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Aspects of Holographic Pseudo Entropy

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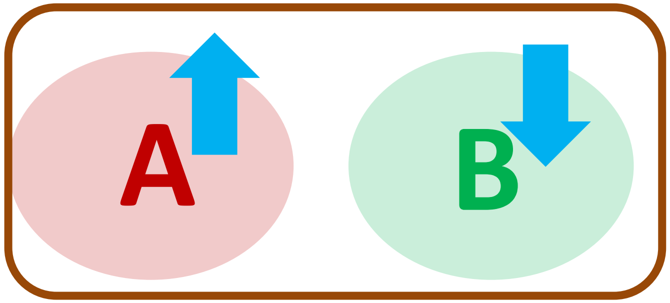
Grant-in-Aid for Transformative Research Areas A
Extreme Universe

① Introduction



Quantum Entanglement (QE)

Two parts (subsystems) A and B in a total system are quantum mechanically correlated.

e.g. Bell state: $|\Psi_{Bell}\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_A \otimes |\downarrow\rangle_B + |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right]$ ➡ **Minimal Unit of Entanglement**

Pure States: Non-zero QE ⇔ $|\Psi\rangle_{AB} \neq |\Psi_1\rangle_A \otimes |\Psi_2\rangle_B$.

Direct Product

The best (or only) measure of quantum entanglement for pure states is known to be **entanglement entropy (EE)**.

EE ＝ # of Bell Pairs between A and B

Entanglement entropy (EE)

Divide a quantum system into two subsystems A and B:

$$H_{tot} = H_A \otimes H_B \quad .$$

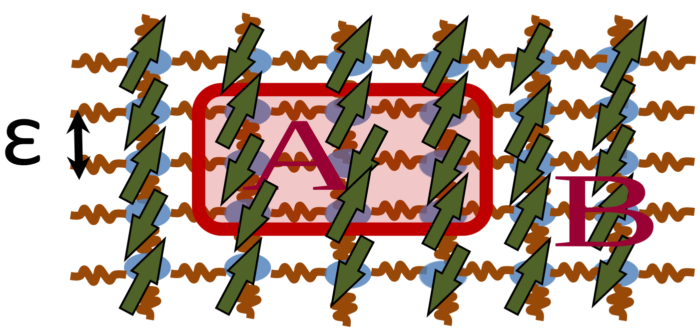
Define the reduced density matrix by $\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$.

The entanglement entropy S_A is defined by

$$S_A = -\mathrm{Tr}_A\, \rho_A \log \rho_A \,.$$

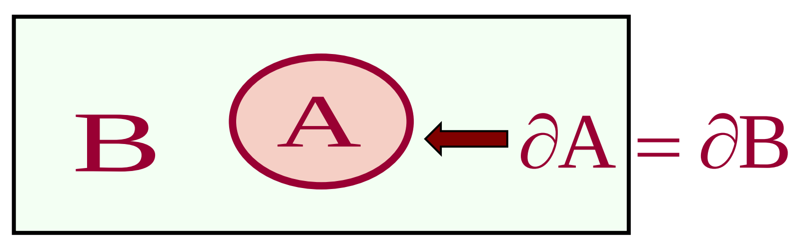
(von-Neumann entropy)

Quantum Many-body Systems



Continuum Limit ε→0

Quantum Field Theories (QFTs)



Measurement of EE in Experiments

Ex.1: Ultracold bosonic atoms in optical lattices

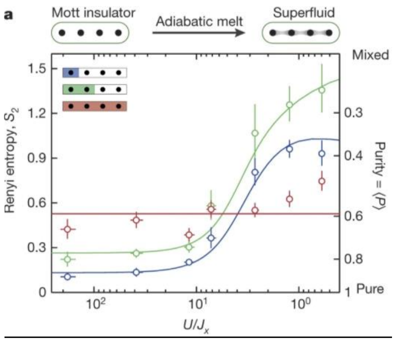
Published: 02 December 2015

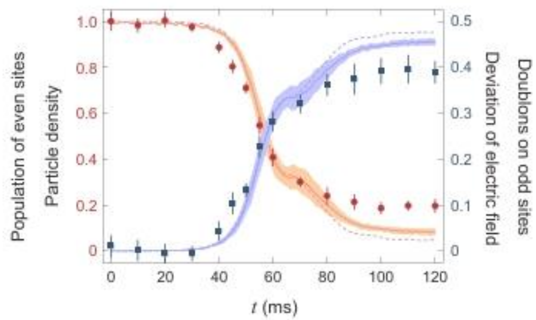
Measuring entanglement entropy in a quantum many-body system

Rajibul Islam, Ruichao Ma, Philipp M. Preiss, M. Eric Tai, Alexander Lukin, Matthew Rispoli & Markus Greiner

Nature 528, 77–83 (2015) | Cite this article

$$H = -J\sum_{\langle i,j\rangle} a_i^\dagger a_j + \frac{U}{2}\sum_i n_i(n_i-1) \qquad (4)$$





Ex2: Trapped-ion quantum simulator

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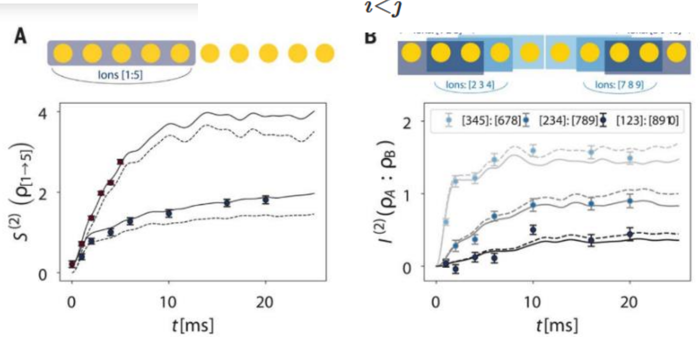
REPORT

Probing Rényi entanglement entropy via randomized measurements

TIFF BRYDGES, ANDREAS ELBEN, PETAR JURCEVIC, BENOÎT VERMERSCH, CHRISTINE MAIER, BEN P. LANYON, PETER ZOLLER, RAINER BLATT, AND CHRISTIAN F. ROOS Authors Info & Affiliations

SCIENCE • 19 Apr 2019 • Vol 364, Issue 6437 • pp. 260-26

$$H_{XY} = \hbar\sum_{i<j} J_{ij}\left(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+\right) + \hbar B\sum_j \sigma_j^z$$



Ex3. Topological EE in superconducting qubits

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SCIENCE • 2 Dec 2021 • Vol 374, Issue 6572 • pp. 1237-1241 • DOI: 10.1126/science.abi8378

RESEARCH ARTICLE | TOPOLOGICAL MATTER

Realizing topologically ordered states on a quantum processor

K. J. SATZINGER, Y.-J LIU, A. SMITH, C. KNAPP, M. NEWMAN, C. JONES, Z. CHEN, C. QUINTANA, X. MI, [...], AND P. ROUSHAN +88 authors

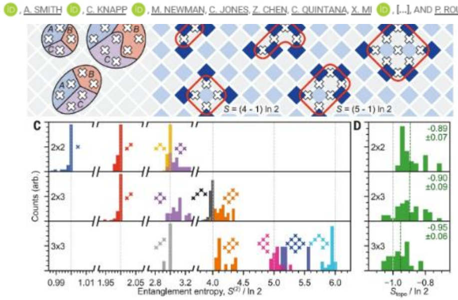

Fig. 2. Topological entanglement entropy.

In this talk, we will introduce a generalization of entanglement entropy, called pseudo entropy (PE).

Motivation 1

Generalize entanglement entropy to post-selection processes

Motivation 2

Generalize holographic entanglement to Euclidean time-dep. AdS

Motivation 3

Non-standard Lorentzian Holography

Dual CFTs are non-Hermitian !

(i) Holographic entanglement for dS/CFT ? ➔ Need PE !

(ii) Traversable wormholes in AdS ➔ PE is a useful probe !

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③ Pseudo Entropy
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⑤ De Sitter Holography and Pseudo Entropy
⑥ Traversable Wormholes
⑦ Conclusions

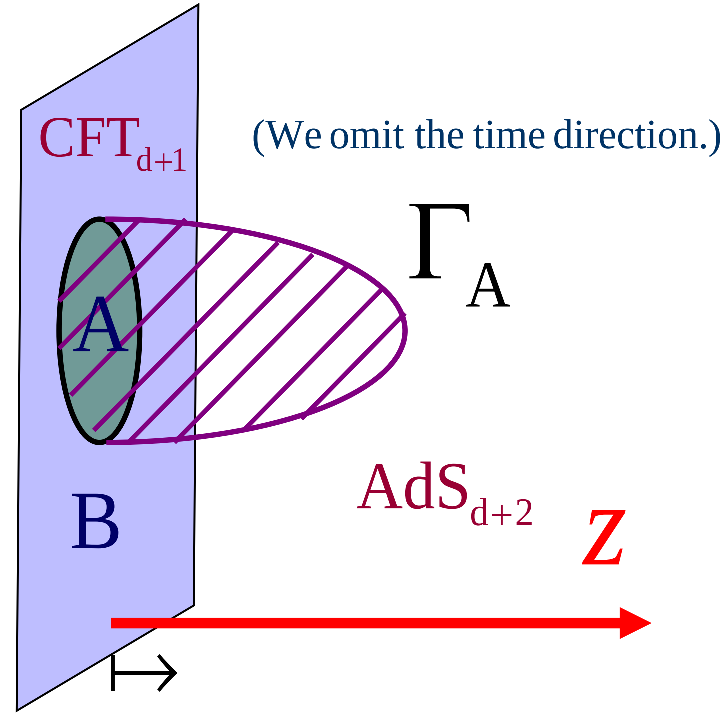
② Ver.3 of Holographic Entanglement Entropy ?

Ver. 1 Holographic EE for Static Spacetimes

[Ryu-TT 06]

For static asymptotically AdS spacetimes:

$$S_A = \underset{\substack{\partial\Gamma_A=\partial A \\ \Gamma_A \approx A}}{\mathrm{Min}} \left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N} \right]$$



Γ_A is the minimal area surface (codim.=2) on the time slice such that

$$\partial A = \partial \gamma_A \text{ and } A \sim \gamma_A \ .$$

homologous

$z > \varepsilon$ (UV cut off)

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

Ver. 2 Covariant Holographic Entanglement Entropy

[Hubeny-Rangamani-TT 07]

A generic Lorentzian asymptotic AdS spacetime is dual to a time dependent state $|\Psi(t)\rangle$ in the dual CFT.

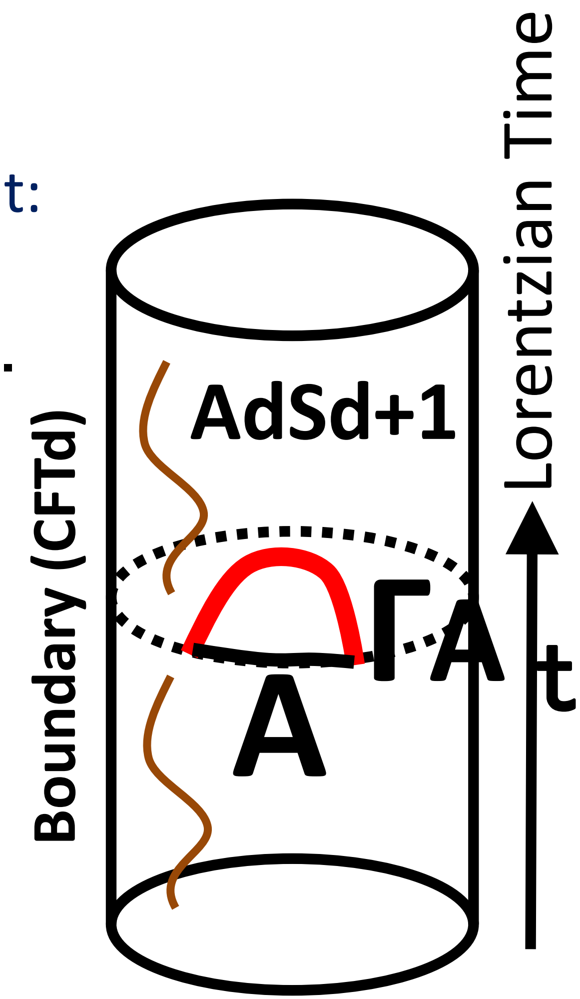
The entanglement entropy gets time-dependent:

$$\rho_A(t) = \mathrm{Tr}_B[|\Psi(t)\rangle\langle\Psi(t)|] \Rightarrow S_A(t).$$

This is computed by the holographic formula:

$$S_A(t) = \mathrm{Min}_{\Gamma_A}\mathrm{Ext}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right]$$

$$\partial A = \partial\gamma_A \text{ and } A \sim \gamma_A \; .$$



Ver 3. Formula ?

Minimal areas in ***Euclidean time dependent***
asymptotically AdS spaces

= What kind of QI quantity (Entropy ?) in CFT ?

The answer is Pseudo Entropy !

[Nakata-Taki-Tamaoka-Wei-TT, 2020]

③ Pseudo Entropy

(3-1) Definition of Pseudo (Renyi) Entropy

Consider two quantum states $|\psi\rangle$ and $|\varphi\rangle$, and define the *transition matrix*:

$$\tau^{\psi|\varphi} = \frac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}.$$

We decompose the Hilbert space as $H_{tot} = H_A \otimes H_B$.

and introduce the reduced transition matrix:

$$\tau_A^{\psi|\varphi} = \mathrm{Tr}_B\left[\tau^{\psi|\varphi}\right]$$

Pseudo Entropy

$$S\left(\tau_A^{\psi|\varphi}\right) = -\mathrm{Tr}\left[\tau_A^{\psi|\varphi}\log\tau_A^{\psi|\varphi}\right].$$

Renyi Pseudo Entropy

$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \frac{1}{1-n}\log\mathrm{Tr}\left[\left(\tau_A^{\psi|\varphi}\right)^n\right].$$

(3-2) Basic Properties of Pseudo Entropy (PE)

- In general, $\tau_A^{\psi|\varphi}$ is not Hermitian. Thus PE is complex valued.

◆For thermal pseudo entropy, Kramers-Kronig relation relates the real part of PE to the imaginary part. [Caputa-Chen-Tsuda-TT 2024]

$$\mathrm{Im}[f(t)] = \frac{1}{\pi} P \int_{-\infty}^{\infty} ds \frac{\mathrm{Re}[f(s)]}{s-t},$$

When does PE become real ? ➡

◆Real valued Euclidean PI= Holographic PE

◆Pseudo Hermiticity [Guo-He-Zhan 2022]

- If either $|\psi\rangle$ or $|\varphi\rangle$ has no entanglement (i.e. direct product state) , then

$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = 0.$$

- We can show $S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \left[S^{(n)}\left(\tau_A^{\varphi|\psi}\right)\right]^{\dagger}.$

- We can show $S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = S^{(n)}\left(\tau_B^{\psi|\varphi}\right).$ →"SA=SB"

(3-3) Thermal Pseudo Entropy [Caputa-Chen-Tsuda-TT 2024]

Consider TFD state under time evolution:

$$|\Psi_\beta(t)\rangle = e^{-iH_L t}|\Psi_\beta\rangle = \frac{1}{\sqrt{Z(\beta)}}\sum_n e^{-\frac{\beta+2it}{2}E_n}|E_n\rangle_L \otimes |E_n\rangle_R .$$

$$\tau = \frac{|\Psi_\beta(t)\rangle\langle\Psi_\beta|}{\langle\Psi_\beta|\Psi_\beta(t)\rangle}, \qquad \tau_L = \mathrm{Tr}_R(\tau) = \frac{1}{Z(\beta+it)}\sum_n e^{-(\beta+it)E_n}|E_n\rangle\langle E_n| .$$

Thermal Pseudo Entropy: $$S^{(n)}(\beta+it) = \frac{1}{1-n}\log\left[\frac{Z(n\beta)}{Z(\beta)^n}\right]_{\beta\to\beta+it},$$

Relation to SFF: $$\overline{\mathrm{Re}\, S^{(n)}(\beta+it)} = \frac{1}{2(1-n)}\left[\log\overline{|Z(n(\beta+it))|^2} - n\log\overline{|Z(\beta+it)|^2}\right].$$

Ex. Behavior of TPE for Chaotic systems

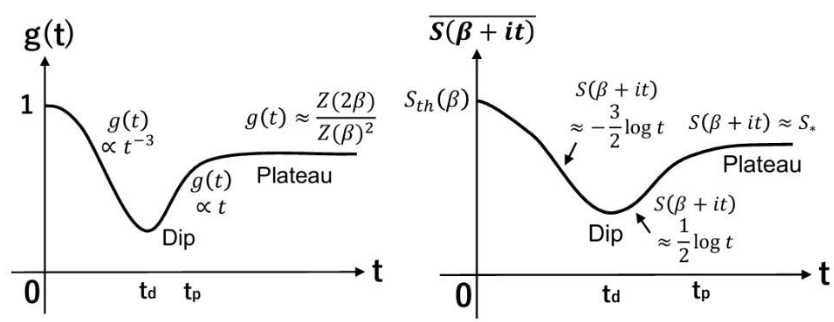


Figure 7: The sketches of evolution of the TPE $g_\beta(t)$ (left) and the averaged TPE $\overline{S(\beta+it)}$ (right) in the random matrix model.

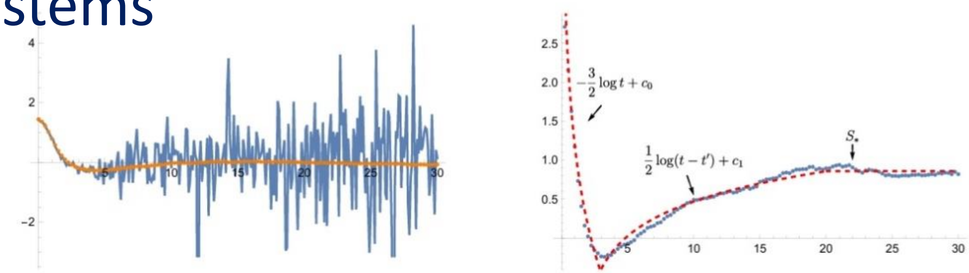


Figure 9: (Left) TPE averages over 200 instances of 100×100 random unitary matrices. Here $\beta = 1$. A time averaged version is shown in orange. (Right) Time-averaged TPE with the same parameters, fit by the logarithmic behaviours for slope and ramp as well as the final plateau in red dashed lines.

(3-4) SVD entropy [Parzygnat–Taki–Wei–TT 2023]

Motivation: Improve PE so that **(i)** it become real and non–negative and **(ii)** it has a better LOCC interpretation.

→ SVD entropy

$$S_{SVD}\left(\tau_A^{\psi|\varphi}\right) = -\mathrm{Tr}\left[|\tau_A^{\psi|\varphi}| \cdot \log|\tau_A^{\psi|\varphi}|\right].$$

here, $|\tau_A^{\psi|\varphi}| \equiv \sqrt{\tau_A^{\dagger\psi|\varphi}\tau_A^{\psi|\varphi}}$

- This is always non–negative and is bounded by log dim H_A.

- From quantum information theoretic viewpoint, this is the number of Bell pairs distilled from the intermediate state:

$\tau_A^{\psi|\varphi}$=U・Λ・V,

$$\frac{\langle\varphi|V^\dagger \sum_k |\mathrm{EPR}_k\rangle\langle\mathrm{EPR}_k| U^\dagger|\psi\rangle}{\langle\varphi|V^\dagger U^\dagger|\psi\rangle} = \sum_k p_k = 1$$

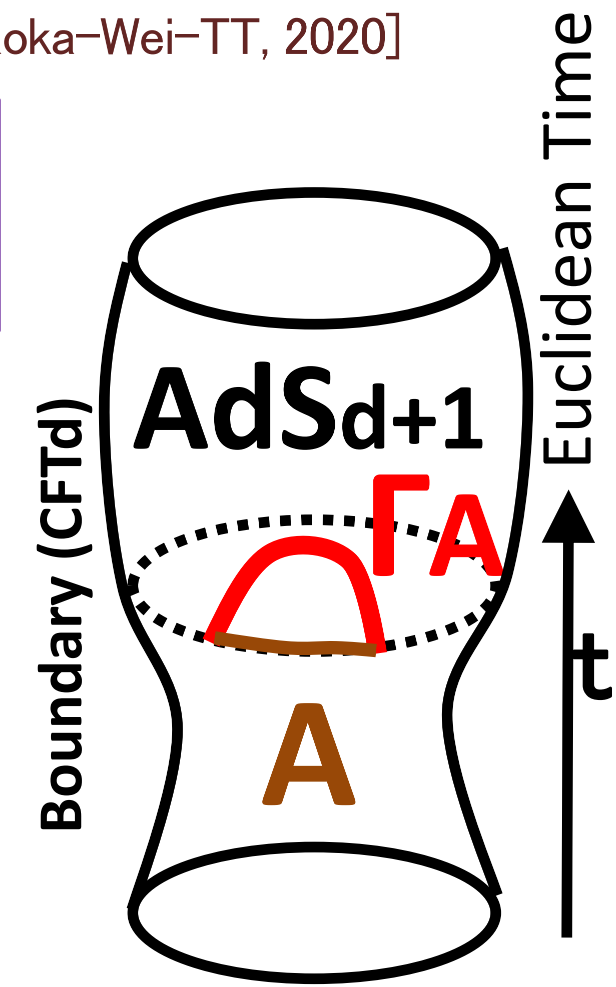
$$\Rightarrow \quad S_{SVD} \approx \sum_k p_k \cdot \#\text{of Bell Pairs in} |\mathrm{EPR}_k\rangle$$

(3-4) Holographic Pseudo Entropy

Holographic Pseudo Entropy (HPE) Formula

[Nakata−Taki−Tamaoka−Wei−TT, 2020]

$$S\left(\tau_A^{\psi|\varphi}\right) = \mathrm{Min}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right]$$



Basic Propertie

(i) If ρ_A *is pure*, $S\left(\tau_A^{\psi|\varphi}\right)$ =0.

(ii) If ψ *or* φ *is not entangled*,
$S\left(\tau_A^{\psi|\varphi}\right)$ =0.

→This follows from AdS/BCFT [TT 2011]

(iii) $S\left(\tau_A^{\psi|\varphi}\right) = S\left(\tau_B^{\psi|\varphi}\right)$. **"SA=SB"**

④ Pseudo Entropy and Quantum Phase Transitions

[Mollabashi–Shiba–Tamaoka–Wei–TT 20, 21]

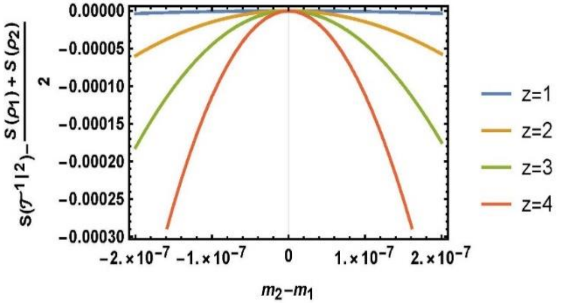
(4-1) Basic Properties of Pseudo entropy in QFTs

[1] Area law

$$S_A \sim \frac{\mathrm{Area}(\partial A)}{\varepsilon^{d-1}} + (\text{subleading terms}),$$

[2] The difference

$$\Delta S = S\left(\tau_A^{1|2}\right) + S\left(\tau_A^{1|2}\right) - S(\rho_A^1) - S(\rho_A^2)$$



is negative if $|\psi_1\rangle$ and $|\psi_2\rangle$ are in a same phase.

PE in a 2 dim. free scalar when we change its mass.

➡ **What happen if they belong to different phases ?**
Can ΔS be positive ?

(4-2) Quantum Ising Chain with a transverse magnetic field

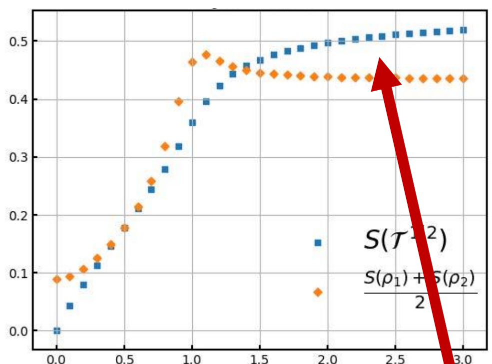
$$H = -J \sum_{i=0}^{N-1} \sigma_i^z \sigma_{i+1}^z - h \sum_{i=0}^{N-1} \sigma_i^x,$$

Ψ1→ vacuum of H(J1)
Ψ2→ vacuum of H(J2)
(We always set h=1)

J<1	Paramagnetic Phase
J>1	Ferromagnetic Phase

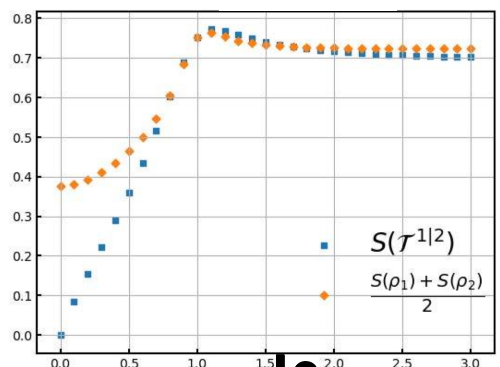
N=16, NA=8

J1=1/2



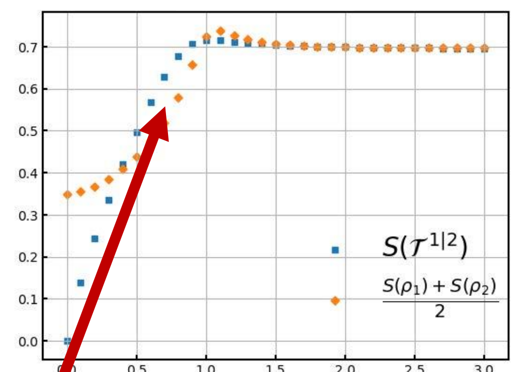
J2

J1=1



J2

J1=2

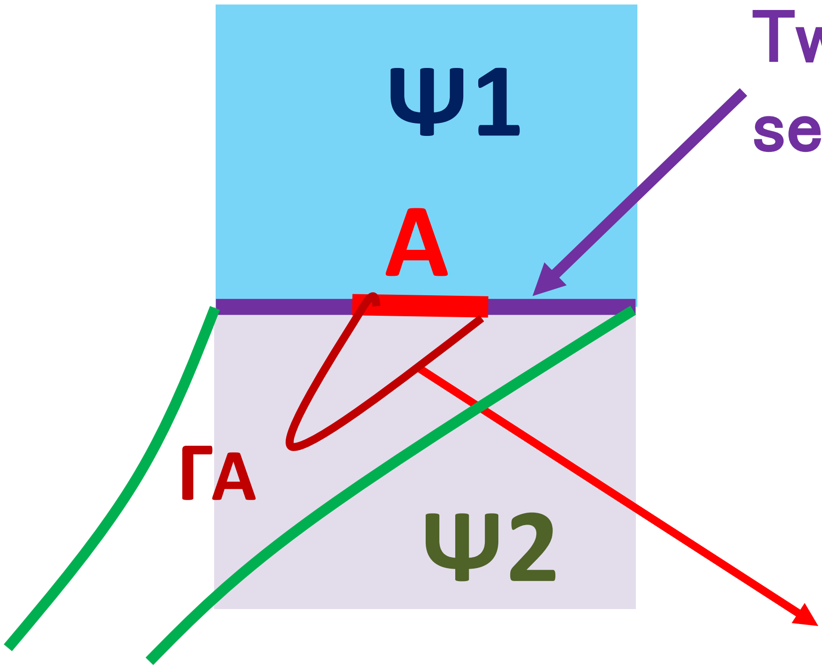


J2

$$\Delta S = S\left(\tau_A^{1|2}\right) + S\left(\tau_A^{1|2}\right) - S(\rho_A^1) - S(\rho_A^2) > 0$$

We find when Ψ1 and Ψ2 are in different phases !

Heuristic Interpretation



Two gapped phases are separated by a gapless phase.

CFT !

PE is enhanced ! $\Delta S > 0$

AdS Dual of
Gapless Interface

The gapless interface (edge state) also occurs in topological orders.

➔Topological pseudo entropy

[Nishioka-Taki-TT 2021, Caputa-Purkayastha-Saha-Sułkowski 2024]

⑤ dS Holography and Pseudo Entropy

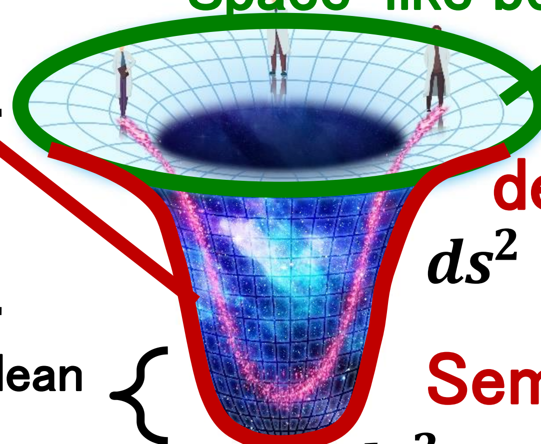
A Sketch of dS/CFT [Strominger 2001, Witten 2001, Maldacena 2002,⋯.]

d+1 dim. Lorentzian de−Sitter spacetime

Dual ⟺

Euclidean d. dim CFT on S^d

Space−like bdy



$t=\infty$

Time

Lorentzian time

Euclidean time

Time emerges from Euclidean CFT !

de Sitter

$$ds^2 = L_{dS}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$$

Semi sphere

$$ds^2 = L_{dS}^2(d\theta^2 + \mathrm{Sin}^2\theta d\Omega^2)$$

$$\theta = it + \frac{\pi}{2}$$

Ψ [dS gravity]$=$Z [CFT]

What we expect for dS/CFT

→Let us assume dS Einstein gravity and extract general expectations.

d+1 dim. (Lorentzian) de-Sitter $ds^2 = L_{dS}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$

S^{d+1} (Euclidean de-Sitter) $ds^2 = L_{dS}^2(d\theta^2 + \mathrm{Sin}^2\theta d\Omega^2)$

$L_{AdS} = iL_{dS}, \ \rho = i\theta$

Euclidean AdS (H^{d+1}) $ds^2 = L_{AdS}^2(d\rho^2 + \mathrm{Sinh}^2\rho d\Omega^2)$

Central charge: $$c \sim \frac{L_{AdS}^{d-1}}{G_N} = i^{d-1} \cdot \frac{L_{dS}^{d-1}}{G_N}$$

We are interested in d=2 case in this talk !

(i) Central charge becomes imaginary for d=even !

(ii) Central charge gets larger in classical gravity limit.

CFT dual of dS in Einstein gravity

[Hikida–Nishioka–Taki–TT, 2021]

Large c limit of SU(2)k WZW model (a 2dim. CFT)

= **Einstein Gravity on 3 dim. de Sitter** (radius L_{dS})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VIEWPOINT

Steps toward Quantum Gravity in a Realistic Cosmos

Jordan Cotler

Society of Fellows, Harvard University, Cambridge, MA, USA

July 18, 2022 • Physics 15, 107

Theorists have modeled an expanding spacetime—akin to our Universe—by taking inspiration from a string theory framework in which spacetime is emergent.

Level

$$k \approx -2 + \frac{4iG_N}{L_{dS}} \longrightarrow \Delta \approx iL_{dS} \cdot E_{dS}$$

Conformal dim. **Energy in dS**

Central charge

$$c = \frac{3k}{k+2} \approx i\frac{3L_{dS}}{2G_N}$$

$$Z[S^3, R_j] = |S_j^0|^2 \approx e^{\frac{\pi L_{dS}}{2G_N}\sqrt{1-8G_N E}}$$

CFT partition function — De Sitter Entropy

[This k=−2 is equivalent to k=∞ via triality in Gaberdiel–Gopakumar 2012]

This non-unitary CFT is essentially equivalent to

the two Liouville CFTs at $b^{-2} \approx \pm \frac{i}{4G_N}$. [Hikida–Nishioka–Taki–TT 2022]

[→Reproduced by Verlinde–Zhang 2024 via the Double Scaled SYK]

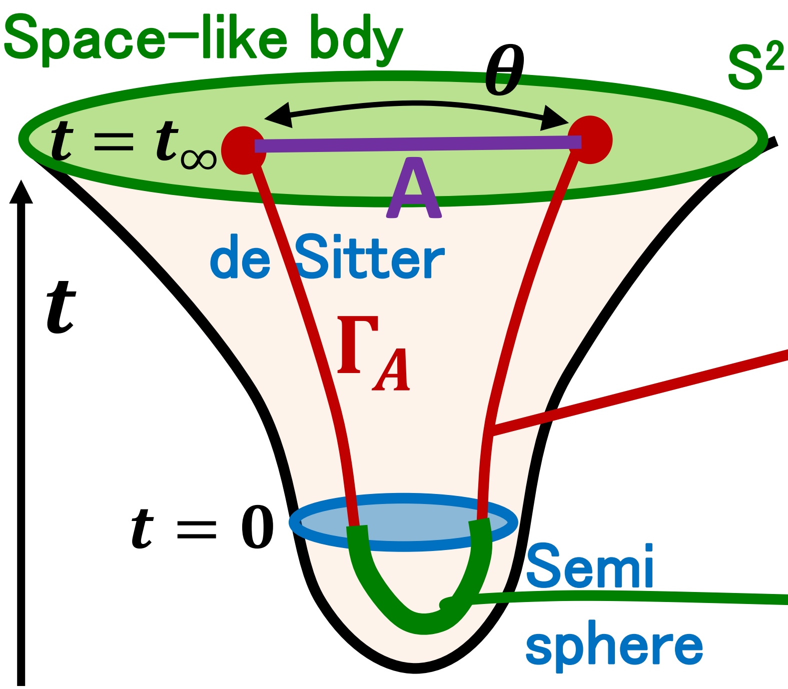
Holographic Pseudo Entropy in dS3/CFT2

[No space-like extreme surface ending on bdy →complex valued EE: Narayan, Sato 2015, Interpretation as PE: Doi-Harper-Mollabashi-Taki-TT 2022]

If we naively apply the HEE in AdS/CFT to dS/CFT, we obtain

$$S_A = \frac{\mathrm{L}(\Gamma_A)}{4G_N} = i\frac{c_{ds}}{3}\log\left(\frac{2}{\epsilon}\mathrm{Sin}\frac{\theta}{2}\right) + \underbrace{\frac{c_{dS}}{6}\pi}_{\mathrm{SdS}/2}.$$

$$ds^2 = L_{ds}^2(-dt^2 + \mathrm{Cosh}^2 t\,(d\theta^2 + Sin^2\theta d\varphi^2))$$



$$\mathrm{L}(\Gamma_A) = \underbrace{2it_\infty + i\log(\mathrm{Sin}^2\frac{\theta}{2})}_{} + \pi$$

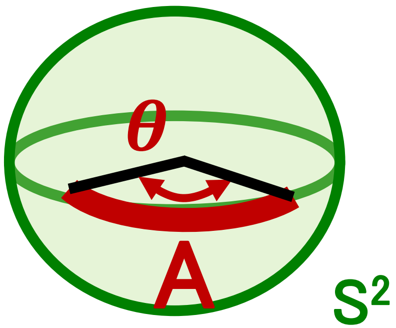
Length of time-like geodesics
➔imaginary value

Space-like geodesic (Semi circle)

This nicely reproduces the familiar 2d CFT result as follows:

$$S_A = \frac{c_{CFT}}{6}\log\left[\frac{\mathrm{Sin}^2\frac{\theta}{2}}{\tilde{\varepsilon}^2}\right],$$ by setting

$$c_{CFT} = ic_{dS}$$ and $$\tilde{\varepsilon} = i\varepsilon = ie^{-t_\infty}.$$



However, one may wonder why the EE is complex valued.

We argue it is more properly considered as the pseudo entropy.

[Doi-Harper-Mollabashi-Taki-TT 2022]

This is because the reduced density matrix ρ_A **is not Hermitian** in the CFT dual to dS, as it is not unitary.

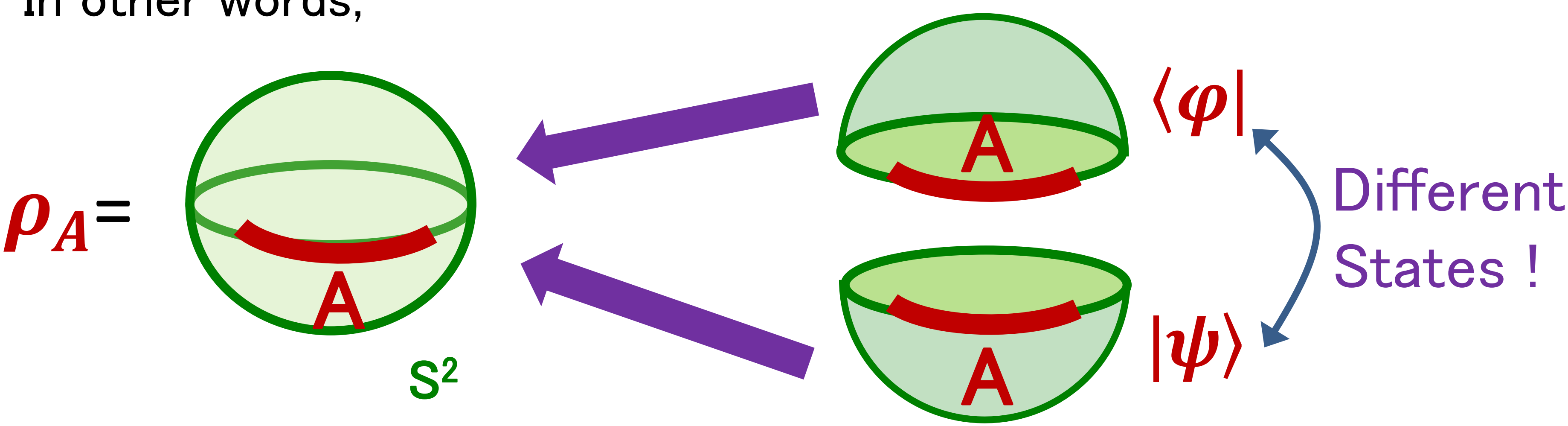
➔For the dual 2d CFT on Σ with metric $h_{ab} = e^{2\phi}\delta_{ab}$, we have

[See e.g. Boruch−Caputa−Ge−TT 2021]

$$Z_{CFT}(S^2) \approx e^{-I_{CFT}[\phi]}, \quad I_{CFT}[\phi] = i\frac{c_{ds}}{24\pi}\int d^2x[(\partial_a\phi)^2 + e^{2\phi}].$$

Complex valued ! ➔ $\rho_A \neq \rho_A^\dagger$

In other words,



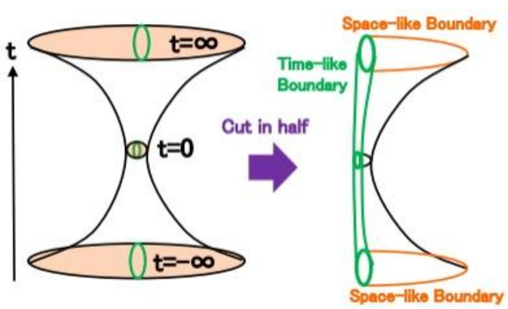
Thus, the emergent time coordinate = imaginary part of PE.

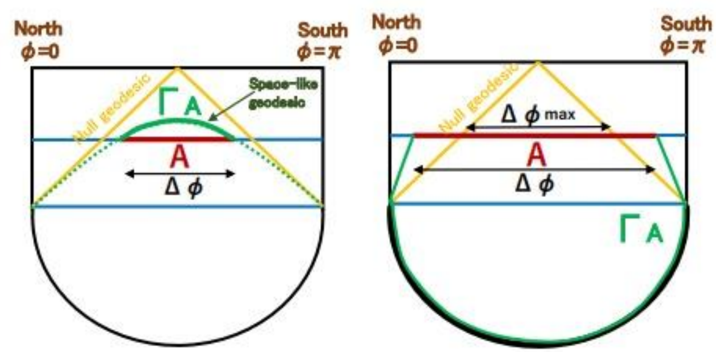
Another approach to holography for de Sitter space

[Kawamoto–Ruan–Suzuki–TT 2023]

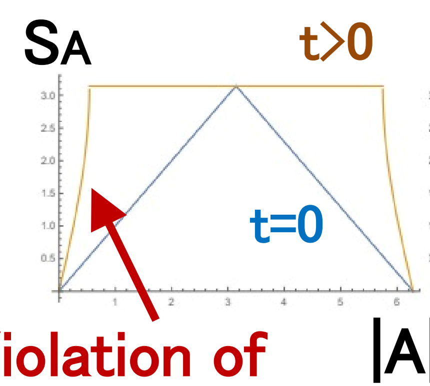
We want a time–like boundary !➔ A half de Sitter space

Non–local QFT on dS_d = Gravity on a half dS_{d+1}

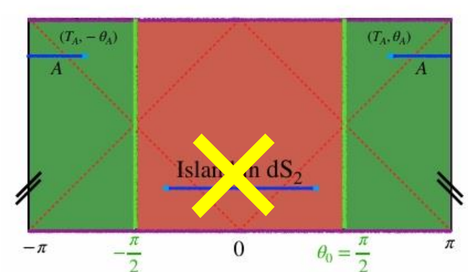




HEE for a half dS



Violation of strong subadditivity !



Moreover, by considering a CFT on a half dS coupled to Gravity on a half dS, we find that the original Island formula does not work !

➔ Non-extremal Island [Hao–Kawamoto–Ruan–TT 2024]

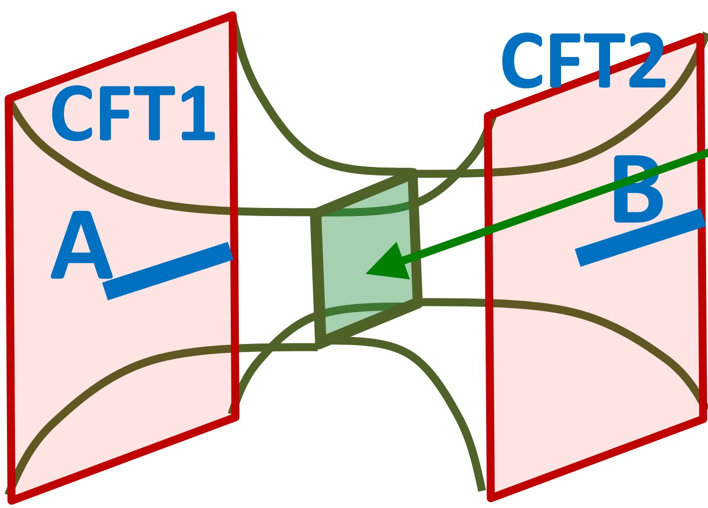
⑥ Traversable Wormholes

[Kawamoto–Maeda–Nakamura–TT, in preparation]

We would like to explore more on the question:

Is pseudo entropy relevant for Lorentzian spacetimes ?

➡ Consider traversable AdS wormholes !



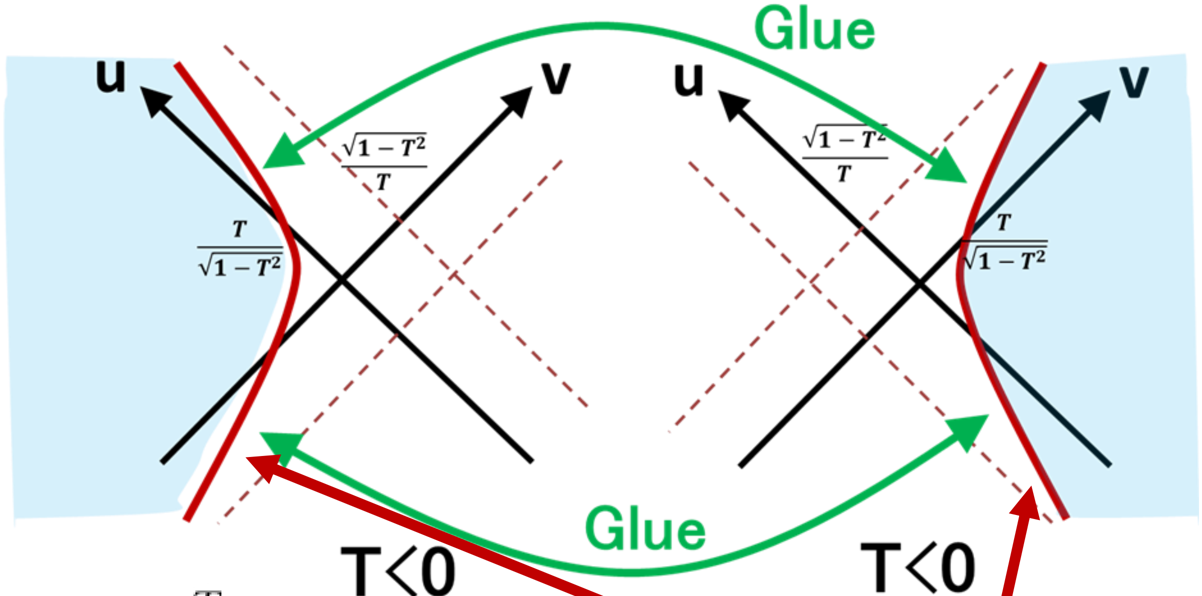
Negative tension brane
(e.g. due to Casimir effect of
the double trace deformation)

[Gao-Jafferis-Wall 16,..]

How does S_{AB} look like ?

Gluing two BTZ BHs along a negative tension brane

Kruskal Coordinate of BTZ: $$ds^2 = \frac{-4dudv + \frac{(1-uv)^2}{a^2}dx^2}{(1+uv)^2}.$$



$$v = \frac{u - \frac{T}{\sqrt{1-T^2}}}{\frac{T}{\sqrt{1-T^2}}u + 1}.$$

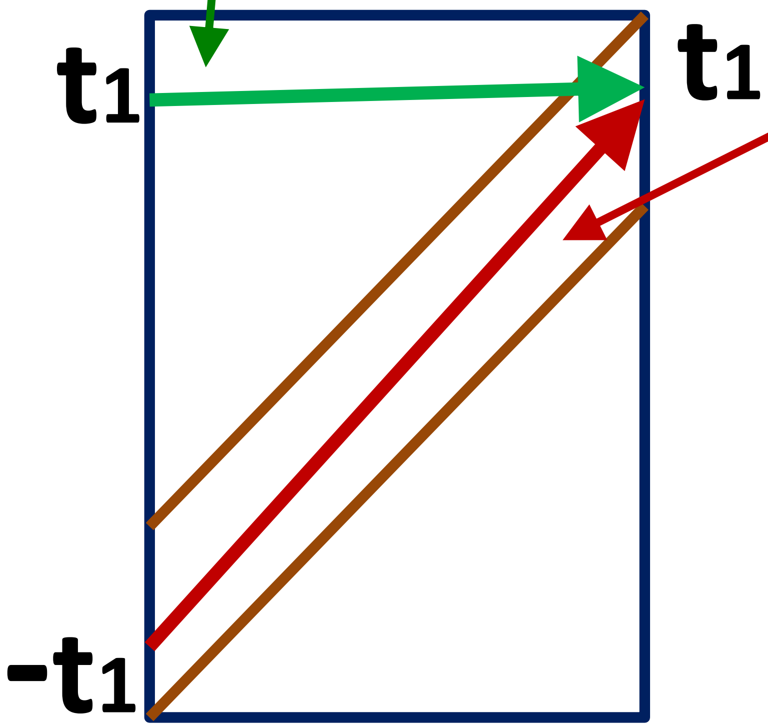
Negative tension brane

$$K_{ab} - K h_{ab} = -T\, h_{ab}$$

Computing Holographic Entropy

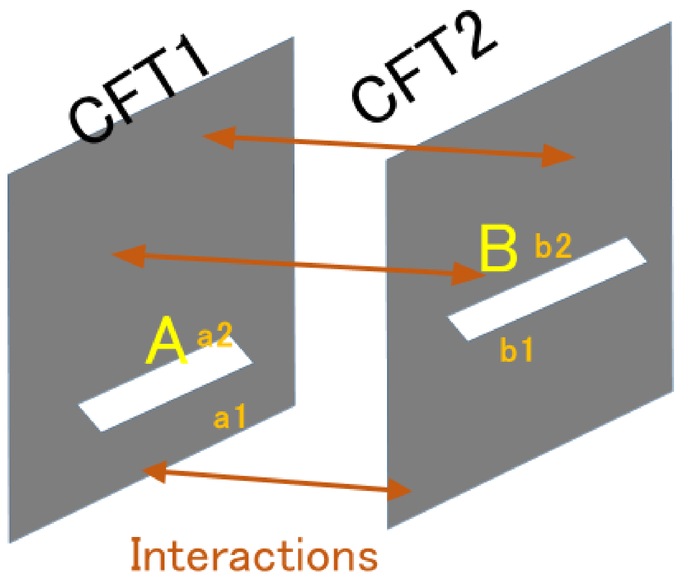
$$SA = \frac{c}{3} \log\left[\frac{2a}{\epsilon}\cosh\left(\frac{t_1}{a}\right)\left(-\lambda+\sqrt{\lambda^2+1}\right)\right]$$

$$SA = \frac{c}{3} \log\left[\frac{2a}{\epsilon}\left(-\lambda\cosh\left(\frac{t_1}{a}\right)+\sqrt{1+\lambda^2}\right)\right]$$



This entropy becomes complex valued for later time !
This is because the geodesic can becomes time-like.

Should this be regarded as pseudo entropy ?

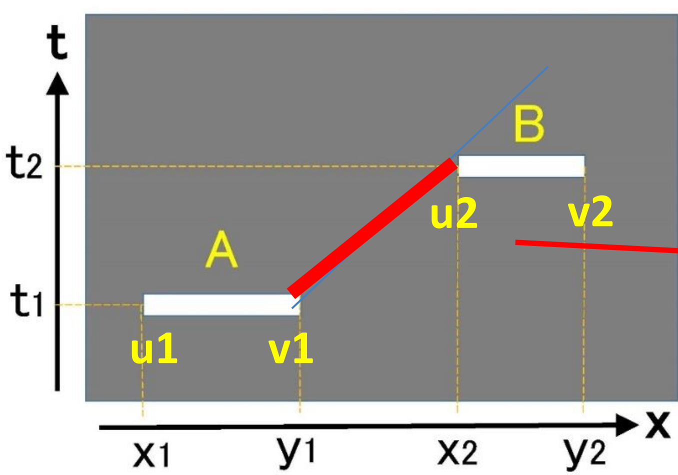


Indeed, we can easily find

$$H_{tot} \neq H_A \otimes H_B \otimes H_{others}$$

because A and B are causally connected.

This is analogous to the following setup in a single CFT:



e.g. Free Dirac fermion CFT c=1

$$S_{AB} = \frac{c}{6}\log\frac{|v_1-u_1|^2}{\epsilon^2} + \frac{c}{6}\log\frac{|v_2-u_2|^2}{\epsilon^2} + \frac{c}{6}\log\frac{|v_1-u_2|^2}{\epsilon^2} + \frac{c}{6}\log\frac{|v_2-u_1|^2}{\epsilon^2} - \frac{c}{6}\log\frac{|u_1-u_2|^2}{\epsilon^2} - \frac{c}{6}\log\frac{|v_1-v_2|^2}{\epsilon^2}.$$

If this interval is time-like, entropy gets complex valued !

Explicit construction from Janus deformation

We start with 3D Janus BH solutions in [Bak-Gutperle-Hirano 07].

The model is given by the 3d gravity action

$$I = \frac{1}{16\pi G_N}\int d^3x \left[R - g^{ab}\partial_a\phi\partial_b\phi + 2\right].$$

The solution ansatz looks like

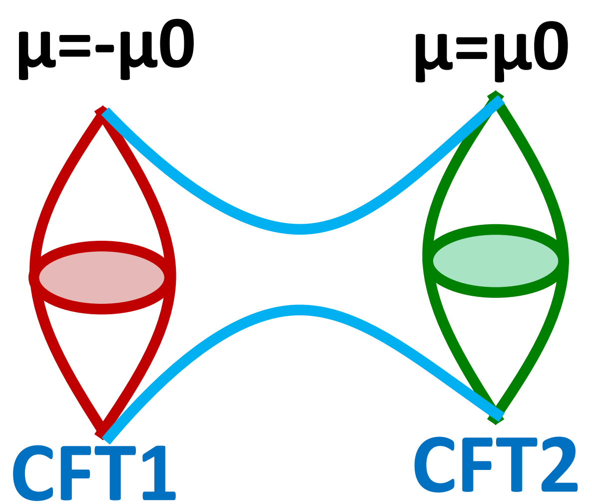
$$ds^2 = f(\mu)(d\mu^2 + ds^2_{AdS2}), \qquad \phi = \phi(\mu).$$

$$ds^2_{AdS2} = -d\tau^2 + r_0^2\cos^2\tau d\theta^2$$

γ is Janus deformation Parameter.

$$\frac{d\phi(\mu)}{d\mu} = \frac{\gamma}{\sqrt{f(\mu)}},$$

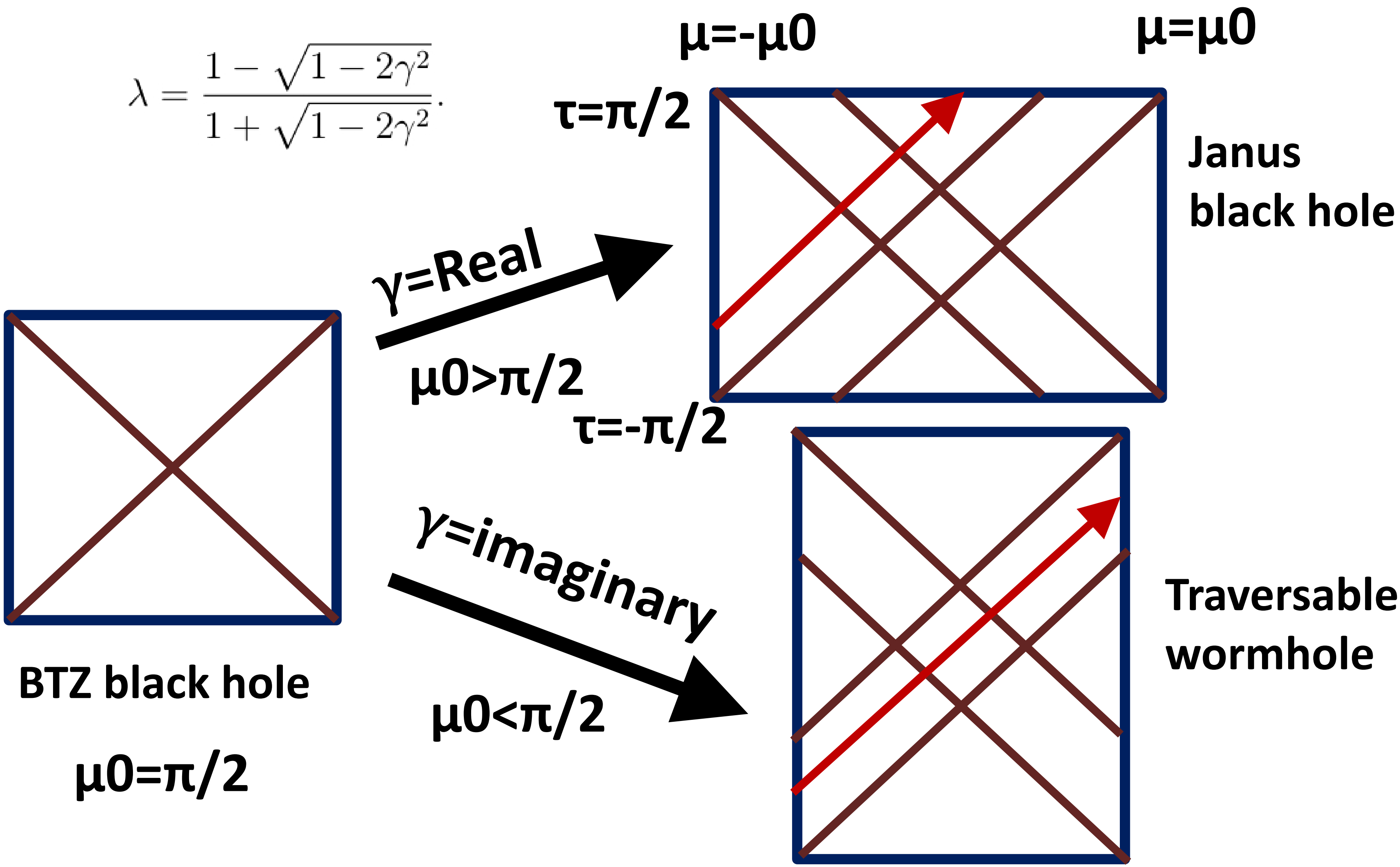
$$\frac{df(\mu)}{d\mu} = \sqrt{f(4f^2 - 4f + 2\gamma^2)}.$$



We now extend this solution to imaginary γ.

$$\mu_0 = \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-\lambda x^2)}},$$

$$\lambda = \frac{1-\sqrt{1-2\gamma^2}}{1+\sqrt{1-2\gamma^2}}.$$

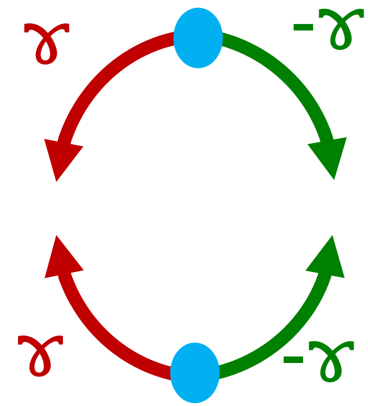


<u>CFT dual of Janus BH (not traversable)</u>

When γ is real, it is dual to an asymmetric TFD state:

$$\langle TFD| = \sum_n e^{-\frac{\beta(E_n^{(1)}+E_n^{(2)})}{4}} \left\langle E_n^{(1)}, \gamma \right| \left\langle E_n^{(2)}, -\gamma \right|$$

$$|TFD\rangle = \sum_n e^{-\frac{\beta(E_n^{(1)}+E_n^{(2)})}{4}} \left| E_n^{(1)}, \gamma \right\rangle \left| E_n^{(2)}, -\gamma \right\rangle$$



[Bak-Gutperle-Karch 07]

The replica method leads to

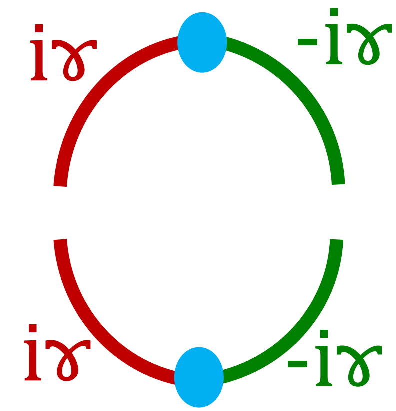
$$\langle TFD|\sigma_n(a)\,\sigma_n(b)|TFD\rangle$$ → Entanglement Entropy is well-defined.

CFT dual of traversable wormhole

When γ is imaginary, it is dual to an asymmetric TFD state:

$$\langle TFD'| = \sum_n e^{-\frac{\beta(E_n^{(1)}+E_n^{(2)})}{4}} \left\langle E_n^{(1)}, i\gamma \right| \left\langle E_n^{(2)}, -i\gamma \right|$$

$$|TFD\rangle = \sum_n e^{-\frac{\beta(E_n^{(1)}+E_n^{(2)})}{4}} |E_n^{(1)}, i\gamma\rangle\, |E_n^{(2)}, -i\gamma\rangle$$



Note: $(|\,TFD\rangle)^\dagger \equiv \langle TFD| \neq \langle TFD'|$.

The replica method leads to

$$\langle TFD'|\sigma_n(a)\,\sigma_n(b)|TFD\rangle$$

This leads to pseudo entropy.

“Causality” is violated.

(→post selection)

⑦ Conclusions

Pseudo entropy (PE) is a generalization of entanglement entropy.

- PE depends on both the initial and final state.
- PE is in general complex valued.
- ΔS for two states in different phases can be positive, while ΔS in the same phase is always non-positive.

  ➡ New quantum order parameter
- In AdS/CFT, PE is equal to the minimal surface area in Euclidean time-dependent asymptotically AdS geometry.

  ➡ Emergence of space from real part of PE
- In dS/CFT, PE becomes complex valued.

  ➡ Emergence of time from imaginary part of PE (Non-Hermitian nature of the dual CFT)
- Traversable wormholes in AdS can be probed by PE.

Future directions

- Quantum information meaning of the complex values of PE ?
- Applications to non-Hermitian cond-mat physics ?
- Implications to quantum gravity ? Emergent time ?
- Holographic dual of SVD entropy ?
- Constraints on QFTs using PE ?

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Thank you very much !