Berry Phase in Axion Physics

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2024.09.09 威海

Berry Phase

$$i\frac{\partial}{\partial t}\left|\psi\right\rangle = \underline{H(t)}\left|\psi\right\rangle$$

Time dependent system

$$\begin{cases} \xi_{\mathrm{dym}} = \int E(t)\mathrm{d}t \\ \xi_{\mathrm{Berry}} = i\oint_C A_\mu \mathrm{d}R^\mu \end{cases}$$

Dynamical Phase

Berry Phase

closed loop | potential | parameter

Example :

$$H(t) = \mathbf{B}(t)\cdot\mathbf{j}$$

Magnetic Field | Spin Operator

Can axions induce the Berry phase?

Berry Phase in Axion Physics

$$\mathscr{L}_{af} = -\frac{1}{2}\frac{g_f}{f_a}\partial_\mu a \bar{f}\gamma^\mu\gamma^5 f$$

$$\mathscr{L}_{a\gamma} = \frac{1}{4}\frac{g_\gamma}{f_a} a F^{\mu\nu}\tilde{F}_{\mu\nu}$$

$$\eta_a = \frac{\mathrm{d}a}{\mathrm{d}t}$$

$$H_f = \frac{g_f}{2f_a}\eta_a \frac{\mathbf{p}\cdot\boldsymbol{\sigma}}{m_f}$$

Same Form !

$$H_\gamma = \frac{g_\gamma}{2f_a}\eta_a \frac{\mathbf{k}\cdot\mathbf{S}}{|\mathbf{k}|}$$

$$H(t) = \mathbf{V}(t)\cdot\mathbf{j}$$

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Berry Phase in Axion Physics

$$H(t) = \mathbf{V}(t) \cdot \mathbf{j}$$

- **Scenario I:** vector's **direction** changes with time
- **Scenario II:** vector's **magnitude** changes with time

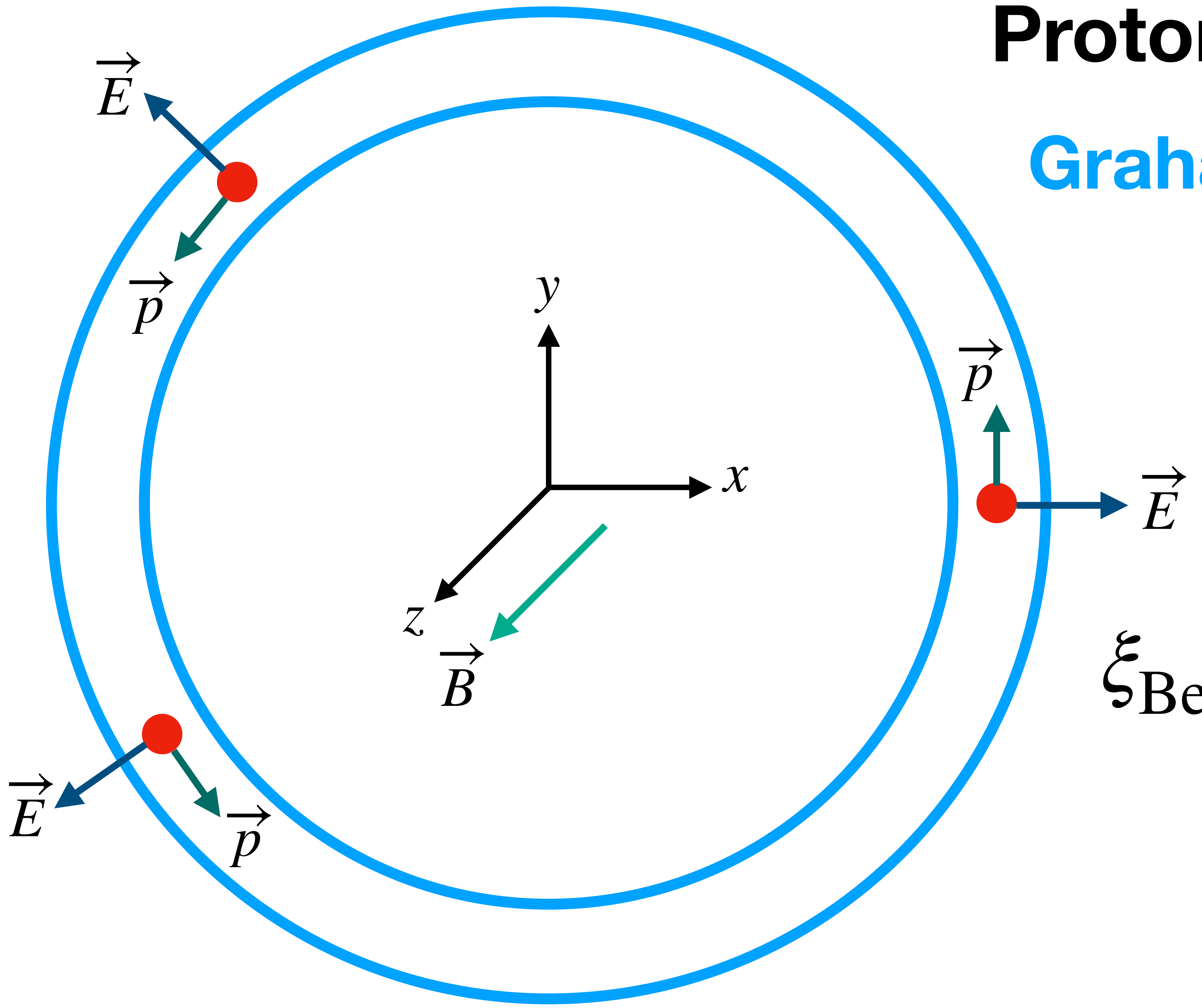
Scenario I: Take the axion-fermion system as an example

Two scenarios are applicable for both systems

Scenario II: Take the axion-photon system as an example

Scenario One: Direction

$$H_f = \frac{g_f}{f_a}\eta_a \frac{\mathbf{p}\cdot\sigma}{m_f} - \frac{gq}{2m_f}\mathbf{B}\cdot\sigma - \frac{gq}{2m_f^2}(\mathbf{E}\times\mathbf{p})\cdot\sigma + (\gamma-1)\frac{\mathbf{a}\times\mathbf{v}}{v^2}\cdot\sigma$$



Proton Ring Experiment

Graham et al. 2017, PRD

Assume η_a is a constant

⇕

Axion mass is very small

$$\xi_{\text{Berry}} \sim \mathcal{O}\left(\frac{g_f^2/f_a^2}{|B-\omega|}\right) \sim 10^{-36}$$

→ **Very small value**

Scenario One: Direction

Q: Why the Berry phase is so small

A: Very large Standard Model background

- **Rotation Effect**
- **Electromagnetism Effect**

$$\xi_{\text{Berry}} \sim \mathcal{O}\left(\frac{g_f^2/f_a^2}{|B-\omega|}\right)$$

Resonance Condition

$$GB + vE\left(G - \frac{1}{\gamma^2 - 1}\right) = 0$$

$$\gamma = \frac{1}{1 - v^2} \qquad G = \frac{g-2}{2}$$

→

$$\xi_{\text{Berry}} = -2\pi m$$

$$m = \pm\frac{1}{2}$$

Scenario Two: Magnitude

- **Focus on the situation where η_a changes with time**

- **Assume photons propagate along the z direction**

$$H_\gamma = \frac{g_\gamma}{2f_a}\eta_a(t)\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \Rightarrow \quad U_\gamma(t) = \begin{pmatrix} \cos\left(\frac{g_\gamma}{2f_a}\Delta a\right) & -\sin\left(\frac{g_\gamma}{2f_a}\Delta a\right) \\ \sin\left(\frac{g_\gamma}{2f_a}\Delta a\right) & \cos\left(\frac{g_\gamma}{2f_a}\Delta a\right) \end{pmatrix}$$

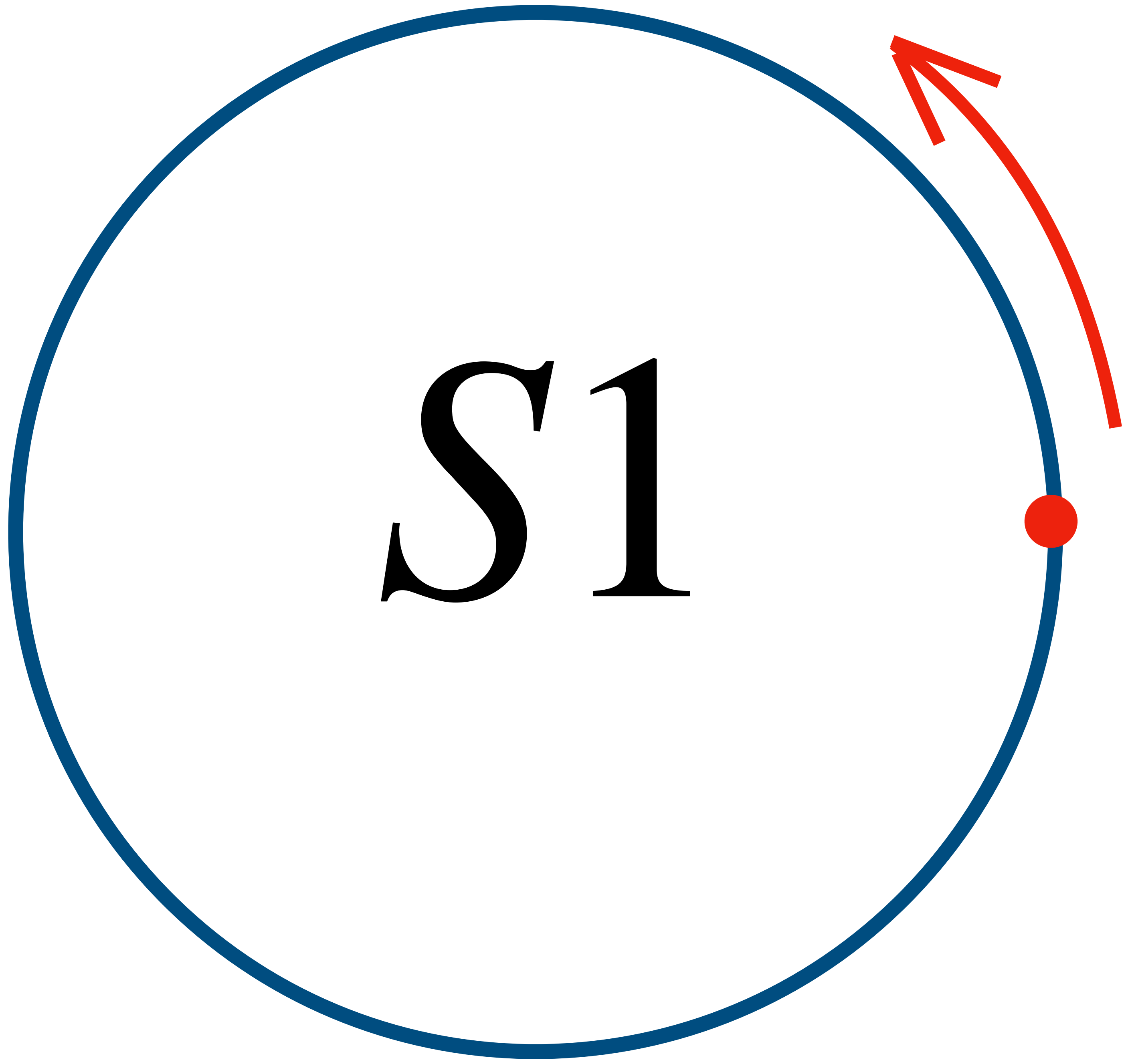
where $\Delta a(t) = \tilde{a}(t) + At$

$$\Rightarrow \quad \xi_{\text{Berry}} = m\frac{g_\gamma}{2f_a}\left[\tilde{a}(T) - \tilde{a}(0)\right],\ m = \pm 1$$

which means the Berry phase must be zero for a closed loop.

- **Quantization of the axion: shift symmetry** Choi et al. 2024, PRL

$$a \sim a + 2\pi f_a$$



$$\tilde{a}(t+T) = \tilde{a}(t) + 2\pi N_w f_a$$

$$\xi_{\rm Berry} = m \frac{g_\gamma}{2f_a}\left[\tilde{a}(T) - \tilde{a}(0)\right]$$

$$\xi_{\rm Berry} = m\pi g_\gamma N_w, \quad m = \pm 1$$

Winding number

The non-zero winding number can be realized by the axion string, axion domain wall, etc.

Jain et al. 2021, JCAP

Application of The Berry Phase

- g_γ **is related to the generalized symmetry**

Choi et al. 2024, PRL

$$\xi_{\text{Berry}} = m\pi g_\gamma N_w,\ m = \pm 1$$

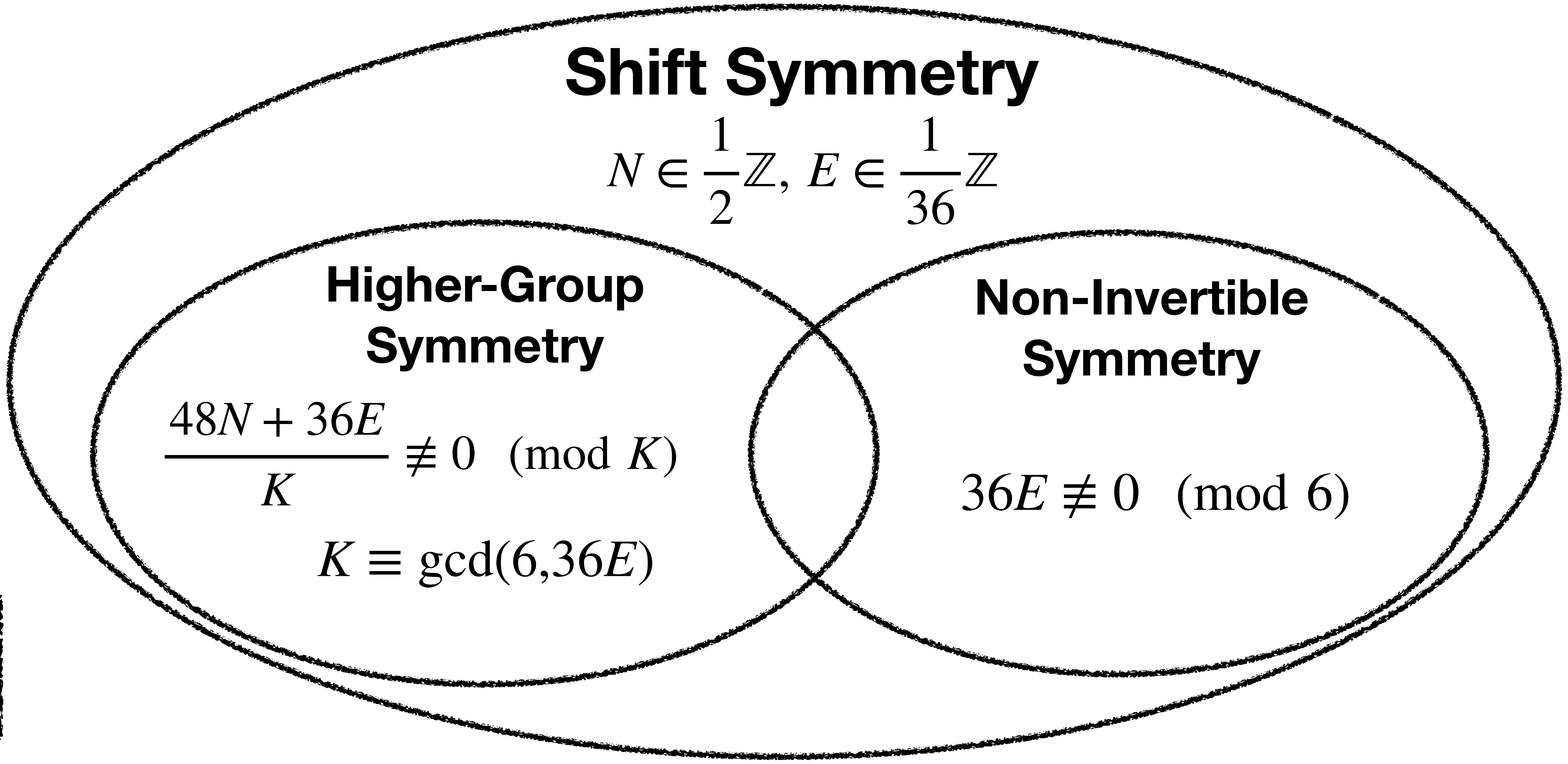
$$g_\gamma = \frac{\alpha}{2\pi}\left(\frac{E}{N} - 1.92\right)$$

Berry phase measurement

↓

Generalized symmetry research

$$G_{\text{SM}} = [SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_1$$



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Conclusion

- **We perform a systematical study on the Berry phase in the axion physics.**
- **We find the unified form of axion-fermion and axion-photon Hamiltonian and research two different scenarios**
- **Measuring the Berry phase which can help us understand the generalized symmetry of the axion.**

Thank You!

Back Up

$$\mathscr{L}_{a\gamma} = \frac{1}{4}\frac{g_\gamma}{f_a} a F^{\mu\nu}\tilde{F}_{\mu\nu} \qquad g_\gamma \sim \frac{\alpha}{2\pi}\frac{E}{N}$$

Fundamental Symmetry : Shift Symmetry $\rightarrow N \in \frac{1}{2}\mathbb{Z},\ E \in \frac{1}{36}\mathbb{Z}$

Generalized Symmetry

- **Higher Group Symmetry** $\rightarrow \frac{48N + 36E}{K} \not\equiv 0 \pmod{K}$
- **Non-Invertible Symmetry** $\rightarrow 36E \not\equiv 0 \pmod{6}$

$K \equiv \gcd(6, 36E)$

Choi et al. 2024, PRL

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- **The Lagrangian in axion physics**

$$\mathscr{L}_{a\gamma} = \frac{1}{4}\frac{g_\gamma}{f_a} a F^{\mu\nu}\tilde{F}_{\mu\nu} \qquad \mathscr{L}_{af} = -\frac{1}{2}\frac{g_f}{f_a}\partial_\mu a \bar{f}\gamma^\mu\gamma^5 f$$

	a	$F^{\mu\nu}\tilde{F}_{\mu\nu}$	$\partial_\mu \bar{f}\gamma^\mu\gamma^5 f$
CP Parity	-1	-1	-1
T Parity	-1	-1	-1

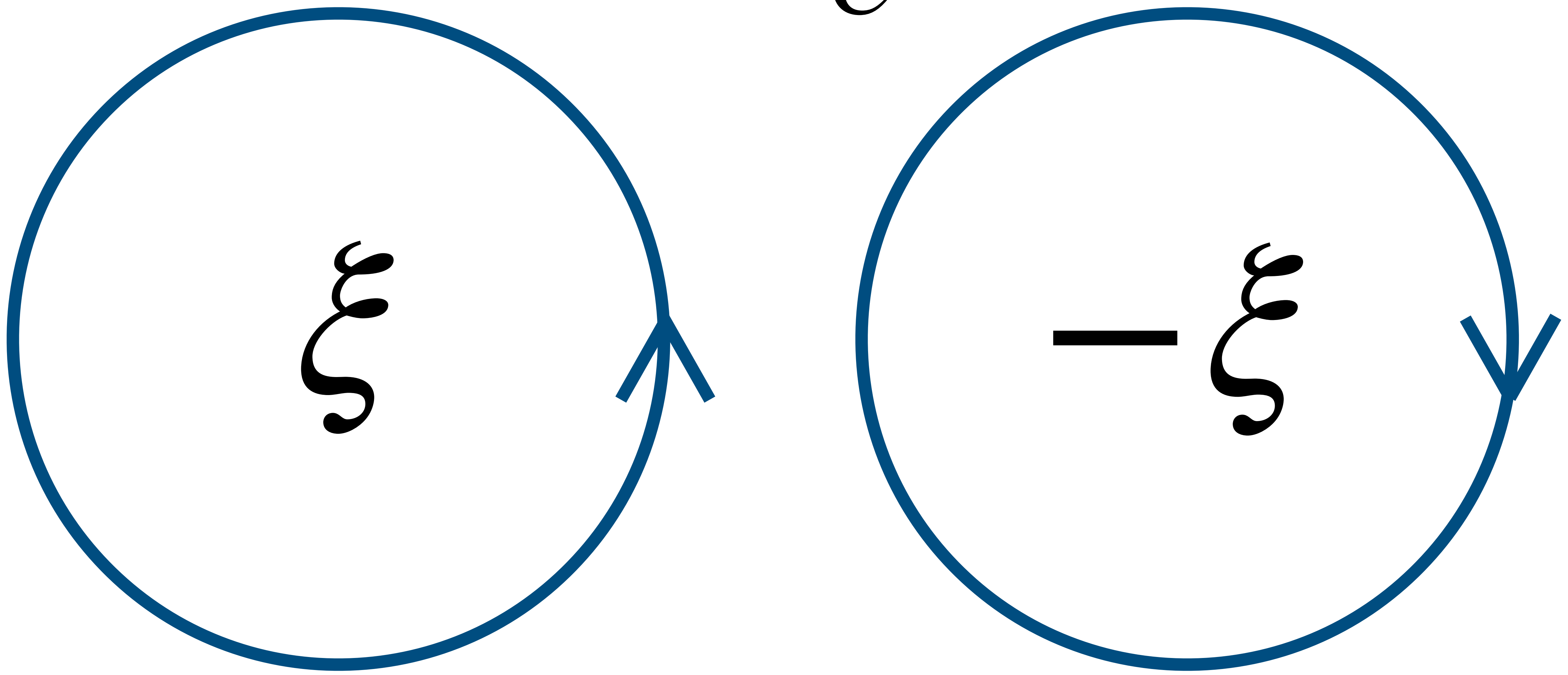
The key property to induce the Berry phase

Back Up

For a non-degenerate quantum system with time reversal symmetry, the Berry phase must be zero.

Baggio et al. 2017, JHEP

$$\xi_{\mathrm{Berry}} = i\oint_C A_\mu \mathrm{d}R^\mu$$



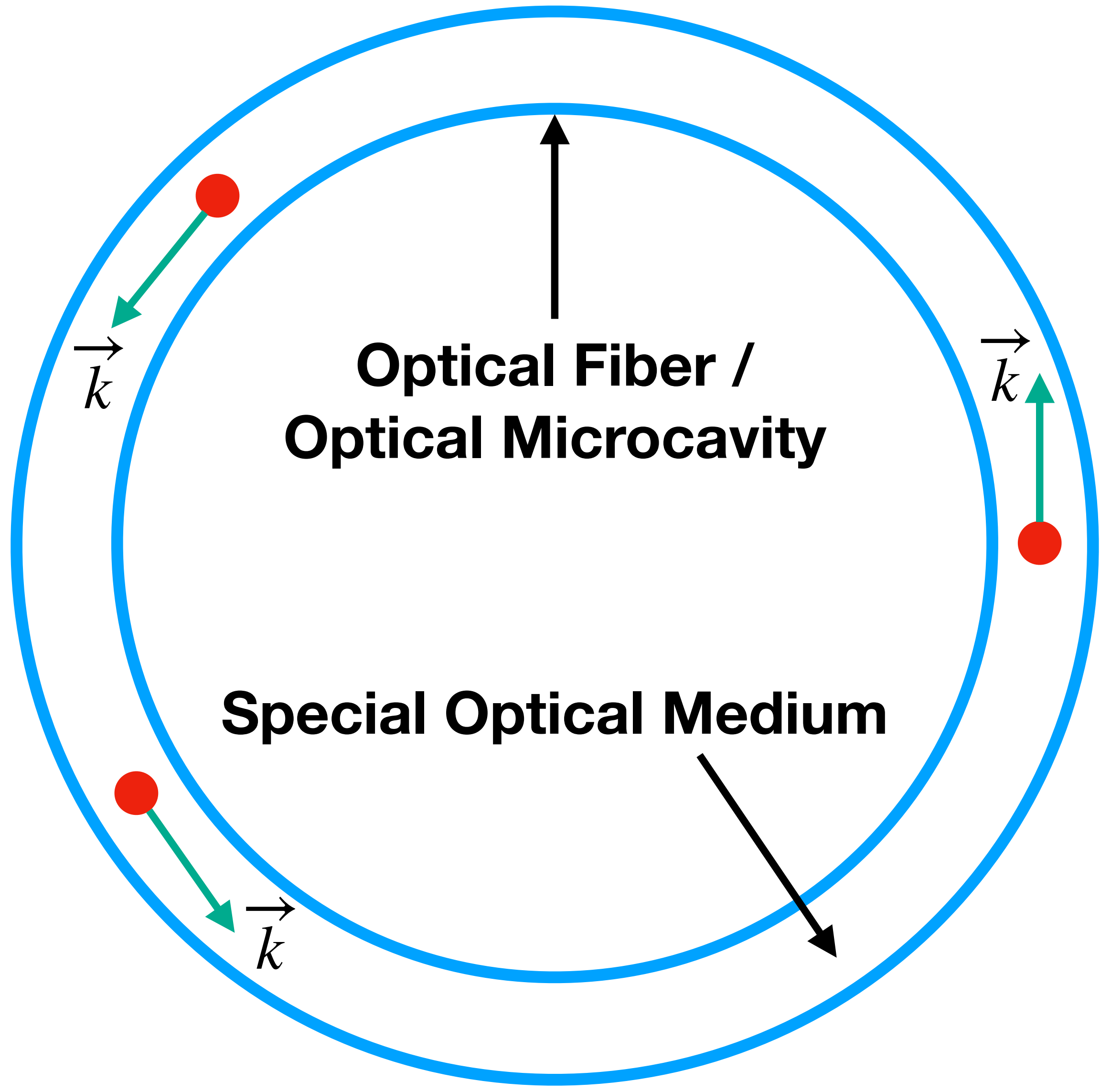
Time reversal symmetry →

$$\xi = -\xi$$

No Berry phase

Back Up

	Proton Ring Exp	Photon Ring Exp
How to make particles move	Electromagnetic Field	Optical Fiber
How to probe the axion field	Mesure protons' spin	Measure photons' polarization
How to satisfy the resonance condition	Electromagnetic Field	Birefringence Medium



- **New Hamiltonian**

$$H_\gamma = -\frac{\omega}{2\varepsilon_n}\overleftrightarrow{\chi} + \frac{\eta_a}{2\varepsilon_n\omega}\mathbf{k}\cdot\mathbf{S} \quad \xrightarrow{\text{Photons' motion}} \quad \tilde{H}_\gamma = -\frac{\omega}{2\varepsilon_n}\overleftrightarrow{\chi} - \Omega S_z + \frac{\eta_a}{2\varepsilon_n\omega}\mathbf{k}\cdot\mathbf{S}$$

If $\overleftrightarrow{\chi}$ is proportional to S_z, the first and second terms of $\tilde{H}_\gamma$ could cancel out, and $\tilde{H}_\gamma$ will be dominated by the axion term

Back Up

$$\overleftrightarrow{\varepsilon} = \varepsilon_n \overleftrightarrow{I} - rS^z = \begin{pmatrix} \varepsilon_n & ir & 0 \\ -ir & \varepsilon_n & 0 \\ 0 & 0 & \varepsilon_n \end{pmatrix}$$

The optical medium we want is just the birefringence medium

Resonance Condition $$\Omega = \frac{\omega r}{2\epsilon_n}$$

All parameters can be tuned experimentally

Ω **: photons' angular velocity**

ω **: photons' energy**

- **Theoretical prediction**

$$\alpha = \frac{\eta_\gamma}{2\sqrt{\epsilon_n}} t \qquad \eta_\gamma = \frac{g_\gamma \partial_t a}{f_a}$$

t **: photons' motion time**

$$\alpha = 1.63 \times 10^{-9}\ \text{rad} \times \left(\frac{g_\gamma / f_a}{10^{-12}\ \text{GeV}^{-1}}\right)\left(\frac{\sqrt{\rho_{\text{DM}}}}{\sqrt{0.3\ \text{GeV}\cdot\text{cm}^{-3}}}\right)\left(\frac{t}{1\ \text{s}}\right)$$

- **Experimental precision :** 10^{-9} **rad** Rowe et al. 2017, Rev.Sci.Instrum.

It is promising to probe the axion by the photon ring experiment !