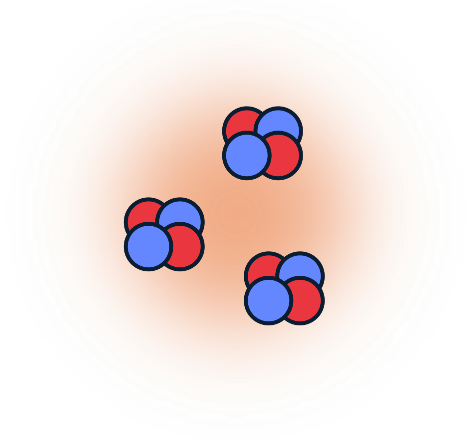
Search for Hoyle-analog state in light nuclei



Bo Zhou (周波)

Fudan University

Nov. 16, 2024

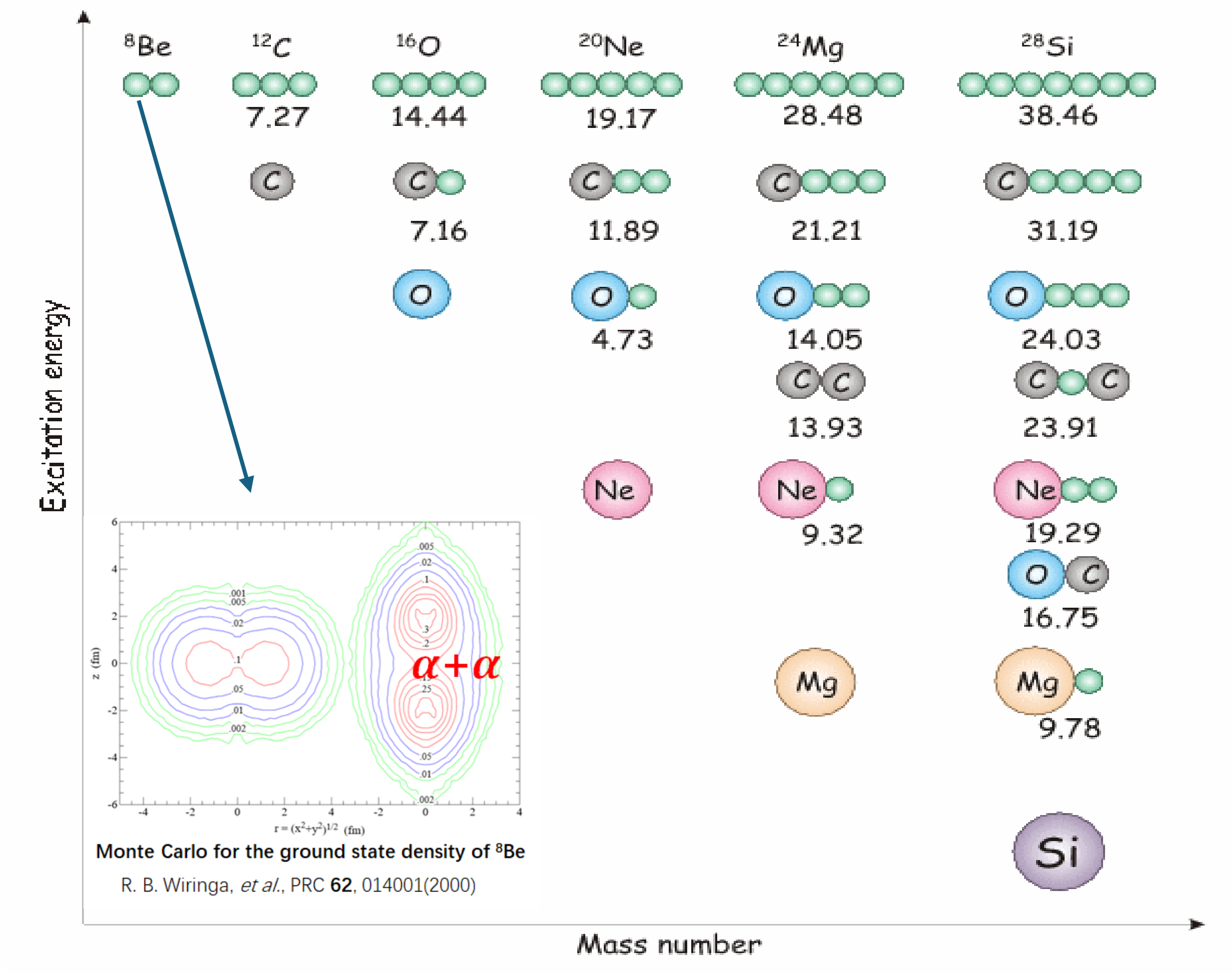
Huizhou, Guangdong, China

outline

- ^{12}C clustering structure
- 5α condensate state
- 3α+p clustering in ^{13}N
- Summary and Prospect

Nuclear Cluster Physics

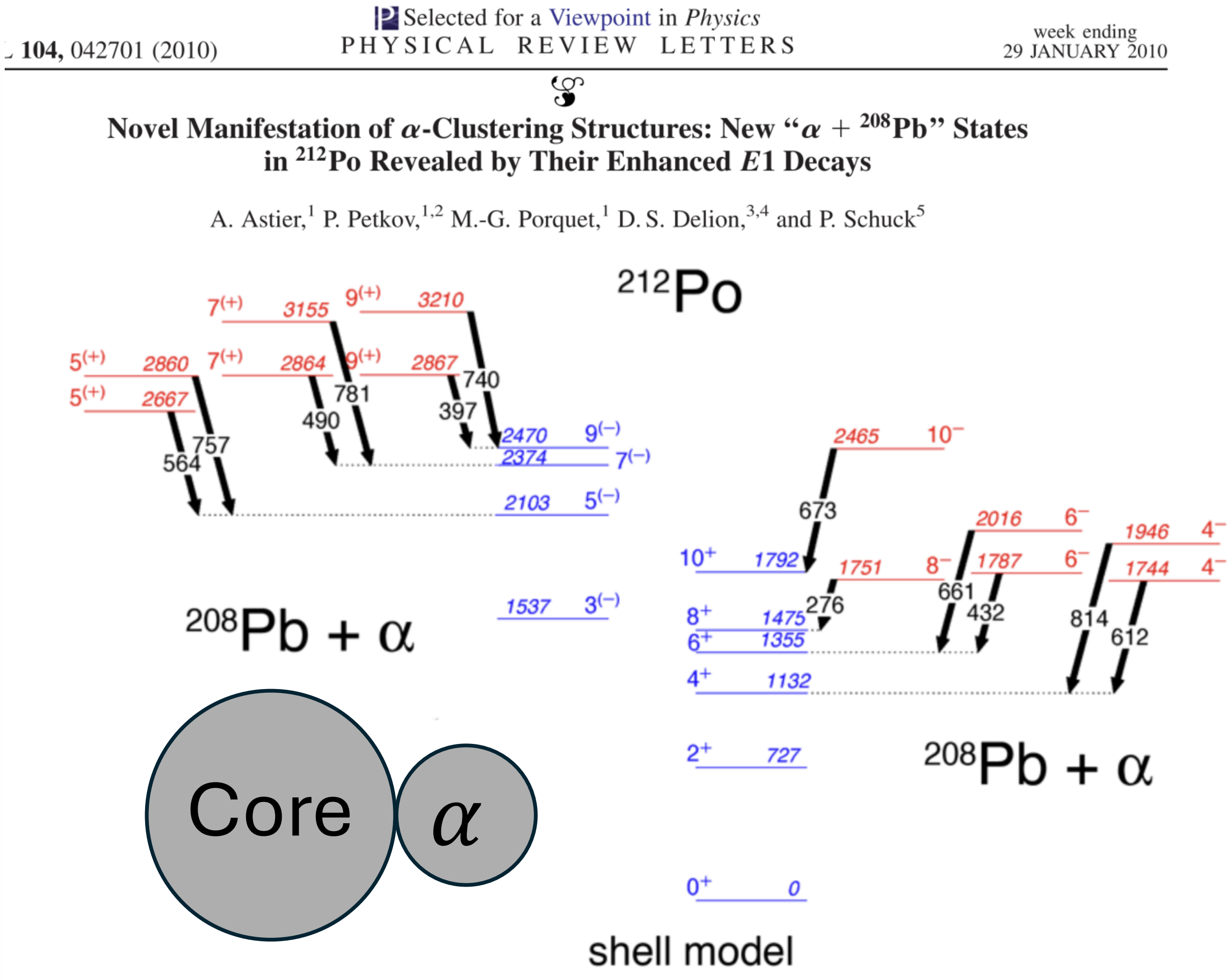
Ikeda diagram of light nuclei



Monte Carlo for the ground state density of ^{8}Be

R. B. Wiringa, *et al.*, PRC **62**, 014001(2000)

Clustering in heavy nuclei ?



Selected for a Viewpoint in *Physics*

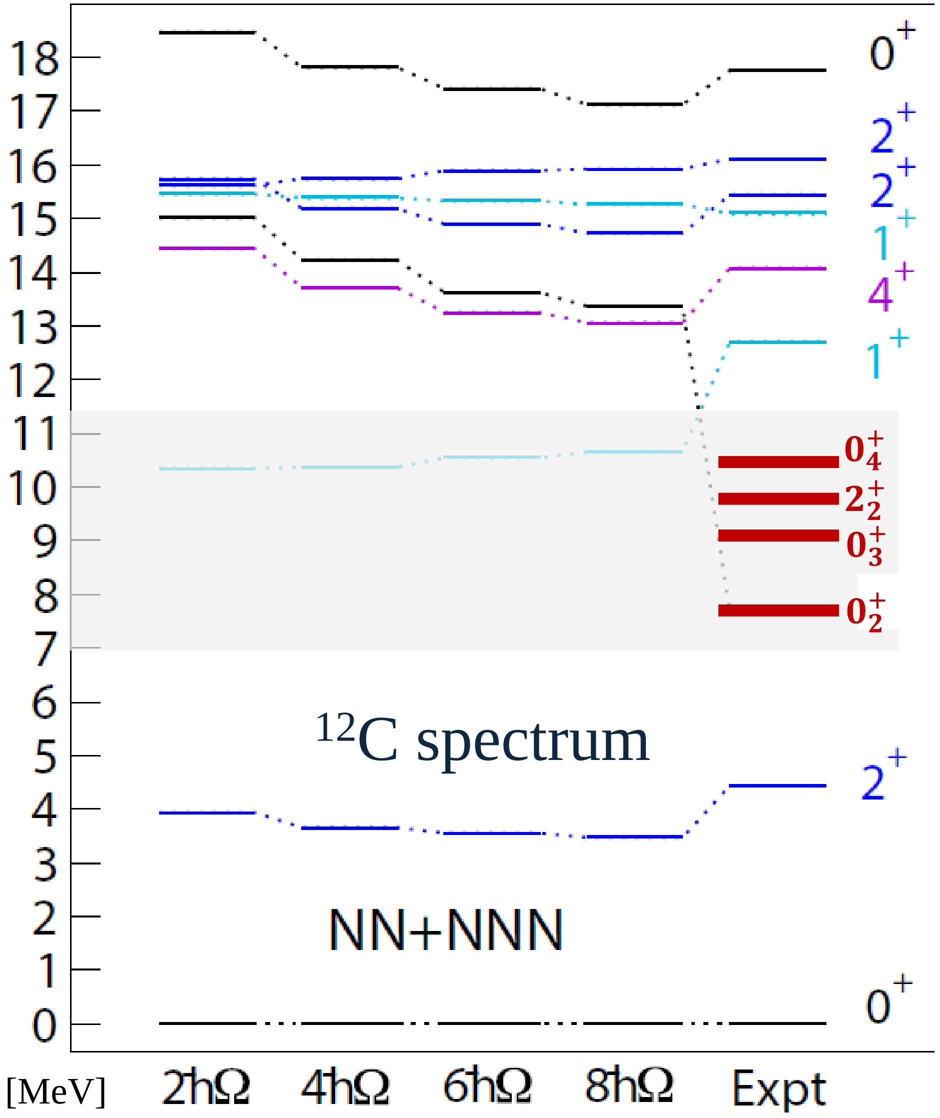
PRL **104,** 042701 (2010) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2010

Novel Manifestation of α-Clustering Structures: New "α + ^{208}Pb" States in ^{212}Po Revealed by Their Enhanced *E*1 Decays

A. Astier,[1] P. Petkov,[1,2] M.-G. Porquet,[1] D. S. Delion,[3,4] and P. Schuck[5]

https://physics.aps.org/articles/v3/8

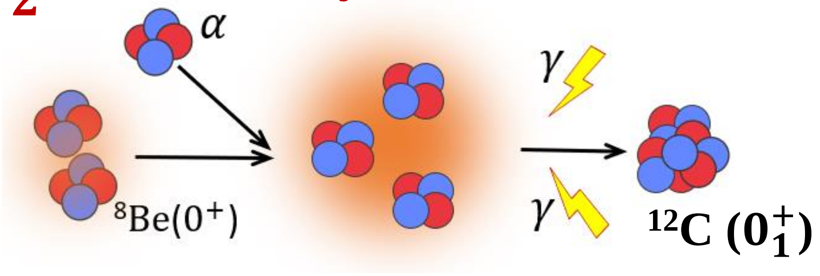
Cluster states of ^{12}C



Recent No-Core-Shell-Model calculations

V.Somà,P.Navrátil, *et al.* PRC,101,014318 (2020)

0_2^+ **Hoyle State**



Hoyle state & Bose-Einstein Condensate.

Rev. Mod. Phys. **89**, 011002 (2017)

$0_{3,4}^+$

Two broad resonance states with large decay width

Phys. Rev. C **84**, 054308 (2011)

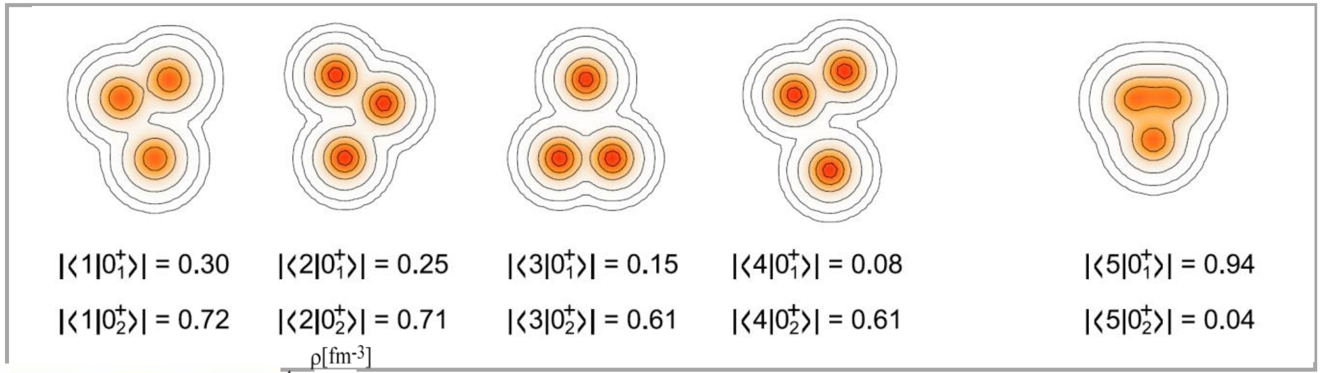
2_2^+

Long puzzle and it now has been confirmed for its existence.

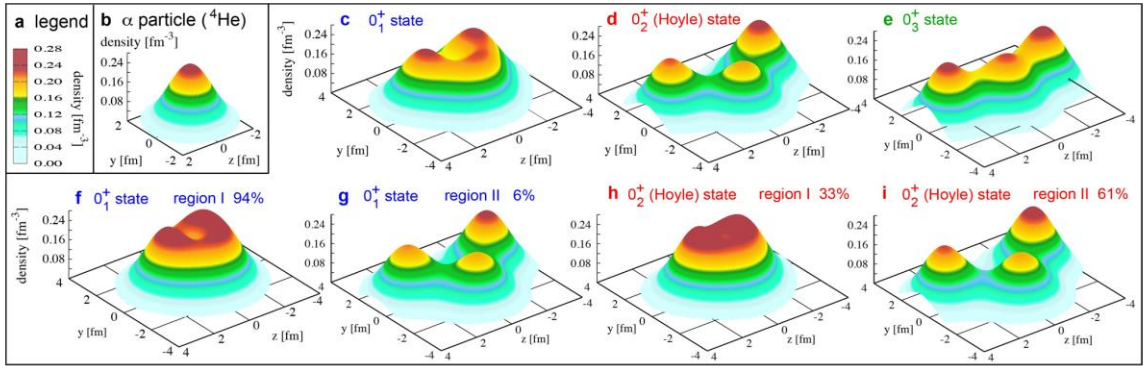
Phys. Rev. Lett. **110**, 152502 (2013)

well-developed clustering states

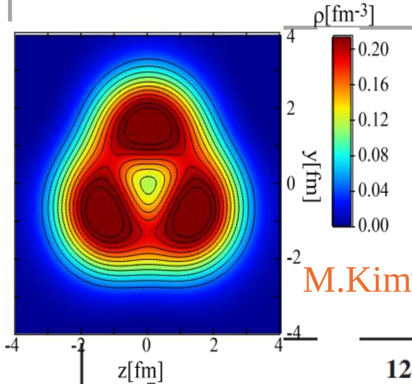
Shape/Structure of the ^{12}C



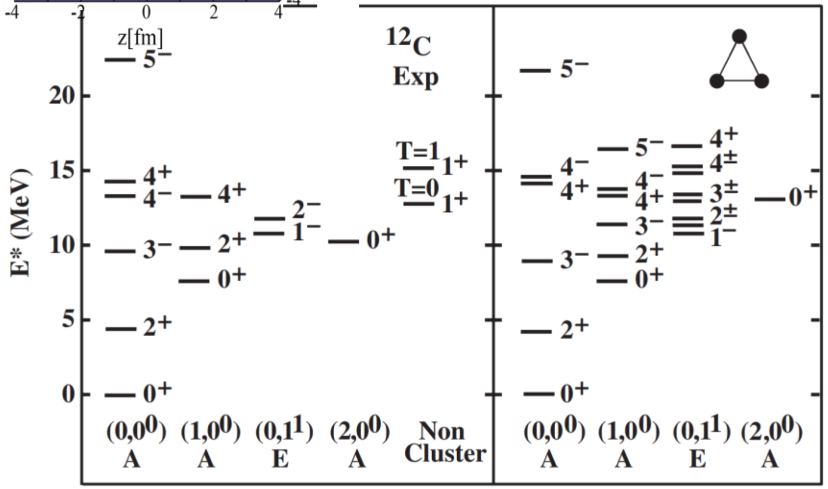
M. Chernykh, et al., PRL 98, 032501 (2007)



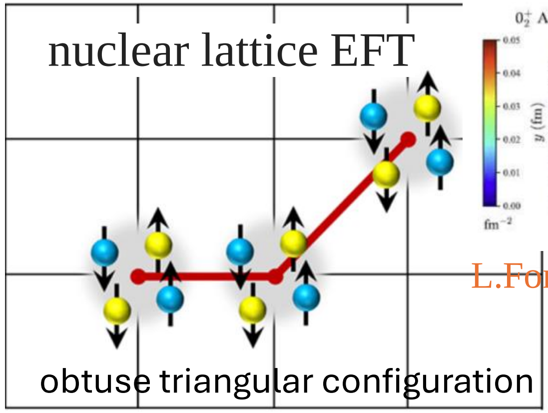
Otsuka, et al., Nat Commun 13, 2234 (2022)



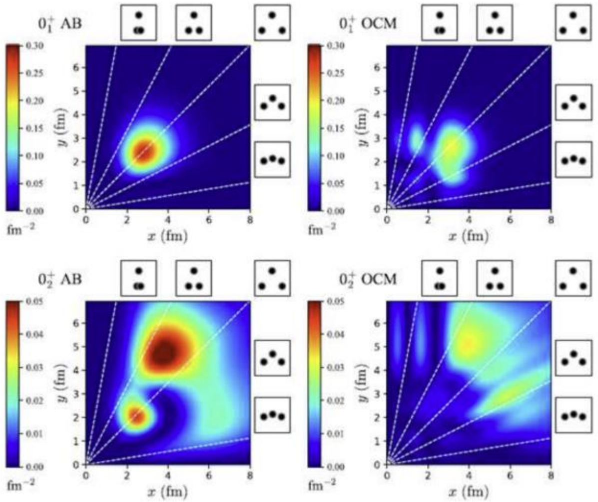
M.Kimura, Eur. Phys. J. A 60, 77 (2024)



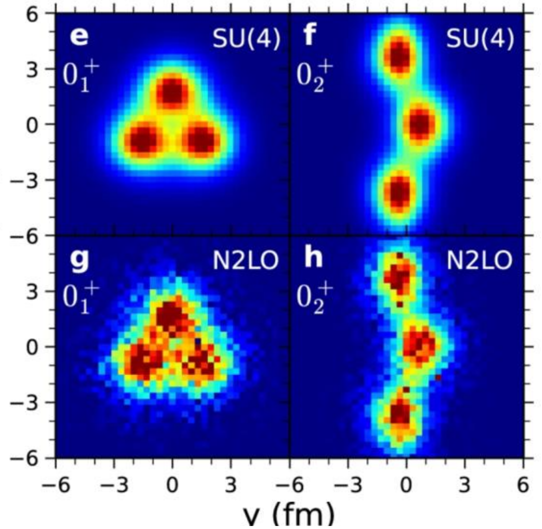
D J Marín-Lámbarri,et al., PRL 113, 012502 (2014)



E.Epelbaum,et al,,PRL 109,252501 (2012)

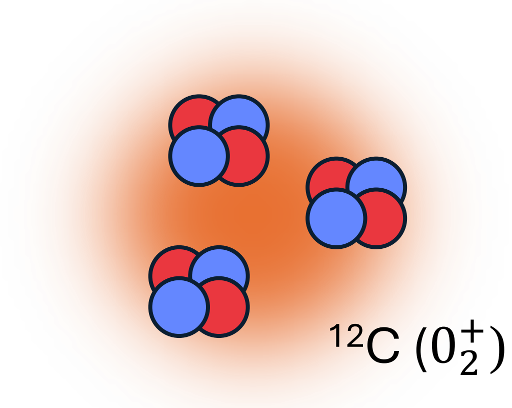


L.Fortunato, *Few-Body Syst* **65**, 1 (2024).



S.Shen, et al., *Nat Commun* **14**, 2777 (2023)

Hoyle state of ^{12}C



3α Bose-Einstein state

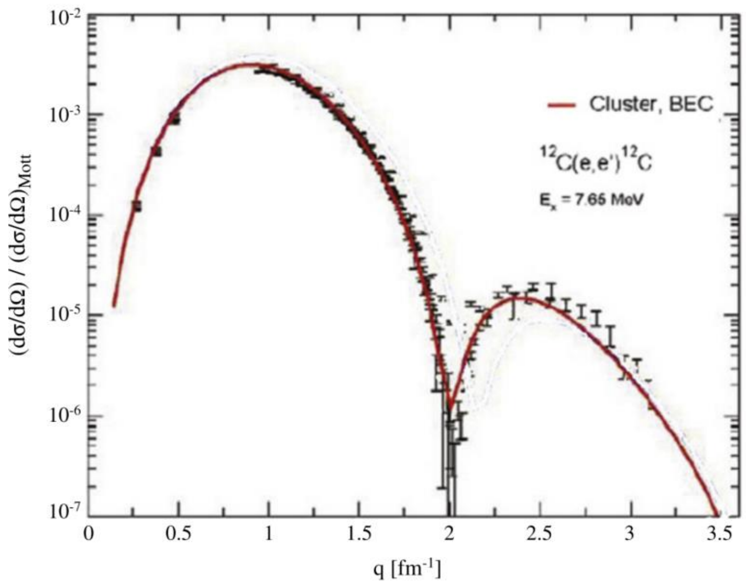
$$\Psi_{3\alpha}^{\mathrm{THSR}} = \mathcal{A}\left\{\exp\left[-\frac{2}{B^2}(\boldsymbol{X}_1^2+\boldsymbol{X}_2^2+\boldsymbol{X}_3^2)\right]\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\}$$

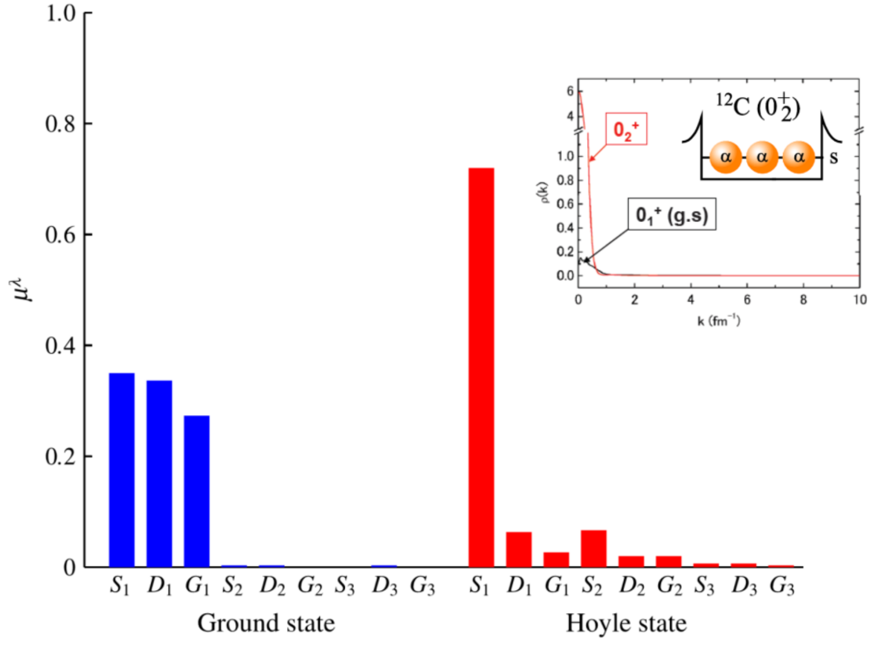
$$= \exp\left(-\frac{6}{B^2}\boldsymbol{\xi}_3^2\right)\mathcal{A}\left\{\exp\left(-\frac{4}{3B^2}\boldsymbol{\xi}_1^2-\frac{1}{B^2}\boldsymbol{\xi}_2^2\right)\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\},$$

$$\boldsymbol{\xi}_1 = \boldsymbol{X}_1 - \frac{1}{2}(\boldsymbol{X}_2+\boldsymbol{X}_3), \qquad \boldsymbol{\xi}_2 = \boldsymbol{X}_2 - \boldsymbol{X}_3, \qquad \boldsymbol{\xi}_3 = \frac{1}{3}(\boldsymbol{X}_1+\boldsymbol{X}_2+\boldsymbol{X}_3)$$

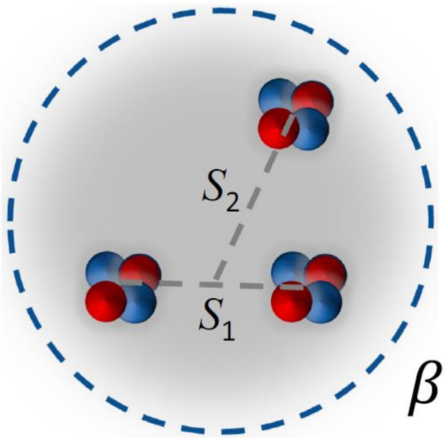
THSR, PRL 87, 192501 (2001)

Y. Funaki et al. / Progress in Particle and Nuclear Physics 82 (2015) 78–132





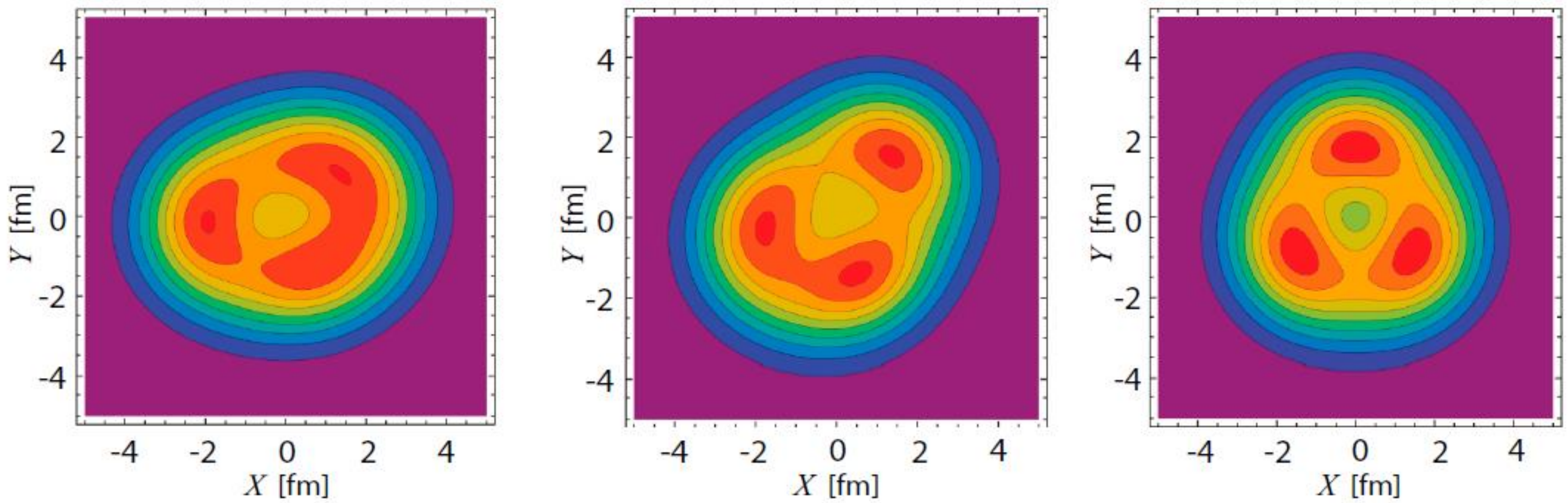
Nonlocalized cluster motion of 3α clusters in ^{12}C



We really obtained the single high-accuracy THSR-type wave functions for 3^- and 4^- states,

$$\propto \mathcal{A}\{\exp[-\frac{(\boldsymbol{\xi}_1-\boldsymbol{S}_1)^2}{b^2+2\boldsymbol{\beta}^2}-\frac{(\boldsymbol{\xi}_2-\boldsymbol{S}_2)^2}{3/4\ (b^2+2\boldsymbol{\beta}^2)}]\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\}$$

Size parameters β obtained by variational calculations.

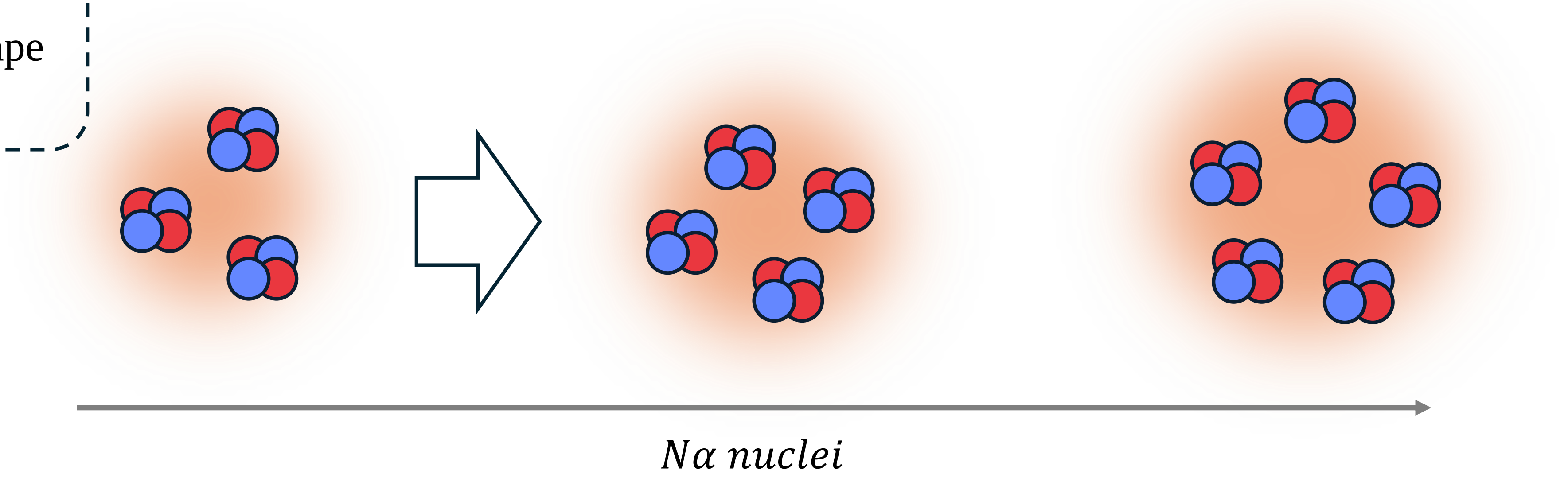


$|\langle\Phi^{3^-}(3/2,3/2,1/2)|\Phi^{3^-}_{\rm GCM}\rangle|^2$=0.94 $|\langle\Phi^{3^-}(1,3/2,3/2)|\Phi^{3^-}_{\rm GCM}\rangle|^2$=0.93 $|\langle\Phi^{3^-}(3/2,0,3/2)|\Phi^{3^-}_{\rm GCM}\rangle|^2$=0.94

$|\langle\Phi^{4^-}(3/2,3/2,1/2)|\Phi^{4^-}_{\rm GCM}\rangle|^2$=0.92 $|\langle\Phi^{4^-}(1,3/2,3/2)|\Phi^{4^-}_{\rm GCM}\rangle|^2$=0.92 $|\langle\Phi^{4^-}(3/2,0,3/2)|\Phi^{4^-}_{\rm GCM}\rangle|^2$=0.92

B. Zhou, et al., Phys. Rev. C **99**, 051303(R) (2019).

- gas-like cluster state
- no-geometry shape
- excited states

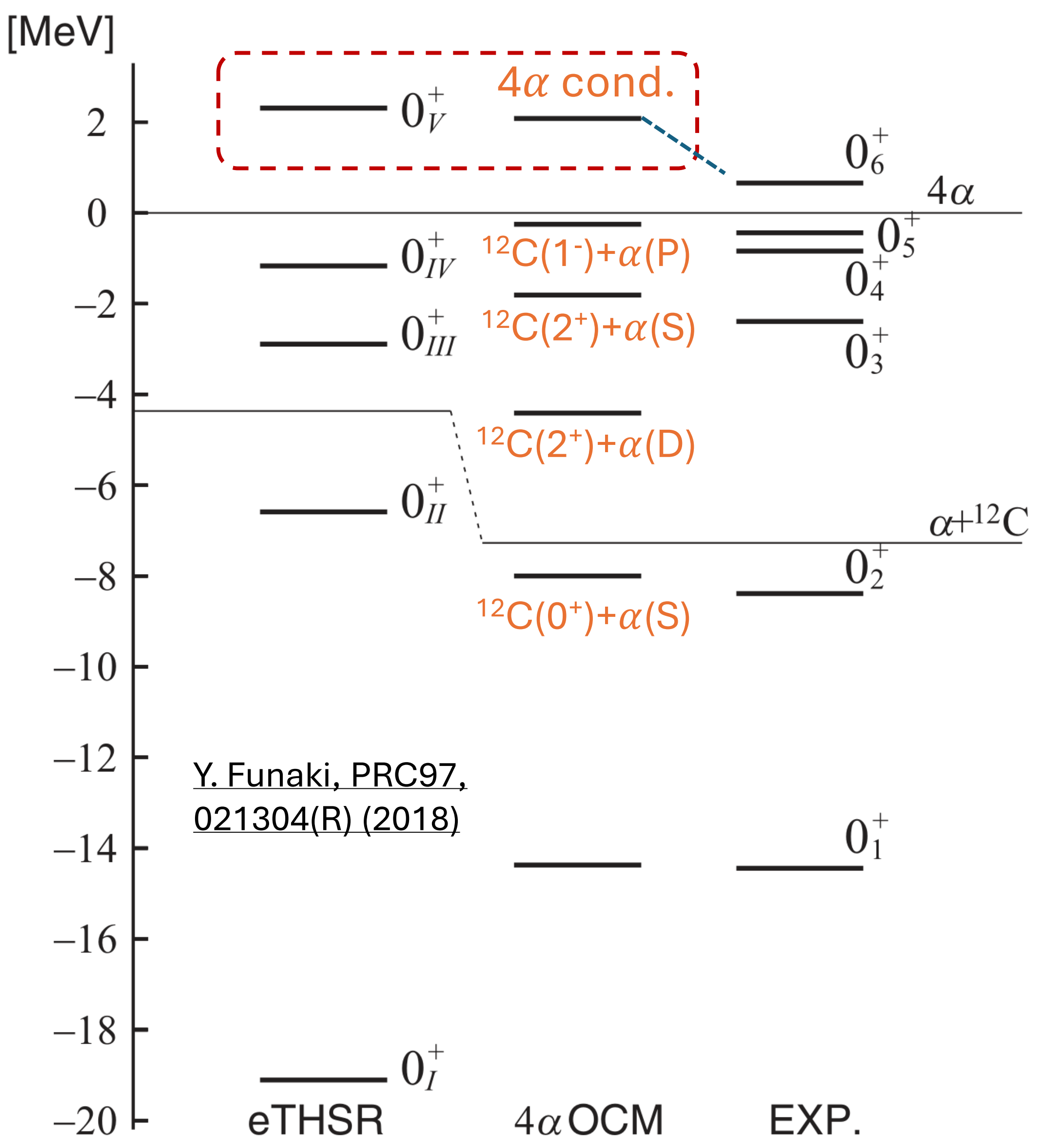


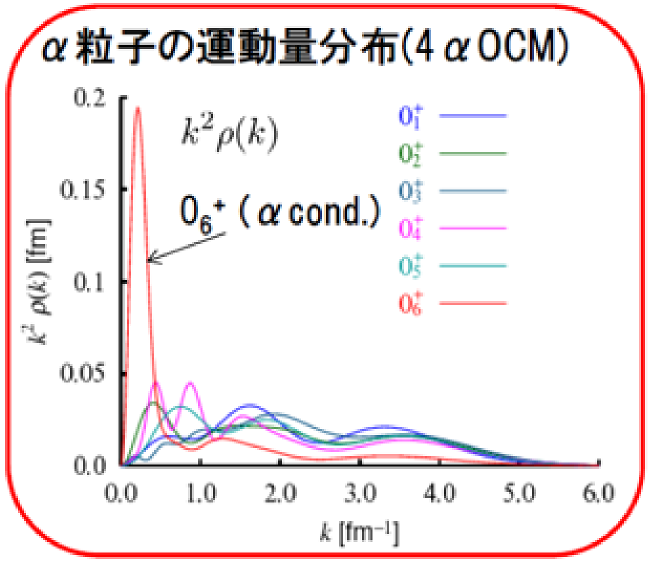
Search for the 5α condensate state

3α condensate	4α condensate	5α condensate
(Hoyle state)	(0_6^+ state)	(?)
2001 (THSR)	2008~ (OCM,THSR)	2019~

study of alpha condensate in finite nuclei

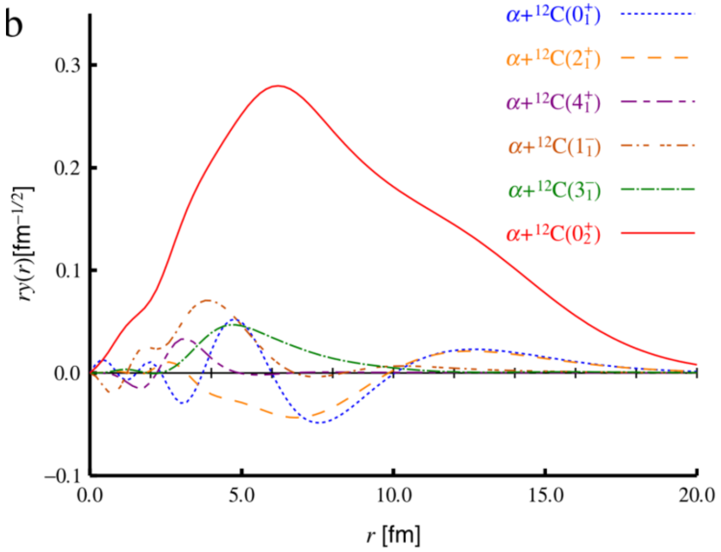
Alpha condensate in ^{16}O





4αOCM Y. F. et al., PRL **101**, 082502 (2008).

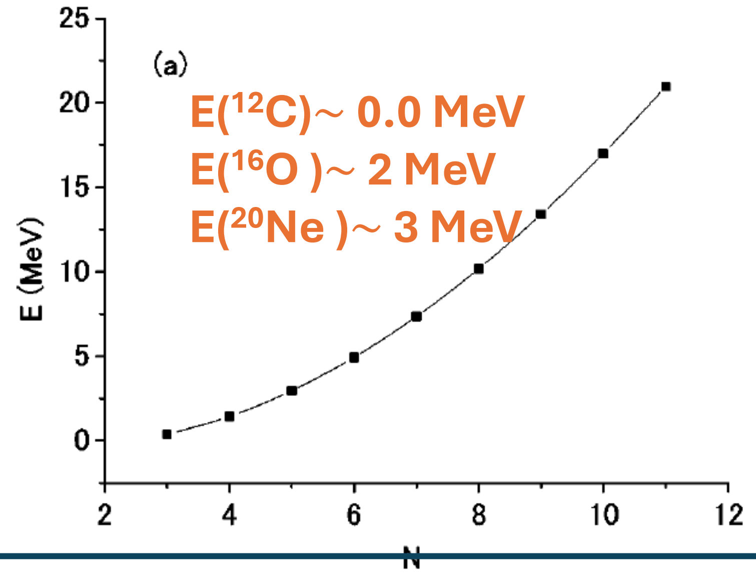
4αTHSR Y. F. et al., PRC **82**, 024312 (2010).



Multi-α condensation

Dilute multi-α cluster condensed states with spherical and axially deformed shapes are studied with the Gross-Pitaevskii equation and Hill-Wheeler equation where the α cluster is treated as a structureless boson,
it is predicted to exist in heavier self-conjugate 4*N* nuclei up to *N*=10.

T. Yamada and P. Schuck, Phys. Rev. C 69, 024309 (2004).



Some candidates for α condensate were found from experiments for ^{12}C and ^{16}O.

Rev. Mod. Phys. **89**, 011002 (2017).

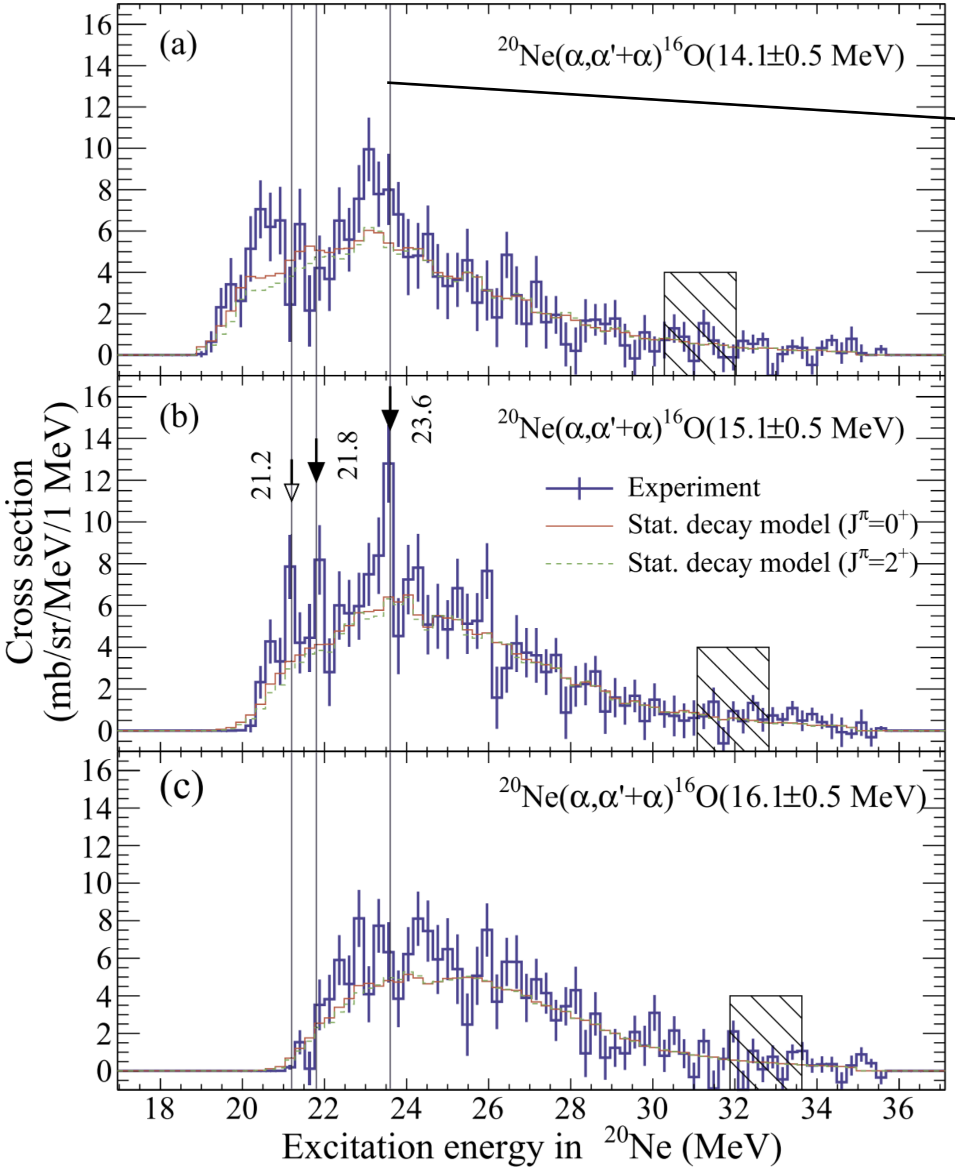
No experimental signatures for α condensation were observed

Phys. Rev. C **100**, 034320 (2019)

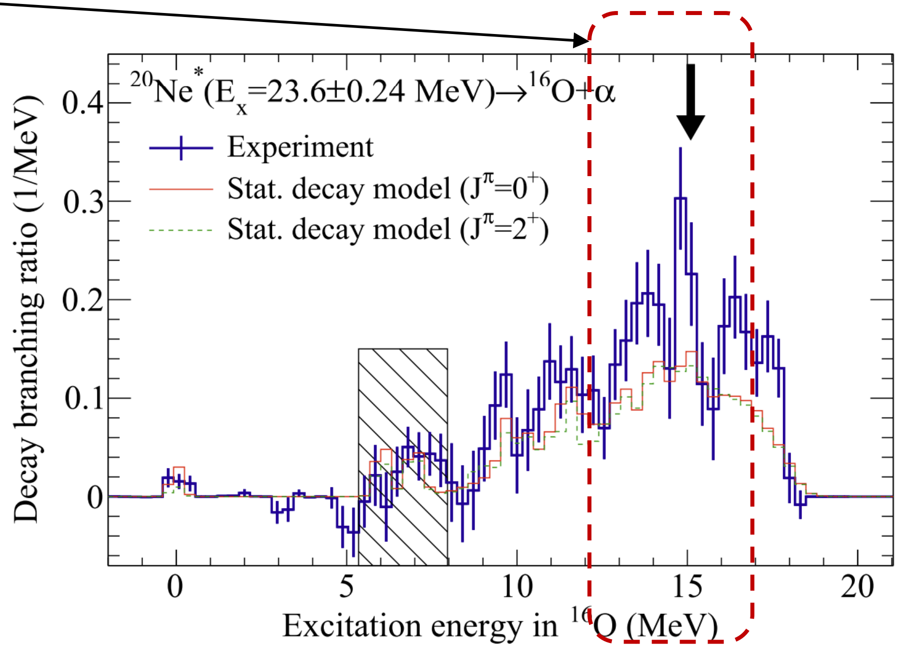
An experimental way of testing Bose-Einstein condensation of clusters in the atomic nucleus is reported. The enhancement of cluster emission and the multiplicity partition of possible emitted clusters could be direct signatures for the condensed states.

PRL **96**, 192502 (2006)

Recent experiment for 5α condensation



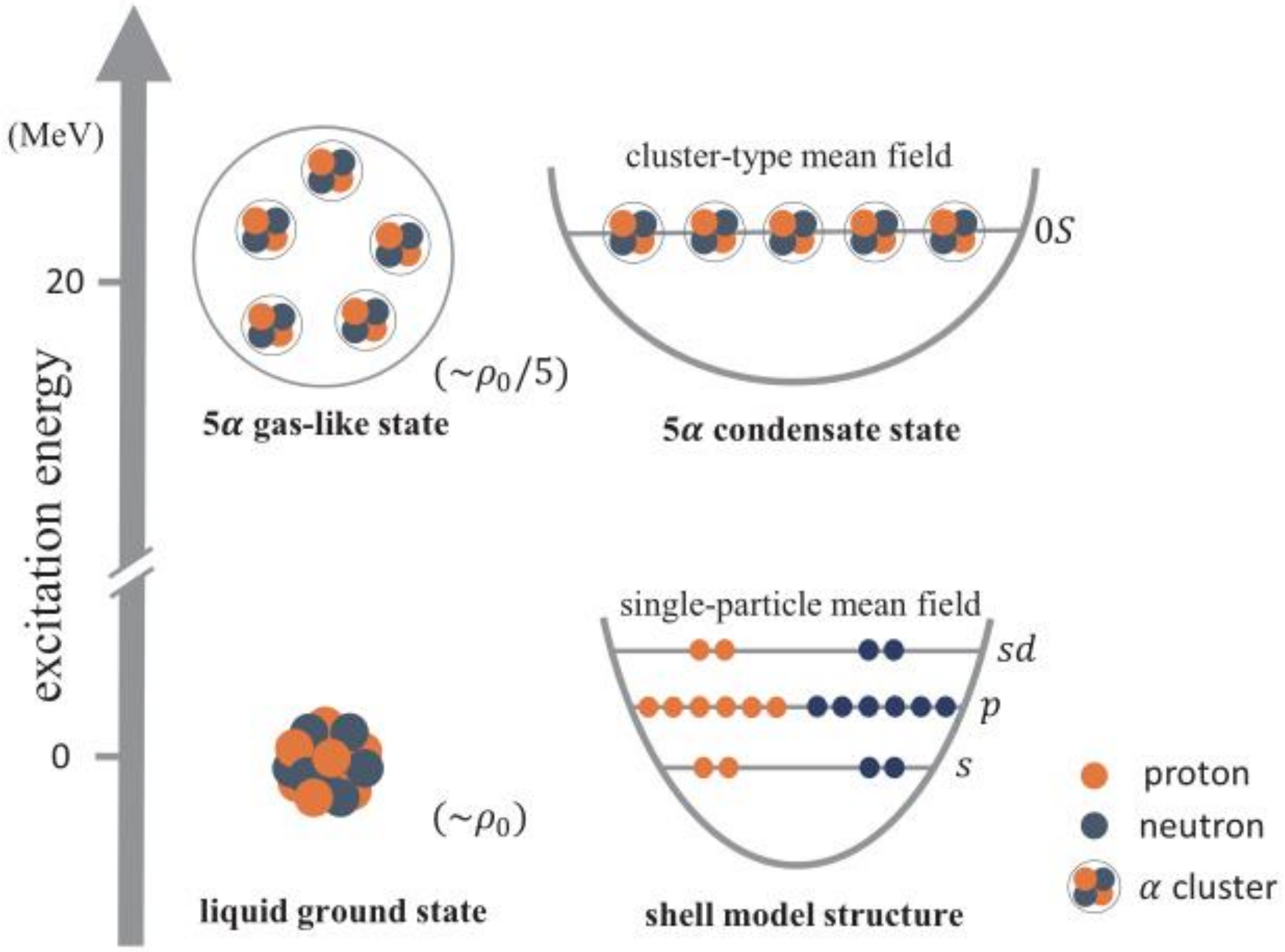
23.6-MeV state enhances in the decay to the ^{16}O (0_6^+) $+\alpha$ channel

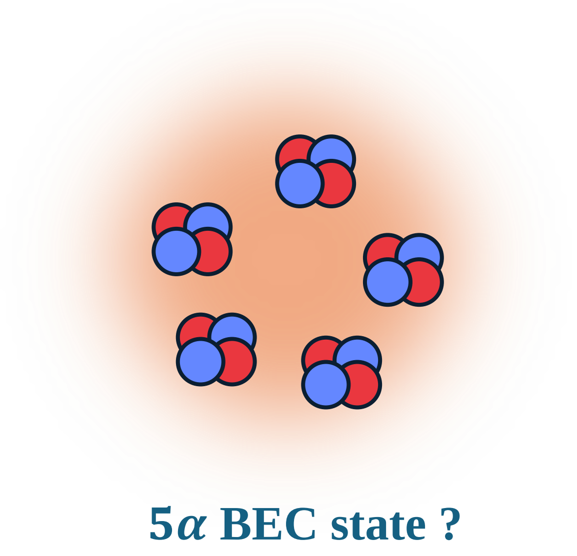


- 3.3 MeV above the 5α threshold
- strongly coupled to ^{16}O(0_6^+) state

S. Adachi et al., *Candidates for the 5α condensed state in ^{20}Ne*, PLB **819**,136411 (2021).

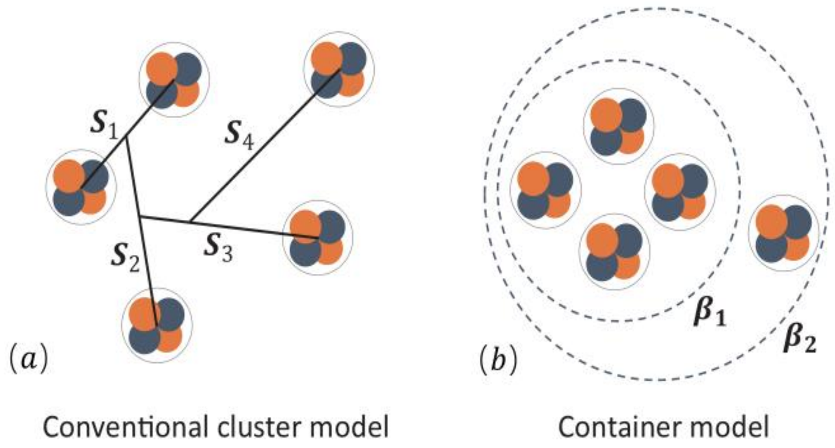
Search for the 5α condensate state





The 5α microscopic wave function

To solve the configurations problem:



| **Schematic illustrations of two distinct microscopic cluster models. a**, The convention cluster model of Φ^{B}, in which the inter-cluster variables $\{S_i\}$ are the Jacobi coordinates of $\{R_i\}$. **b** Container picture for 4α + α cluster structure of ^{20}Ne. The β_1 is the size variable for the descripti 4α and β_2 for the description of the relative motion between 4α and α clusters.

$$\Psi(\beta_1,\beta_2) = \int d^3R_1 d^3R_2 d^3R_3 d^3R_4 d^3R_5 \,\mathrm{Exp}\left[-\frac{1/2S_1^2+2/3S_2^2+3/4S_3^2}{\beta_1^2}-\frac{4/5S_4^2}{\beta_2^2}\right]\Phi^{B}(R_1,R_2,R_3,R_4,R_5)$$

$$= n_0\,\mathscr{A}\left\{\mathrm{Exp}\left[-\frac{2\xi_1^2+8/3\xi_2^2+3\xi_3^2}{2(b^2+2\beta_1^2)}\right]\mathrm{Exp}\left[-\frac{16/5\xi_4^2}{2(b^2+2\beta_2^2)}\right]\prod_{i=1}^{5}\varphi_i^{\mathrm{int}}(\alpha)\right\},$$

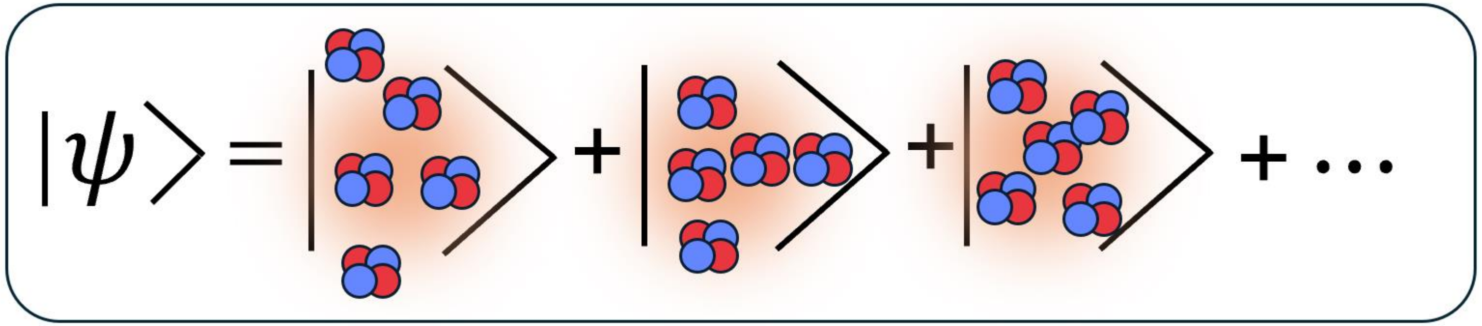
where the conventional Brink cluster wave function Φ^{B},

$$\Phi^{B}(R_1,R_2,R_3,R_4,R_5) = \frac{1}{\sqrt{20!}}\mathscr{A}[\phi_1(R_1)...\phi_5(R_2)...\phi_{20}(R_5)],$$

$$\propto \phi_g\,\mathscr{A}\left\{\mathrm{Exp}\left[-\frac{2(\xi_1-S_1)^2+8/3(\xi_2-S_2)^2+3(\xi_3-S_3)^2}{2b^2}\right]\mathrm{Exp}\left[-\frac{16/5(\xi_4-S_4)^2}{2b^2}\right]\prod_{i=1}^{5}\varphi_i^{\mathrm{int}}(\alpha)\right\},$$

with the single-nucleon wave function,

$$\phi_i(R_k) = \left(\frac{1}{\pi b^2}\right)^{\frac{3}{4}} e^{-\frac{1}{2b^2}(r_i-R_k)^2}\chi_i\tau_i.$$



Three-body effective interaction

To solve the interaction problem:

The Hamiltonian for ^{20}Ne in this work can be written as:

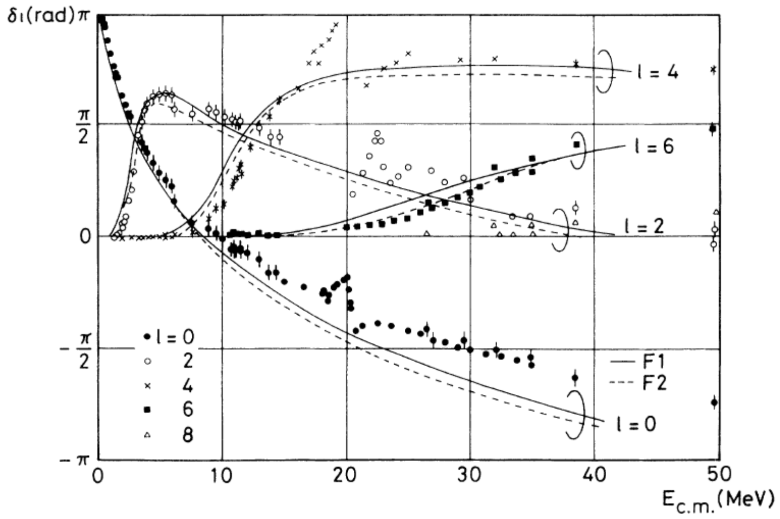
$$\mathcal{H} = -\frac{\hbar^2}{2M}\sum_i \nabla_i^2 - T_G + \sum_{i<j} V_{ij}^C + \sum_{i<j} V_{ij}^{(2)} + \sum_{i<j<k} V_{ijk}^{(3)},$$

The effective nucleon-nucleon potential part is taken a Gaussian form, which is expressed as:

$$V_{ij}^{(2)} = \sum_n v_n^{(2)} \exp\left\{-\left(\frac{r_{ij}}{r_n^{(2)}}\right)^2\right\}(W_n^{(2)} + M_n^{(2)} P_{ij})$$

and

$$V_{ijk}^{(3)} = \sum_n v_n^{(3)} \exp\left\{-\left(\frac{r_{ij}}{r_n^{(3)}}\right)^2 - \left(\frac{r_{jk}}{r_n^{(3)}}\right)^2\right\} \times (W_n^{(3)} + M_n^{(3)} P_{ij})(W_n^{(3)} + M_n^{(3)} P_{jk}),$$



A. Tohsaki,Phys. Rev. C **49**, 1814 (1994).

Tohsaki F1 three-body interaction was used.

Radius-Constraint Method + Stabilization Method

To solve the resonance problem:

Radius-Constraint Method,

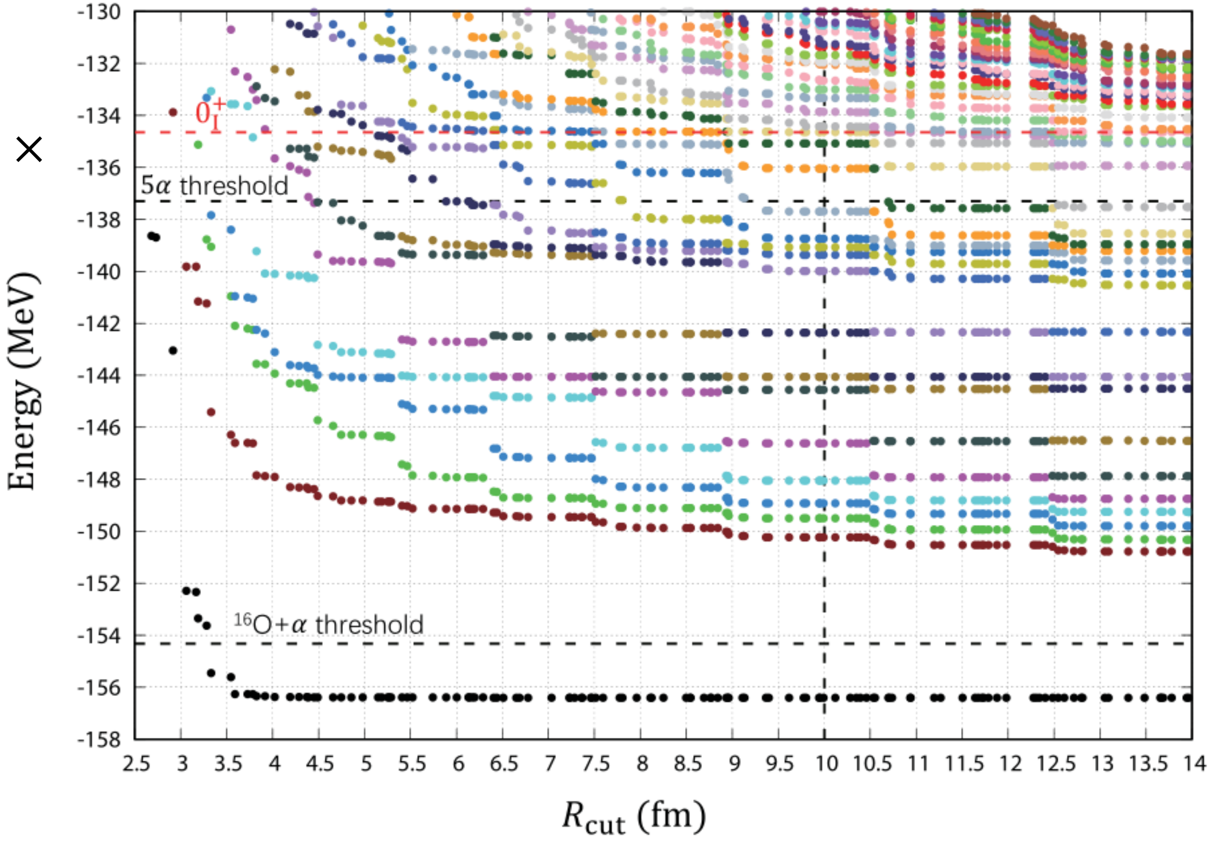
$$\sum_{\beta_1',\beta_2'} \left\langle \hat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1,\beta_2) \middle| \sum_{i=1} \frac{1}{20}(r_i - X_G)^2 \middle| \hat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1',\beta_2') \right\rangle \times g^{(\gamma)}(\beta_1',\beta_2')$$

$$= \{R^{(\gamma)}\} g^{(\gamma)}(\beta_1,\beta_2) \left\langle \hat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1,\beta_2) \middle| \hat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1',\beta_2') \right\rangle$$

$$\Psi^{0^+}_{GCM} = \sum_{\beta_1,\beta_2} g^{(\gamma)}(\beta_1,\beta_2) \hat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1',\beta_2')$$

Here ,$R^{(\gamma)} \leq R_{cut}$ and R_{cut} is the radius cut-off parameter.

Stabilization Method

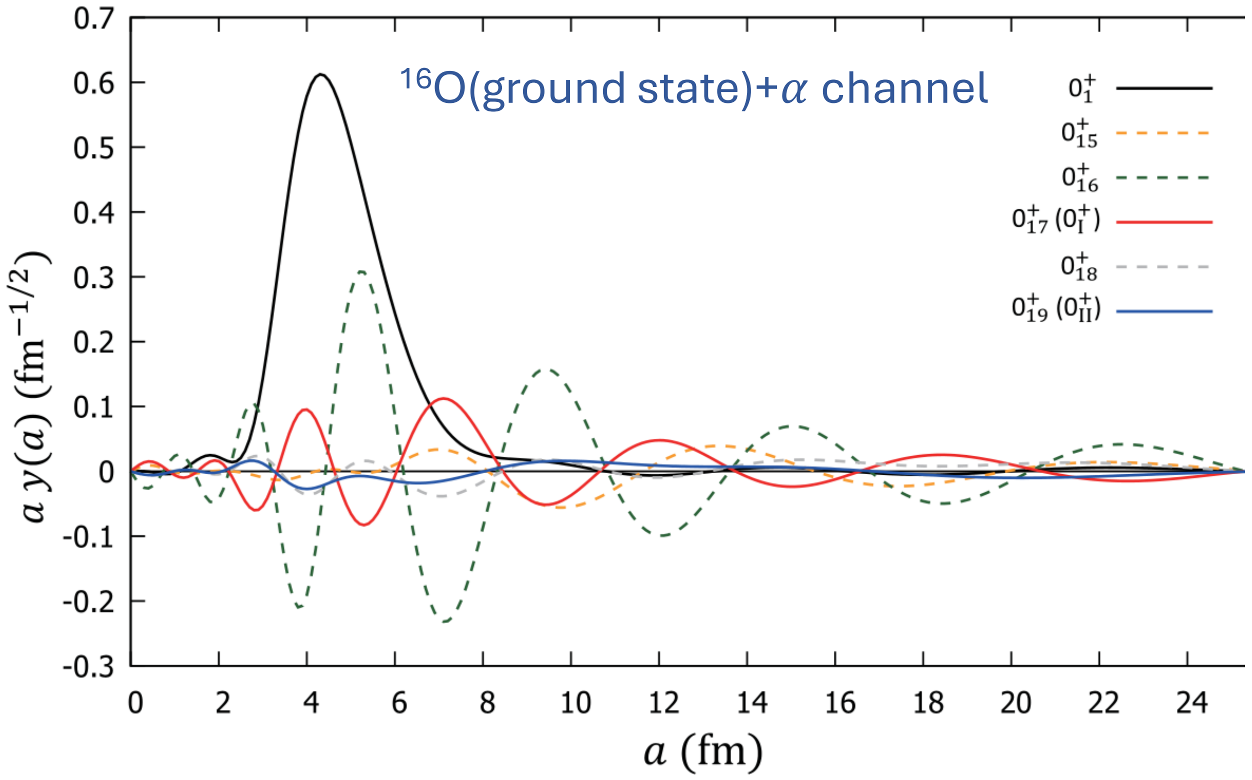


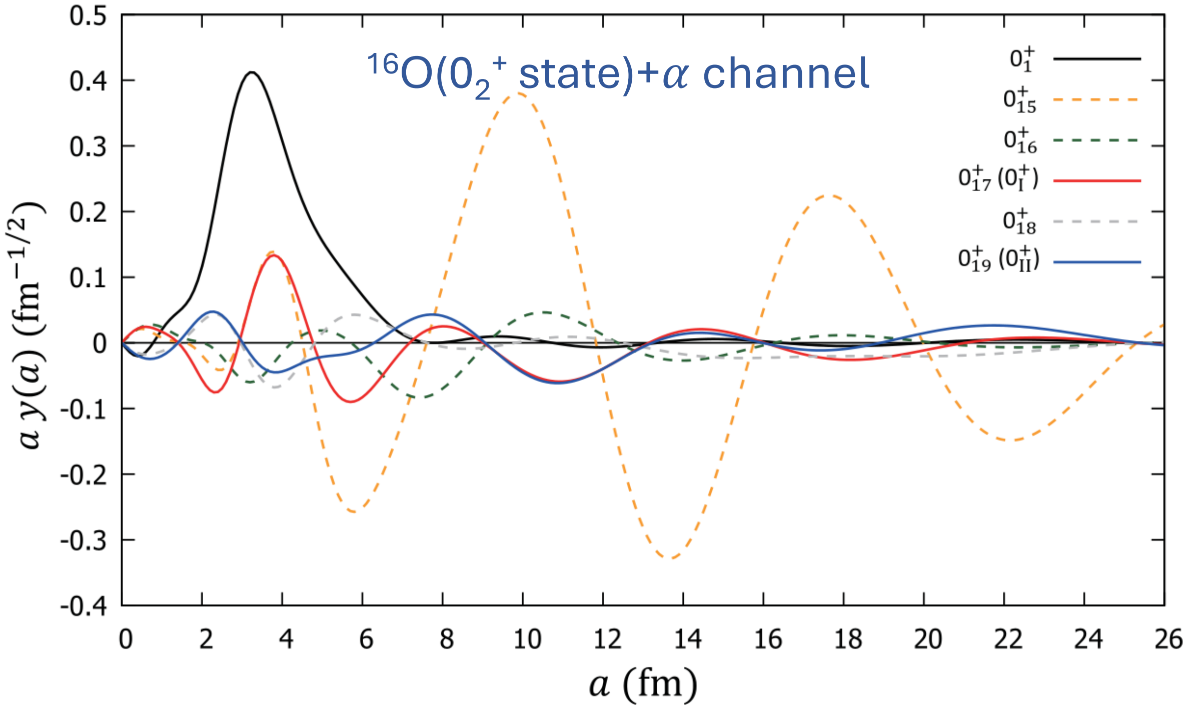
Above the 5alpha threshold: $\mathbf{0^+_{15}} \sim \mathbf{0^+_{19}}$

B. Zhou, Y. Funaki, H. Horiuchi, Y-G Ma, G. Röpke, P. Schuck, A. Tohsaki & T. Yamada, *Nat. Commun.* **14**, 8206 (2023).

Five 0^+ states above the threshold

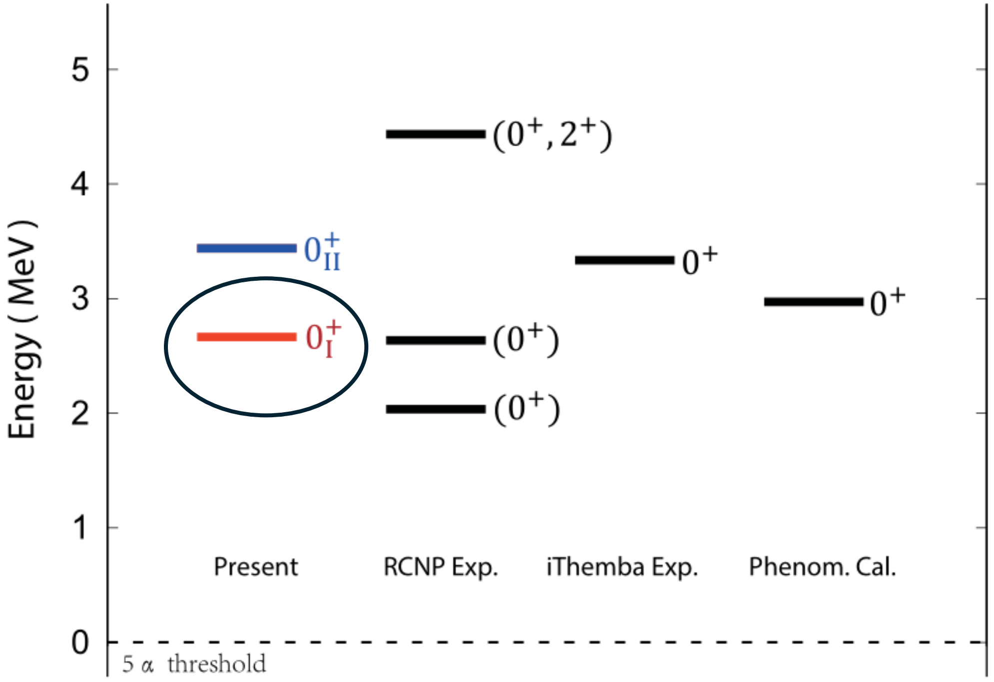
To solve the resonance problem



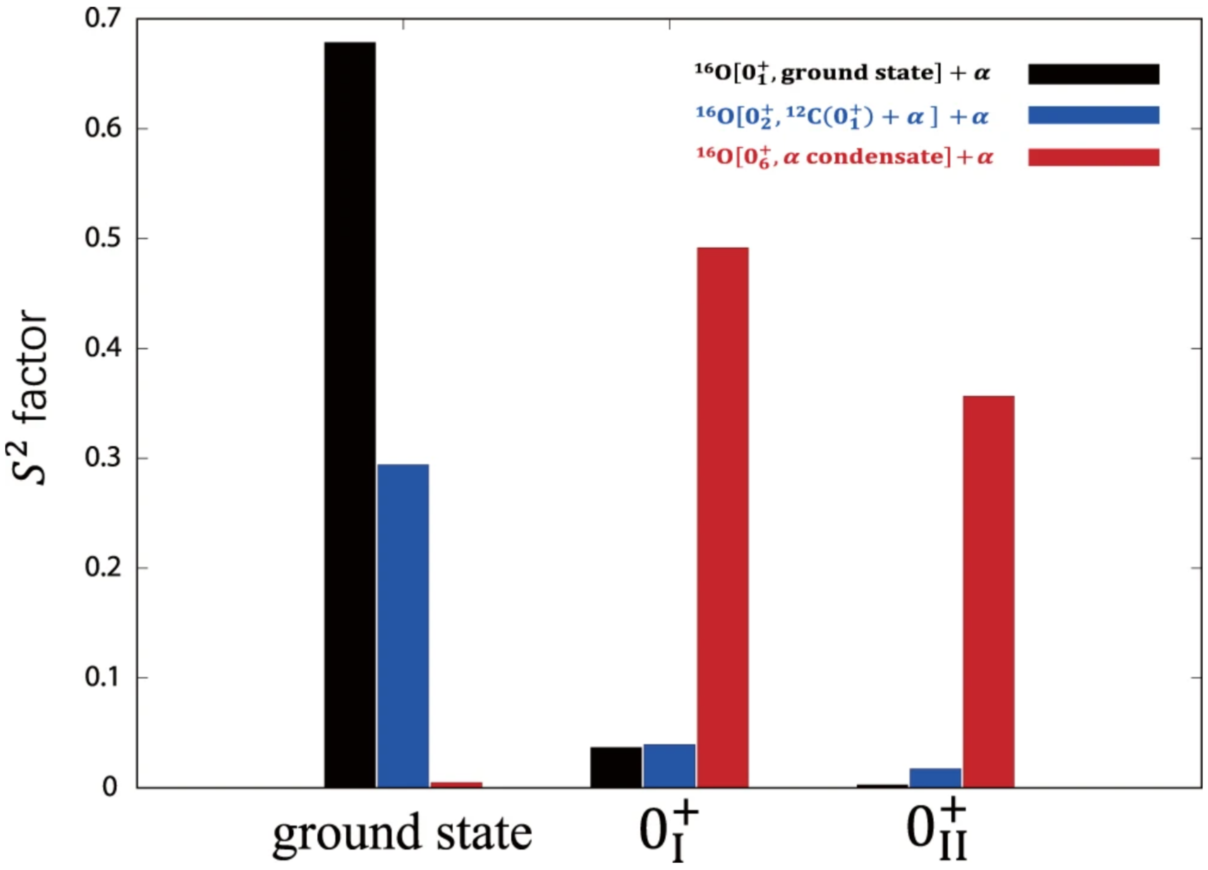


$$y(a) = \sqrt{\frac{20!}{4!16!}} \left\langle \left[[\Psi^{0^+_s}_{\mathrm{gcm}}(^{16}\mathrm{O})\varphi_5(\alpha)]_{0^+} Y_{00}(\hat{\xi}_4) \right]_{0^+} \frac{\delta(\xi_4 - a)}{\xi_4^2} \middle| \Psi^{0^+_\lambda}_{\mathrm{gcm}}(^{20}\mathrm{Ne}) \right\rangle$$

Five 0^+ states above the threshold



Two 0^+ states around 3 MeV are found.

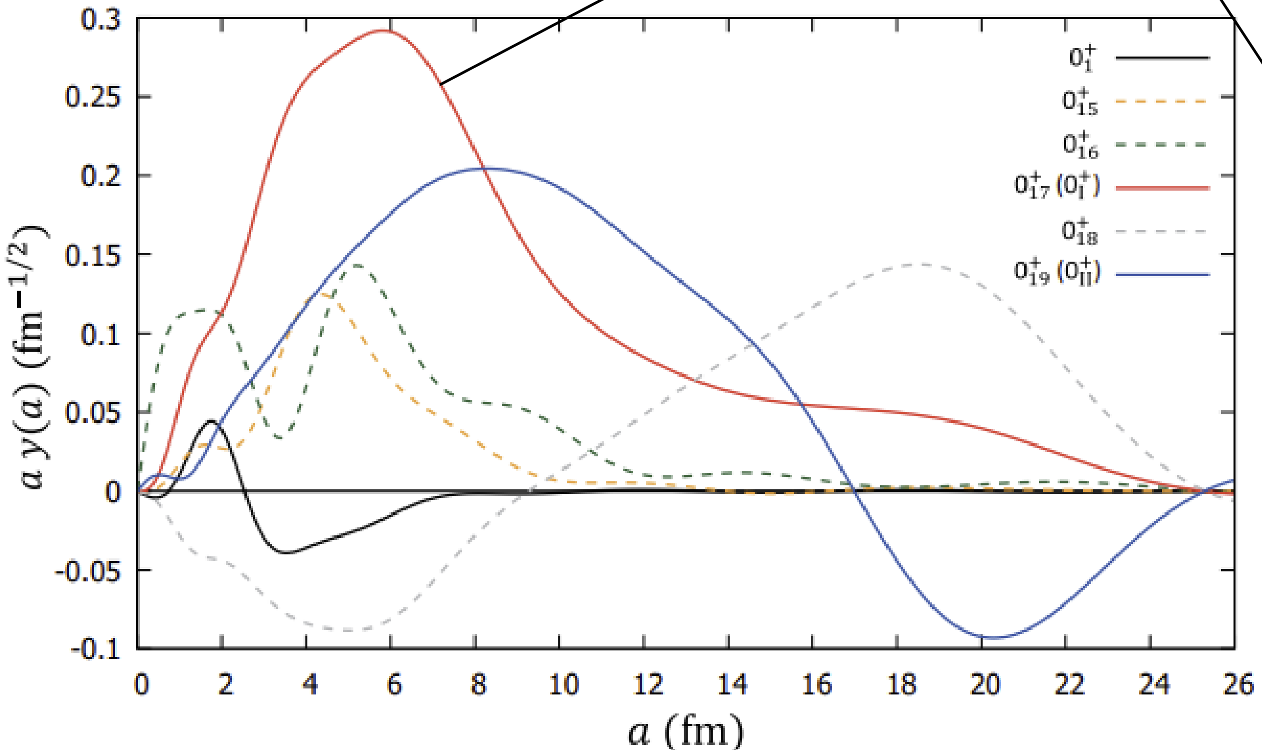


B. Zhou, Y. Funaki, H. Horiuchi, Y-G Ma, G. Röpke, P. Schuck, A. Tohsaki & T. Yamada, *Nat. Commun.* **14**, 8206 (2023).

The 5α condensate state

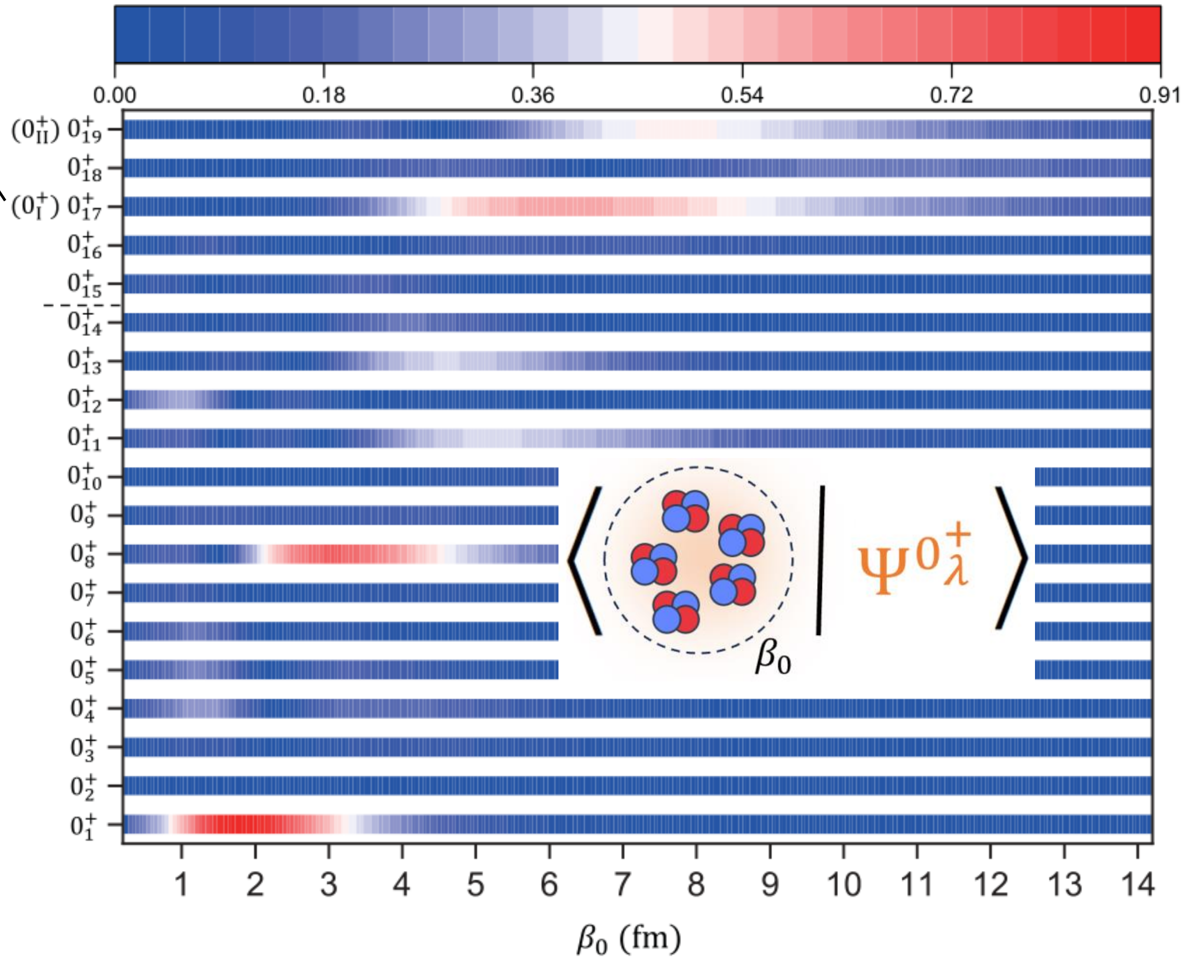
very larger amplitude

Reduced width amplitude



$$y(a) = \sqrt{\frac{20!}{4!16!}} \left\langle \left[\left[\Psi_{\mathrm{gcm}}^{0_s^+}(^{16}\mathrm{O})\varphi_5(\alpha)\right]_{0^+} Y_{00}(\hat{\xi}_4)\right]_{0^+} \frac{\delta(\xi_4 - a)}{\xi_4^2} \middle| \Psi_{\mathrm{gcm}}^{0_\lambda^+}(^{20}\mathrm{Ne})\right\rangle$$

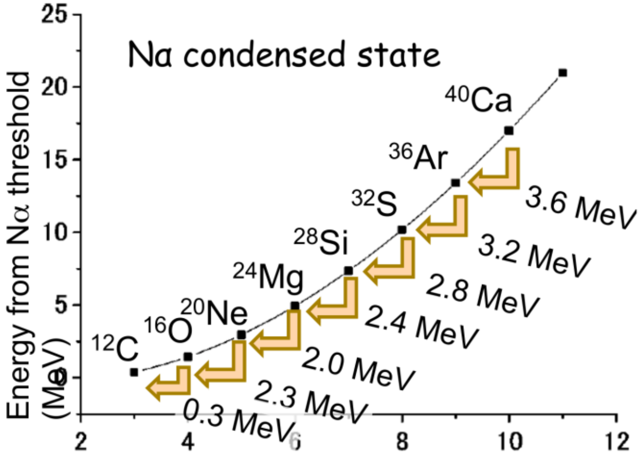
Simple way to confirm condensate state



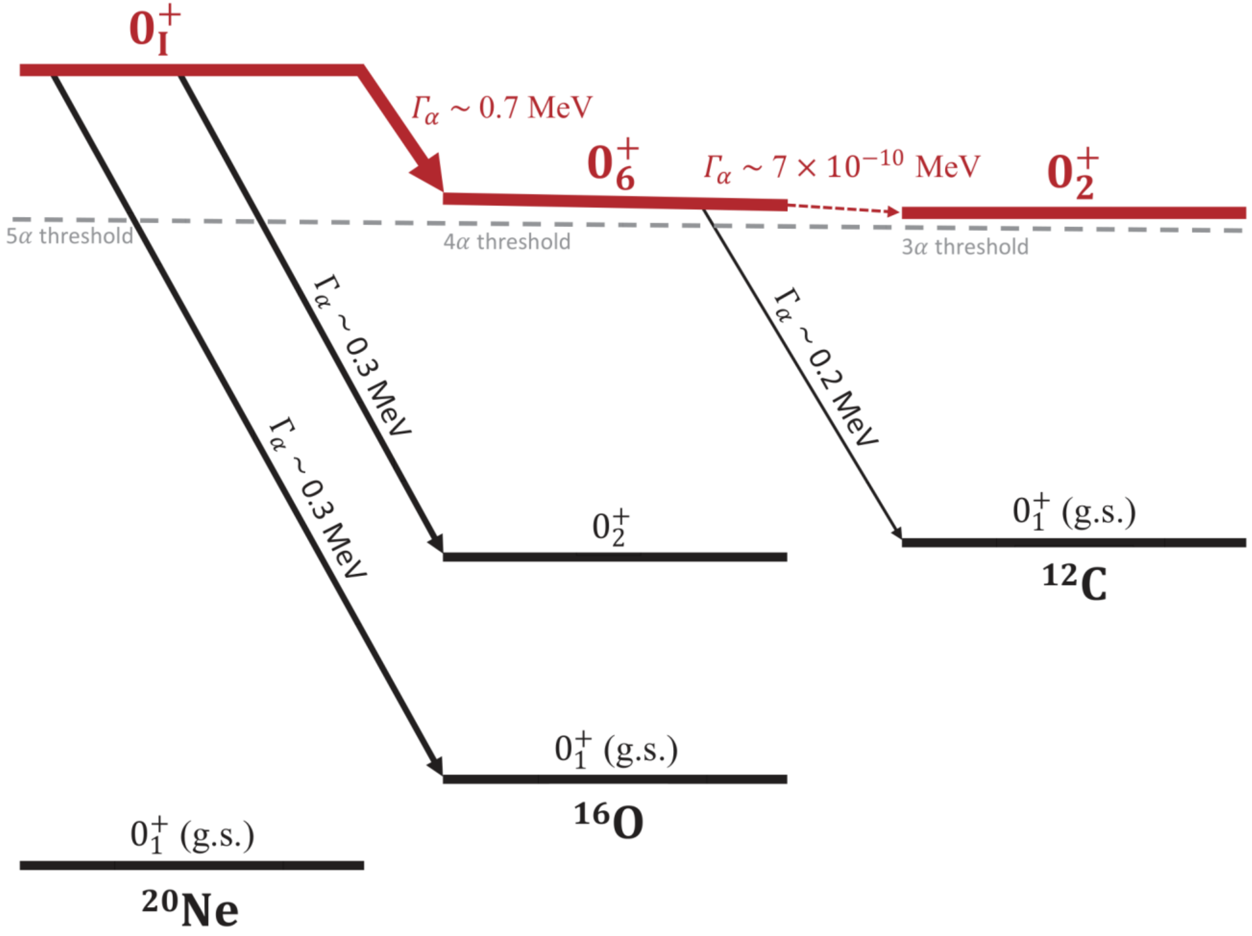
B. Zhou, Y. Funaki, H. Horiuchi, Y-G Ma, G. Röpke, P. Schuck, A. Tohsaki & T. Yamada, *Nat. Commun.* **14**, 8206 (2023).

The decay scheme and connections

Exotic clustering structure ?



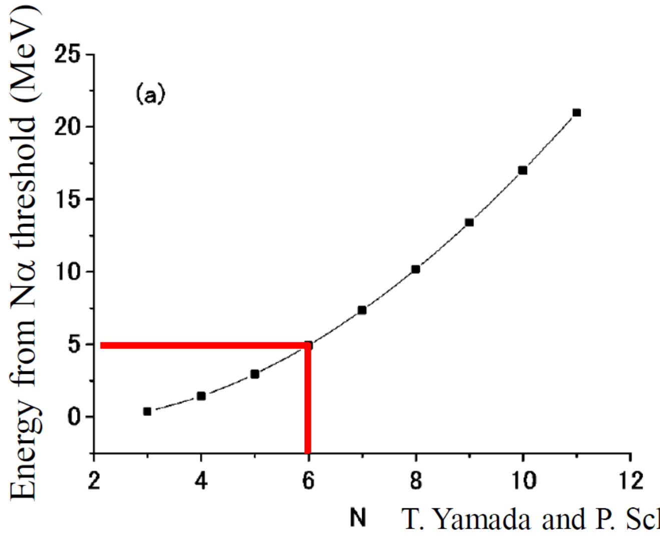
T. Yamada and P. Schuck, PRC 69, 024309 (2004).



B. Zhou, Y. Funaki, H. Horiuchi, Y-G Ma, G. Röpke, P. Schuck, A. Tohsaki & T. Yamada, *Nat. Commun.* **14**, 8206 (2023).

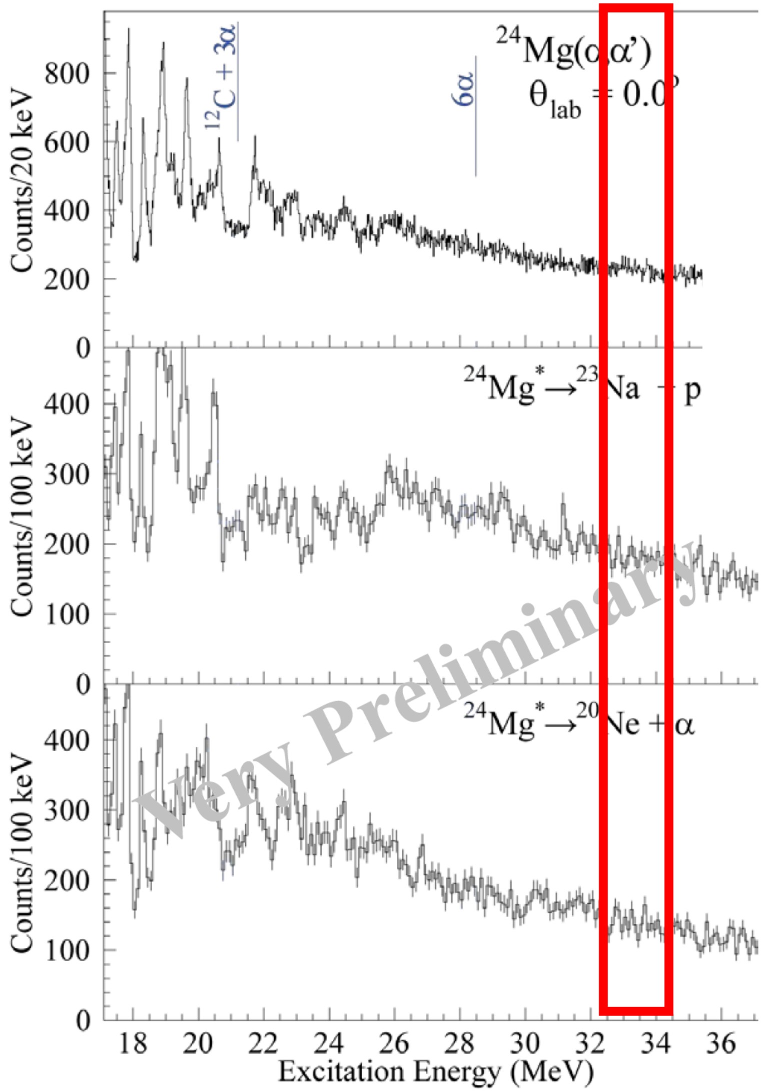
The 6α clustering structure probed by Inelastic Scattering

6α condensed state was searched for in the highly excited region.



T. Yamada and P. Schuck, Phys. Rev. C **69**, 024309 (2004).

- 6α condensed state is expected
  at 5 MeV above the 6α threshold.
  - $E_x \sim 28.5 + 5 = 33.5$ MeV
- No significant structure suggesting the 6α condensed state.
  - Several small structures indistinguishable from the statistical fluctuation. → Need more statistics.



PLB,848 (2024)

A. Tohsaki et al. / Nuclear Physics A738 (2004) 259–263 261

Table 1
The independent number of permutations for each kernel. Here, the case of the norm kernel for ^{24}Mg is added. The final row shows a full number of permuations without any reduction for the norm kernel.

	^{8}Be(2α)	^{12}C(3α)	^{16}O(4α)	^{20}Ne(5α)	^{24}Mg(6α)
norm	3	9	35	185	1614
kinetic	7	34	242	2546	
two-body	9	58	669	10912	
three-body	40	366	6773	156617	
$(n!)^4$	16	1296	3.32×10^5	2.07×10^8	2.79×10^{11}

$$\langle \Psi_{n\alpha}^{\mathrm{THSR}}(\beta)|\mathcal{O}|\Psi_{n\alpha}^{\mathrm{THSR}}(\beta')\rangle = \sum_{p=0}^{m_p^{(1)}-1} W_p^{(1)} I_p^{(1)}$$

Remains challenging in theoretical calculations

PHYSICAL REVIEW LETTERS **131**, 242501 (2023)

Editors' Suggestion **Featured in Physics**

Observation of the Exotic 0_2^+ Cluster State in ^{8}He

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...ai Research Center for ...
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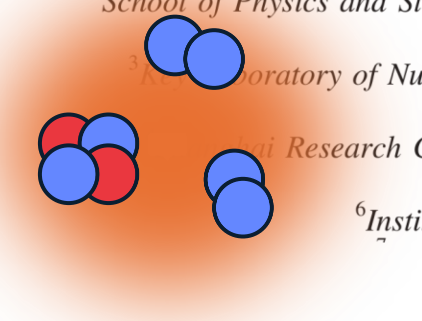
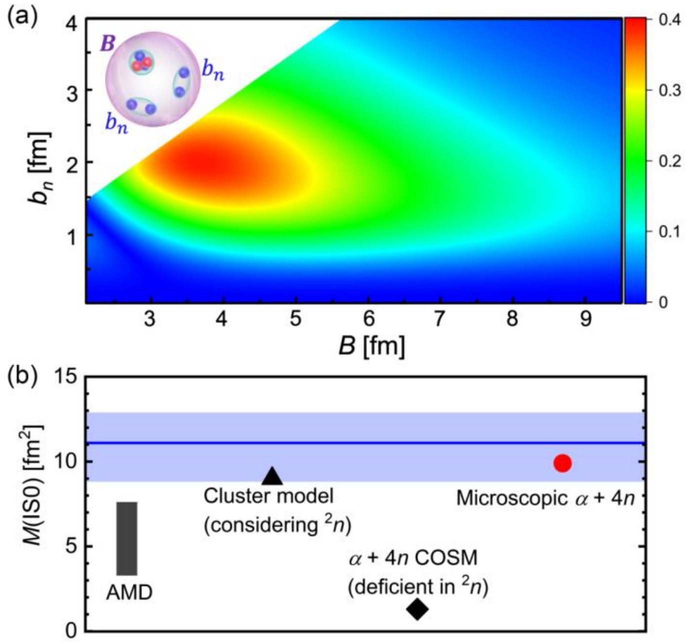


$$\Phi(\boldsymbol{B}, b_n) \propto \mathcal{A}\left\{\exp\left[-\frac{4\xi_1^2}{3B^2} - \frac{3\xi_2^2}{2B^2}\right] \times \phi_\alpha(b_\alpha)\phi_{1^2n}(b_n)\phi_{2^2n}(b_n)\right\},$$



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Letter

New trial wave function for the nuclear cluster structure of nuclei

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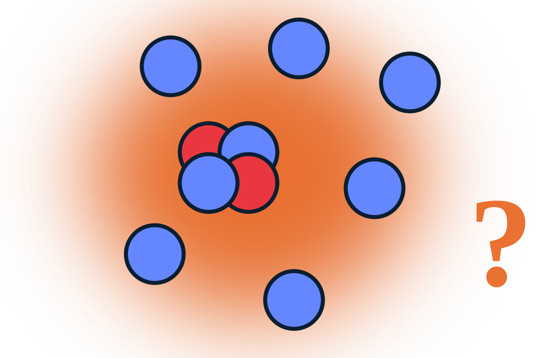
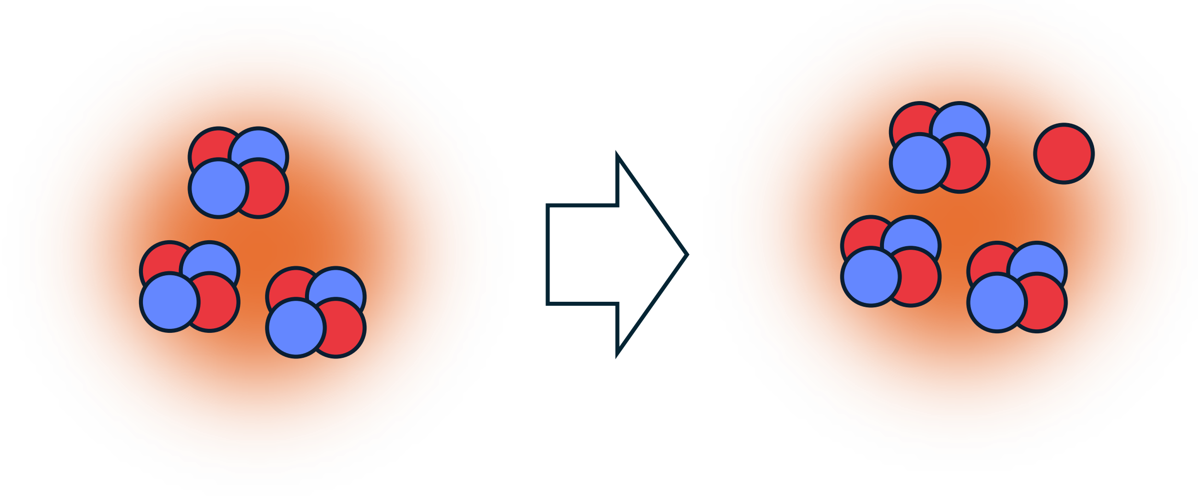
A new trial wave function is proposed for nuclear cluster physics, in which an exact solution to the long-standing center-of-mass problem is given. In the new approach, the widths of the

$$\Psi(\mathbf{r}) = \Phi_{\rm g}(\mathbf{r}_g)\Phi_{\rm int}(\mathbf{r}_i - \mathbf{r}_j)$$

$$\begin{aligned}\Psi_{\rm new} &= \widehat{L}_{n-1}(\beta)\widehat{G}_n(\beta_0)\widehat{D}(\boldsymbol{Z})\Phi_0(\boldsymbol{r}) \\ &= \int d^3\widetilde{T}_1\cdots d^3\widetilde{T}_{n-1}\exp[-\sum_{i=1}^{n-1}\frac{\widetilde{T}_i^2}{\beta_i^2}]\int d^3R_1\cdots d^3R_n\exp[-\sum_{i=1}^{n}(\frac{A_i}{\beta_0^2-2b_i^2})(\boldsymbol{R}_i-\boldsymbol{Z}_i-\boldsymbol{T}_i)^2]\Phi_0(\boldsymbol{r}-\boldsymbol{R}) \\ &= n_0\exp[-\frac{A}{\beta_0^2}X_g^2]\,\mathcal{A}\{\prod_{i=1}^{n-1}\exp[-\frac{1}{2B_i^2}(\boldsymbol{\xi}_i-\boldsymbol{S}_i)^2]\prod_{i=1}^{n}\phi_i^{\rm int}(b_i)\}.\end{aligned}$$

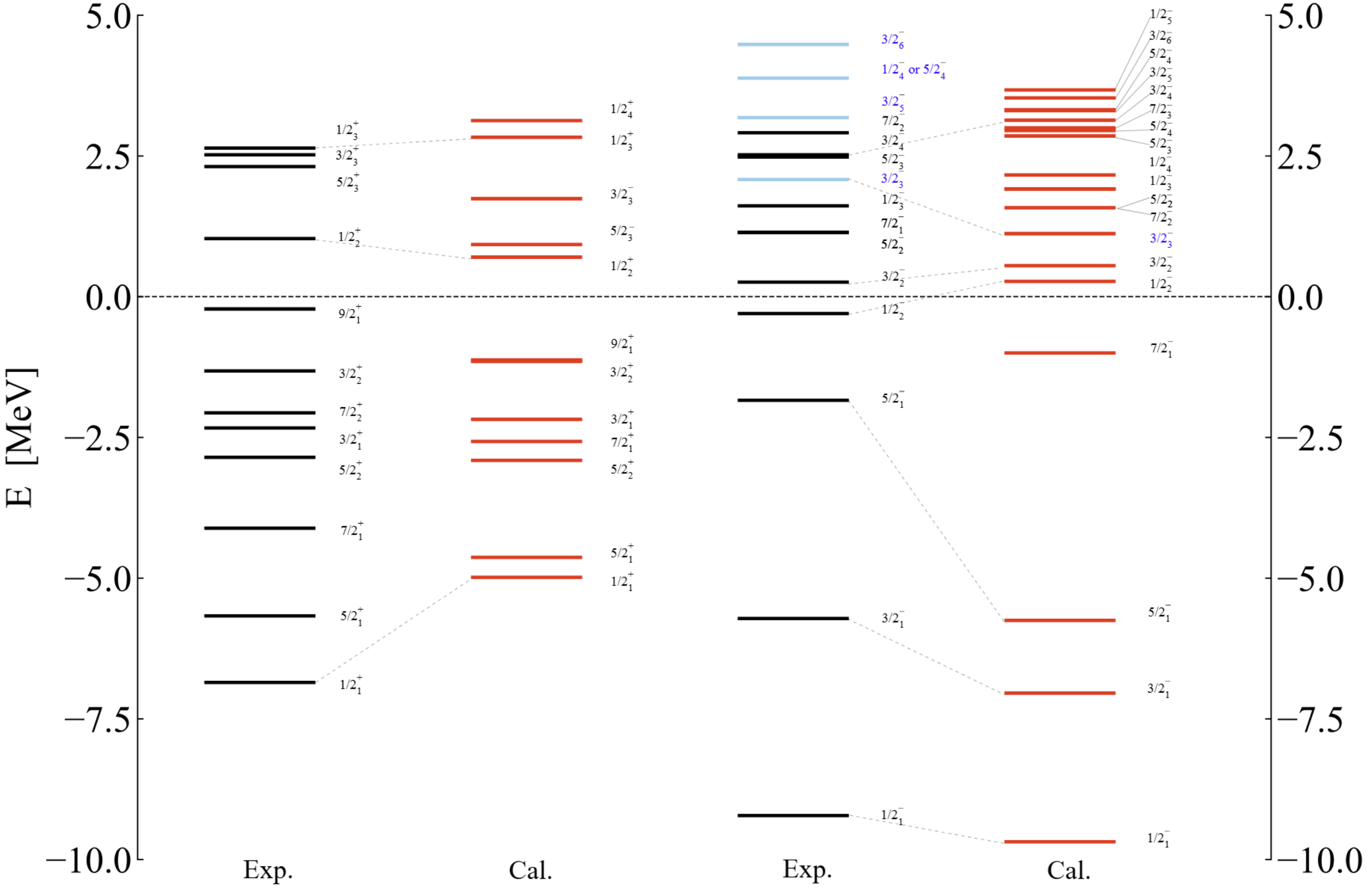


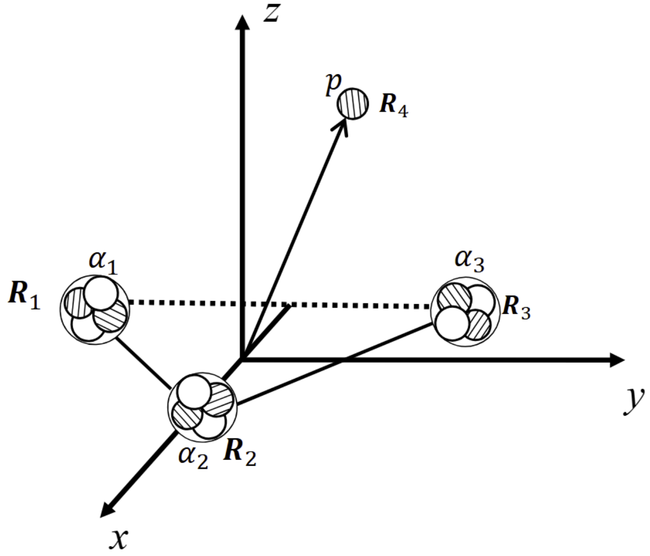
a tool for studying the cluster correlations



Clustering structure of $3\alpha+p$ in ^{13}N

Hoyle-analog state in ^{13}N





Transition	Present	Experiment
$B(E1, 3/2_1^- \to 1/2_1^+)$	0.016	0.036
$B(E1, 1/2_1^+ \to 1/2_1^-)$	0.0007	0.036 ± 0.004
$B(E2, 3/2_1^- \to 1/2_1^-)$	4.87	
$B(E2, 5/2_1^- \to 1/2_1^-)$	3.71	
$B(E2, 1/2_1^- \to 3/2_1^+)$	21.58	

to be submitted

Hoyle-analog state in ^{13}N

PHYSICAL REVIEW C **109**, 054308 (2024)

Cluster structure of $3\alpha + p$ states in ^{13}N

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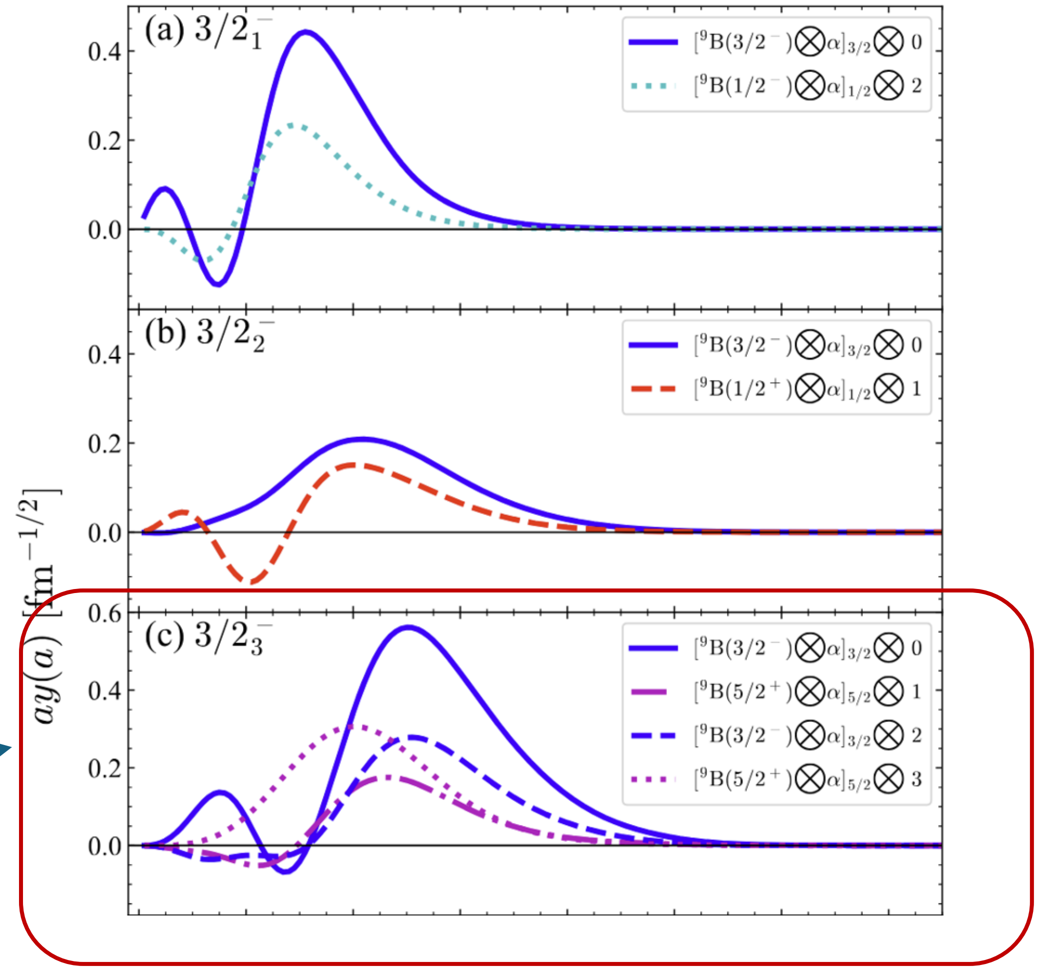
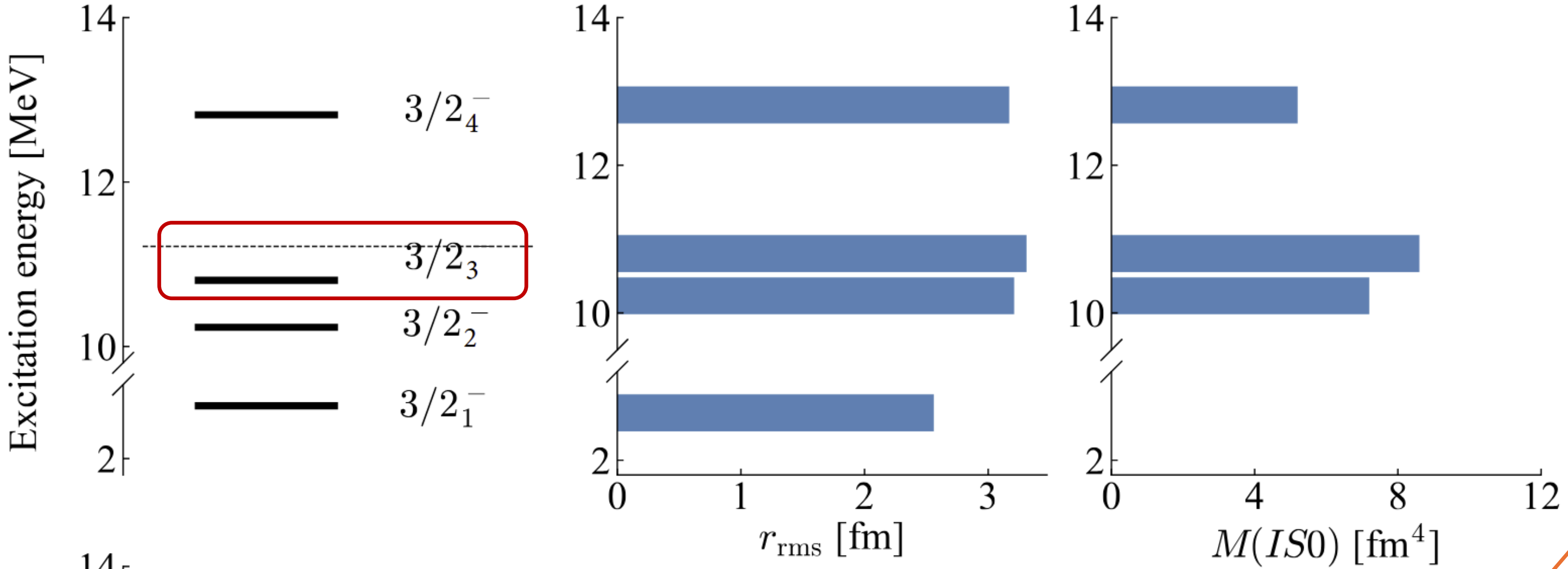
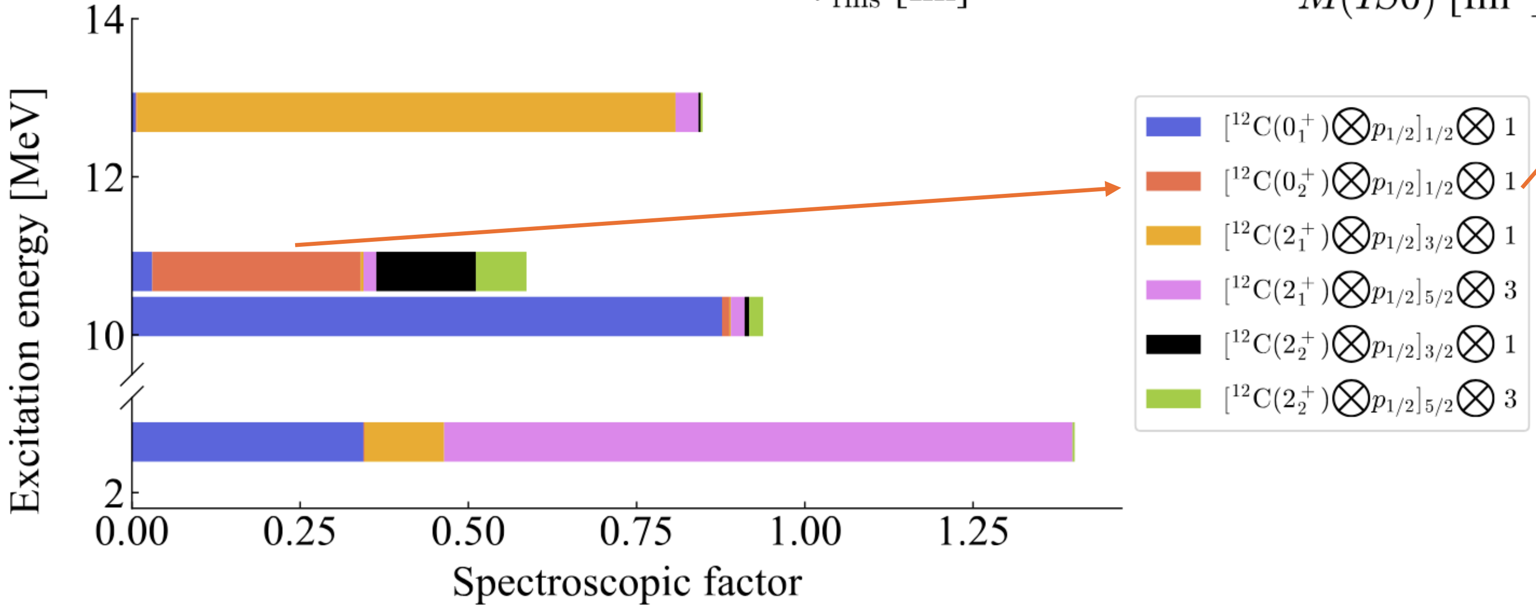
Background: Cluster states in ^{13}N are extremely difficult to measure due to the unavailability of ^{9}B $+\alpha$ elastic-scattering data.
Purpose: Using β-delayed charged-particle spectroscopy of ^{13}O, clustered states in ^{13}N can be populated and measured in the $3\alpha + p$ decay channel.
Methods: One-at-a-time implantation and decay of ^{13}O was performed with the Texas Active Target Time Projection Chamber. 149 $\beta 3\alpha p$ decay events were observed and the excitation function in ^{13}N reconstructed.
Results: Four previously unknown α-decaying excited states were observed in ^{13}N at an excitation energy of 11.3, 12.4, 13.1, and 13.7 MeV decaying via the $3\alpha + p$ channel.
Conclusions: These states are seen to have a $[^{9}\mathrm{B(g.s)} \otimes \alpha / p + {}^{12}\mathrm{C}(0_2^+)]$, $[^{9}\mathrm{B}(\frac{1}{2}^+) \otimes \alpha]$, $[^{9}\mathrm{B}(\frac{5}{2}^+) \otimes \alpha]$, and $[^{9}\mathrm{B}(\frac{5}{2}^+) \otimes \alpha]$ structure, respectively. A previously seen state at 11.8 MeV was also determined to have a $[p + {}^{12}\mathrm{C(g.s.)} / p + {}^{12}\mathrm{C}(0_2^+)]$ structure. The overall magnitude of the clustering is not able to be extracted, however, due to the lack of a total width measurement. Clustered states in ^{13}N (with unknown magnitude) seem to persist from the addition of a proton to the highly α-clustered ^{12}C. Evidence of the $\frac{1}{2}^+$ state in ^{9}B was also seen to be populated by decays from ^{13}N*.

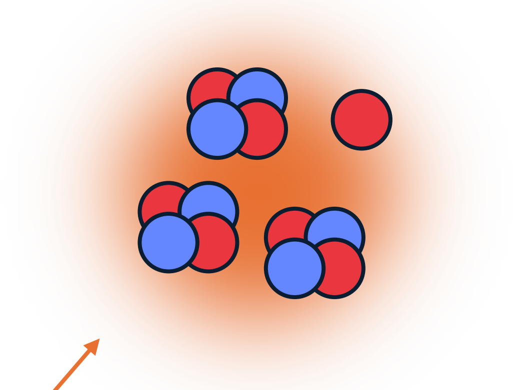


This obtained state corresponds to the state observed at 11.3 MeV

Hoyle-analog state in ^{13}N





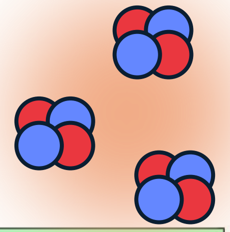


$\langle \mathrm{Hoyle} + p | {}^{13}\mathrm{N\ state} \rangle$

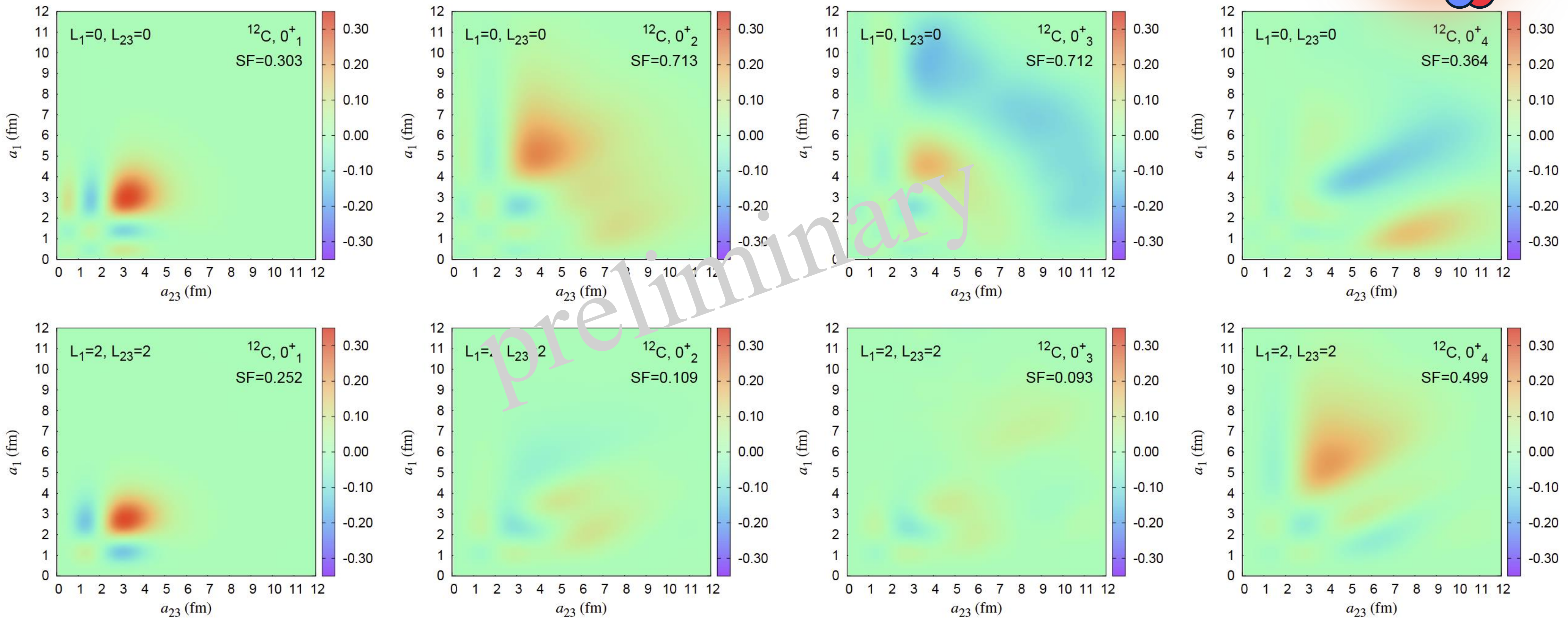
$$y^{J\pi}_{j_1\pi_1 j_2\pi_2 j_{12} l}(a) = \sqrt{\frac{A!}{(1+\delta_{C_1C_2})\, C_1! C_2!}} \times \left\langle \frac{\delta(r-a)}{r^2} \left[Y_l(\hat{r}) \left[\Phi^{j_1\pi_1}_{C_1} \Phi^{j_2\pi_2}_{C_2} \right]_{j_{12}} \right]_{JM} \middle| \Psi^{J\pi}_M \right\rangle$$

to be submitted

Two-body overlap function (Two-body RWA)



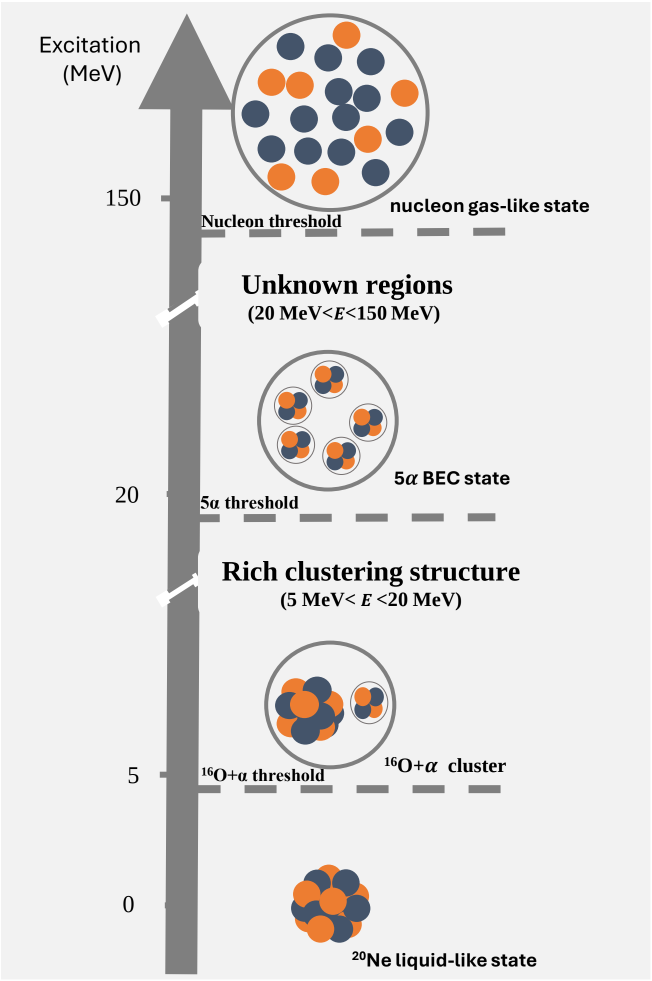
$[\alpha \otimes [\alpha \otimes \alpha]_0]_0 \otimes [0 \otimes 0]_0$



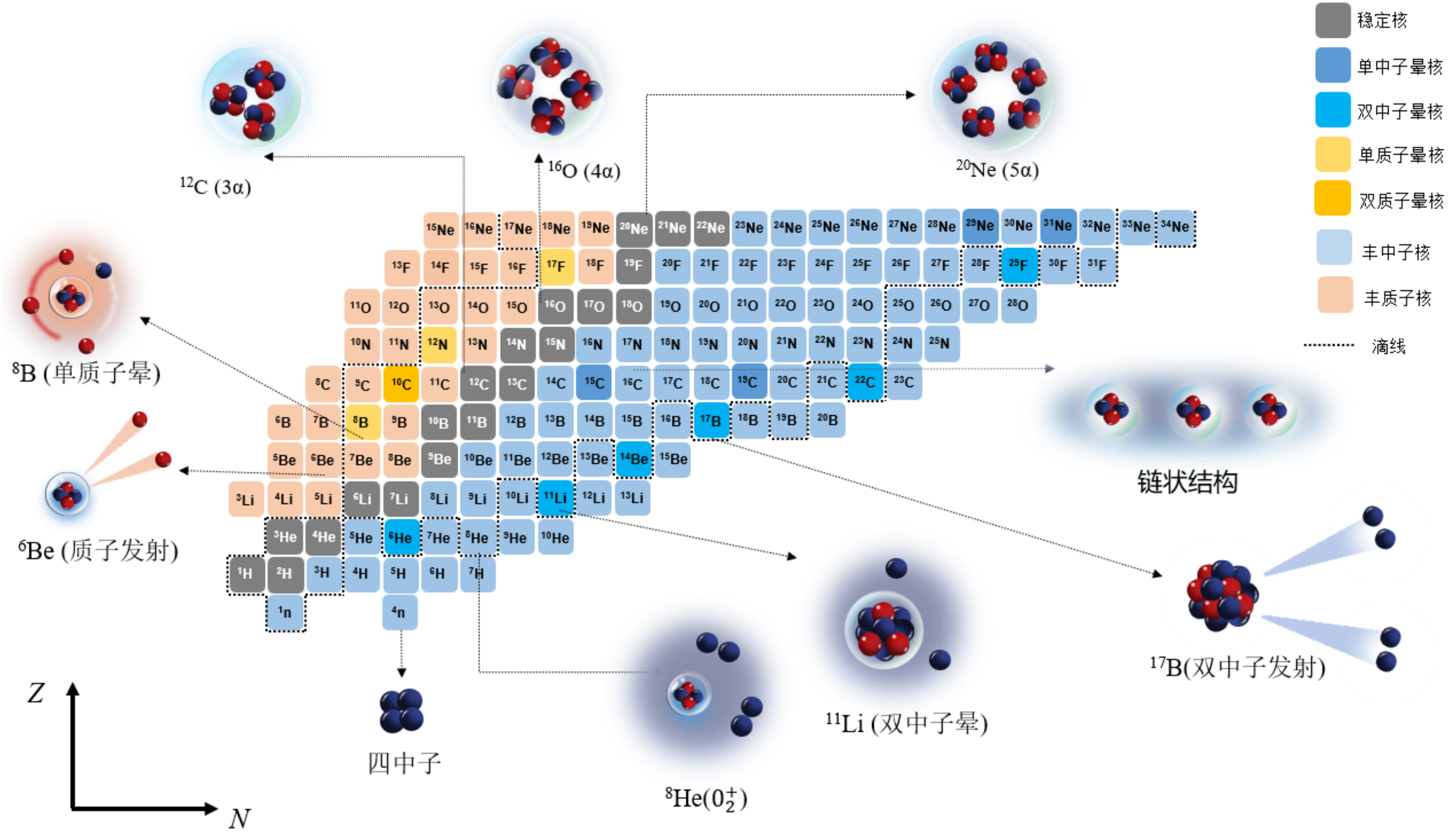
$$\mathcal{Y}_c^{J\pi}(a_1,a_2)=\sqrt{\frac{A!}{C_1!C_2!C_3!}}\left\langle \frac{\delta(r_1-a_1)\delta(r_2-a_2)}{r_1^2 r_2^2}\left[[Y_{l_1}(\hat{r}_1)\otimes Y_{l_2}(\hat{r}_2)]_L\otimes\left[\Phi_{C_1}^{j_1\pi_1}\otimes\left[\Phi_{C_2}^{j_2\pi_2}\otimes\Phi_{C_3}^{j_3\pi_3}\right]_{j_{23}}\right]_{j_{123}}\right]_{JM}\middle|\Psi_M^{J\pi}\right\rangle$$

Summary and Prospect

rich clustering structure



evolution of structure of ^{20}Ne



explore novel clustering structure of light nuclei

Thanks for my collaborators

and your attentions.