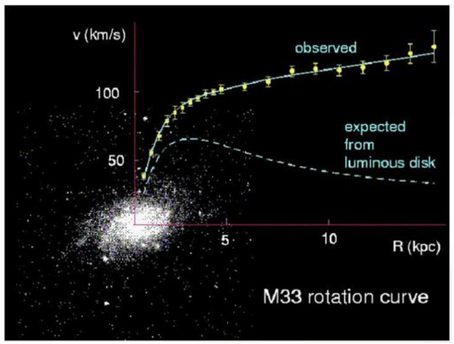
Soft Scattering Evaporation of Dark Matter Subhalos by Galactic Gases

Yugen Lin
（林宇根）
Insitute of Theoretical Physics, Chinese Academy of Sciences
2024.09.09

Based on Eur.Phys.J.C 83 (2023) 9, 808 and arXiv: 2311.11584

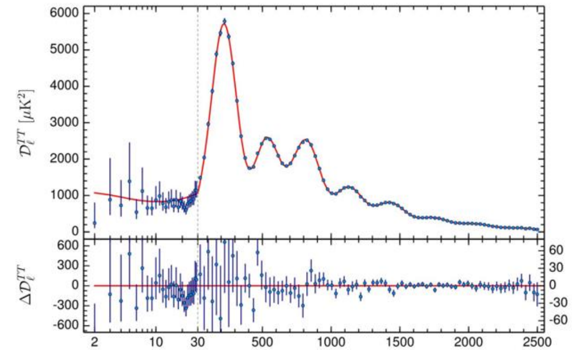
Introduction for dark matter (DM)

- Observational evidence:



Galaxy rotation curve

Bullet Cluster



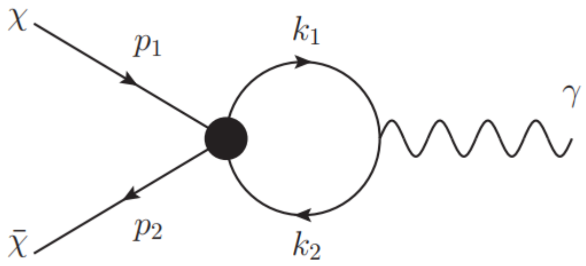
CMB

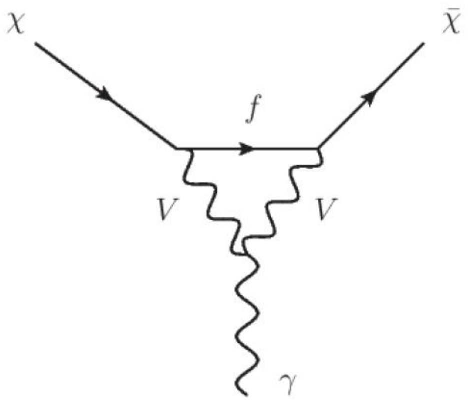
- Basic features of DM:

1. Electric neutrality
2. Don’t interact with baryon matter
3. Stable and long-life
4. moves slowly compared to the speed of light (Cold dark matter)
5. …

Electromagnetic factor of DM

- Electrically neutral DM (such as WIMP) can acquire an effective coupling to photons via loop effects





- For Dirac fermion χ , the EM effective operator has the following form:

Dim-5: electric dipole moment (EDM) : $\mathcal{L} \supset \mathcal{D}\bar{\chi}\sigma^{\mu\nu}\gamma^5\chi F_{\mu\nu}$

magnetic dipole moment (MDM) : $\mathcal{L} \supset \mu\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu}$

Dim-6: anapole moment (AM) : $\mathcal{L} \supset a\bar{\chi}\gamma^{\mu}\gamma^5\chi\partial^{\nu}F_{\mu\nu}$

charge radius: $\mathcal{L} \supset b\bar{\chi}\gamma^{\mu}\chi\partial^{\nu}F_{\mu\nu}$

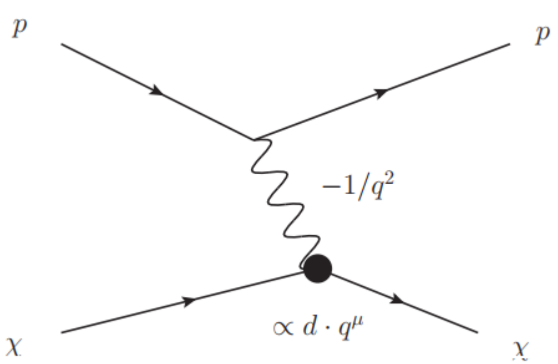
Dim-7: Rayleigh operator: $\mathcal{L} \supset c\bar{\chi}\chi F^{\mu\nu}F_{\mu\nu}$ or $\mathcal{L} \supset c\bar{\chi}\gamma^5\chi\tilde{F}^{\mu\nu}F_{\mu\nu}$ …

DM with EM Dipole Moment

- The Leading effective operator is the dimension-5 EM dipole operator ($\mathcal{D}$, μ represent electric and magnetic dipole moments)

$$\Delta\mathcal{L} = -\frac{i}{2}\bar{\chi}\sigma_{\mu\nu}(\mu + \gamma_5\mathcal{D})\chi F^{\mu\nu}$$

- Such effective electromagnetic (EM) operators allow for efficient soft scattering between dark matter and charged particles



➢ The typical feature for dipole-charge scattering is there is q^{-1} dependence in scattering amplitude and the cross-section has a well-known q^{-2} divergence

➢ Dipole-charge scattering is the last infrared divergent diagram with EM operators.

Dipole-charge scattering cross-section

- Non-relativistic scattering (DM and hot gas)

$$\frac{d\sigma}{d\cos\theta} = \begin{cases} \alpha\mathcal{D}^2 \frac{1}{v^2(1-\cos\theta)} & \text{(EDM)} \\ \alpha\mu^2 \frac{3m_\chi^2+2m_\chi m_p+2m_p^2}{2(m_\chi+m_p)^2(1-\cos\theta)} & \text{(MDM)} \end{cases}$$

$$\sigma_T(v) \equiv \int d\cos\theta \frac{d\sigma}{d\cos\theta}(1-\cos\theta)$$

$$= \begin{cases} 2\alpha\mathcal{D}^2 v^{-2} & \text{(EDM)} \\ \alpha\mu^2 \frac{3m_\chi^2+2m_\chi m_p+2m_p^2}{(m_\chi+m_p)^2} & \text{(MDM)} \end{cases}$$

v is relative velocity between DM and gas. For **EDM**, there is a explicit v^{-2} dependence
For **MDM**, the leading term is finite and not enhanced by v^{-2} .

- Relativistic scattering (DM and cosmic ray)

$$\frac{d\sigma}{dT_\chi} = \begin{cases} \frac{e^2\mathcal{D}^2}{8\pi T_\chi |\mathbf{p}|^2}(2E^2 - 2ET_\chi - m_\chi T_\chi) & \text{(EDM)} \\ \frac{e^2\mu^2}{8\pi T_\chi |\mathbf{p}|^2}(2|\mathbf{p}|^2 - 2ET_\chi + m_\chi T_\chi) & \text{(MDM)} \end{cases}$$

E is the total energy of proton
p is the incident proton's 3-momentum
T_χ is the DM kinetic energy after collision

$$\sigma_T = \begin{cases} \alpha\mathcal{D}^2\left[1 + m_p^2\left(\frac{1}{(m_\chi+m_p)^2+2m_\chi T_p} + \frac{2}{2m_pT_p+T_p^2}\right)\right] & \text{(EDM)} \\ \alpha\mu^2\left[1 + \frac{2m_\chi^2+m_p^2}{(m_\chi+m_p)^2+2m_\chi T_p}\right] & \text{(MDM).} \end{cases}$$

Boltzmann Equation

- For a flat FRW metric, the Boltzmann equation is

$$E\left(\partial_t - H\mathbf{p}\cdot\nabla_{\mathbf{p}}\right) f = C[f].$$

$$C[f] = \gamma(T) m_\chi \left[m_\chi T \nabla_{\mathbf{p}}^2 + \mathbf{p}\cdot\nabla_{\mathbf{p}} + 3\right] f(\mathbf{p})$$

$\gamma(T)$ represents the momentum exchange rate between DM and relativistic SM particles, which can be written as

$$\gamma(T) = \sum_i \frac{g_{\mathrm{SM}}}{6(2\pi)^3 m_\chi^3 T} \int dk k^5 \omega^{-1} g^{\pm}\left(1 \mp g^{\pm}\right) \frac{1}{8k^4} \int_{-4k^2}^{0} dt(-t)\overline{|\mathcal{M}|}^2.$$

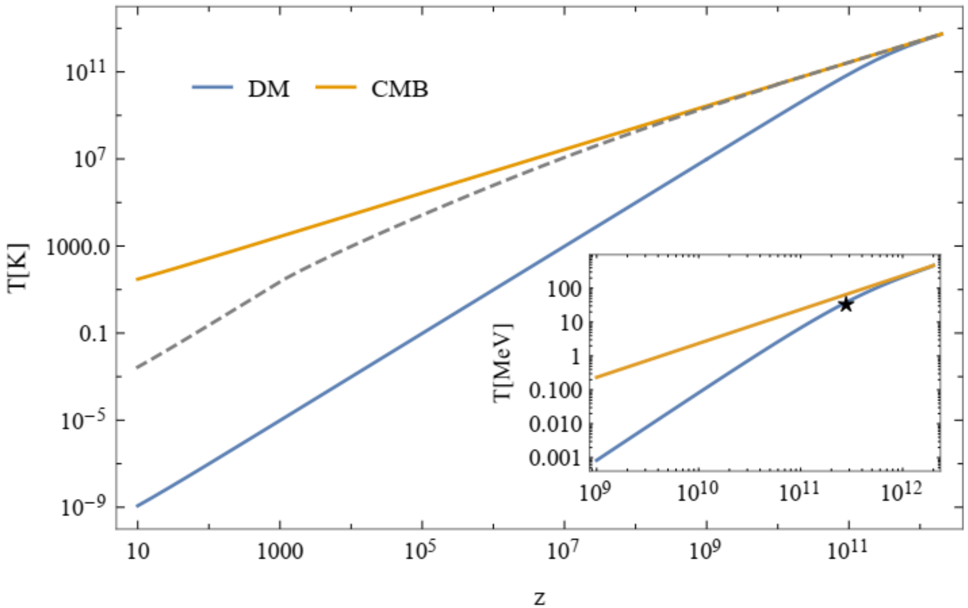
- When DM scatter with non-relativistic particles, the momentum transfer rate can be expressed as

$$\gamma = \begin{cases} \frac{8\alpha\mathcal{D}^2 m_\chi \rho_i}{\sqrt{2\pi}(m_i+m_\chi)^2}\left(\frac{T_\chi}{m_\chi} + \frac{T_i}{m_i}\right)^{-1/2} & \text{(EDM)} \\ \frac{12\alpha\mu^2 m_\chi \rho_i}{\sqrt{2\pi}(m_i+m_\chi)^2}\left[1 - \frac{m_i(m_i+4m_\chi)}{3(m_i+m_\chi)^2}\right]\left(\frac{T_\chi}{m_\chi} + \frac{T_i}{m_i}\right)^{1/2} & \text{(MDM)} \end{cases}$$

DM temperature evolution

The DM temperature evolution equation

$$(1+z)\frac{dT_\chi}{dz} = 2T_\chi + \frac{\gamma(T)}{H(z)}(T_\chi - T)$$



The two asymptotic behaviour of DM temperature:
At high temperature, $T_\chi = T_{CMB} \propto a^{-1}$
At low temperature, $T_\chi \propto a^{-2}$

The kinetic decoupling occurs when $H(T_{kd}) \sim \gamma(T_{kd})$

A larger coupling (gray dashed) makes decoupling slow and the kink at $z \approx 10^3$ is due to the decrease of cosmic ionization fraction

The choice of parameter: $m_\chi = 1\ \mathrm{GeV}$
$\mathcal{D} = 10^{-6}\mathrm{GeV}^{-1}$(blue solid) , $10^{-3}\mathrm{GeV}^{-1}$(gray dashed)
The decoupling temperature: around $30\mathrm{MeV}$ (**asterisk**)

Free-streaming scale and protohalo size

- After t_{kd}, DM can free stream from areas of high to low density, erasing the perturbations on scales smaller than the free-streaming length

$$\lambda_{fs} = a(t_0) \int_{t_{kd}}^{t_0} \frac{v(t)}{a(t)} dt.$$

The free-streaming length is the distance that DM can travel freely from the time of kinetic decoupling to present time t_0

- The smallest protohalos from free-streaming effects can be estimated as the DM mass contained inside a sphere of radius $\lambda_{fs}/2$

$$M_{fs} = \frac{4\pi}{3} \rho_m(t_0) (\frac{\lambda_{fs}}{2})^3.$$

- For GeV scale dark matter and a kinetic decoupling temperature around 30 MeV, the corresponding protohalo mass is around $10^{-7} M_{\odot}$

Jeans scale and Jeans mass

- The Jeans scale is a system's typical size for gravitational instability appearance and it is related to the DM temperature

$$\lambda_J = c_s \sqrt{\frac{\pi}{G\rho_m}}.$$

C_s is the sound speed, $C_s \approx \sqrt{T/m}$

- When $\lambda > \lambda_J$, the system will become unstable and gravitational perturbation can sustainingly grow. The DM mass contained inside a sphere of radius $\lambda_J/2$ is the Jeans mass

$$M_J = \frac{4\pi}{3}\rho_m\left(\frac{1}{2}\lambda_J\right)^3 = \frac{\pi^{5/2}}{6}\frac{c_s^3}{G^{3/2}\rho_m^{1/2}}.$$

- The DM temperature during structure formation ($z \sim 20 - 30$) is around $10^{-8}\mathrm{K}$, and the corresponding Jeans mass is around $10^{-10} M_\odot$

- As a conservative choice, we may adopt the larger of the two as the small-scale structure cut-off, i.e. the free-streaming scale to give the smallest protohalo mass.

Subhalo heating rate due to hot gas

When subhalo colliders with ionized hot gas, the thermally averaged energy transfer rate of per unit time is

$$\frac{\mathrm{d}\Delta E_p}{\mathrm{d}t} = \frac{m_\chi \rho_p}{(m_\chi + m_p)} \int d^3 v_p f_p(v_p) \int d^3 v_\chi f_\chi(v_\chi) \times \sigma_T(|\vec{v}_\chi - \vec{v}_p|) |\vec{v}_\chi - \vec{v}_p| [\vec{v}_{\rm cm} \cdot (\vec{v}_p - \vec{v}_\chi)]$$

$$f_\chi(\vec{v}_\chi) = \frac{1}{n} e^{-|\vec{v}_\chi - \vec{v}_0|^2/\sigma_v^2}$$

$$f_p(\vec{v_p}) = \frac{1}{n} e^{-m_p|\vec{v}_p - \vec{v}_{p0}|^2/2k_B T}$$

In the limit that energy transfer rate is dominated by their relative velocity v

$$\frac{\mathrm{d}\Delta E_\chi}{\mathrm{d}t} = \begin{cases} \frac{2\alpha \mathcal{D}^2 m_p m_\chi \rho_p v}{(m_p+m_\chi)^2} & \text{(EDM)} \\ 3\alpha\mu^2 \left[1 - \frac{m_p(m_p+4m_\chi)}{3(m_p+m_\chi)^2}\right] \frac{m_p m_\chi \rho_p v^3}{(m_p+m_\chi)^2}. & \text{(MDM)} \end{cases}$$

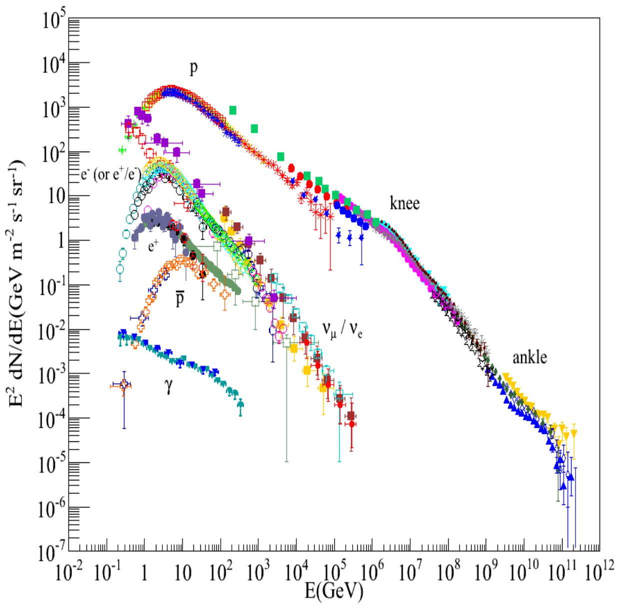
- For EDM, the heating rate is proportional to relative veolocity v between DM and gas
- For MDM, the heating rate is proportional to v^3

Subhalo heating rate due to cosmic ray

When subhalo colliders with cosmic ray, the heating rate is obtained by integrating transfer cross-section σ_T with the cosmic ray flux intensity Φ

$$\frac{\mathrm{d}\Delta E_\chi}{\mathrm{d}t} = \int \Delta E_\chi n v \, \mathrm{d}\sigma$$
$$= \int \mathrm{d}T_i \mathrm{d}\Omega \left(\frac{\mathrm{d}\Phi}{\mathrm{d}T_i \mathrm{d}\Omega} \right) \int \frac{\mathrm{d}\sigma}{\mathrm{d}T_\chi} T_\chi \mathrm{d}T_\chi$$

- The proton energy spectrum is an approximate $E^{-2.7}$ power-law above the GeV scale.
- So far the cosmic ray energy spectrum has only been measured locally at the Earth.



The relative intensity distribution in other location can be modeled as

$$\frac{I(r,z)}{I(r_\odot,0)} = \frac{\mathrm{sech}(r/r_{\mathrm{CR}})}{\mathrm{sech}(r_\odot/r_{\mathrm{CR}})} \cdot \mathrm{sech}(z/z_{\mathrm{CR}})$$

The volume-averaged proton flux within 1 kpc from the galactic center is about 2.1 times of that at the Sun's location.

Escaped time scale

- The time scale for an average DM particle to be heated to its host subhalo's escaped velocity can be estimated as

$$\tau_{\rm esc.} = \frac{1}{2} m_\chi \left(v_{\rm esc}^2 - v_{\rm rms}^2\right) \cdot \left(\frac{{\rm d}\Delta E_\chi}{{\rm d}t}\right)^{-1}$$

$$\tau_{\rm esc.} = \frac{(m_\chi + m_p)^2}{m_p^2} \cdot \left(\frac{v_{\rm esc}^2 - v_{\rm rms}^2}{2v^2}\right) (\sigma_T v n_p)^{-1} \, .$$

The time scale in term of σ_T

- The Stability of subhalos would require $\tau_{esc} > 10^{10}$ yr by collision with either gas or cosmic rays . By assuming the survival of subhalos, we can get an upper limit on the DM's dipole form factor.

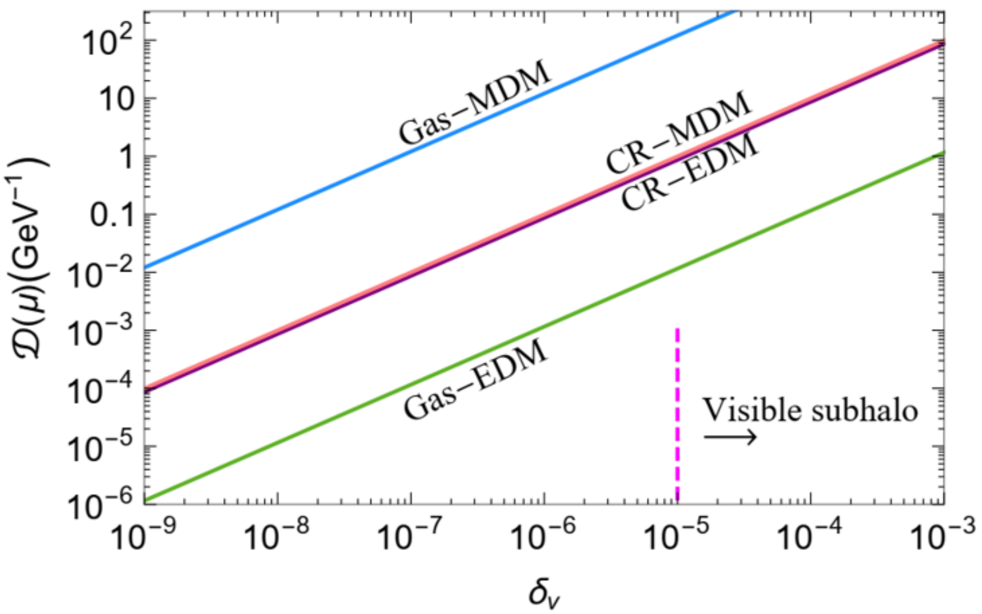
- DM particle's root-mean-square velocity v_{rms} , escaped velocity v_{esc} and velocity dispersion δ_v depend on the subhalo size

$$\delta_v \approx 3.9 \ {\rm km/s} \left(\frac{M}{10^6 M_\odot}\right)^{1/3}$$

For a Maxwellian distribution:
v_{rms}=1.73 δ_v , v_{esc}= 2.44 δ_v

Dipole moment limits

$m_\chi = 1\mathrm{GeV}$

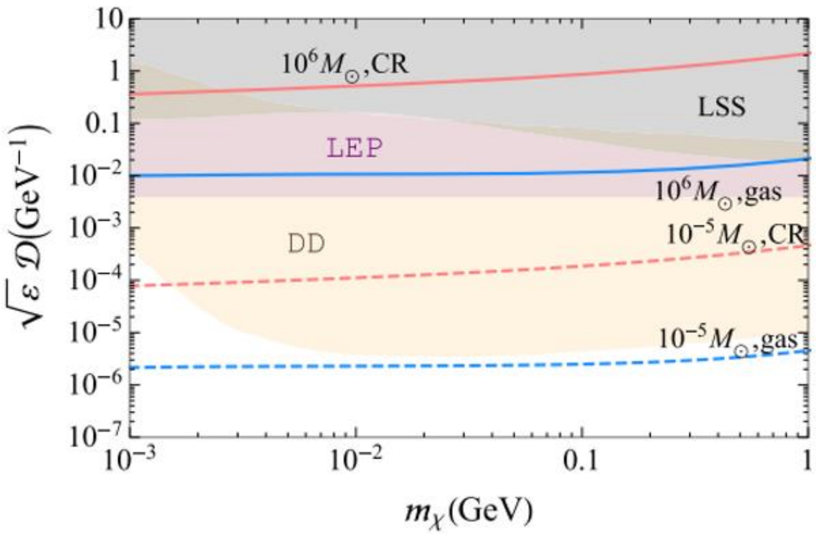


- Cosmic ray collisions are insensitive to subhalo's velocity and their limits on EDM and MDM are comparable.
- In non-relativistic gas-DM collision, the v^{-2} dependence in EDM σ_T leads to faster heating than MDM and a significantly more stringent limit.

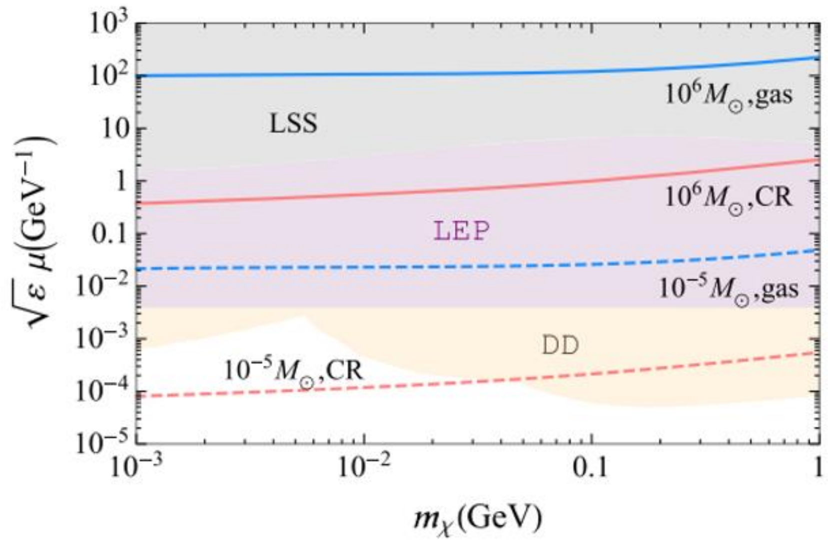
Dipole moment $\mathcal{D}$ and μ limits for different subhalo's velocity dispersion (corresponding to different subhalo size) that leads to 10^{10} yr evaporation.

Dipole moment limits

EDM limit



MDM limit



- Dark matter EDM and MDM limits that leads to 10^{10} yr evaporation versus the different DM particle mass from soft collisional heating on gas(blue line) with $v = 10^{-4}$ and relativistic scattering with cosmic rays(red line).

- The solid and dashed line respectively represent the evaporation limits of **visible** subhalo ($10^6 M_\odot$)and a much lower **invisible** subhalo ($10^{-5} M_\odot$) that allows the dipole moment sensitivity dips below the current direct-search dipole limits.

DM with Coulomb-like Interaction

- The typical scenario to realize DM Coulomb-like interaction is the kinetically mixed $U(1)'$model

$$\mathcal{L} \supset -eA_\mu \bar{p}\gamma^\mu p + \epsilon g_\chi A_\mu \bar{\chi}\gamma^\mu \chi$$

- We use the Debye screening length to be the maximal impact parameter to regulate the forward scattering singularity of momentum-transfer integral

$$\lambda_D = \sqrt{\frac{T_p}{e^2 x_e n_p}}, \quad \Longrightarrow \quad \theta_{\min} = \arctan \frac{\epsilon e g_\chi}{4\pi \mu_{\chi p} v^2 \lambda_{\rm D}} \approx \frac{\epsilon e g_\chi}{6\pi T_p \lambda_{\rm D}}$$

- In terms of the cutoff scattering angle, the momentum-transfer cross section is

$$\sigma_{\rm T} = \frac{2\pi\epsilon^2 \alpha \alpha_\chi}{\mu_{\chi p}^2 v^4} \ln\left[\csc^2\left(\frac{\theta_{\min}}{2}\right)\right] \approx \frac{2\pi\epsilon^2 \alpha \alpha_\chi}{\mu_{\chi p}^2 v^4} \ln\left(\frac{9T_p^3}{4\pi\epsilon^2\alpha^2\alpha_\chi x_e n_p}\right)$$

- Be different from dipole-charge scattering, such cross-section $\sigma = \sigma_0 \boldsymbol{v}^{-4}$

Evaporation of DM Subhalos

$$\frac{\mathrm{d}\Delta E_p}{\mathrm{d}t} = \frac{m_\chi \rho_p}{(m_\chi + m_p)} \int d^3 v_p f_p(v_p) \int d^3 v_\chi f_\chi(v_\chi) \times \sigma_T(|\vec{v}_\chi - \vec{v}_p|)\, |\vec{v}_\chi - \vec{v}_p|\, [\vec{v}_{\mathrm{cm}} \cdot (\vec{v}_p - \vec{v}_\chi)]$$

$$f_\chi(\vec{v}_\chi) = \frac{1}{n} e^{-|\vec{v}_\chi - \vec{v}_0|^2/\sigma_v^2}$$

$$f_p(\vec{v}_p) = \frac{1}{n} e^{-m_p|\vec{v}_p - \vec{v}_{p0}|^2/2k_B T}$$

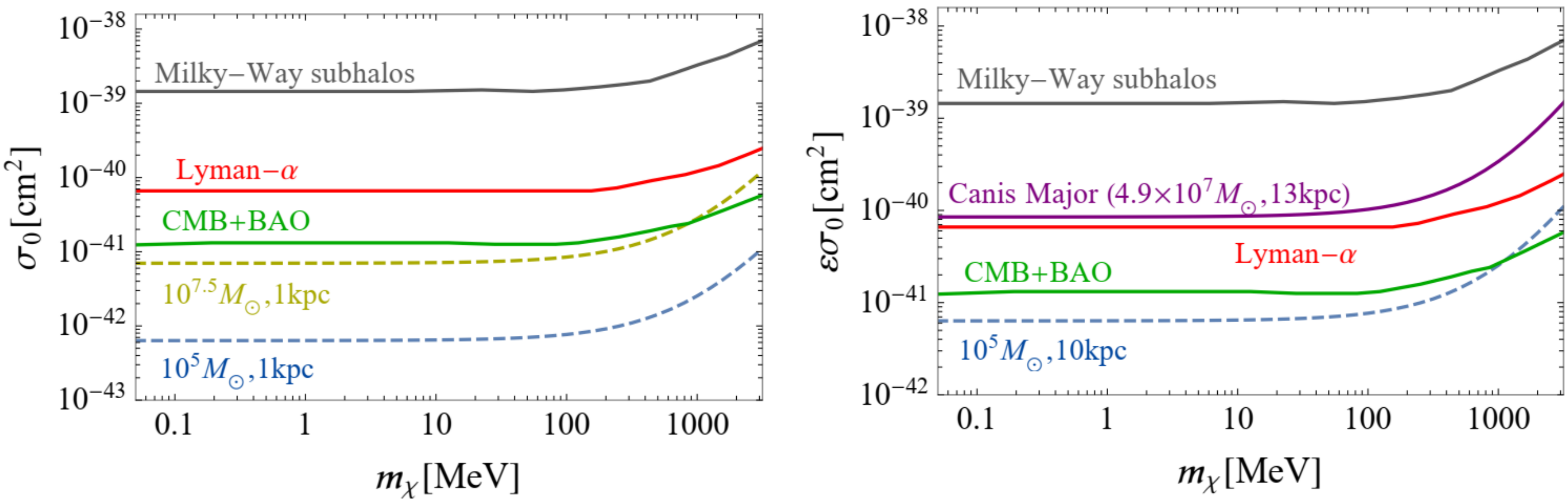
$$\frac{d\Delta E_\chi}{dt} = \frac{m_\chi \rho_p \sigma_0}{(m_\chi + m_p)^2 \sqrt{2\pi} u_{\mathrm{th}}} \left[2 \frac{T_p - T_\chi}{u_{\mathrm{th}}^2} e^{-\frac{r^2}{2}} + m_p \frac{F(r)}{r} \right]$$

$$F(r) = \mathrm{erf}\left(\frac{r}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}} e^{-r^2/2} r \qquad u_{th}^2 = \frac{T_p}{m_p} + \frac{T_\chi}{m_\chi},\ r = \frac{v}{u_{th}}$$

In the limit that energy transfer rate is dominated by their relative velocity v, the heating rate can take a much simpler expression

$$\frac{\mathrm{d}\Delta E_\chi}{\mathrm{d}t} = \frac{m_\chi m_p \rho_p \sigma_0}{(m_\chi + m_p)^2 v}$$

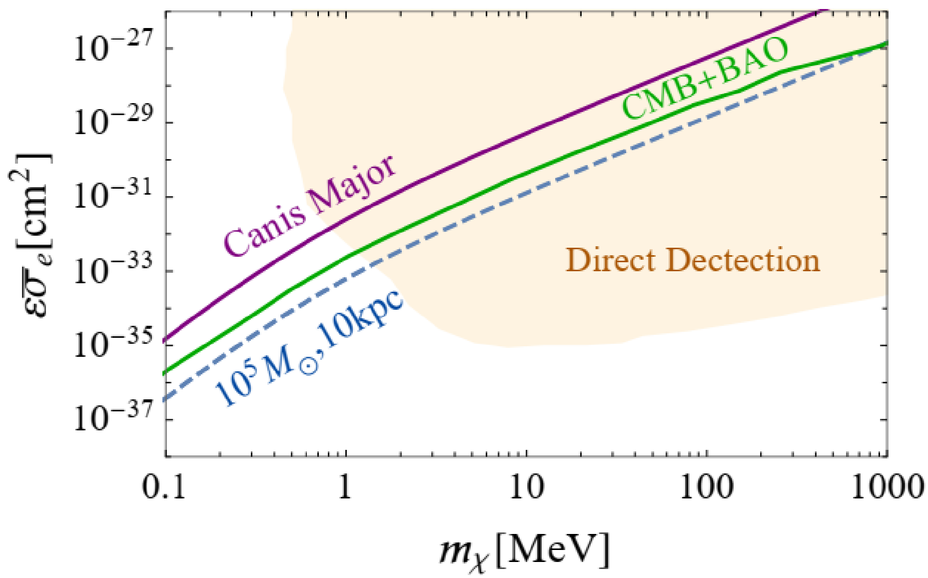
Evaporation Constraint



- For subhalos at a 1 kpc orbit radius from the center of the Milky Way , the corresponding evaporation limit on σ_0 is one order of magnitude stronger than previous CMB+BAO constraint for a DM mass range below GeV scale.

- Fully ionized galactic gas can evaporate subhalos below $10^{7.5} M_{\odot}$ at a 1 kpc orbit radius, at a larger 10 kpc orbit, the affected subhalo mass will drop to $10^{5} M_{\odot}$. The evaporation limit of Canis Major substructure is comparable with Lyman-α.

Evaporation Constraint



$$\bar{\sigma}_e \equiv \frac{\overline{|\mathcal{M}|^2}\Big|_{q=q_{\mathrm{ref}}}}{16\pi\,(m_e+m_\chi)^2} = \frac{\mu_{\chi e}^2\,\overline{|\mathcal{M}|^2}\Big|_{q=q_{\mathrm{ref}}}}{16\pi m_\chi^2 m_e^2}$$

$$\overline{|\mathcal{M}|^2} = \frac{16\epsilon^2 e^2 g_\chi^2 m_e^2 m_\chi^2}{t^2}$$

$$\bar{\sigma}_e \equiv \frac{1}{16\pi\,(m_e+m_\chi)^2}\overline{|\mathcal{M}|^2}\Big|_{q=q_{\mathrm{ref}}} = \frac{16\pi\epsilon^2\alpha\alpha_\chi\mu_{\chi e}^2}{q_{\mathrm{ref}}^4}.$$

$\sigma_0 \to \bar{\sigma}_\mathrm{e}$: $\sigma_0 = \bar{\sigma}_e \times \dfrac{\alpha^4 m_e^4 \xi}{8\mu_{\chi p}^4}$ ξ is the Debye logarithm in transfer cross section.

- The direct-detection experiments can also constrain the same t-channel scattering process, which can be the complementarity of cosmological probes.
- The evaporation limits of subhalos can readily extend sub-MeV DM mass range where the direct detection experiments lose sensitivity.

Summary

- DM can acquire a electromagnetic factor which allows for efficient soft scattering between DM and environmental charged particles.

- We have studied DM kinetic decoupling in the Early Universe and temperature evolution, then evaluate the corresponding smallest protohalo mass within current direct detection limits.

- In the late Universe, for subhalos located in inner galactic regions, we calculate their evaporation rate by colliding with galactic ionized particles including hot gas and cosmic ray.

- For dipole-charge scattering, when satisfying the current sub-GeV direct detection limits, the galaxy's ionized particles are capable of evaporating subhalos below $10^{-5} M_{\odot}$.

- For Coulomb-like scattering, when satisfying the current sub-GeV CMB+BAO limits, the galaxy's ionized particles are capable of evaporating subhalos below $10^{7.5} M_{\odot}$.