Kinematical higher-twist contributions in two-photon reactions and hadron gravitational form factors

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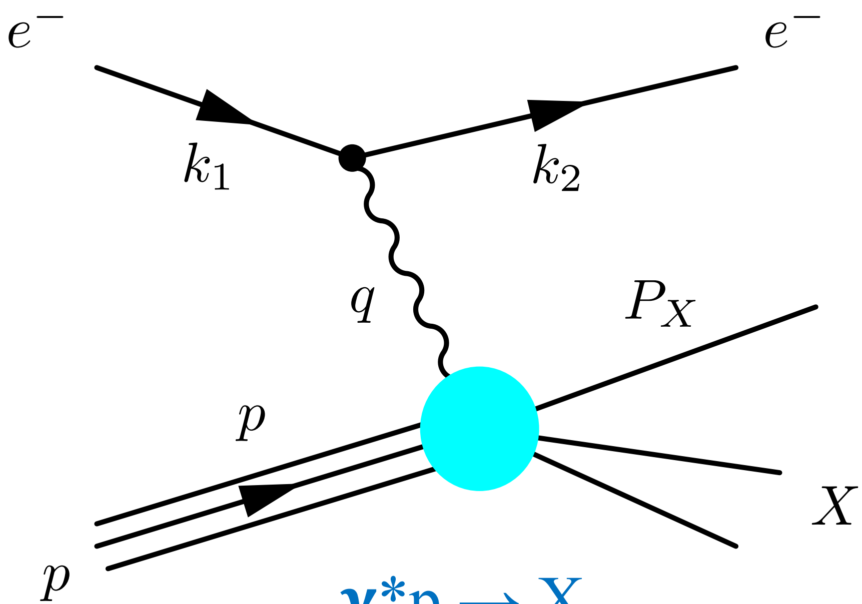
2024年5月24日，强子物理在线论坛

References:
C. Lorce, B. Pire and Qin-Tao Song, Phys. Rev. D106 (2022), 094030.
B. Pire and Qin-Tao Song, Phys. Rev. D 107 (2023), 114014.
B. Pire and Qin-Tao Song, Phys. Rev. D 109 (2024), 074016

Outline

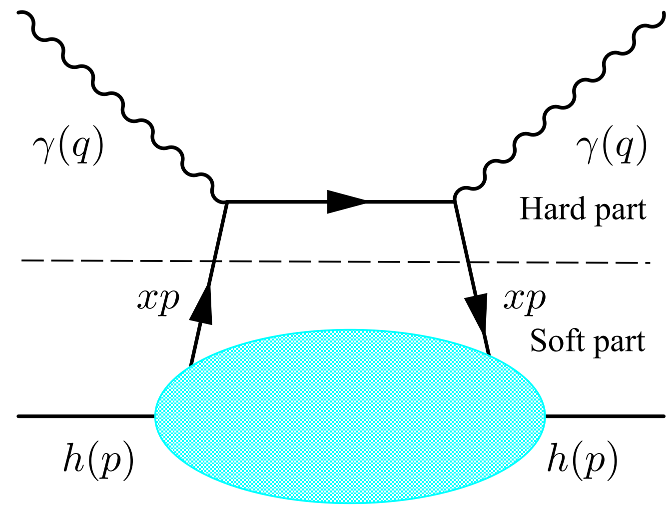
- PDFs, GPDs and GDAs of hadrons
- Gravitational (EMT) FFs of hadrons
- Kinematical higher-twist corrections
- Exotic hybrid mesons and shear viscosity term (a new gravitational FF) at BESIII and Belle II

Parton distribution function(部分子函数)



$\gamma^*p \rightarrow X$

Deep Inelastic Scattering
(DIS，深度非弹性散射)

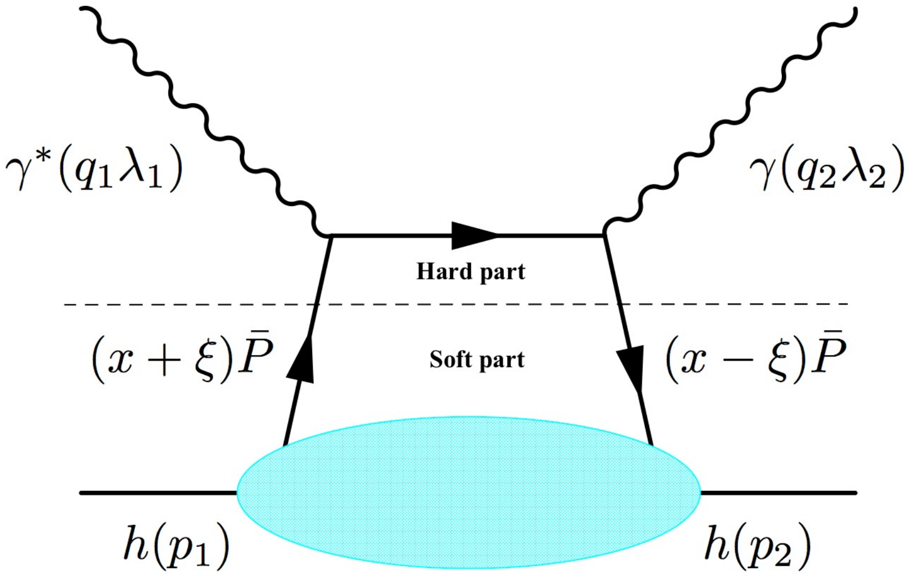


Hadron tensor (强子张量, amplitude squared) of DIS at leading order

Unpolarized PDF: $f(x) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} \mathrm{e}^{-\mathrm{i}xp^+z^-} \langle \mathrm{h}(p)|\bar{q}(z^-)\gamma^+ q(0)|\mathrm{h}(p)\rangle$

It describes the probability of finding a parton with momentum xp in the hadron.

Generalized Parton distributions (GPDs，广义部分子函数)



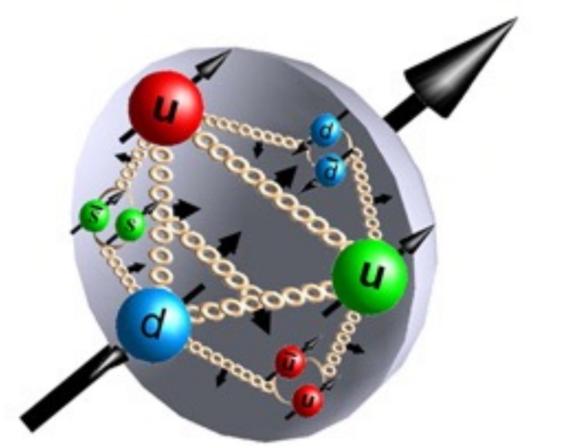
$\gamma^*h \rightarrow \gamma\ h$

Deeply Virtual Compton Scattering (DVCS，深度虚康普顿散射)

Spin puzzle of proton:

$$\Delta u^+ + \Delta d^+ + \Delta s^+ \approx 0.3$$

$$\Delta g + \Delta L \neq 0$$



Generalized Parton Distributions (GPDs) provide information on ΔL to solve the proton spin puzzle!

X.D. Ji, PRL 78(1997), 610.

Generalized Parton distributions (GPDs)

GPDs

- Proton spin puzzle
- Energy momentum tensor (EMT) form factors of hadrons
- mass radius, mass distribution, pressure distribution and shear force distribution

Interesting, little known

↓

Recent Reviews:

M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) no.26, 1830025.

V. D. Burkert, L. Elouadrhiri, F. Girod, C. Lorce, P. Schweitzer and P. Shanahan, Rev. Mod. Phys. 95 (2023), 041002.

EMT form factors and mass radius of pions?

The GPDs of pions cannot be accessed by DVCS, since there is currently no such a facility. $\gamma^* + \pi \to \gamma + \pi$

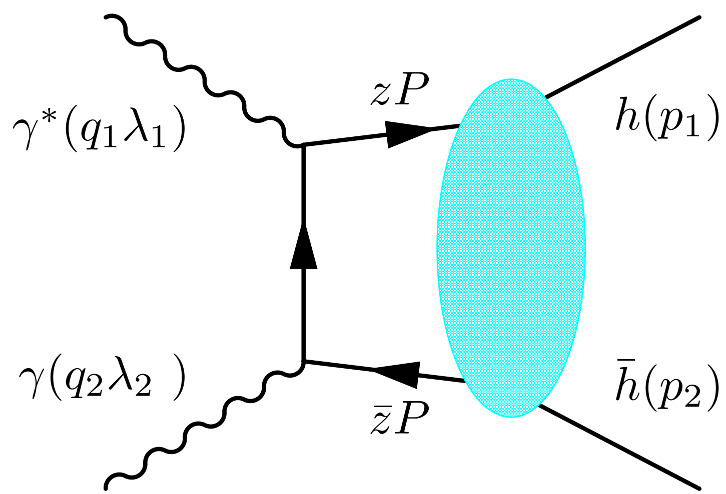
How to obtain EMT form factors of pions?

Option 1: Model calculations of EMT form factors.

Option 2: EMT form factors can be obtained from generalized distribution amplitudes (GDAs) of pions

M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003.
S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

GDAs in $\gamma^* + \gamma \to h + \bar{h}$:



Hard part: $\gamma^* + \gamma \to q + \bar{q}$
Soft part: $q + \bar{q} \to h + \bar{h}$, GDAs.

Quark GDA of a scalar meson is defined as:

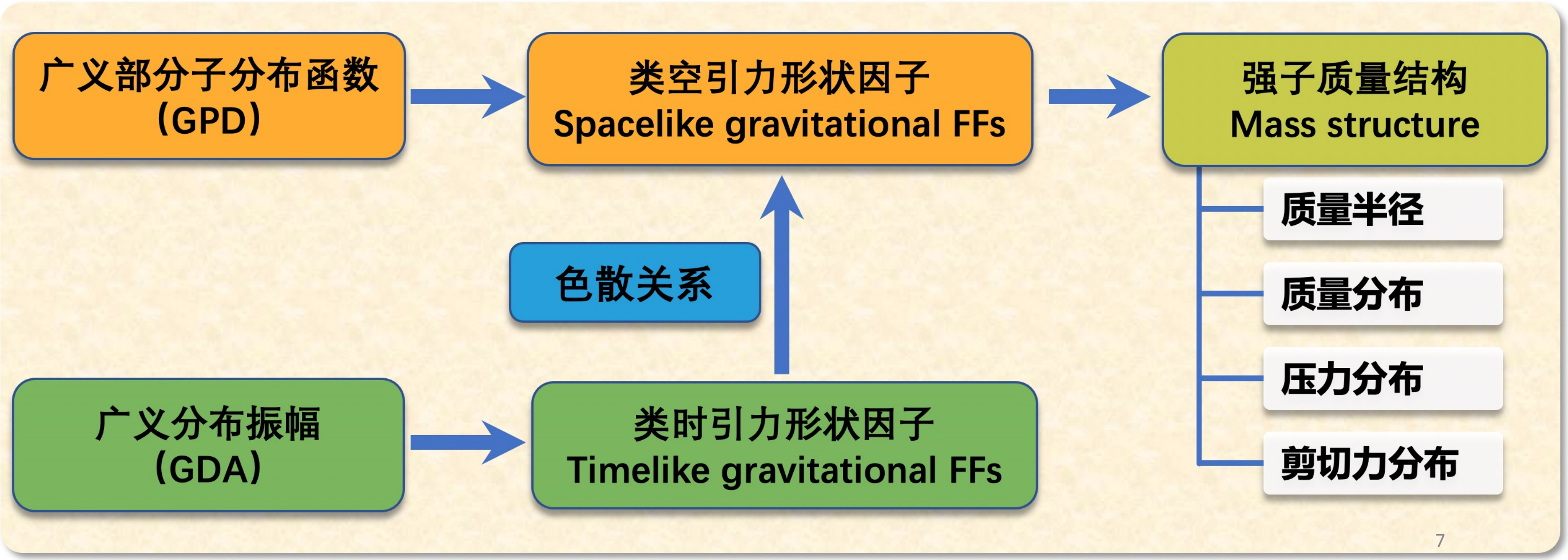
$$\Phi(z, \cos\theta, s) = \int \frac{dx^-}{2\pi} e^{-iP^+x^-} \langle h(p)\bar{h}(p') | \bar{q}(x^-)\gamma^+ q(0) | 0 \rangle$$

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL **81** (1998) 1782.
M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.
M. V. Polyakov, NPB **555** (1999) 231.

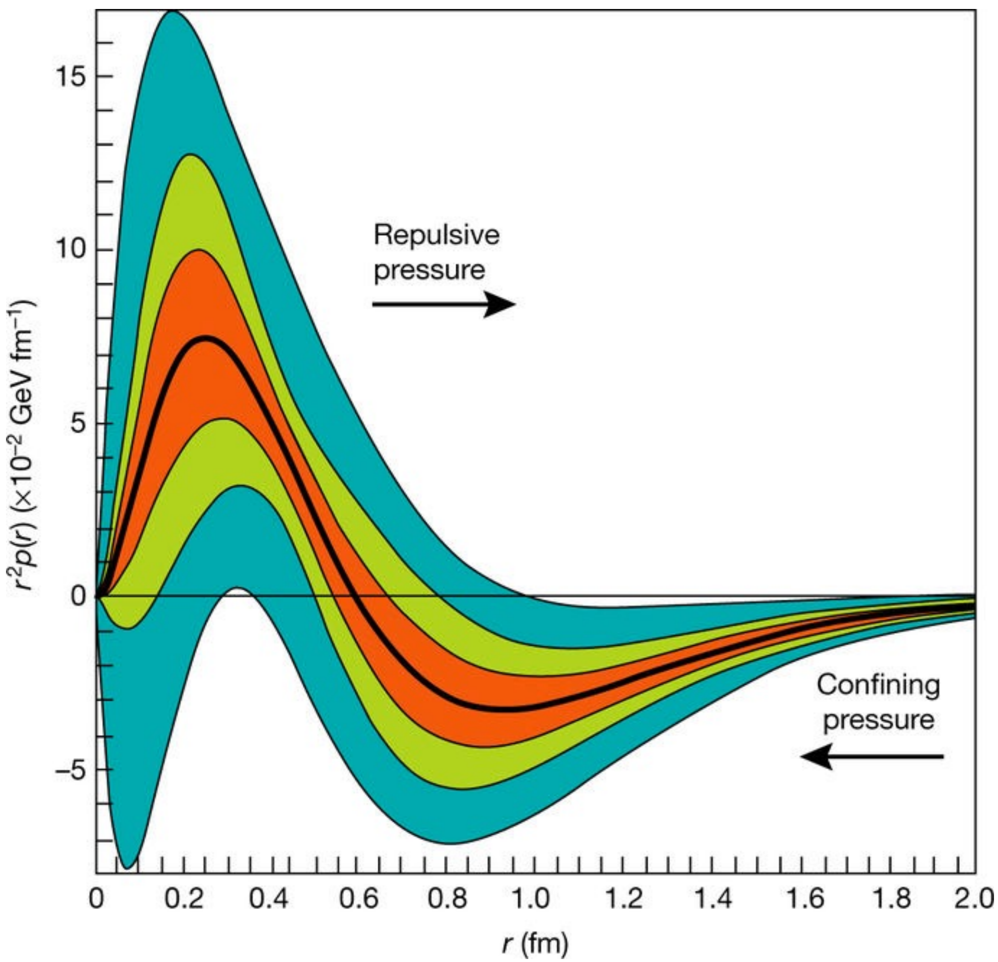
GDAs are also important inputs for decays of B mesons.

W. F. Wang, H. N. Li, W. Wang and C. D. Lu, PRD 91 (2015), 094024.
Y. Li, A. J. Ma, W. F. Wang and Z. J. Xiao, PRD 95 (2017), 056008.
M. K. Jia, C. Q. Zhang, J. M. Li and Z. Rui, PRD 104 (2021), 073001.

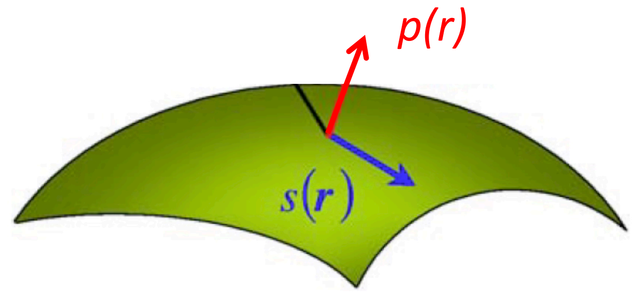
From GPDs and GDAs to hadron gravitational FFs:



First measurement of pressure distribution in proton



Pressure distribution p(r) of the proton



Pressure distribution p(r) and shear force (剪切力) distribution can be obtained from gravitational FFs.

DVCS meaments: **γ***+proton → **γ**+ proton at JLAB

V. D. Burkert, L. Elouadrhiri and F. X. Girod, Nature 557 (2018) 7705，396.

Other recent papers on EMT form factors (incomplete)

Pressure distribution of proton:

Krešimir Kumerički, Nature 570 (2019) 7759, E1-E2.
C. Lorce, H. Moutarde and A. P. Trawinski, EPJC 79 (2019), 89.
P. E. Shanahan and W. Detmold, PRL 122 (2019), 072003.

EMT FFs of proton:

D. Chakrabarti *et.al*, PRD 102 (2020), 113011.
X. B. Tong, J. P. Ma and F. Yuan, JHEP 10 (2022), 046.
O. V. Selyugin and O. V. Teryaev, PRD 79 (2009), 033003.
B. Duran et al., Nature **615**, 813–816 (2023).

Mass radius of proton:

R. Wang, W. Kou, C. Han, J. Evslin and X. Chen, PRD 104 (2021), 074033.
X. Y. Wang, F. Zeng and Q. Wang, PRD 105 (2022), 096033.

EMT FFs of pion meson:

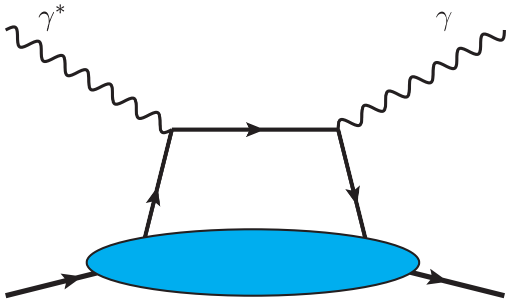
S. Kumano, Qin-Tao Song and O. Teryaev, PRD 97 (2018), 014020.
A. Freese and I. C. Cloet, PRC 100 (2019), 015201.
J. L. Zhang *et. al*., PLB 815 (2021), 136158.

EMT FFs of rho meson:

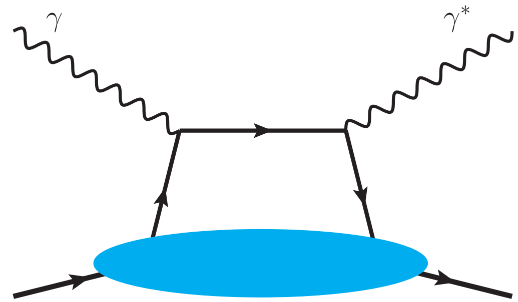
B. D. Sun and Y. B. Dong, PRD 101 (2020), 096008.

GPDs and GDAs are measured in two-photon reactions

GPDs:

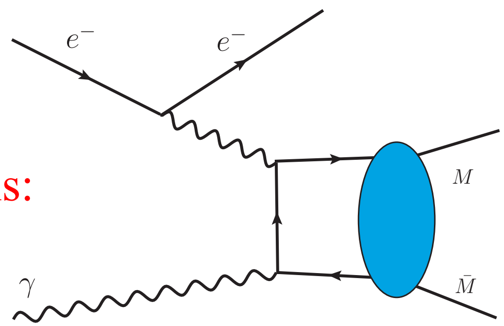


DVCS@JLAB, Compass,
EIC-US, EicC



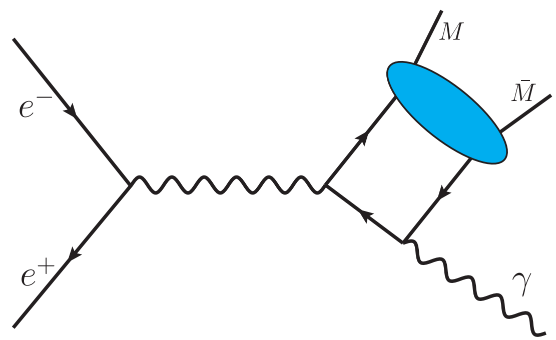
Timelike Compton scattering(TCS)
@JLAB, EIC-US, EicC
First measurement of TCS: PRL 127 (2021), 262501

GDAs:



$e^- + \gamma \to e^- + \mathrm{M} + \bar{\mathrm{M}}$
@Belle and Belle II

M. Masuda et al. [Belle], PRD 93 (2016), 032003.
M. Masuda et al. [Belle], PRD 97 (2018), 052003.

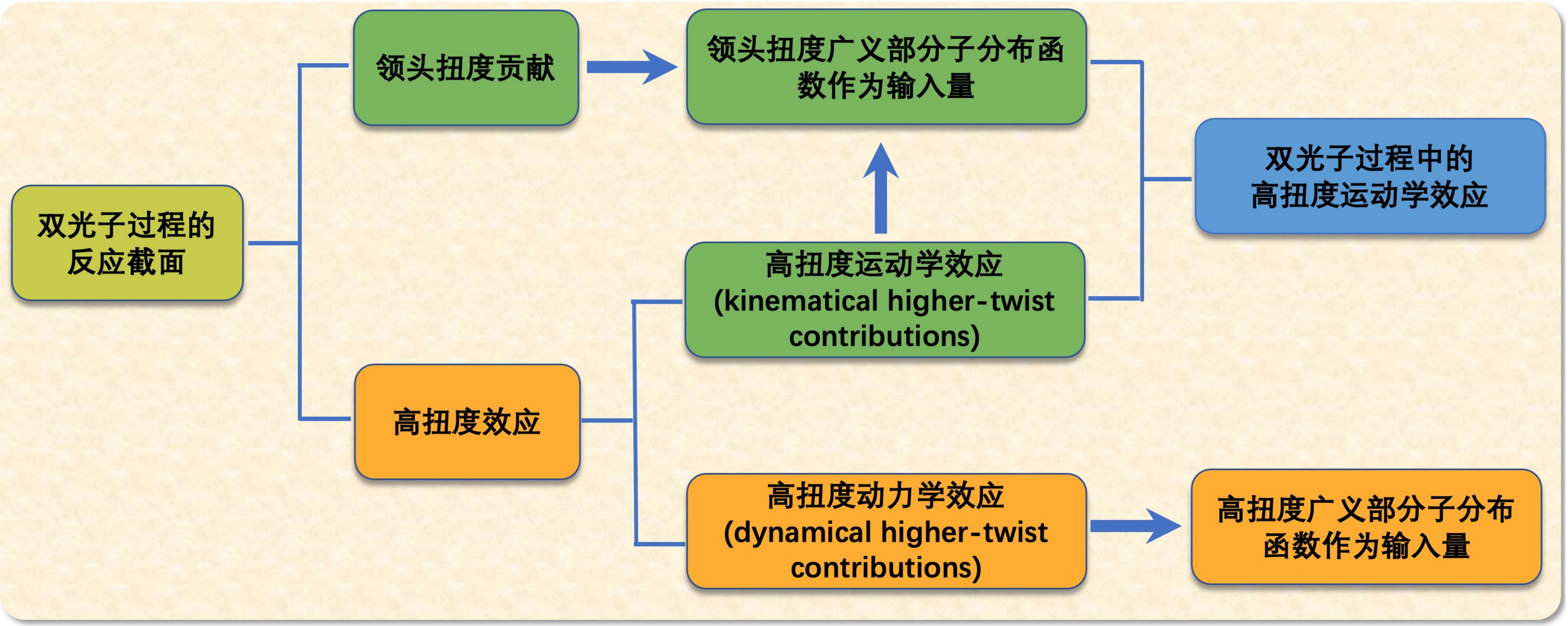


$\mathrm{e}^+\mathrm{e}^- \to \gamma^* \to \mathrm{M} + \bar{\mathrm{M}} + \gamma$@BESIII
@STCF

Higher-twist contributions of order s/Q^2 and m^2/Q^2 are important in the measurements.
Kinematical higher-twist corrections of the above reactions are discussed in this talk.

Kinematical higher-twist contributions

DVCS: $\gamma^* + h \to \gamma + h$



The kinematical higher-twist contributions can give a better description of experimental measurements without introducing genuine higher-twist GPDs.

Higher-order corrections of α_s also introduce new distributions (gluon GPDs).

Kinematical higher-twist contributions in DIS

Kinematical higher-twist contributions in DVCS can be considered as a general case of the target mass corrections in DIS, $\sim m^2/Q^2$.

Target masss corrections were already included when PDFs were extracted.

N. Sato et al. [Jefferson Lab Angular Momentum], PRD 93 (2016), 074005.

The total derivative of the leading twist operators contribute in DVCS.

$$\left[iP^{\mu}, \left[iP_{\mu}, \mathcal{O}^{t=2}\right]\right]$$

$$\left[iP^{\mu}, \frac{\partial}{\partial x^{\mu}}\mathcal{O}^{t=2}\right]$$

$\xrightarrow{\text{DVCS}}$ $\sim t/Q^2, \sim m^2/Q^2$ corrections

Twist-2 operator:

$$\mathcal{O}^{t=2}(z_1 x, z_2 x) = \frac{1}{2}\left[O_V(z_1 x, z_2 x) - O_V(z_2 x, z_1 x) - O_A(z_1 x, z_2 x) - O_A(z_2 x, z_1 x)\right]$$

In DIS, the matrix elements of total derivative operators vanish, only target mass corrections of m^2/Q^2 are available.

Kinematical higher-twist contributions in DVCS

Theory :

A separation of kinematical and dynamical contributions in the operator product of two electromagnetic currents was proven by Braun *et. al*.

V. M. Braun and A. N. Manashov, PRL 107(2011), 202001;JHEP 01 (2012), 085; PPNP 67 (2012), 162–167.

The kinematical corrections of order t/Q^2 and m^2/Q^2 were estimated for DVCS.

Scalar meson:

$\gamma^* + \pi \rightarrow \gamma + \pi$

V. M. Braun, A. N. Manashov, and B. Pirnay, PRD 86 (2012), 014003.

Proton case:

$\gamma^* + P \rightarrow \gamma + P$

V. M. Braun, A. N. Manashov, and B. Pirnay, PRL 109 (2012), 242001.
V. M. Braun, A. N. Manashov, D. Müller, and B. M. Pirnay, PRD 89 (2014), 074022.

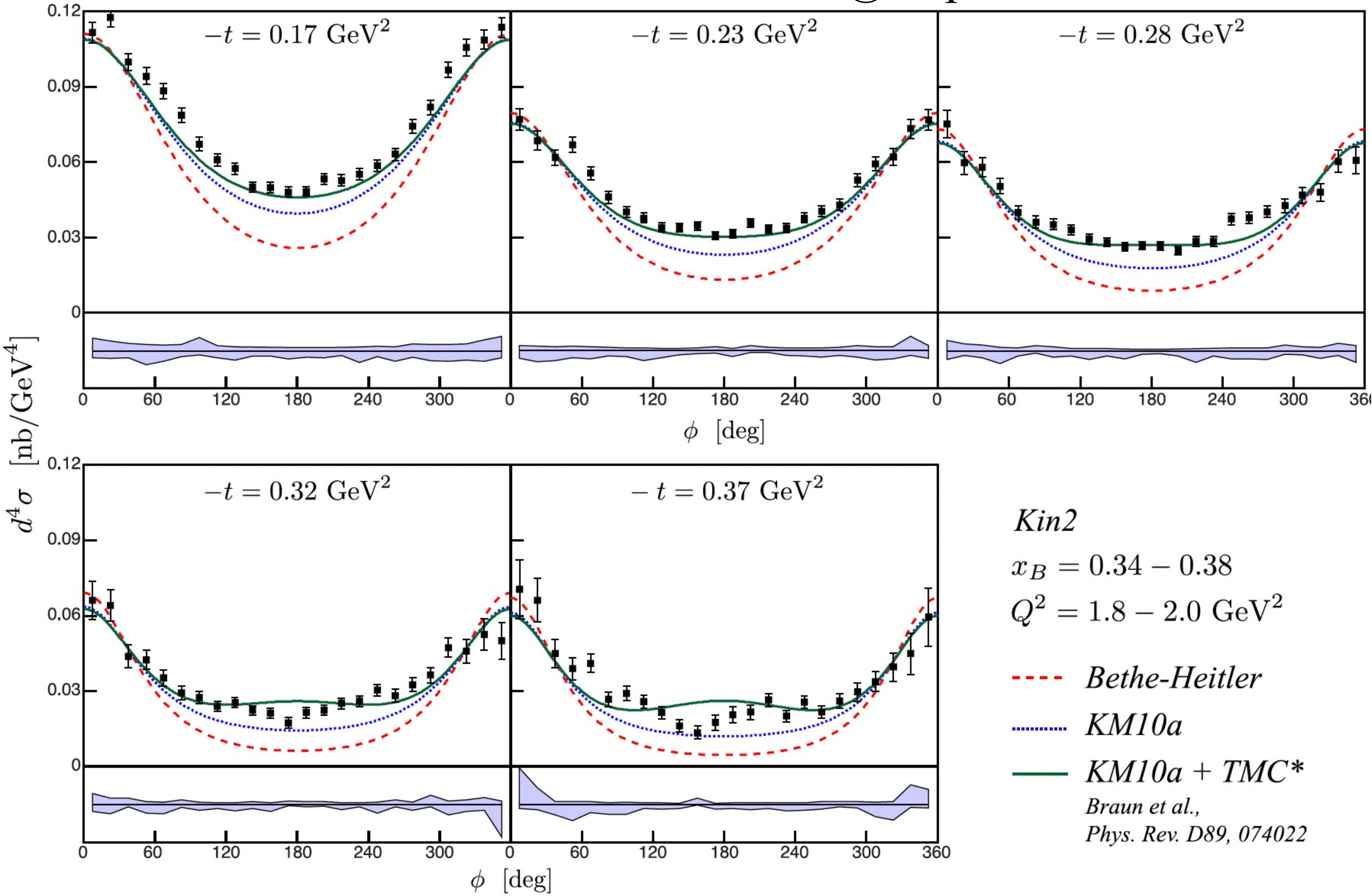
Experiment:

The kinematical corrections are included in recent experiment measurements.

DVCS measurements:

F. Georges et al. [Jefferson Lab Hall A], PRL. 128 (2022), 252002.
M. Defurne et al., Nature Communication 8(2017), 1408.
M. Defurne et al., Hall A collaboration, PRC92 (2015) no.5, 055202

Picture taken from slides of M. Defurne @3Dpartons 2022



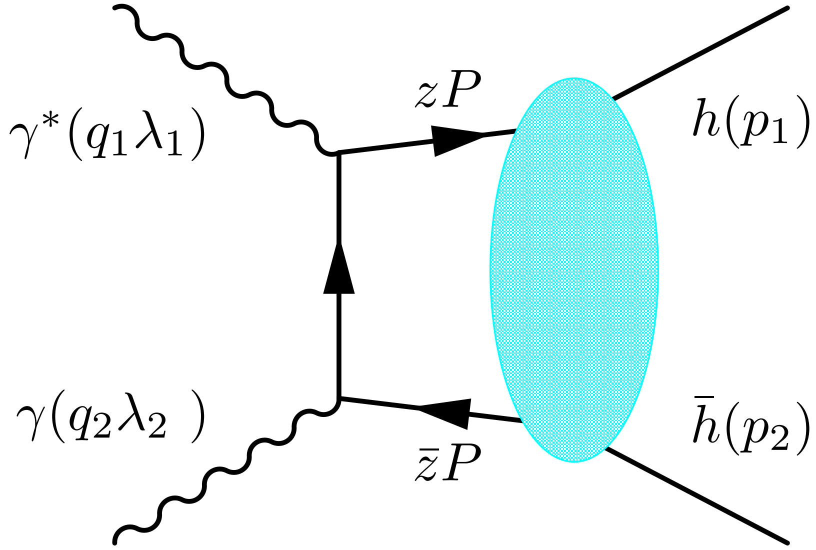
Kinematical power corrections seems to explain the gap.

Kinematical higher-twist corrections in $\gamma^* + \gamma \to \mathrm{M} + \bar{\mathrm{M}}$

Kinematical contributions in $\gamma^* + \gamma \to M + \bar{M}$

We can also calculte the amplitudes of $\gamma^* + \gamma \to M + \bar{M}$ by using the operator results of the kinematical contributions in two electromagnetic currents.

$$T_{\mu\nu} = \mathrm{T}\{j_\mu(z_1 x) j_\nu(z_2 x)\}$$



Helicity amplitudes of a scalar meson:

$$A_{\lambda_1\lambda_2} = T_{\mu\nu}\epsilon^\mu(\lambda_1)\epsilon^\nu(\lambda_2)$$

There are three independent helicity amplitudes: A_{++} , A_{0+} and A_{+-}.

Leading twist amplitude: A_{++}
Higher twist amplitudes: A_{0+} and A_{+-}.

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL **81** (1998) 1782.
M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.
M. V. Polyakov, NPB **555** (1999) 231.

Helicity amplitudes (up to twist 4):

$$A^{(0)} = \chi \left\{ \left(1 - \frac{s}{2Q^2}\right) \int_0^1 dz \, \frac{\Phi(z,\eta,s)}{1-z} - \frac{s}{Q^2} \int_0^1 dz \, \frac{\Phi(z,\eta,s)}{z} \ln(1-z) \right.$$
$$\left. - \left(\frac{2s}{Q^2}\eta + \frac{\Delta_T^2}{\beta_0^2 Q^2} \frac{\partial}{\partial \eta} \right) \frac{\partial}{\partial \eta} \int_0^1 dz \, \frac{\Phi(z,\eta,s)}{z} \left[\frac{\ln(1-z)}{2} + \mathrm{Li}_2(1-z) - \mathrm{Li}_2(1) \right] \right\},$$
$$A^{(1)} = \frac{2\chi}{\beta_0 Q} \frac{\partial}{\partial \eta} \int_0^1 dz \, \Phi(z,\eta,s) \, \frac{\ln(1-z)}{z},$$
$$A^{(2)} = -\frac{2\chi}{\beta_0^2 Q^2} \frac{\partial^2}{\partial \eta^2} \int_0^1 dz \, \Phi(z,\eta,s) \, \frac{2z-1}{z} \ln(1-z),$$

$\eta = \cos\theta$

C. Lorce, B. Pire and Qin-Tao Song, PRD 106 (2022) , 094030

$A_{++} = A^{(0)}$

$A_{0+} = -A^{(1)} \Delta \cdot \epsilon^{\mu}(-) \longrightarrow \propto \Delta_T$

$A_{-+} = -A^{(2)} [\Delta \cdot \epsilon^{\mu}(-)]^2 \longrightarrow \propto (\Delta_T)^2$

Δ is the relative momentum of final meson pair.

Asymptotic form of pion GDAs:

$$\Phi(z, \cos\theta, s) = 18z(1-z)(2z-1)[\tilde{B}_{10}(s) + \tilde{B}_{12}(s) \, P_2(\cos\theta)]$$

The nonvanishing helicity-flip amplitudes A_{0+} and A_{+-} indicate the existence of the D-wave GDAs.

Numerical estimate of the cross section

$$\frac{d\sigma(e+\gamma\to e+M+\bar{M})}{dQ^2 ds d\cos\theta}=\frac{\alpha^3\beta_0}{8s_{e\gamma}^2}\frac{1}{Q^2(1-\varepsilon)}\left(|A_{++}|^2+|A_{-+}|^2+2\varepsilon|A_{0+}|^2\right)$$

M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.

Three types of GDAs are employed to estimate the cross section

- π π GDA extracted from Belle measurements

S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.
M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003.

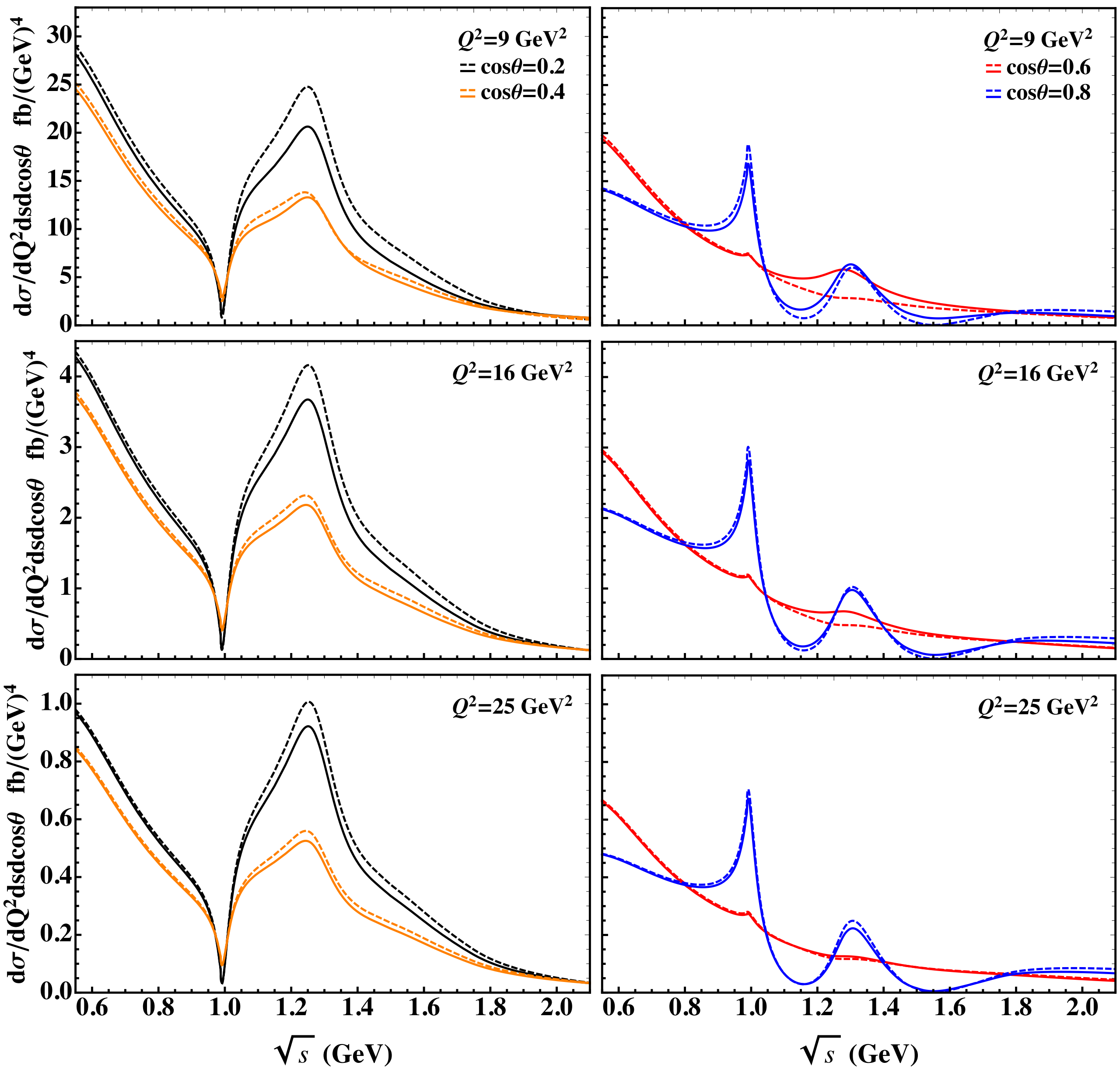
- Asymptotic π π GDA

M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.

- Model for $\eta\ \eta$ GDA

The range of kinematics in the following plots are same with that of Belle measurements.

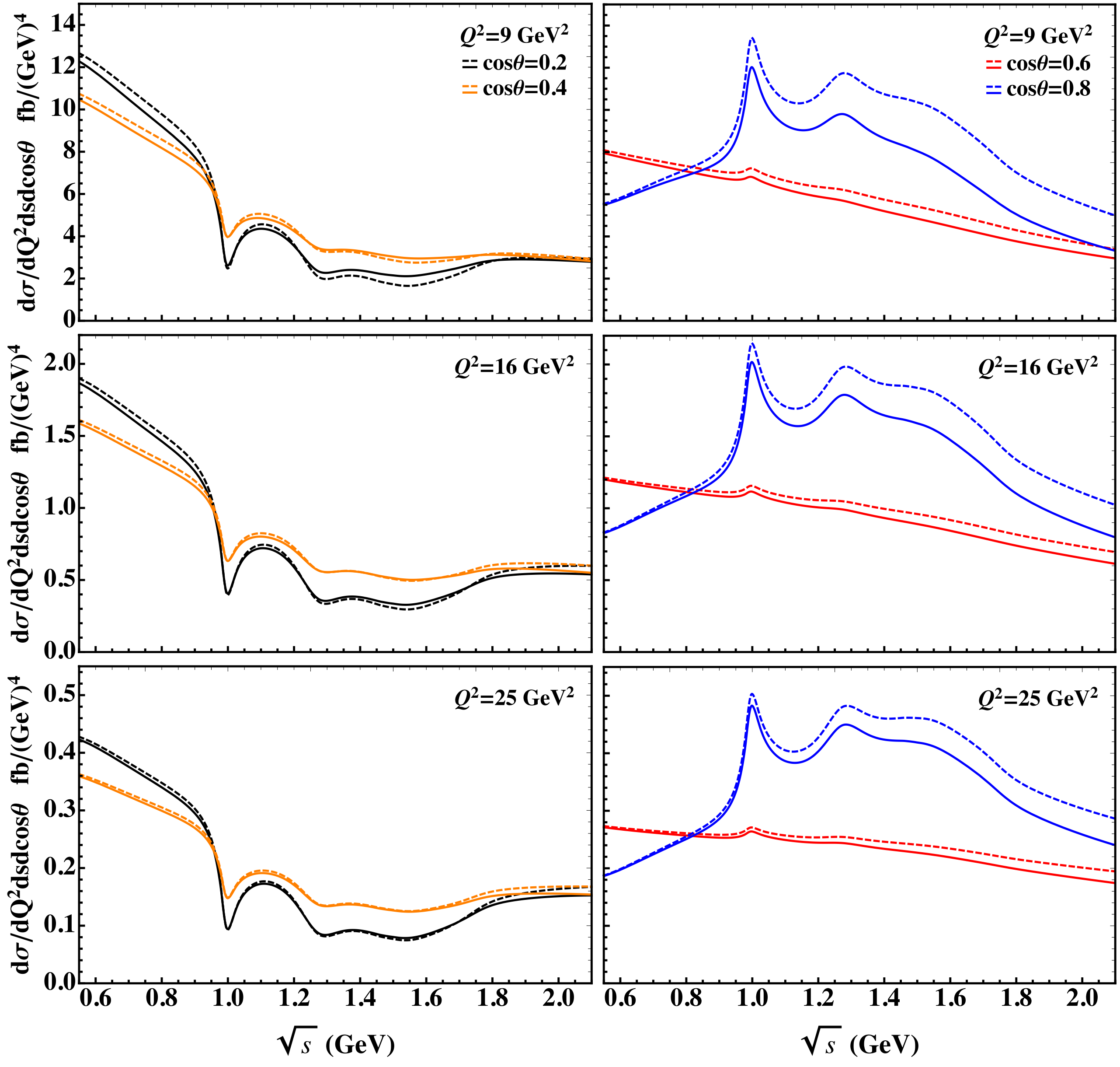
Cross section by using the extracted π π GDA



Solid lines:
cross section with kinematical contributions, twist 2+twist 3+twist 4.

Dashed lines:
twist-2 cross section

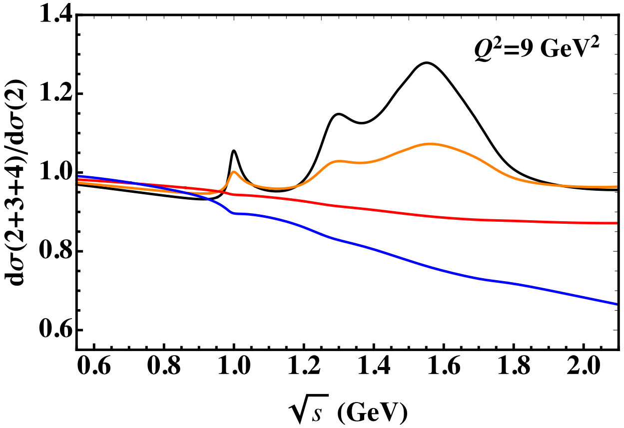
Cross section by using the asymptotic π π GDA

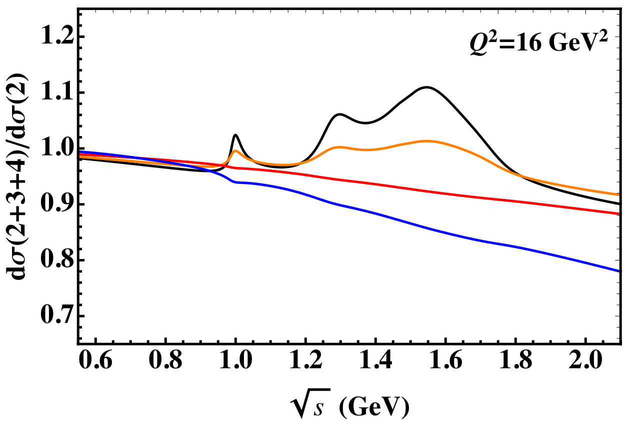


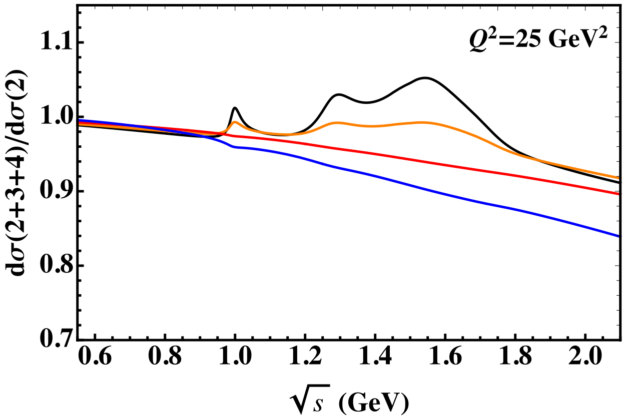
Cross section of $e + \gamma \rightarrow e + \pi + \pi$ is calculated at higher Q^2, the kinematical contributions become less important as Q^2 increases.

Ratios are estimated with the asymptotic π π GDA

Ratio=(twist 2+twist 3+twist 4)/ twist 2







Both types of π π GDAs indicate that the higher-twist kinematical contributions cannot be neglected if s > 1 GeV^2

GDAs ⟶ Timelike EMT form factors

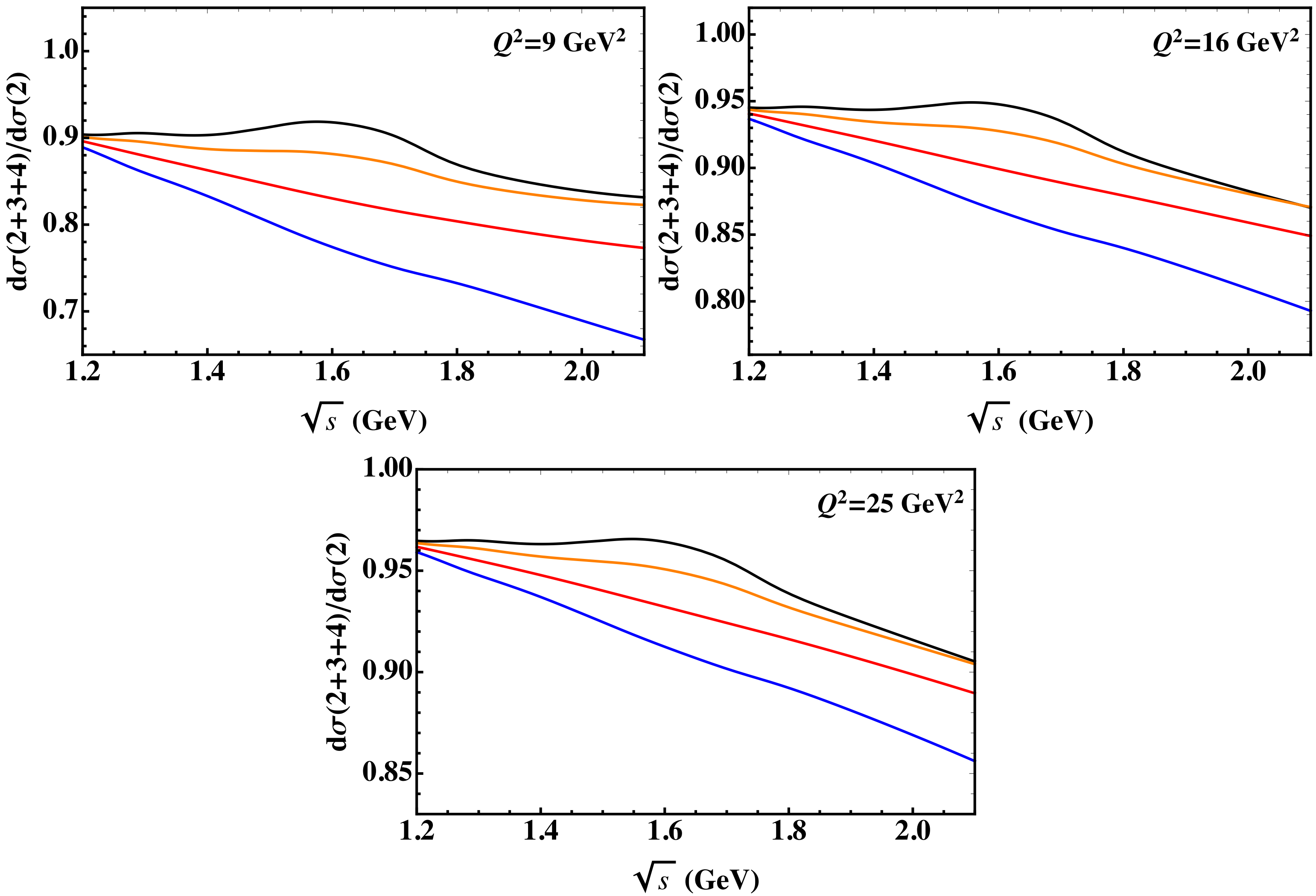
$\Lambda \geq 3$ GeV^2 is necessary for pion EMT form factor, PRD 97 (2018) 014020.

Dispersion relation: Spacelike form factor t<0

$$F(t) = \int_{4m^2}^{\Lambda} \frac{ds}{\pi} \frac{\mathrm{Im}[F(s)]}{s - t - i\varepsilon}$$

Timelike form factor s>0

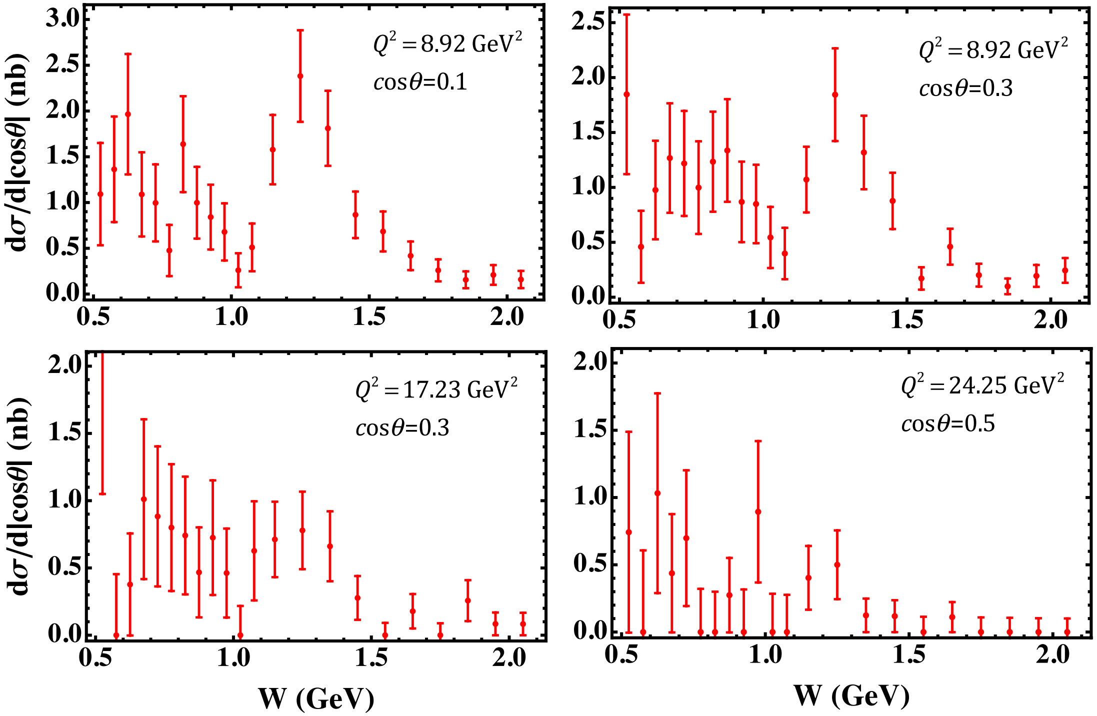
Ratios are estimated with the model for $\eta\,\eta$ GDA



The kinematical higher-twist contributions have a significant impact on the cross section even in the region which is close to the ηη threshold.

C. Lorce, B. Pire and Qin-Tao Song, PRD 106 (2022) , 094030

Future measurements of $\gamma^* + \gamma \to \mathrm{M} + \bar{\mathrm{M}}$ at Belle II



Belle measurements on $\boldsymbol{\gamma^*\gamma} \to \pi^0\pi^0$ in 2016

$8\ \mathrm{GeV}^2 < \mathrm{Q}^2 < 24\ \mathrm{GeV}^2$

$0.2\ \mathrm{GeV}^2 < s < 4\ \mathrm{GeV}^2$

$\sim s/Q^2$, $\sim m^2/Q^2$

kinematical corrections

The errors are large, and statistical errors are dominant, however, this situation can be improved by Belle II.

Luminosity: $2\times 10^{34}\ \mathrm{cm}^{-2}s^{-1} \to 8\times 10^{35}\ \mathrm{cm}^{-2}s^{-1}$

Previous measurements at Belle focused on EM FFs, however, the extraction of EMT FFs will be the main physical target for measurements of two-photon reactions at Belle II.

See talk of Dr. Masuda at Joint Meeting the APS and JPS 2023.

Kinematical higher-twist corrections in $e^+e^- \to \gamma^* \to M + \bar{M} + \gamma$: neutral meson pair

Helicity amplitudes (up to twist 4):

The leading-twist amplitude: Z. Lu and I. Schmidt, PRD 73 (2006), 094021

Kinematical higher-twist helicity amplitudes: B. Pire and Q. T. Song, arXiv:2304.06389.

$$A^{(0)} = \chi \left\{ \left(1 + \frac{\hat{s}}{2s}\right) \int_0^1 dz \, \frac{\Phi(z,\eta,\hat{s})}{1-z} + \frac{\hat{s}}{s} \int_0^1 dz \, \frac{\Phi(z,\eta,\hat{s})}{z} \ln(1-z) + \left(\frac{2\hat{s}}{s}\eta + \frac{\Delta_T^2}{\beta_0^2 s}\frac{\partial}{\partial\eta}\right) \frac{\partial}{\partial\eta} \int_0^1 dz \, \frac{\Phi(z,\eta,\hat{s})}{z} \left[\frac{\ln(1-z)}{2} + \mathrm{Li}_2(1-z) - \mathrm{Li}_2(1)\right] \right\},$$

$$A^{(1)} = -\frac{2\chi}{\beta_0\sqrt{s}} \frac{\partial}{\partial\eta} \int_0^1 dz \, \Phi(z,\eta,\hat{s}) \, \frac{\ln(1-z)}{z},$$

$\eta = \cos\theta$

$$A^{(2)} = \frac{2\chi}{\beta_0^2 s} \frac{\partial^2}{\partial\eta^2} \int_0^1 dz \, \Phi(z,\eta,\hat{s}) \, \frac{2z-1}{z} \ln(1-z),$$

$$A_{++} = A^{(0)}$$

$$A_{0+} = -A^{(1)} \Delta \cdot \epsilon^{\mu}(-) \longrightarrow \propto \Delta_T$$

$$A_{-+} = -A^{(2)} [\Delta \cdot \epsilon^{\mu}(-)]^2 \longrightarrow \propto (\Delta_T)^2$$

Δ is the relative momentum of final meson pair.

Asymptotic form of pion GDAs:

$$\Phi(z,\cos\theta, s) = 18z(1-z)(2z-1)[\tilde{B}_{10}(s) + \tilde{B}_{12}(s) \, P_2(\cos\theta)]$$

Numerical estimate of the cross section

$$\frac{d\sigma(e^+ + e^- \to M + \bar{M} + \gamma)}{dW^2 du d\cos\theta} = \frac{\alpha^3 \beta_0}{8s^3} \frac{1}{(1+\varepsilon)} \left(|A_{++}|^2 + |A_{-+}|^2 + 2\varepsilon |A_{0+}|^2 \right)$$

M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.

W is center-of-mass energy of meson pair

Three types of GDAs are employed to estimate the cross sections.

- $\pi\,\pi$ GDA extracted from Belle measurements

S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.
M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003.

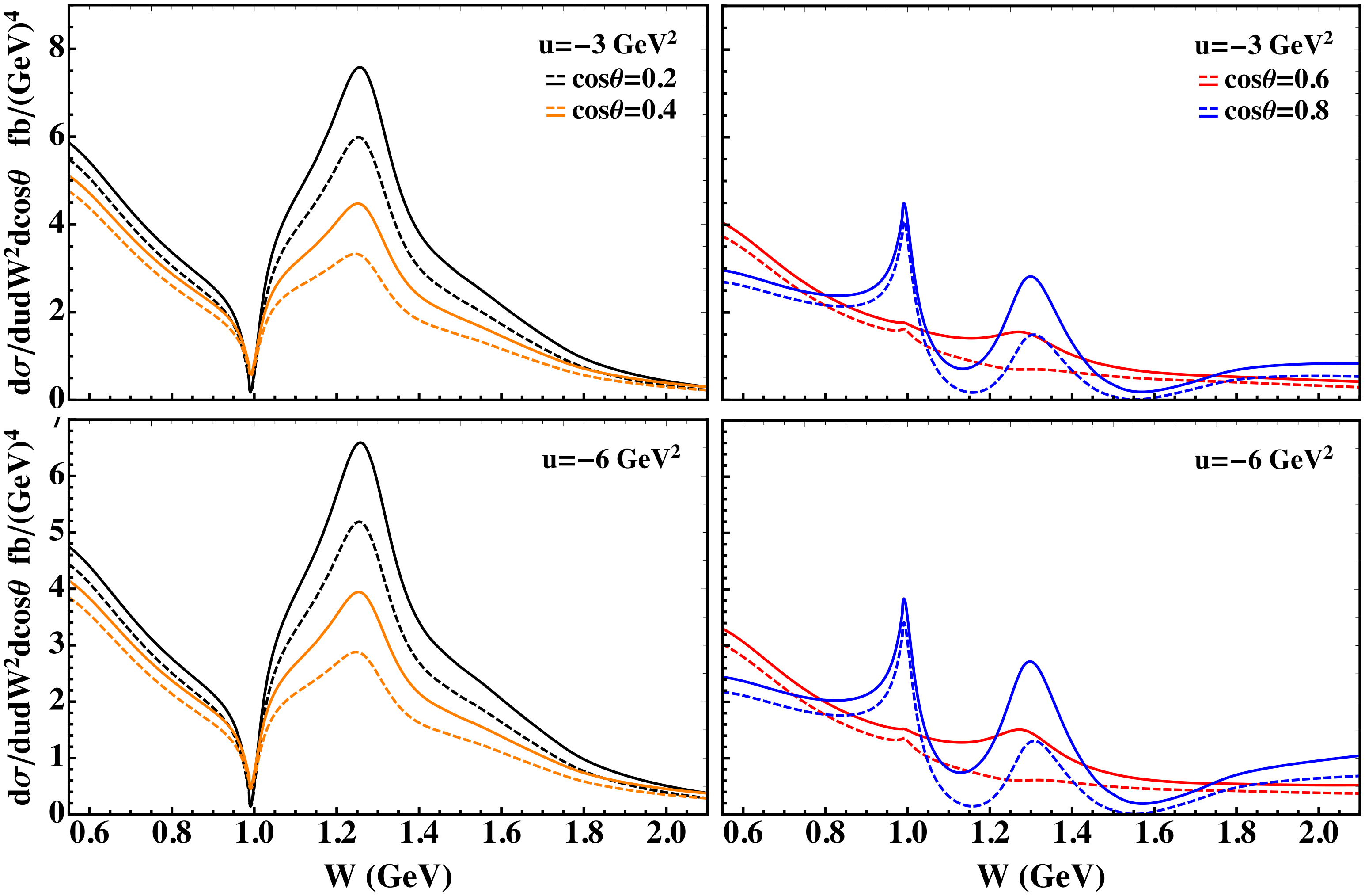
- Asymptotic $\pi\,\pi$ GDA

M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.

- Model for $\eta\,\eta$ GDA

The range of kinematics in the following plots is chosen according to BESIII

Cross section by using the extracted π π GDA



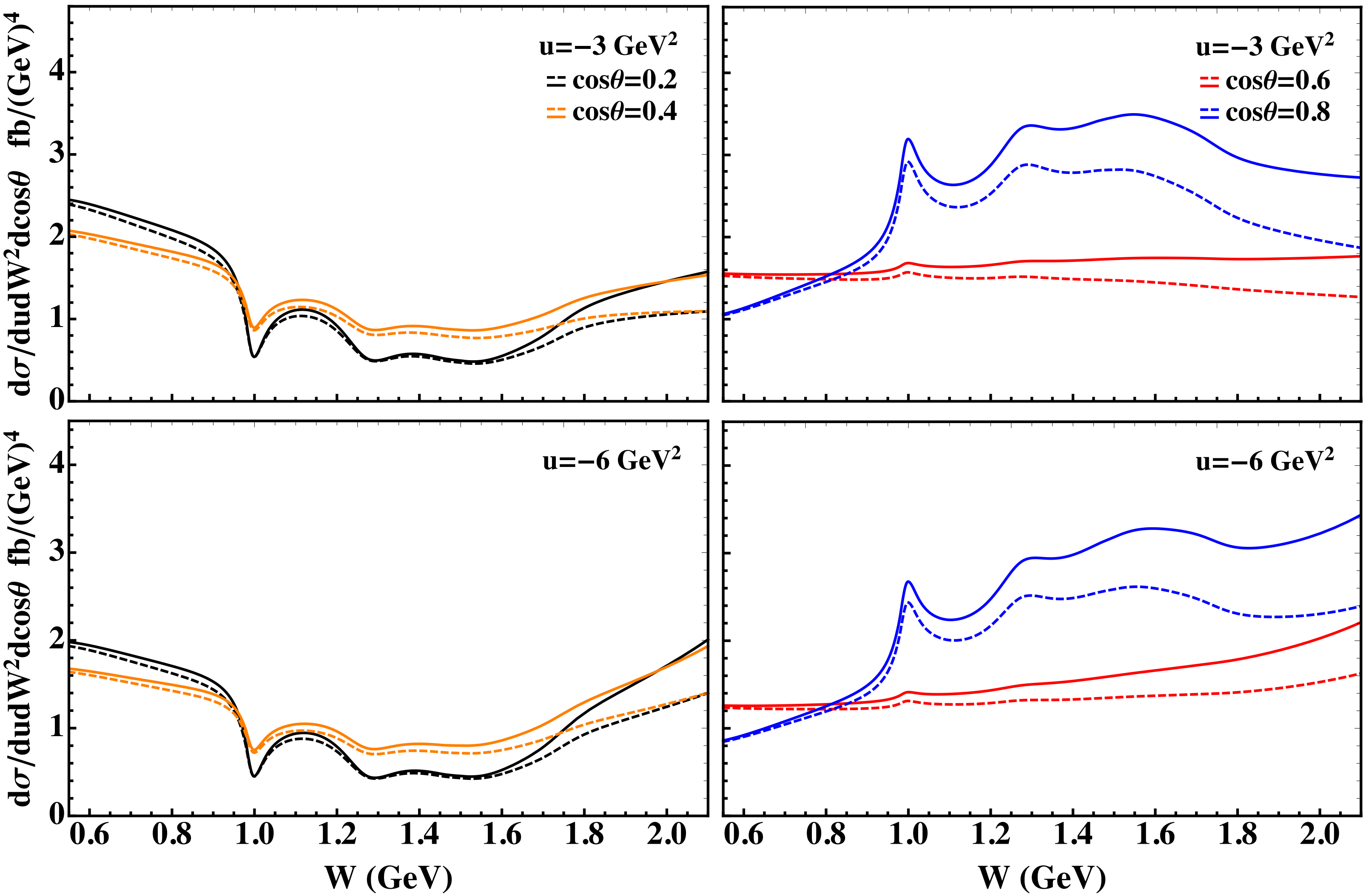
Solid lines:

cross section with kinematical contributions, twist 2+twist 3+twist 4.

Dashed lines:

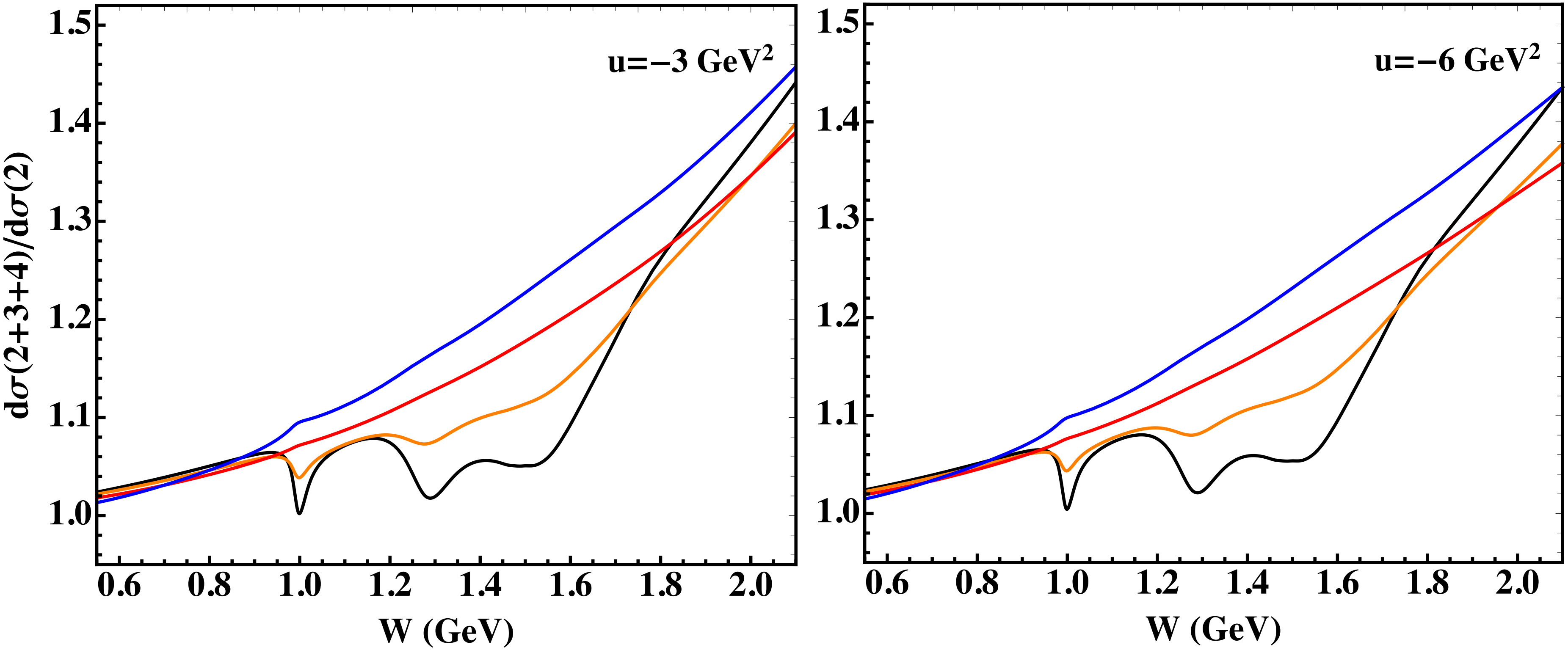
twist-2 cross section

Cross section by using the asymptotic π π GDA



Ratios are estimated with the asymptotic π π GDA

Ratio=(twist 2+twist 3+twist 4)/ twist 2



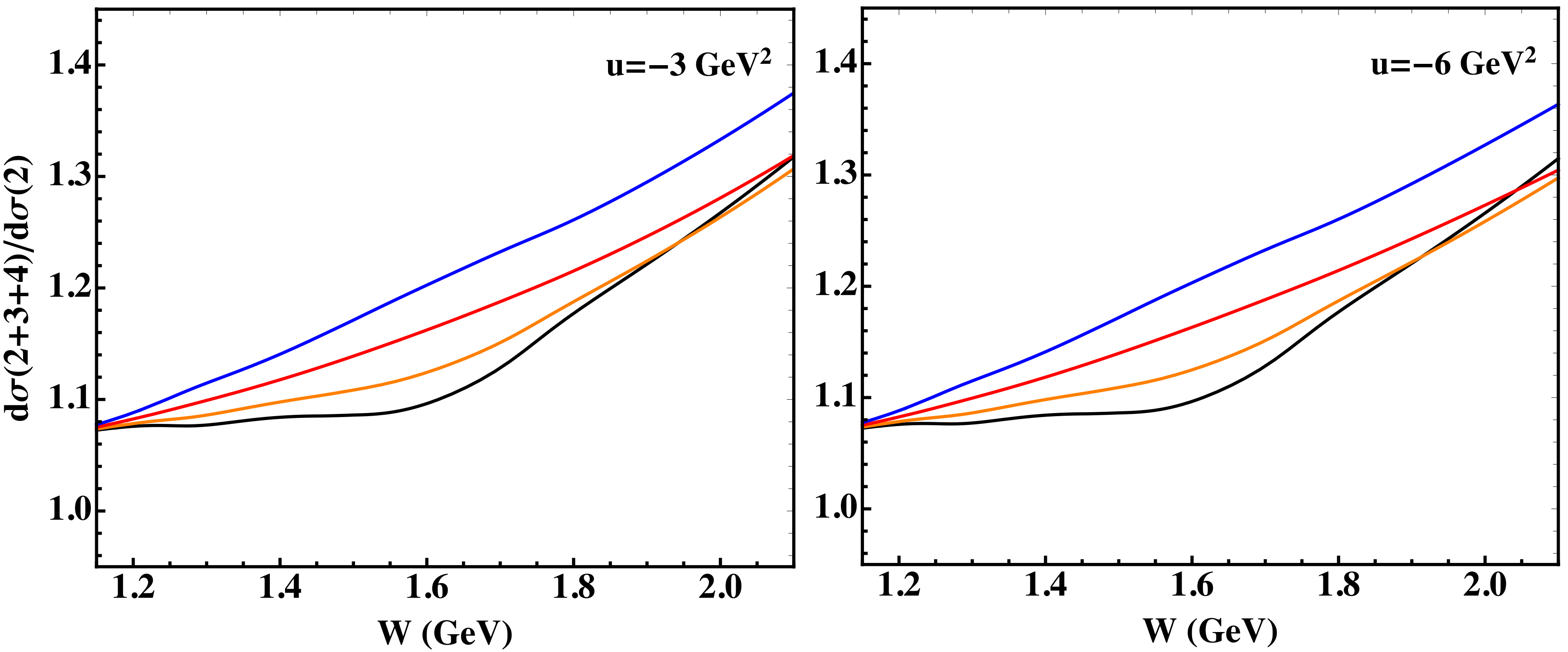
Both types of π π GDAs indicate that the higher-twist kinematical contributions cannot be neglected if W > 1 GeV.

GDAs ⟶ Timelike EMT form factors

Spacelike EMT form factors

Dispersion relation: the region of W > 1 GeV is necessary.

Ratios are estimated with the model for $\eta\eta$ GDA



The kinematical higher-twist contributions increase as W goes up.

B. Pire and Q. T. Song, PRD 107 (2023), 114014.

Kinematical higher-twist corrections in $e^+e^- \to \gamma^* \to M_1 + M_2 + \gamma$ and $e + \gamma \to e + M_1 + M_2$

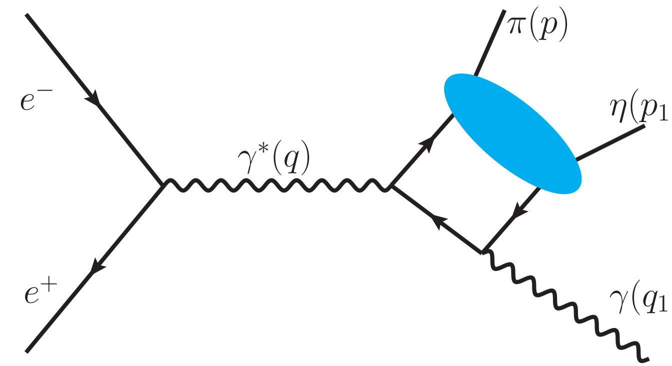
Exotic hybrid mesons

One can search for the candidates of the hybrid mesons from the P-wave of $M_1 M_2$ in $\gamma^* \to M_1 + M_2 + \gamma$ and $\gamma^* + \gamma \to M_1 + M_2$.

Isovector hybrid mesons

$M_1 M_2$: $\pi\eta, \pi\eta'$ $\quad I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1400), \pi_1(1600)$



Isoscalar hybrid mesons

$M_1 M_2$: $\eta\eta'$ $\quad I^G(J^{PC}) = 0^+(1^{-+})$

$\eta_1(1855)$

$\gamma^* \to \pi + \eta + \gamma$ at BESIII

The exotic quantum number($J^{PC} = 1^{-+}$) does not exit in quark model.

$\eta_1(1855)$ was observed by BESIII in $J/\psi \to \eta + \eta' + \gamma$ recently.

M. Ablikim et al. [BESIII], PRL 129 (2022), 192002.
M. Ablikim et al. [BESIII], PRD 106 (2022), 072012.

$J/\psi \to \gamma^*$: $\gamma^* \to \eta + \eta' + \gamma$ can be also measured by BESIII.

B. Pire and Q. T. Song, PRD 107 (2023), 114014.

Shear viscosity term (a new gravitational FF)

If the hybrid mesons are observed in $\gamma^* \to M_1 + M_2 + \gamma$ and $\gamma^* + \gamma \to M_1 + M_2$, it will indicate the existence of a new EMT FF.

$$\langle M_2(p_2) M_1(p_1) | T_q^{\mu\nu} | 0 \rangle \sim E_q(s) P^\mu \Delta^\nu$$

O. Teryaev, JPS Conf. Proc. 37(2022), 020406.

The shear viscosity term could exist in matrix element of EMT.

Its sum over quarks and gluons should be zero which is a consequence of the conserved EMT, however, it will exist for a single flavor q on condition that there is P-wave GDA.

The study of shear viscosity term and hybrid mesons by GDAs at BESIII and Belle II.

$\gamma^* \to M_1 M_2 \gamma$
$\gamma^* \gamma \to M_1 M_2$

→ P wave $M_1 M_2$ →

Shear viscosity term exists

→ Resonances in P wave →

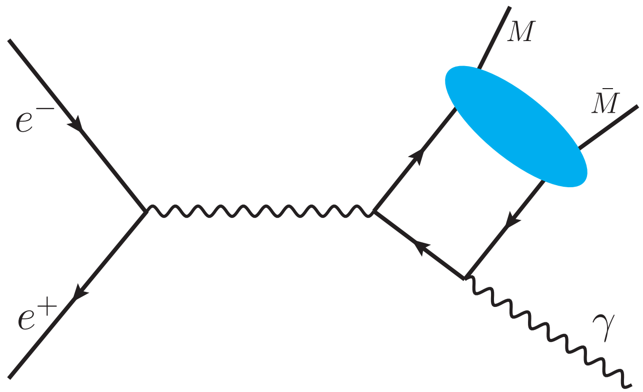
Exotic (hybrid) mesons observed

↑ Extracted from cross sections ↑

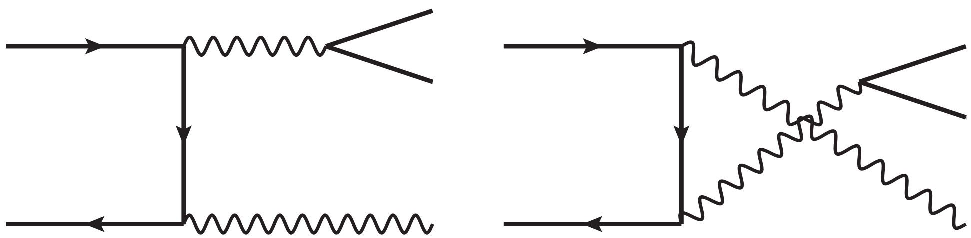
GDAs and EMT FFs

Kinematical higher-twist corrections in $e^+e^- \to M + \bar{M} + \gamma$: charged meson pair

Charged meson pair: $\pi^+\pi^-$ and K^+K^-



GDA process
C even meson pair



ISR process: meson EM FFs
C odd pair

Three types of contribution in the cross section:

Interference term

$$d\sigma_{\mathrm{G}} : d\sigma_{\mathrm{I}} : d\sigma_{\mathrm{ISR}} \sim \hat{s} : \sqrt{\hat{s}s} : s$$

GDA process, same as neutral meson pair

ISR process, largest, no GDAs

Advantages of interference term in charged meson production

- Larger cross section

- Extraction of the complete information of GDAs

$$d\sigma_{\rm I} \propto {\rm Re}(A_{ij} F_M^*(\hat{s}))$$

$$d\sigma_{\rm G} \propto {\rm Re}(A_{ij} A_{kl}^*) \quad \longrightarrow$$

Imaginary phases of GDAs cannot be extracted.

$$\frac{d\sigma_{\rm I}}{d\hat{s}\, du\, d(\cos\theta)\, d\varphi} = \frac{\alpha_{\rm em}^3 \beta_0}{8\pi s^2} \frac{\sqrt{2}\beta_0}{\sqrt{\hat{s}s\epsilon(1+\epsilon)}} \left[C_0 + C_1 \cos\varphi + C_2 \cos(2\varphi) + C_3 \cos(3\varphi)\right]$$

$$C_0 = -\,{\rm sgn}(\rho)\sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}{\rm Re}(A_{++}F_M^*)\cos\theta + {\rm sgn}(\rho)\,(x-1)\sqrt{\epsilon(1-\epsilon)}{\rm Re}(A_{0+}F_M^*)\sin\theta,$$

$$C_1 = -\,[1-(1-x)(1-\epsilon)]\,{\rm Re}(A_{++}F_M^*)\sin\theta + 2\epsilon\sqrt{2x(x-1)}{\rm Re}(A_{0+}F_M^*)\cos\theta + (x-1){\rm Re}(A_{-+}F_M^*)\sin\theta,$$

$$C_2 = {\rm sgn}(\rho)\sqrt{\epsilon(1-\epsilon)}x{\rm Re}(A_{0+}F_M^*)\sin\theta + {\rm sgn}(\rho)\sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}{\rm Re}(A_{-+}F_M^*)\cos\theta,$$

$$C_3 = -\,\epsilon x{\rm Re}(A_{-+}F_M^*)\sin\theta.$$

B. Pire and Qin-Tao Song, PRD 109 (2024), 074016

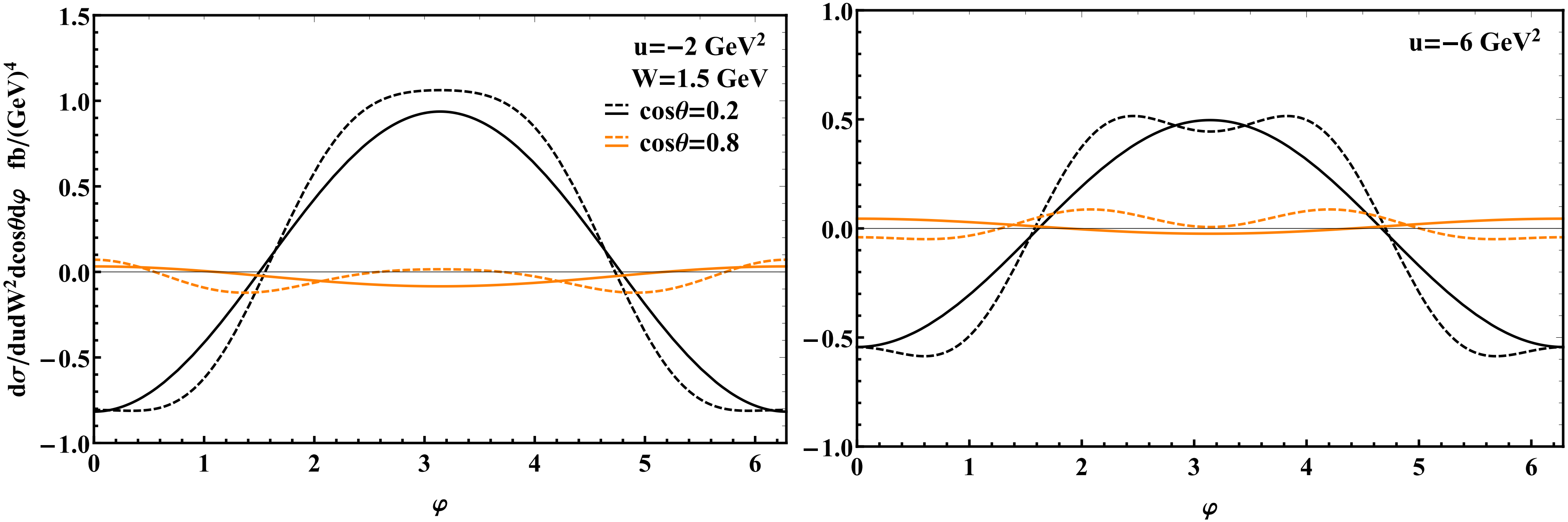
Only the interference term remains if one interchanges meson pair

$$d\sigma(M\bar{M}) - d\sigma(\bar{M}M) = 2d\sigma_{\rm I}$$

BaBar measurement of pion meson pair : PRD 92 (2015), 072015.

Numerical estimate of interference term

The dashed curves denote the twist-2 cross sections, and the solid ones include the kinematical higher-twist contributions, s=12 GeV^2 for BESIII.



The higher-twist kinematical contributions cannot be neglected.

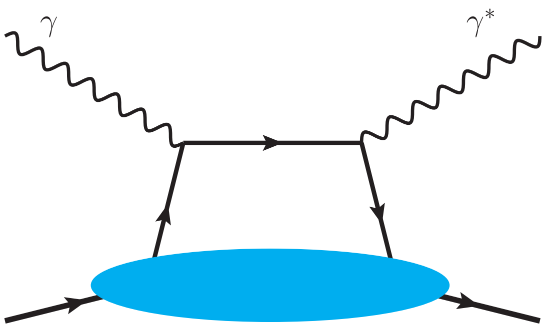
B. Pire and Qin-Tao Song, PRD 109 (2024), 074016

Future plan on kinematical higher-twist corrections

(1) Timelike Compton scattering(TCS) for proton
@JLAB, EIC-US, EicC

TCS is one of three golden channels to access GPDs by experiment.

First measurement of TCS in 2021:
PRL 127 (2021), 262501



TCS: $\boldsymbol{\gamma}$p →$\boldsymbol{\gamma}$*p

(2) DVCS of $\boldsymbol{\gamma}$*p → $\boldsymbol{\gamma}$Δ:
Transition GPDs and transition gravitational FFs
@JLAB, EIC-US, EicC.

Preliminary results are obtained by JLAB: see the talk by Kyungseon JOO at Strong QCD 2024 (Nanjing).

(3) DVCS for deuteron
@JLAB, EIC-US, EicC

Summary

- GDAs can be considered as an alternative way to investigate the EMT form factors of pions.

- Kinematical higher-twist contributions are calculated for $\gamma^* + \gamma \to \mathrm{M} + \bar{\mathrm{M}}$ and $\gamma^* \to \mathrm{M} + \bar{\mathrm{M}} + \gamma$ from which the GDAs can be extracted. The numerical calculation indicates that kinematical contributions are significant for Belle (II) and BESIII (STCF).

- The measurements of $\gamma^* \to \mathrm{M} + \bar{\mathrm{M}} + \gamma$ at BESIII (STCF) can be a new research direction.

- In future, one can search for exotic hybrid mesons and study the new EMT FF (shear viscosity) in $\gamma^* \to M_1 + M_2 + \gamma$ and $\gamma^* + \gamma \to M_1 + M_2$.

- Kinematical higher-twist contributions for DVCS and TCS are in progress, which can be measured at JLab, EIC-US, and EicC.

Thank you very much