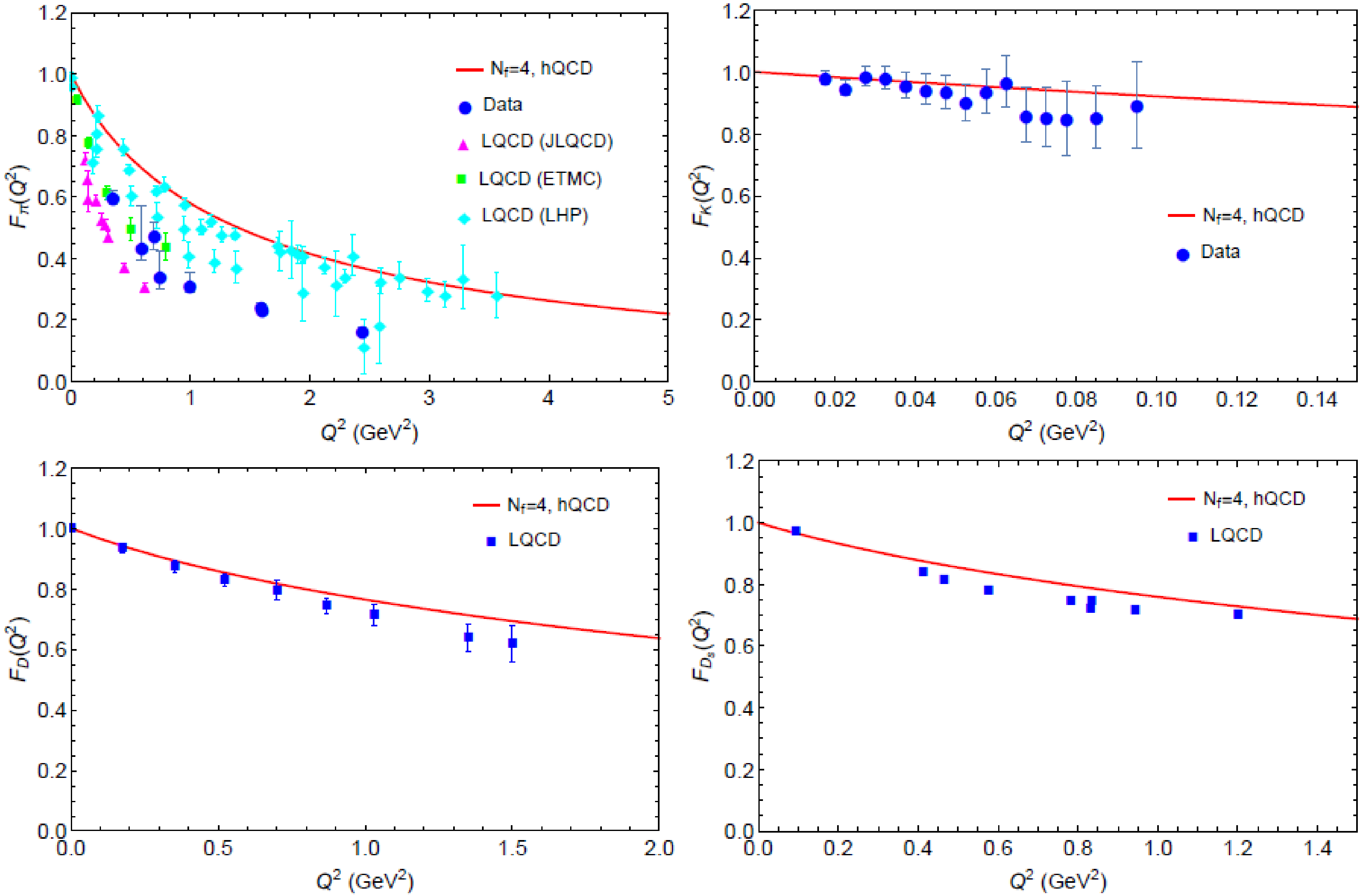
$$\left\langle 0\left|\mathcal{T}\left\{J_{V\perp}^{\mu a}(x)J_{V\perp}^{\nu b}(y)J_{V\perp}^{\alpha c}(w)\right\}\right|0\right\rangle = -\frac{\delta^3\, \mathrm{S}(VVV)}{\delta V_{\perp\mu}^{0a}(x)\delta V_{\perp\nu}^{0b}(y)\delta V_{\perp\alpha}^{0c}(w)},$$

$$\left\langle 0\left|\mathcal{T}\left\{J_{A\perp}^{\alpha a}(x)J_{V\perp}^{\mu b}(y)J_{A\perp}^{\beta c}(w)\right\}\right|0\right\rangle = -\frac{\delta^3\, \mathrm{S}(VAA)}{\delta A_{\perp\alpha}^{0a}(x)\delta V_{\perp\mu}^{0b}(y)\delta A_{\perp\beta}^{0c}(w)},$$

H.Ameld, Y.D. Chen, M.H.arXiv:2308.14975 *Phys.Rev.D 108 (2023) 8, 086034)*, arXiv:2309.06156

Psudoscalar mesons

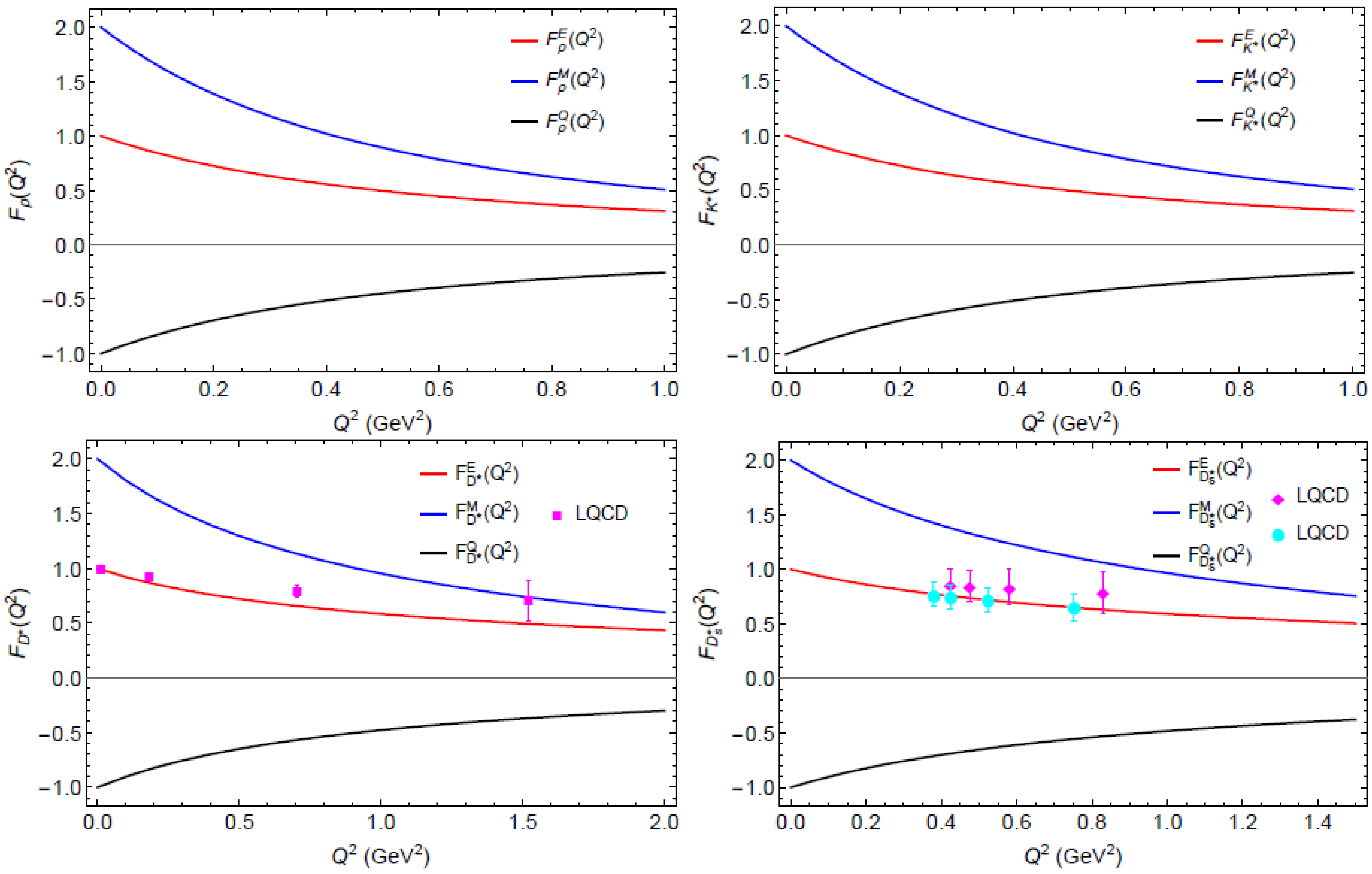
H.Ameld, Y.D. Chen, M.H.arXiv:2308.14975 *Phys.Rev.D 108 (2023) 8, 086034)*, arXiv:2309.06156



Data(π)(G. M. Huber et al., prc(2008)), LQCD(JLQCD)(T. Kaneko et al., pos (2008)), LQCD(ETMC)(R. Frezzotti et al., prd(2009)), LQCD(LHP)(F. D. R. Bonnet et al., prd(2005)), Data(K)(S. R. Amendolia et al., plb(1986)), LQCD(D)(K. U. Can et al., PLB(2013)), LQCD(D_s)(N. Li, Y.-J. Wu, EPJA(2017))

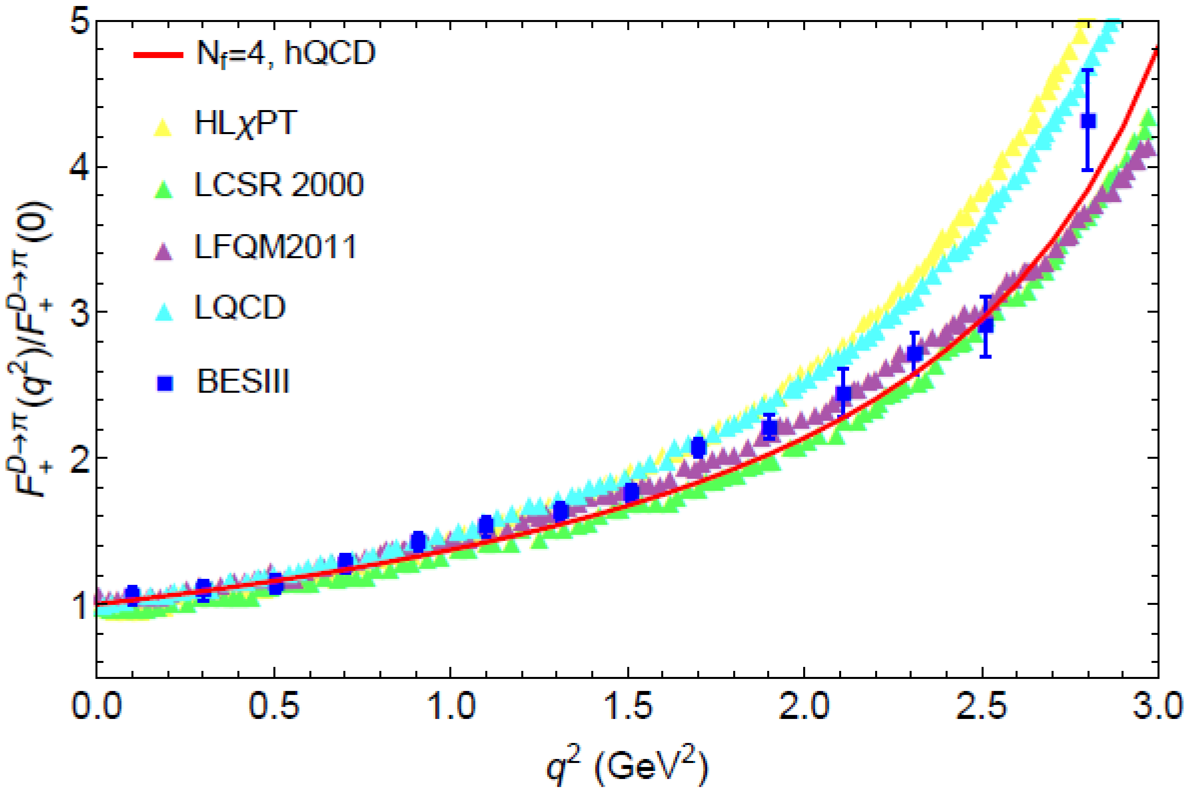
Vector mesons

H.Ameld, Y.D. Chen, M.H.arXiv:2308.14975, arXiv:2309.06156



H.Ameld, Y.D. Chen, M.H. arXiv:2309.06156

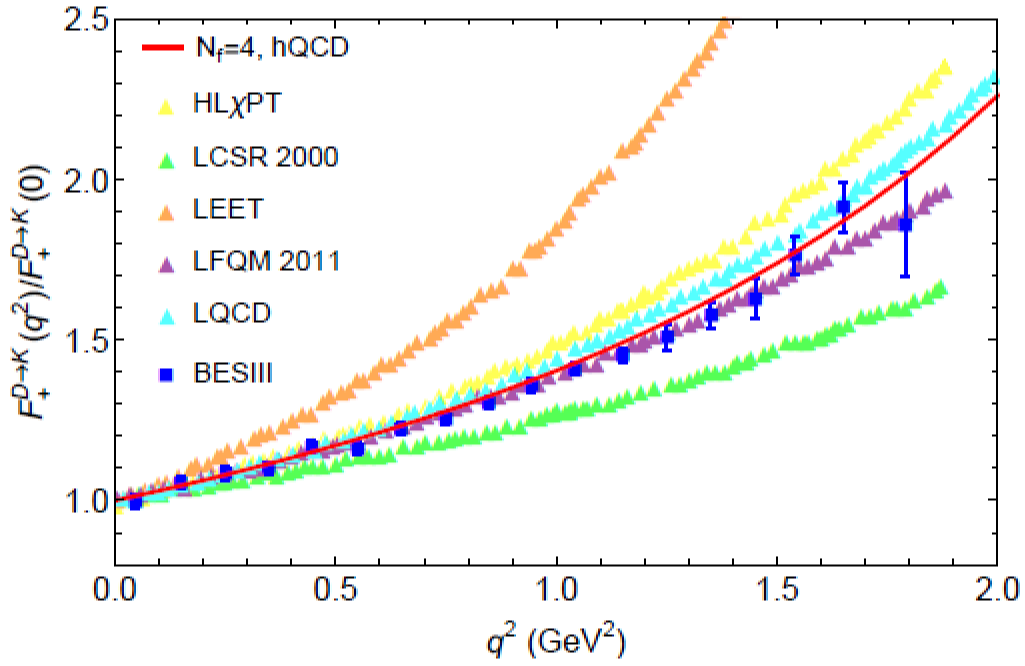
The semileptonic form factor $F_+(q^2)$ for $D \to \pi l^+ \nu_l$.

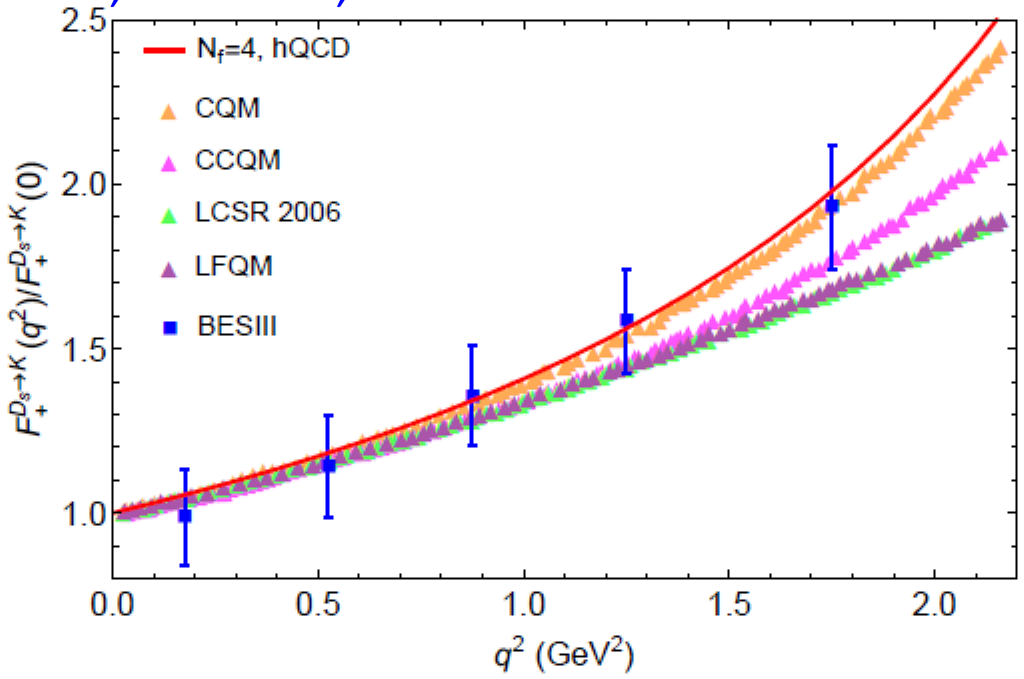


- HLχPT: S. Fajfer and J. F. Kamenik, prd(2005)
- LSCR: A. Khodjamirian et al., prd(2000)
- LFQM: R. C. Verma, J. Phys. G (2012)
- LQCD: C. Aubin et al, prl(2005)
- BESIII: M. Ablikim et al., prd(2015)

Results of $F_+(q^2)$ for the decays $D \to K l^+ \nu_l$ and $D_s \to K l^+ \nu_l$.

H.Ameld, Y.D. Chen, M.H.arXiv:2309.06156



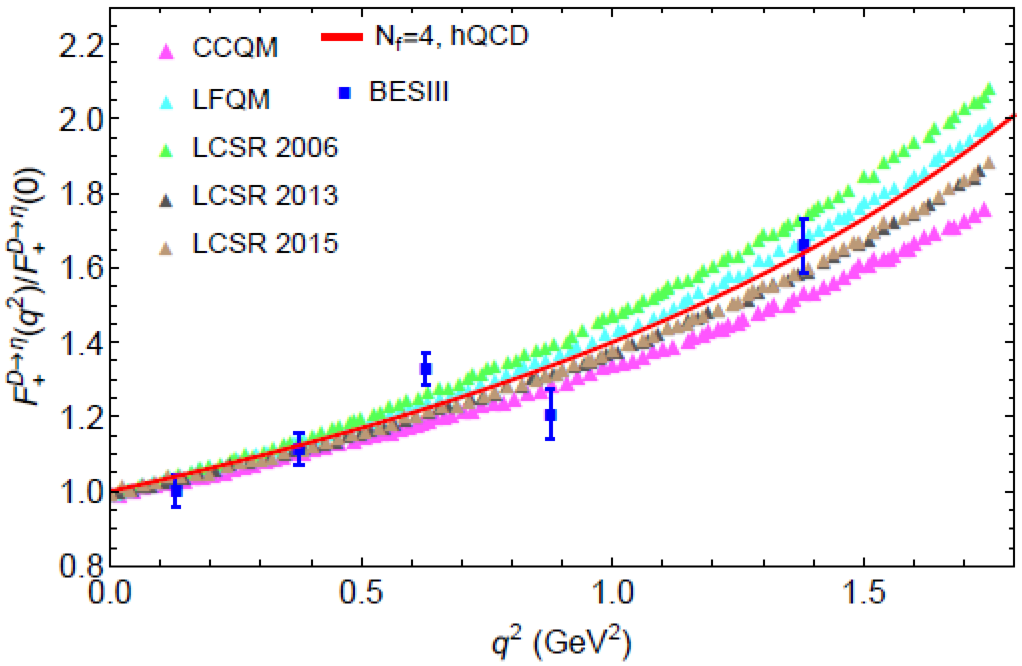


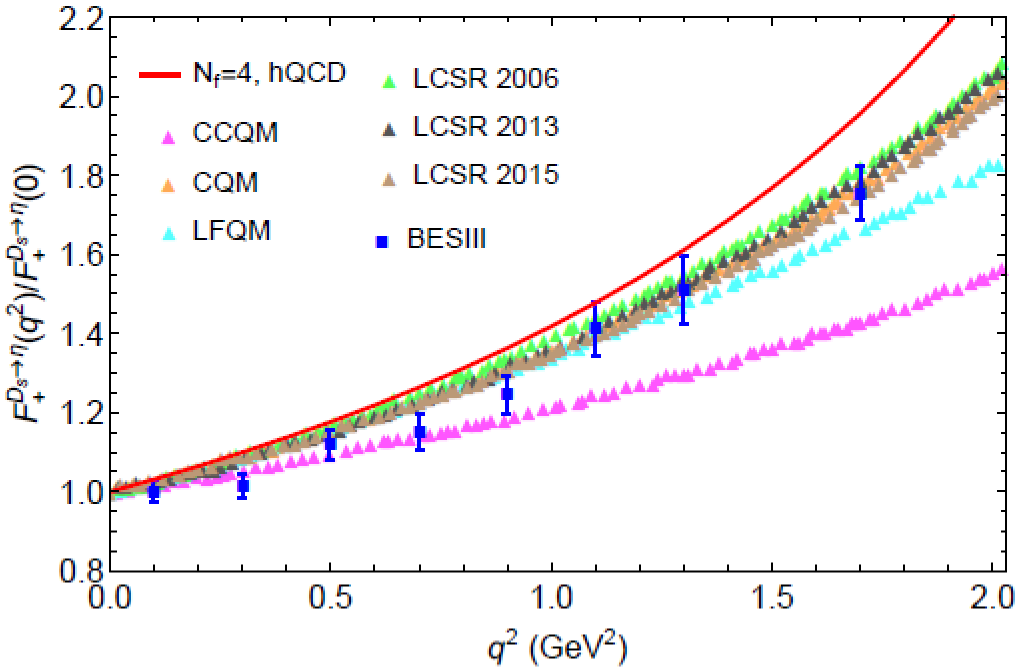
- HLχPT: S. Fajfer and J. F. Kamenik, prd(2005)
- LSCR: A. Khodjamirian et al., prd(2000)
- LEET: J. Charles et al., prd(1999)
- LFQM: R. C. Verma, J. Phys. G (2012)
- LQCD: C. Aubin et al, prl(2005)
- BESIII: M. Ablikim et al., prd(2015)

- CQM: D. Melikhov and B. Stech, prd(2000)
- CCQM: N. R. Son et al., prd(2018)
- LSCR: Y. L. Wu, et al., IJMPA(2006)
- LFQM: R. C. Verma, J. Phys. G (2012)
- BESIII: M. Ablikim et al., prl(2019)

Form factor $F_+(q^2)$ for $D^+_{(s)} \to \eta$

H.Ameld, Y.D. Chen, M.H.arXiv:2309.06156





- CCQM: N. R. Son et al., prd(2018)
- LFQM: R. C. Verma, J. Phys. G (2012)
- LSCR2006: Y. L. Wu, et al., IJMPA(2006)
- LSCR2013: N. Offen et al., prd(2013)
- LSCR2015: G. Duplancic and B. Melic, JHEP(2015)
- BESIII: M. Ablikim et al., prl(2020)

- CCQM: N. R. Son et al., prd(2018)
- CQM: D. Melikhov and B. Stech, prd(2000)
- LFQM: R. C. Verma, J. Phys. G (2012)
- LSCR2006: Y. L. Wu, et al., IJMPA(2006)
- LSCR2013: N. Offen et al., prd(2013)
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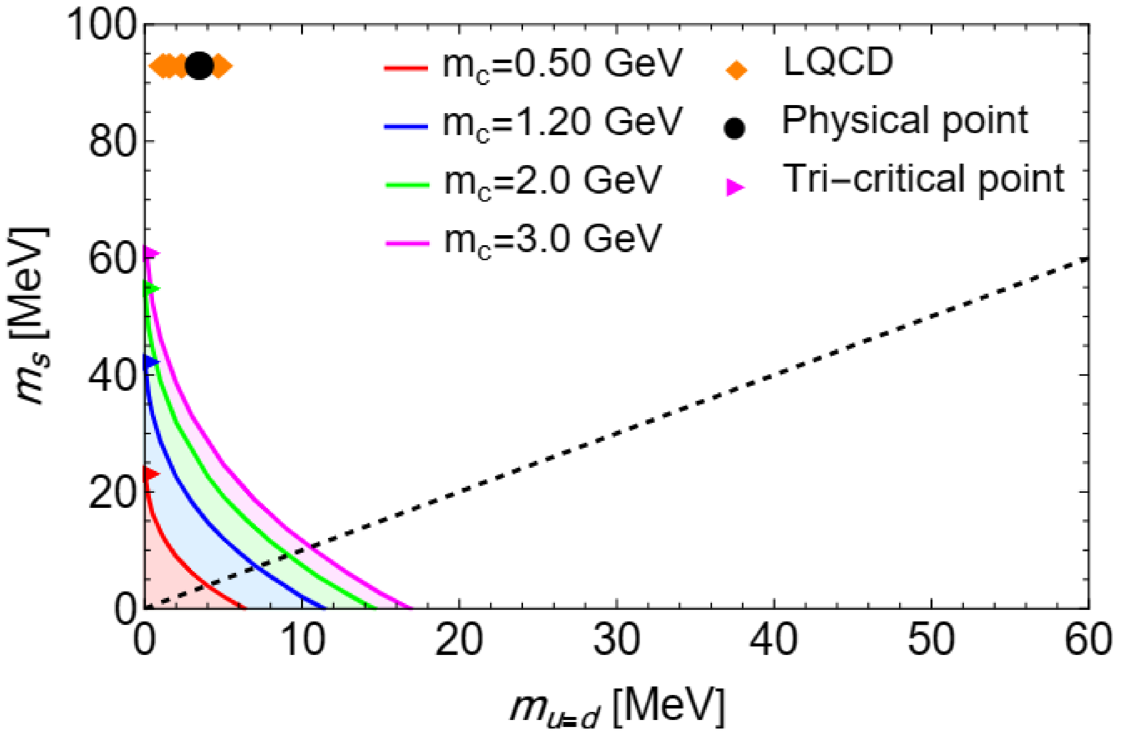
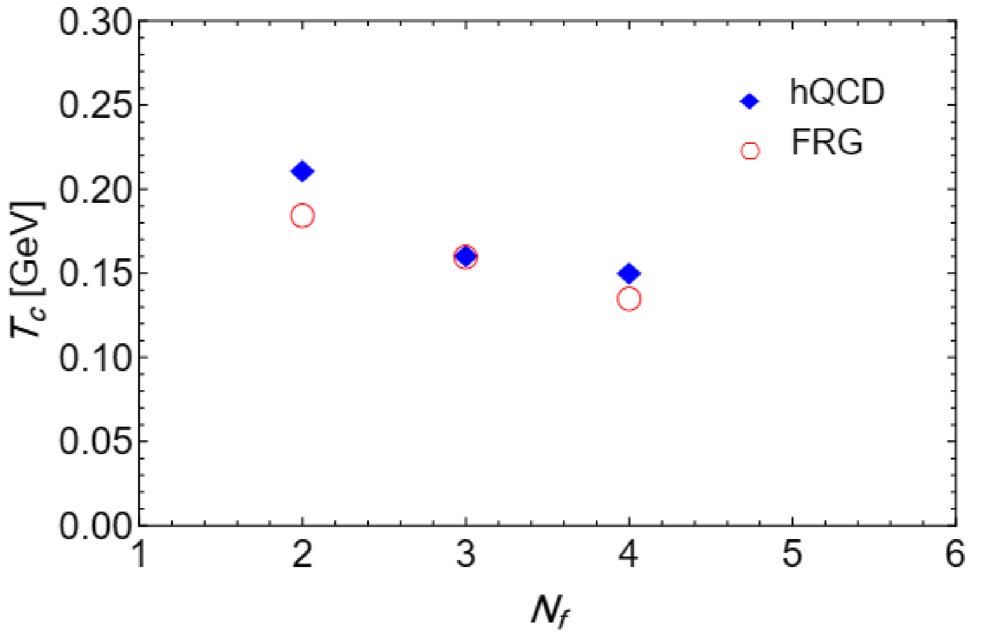
- BESIII: M. Ablikim et al., prl(2019)

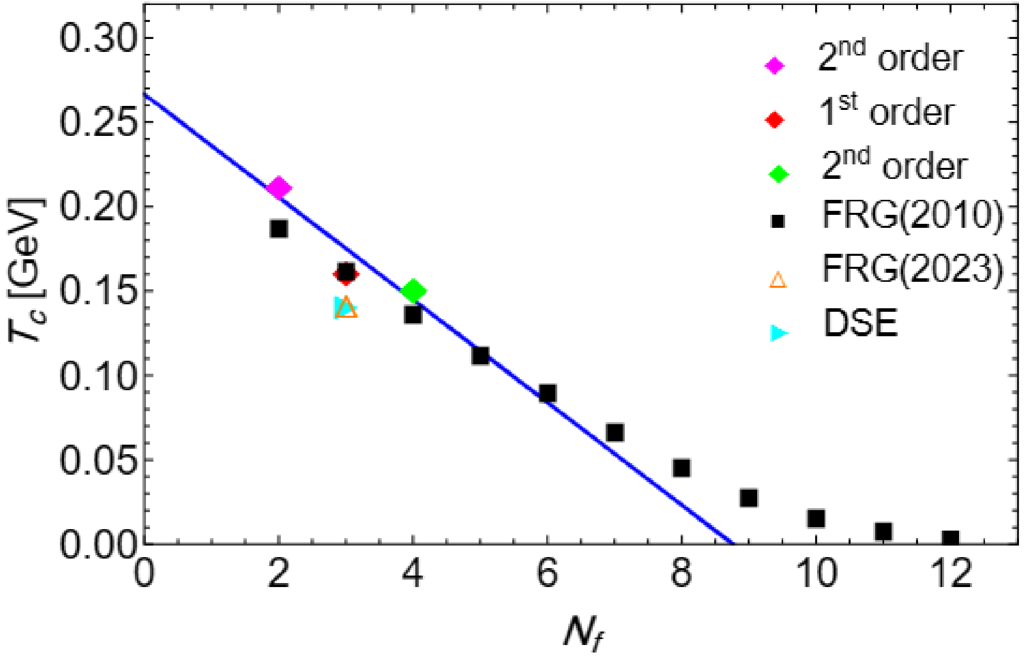
Heavy flavor effect on chiral phase transition

Hiwa Ameld, Mamiya Kawaguchi, Phys.Rev.D 110 (2024) 4, 046002



1st order chiral phase transition occurs with charm quark enters in.





EOS for cold QCD matter

The Einstein-Maxwell-dilaton system at finite baryon density

$$
\begin{aligned}
S &= S_b + S_m, \\
S_b &= \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^s}e^{-2\phi}[R^s + 4\partial_\mu\phi\partial^\mu\phi - V_s(\phi) - \frac{h(z)}{4}e^{\frac{4\phi}{3}}F_{\mu\nu}F^{\mu\nu}], \\
S_m &= -\int d^5x\sqrt{-g^s}e^{-\phi}Tr[\nabla_\mu X^\dagger\nabla^\mu X + V_X(|X|, F_{\mu\nu}F^{\mu\nu})].
\end{aligned}
$$

the components of the vector field AM(z) are zero except the t component At(z). $\mu = A_t(z=0),$

$$S_m^s = -\beta \int \mathrm{d}^5x\sqrt{-g^s}\mathrm{e}^{-\Phi}\,\mathrm{Tr}\Big\{|D_M X|^2 + V_X^s(X_M) + \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\Big\}$$

$$ds^2 = \frac{L^2 e^{2A_E(z)}}{z^2}\Big(-f(z)dt^2 + \frac{dz^2}{f(z)} + dy_1^2 + dy_3^2 + dy_3^2\Big),$$

$$R_{MN}^E - \frac{1}{2}g_{MN}^E R^E - T_{MN} = 0,$$

$$\nabla_M\left[h_\phi(\phi)F^{MN}\right] = 0,$$

$$\partial_M\left[\sqrt{-g}\partial^M\phi\right] - \sqrt{-g}\left(\frac{\mathrm{d}V_\phi(\phi)}{\mathrm{d}\phi} + \frac{F^2}{4}\frac{\mathrm{d}h_\phi(\phi)}{\mathrm{d}\phi}\right) = 0,$$

$$Q_G = \frac{\mathrm{e}^{A_E(z)}}{z}h_\phi(\phi)\frac{\mathrm{d}}{\mathrm{d}z}A_t(z).$$

$$n_b = -\frac{1}{2\kappa_5^2}\frac{\mathrm{e}^{A_E(z_h)}}{z_h}h_\phi(\phi=\phi(z_h))A_t{}'(z_h).$$

$$A_t'' + A_t'\left(-\frac{1}{z} + \frac{h_\phi{}'}{h_\phi} + A_E{}'\right) = 0,$$

$$f'' + f'\left(-\frac{3}{z} + 3A_E{}'\right) - \frac{e^{-2A_E}A_t'^2 z^2 h_\phi}{L^2} = 0,$$

$$A_E'' + \frac{f''}{6f} + A_E'\left(-\frac{6}{z} + \frac{3f'}{2f}\right) - \frac{1}{z}\left(-\frac{4}{z} + \frac{3f'}{2f}\right.$$

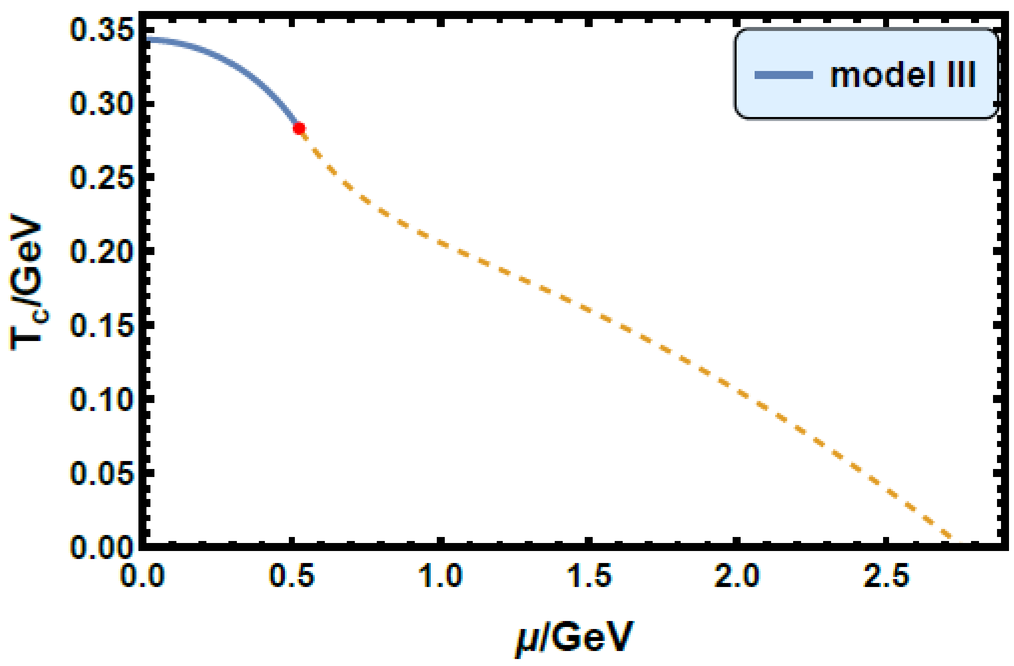
$$+ 3A_E{}'^2 + \frac{L^2 e^{2A_E}V_\phi}{3z^2 f} = 0,$$

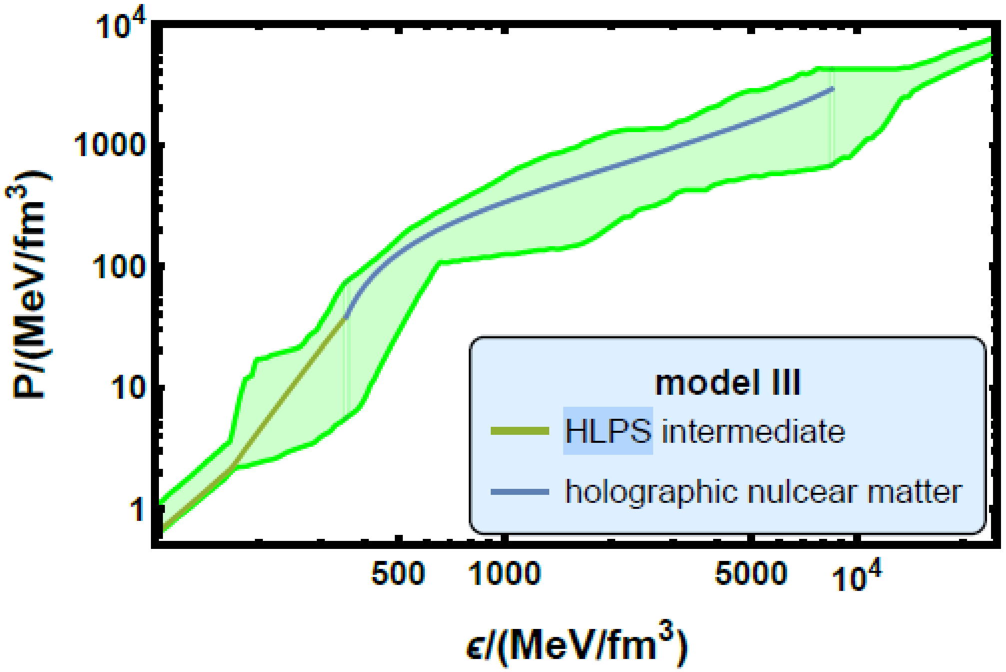
$$A_E'' - A_E'\left(-\frac{2}{z} + A_E'\right) + \frac{\phi'^2}{6} = 0,$$

$$\phi'' + \phi'\left(-\frac{3}{z} + \frac{f'}{f} + 3A_E'\right) - \frac{L^2 e^{2A_E}}{z^2 f}\frac{\mathrm{d}V_\phi(\phi)}{\mathrm{d}\phi} + \frac{z^2 e^{-2A_E}A_t'^2}{2L^2 f}\frac{\mathrm{d}h_\phi(\phi)}{\mathrm{d}\phi} = 0.$$

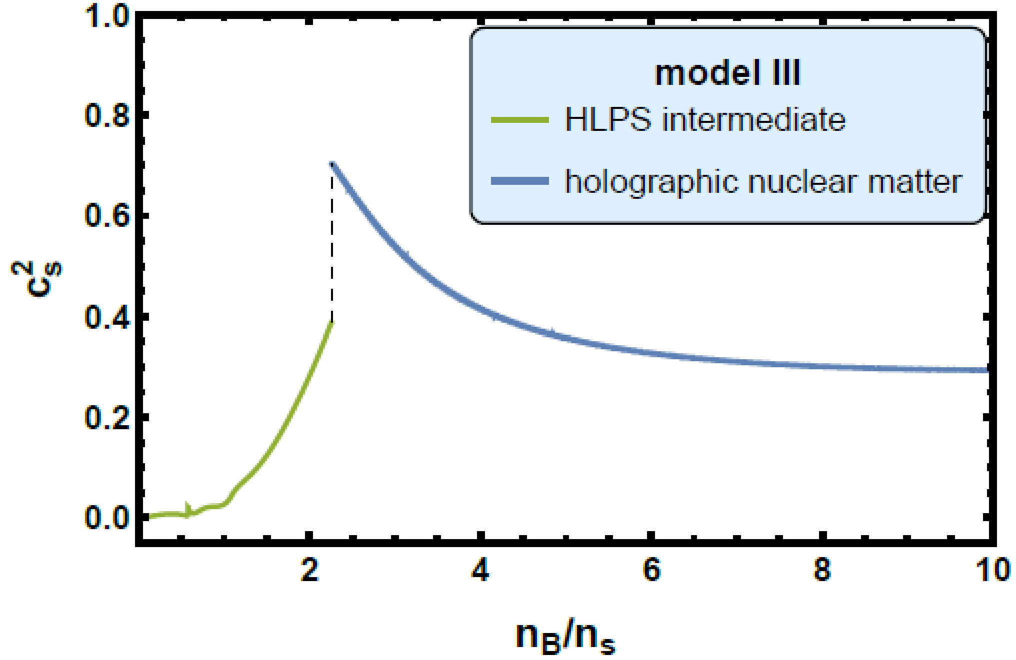
$$\epsilon_b = Ts_b - P_b + \mu n_b,$$

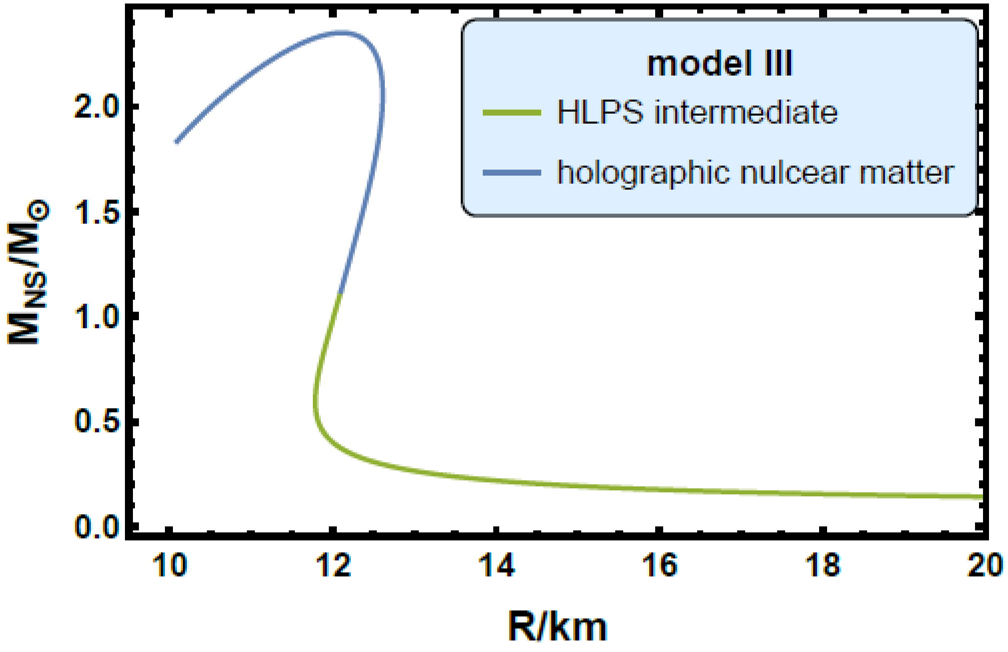
$$\mathcal{F}_b = -P_b = \epsilon_b - Ts_b - \mu n_b.$$

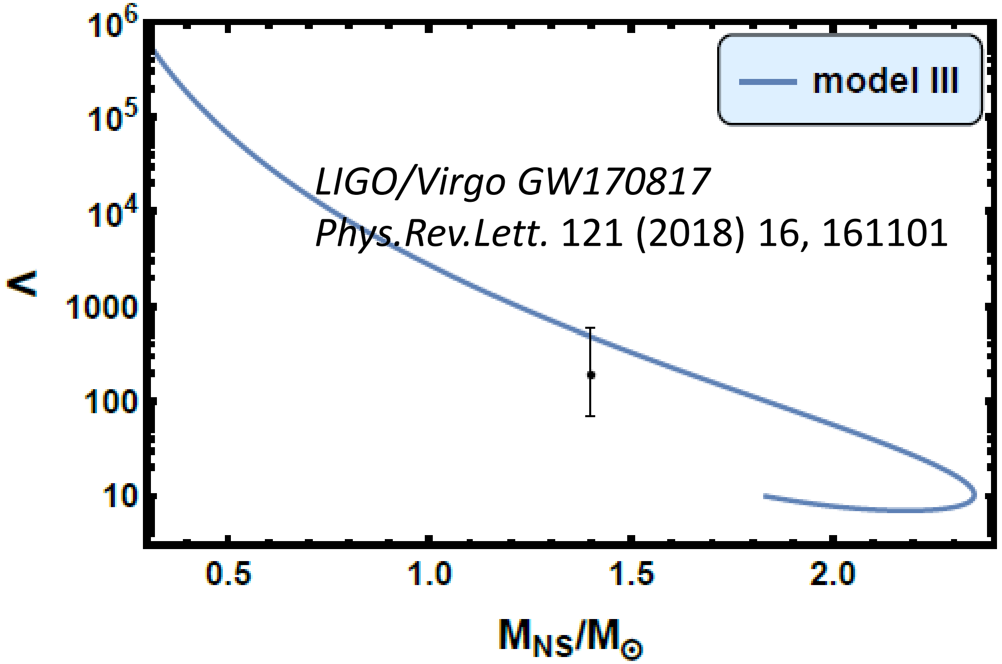




Lin Zhang, M.H.,
Phys.Rev.D 106 (2022) 9, 096028







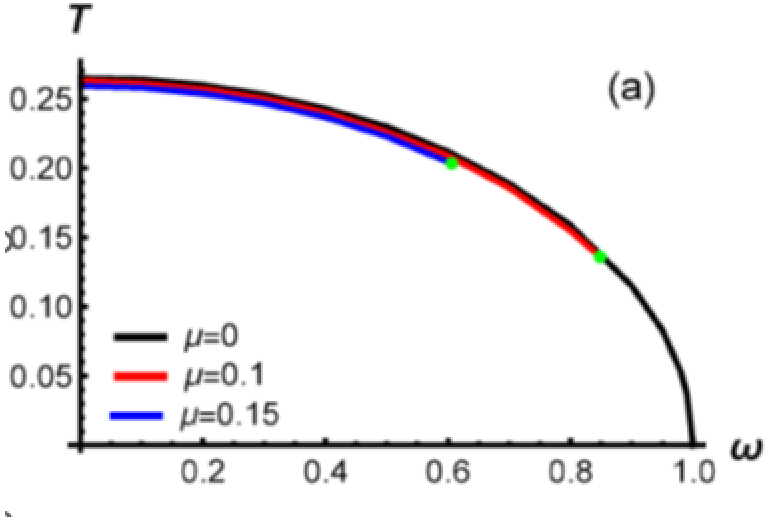
Lin Zhang, M.H.,
Phys.Rev.D 106 (2022) 9, 096028

Deconfinement phase transition under rotation

Pure gluon

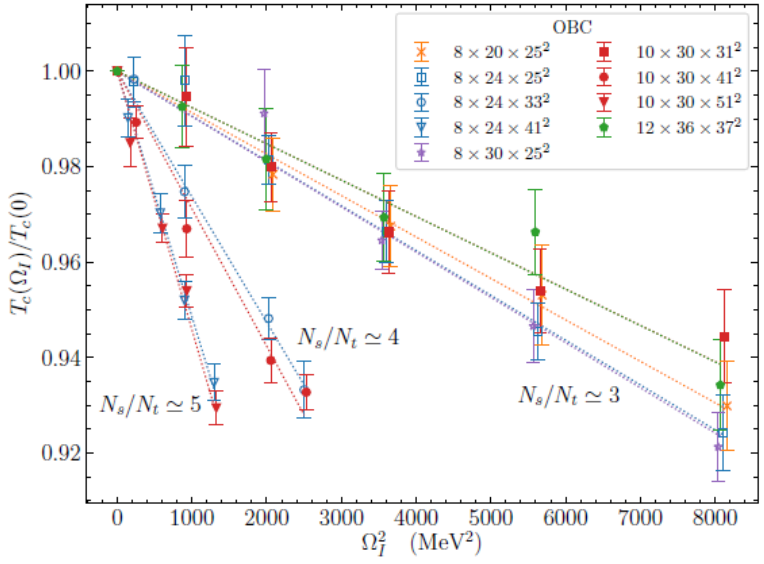
$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}[R - \frac{h(\phi)}{4}F^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)].$$

$$t \to \frac{1}{\sqrt{1-\omega^2}}(t+\omega L\phi), \phi \to \frac{1}{\sqrt{1-\omega^2}}(\phi + \frac{\omega}{L}t),$$



Xun Chen, Lin Zhang, Danning Li, Defu Hou, M.H. arXiv: 2010.14478

Also confirmed by other hQCD results!



V.V. Braguta, et.al. Phys.Rev.D 103 (2021) 9, 094515, e-Print: 2102.05084

Ji-Chong Yang,Xu-Guang Huang e-Print: 2307.05755 [hep-lat]

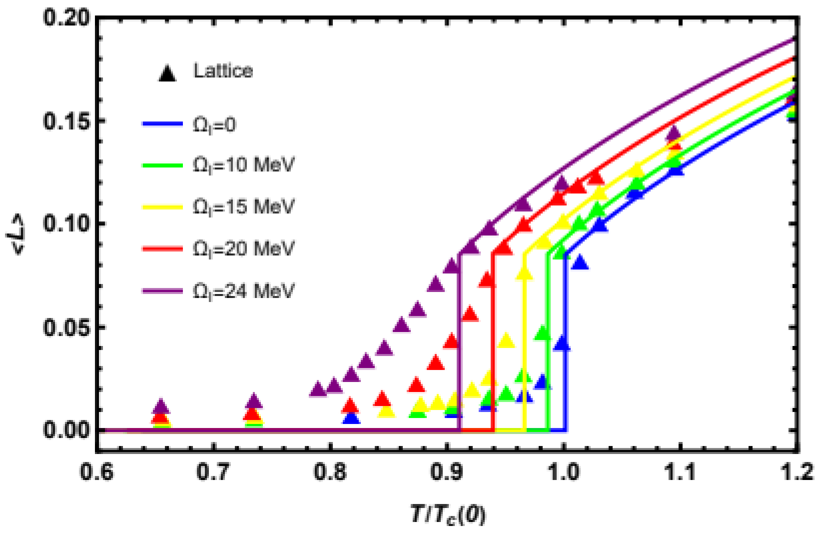
Opposite results on the effect of rotation on the critical temperature of deconfinement phase transition in hQCD and lattice has attracted much attention in recent years! Further studies in progress!

hQCD:Anisotropic background under rotation

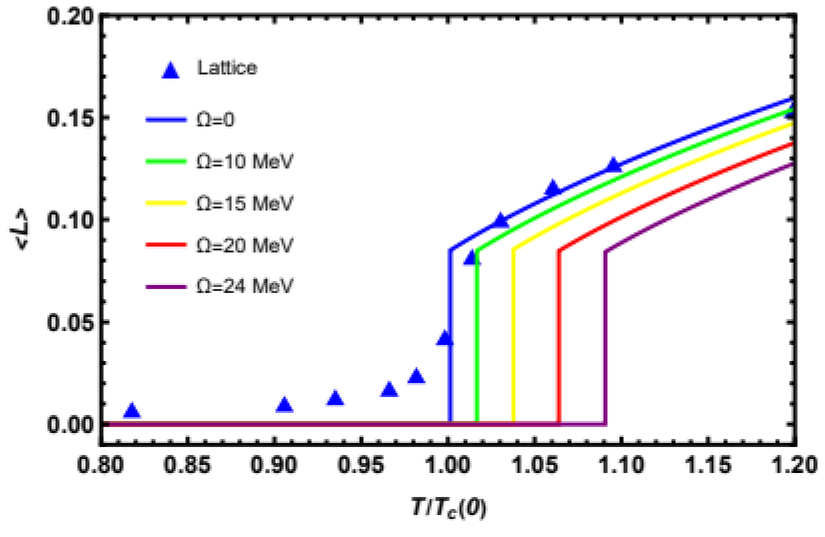
$$ds^2 = \frac{L^2 e^{2A_e(z)}}{z^2}[-f(z)dt^2 + \frac{dz^2}{f(z)} + e^{B(z)}dr^2 + r^2 e^{B(z)}d\theta^2 + e^{-2B(z)}dx_3^2], \qquad A_M = (A_t, 0, 0, A_\theta, 0), \qquad A_\theta = \Omega r^2,$$

Polarized gluonic background

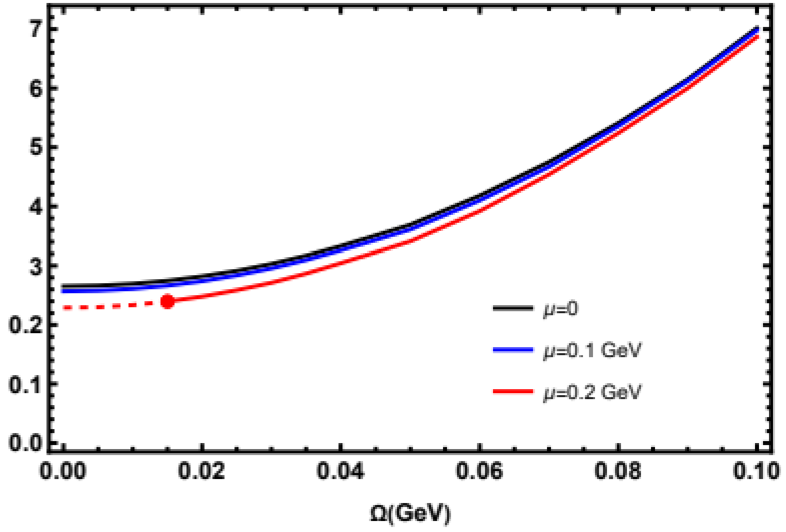
$$\Phi = (\mu_G + \mu_\Omega \Omega^2)^2 z^2 \tanh(\mu_{G^2}^4 z^2/(\mu_G + \mu_\Omega \Omega^2)^2).$$



(a) imaginary rotation



(b) real rotation

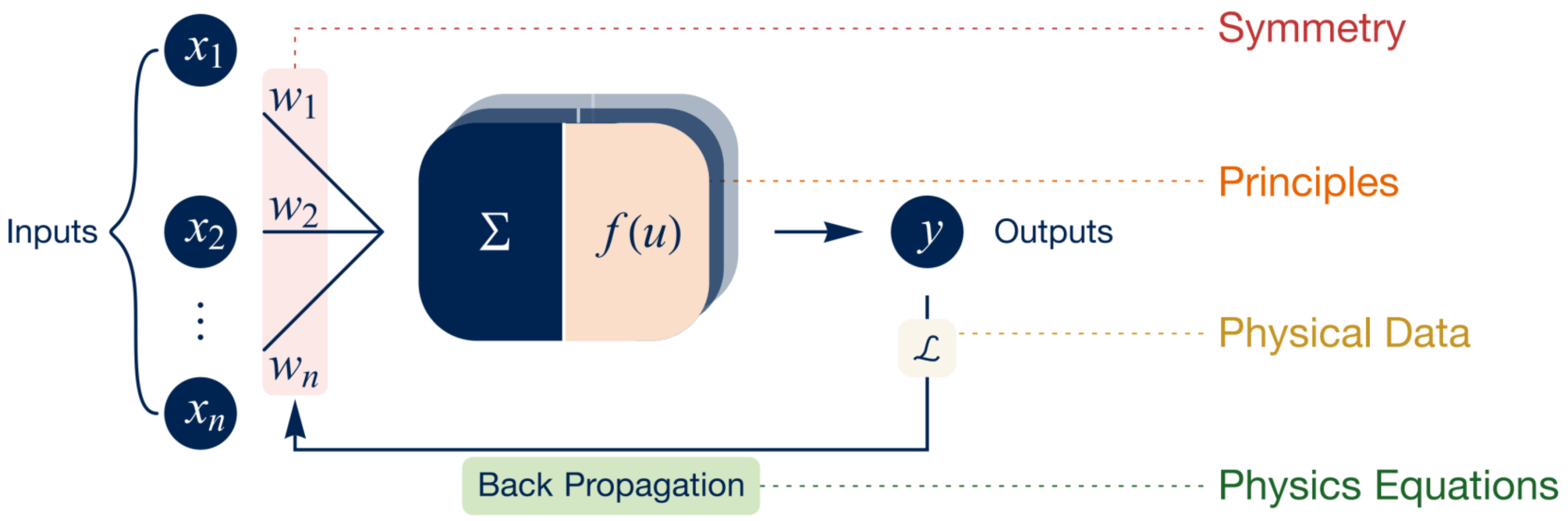


Yidian Chen,Xun Chen,Danning Li,Mei Huang, e-Print: 2405.06386 [hep-ph]

Machine learning hQCD from data,

see Xun Chen's talk!

Physics-Driven Learning for Solving Inverse Problems towards QCD Physics



Using Machine learning to extract the deformed metric,

To get the analytical solutions, we have two assumptions:

$$A(z) = d\ln(az^2 + 1) + d\ln(bz^4 + 1)$$

$$f(z) = e^{cz^2 - A(z) + k}$$

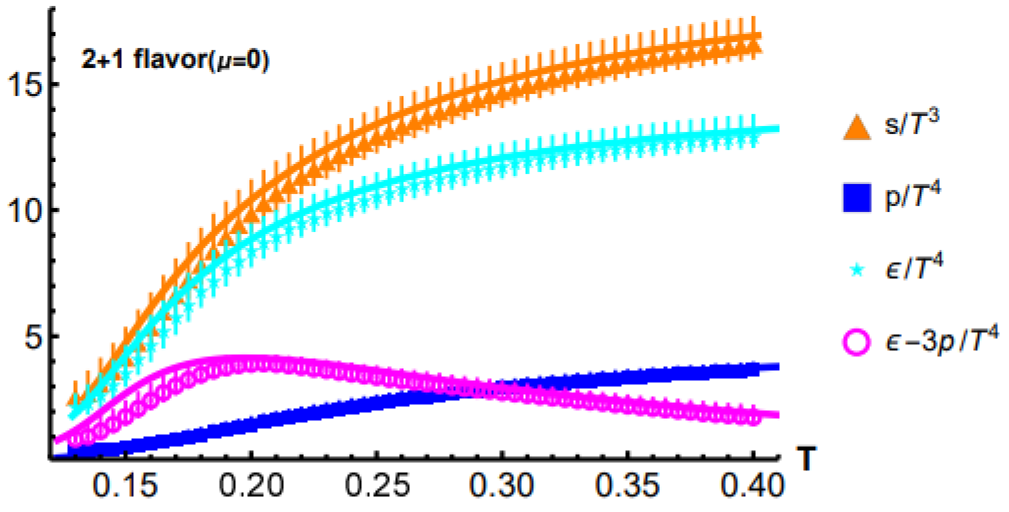
Black hole entropy: $S_{\mathrm{BH}} = \frac{e^{3A(z_h)}}{4G_5 z_h^3}$

hQCD:Xun Chen, M.H., Phys.Rev.D 109 (2024) 5, L051902, e-Print:2401.06417

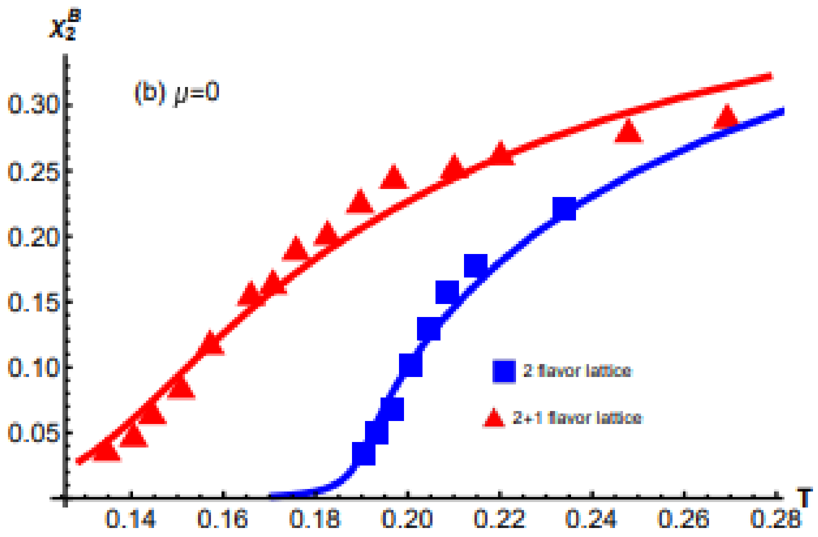
	a	b	c	d	k	G_5	T_c
$N_f = 0$	0	0.072	0	-0.584	0	1.326	0.265
$N_f = 2$	0.067	0.023	-0.377	-0.382	0	0.885	0.189
$N_f = 2+1$	0.204	0.013	-0.264	-0.173	-0.824	0.400	0.128

Strategy of model calculations (rPNJL model, DSE-fRG,fRG,holographic QCD models):

1, Fit model paraters with Lattice QCD EOS and baryon number susceptibility at zero chemical potential;
2,Predictions at finite baryon number chemical potential.



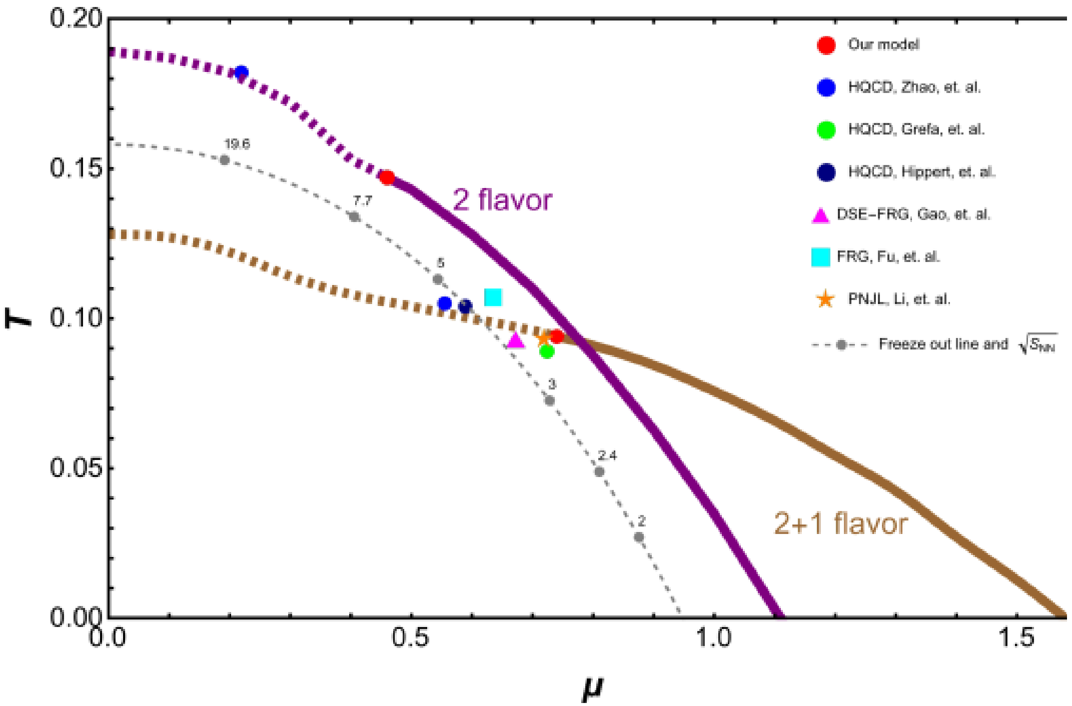
Determine Tc at mu=0

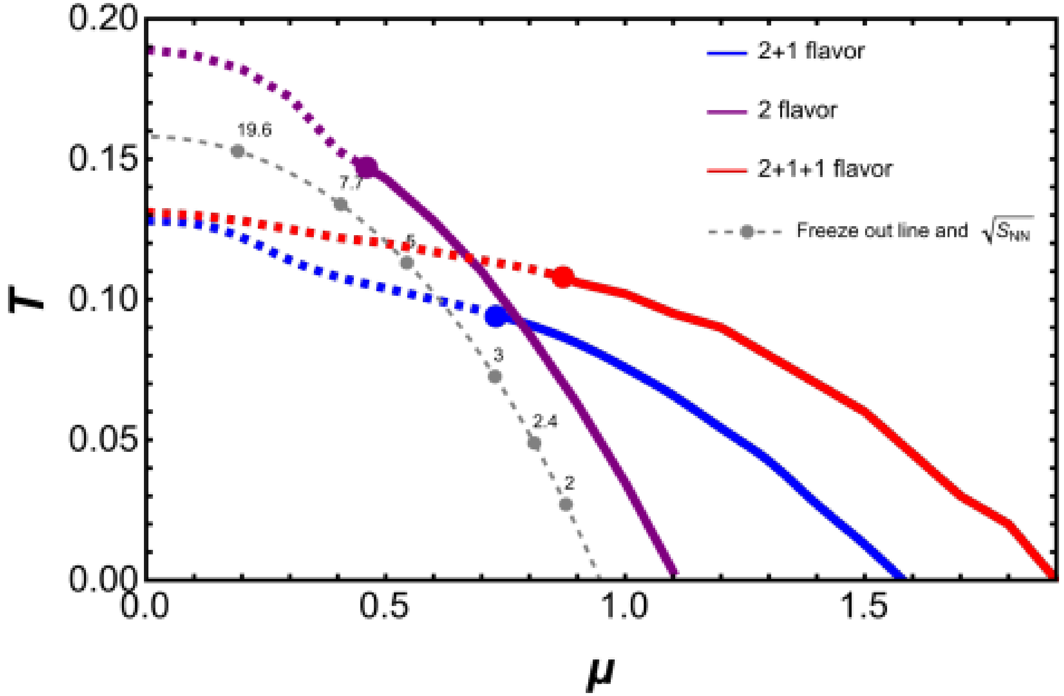


Determine mu dependent Tc

hQCD:Xun Chen, M.H., Phys.Rev.D 109 (2024) 5, L051902, e-Print:2401.06417

Locations of CEP from rPNJL model,holographic QCD models, DSE-fRG,fRG **converge** at around (Tc~100MeV, mu_B^c~700 MeV)

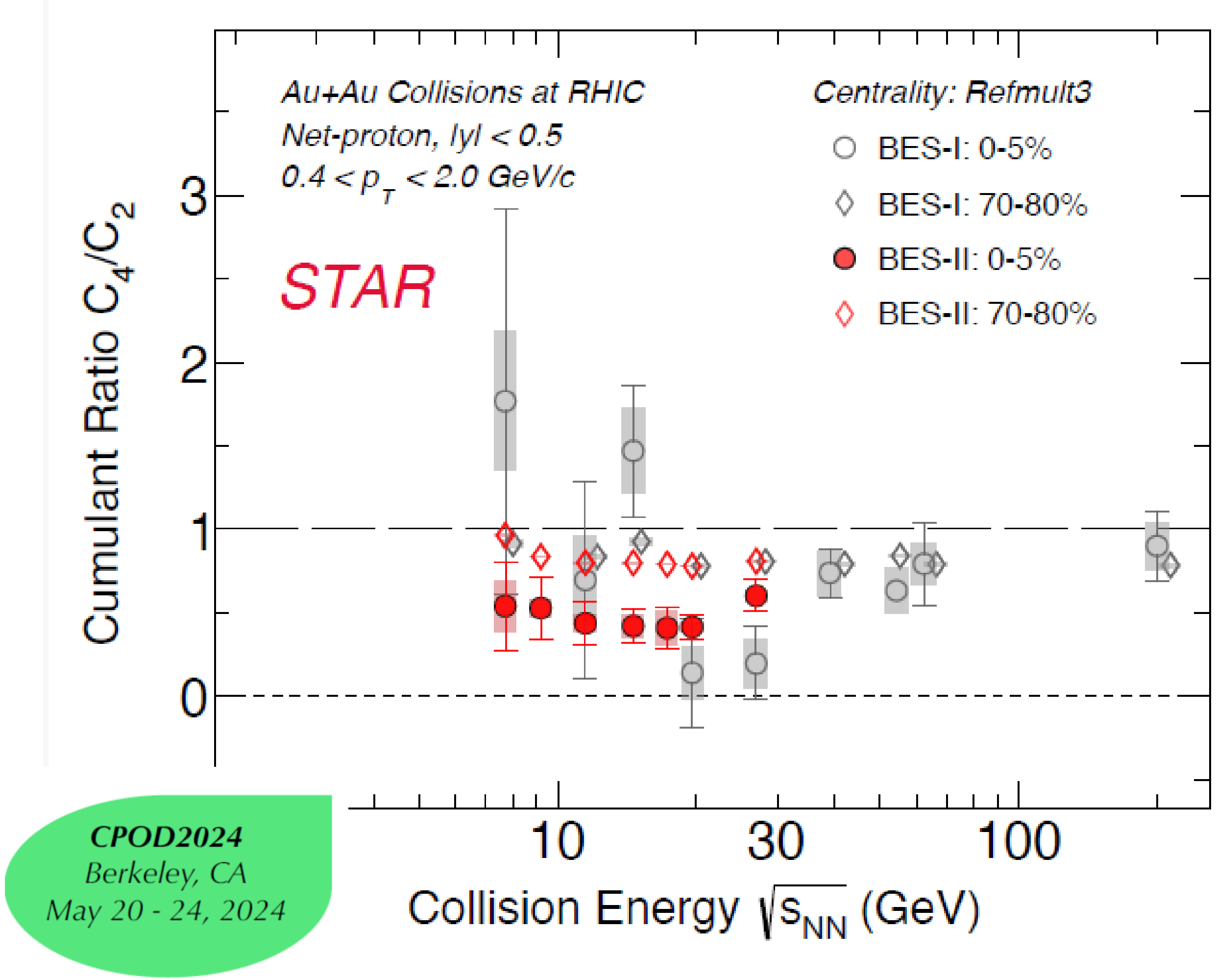




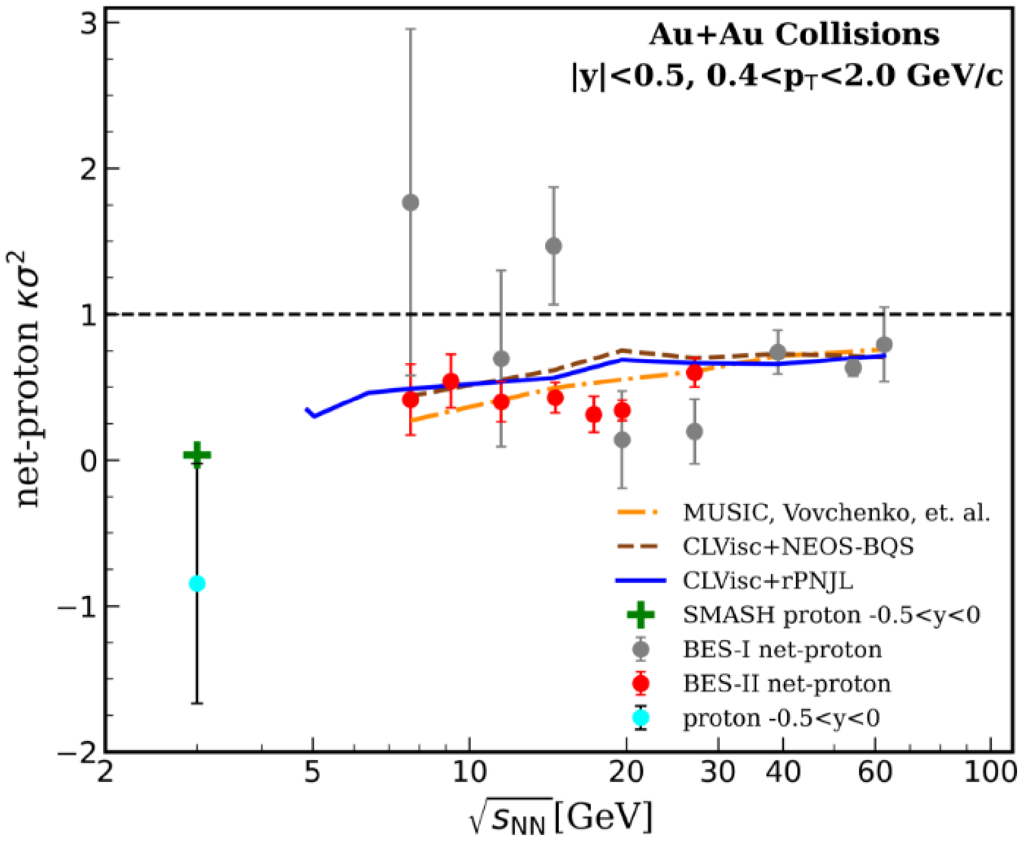
hQCD:Xun Chen, M.H., Phys.Rev.D 109 (2024) 5, L051902, e-Print:2401.06417 [hep-ph];e-Print: 2405.06179 [hep-ph]
J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, and R. Rougemont, Phys. Rev. D 104, 034002 (2021), arXiv:2102.12042 [nucl-th]; M. Hippert, J. Grefa, T. A. Manning, J. Noronha, J. Noronha-Hostler, I. Portillo Vazquez, C. Ratti, R. Rougemont, and M. Trujillo (2023) arXiv:2309.00579 [nucl-th], Y.-Q. Zhao, S. He, D. Hou, L. Li, and Z. Li, JHEP 04, 115 (2023), arXiv:2212.14662 [hep-ph]
rPNJL:Zhibin Li, Kun Xu, Xinyang Wang and MH, arXiv:1801.09215
DSE-fRG: *F. Gao and J. M. Pawlowski, Phys. Rev. D 102, 034027 (2020), arXiv:2002.07500 [hep-ph].*
fRG:W.-j. Fu, J. M. Pawlowski, and F. Rennecke, Phys. Rev. D 101, 054032 (2020), arXiv:1909.02991 [hep-ph].

Disappearance of the nonmonotonic structure above 7.7 GeV and EOS independent!

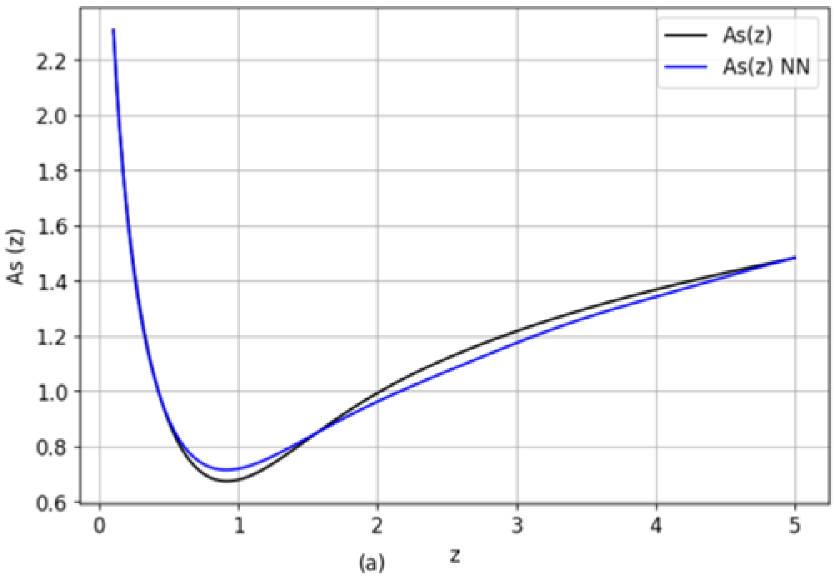
Signature washed out through hadronization? Other signals not sensitive to hadronization are needed!

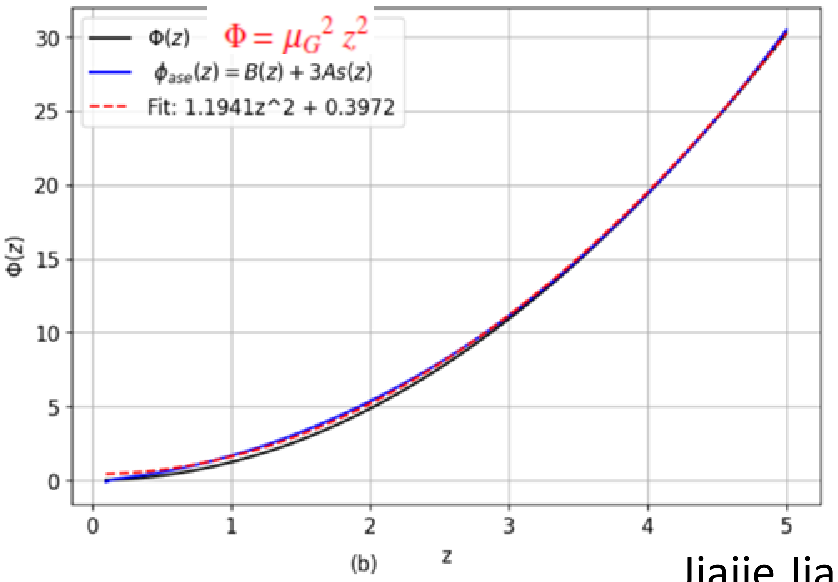


Ashish Pandav for STAR Collaboration
Lawrence Berkeley National Laboratory
May 21, 2024



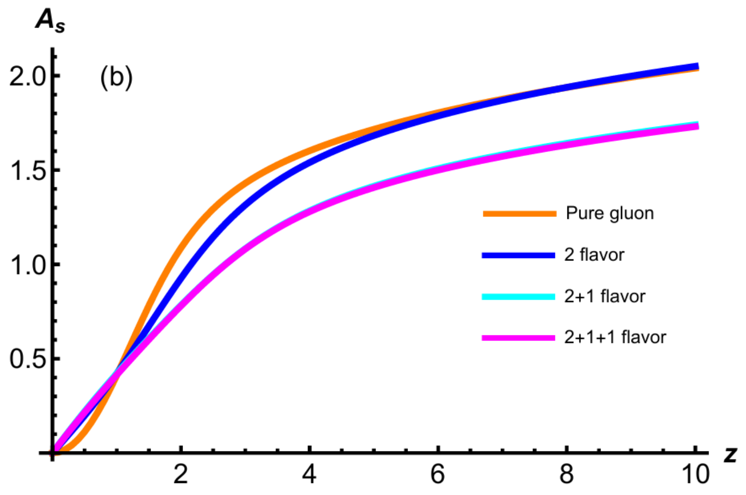
Yifan Shen, Wei Chen, Xiangyu Wu, Kun Xu, MH,
arXiv:e-Print: 2404.02397 [hep-ph]

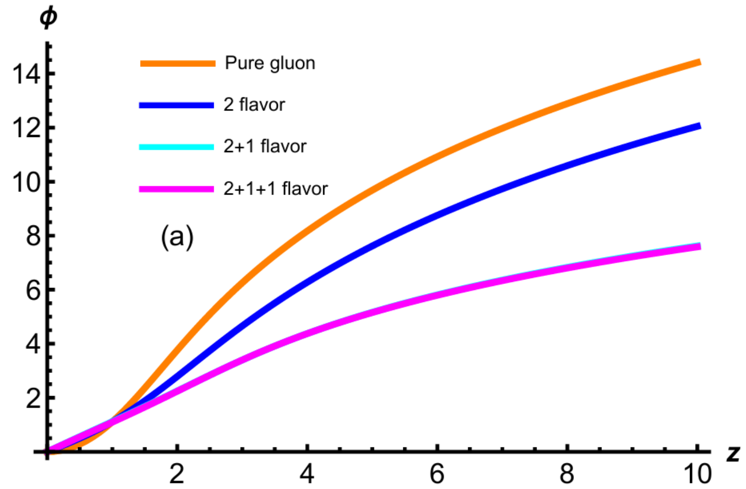




Learning scalar glueball spectra

Jiajie Jiang, … in progress

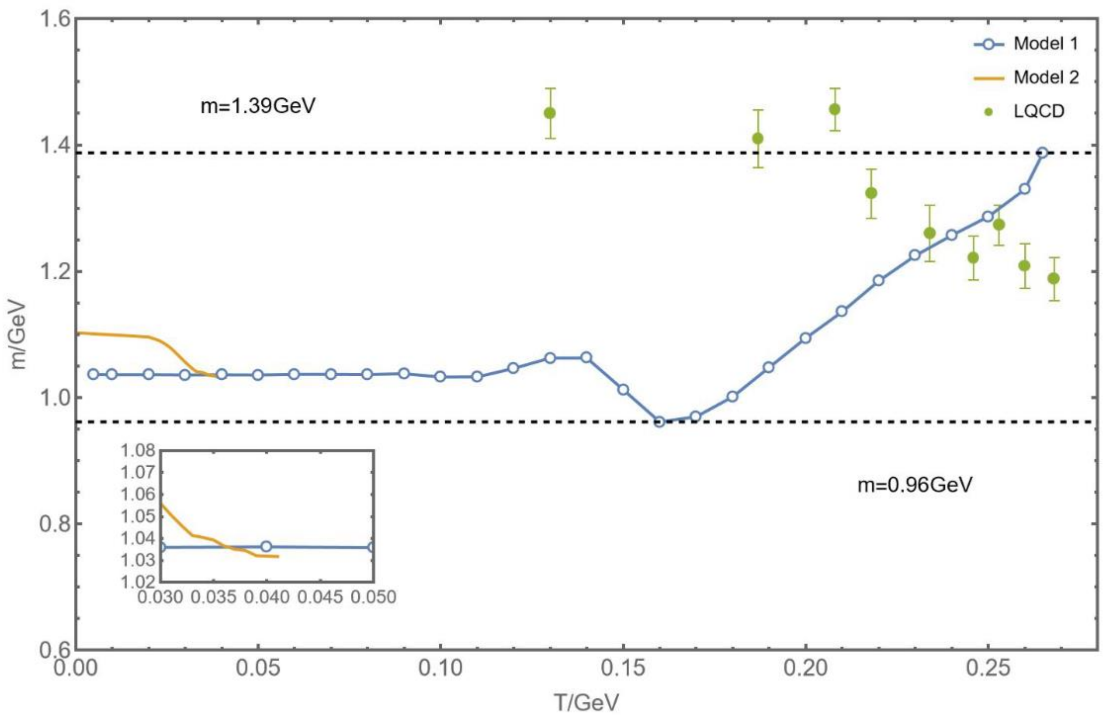




Learning EOS

Xun Chen, M.H., e-Print: 2405.06179 [hep-ph]

The machine learning hQCD from glueball spectra is different from the that from EOS.



Ruixiang Chen, … in progress

The machine learning hQCD from EOS cannot get scalar glueball mass correct at zero and low temperature !

Missing something between zero temperature and finite temperature? The way of introducing the BH?

Conclusion and outlook

1, 5D DhQCD offers a systematic framework to describe the emergent real world from QCD theory, including Linear confinement and chiral symmetry breaking; hadron spectra (glueball spectra, light-flavor and heavy flavor spectra) and form factors, QCD phase transitions, thermodynamical and transport propertities, and

More: 1)Nonequilibrium evolution, inflation,GW

2) nucleon structure,PDF

Future： Machine learning will accelerate the building of HQCD! we will have a unified hQCD framework within several years!

Thanks for your attention!