

The 2024 ANPhA meeting and ANPhA symposium

2024.11.13-17, Huizhou

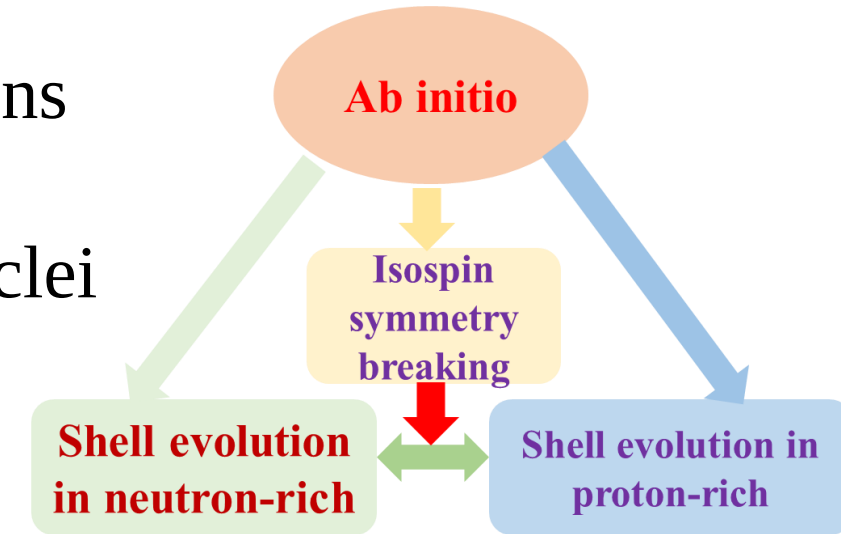
Ab initio calculations for the nuclear shell evolution

Jianguo Li

Institute of Modern Physics, Chinese Academic of Science

Collaboration: W. Zuo, Q. Yuan, H. H. Li, P.Y. Wang, M. R. Xie, N. Chen, K.H. Li, X.P. Wang ...

1. Introduction of ab initio calculations
2. Shell evolution in neutron-rich nuclei
3. Isospin symmetry breaking
4. Isospin symmetry breaking for the shell evolution in proton-rich nuclei
5. Summary





1. Introduction of ab initio calculations

What is *ab initio* calculations?



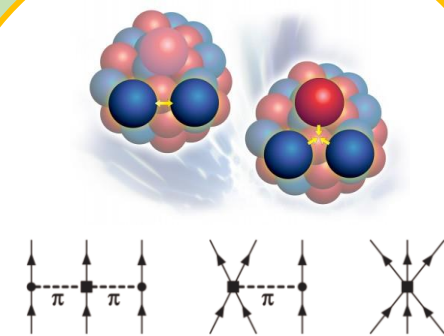
Nuclear Force

Freedom: Pion and Nucleon

Requirement: Reproduce nucleon-nucleon scattering phase shifts

Properties: Short-range repulsion and long-range attraction, strong repulsion

Typical Forces: CD-Bonn, AV18, χ EFT



3NF

Renormalization

Objective: To decouple high and low momentum, soften the nuclear force, and accelerate the convergence speed of many-body calculations.

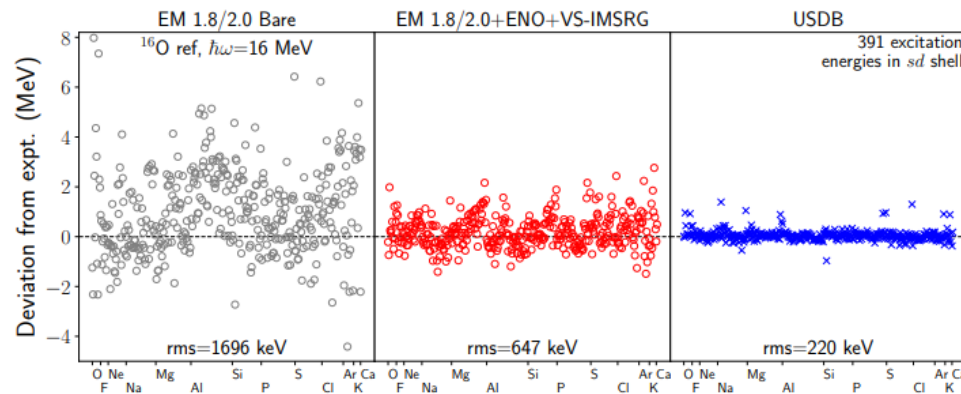
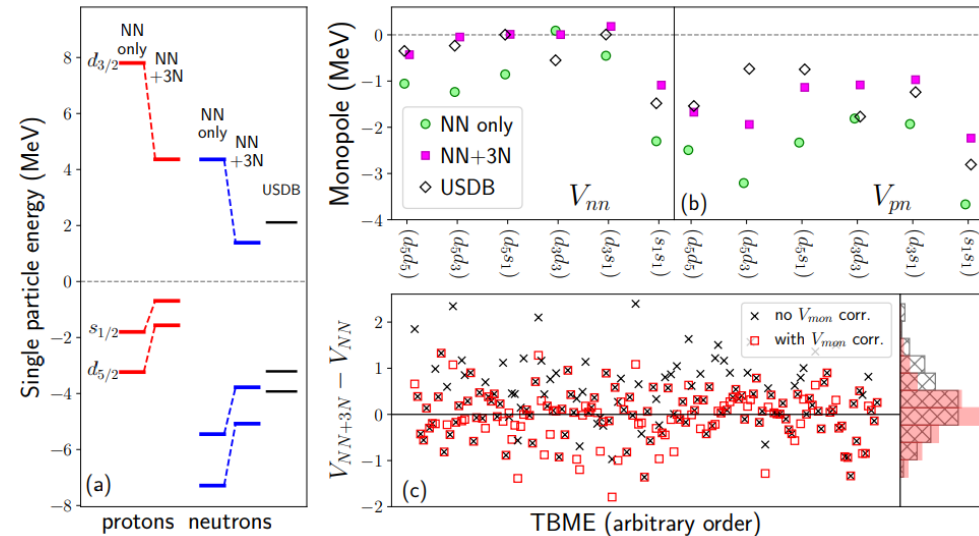
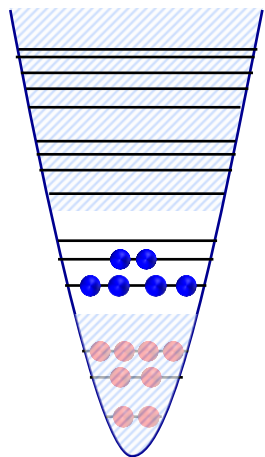
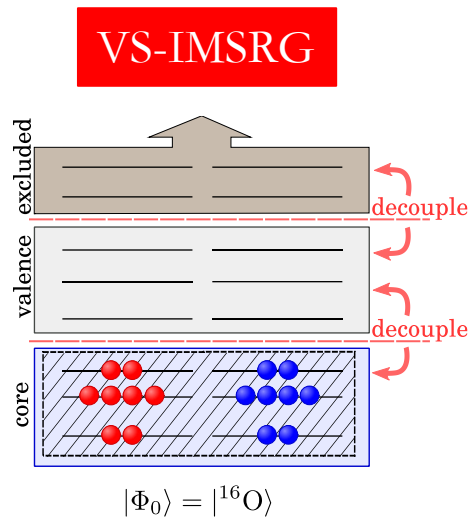
Representatives: G-Matrix, $V_{\text{low-k}}$, SRG

Many-Body Methods

Objective: To solve the Hamiltonian of strongly interacting many-body nuclear systems

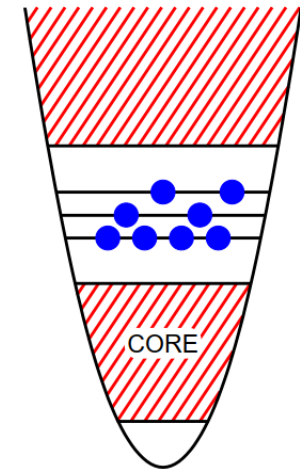
Representatives: NCSM ($A \leq 16$), MBPT ($A \sim 100$), CC ($A \sim 100$, 208Pb), IMSRG ($A \sim 100$, 208Pb), ACSE (We are developing this method)...

VS-IMSRG realistic interaction vs USDB phenomenological shell model interaction



S. Ragnar Stroberg, Scott K. Bogner, Heiko Hergert, Jason D. Holt
Annual Review of Nuclear and Particle Science, 69 307-362 (2019)

USDB SM



Many-body problem: $H|\Psi\rangle = E|\Psi\rangle$

1. A-body Hamiltonian H

$$H = \sum_{i=1}^A \left(1 - \frac{1}{A}\right) \frac{p_i^2}{2m} + \sum_{i<j}^A \left(v_{ij}^{\text{NN}} - \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{mA} \right) + \sum_{i<j<k}^A v_{ijk}^{\text{3N}}$$

2. Normal-ordered many-body Hamiltonian (Normal Order)

$$\begin{aligned}
 E &= \left(1 - \frac{1}{A}\right) \sum_i \langle i | T^{(1)} | i \rangle n_i + \frac{1}{2} \sum_{ij} \langle ij | T^{(2)} + V^{(2)} | ij \rangle n_i n_j \\
 &\quad + \frac{1}{6} \sum_{ijk} \langle ijk | V^{(3)} | ijk \rangle n_i n_j n_k \\
 H_{NO} &= \underbrace{E}_{\text{red circle}} + \sum_{ij} \underbrace{f_{ij}}_{\text{blue circle}} : a_i^\dagger a_j : + \frac{1}{4} \sum_{ijkl} \underbrace{\Gamma_{ijkl}}_{\text{green circle}} : a_i^\dagger a_j^\dagger a_l a_k : + \frac{1}{36} \sum_{ijklmn} \underbrace{W_{ijklmn}}_{\text{orange circle}} : a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l : \\
 f_{ij} &= \left(1 - \frac{1}{A}\right) \langle i | T^{(1)} | j \rangle + \sum_a \langle ia | T^{(2)} + V^{(2)} | ja \rangle n_a \\
 &\quad + \frac{1}{2} \sum_{ab} \langle iab | V^{(3)} | jab \rangle n_a n_b \\
 \Gamma_{ijkl} &= \langle ij | T^{(2)} + V^{(2)} | kl \rangle + \sum_a \langle ija | V^{(3)} | kla \rangle n_a \\
 W_{ijklmn} &= \langle ijk | V^{(3)} | lmn \rangle
 \end{aligned}$$

Normal-ordered two-body interaction capture the main contribution of three-body forces

Many-body problem: $H|\Psi\rangle = E|\Psi\rangle$

1. A-body Hamiltonian H

$$H = \sum_{i=1}^A \left(1 - \frac{1}{A}\right) \frac{p_i^2}{2m} + \sum_{i<j}^A \left(v_{ij}^{\text{NN}} - \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{mA} \right) + \sum_{i<j<k}^A v_{ijk}^{\text{3N}}$$

2. Normal-ordered many-body Hamiltonian (Normal Order)

$$E = \left(1 - \frac{1}{A}\right) \sum_i \langle i | T^{(1)} | i \rangle n_i + \frac{1}{2} \sum_{ij} \langle ij | T^{(2)} + V^{(2)} | ij \rangle n_i n_j + \frac{1}{6} \sum_{ijk} \langle ijk | V^{(3)} | ijk \rangle n_i n_j n_k$$

$$H_{NO} = \underbrace{E}_{\text{core}} + \sum_{ij} \underbrace{f_{ij}}_{\text{core}} : a_i^\dagger a_j : + \frac{1}{4} \sum_{ijkl} \underbrace{\Gamma_{ijkl}}_{\text{core}} : a_i^\dagger a_j^\dagger a_l a_k : + \frac{1}{36} \sum_{ijklmn} \underbrace{W_{ijklmn}}_{\text{core}} : a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l :$$

$$f_{ij} = \left(1 - \frac{1}{A}\right) \langle i | T^{(1)} | j \rangle + \sum_a \langle ia | T^{(2)} + V^{(2)} | ja \rangle n_a + \frac{1}{2} \sum_{ab} \langle iab | V^{(3)} | jab \rangle n_a n_b$$

$$W_{ijklmn} = \langle ijk | V^{(3)} | lmn \rangle$$

^{16}O core

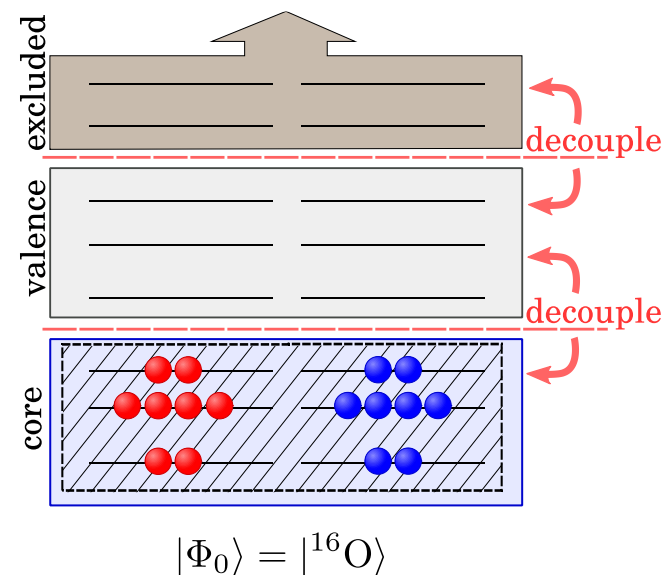
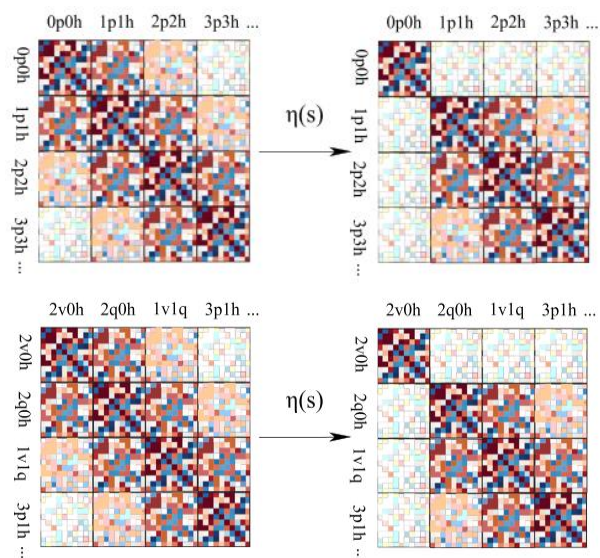
^{16}O
core

Normal-ordered two-body interaction capture the main contribution of three-body forces

3. Valence-space Hamiltonian can be constructed by using the unity transformation

$$H_{eff}(s) = U(s)H_{NO}U^\dagger(s), (UU^\dagger = 1)$$

- **Flow equation** $\frac{d}{ds}H_{eff}(s) = [\eta(s), H(s)]$
- **Generator**: $\eta(s) = \frac{dU(s)}{ds}U^\dagger(s) = -\eta^\dagger(s)$
- **Unity transformation**: $U(s) = e^{\Omega(s)} = e^{\int \eta(s)ds}$

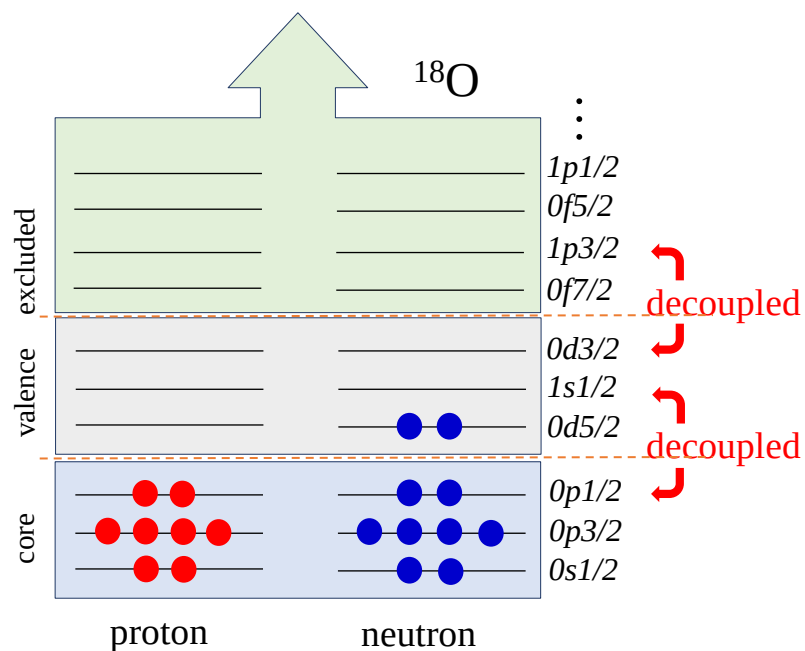
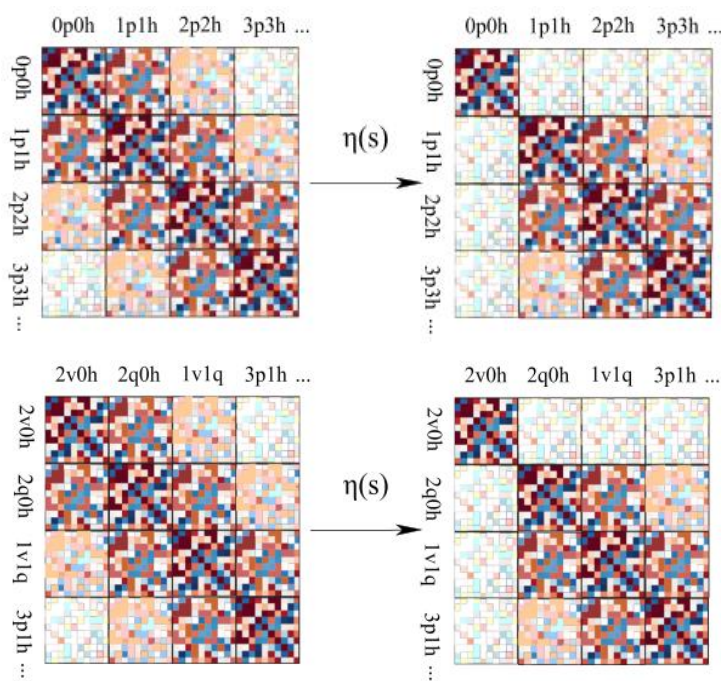


Ab initio Valence-Space In-Medium Similarity Renormalization Group(*Ab initio* VS-IMSRG)

➤ Mainly ideas

$$H(s) = U(s)H(0)U^+(s)$$

$$= H^d(s) + H^{od}(s) \rightarrow H^d(\infty)$$



➤ Advantage:

- Realistic nuclear interaction (NN+3N)
 - Including CSB + CIB
 - Isospin symmetry breaking in contact and exchange terms
- No parameter introduced during the calculations
- “Exact” calculations

VS-IMSRG works for nuclei covered by shell model calculations

2. Shell evolution in neutron-rich nuclei

- ① [J. G. Li*](#), Phys. Lett. B 840 137892(2023)
- ② Q. Yuan, [J. G. Li*](#), H. H. Li, PLB 848 138331 (2024)
- ③ H. H. Li, [J. G. Li*](#), M. R. Xie, W. Zuo, PRC (Letter) 109, L061304 (2024).
- ④ N. Chen, [J. G. Li*](#), and H. H. Li, Phys. Rev. C 110, 034316(2024)
- ⑤ M. R. Xie, L. Y. Shen, [J. G. Li*](#), H. H. Li, Q. Yuan and W. Zuo, CPC 48 074106(2024)
- ⑥ Q. Yuan, [J. G. Li*](#), and W. Zuo, Phys. Rev. C(Letter) 109, L041301 (2024)
- ⑦ X. P. Wang, [J. G. Li*](#), N. Chen, submitted to PRC

Ab initio

**Shell evolution
in neutron-rich**

Shell evolution is hot topic in nuclear physics



The disappearance of traditional magic number and appearance of new magic number

Disappearance of magic number



Island of Inversion (IOI)

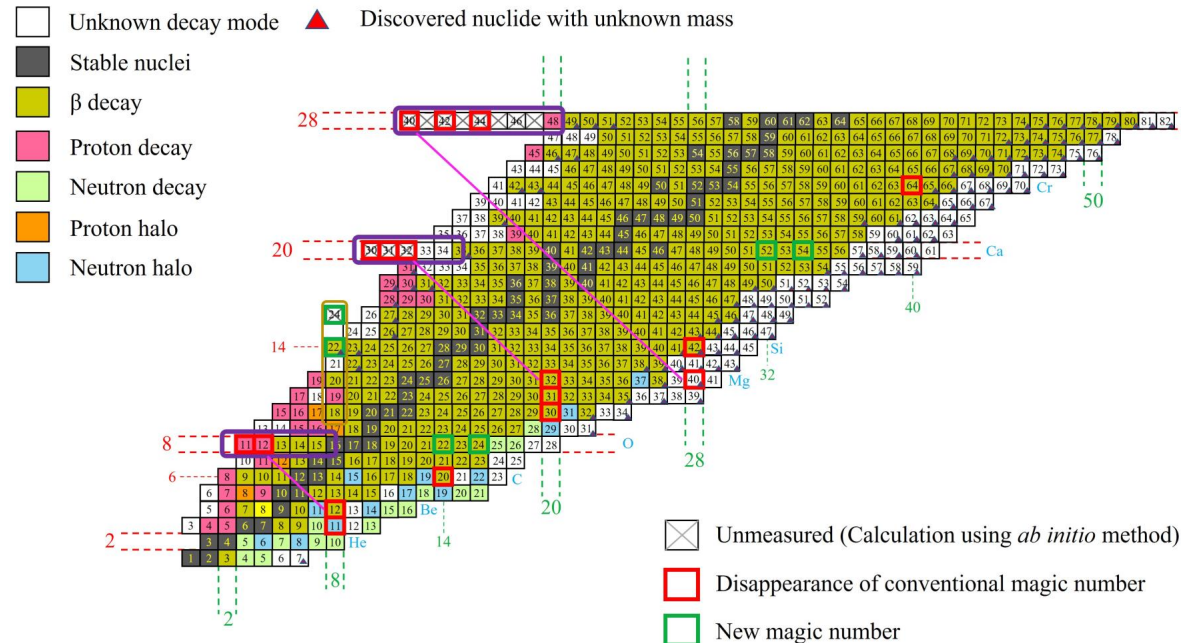
Neutron-rich nuclei in the vicinity of $N=8, 20, 28$, and 40 are deformed

$N=8$: ^{12}Be

$N=20$: $^{30}\text{Ne}, ^{32}\text{Mg}$

$N=28$: $^{42}\text{Si}, ^{40}\text{Mg}$

$N=40$: ^{64}Cr



appearance of new magic number

New magic number
 $N=14, 16, 32$ and 34

$N=14$: ^{22}O

$N=16$: ^{24}O

$N=32$: ^{52}Ca

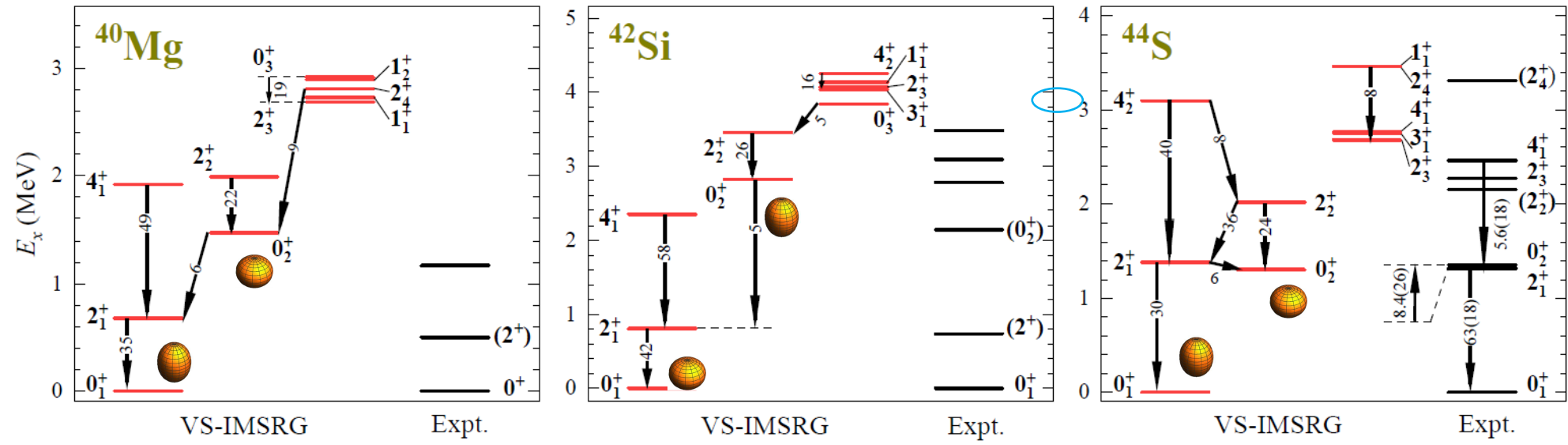
$N=34$: ^{54}Ca

Deep understanding of the shell evolution is important for the study of properties of heavy nuclei

1.Shape transition in ^{40}Mg , ^{42}Si and ^{44}S near N = 28



Ground state



Nuclei	Q (efm ²)		n _v [0f _{7/2}]		n _π [0d _{5/2}]	
	2 ₁ ⁺	2 ₂ ⁺	0 ₁ ⁺	0 ₂ ⁺	0 ₁ ⁺	0 ₂ ⁺
^{40}Mg	-12.0	9.6	4.91	4.52	3.06	2.96
^{42}Si	13.2	-10.2	4.86	5.30	4.36	4.38
^{44}S	-6.0	4.7	6.04	5.74	5.26	5.35

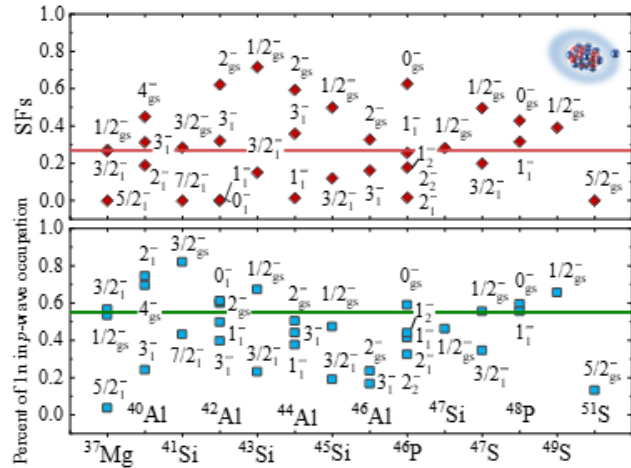
- ▣ Rotational band of ^{40}Mg , ^{42}Si , ^{44}S constructed via BE2
- ▣ shape coexistence and shape transition in ^{40}Mg , ^{42}Si , and ^{44}S
- ▣ ^{44}S : 4₁⁺ high-K isomer

Q. Yuan, [J. G. Li*](#), H. H. Li, PLB 848 138331 (2024)

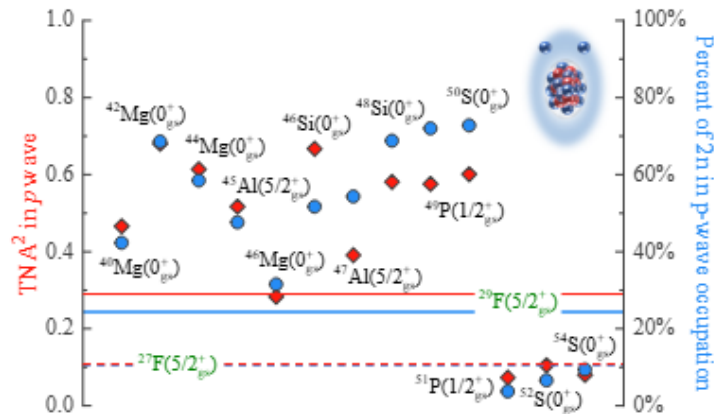
2. Ab initio predictions of halo nuclei in medium-mass region



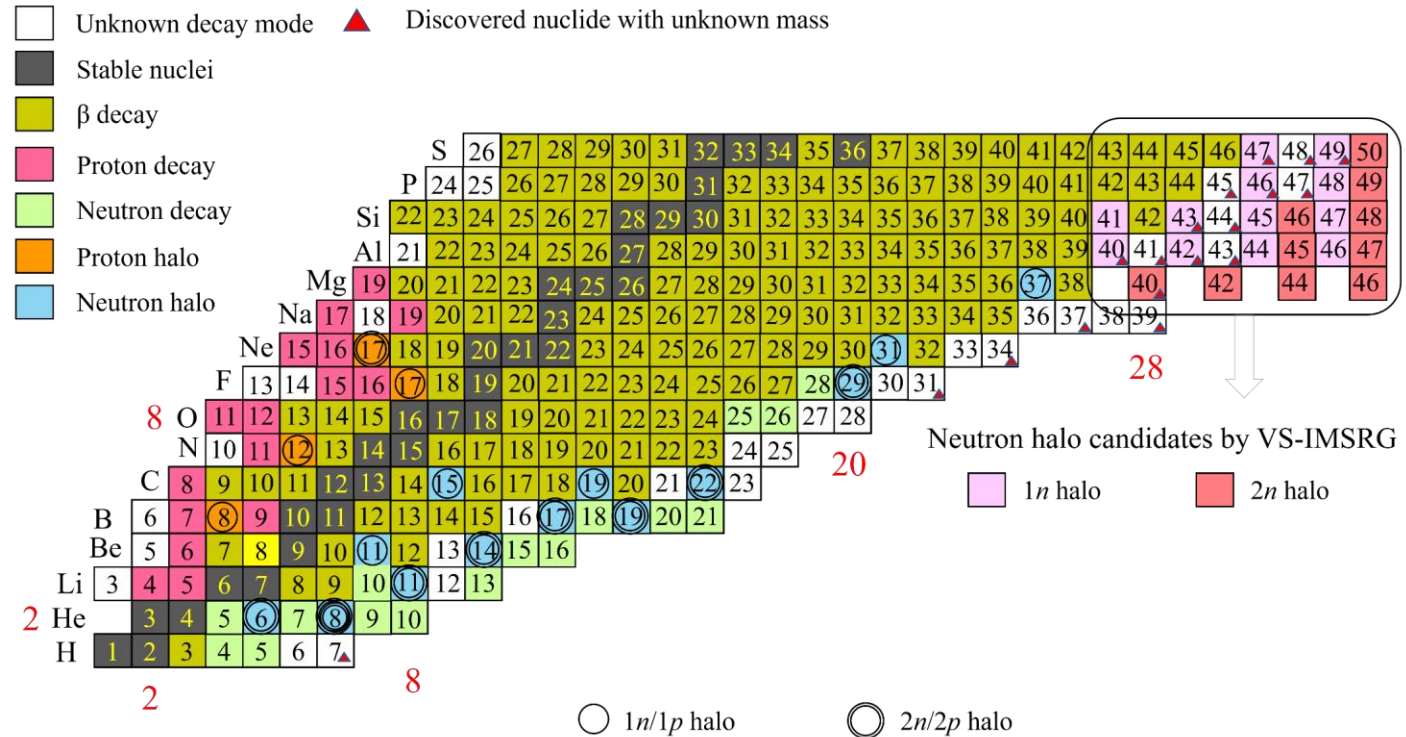
SFs+ occupations



TNA + occupations



Based on the ab initio calculations, we use the spectroscopic factor (SF) and two-nucleon amplitude (TNA) to predict the one- and two-neutron halo, respectively



H. H. Li, [J. G. Li*](#), M. R. Xie, W. Zuo, PRC (Letter) 109, L061304 (2024).

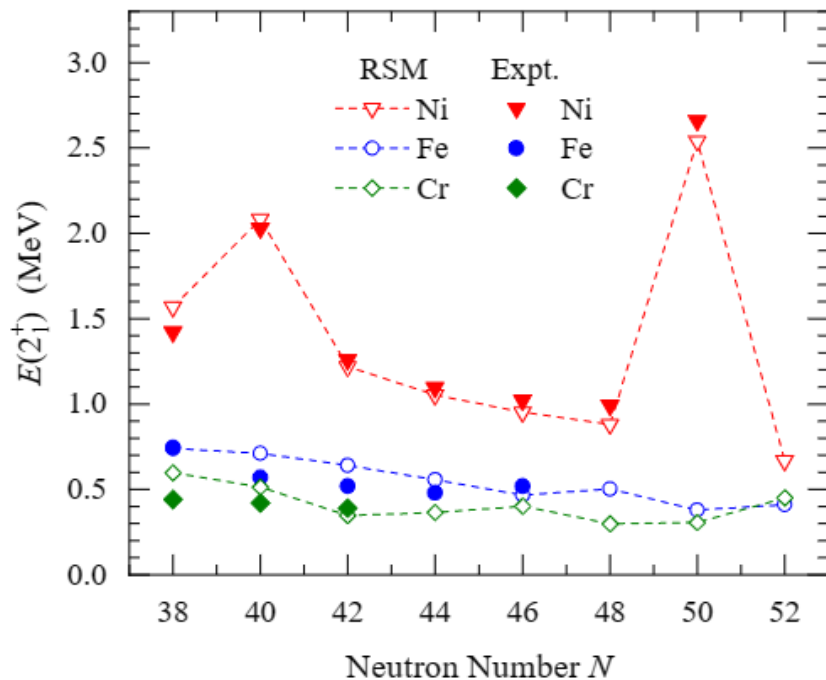
3. Prediction of N=50 Island of inversion



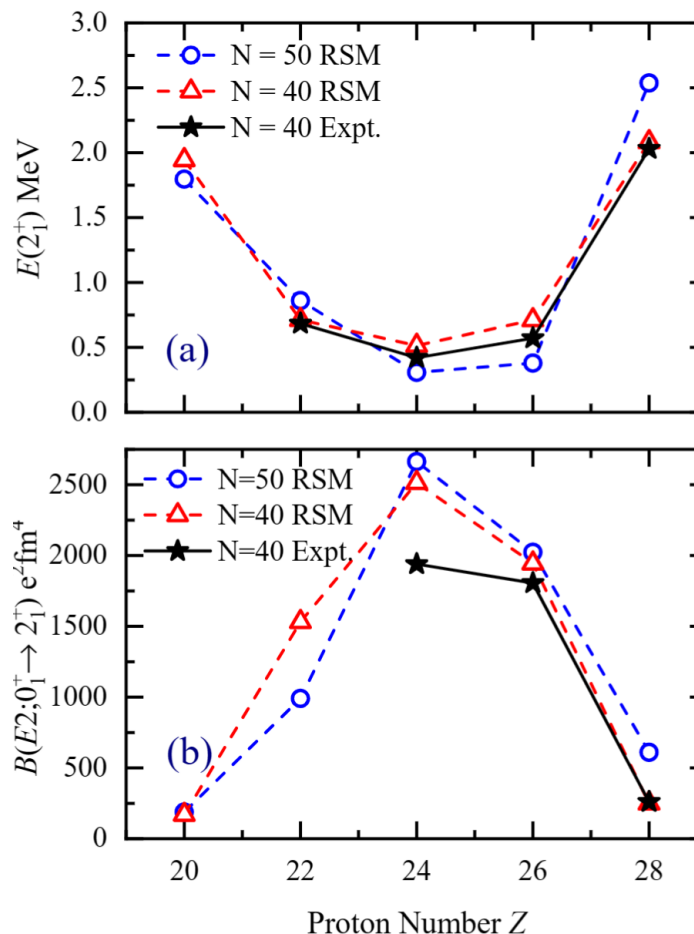
Merging of the island of inversion at $N = 40$ and $N = 50$

J.G. Li^{a,b,*}

^a CAS Key Laboratory of High Precision Nuclear Spectroscopy, Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
^b School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China



- ✓ Reproduce of properties of N=40 nuclei
- ✓ Predict the existence of N=50 IoI



N=40 and N=50 Isotones exhibit similar properties

Cited by

nature physics

Article

<https://doi.org/10.1038/s41567-024-02680-0>

In-beam spectroscopy reveals competing nuclear shapes in the rare isotope ^{62}Cr

Received: 17 March 2024

Accepted: 24 September 2024

Published online: 18 October 2024

Check for updates

Alexandra Gade^{1,2,✉}, Brenden Longfellow³, Robert V. F. Janssens^{4,5}, Duc D. Dao⁶, Frédéric Nowacki⁶, Jeffrey A. Tostevin⁷, Akaa D. Ayangeakaa^{4,5}, Marshall J. Basson^{1,2}, Christopher M. Campbell⁸, Michael P. Carpenter⁹, Joseph Chung-Jung^{1,2}, Heather L. Crawford⁸, Benjamin P. Crider¹⁰, Peter Farris^{1,2}, Stephen Gillespie¹, Ava M. Hill^{1,2}, Silvia M. Lenzi¹, Shumpei Noji¹, Jorge Pereira¹, Carlotta Porzio⁸, Alfredo Poves¹, Elizabeth Rubino¹ & Dirk Weisshaar¹

Motivations for Early High-Profile FRIB Experiments

B. Alex Brown¹, Alexandra Gade¹, S. Ragnar Stroberg², Jutta Escher³, Kevin Fosse⁴, Pablo Giuliani^{5,6}, Calem R. Hoffman⁷, Witold Nazarewicz¹, Chien-Yeah Seng^{5,8}, Agnieszka Sorensen⁹, Nicole Vassh¹⁰, Daniel Bazin¹, Kyle W. Brown¹¹, Mark A. Caprio², Heather Crawford¹², Pawel Danielewicz¹, Christian Drischler¹³, Ronald F. Garcia Ruiz¹⁴, Kyle Godbey⁵, Robert Grzywacz¹⁵, Jeremy W. Holt¹⁶, Hiro Iwasaki¹, Dean Lee¹⁷, Silvia M. Lenzi¹⁸, Sean Liddick¹¹, Rebeka Lubna⁵, Augusto O. Macchiavelli¹⁹, Gabriel Martínez Pinedo^{20,21,22}, Anna McCoy^{5,23}, Alexis Mercenne²⁴, Kei Minamisono¹, Belen Montegudo²⁵, Petr Navrátil¹⁰, Ryan Ringle¹, Grigor Sargsyan^{3,5}, Hendrik Schatz¹, Mark-Christoph Spieker⁴, Alexander Volya⁴, Remco G. T. Zegers¹, Vladimir Zelevinsky¹ and Xilin Zhang⁵

Phys. Lett. B 855 (2024) 138841



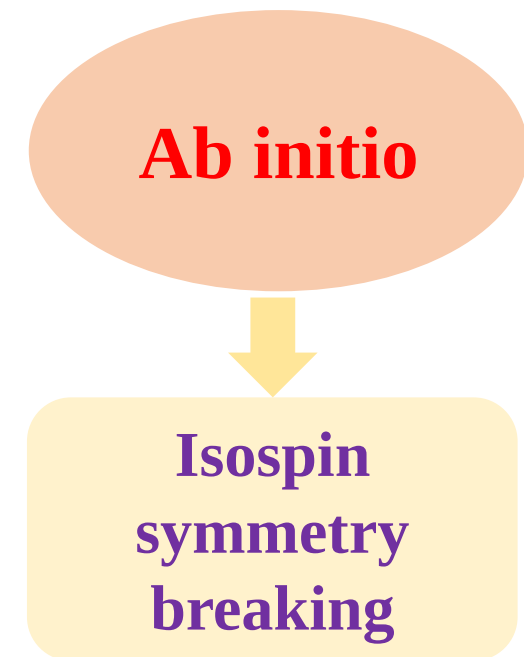
Letter

Spectroscopy of $N = 50$ isotones with the valence-space density matrix renormalization group

A. Tichai^{a,b,c,d,e}, K. Kapás^{d,e}, T. Miyagi^{a,b,c,f}, M.A. Werner^{a,b,c,f}, Ö. Legeza^{d,h}, A. Schwenk^{a,b,c,g}, G. Zarand^{d,g}

3. Isospin symmetry breaking in nuclear systems

- ① H. H. Li, Q. Yuan, [J. G. Li*](#), M. R. Xie, S. Zhang, Y. H. Zhang, X. X. Xu, N. Michel, F. R. Xu, and W. Zuo, Phys. Rev. C 107, 014302(2023)
- ② H. H. Li, [J. G. Li](#)*, M. R. Xie, W. Zuo, Chinese Physics C 47, 12 (2023) 124101
- ③ M.Z. Sun, Y. Yu, X.P. Wang, M. Wang, [J.G. Li*](#), Y.H. Zhang*, et al., Chin. Phys. C(2024)
- ④ Y. Yu, Y. M. Xing, Y. H. Zhang, M. Wang, X. H. Zhou, [J. G. Li](#), H. H. Li, et al., PRL(Accepted)



Nuclear isospin symmetry breaking

➤ Particle physics level

- ✓ mass of u-d quark is difference
- ✓ Coulomb forces between quarks

1. p-n mass difference:

$$m_p = 1.00782503 u, m_n = 1.00866491 u$$

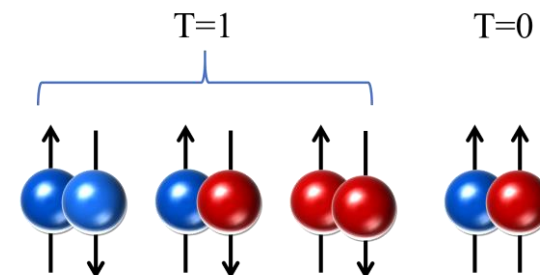
2. p-n charge difference:

$$e_p = e, e_n = (-4.3 \pm 7.1) \times 10^{-20} e$$

➤ Nuclear physics level

- ✓ Coulomb forces
- ✓ Nucleon-nucleon interaction

- Charge symmetry breaking—CSB $V_{nn} - V_{pp}$ (isospin vector)
- Charge independent breaking--CIB) $2V_{np} - (V_{nn} + V_{pp})$ (isospin tensor)



Isospin non-conserving force--INC

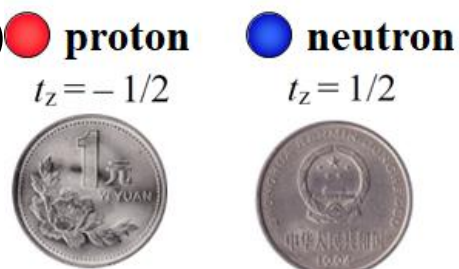
✓ Weakly-bound effects

Study the mechanism of isospin symmetry breaking: Coulomb, nuclear forces, and weakly-bound effects...

Status of study of isospin symmetry breaking



➤ **Isospin: (approximation)**

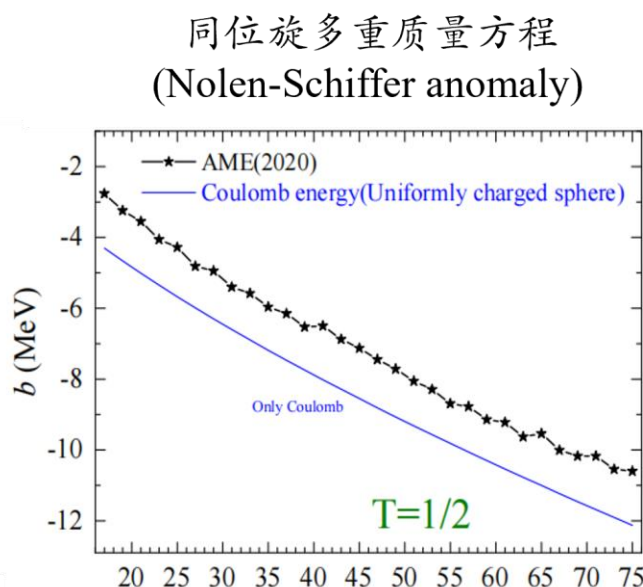
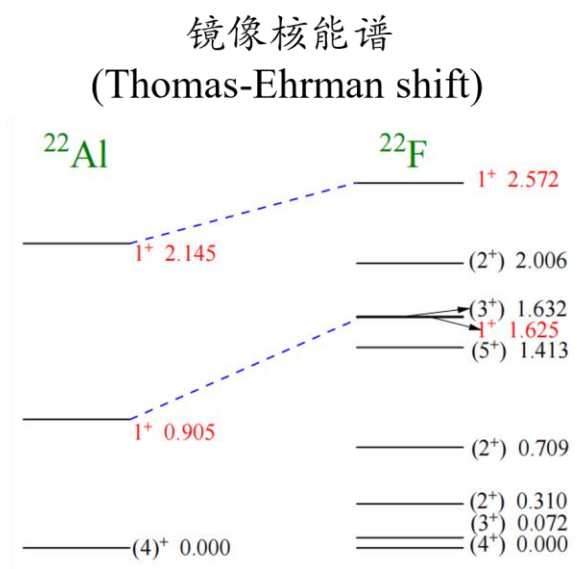


➤ **Theoretical calculations: shell model**

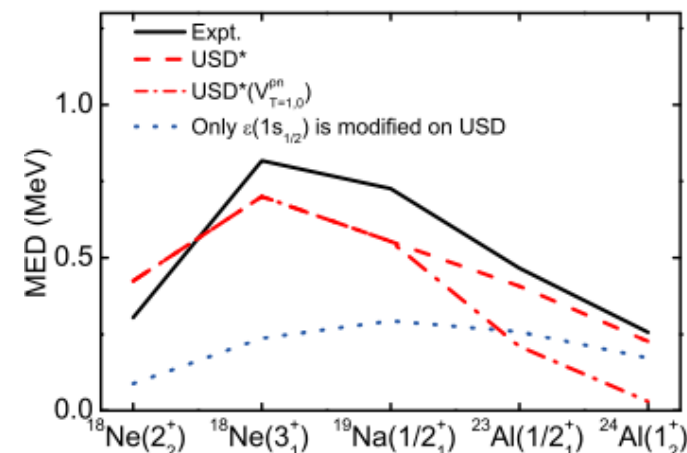
Phenomenological ways:

Adjusting the matrix elements related to $1s_{1/2}$ to reproduce the experimental data.

Isospin symmetry breaking: Experimental results



J. Lee, X.X. Xu, et al., PRL 125, 192503 (2020) *Y. H. Zhang et al., PRL 109, 102501 (2012)*



No parameter needs to be adjusted within ab initio calculations, which is suitable for the investigations of isospin symmetry breaking

Mirror Energy difference (MED)

$$\text{MED} = E_x(J, T, T_Z = -T) - E_x(J, T, T_Z = T),$$

Thomas-Ehrman Shift

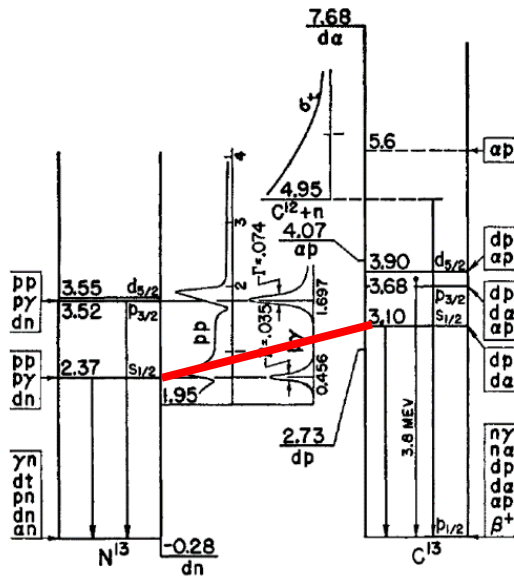


FIG. 1. The energy levels of ^{13}C and ^{13}N below 6 MeV.

Phys. Rev. 88,1109 (1952)

mirror asymmetry parameter (δ) :

$$\delta = \frac{ft^+}{ft^-} - 1 \quad \rightarrow \quad \delta \gg 0$$

isospin symmetry breaking

PHYSICAL REVIEW LETTERS 125, 192503 (2020)

Isospin Asymmetry in $^{22}\text{Si}/^{22}\text{O}$ Mirror Gamow-Teller Transitions Reveals the Halo Structure of ^{22}Al

$^{22}\text{Si} \rightarrow ^{22}\text{Al} \ Q_{\text{EC}} = 13963 \text{ keV}$						$^{22}\text{O} \rightarrow ^{22}\text{F} \ Q_{\beta-} = 6490 \text{ keV}$						δ (%)	
Experiment				Calculations		Experiment			Calculations				
I^π_i	E_x (MeV)	$br\%$	$\log(ft^+)$	E_x (MeV)	$\log(ft^+)$	E_x (MeV)	$br\%$	$\log(ft^-)$	E_x (MeV)	$\log(ft^-)$	Experiment	Calculations	
$1^+_{1/2}$	0.905	5.3 (10)	5.09 (9)	1.12 [1.69]	4.81 [4.52]	1.625	29 (4)	4.6 (1)	1.98 [1.56]	4.32 [4.56]	209 (96)	212 [−7]	
$1^+_{3/2}$	2.145	56.5 (51)	3.83 (5)	2.43 [2.55]	3.71 [3.72]	2.572	68 (6)	3.8 (1)	2.58 [2.51]	3.72 [3.68]	7 (28)	−3.4 [10]	

Comparing the mirror β decay of $^{22}\text{Si}/^{22}\text{O}$, we found **the largest** value of mirror asymmetry ($\delta = 209(96) \%$) in low-lying states by far, in the transition to the first 1^+ excited state.

1. Ab initio calculations for the mirror energy difference



Adopting NN(bare) and NN+3N(1.8/2.0) interaction, VS-IMSRG calculations for the mirror states

PHYSICAL REVIEW C **107**, 014302 (2023)

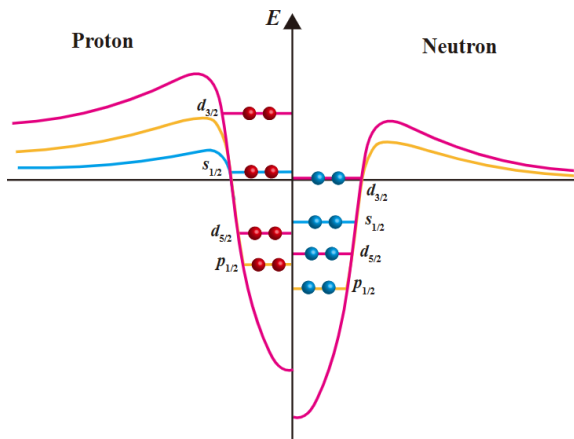
Investigation of isospin-symmetry breaking in mirror energy difference and nuclear mass with *ab initio* calculations

H. H. Li,^{1,2} Q. Yuan,³ J. G. Li,^{1,2,*} M. R. Xie,¹ S. Zhang,³ Y. H. Zhang,^{1,2} X. X. Xu,^{1,2}
N. Michel,^{1,2} F. R. Xu,³ and W. Zuo^{1,2}

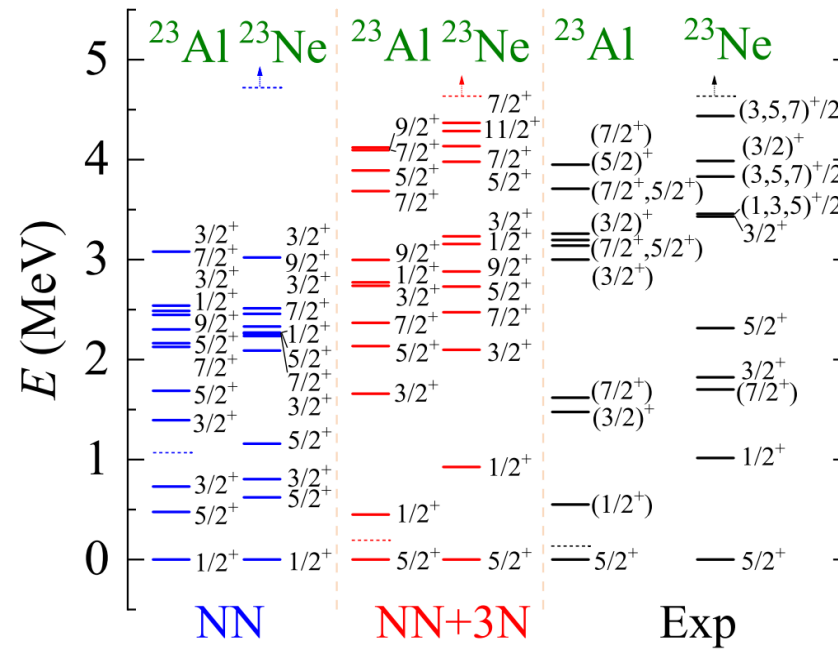
¹CAS Key Laboratory of High Precision Nuclear Spectroscopy, Institute of Modern Physics,
Chinese Academy of Sciences, Lanzhou 730000, China

²School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China

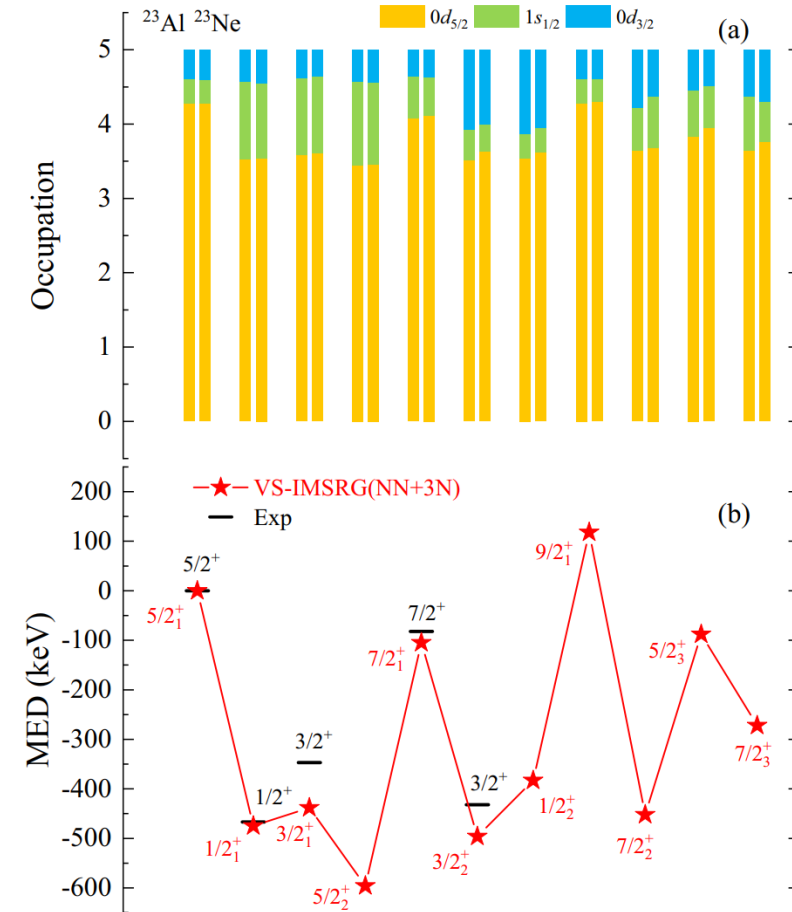
³School of Physics, and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China



Three-Body forces are important for spectra



MED $\rightarrow$ isospin symmetry breaking
(MED) = $E_x(n_rich) - E_x(p_rich)$



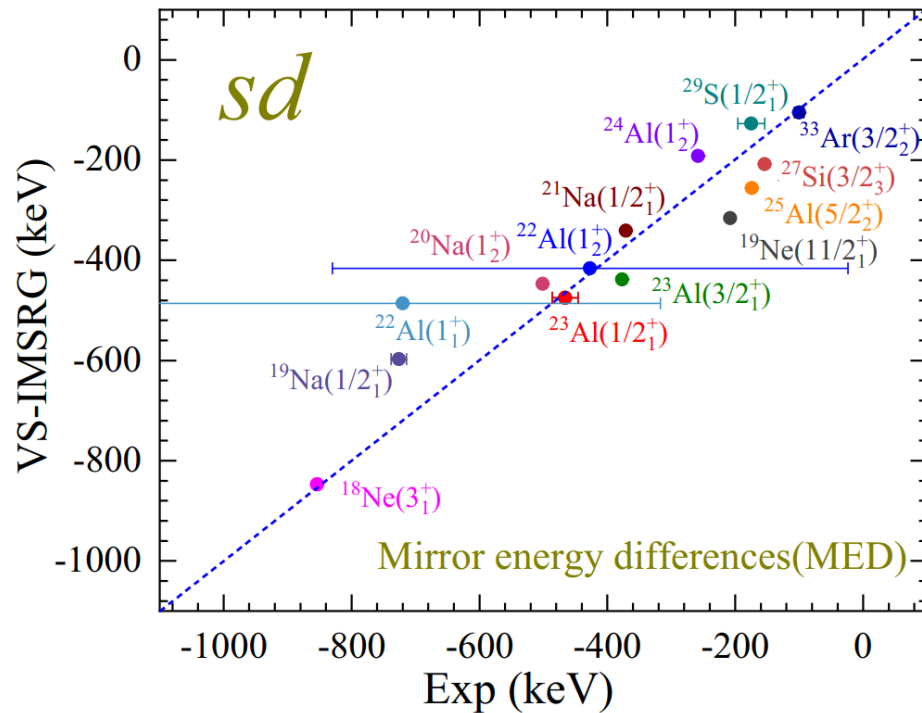
Large MED is mainly caused by the occupying of weakly-bound 1s1/2 orbital

1. Ab initio calculations for the mirror energy difference



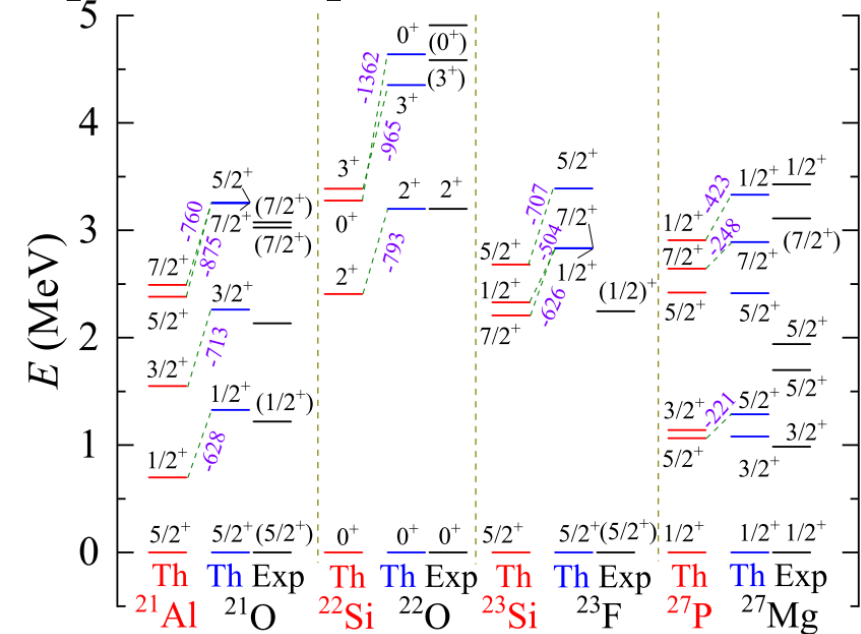
- ① Calculations of mirror states with large MED.

Ab initio calculations well reproduce data



Ab initio VS-IMSRG can be used to predict the spectra for unknown in experiments.

- ② Predictions of low-lying states in *sd*-shell proton-dripline nuclei.



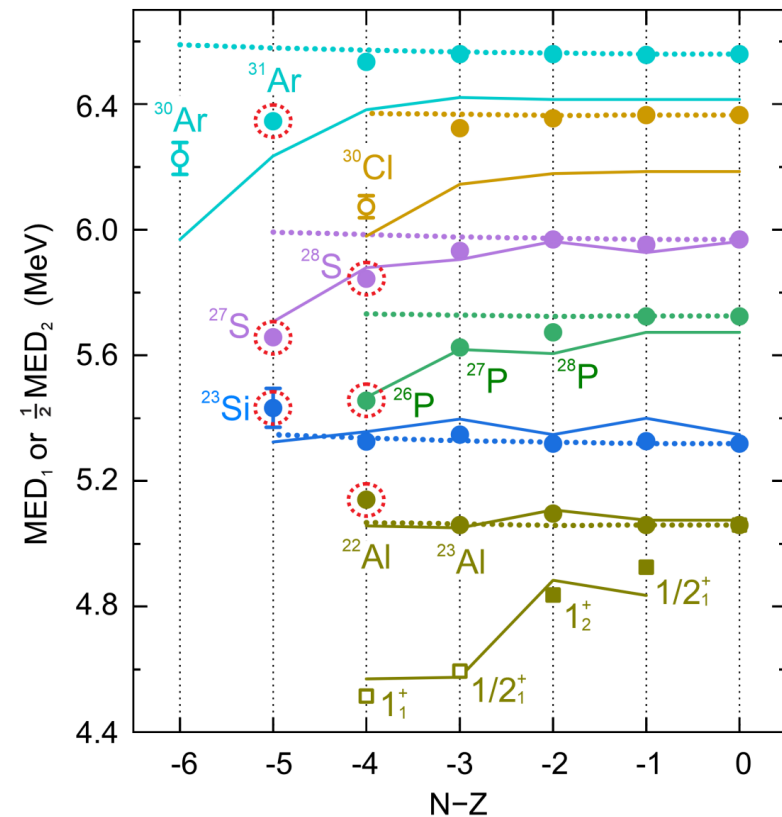
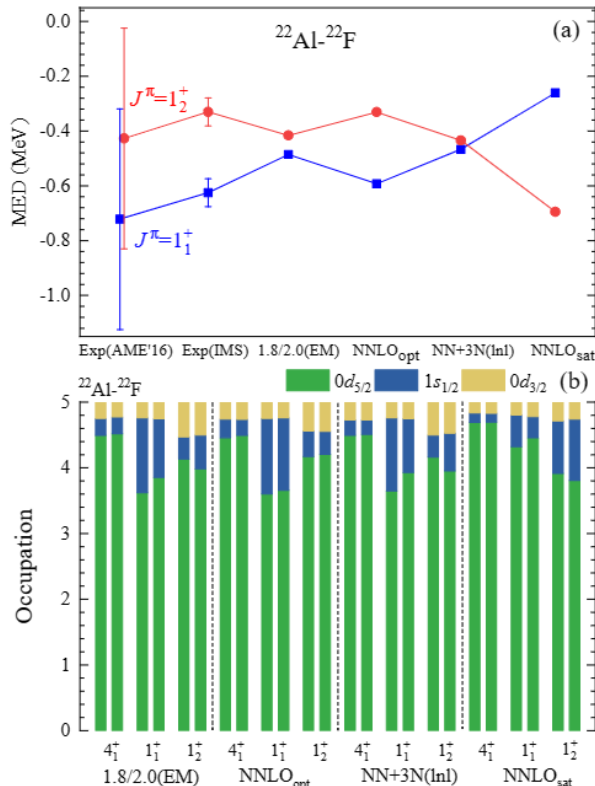
- Low-lying spectra of neutron-rich nuclei can be well produced.
- The reliability of our prediction for proton-rich nuclei.

2. Isospin symmetry—halos—mass measurement



Collaborations with IMP mass measurement group

see Prof. Yuhu Zhang's talk



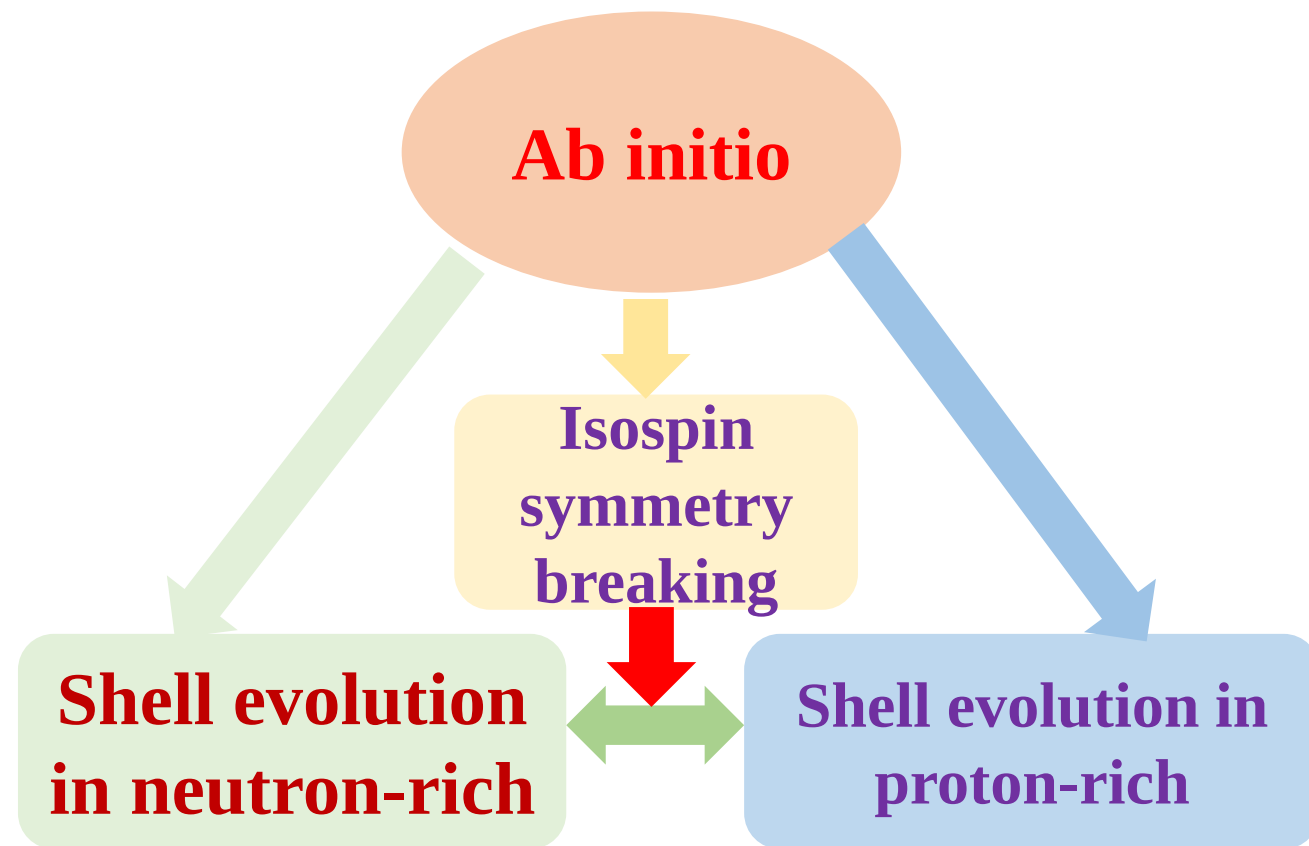
- ✓ Significant isospin symmetry breaking in the excited states of Al isotopes
- ✓ The ground state of ^{26}P , ^{27}P , ^{27}S , ^{31}Ar dripline exhibit significant isospin symmetry breaking
- ✓ The similar mechanism of significant isospin symmetry breaking and halo structure
- ✓ ^{26}P , ^{27}P , ^{27}S , ^{31}Ar are halo nuclei

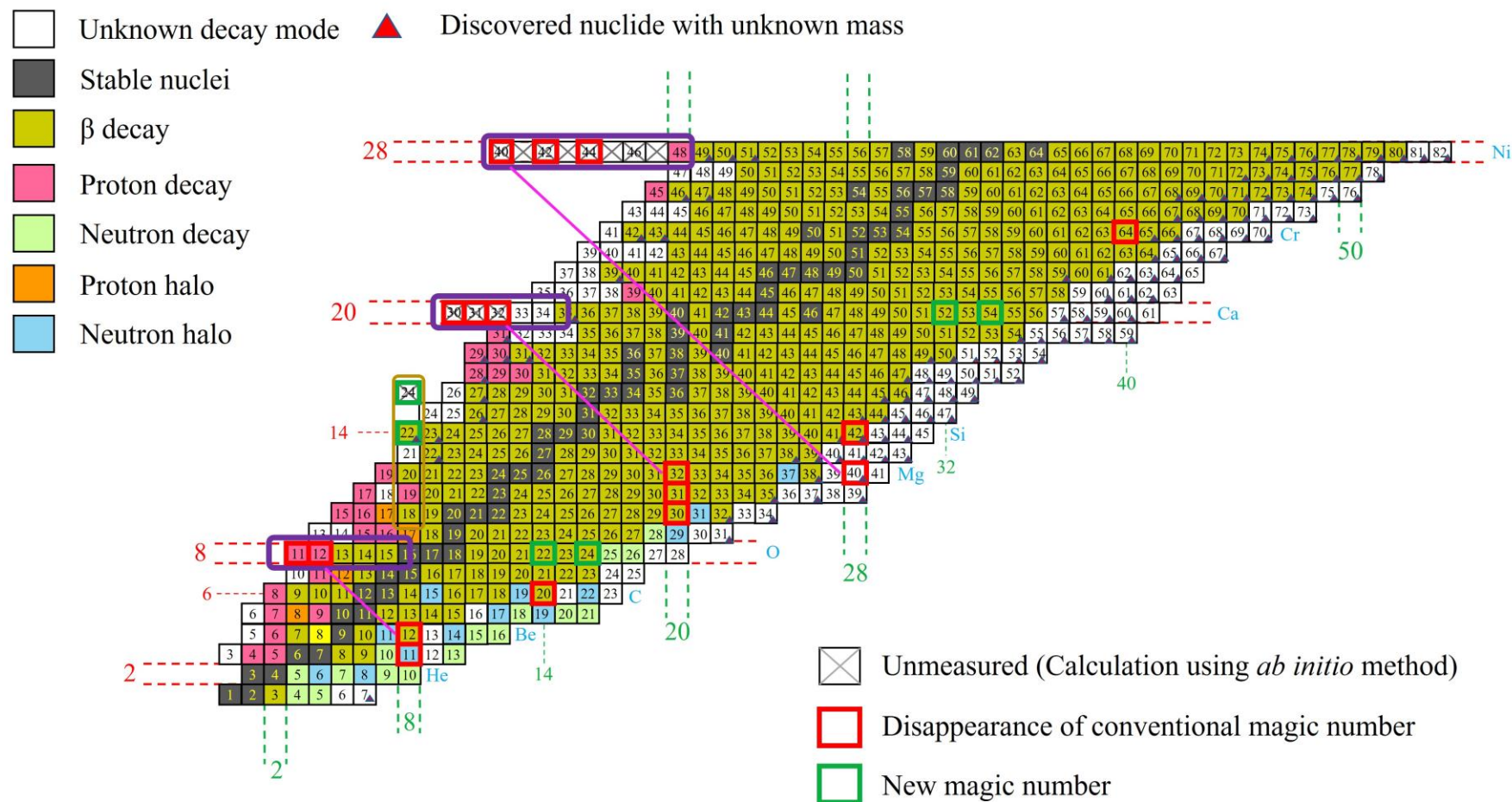
M.Z. Sun, Y. Yu, X.P. Wang, M. Wang, [J.G. Li*](#), Y.H. Zhang*, et al., Chin. Phys. C(2024)

Y. Yu, Y. M. Xing, Y. H. Zhang, M. Wang, X. H. Zhou, [J. G. Li](#), H. H. Li, et al., PRL(Accepted)

4. Isospin symmetry breaking for the shell evolution in proton-rich nuclei

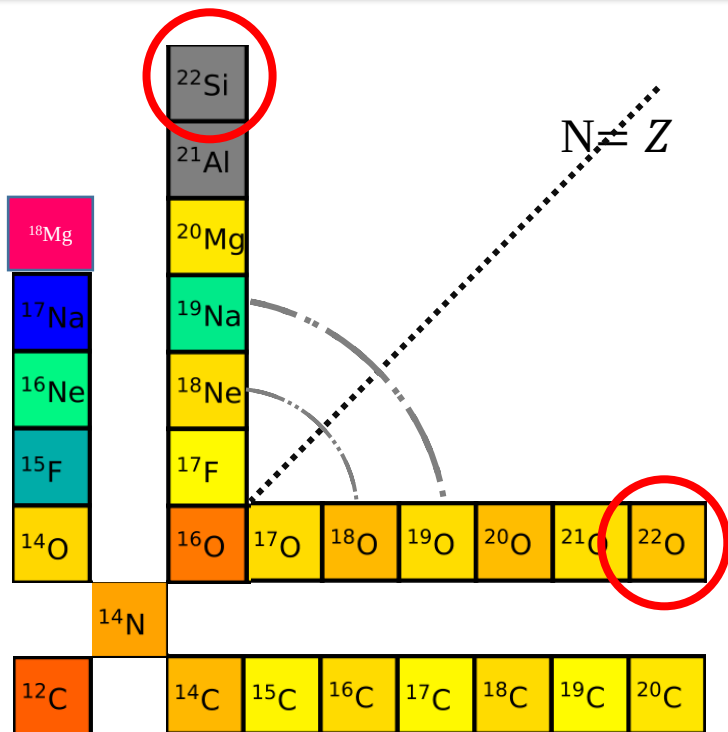
- ① [J.G. Li*](#), H. H. Li, S. Zhang, Y. M. Xing,* W. Zuo,*
Phys. Lett. B 846, 138197 (2023)
- ② H. H. Li, [J. G. Li*](#), M. R. Xie, W. Zuo, in preparation





- ✓ The disappearance of traditional magic number and appearance of new magic number
- ✓ What about the shell evolutions in proton-rich nuclei? The effects of *isospin symmetry breaking*.

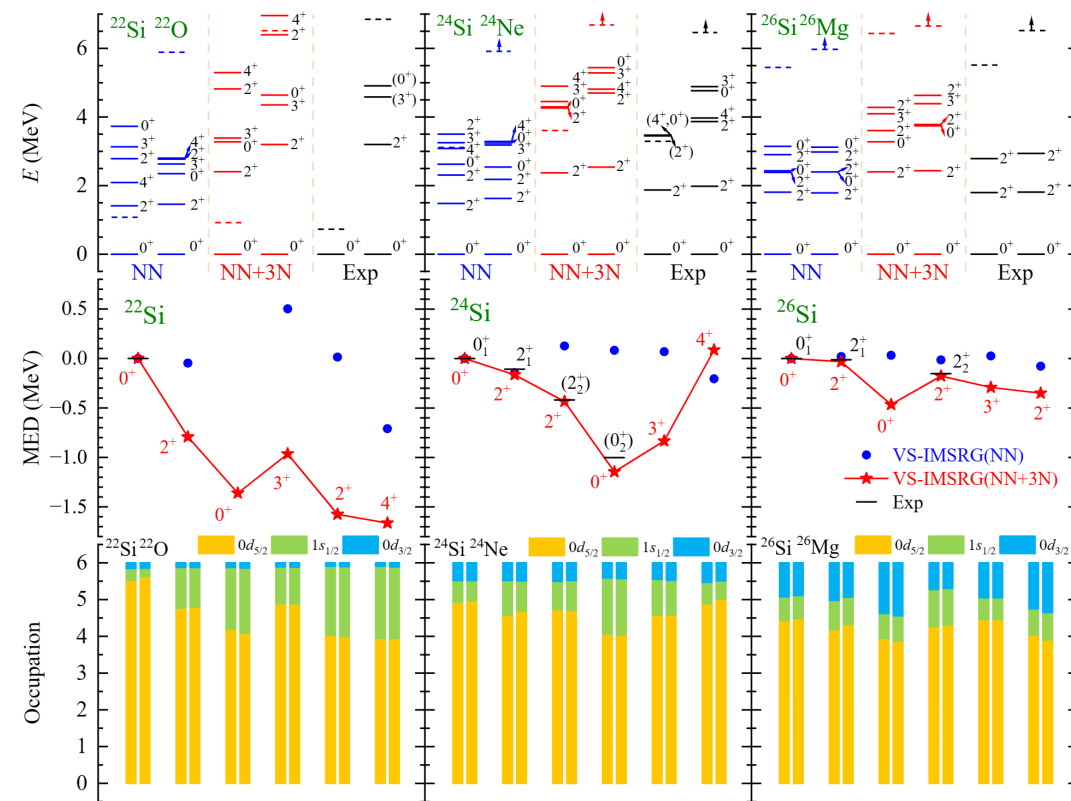
1. ^{22}Si double magic nuclei---new magic number



FRIB and RIKEN
plan to done the
experiments of
 ^{22}Si

- ✓ Neutron-rich double magic nuclei: ^{22}O
- ✓ Isospin symmetry breaking for the magicity

Whether ^{22}Si is a double magic nuclei?



- The excitation energies of ^{22}O is well reproduced with 3.2 MeV; Due to Thomas-Ehrman shift effect, the $E(21^+)$ of ^{22}Si is 2.4 3.2 MeV
- Although the $E(21^+)$ of ^{22}Si is small, but the ground state properties of $^{22}\text{Si}/^{22}\text{O}$ is similar.

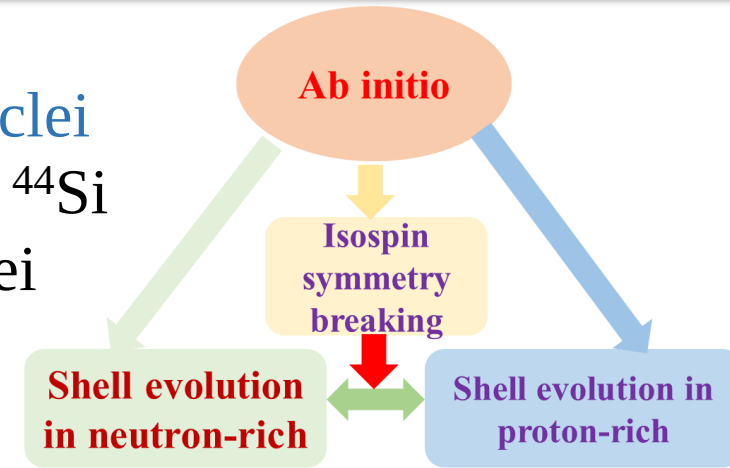
[J.G. Li](#),* H. H. Li, S. Zhang, Y. M. Xing,* W. Zuo,* *Phys. Lett. B* 846, 138197 (2023)

Summary



1. Ab initio calculations for island of inversion in neutron-rich nuclei

- shape coexistence and shape transition in ^{40}Mg , ^{42}Si and ^{44}Si
- Predicted halo in Mg, Al, Si, P and S neutron-rich nuclei
- The study of N=40 and N=50 Island of Inversion



2. Isospin symmetry breaking in mirror nuclei—mirror energy difference

large MED (Thomas-Ehrman shift) is mainly caused by the occupation of $1s_{1/2}$

3. Isospin symmetry breaking for shell evolution in proton-rich nuclei

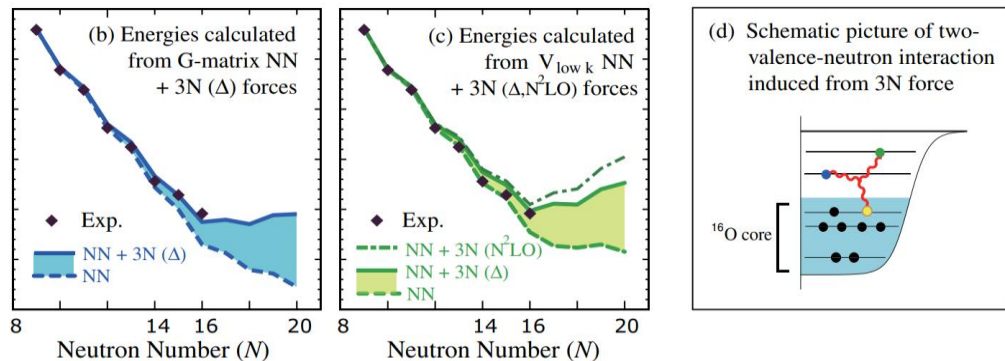
- Predicted doubly magic nuclei ^{22}Si with small 2^+ excitation energy
- *Proton-rich nuclei near $Z=8$, 20 and 28 show deformation, which belong to IOI*

Thank you for your attention!

三体力效应-质量-第一性原理计算

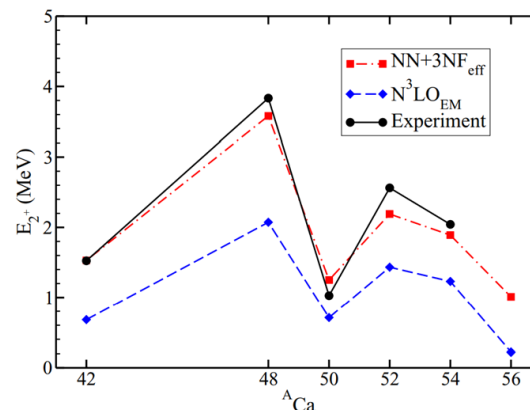


滴线位置



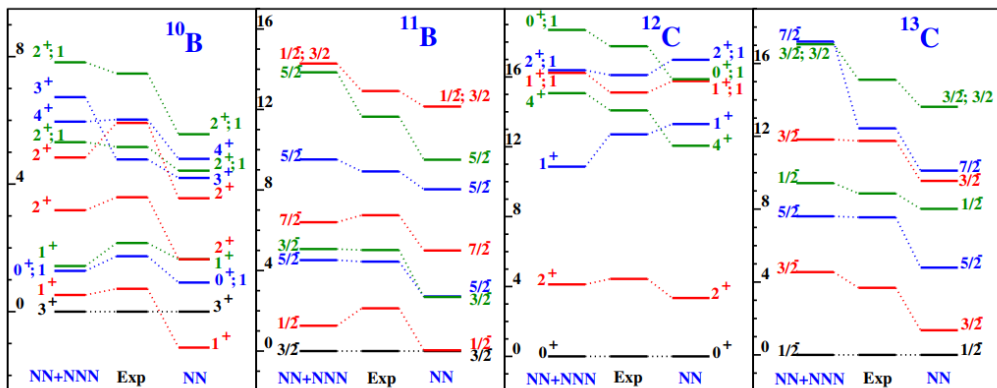
T. Otsuka, *et al.*, Phys. Rev. Lett. 105, 032501 (2010)

壳演化



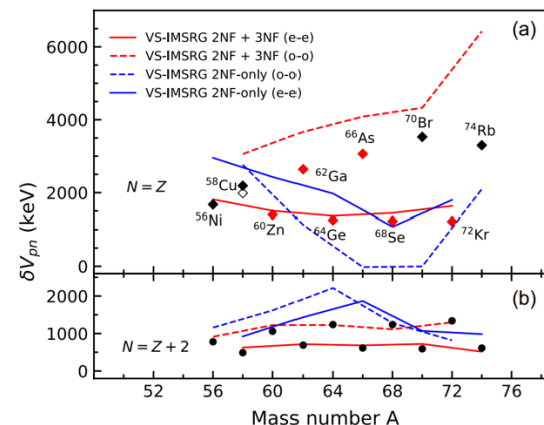
G. Hagen, *et al.*, Phys. Rev. Lett. 109, 032502 (2012)

能谱结构



P. Navrátil, *et al.*, Phys. Rev. Lett. 99, 042501 (2013)

质子中子关联

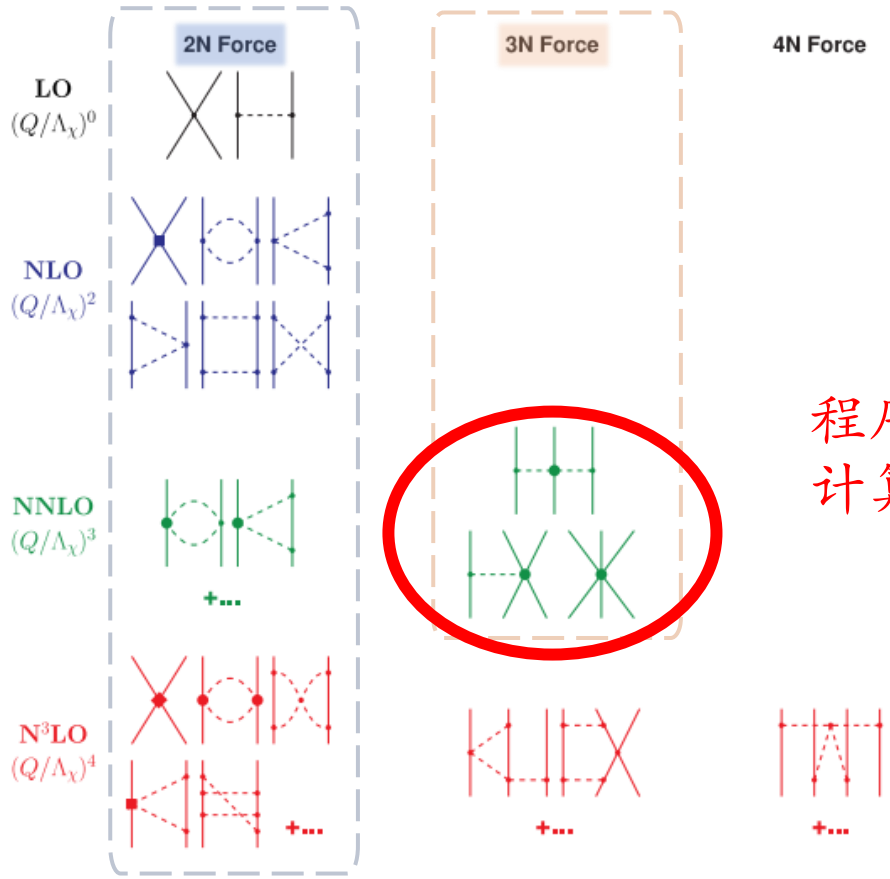


M. Wang, *et al.*, Phys. Rev. Lett. 130, 192501 (2023)

三体力效应在原子核性质中扮演着非常重要的角色

手征有效场论

现实核力 (NN+3N)



程序复杂
计算量非常大

N^3LO 两体力+ N^2LO 三体力

同位旋对称性破缺: 接触项和 π 介子交换项

Isospin breaking contributions to the NN interaction.

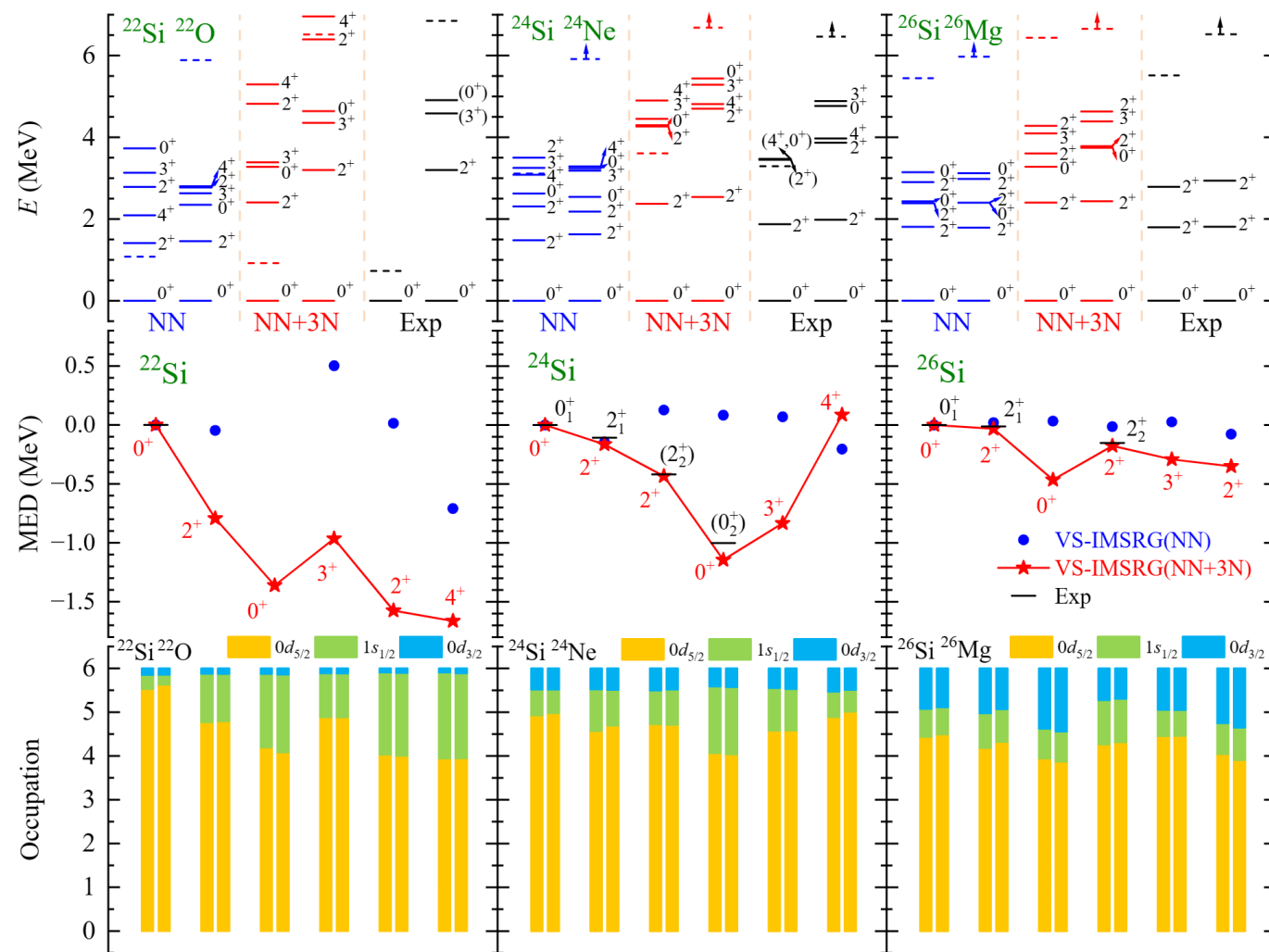
Order	Contributions
NL $\bar{0}$ ($\nu = 2$)	Pion-mass splitting in 1PE, Static Coulomb potential.
NNL $\bar{0}$ ($\nu = 3$)	CSB contacts without derivatives, Charge dependence of the pion-nucleon coupling constant in 1PE ($\sim \epsilon m_\pi^2 / \Lambda_\chi^2$).
N 3 L $\bar{0}$ ($\nu = 4$)	CIB contacts without derivatives, Charge dependence of the pion-nucleon coupling constant in 1PE [$\sim e^2 / (4\pi)^2$], Pion-mass splitting in NLO 2PE, Nucleon-mass splitting in NLO 2PE and LS equation, $\pi\gamma$ exchange, Relativistic corrections to the Coulomb potential ($\sim e^2 Q^2 / M_N^2$), Further electromagnetic corrections.

Table F.2
Partial-wave LECs for two N 3 LO fits by the Idaho group [68] using $\Lambda = 500$ and 600 MeV in the regulator function $f(p', p)$, Eq. (4.63). The $\bar{C}_i$ of the zero order partial-wave counterterms given in Eq. (4.39) are in units of 10^4 GeV^{-2} ; the C_i , Eq. (4.41), in 10^3 GeV^{-2} ; and the D_i , $\bar{D}_i$, Eq. (E.1), in 10^4 GeV^{-6} . The last column lists the exponent n of the regulator function, which is applied to the corresponding partial-wave counterterm.

Partial-wave LEC	$\Lambda = 500 \text{ MeV}$	$\Lambda = 600 \text{ MeV}$	n
$\bar{C}_{10}^{pp}$	-0.145286	-0.151165	3
$\bar{C}_{10}^{nn}$	-0.146285	-0.151467	3
$\bar{C}_{10}^{pn}$	-0.147167	-0.151745	3
C_{10}	2.380	2.200	2
$\bar{D}_{10}$	-2.545	-4.890	2
D_{10}	-16.00	-5.84	2
$C_{1\rho}$	1.487	1.548	2
$D_{1\rho}$	0.245	-0.215	3
$C_{1\pi}$	0.656	0.790	2
$D_{1\pi}$	5.25	4.40	2
C_{20}	-0.630	-0.488	2
D_{20}	2.35	3.24	4
$\bar{C}_{21}$	-0.118972496	-0.116210	3
C_{21}	0.760	0.775	2
$\bar{D}_{21}$	7.00	4.8004	2
D_{21}	6.55	10.8654	2
$D_{2\rho}$	-2.80	-2.35	2
$C_{21-2\rho_1}$	0.826	0.796	2
$\bar{D}_{21-2\rho_1}$	2.25	2.86	2
$D_{21-2\rho_1}$	6.61	5.58	2
D_{22}	-1.770	-1.764	4
$\bar{D}_{22}$	-1.46	-1.27	2
C_{22}	0.54	0.54	2
D_{22}	2.55	2.55	2
$D_{2\rho-2\pi_2}$	0.5	0.5	2
D_{22}	5.66	6.26	2, 3 ^a

美国FRIB与日本RIKEN RIB 均在计划测量 ^{22}Si 激发态

^{22}Si 双幻核结构---新幻数



- ✓ NN + 3N计算的 $^{24}\text{Si}/^{24}\text{Ne}$ 和 $^{26}\text{Si}/^{26}\text{Mg}$ 的能谱与MED与实验符合, 尤其计算的MED
- ✓ MED值较大, 表明此态具有较明显的同位旋对称性破缺, 同时此态的 $1s_{1/2}$ 轨道的占据较大, 如 ^{24}Si 的 0_2^+ 态
- ✓ 计算给出的 ^{22}Si 与 ^{22}O 的 $E(2_1^+)$ 的位置分别为2.4 与 3.2 MeV, Thomas-Ehrman shift 效应
- ✓ 计算显示 $^{22}\text{Si}/^{22}\text{O}$ 的 0^+ 与 2^+ 的组态非常接近
- ✓ 尽管 ^{22}Si 的 $E(2_1^+)$ 较低, 但是与 ^{22}O 相近, ^{22}Si 依然表现幻数性质。

美国FRIB与日本RIKEN RIB 均在计划测量 ^{22}Si 激发态

[J.G. Li](#),* H. H. Li, S. Zhang, Y. M. Xing,* W. Zuo,* *Phys. Lett. B* 846(1):138197(2023)

