(particle)Dark matter and gravitational wave

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Outline

1. Motivation ,Gravitational wave (GW) and electroweak strong first-order phase transition (SFOPT) in a nutshell

2. Heavy dark matter from SFOPT and GW signals

Case I: Dark matter (DM) induced SFOPT (wall velocity)

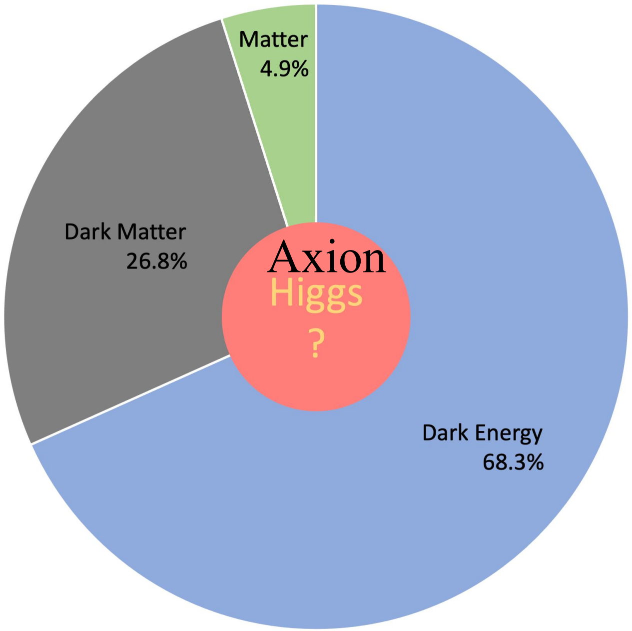
Case II: anti-filtered (gauged)Q-ball DM

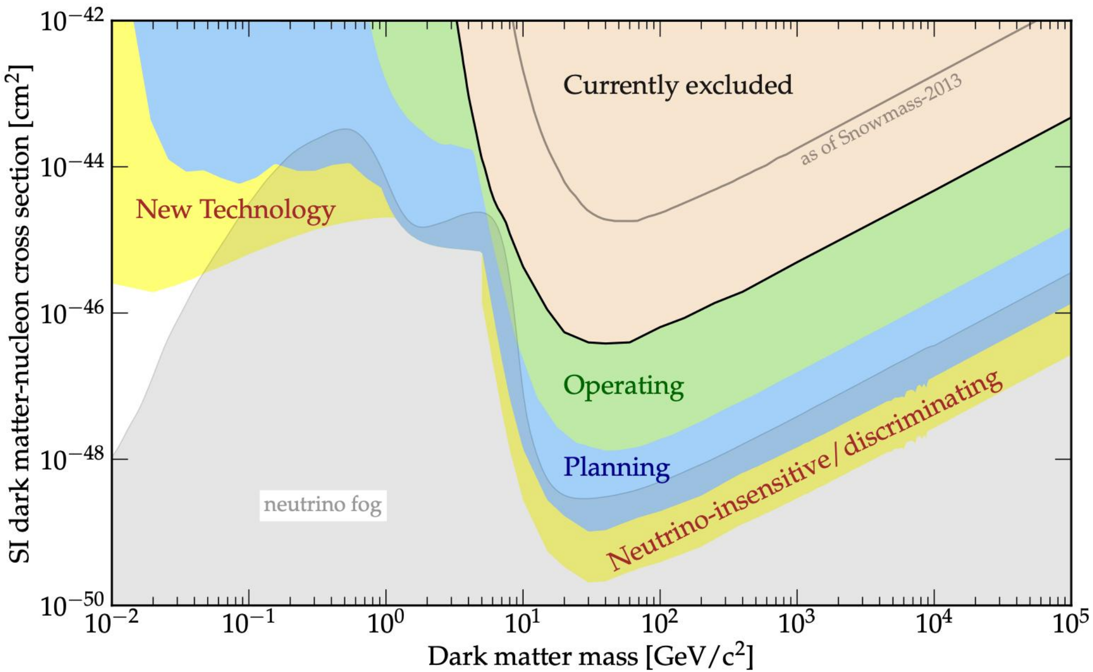
Case III: filtered DM

3. Ultralight (axion) DM and GW detection

4.Summary and outlook

Motivation DM theory and experiments status





迄今没有看到预期的信号，对于WIMP 模型剩余的参数空间往往不够自然.

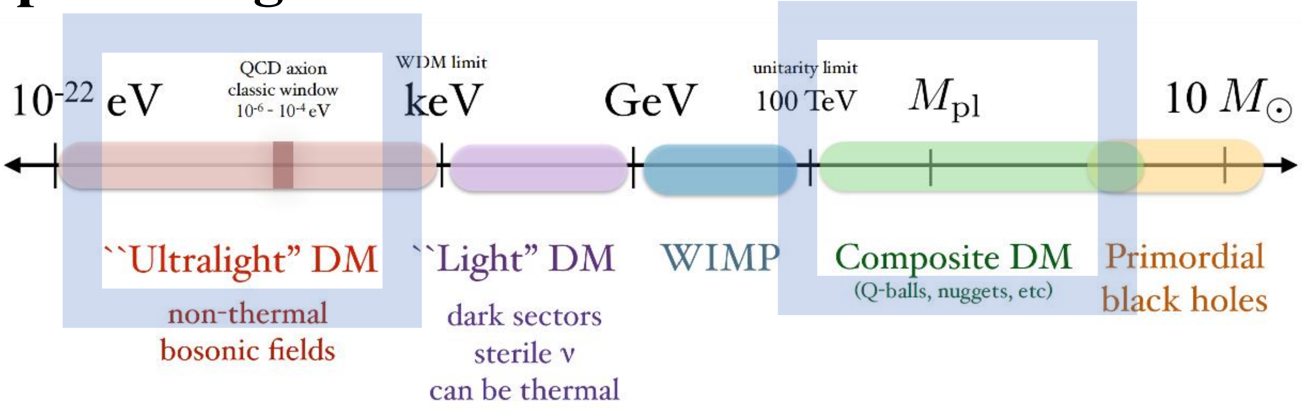
因此近年来以轴子、类轴子为代表的其他类型的暗物质模型也开始更多地受到关注。

新的暗物质产生机制？新的暗物质探测方法？

Motivation DM theory and experiments status

What is the microscopic nature of DM?

No expected signals at LHC and DM direct search.



arXiv: 1904:07915

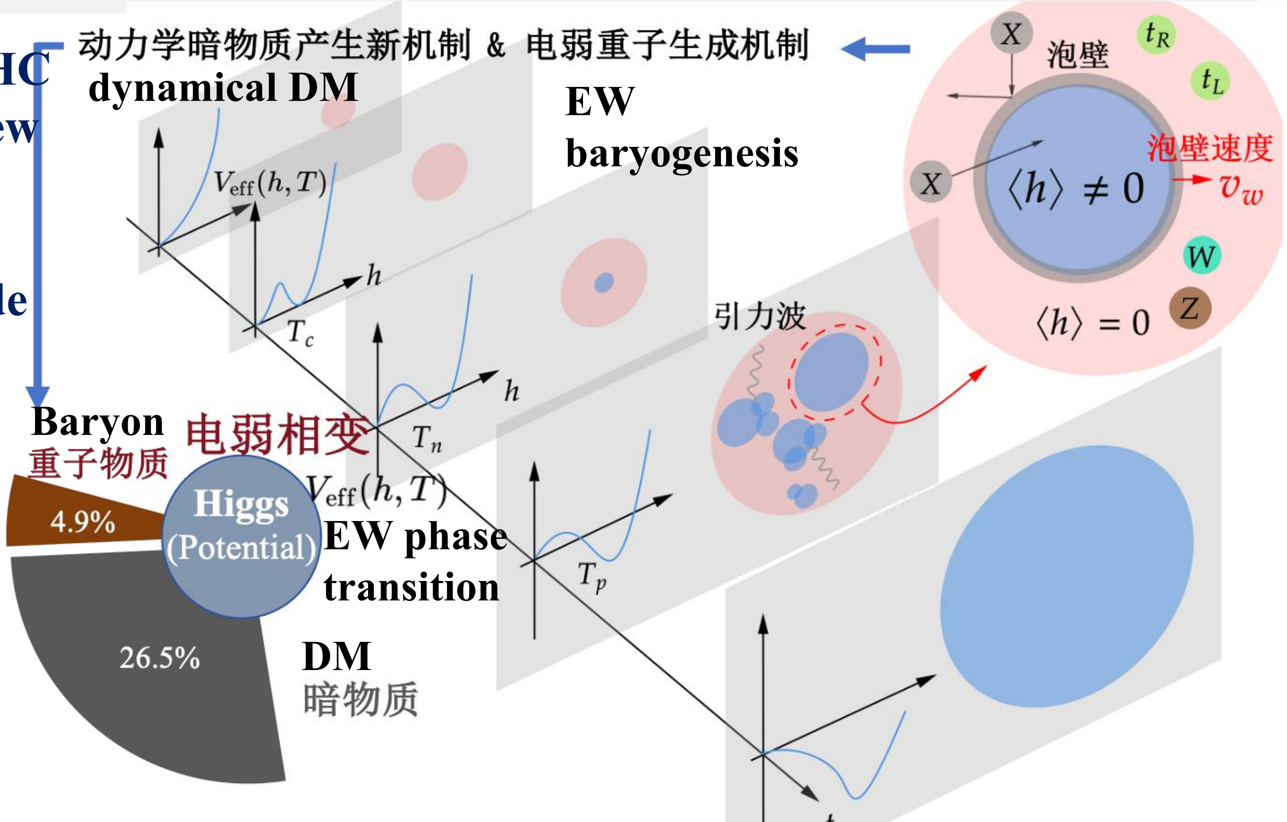
- **new DM mechanism beyond freeze out/in: cosmic phase transition**
- **new detection method: GW detector (LISA, TianQin, Taiji, aLIGO, FAST, SKA, NanoGrav, Cosmic Explorer…)**

Motivation DM in post-Higgs and GW Era

The observation of Higgs@LHC and GW@LIGO initiates a new era of exploring DM by GW.

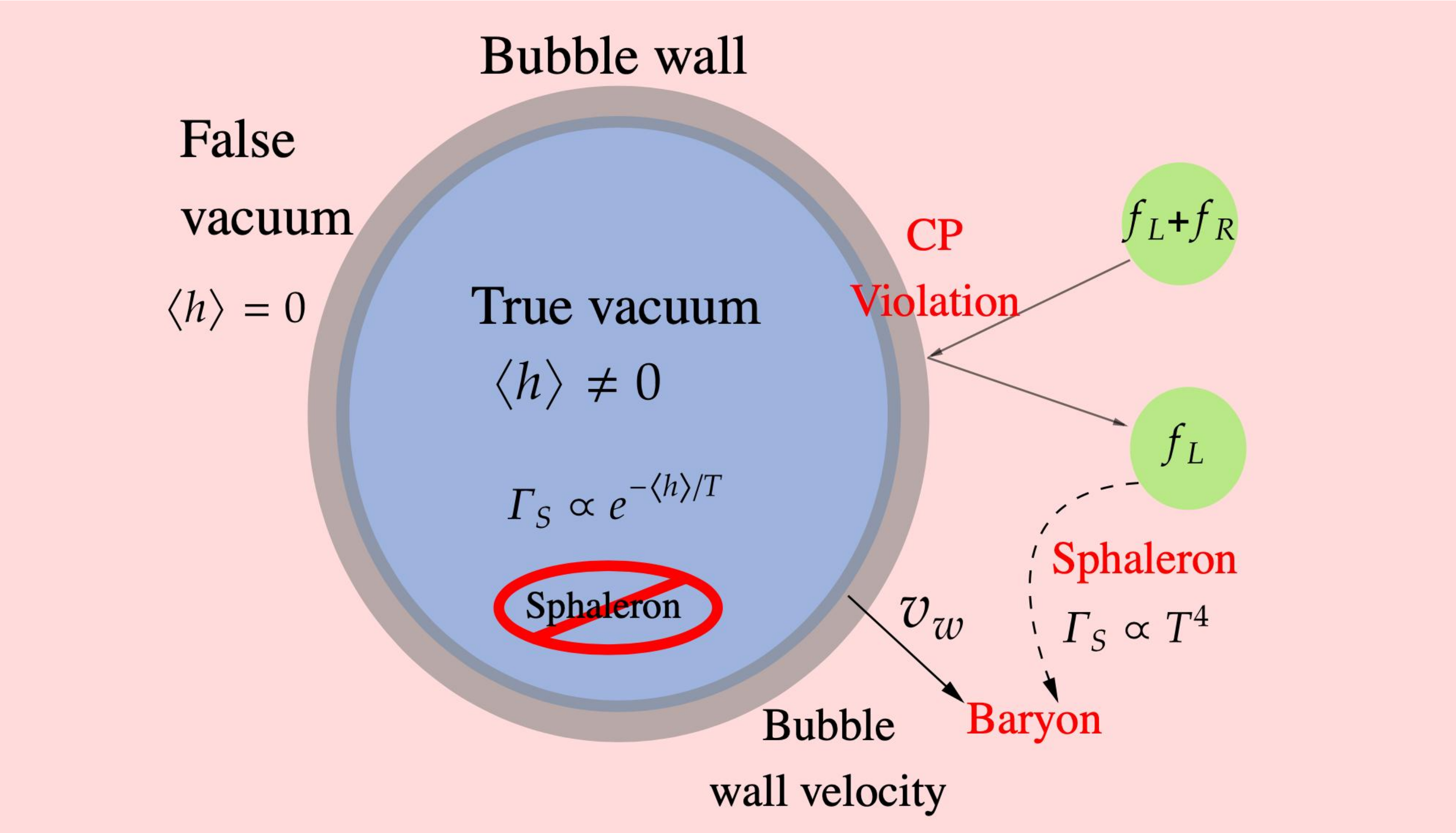
SFOPT by Higgs could provide a new approach for DM production.

Higgs' deep connections to cosmology, such as EW baryogenesis, **DM** testable **by GW signals.**



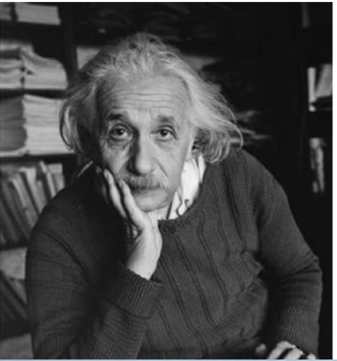
What is the role of Higgs in the early universe?

为什么要研究相变引力波

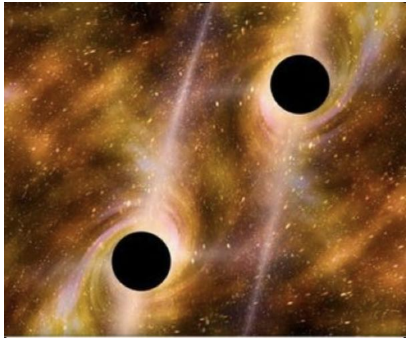


What is GW ?

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$



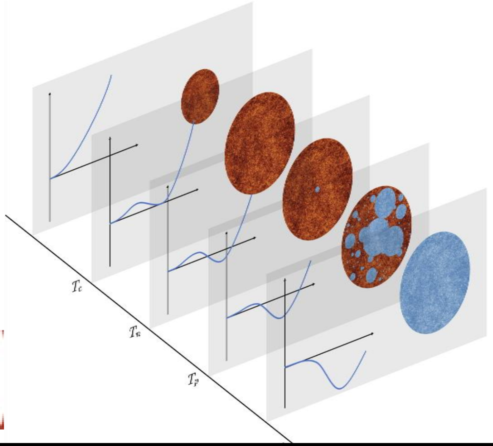
Isolated sources: quadrupole radiation

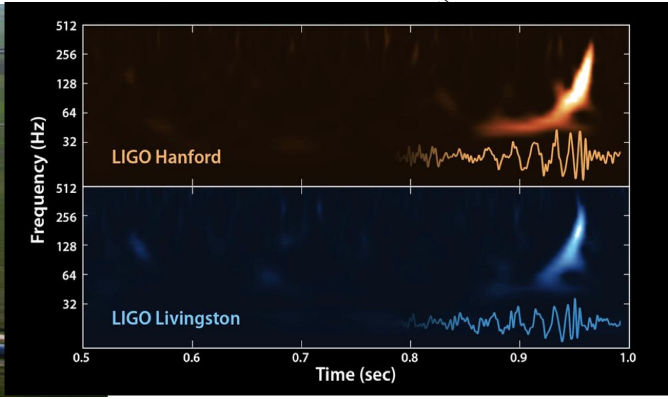


$$h_{ij} \simeq \frac{2G}{c^4 r} \ddot{Q}_{ij}^{TT}(t - r/c)$$

Stochastic sources: anisotropic stress tensor

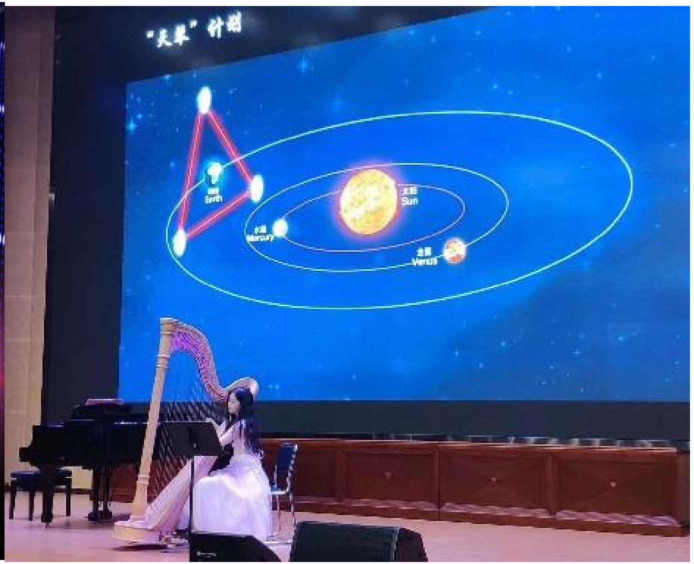
$$\Pi_{ij}(\mathbf{x}, t)$$

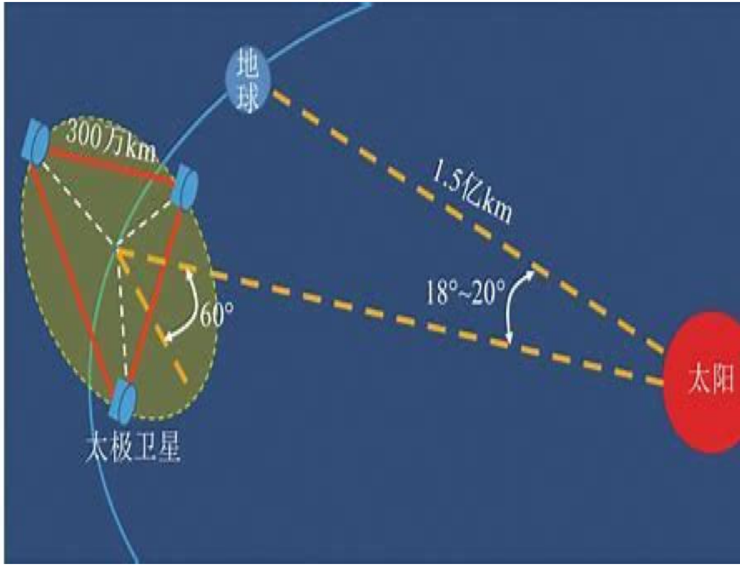




GW experiments

LISA/TianQin/Taiji ~2034



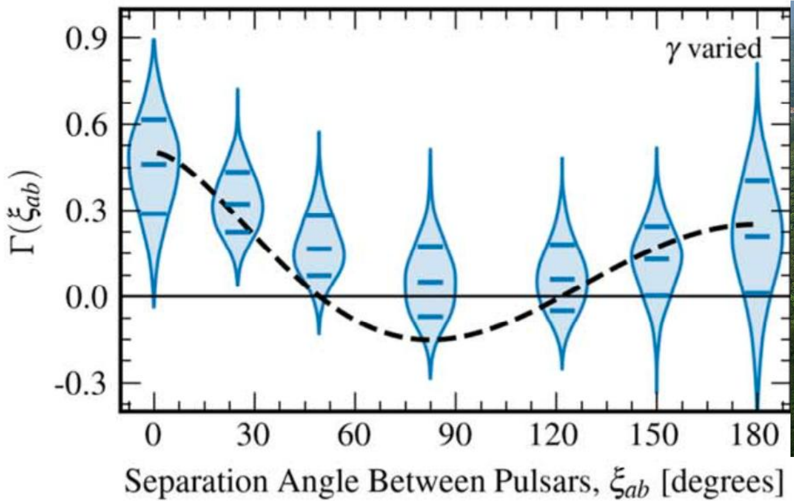


"天琴"

"Harpe in space"

Radio telescope and pulsar timing array

2023 June 29th： NANOGrav, EPTA, InPTA, Parkes PTA, CPTA



Hellings-Downs correlation curve

First observation of stochastic GW

FAST

SKA

High sensitivity sub μJy

GW in a nutshell

The quadruple nature of GW !

EM wave radiation

$$\ddot{\boldsymbol{d}} = e\ddot{\boldsymbol{x}}$$

GW radiation

$$\ddot{\boldsymbol{d}} = \sum_{\text{particles } A} m_A \ddot{\boldsymbol{x}}_A = \dot{\boldsymbol{p}} = 0$$

momentum conservation

$$L_{\text{electric quadrupole}} = \frac{1}{20}\dddot{Q}^2 \equiv \frac{1}{20}\dddot{Q}_{jk}\dddot{Q}_{jk}$$

$$Q_{jk} \equiv \sum_A e_A \left(x_{Aj}x_{Ak} - \frac{1}{3}\delta_{jk}r_A{}^2\right)$$

$$L_{\text{mass quadrupole}} = \frac{1}{5}\langle \dddot{I}^2 \rangle \equiv \frac{1}{5}\langle \dddot{I}_{jk}\dddot{I}_{jk} \rangle$$

$$I_{jk} \equiv \sum_A m_A \left(x_{Aj}x_{Ak} - \frac{1}{3}\delta_{jk}r_A{}^2\right)$$

General GW in the early universe

$$\ddot{h}_{ij}(\mathbf{x},t) + 3H\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}h_{ij}(\mathbf{x},t) = 16\pi G\,\Pi_{ij}(\mathbf{x},t)$$

各向异性
剪切应力张量

- ✓ phase transition:TeV physics (focus)
- ✓ cosmic defects: cosmic string, domain wall…

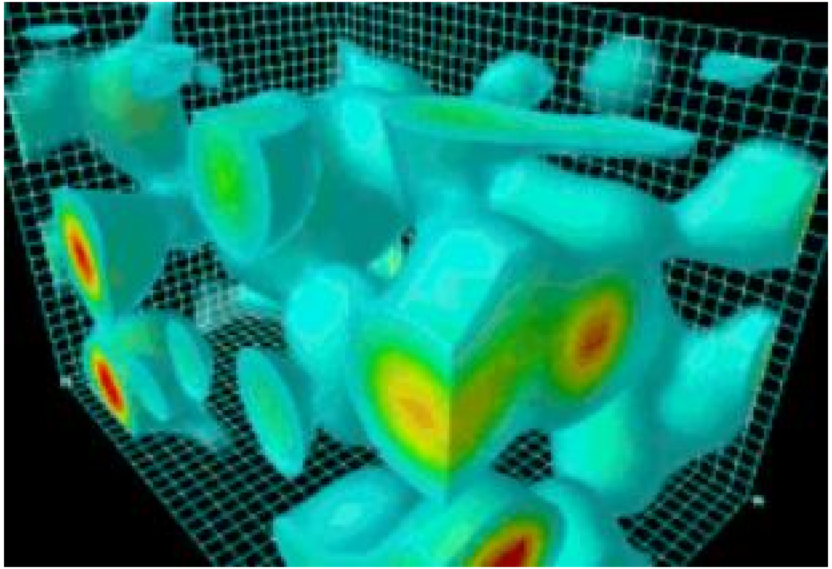
Possible sources of tensor anisotropic stress in the early universe

- Scalar field gradients $\Pi_{ij} \sim [\partial_i\phi\partial_j\phi]^{TT}$ eg. Collisions of bubble walls, cosmic string
- Bulk fluid motion $\Pi_{ij} \sim [\gamma^2(\rho+p)v_iv_j]^{TT}$ eg. Sound waves and turbulence in the fluid
- Gauge fields $\Pi_{ij} \sim [-E_iE_j - B_iB_j]^{TT}$ eg. Primordial magnetic fields (MHD turbule
- Second order scalar perturbations, Π_{ij} from a combination of $\partial_i\Psi, \partial_i\Phi$
- … arXiv:1801.04268

原初引力波——量子涨落

- Inflation provides the primordial seeds of our Universe.

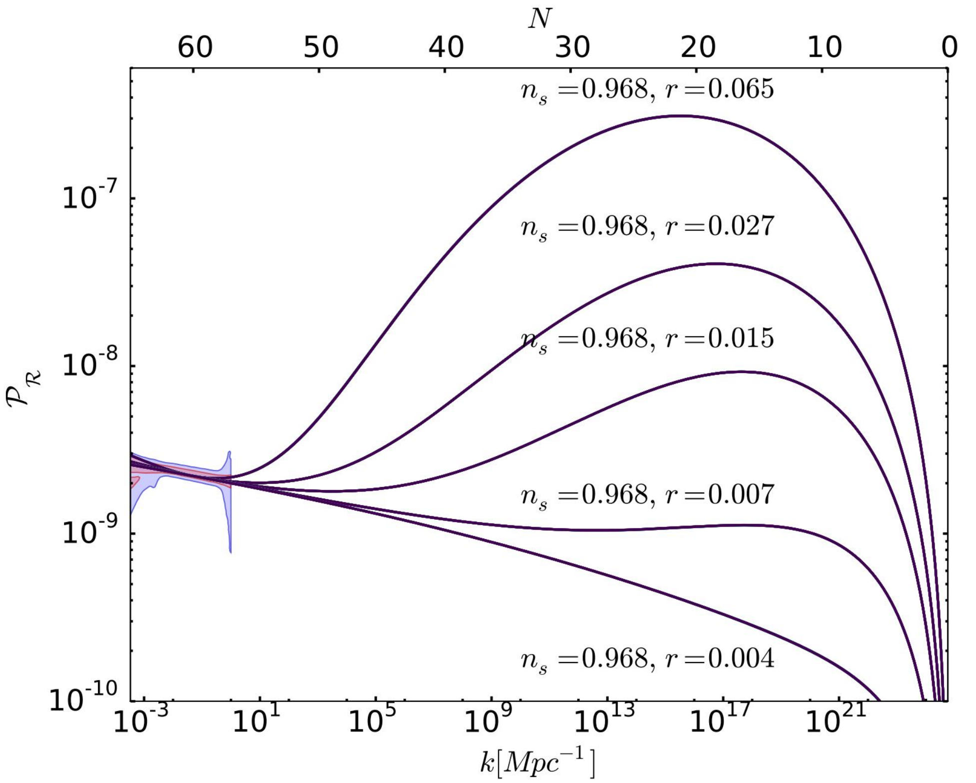
- 引力波几乎每天都被LIGO和其他引力波探测器探测到，但原初引力波信号比这些探测器能检测到的信号弱几个数量级。预计下一代探测器将具有足够的灵敏度，以捕捉到这些最早的涟漪。

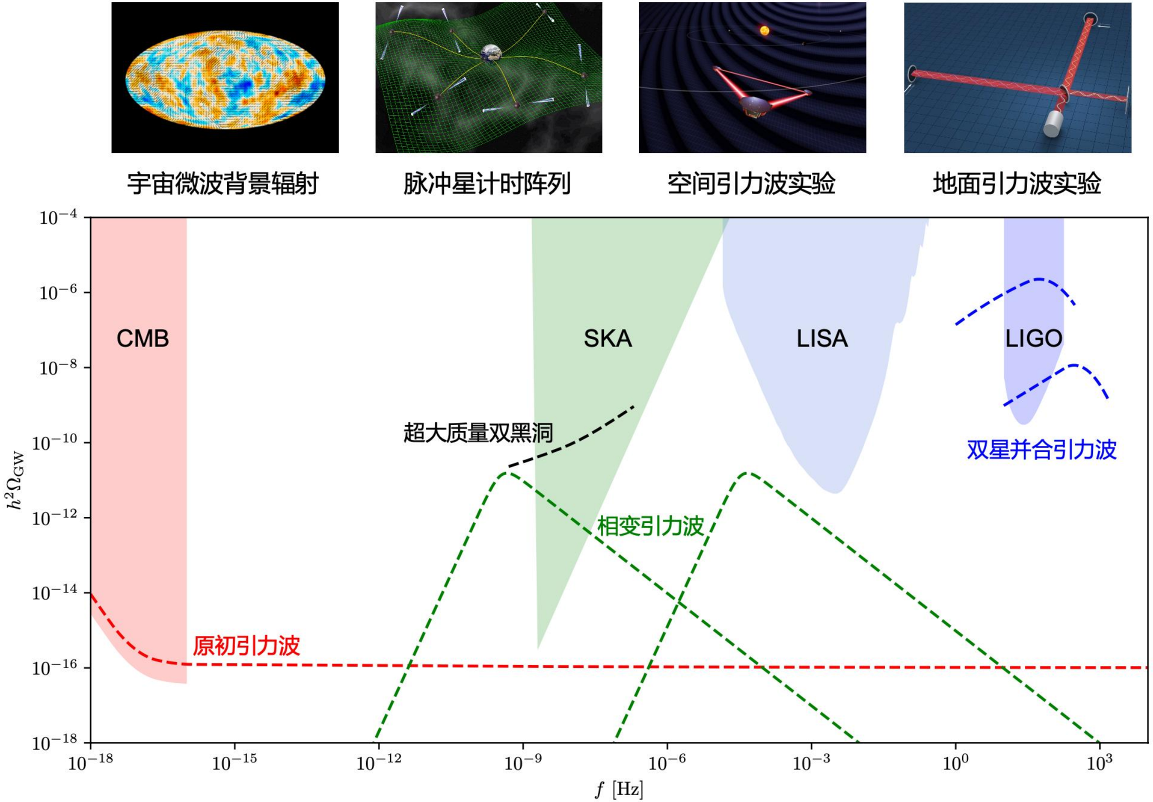


$$\Delta x \Delta p \geq \hbar$$

原初引力波

- CMB-S4 using the resulting establishment of the current reference design of the primordial gravitational-wave component of the Stage-4 experiment, optimized to achieve our science goals of detecting primordial gravitational waves for r>0.003 at greater than 5σ, or, in the absence of a detection, of reaching an upper limit of r<0.001 at 95% CL.

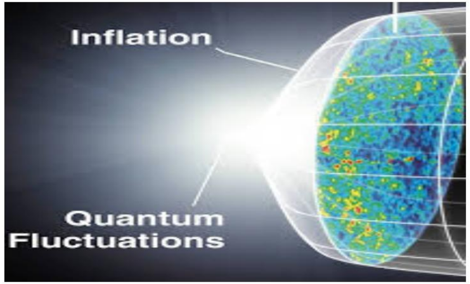




引力波能谱以及引力波的多波段观测示意图. 虚线为不同波源产生的引力波的能谱

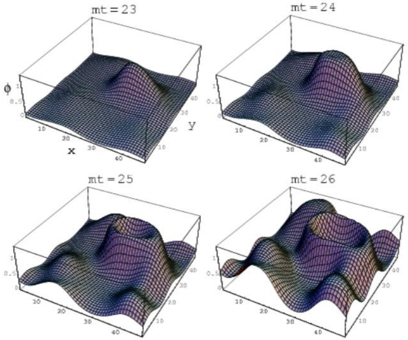
宇宙学起源的引力波

Inflationary Period



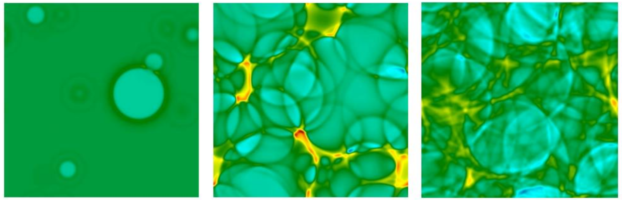
(Image: Google Search)

(p)Reheating



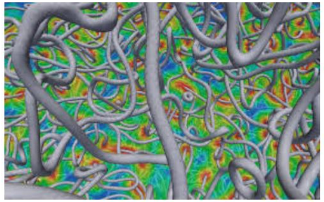
(Fig. credit: Phys.Rev. D67 103501)

Phase Transitions



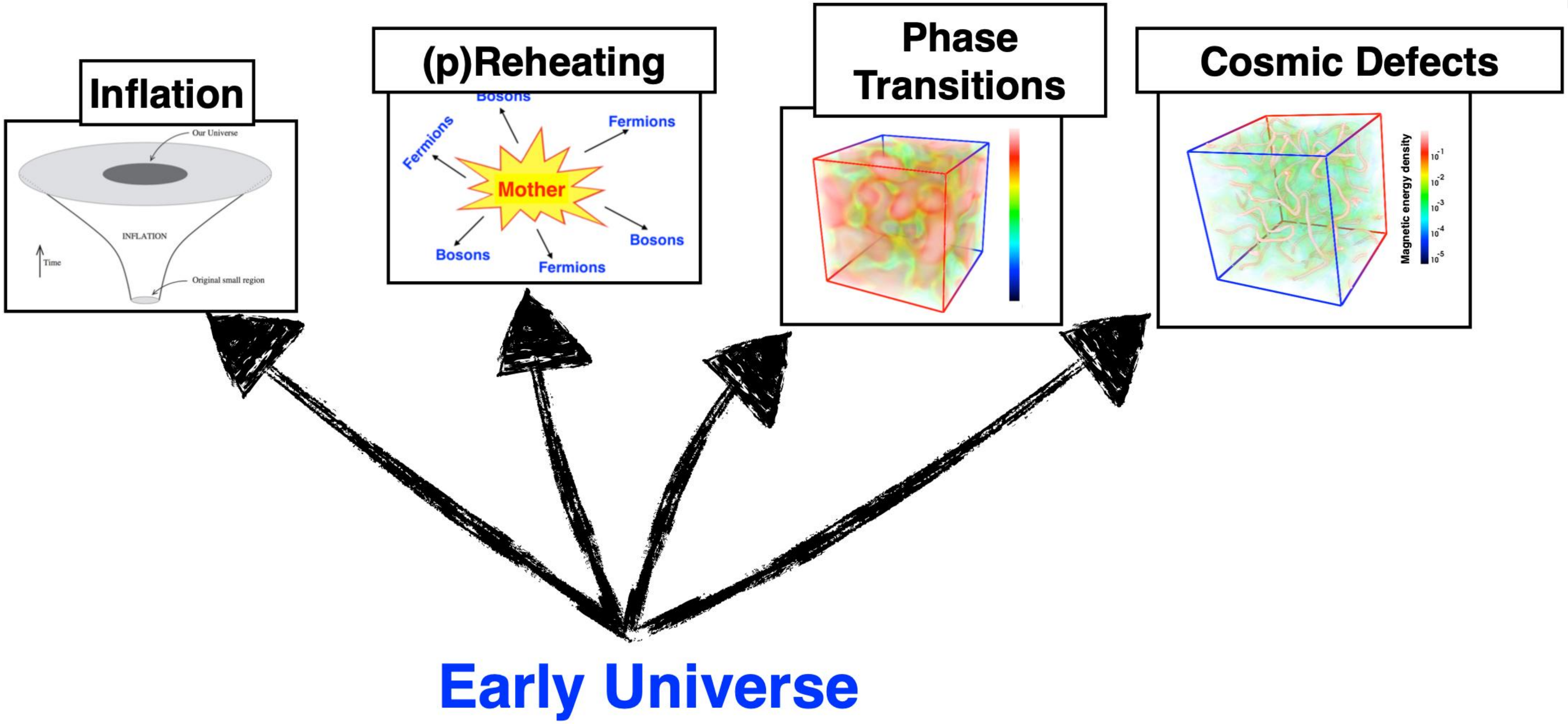
(Image: PRL 112 (2014) 041301)

Cosmic Defects

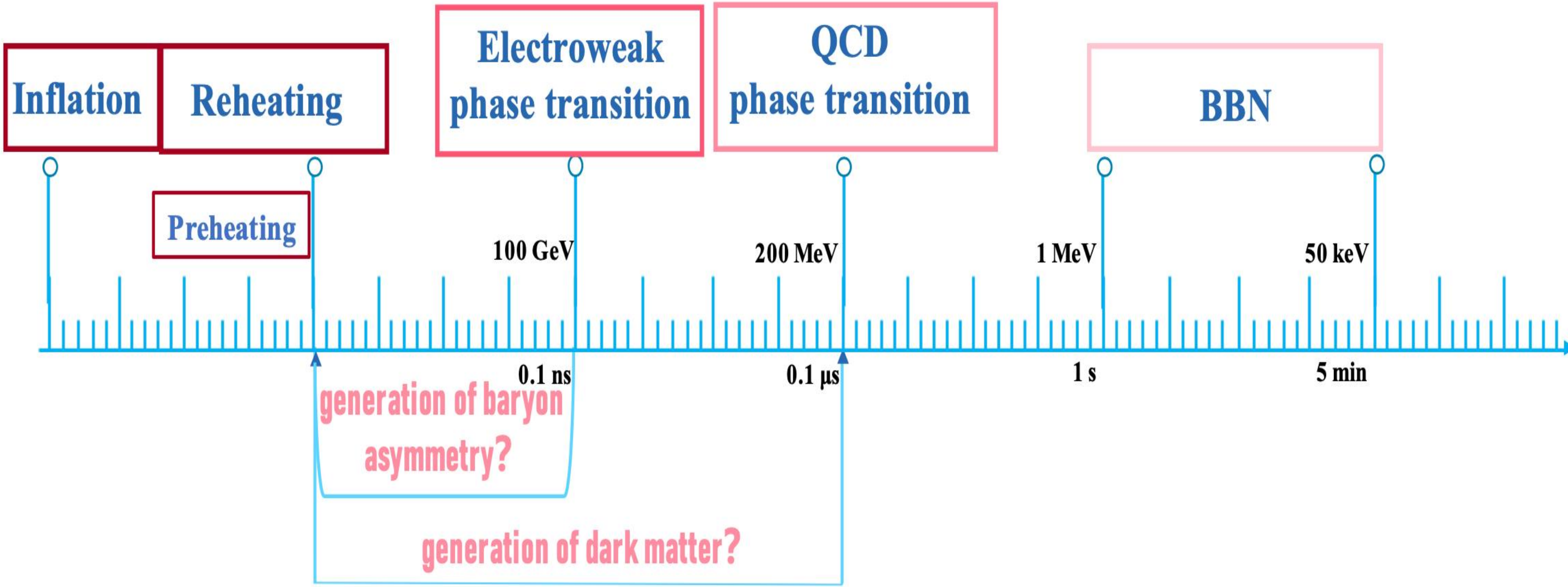


(Image: Daverio et al, 2013)

宇宙学起源的引力波

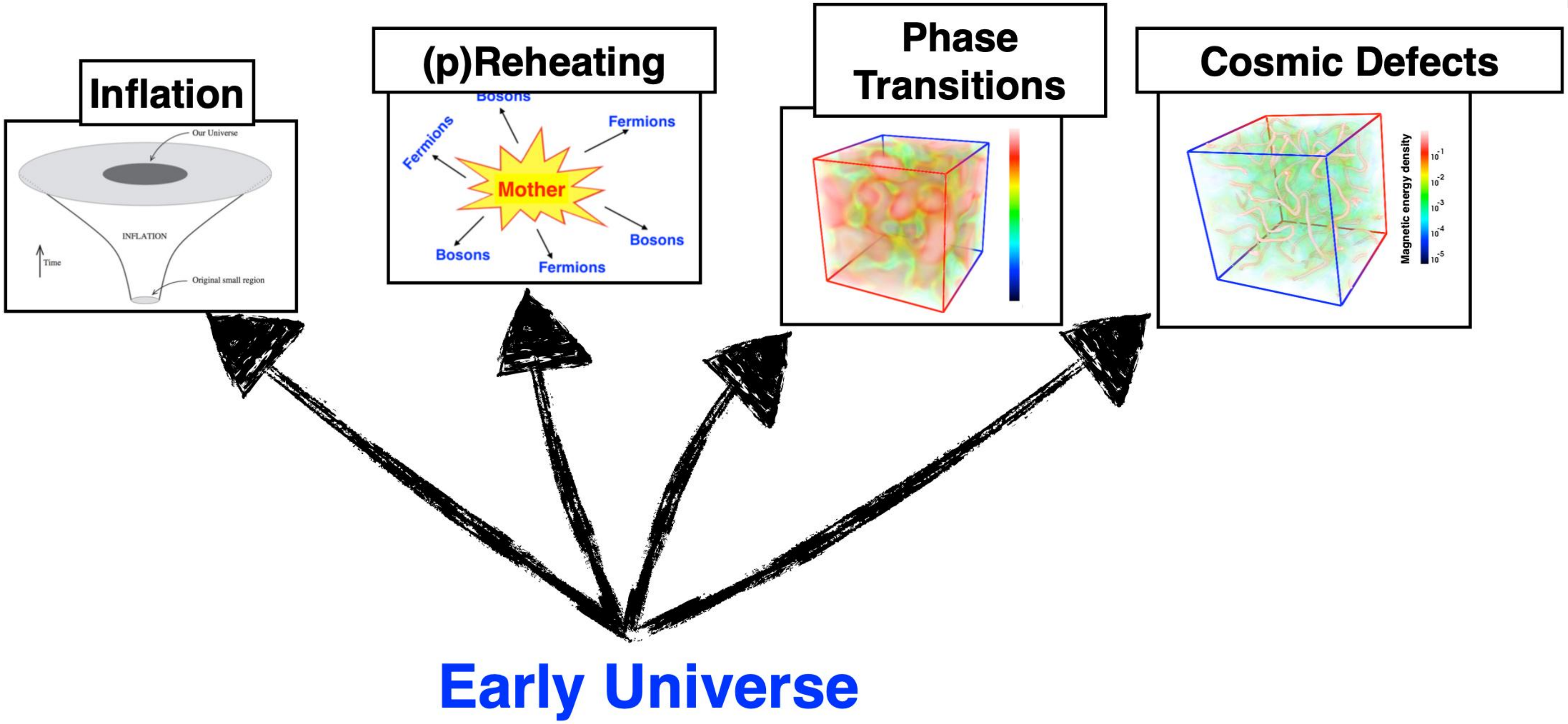

Inflation
Our Universe
INFLATION
Time
Original small region
(p)Reheating
Bosons
Fermions
Fermions
Mother
Bosons
Bosons
Fermions
Phase Transitions
Cosmic Defects
Magnetic energy density
Early Universe

Dark matter generation



2024-9-8

宇宙学起源的引力波——相变引力波



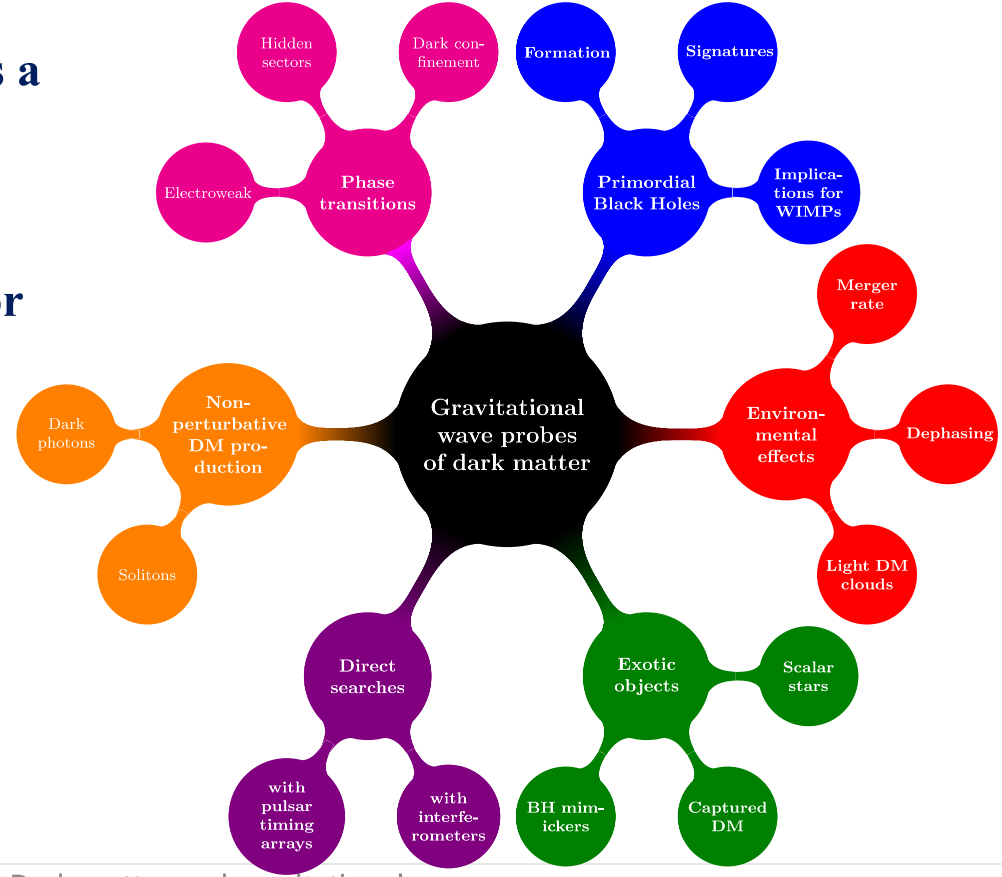
DM and GW

➢ The observation of GW@LIGO initiates a new era of exploring DM by GW.

➢ DM can trigger a SFOPT in the early universe and detectable GW signals.

➢ SFOPT could provide a new approach for DM production.

See Haipeng An, Ligong Bian, Shou-Shan Bao, Yong Tang, Hong Zhang…for more detailed works



Credit: Gianfranco Bertone et. al.

Phase transition GW in a nutshell

characteristic frequency of the GW signal

$$f_* = \frac{1}{\ell_*} \geq H_*$$

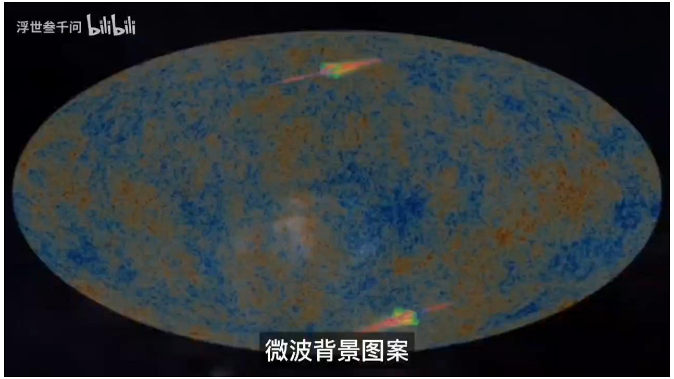
$$\epsilon_* = \ell_* H_*$$

Ratio of the typical length-scale of the GW sourcing process (size of the anisotropic stresses) and the Hubble scale at the generation time

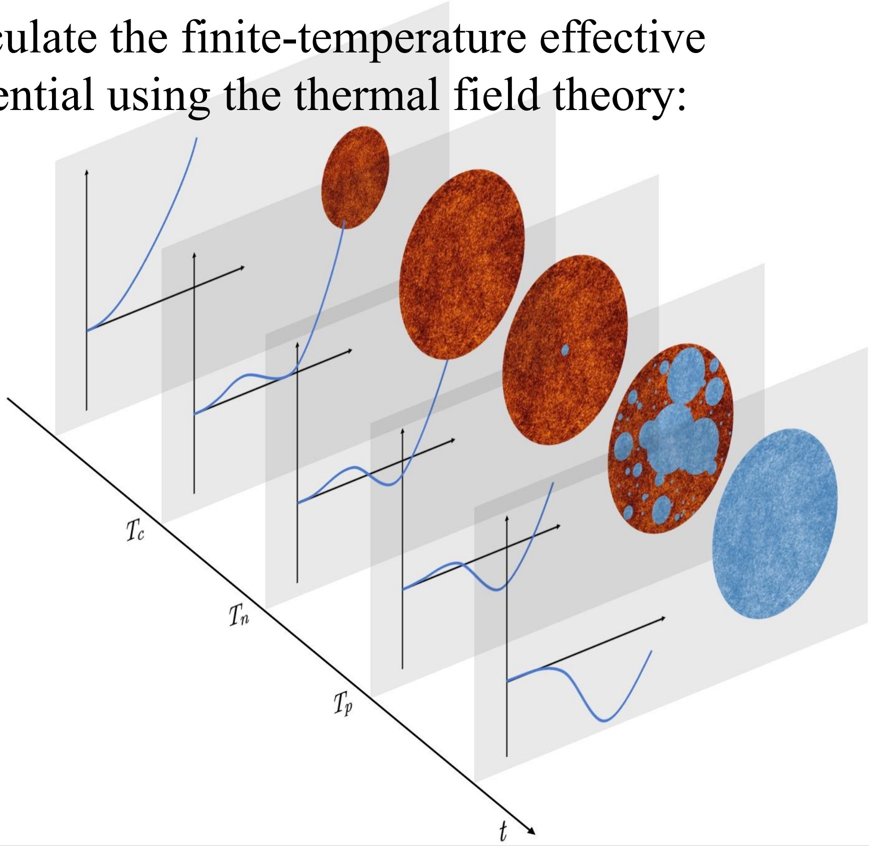
$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-7}}{\epsilon_*} \left(\frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{\text{GeV}} \text{Hz}$$

电弱相变对应的峰值频率在mHz附近，刚好也在空间引力波实验(LISA、天琴、太极)的探测区间

Phase transition in a nutshell



calculate the finite-temperature effective potential using the thermal field theory:



这世上的热闹，源自隧穿

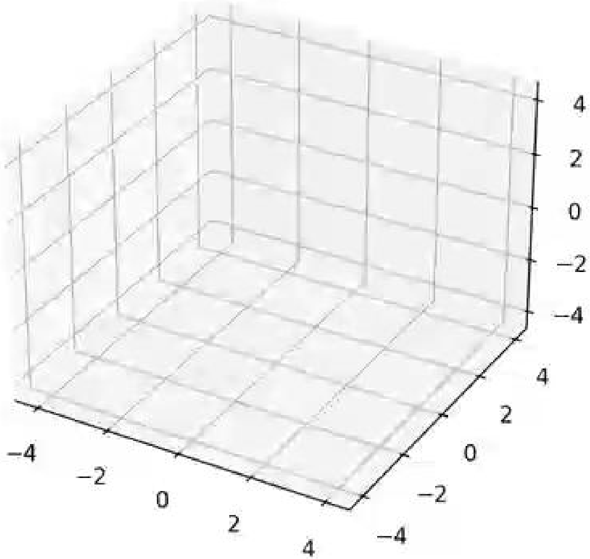
$$\Gamma = \Gamma_0 e^{-S(T)}$$

$$S(T) = \int d^4x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 + V_{\text{eff}}(\phi, T) \right]$$

$$V_{\text{eff}}^{(1)}(\bar{\phi}) = \sum_i n_i \Big[\int \frac{\mathrm{d}^D p}{(2\pi)^D} \ln\big(p^2 + m_i^2(\bar{\phi})\big) + J_{\mathrm{B,F}}\Big(\frac{m_i^2(\bar{\phi})}{T^2}\Big) \Big]$$

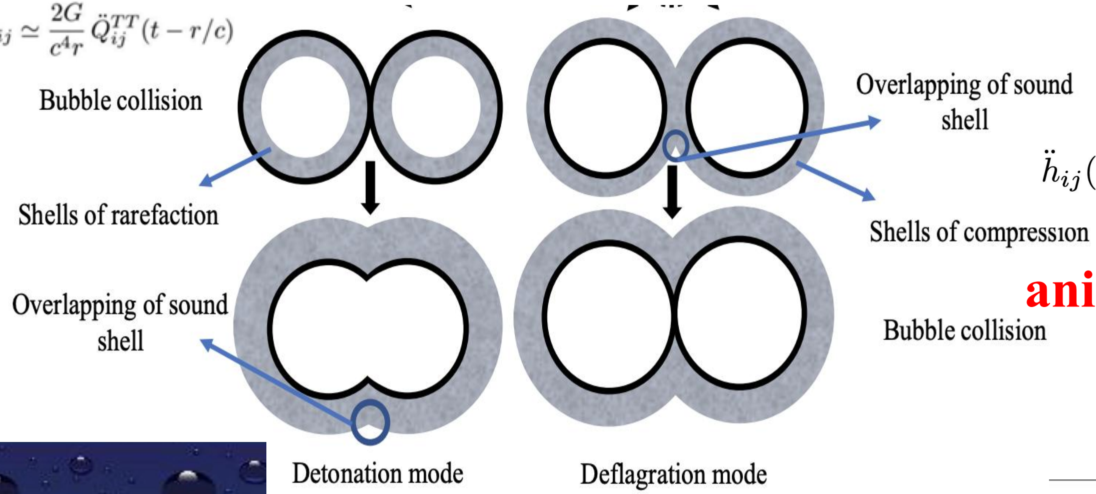
Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

Phase transition GW in a nutshell



Phase transition GW in a nutshell

$h_{ij} \simeq \frac{2G}{c^4 r}\ddot{Q}^{TT}_{ij}(t - r/c)$



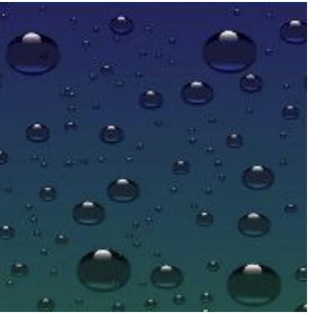
$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} + \Lambda\,g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

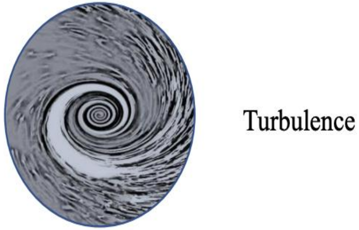
$$\ddot{h}_{ij}(\mathbf{x},t) + 3\,H\,\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}h_{ij}(\mathbf{x},t) = 16\pi G\,\Pi_{ij}(\mathbf{x},t)$$

anisotropic stress tenor: source of GW

E. Witten, Phys. Rev. D 30, 272 (1984)
C. J. Hogan, Phys. Lett. B 133, 172 (1983);
M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994))

EW phase transition GW becomes more interesting and realistic after the discovery of Higgs by LHC and GW by LIGO.





General form Π_{ij}
$[\partial_i\phi\partial_j\phi]^{TT}$
$[\gamma^2(\rho+p)v_iv_j]^{TT}$
$[-E_iE_j - B_iB_j]^{TT}$
$\partial_i\Psi, \partial_i\Phi$

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

Phase transition dynamics

Theory: The most important and difficult phase transition parameter for GW, dynamical DM, baryogenesis is bubble wall velocity v_w

Finite-temperature effective potential

$V_{eff}(\phi, T)$ α T_p $R_* H_*$

(1). Daisy resummation problem: Pawani scheme vs. Arnold scheme
(2). Gauge dependence problem: see Michael J. Ramsey-Musolf's works
(3). No perturbative calculations: lattice calculations and dim-reduction method: by D. Weir, Michael J. Ramsey-Musolf et.al

Bubble wall velocity this talk v_w

Energy budget κ

Experiment: GW experiment is most sensitive to bubble wall velocity v_w

arXiv: 2404.18703
Aidi Yang, **FPH**

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang , arXiv:2007.10343 ,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith, arXiv:2009.14295v2
Xiao Wang, **FPH,** Xinmin Zhang,arXiv:2011.12903
Siyu Jiang, **FPH**, xiao wang,
Phys.Rev.D 107 (2023) 9, 095005

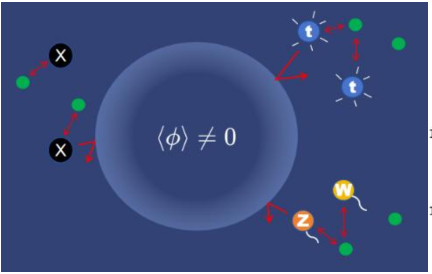
F. Giese, T. Konstandin, K. Schmitz and J. van de ,arXiv:2010.09744
Xiao Wang, **FPH** and Xinmin Zhang,
Phys.Rev.D 103 (2021) 10, 103520
Xiao Wang, Chi Tian, **FPH**, JCAP 07 (2023) 006

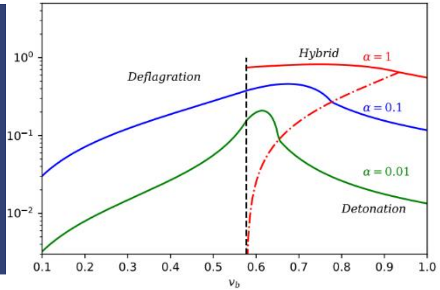
Bubble wall is essential (like a filter)

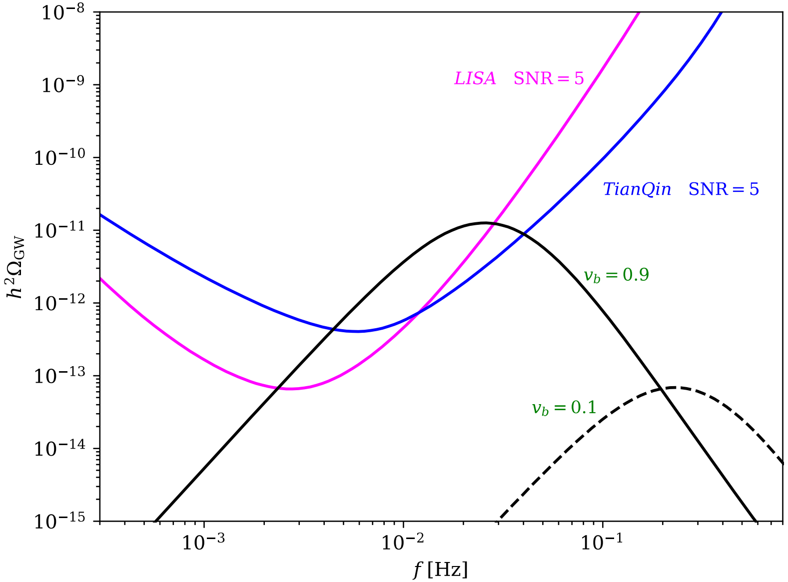
The most essential parameter for phase transition GW, phase transition DM, baryogenesis v_w

GW detection favor lager v_w
EW baryogenesis favor smaller v_w
Dynamical DM is sensitive to v_w

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang, arXiv:2007.10343,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith, arXiv:2009.14295v2
Xiao Wang, **FPH,** Xinmin Zhang,arXiv:2011.12903
Siyu Jiang, **FPH**, xiao wang, Phys.Rev.D 107 (2023) 9, 095005







$$\rho_{DM}^4 v_w^{3/4} = 73.5(2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;

Phase transition dynamics

Systematically calculation of bubble wall velocity in specific model:

Standard Model (small Higgs mass):

Guy D. Moore,Tomislav Prokopec, How fast can the wall move? A Study of the electroweak phase transition dynamics, Phys.Rev.D 52 (1995) 7182-7204

Minimal Supersymmetric Standard Model:

P. John, M.G. Schmidt, Do stops slow down electroweak bubble walls?, Nucl.Phys.B 598 (2001) 291-305

Higgs + scalar singlet:

Jonathan Kozaczuk, Bubble Expansion and the Viability of Singlet-Driven Electroweak Baryogenesis, JHEP 10 (2015) 135

Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith, Wall speed and shape in singlet-assisted strong electroweak phase transitions, Phys.Rev.D 103 (2021) 5, 055020

Inert Doublet Model:

Siyu Jiang, FPH, Xiao Wang, Bubble wall velocity during electroweak phase transition in the inert doublet model, Phys.Rev.D 107 (2023) 9, 095005

Phase transition dynamics

The Guy Moore's method would be invalid at around sound velocity, there are some other solutions:

New ansatz:

Benoit Laurent, James M. Cline, Phys.Rev.D 102 (2020) 6, 063516
James M. Cline, Avi Friedlander, Dong-Ming He, Kimmo Kainulainen, Benoit Laurent, Phys.Rev.D 103 (2021) 12, 123529
Marek Lewicki, Marco Merchand, Mateusz Zych, JHEP 02 (2022) 017
Benoit Laurent, James M. Cline, Phys.Rev.D 106 (2022) 2, 023501
Stefania De Curtis, Luigi Delle Rose, Andrea Guiggiani, Ángel Gil Muyor, Giuliano Panico, JHEP 03 (2022) 163

Higher order corrections in Guy Moore's ansatz

Glauber C. Dorsch, Stephan J. Huber, Thomas Konstandin, JCAP 04 (2022) 04, 010
Glauber C. Dorsch, Daniel A. Pinto, arXiv:2312.02354

Phenomenological parametrization of friction (friction=ηv_w)

Ariel Megevand, et.al, Nucl.Phys.B 820 (2009) 47-74, Nucl.Phys.B 825 (2010) 151-176 …

Phase transition dynamics

Hydrodynamical backreaction:

Marc Barroso Mancha, Tomislav Prokopec, Bogumila Swiezewska, JHEP 01 (2021) 070
Wen-Yuan Ai, Bjorn Garbrecht, Carlos Tamarit, JCAP 03 (2022) 03, 015
Wen-Yuan Ai, Benoit Laurent, Jorinde van de Vis, JCAP 07 (2023) 002
Shao-Jiang Wang, Zi-Yan Yuwen, Phys.Rev.D 107 (2023) 2, 023501
Jun-Chen Wang, Zi-Yan Yuwen, Yu-Shi Hao, Shao-Jiang Wang, arXiv:2310.07691
Tomasz Krajewski, Marek Lewicki, Mateusz Zych, Phys.Rev.D 108 (2023) 10, 103523

Bubble wall velocity for ultra-relativistic bubble walls (run-away criterion):

Dietrich Bodeker, Guy D. Moore, JCAP 05 (2009) 009
Dietrich Bodeker, Guy D. Moore, JCAP 05 (2017) 025
Stefan Höche, Jonathan Kozaczuk, Andrew J. Long, Jessica Turner, Yikun Wang, JCAP 03 (2021) 009
Aleksandr Azatov, Miguel Vanvlasselaer, JCAP 01 (2021) 058
Yann Gouttenoire, Ryusuke Jinno, Filippo Sala, JHEP 05 (2022) 004
Wen-Yuan Ai, JCAP 10 (2023) 052
见康召丰老师的报告

Case I:DM induced SFOPT (wall velocity)

Inert Doublet Models (example)

$$V_0 = M_D^2 D^\dagger D + \lambda_D (D^\dagger D)^2 + \lambda_3 \Phi^\dagger \Phi D^\dagger D + \lambda_4 |\Phi^\dagger D|^2 + (\lambda_5/2)[(\Phi^\dagger D)^2 + h.c.],$$

mixed singlet-doublet model

$$V_0 = \frac{1}{2} M_S^2 S^2 + M_D^2 H_2^\dagger H_2 + \frac{1}{2}\lambda_S S^2 |\Phi|^2 + \lambda_3 \Phi^\dagger \Phi H_2^\dagger H_2 + \lambda_4 |\Phi^\dagger H_2|^2 + \frac{\lambda_5}{2}[(\Phi^\dagger H_2)^2 + \text{H.c.}] + A[S\Phi H_2^\dagger + \text{H.c.}].$$

mixed singlet-triplet model

$$V_0 = \frac{1}{2} M_S^2 S^2 + M_\Sigma^2 \text{Tr}(H_3^2) + \kappa_\Sigma \Phi^\dagger \Phi \text{Tr}(H_3^2) + \frac{\kappa}{2} |\Phi|^2 S^2 + \xi S \Phi^\dagger H_3 \Phi.$$

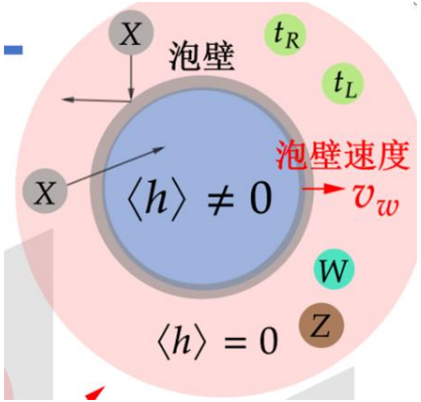
provide natural DM candidate

produce SFOPT and phase transition GW

FPH, Jiang-Hao Yu, Phys.Rev. D98 (2018) no.9, 095022

Yan Wang, Chong Sheng Li, and **FPH**, *Phys.Rev.D* 104 (2021) 5, 053004;

How to calculate wall velocity?



基于有效势，通过能动张量守恒得到早期宇宙plasma中的希格斯场运动方程

$$(1-v_w^2)h'' + \sum_i \frac{dm_i^2}{dh} \int \frac{d^3p}{(2\pi)^3 2E_i} \delta f_i(x,p) + \frac{\partial V_{\rm eff}(h,T)}{\partial h} = 0$$

求解玻尔兹曼方程得到粒子偏离热平衡的扰动分布

$$\frac{d}{dt}(f_i^{\rm eq} + \delta f_i) = \left(\left(v_w + \frac{p_z}{E} \right) \frac{\partial}{\partial z} - \frac{(m_i^2)'}{2E} \frac{\partial}{\partial p_z} \right) (f_i^{\rm eq} + \delta f_i) = -C[(f_i^{\rm eq} + \delta f_i)]$$

用量子场论的方法计算碰撞项C

采用合理的flow ansatz以及truncation scheme，求解扰动的演化方程组

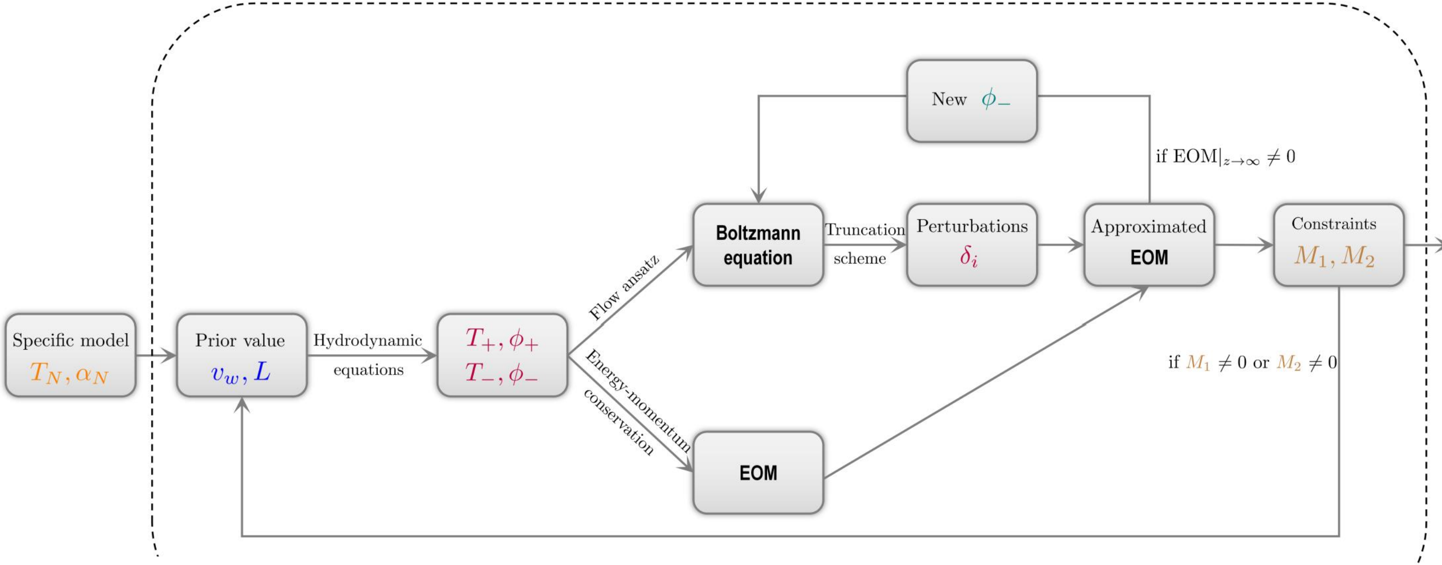
计算粒子在泡壁处的散射

δf_i

将扰动代入运动方程，数值求解提取出泡壁速度和泡壁厚度 v_w, L_w
进一步可讨论相变动力学暗物质、电弱重子生成和相变引力波信号

Phase transition dynamics

A simple DM Model: **Bubble wall velocity in inert doublet model**



← More accurate processes

$$V_0 = \mu_1^2|\Phi|^2 + \mu_2^2|\eta|^2 + \frac{1}{2}\lambda_1|\Phi|^4 + \frac{1}{2}\lambda_2|\eta|^4$$
$$+ \lambda_3|\Phi|^2|\eta|^2 + \lambda_4|\Phi^\dagger\eta|^2 + \frac{1}{2}\{\lambda_5(\Phi^\dagger\eta)^2 + \text{H.c.}\} ,$$

Siyu Jiang, **FPH**, Xiao Wang,
Phys.Rev.D 107 (2023) no.9, 095005

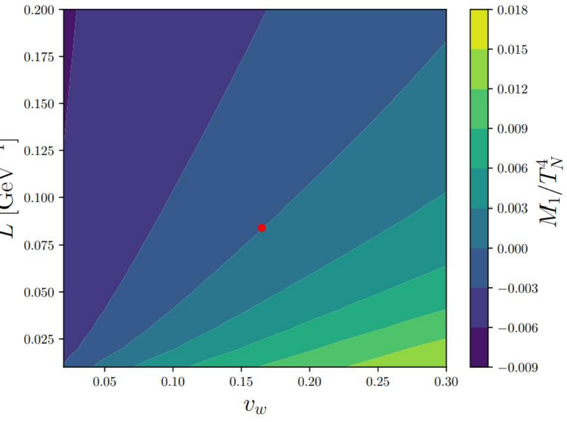
Phase transition dynamics

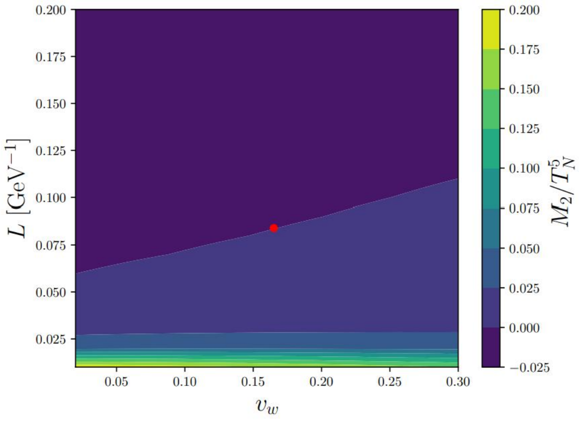
Solving the EOM: bubble wall pressure difference is 0; bubble wall thickness fixed

$$S_{\mathrm{EOM}} \equiv (1-v_w^2)\,\phi'' + \frac{\partial V_{\mathrm{eff}}(\phi, T_+)}{\partial \phi} + \frac{N_t T_+}{2}\frac{dm_t^2}{d\phi} \times \left(c_1^t \mu_t + c_2^t(\delta T_t + \delta T_{bg})\right)$$

$$+ \sum_b \frac{N_b T_+}{2}\frac{dm_b^2}{d\phi}\left(c_1^b \mu_b + c_2^b(\delta T_b + \delta T_{bg})\right) = 0\ ,$$

$$M_1 = \int S_{\mathrm{EOM}}\phi' dz = 0, \quad M_2 = \int S_{\mathrm{EOM}}(2\phi - \phi_-)\phi' dz = 0\ .$$





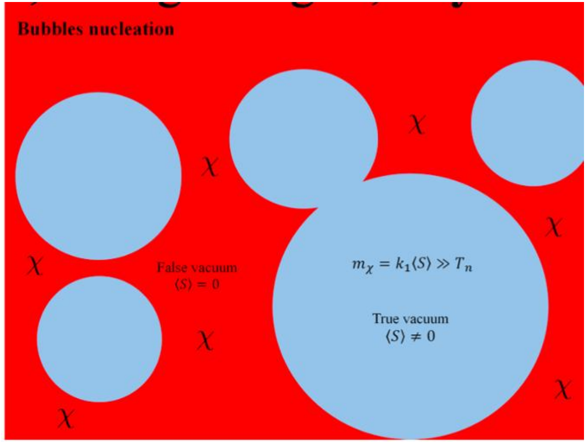
In the allowed parameter spaces, the wall velocity is around 0.165. The basic procedure in this work can also be used for any other SFOPT and dynamical DM model.

Case II: anti-filtered Q-ball DM

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;

Gauged Q-ball dark matter through a cosmological first-order phase transition, Siyu Jiang, FPH, Pyungwon Ko, arXiv:2404.16509

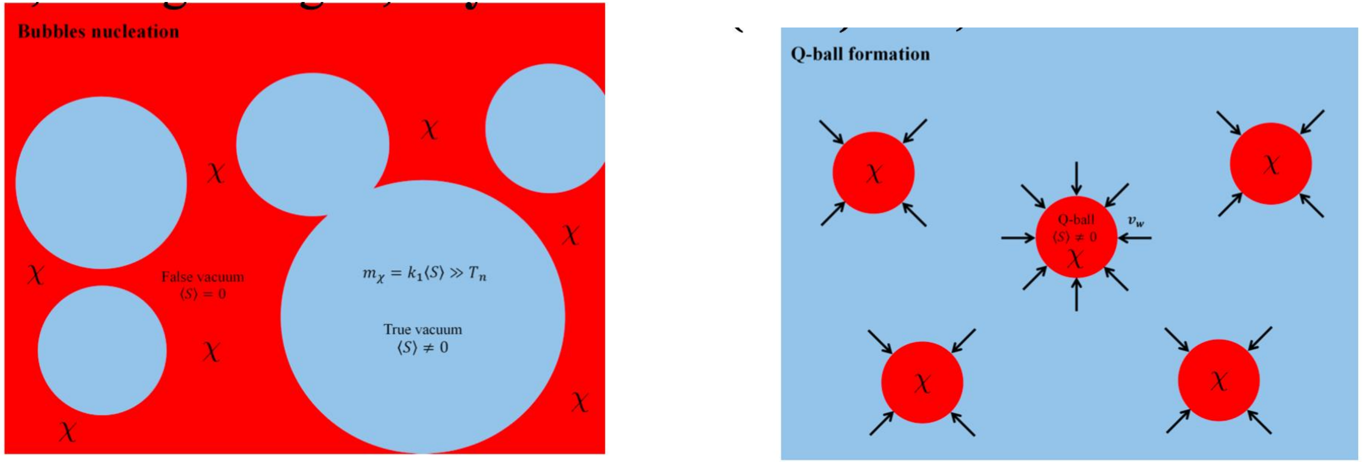
DM



Case II: anti-filtered Q-ball DM

Global Q-ball DM: **The cosmic phase transition with Q-balls production can explain baryogenesis and DM simultaneously..**

$$\rho_{DM}^4 v_w^{3/4} = 73.5(2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$



(a) Bubble nucleation: χ particles trapped in the false vacuum due to Boltzmann suppression

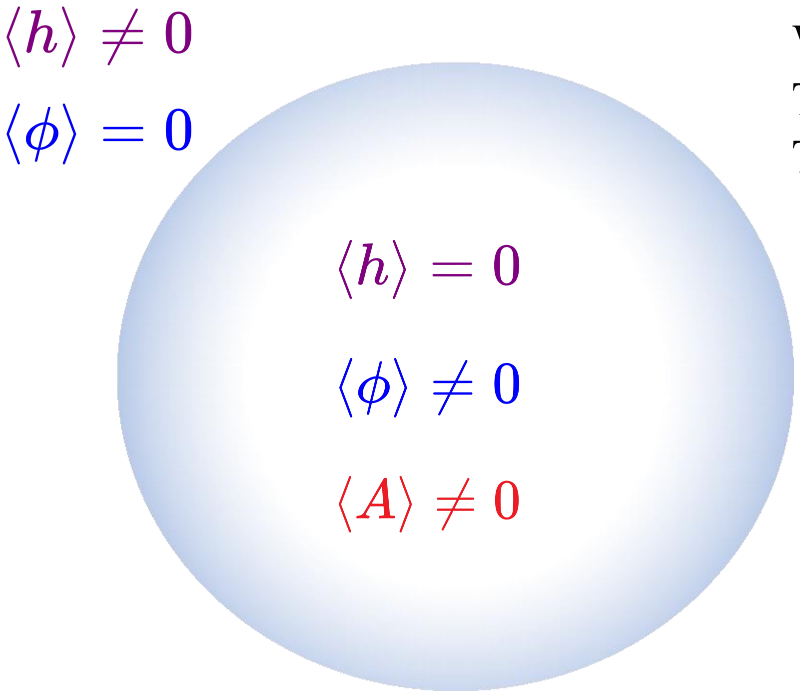
(b) Q-ball formation: After the formation of Q-balls, they should be squeezed by the true vacuum

New DM production scenario filtered by the bubbles

The global Q-ball model proposed by T.D. Lee

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;

Gauged Q-ball DM



When the conseved U(1) symmetry is local,
This introduces an extra gauge field A.
The minimal model achieving gauged Q-ball formation

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) + \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{4}\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu} - V(\phi, h)$$

$$V(\phi, h) = \frac{\lambda_{\phi h}}{2} h^2 |\phi|^2 + \frac{\lambda_h}{4}\left(h^2 - v_0^2\right)^2$$

Interestingly, this portal coupling also naturally induces strong electroweak phase transition.

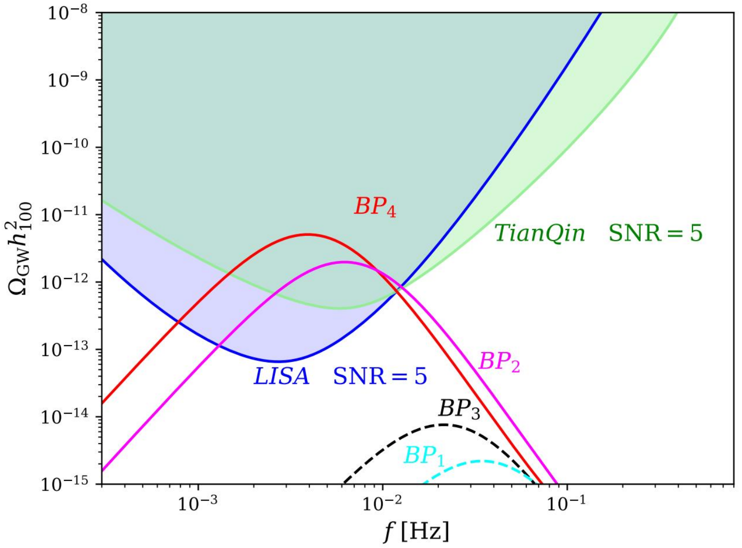
$$J_\mu = i\left(\phi^\dagger \overleftrightarrow{\partial}_\mu \phi + 2i\tilde{g}\tilde{A}_\mu |\phi|^2\right)$$

Conserved charge $\quad Q = \int d^3x J^0$

Gauged Q-ball dark matter through a cosmological first-order phase transition, Siyu Jiang, FPH, Pyungwon Ko, arXiv:2404.16509,JHEP

Gauged Q-ball DM

	$\lambda_{\phi h}$	T_p [GeV]	α_p	β/H_p	v_w	$F_\phi^{\rm trap}$	η_ϕ/η_L	$\delta\sigma_{Zh}$	GW
BP_1	6.8	69.8	0.12	540	0.1	0.932	0.48	-0.36%	●
BP_2	6.8	70.4	0.12	578	0.6	0.805	3.0	-0.36%	●
BP_3	7.0	63.0	0.15	372	0.1	0.965	3.4	-0.37%	●
BP_4	7.0	63.9	0.15	403	0.6	0.858	20.8	-0.37%	●



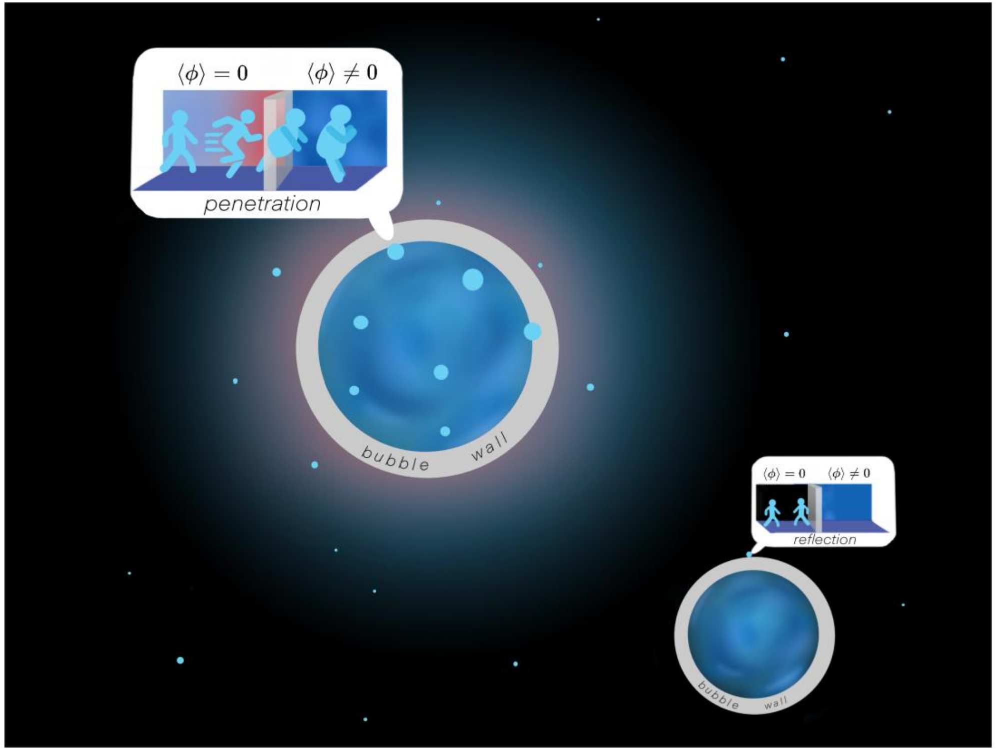
Gauged Q-ball dark matter through a cosmological first-order phase transition, Siyu Jiang, FPH, Pyungwon Ko, arXiv:2404.16509, JHEP

Case III: filtered DM

DM

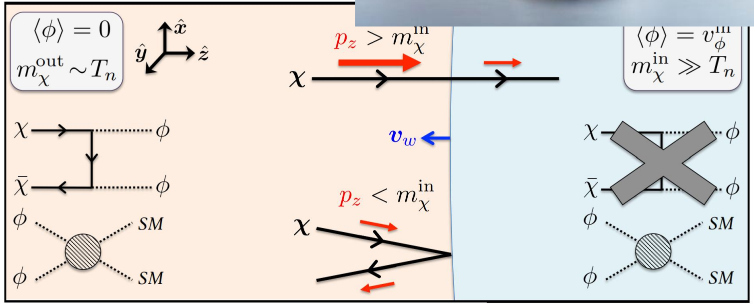
Bubble wall dynamics plays an essential role in the filtered DM mechanism.

Siyu Jiang, FPH, Chong Sheng Li, Phys.Rev.D 108 (2023) 6, 063508



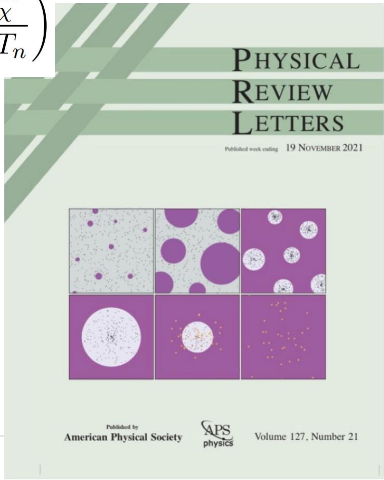
Case III: filtered DM

In recent years, this dynamical DM formed by phase transition has became a new idea and attracted more and more attentions. Namely, bubble in SFOPT can be the “filter” to packet the needed heavy DM.



$$\Omega_{\rm DM}h^2 \approx 0.17\left(\frac{T_n}{\rm TeV}\right)\left(\frac{m_\chi^\infty}{30T_n}\right)^{-\frac{5}{2}}\exp\left(-\frac{m_\chi^\infty}{30T_n}\right)$$

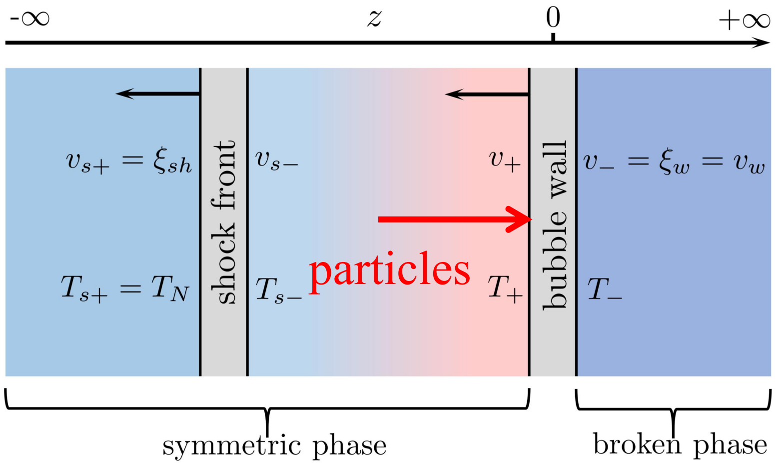
FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028
arXiv:1912.04238, Dongjin Chway, Tae Hyun Jung, Chang Sub Shin
arXiv:1912.02830, Phys.Rev.Lett. 125 (2020) 15, 151102 , Michael J. Baker, Joachim Kopp,and Andrew J. Long
arXiv:2012.15113, Wei Chao, Xiu-Fei Li, Lei Wang
arXiv:2101.05721, Aleksandr Azatov, Miguel Vanvlasselaer, Wen Yin
arXiv:2103.09827, Pouya Asadi , Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov
arXiv:2103.09822, Pouya Asadi , Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov
arXiv:2008.04430 Jeong-Pyong Hong， Sunghoon Jung, Ke-pan Xie
Haipeng An, et.al, arXiv: 2208.14857
Siyu Jiang, FPH, Chong Sheng Li, arXiv:2305.02218
more and more new works…



Case III: filtered DM

Original work:

$$\tilde{v}_{\rm pl} = v_w, \quad T = T' = T_n$$



$$\tilde{v}_{\rm pl} = \tilde{v}_+, \quad T = T_+, \quad T' = T_- \qquad \text{(this work with hydrodynamic effects)}\ .$$

$$J_w^{\rm in} = \frac{g_\chi}{(2\pi)^2}\int_0^{-1} d\cos\theta\cos\theta \int_{-\frac{m_\chi^{\rm in}}{\cos\theta}}^{\infty} dp \frac{p^2}{e^{\tilde{\gamma}_+(1+\tilde{v}_+\cos\theta)p/T_+}} = \frac{g_\chi T_+^3\left(1+\tilde{\gamma}_+ m_\chi^{\rm in}(1-\tilde{v}_+)/T_+\right)}{4\pi^2\tilde{\gamma}_+^3(1-\tilde{v}_+)^2} e^{-\tilde{\gamma}_+ m_\chi^{\rm in}(1-\tilde{v}_+)/T_+}.$$

$$n_\chi^{\rm in} = \frac{J_w^{\rm in}}{\gamma_w v_w} \qquad \Omega_{\rm DM}^{\rm (hy)}h^2 = \frac{m_\chi^{\rm in}(n_\chi^{\rm in}+n_{\bar\chi}^{\rm in})}{\rho_c/h^2}\frac{g_{\star 0}T_0^3}{g_\star(T_-)T_-^3} \simeq 6.29\times 10^8 \frac{m_\chi^{\rm in}}{\rm GeV}\frac{(n_\chi^{\rm in}+n_{\bar\chi}^{\rm in})}{g_\star(T_-)T_-^3}$$

Case III: filtered DM

$$T_\phi^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2 - V_{T=0}(\phi)\right]$$

Energy-momentum tensor of scalar field

$$T_{\rm pl}^{\mu\nu} = \sum_i \int \frac{d^3k}{(2\pi)^3 E_i} k^\mu k^\nu f_i^{\rm eq}(k)$$

Energy-momentum tensor of fluid

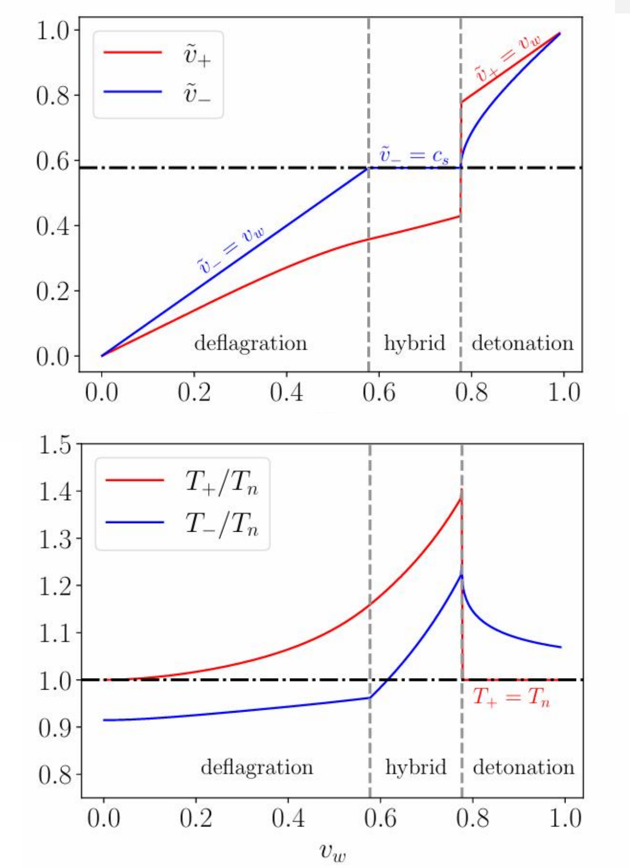
$$T_{\rm fl}^{\mu\nu} = T_\phi^{\mu\nu} + T_{\rm pl}^{\mu\nu} = \omega u^\mu u^\nu - p g^{\mu\nu}$$

Energy-momentum conservation

$$\omega_+\tilde{v}_+^2\tilde{\gamma}_+^2 + p_+ = \omega_-\tilde{v}_-^2\tilde{\gamma}_-^2 + p_-, \quad \omega_+\tilde{v}_+\tilde{\gamma}_+^2 = \omega_-\tilde{v}_-\tilde{\gamma}_-^2$$

$$\alpha_+ \equiv \epsilon/\left(a_+T_+^4\right) \qquad r_\omega = \omega_+/\omega_- = (a_+T_+^4)/(a_-T_-^4)$$

$$\nabla_\mu T^{\mu\nu} = 0 \quad \longleftrightarrow \quad j\frac{v}{\xi} = \gamma^2(1-v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v, \quad \frac{\partial_\xi\omega}{\omega} = \left(1 + \frac{1}{c_s^2}\right)\gamma^2\mu\partial_\xi v\ .$$



Case III: filtered DM

General phase-transition model

$$V_{\text{eff}}(\phi,T)=\frac{\mu^2+DT^2}{2}\phi^2-CT\phi^3+\frac{\lambda}{4}\phi^4-\frac{g_\star\pi^2T^4}{90}$$

$$\langle\phi\rangle=0,\quad \frac{3CT}{2\lambda}\left[1+\sqrt{1-\frac{4\lambda\left(\mu^2+DT^2\right)}{9C^2T^2}}\right]$$

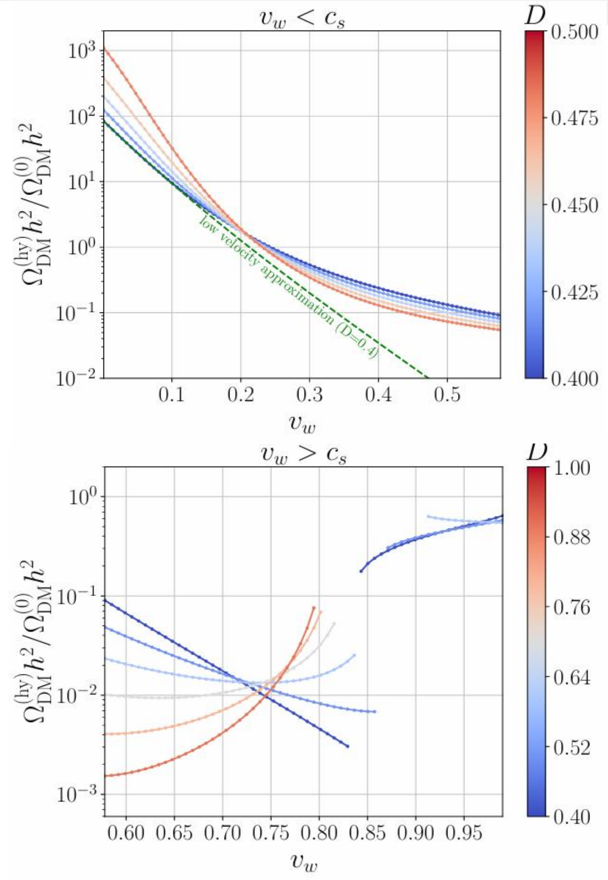
$$m_\chi^{\text{in}}=\frac{y_\chi\phi_-}{\sqrt{2}}\qquad\qquad \phi_-=\phi(T_-)$$

$$n_\chi^{\text{in}}\simeq\frac{g_\chi T_+^3}{\gamma_w v_w}\left(\frac{\tilde{\gamma}_+\left(1-\tilde{v}_+\right)m_\chi^{\text{in}}/T_++1}{4\pi^2\tilde{\gamma}_+^3\left(1-\tilde{v}_+\right)^2}\right)e^{-\tilde{\gamma}_+(1-\tilde{v}_+)m_\chi^{\text{in}}(T_-)/T_+}$$

We found:

(a) for $v_w \lesssim 0.2$: the DM relic density is enhanced

(b) for $v_w \gtrsim 0.2$: the DM relic density is reduced



Case III: filtered DM

Boltzmann equation

$$\mathbf{L}[f_\chi] = \mathbf{C}[f_\chi]$$

$$f_\chi = \mathcal{A}(z, p_z)\, f^{\rm eq}_{\chi,+} = \mathcal{A}(z, p_z) \exp\left(-\frac{\tilde{\gamma}_+(E - \tilde{v}_+ p_z)}{T_+}\right)$$

$$\mathbf{L}[f_\chi] = \frac{p_z}{E}\frac{\partial f_\chi}{\partial z} - \frac{m_\chi}{E}\frac{\partial m_\chi}{\partial z}\frac{\partial f_\chi}{\partial p_z} \qquad m_\chi(z) \equiv \frac{m_\chi^{\rm in}(\phi_-)}{2}\left(1 + \tanh\frac{2z}{L_w}\right)$$

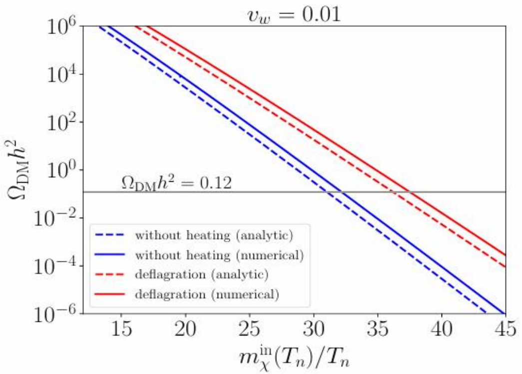
$$g_\chi \int \frac{dp_x dp_y}{(2\pi)^2}\mathbf{L}[f_\chi] \approx \left[\left(\frac{p_z}{m_\chi}\frac{\partial}{\partial z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\partial}{\partial p_z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\tilde{\gamma}_+\tilde{v}_+}{T_+}\right)\mathcal{A}(z, p_z)\right]\frac{g_\chi m_\chi T_+}{2\pi\tilde{\gamma}_+} e^{\tilde{\gamma}_+\left(\tilde{v}_+ p_z - \sqrt{m_\chi^2 + p_z^2}\right)/T_+}$$

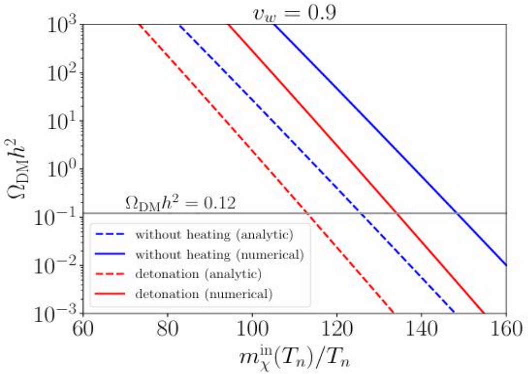
including $\chi\bar{\chi} \leftrightarrow \phi\phi, \chi\phi \leftrightarrow \chi\phi, \chi\chi \leftrightarrow \chi\chi, \chi\bar{\chi} \leftrightarrow \chi\bar{\chi}, \ldots$

$$\begin{aligned} g_\chi \int \frac{dp_x dp_y}{(2\pi)^2}\mathbf{C}[f_\chi] &= -g_\chi g_{\bar{\chi}} \int \frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}} d\Pi_{q^{\mathcal{P}}} 4F\sigma_{\chi\bar{\chi}\to\phi\phi}\left[f_{\chi_p} f^{\rm eq}_{\bar{\chi}_q,+} - f^{\rm eq}_{\chi_p} f^{\rm eq}_{\bar{\chi}_q}\right] \\ &= -g_\chi g_{\bar{\chi}} \int \frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}} d\Pi_{q^{\mathcal{P}}} 4F\sigma_{\chi\bar{\chi}\to\phi\phi}\left[\mathcal{A} f^{\rm eq}_{\chi_p,+} f^{\rm eq}_{\bar{\chi}_q,+} - f^{\rm eq}_{\chi_p} f^{\rm eq}_{\bar{\chi}_q}\right] \\ &\equiv \Gamma_{\rm P}(z, p_z)\mathcal{A}(z, p_z) - \Gamma_{\rm I}(z, p_z)\ , \end{aligned}$$

Case III: filtered DM

$$n_\chi^{\rm in} = \frac{T_+}{\gamma_w \tilde{\gamma}_+} \int_0^\infty \frac{dp_z}{(2\pi)^2} \mathcal{A}(z \gg L_w, p_z) \exp\left[\tilde{\gamma}_+ \left(\tilde{v}_+ p_z - \sqrt{p_z^2 + (m_\chi^{\rm in})^2}\right)/T_+\right] \left(\sqrt{p_z^2 + (m_\chi^{\rm in})^2} + \frac{T_+}{\tilde{\gamma}_+}\right)$$





$v_w = 0.01$

	analytic		numerical	
	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)}h^2/\Omega_{\rm DM}^{(0)}h^2$	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)}h^2/\Omega_{\rm DM}^{(0)}h^2$
BP_1	31	66	32	71
BP_2	31.1	7.9	32.2	8.1
BP_3	30.8	778.8	31.9	858.5
BP_4	$\star$	$\star$	$\star$	$\star$

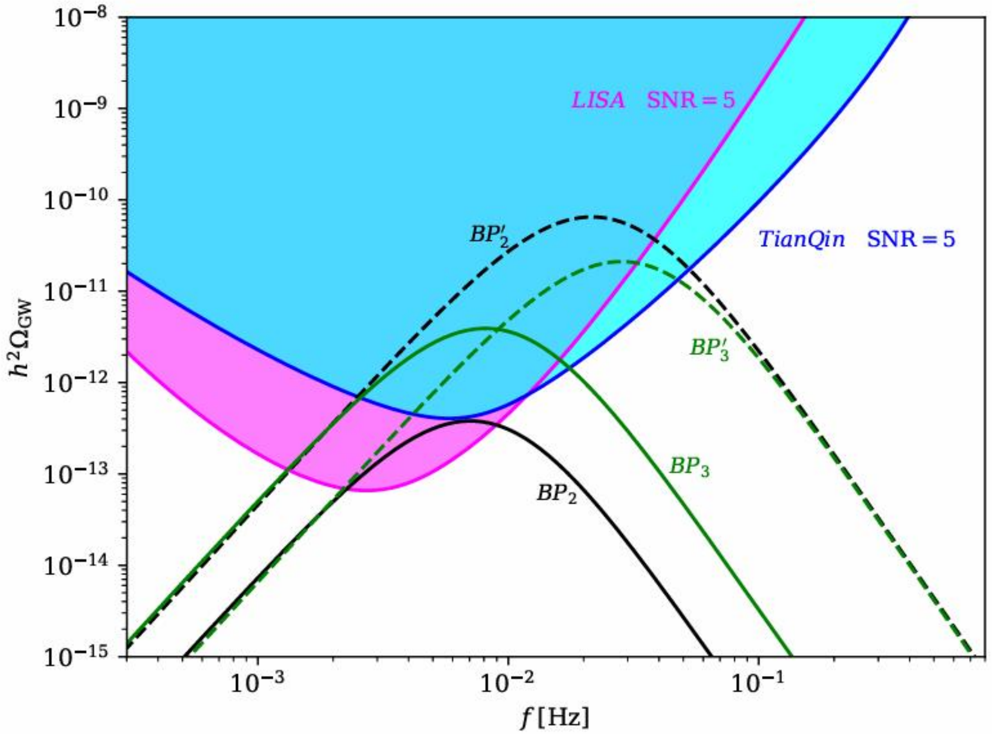
$v_w = 0.9$

	analytic		numerical	
	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)}h^2/\Omega_{\rm DM}^{(0)}h^2$	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)}h^2/\Omega_{\rm DM}^{(0)}h^2$
BP_1	125.3	1/19	147.8	1/27
BP_2	125.9	1/7	148.7	1/9
BP_3	124.6	1/10	147.3	1/12
BP_4	123.8	$1/(1.2\times 10^{13})$	146.5	$1/(2.2\times 10^{15})$

Case III: filtered DM

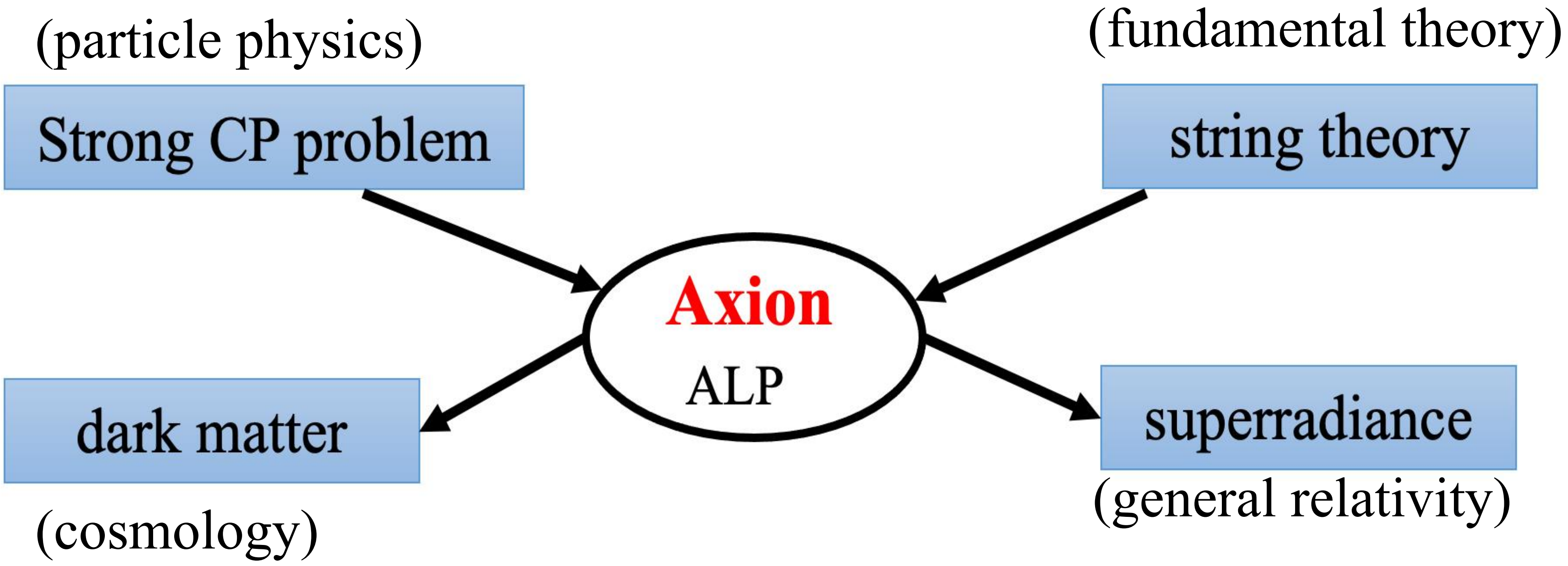
The hydrodynamic effects play essential roles in the filtered DM mechanism. For the deflagration mode with low bubble wall velocity, the hydrodynamic effects significantly enhance the relic density. In contrast, for the detonation mode, the relic density is obviously reduced. For the hybrid mode, the hydrodynamic correction is extremely large.

Precise calculation of filtered DM relic density can help to decide the phase-transition parameters precisely. This gives more accurate phase-transition GW spectra.

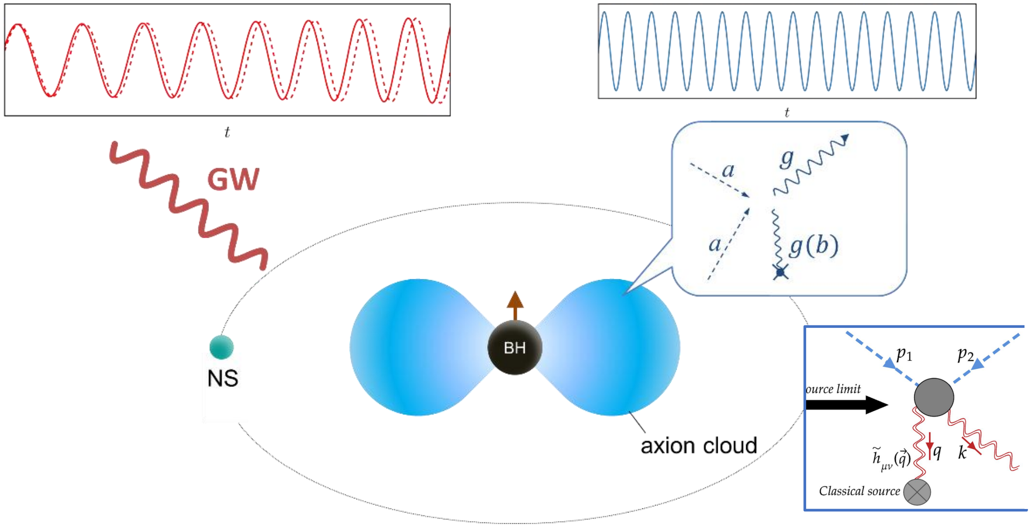


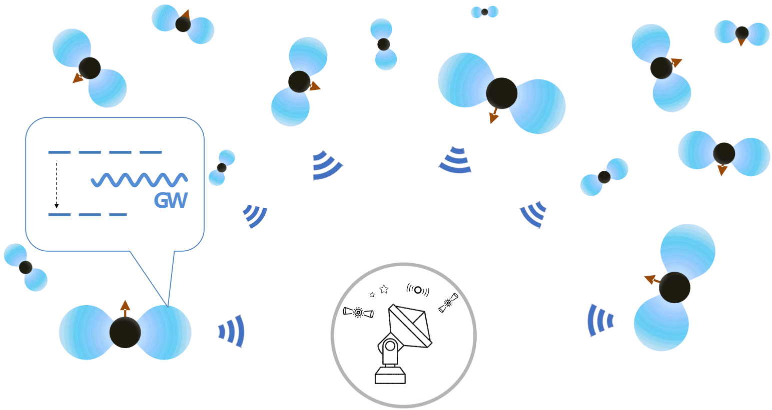
Axion particle cosmology

Ultralight axion is a promising DM candidate.



GW signals of ultralight axions DM





Jing Yang, Ning Xie, FPH arXiv:2306.17113, arXiv:2404.18703**Aidi Yang, FPH**

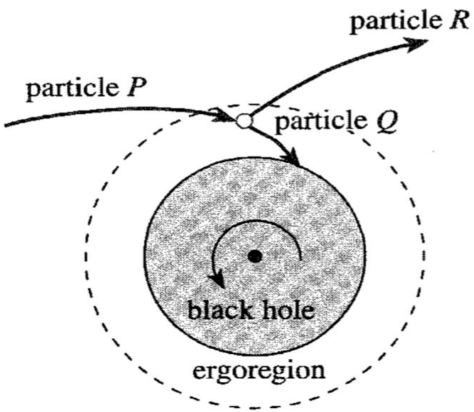
Ning Xie, FPH, SCPMA Vol.66, No.1(2024);

Jing Yang, FPH Phys.Rev.D 108 (2023) 10, 103002

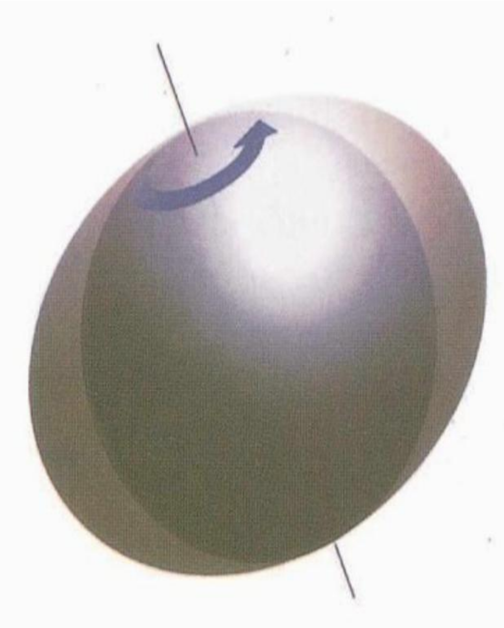
What is superradiance?

When Klein (-Gordon) meets Kerr——superradiance

$$\Delta \frac{d}{dr}\left(\Delta \frac{dR}{dr}\right) + \left[\omega^2\left(r^2+a^2\right)^2 - 4aMrm\omega + a^2m^2 - \Delta\left(m_a^2 r^2 + a^2\omega^2 + \lambda\right)\right] R = 0$$



Penrose '69 '71
Zel'dovich '72
Starobinsky '73



S. Hawking

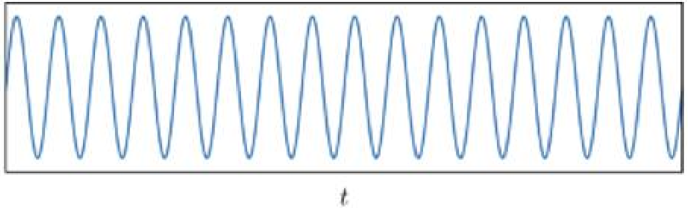
Exponential growth solution of Klein-Gordon equation due to the boundary condition at the horizon of Kerr BH. Ultralight axion can form axion cloud around rotating BH， Gravitational atom (GA).

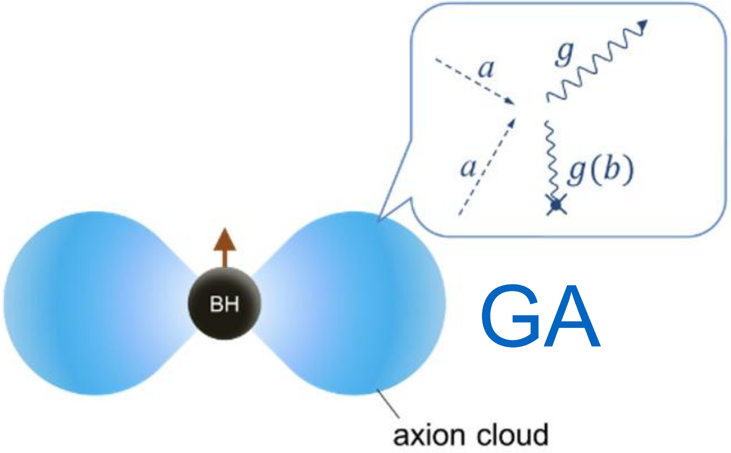
GW of ultralight DM from black hole

Axions can annihilate to GW

A. Arvanitaki and S. Dubovsky, Phys. Rev. D 83, 044026 (2011)
R. Brito, V. Cardoso and P. Pani, Class. Quant. Grav. 32, no.13, 134001 (2015)
H. Yoshino and H. Kodama, PTEP 2014, 043E02 (2014)

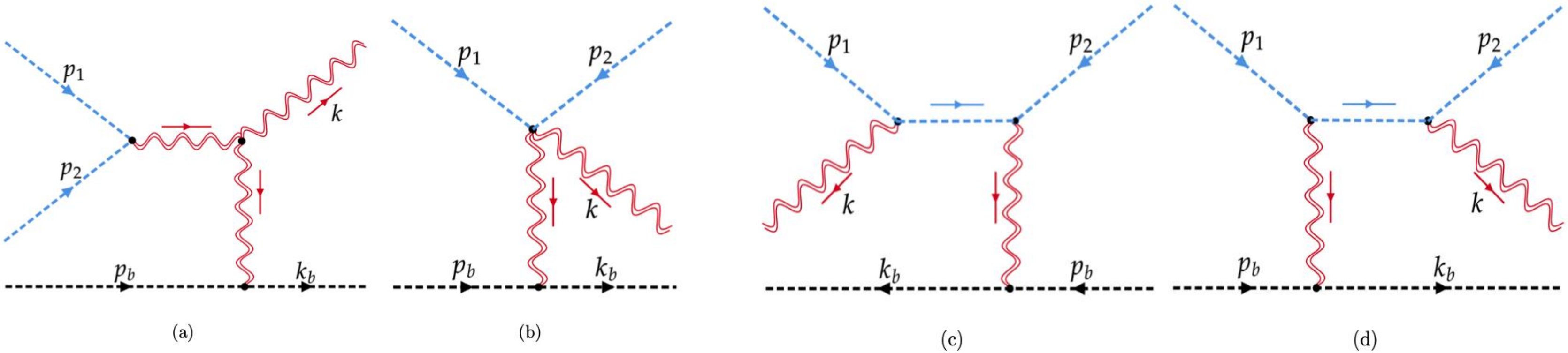
Jing Yang, FPH, Phys.Rev.D 108 (2023) 10, 103002



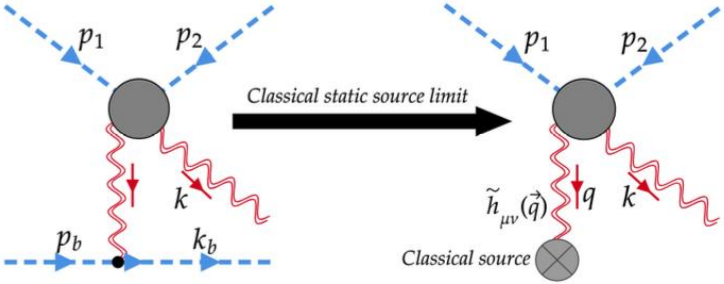


GA

Microscopic physics

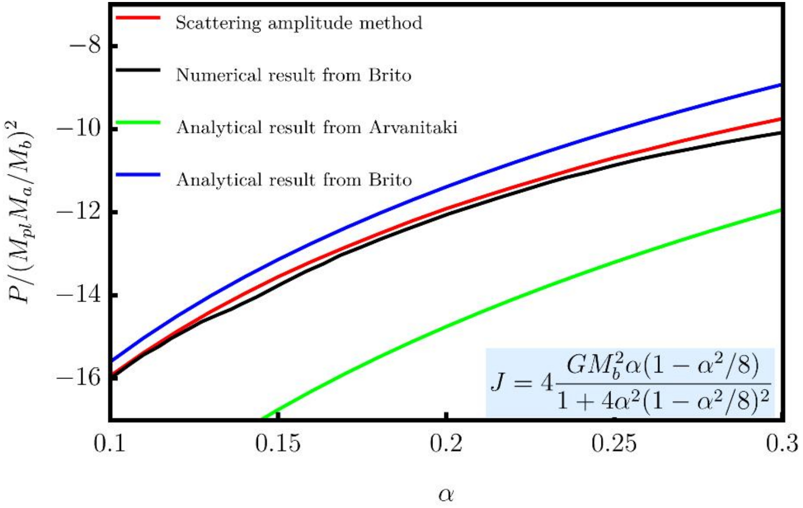


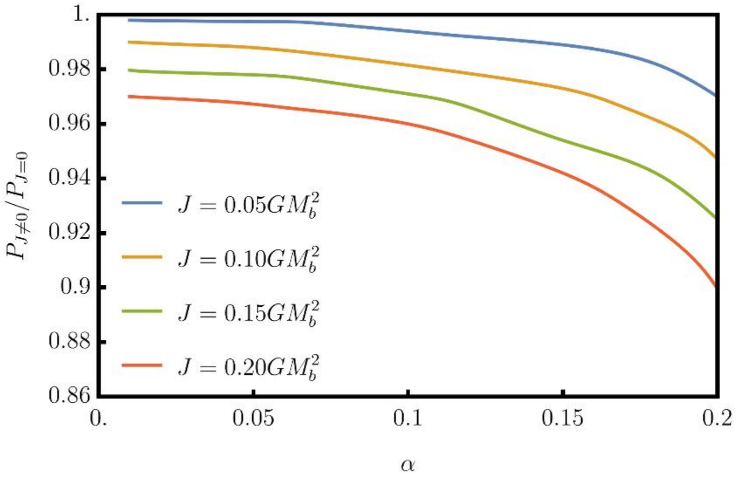
$$M(\boldsymbol{p_b}, \boldsymbol{p_1}, \boldsymbol{p_2} \to \boldsymbol{k}, \boldsymbol{k_b})$$



Jing Yang, FPH,
Phys.Rev.D 108 (2023) 10, 103002

GW radiation from axion annihilation





$\alpha = GM_b m_a \quad M_b = 100M_{sun} \quad M_a = M_{sun}$

Jing Yang, FPH,
Phys.Rev.D 108 (2023) 10, 103002

- ✓ monochromatic GW signal $\omega_{ann} \sim 2\ m_a$
- ✓ gradually depletion of axion cloud (DC) and reduce GA mass

GW radiation from axion annihilation

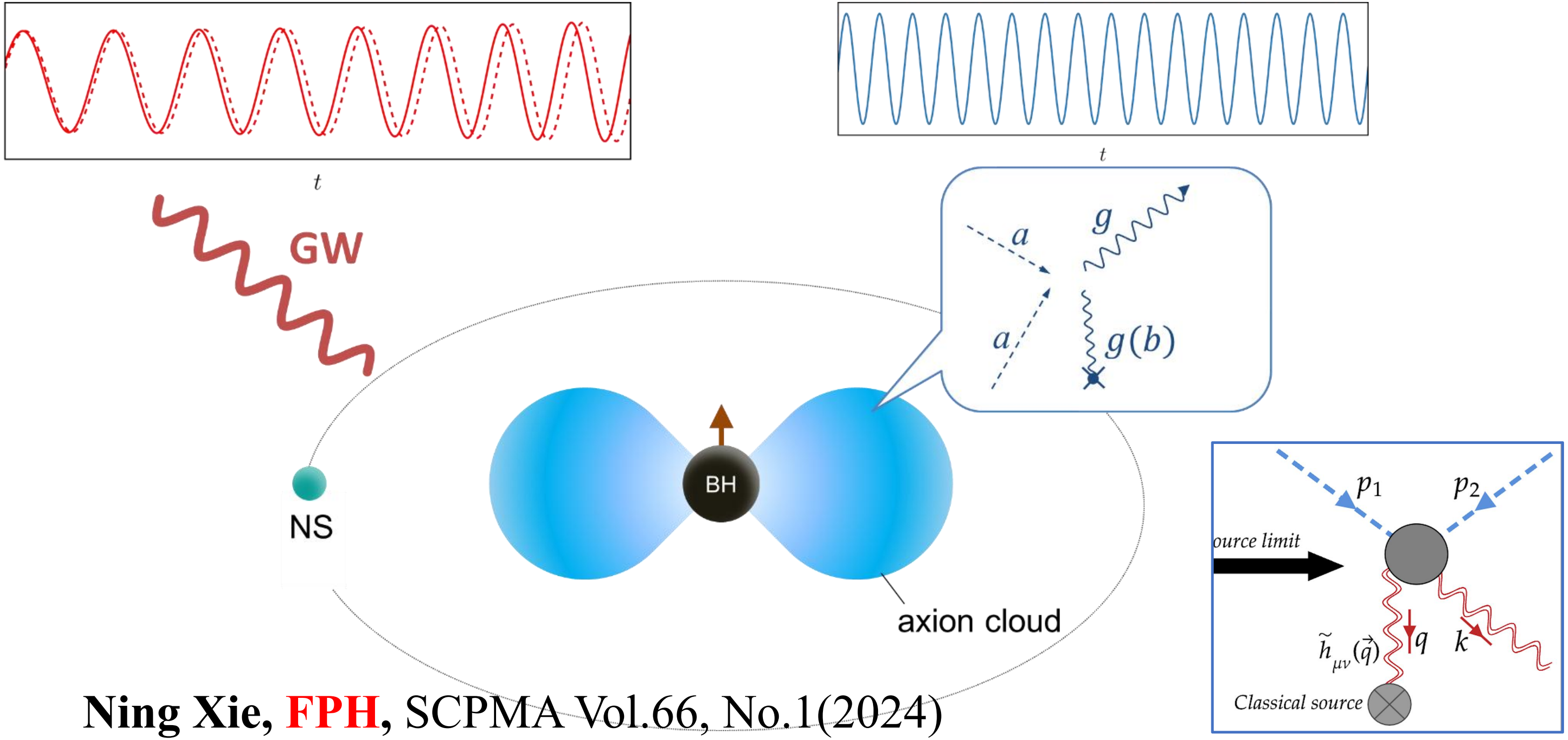
- ✓ Simple and straightforward.
- ✓ Easy to include Kerr metric effects.
- ✓ Microscopic physics is intuitive.
- ✓ It is clearly and simple to demonstrate the analytic approximation formulae.

$$\begin{aligned} P = & \frac{(M_a/\mathrm{GeV})^2\alpha^{14}}{(M_b/\mathrm{GeV})^6(2+\alpha^2)^{11}(4+\alpha^2)^4}\Big[(M_b/\mathrm{GeV})^4(9.671\times10^{41}+5.577\times10^{42}\alpha^2 \\ & + 1.474\times10^{43}\alpha^4+2.361\times10^{43}\alpha^6)+J(M_b/\mathrm{GeV})^2\alpha(-3.839\times10^{80} \\ & - -2.111\times10^{81}\alpha^2-5.329\times10^{81}\alpha^4-8.165\times10^{81}\alpha^8)+J^2\alpha^2(3.809\times10^{118} \\ & + 2.184\times10^{119}\alpha^2+5.799\times10^{119}\alpha^4+9.450\times10^{119}\alpha^6)\Big]\ \mathrm{GeV}^2. \end{aligned}$$

Important for the GW and axion search. More precise calculations and more broad applications are working in progress.

Jing Yang, FPH,
Phys.Rev.D 108 (2023) 10, 103002

Imprints of axions on GW



Ning Xie, FPH, SCPMA Vol.66, No.1(2024)

Imprints of axions on GW

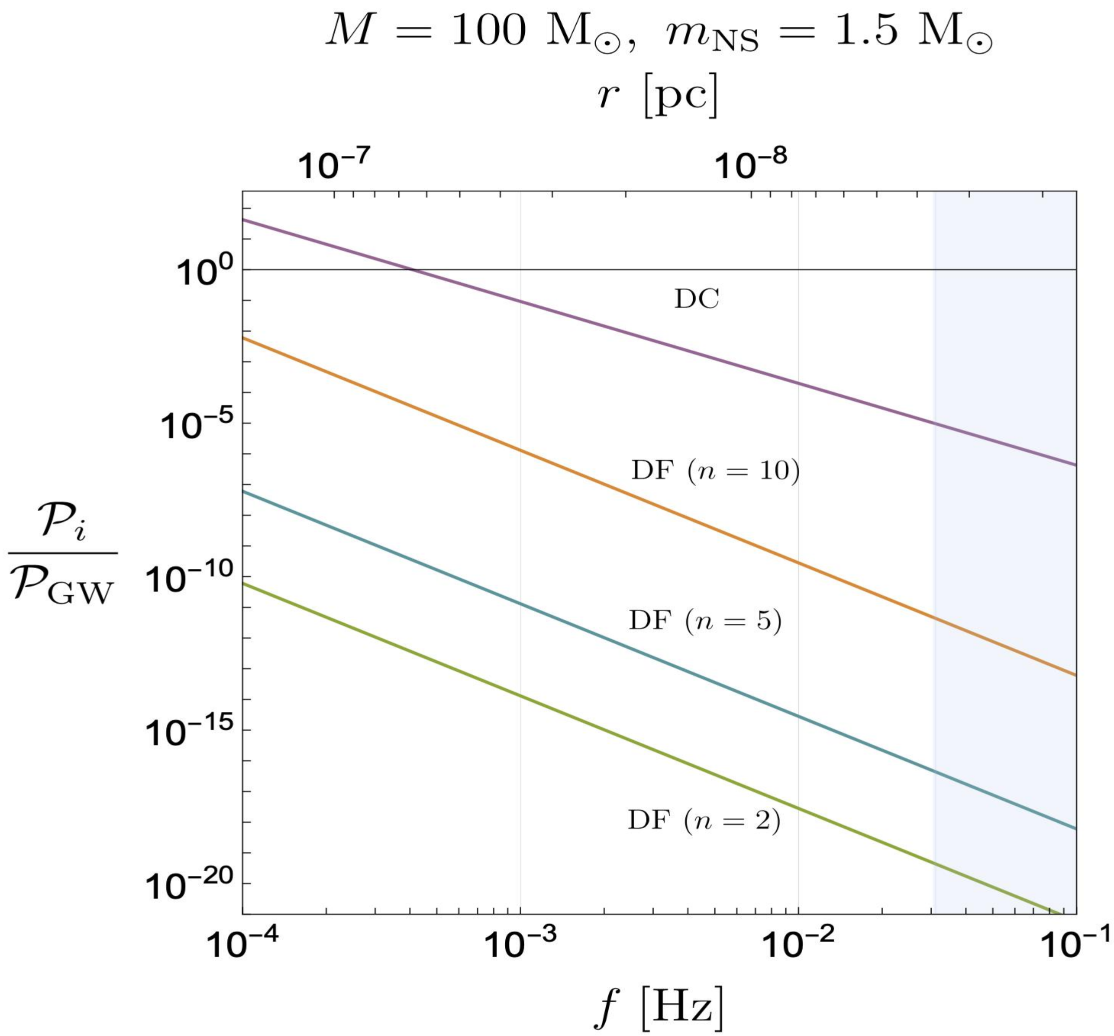
Without ultralight axions $-\frac{\mathrm{d}E_0}{\mathrm{d}t}=\mathcal{P}_{\mathrm{GW}}$ $\mathcal{P}_{\mathrm{GW}}=\frac{32}{5}\mu^2 r^4\omega^6$

With ultralight axions

$$-\frac{dE}{dt}=(\mathcal{P}_{\mathrm{GW}}+\mathcal{P}_{\mathrm{DC}}+\mathcal{P}_{\mathrm{DF}}+\mathcal{P}_{\mathrm{DR}})$$

dynamical friction (DF), depletion of axion cloud (DC), dipole radiation(DR)

Ning Xie, **FPH**, SCPMA Vol.66, No.1(2024)


$M = 100\ \mathrm{M}_\odot,\ m_{\rm NS} = 1.5\ \mathrm{M}_\odot$
r [pc]
10^{-7}
10^{-8}
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
$\frac{\mathcal{P}_i}{\mathcal{P}_{\rm GW}}$
DC
DF ($n = 10$)
DF ($n = 5$)
DF ($n = 2$)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
f [Hz]

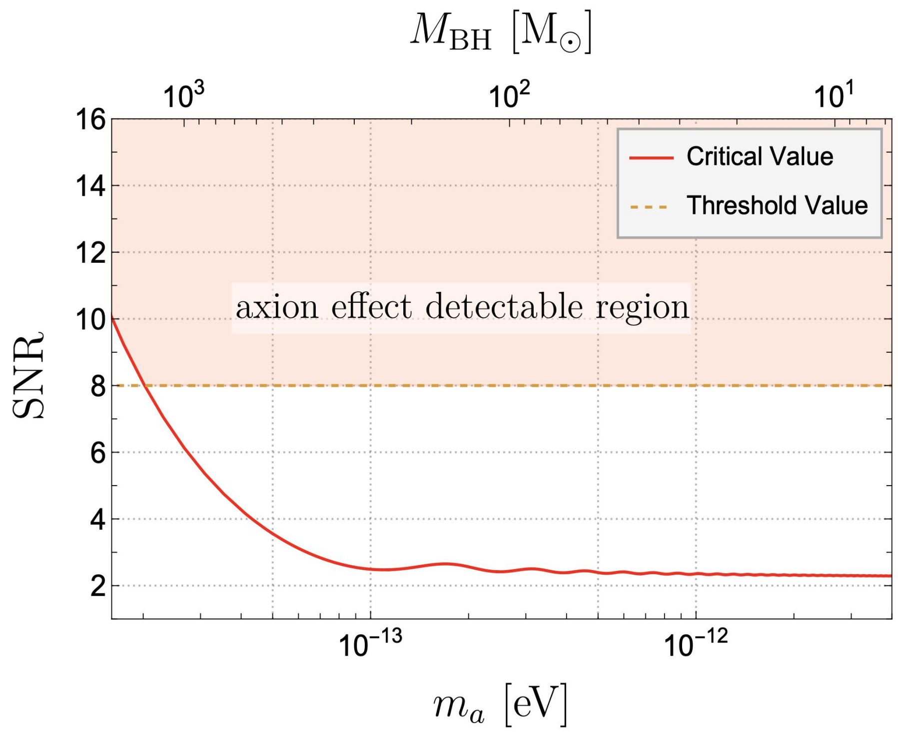
Imprints of axions on GW

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \left(-\frac{M m_{\mathrm{NS}}}{2r^2}\right)^{-1} (\mathcal{P}_{\mathrm{GW}} + \mathcal{P}_{\mathrm{DC}} + \mathcal{P}_{\mathrm{DF}} + \mathcal{P}_{\mathrm{DR}})$$

$$\Delta\phi \sim 15\pi \left(\frac{m_a}{10^{-12}\ \mathrm{eV}}\right)\left(\frac{f_T}{10^{-2}\ \mathrm{Hz}}\right)\left(\frac{T}{5\ \mathrm{yrs}}\right)^2$$

Ning Xie, FPH, SCPMA Vol.66, No.1(2024)

Complementary search: GW+PTA



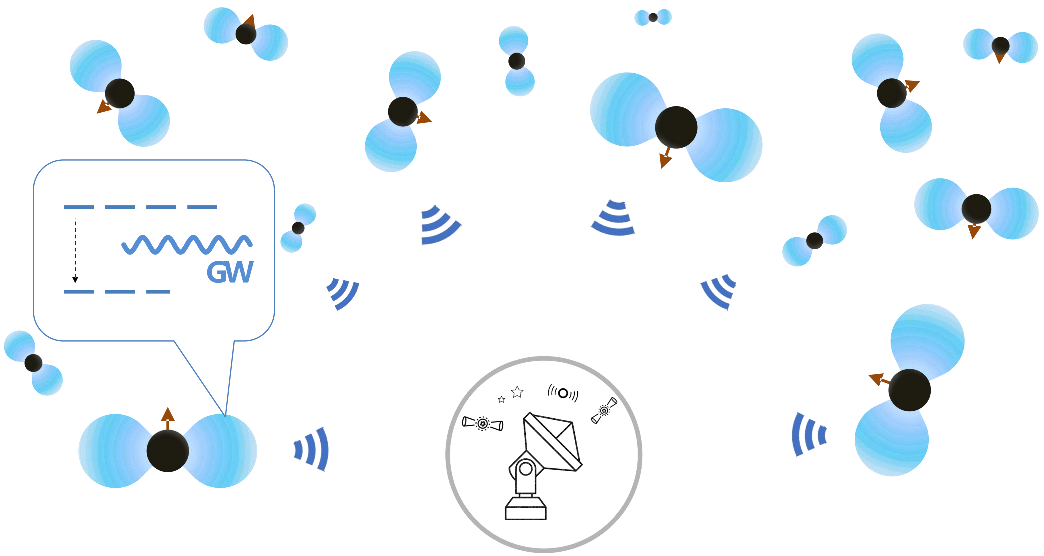
Axions modify the rate of binary period change

$$\Delta\dot{P} = \left|\dot{P} - \dot{P}_{\rm vac}\right| \approx 10^{-12}\ {\rm s/s}$$

Future Pulsar timing measurement precision, such as SKA

$$10^{-15}\ {\rm s/s}$$

Fuzzy axion (DM) particles

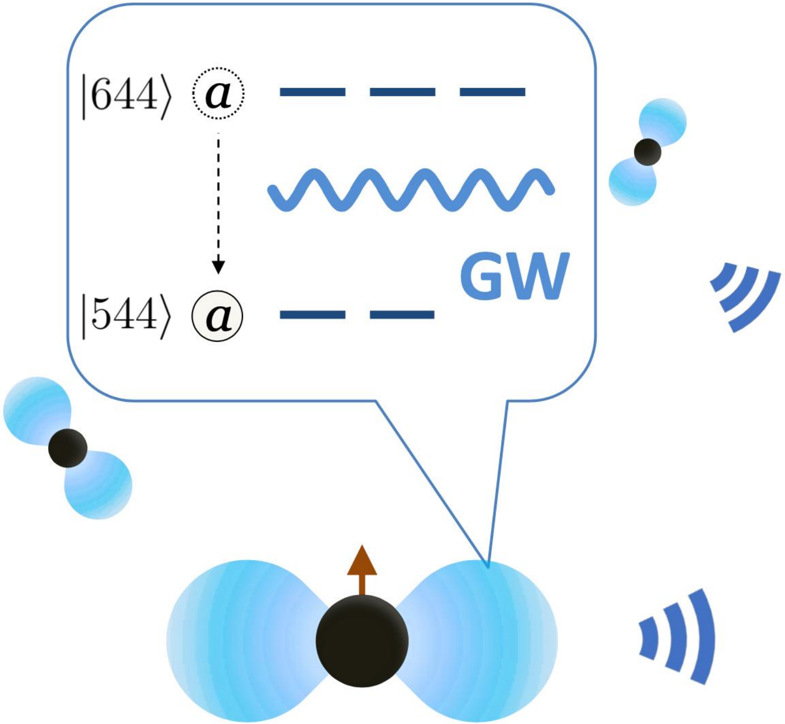


The cosmic populated SMBHs dressed with axion cloud as a natural source of nano-Hertz GW. The energy level transition process can radiate GWs continuously, which naturally fall in nano-Hertz frequency band.

Consequently, the PTA could detect this new source which provides a new approach to probe ultralight axion DM and isolated BHs.

Jing Yang, Ning Xie, FPH, arXiv:2306.17113

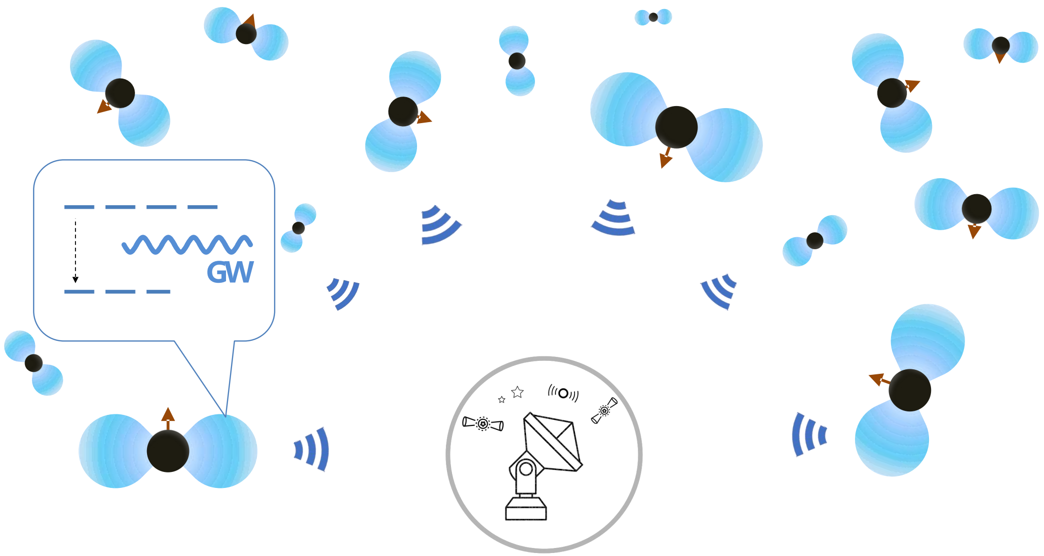
Fuzzy axion (DM) particles



$$\Delta\omega = \frac{11}{1800}\alpha^2 m_a$$

$$P = -\frac{dE}{dt} = \frac{dN_5(t)}{dt}\Delta\omega$$

Fuzzy axion (DM) particles



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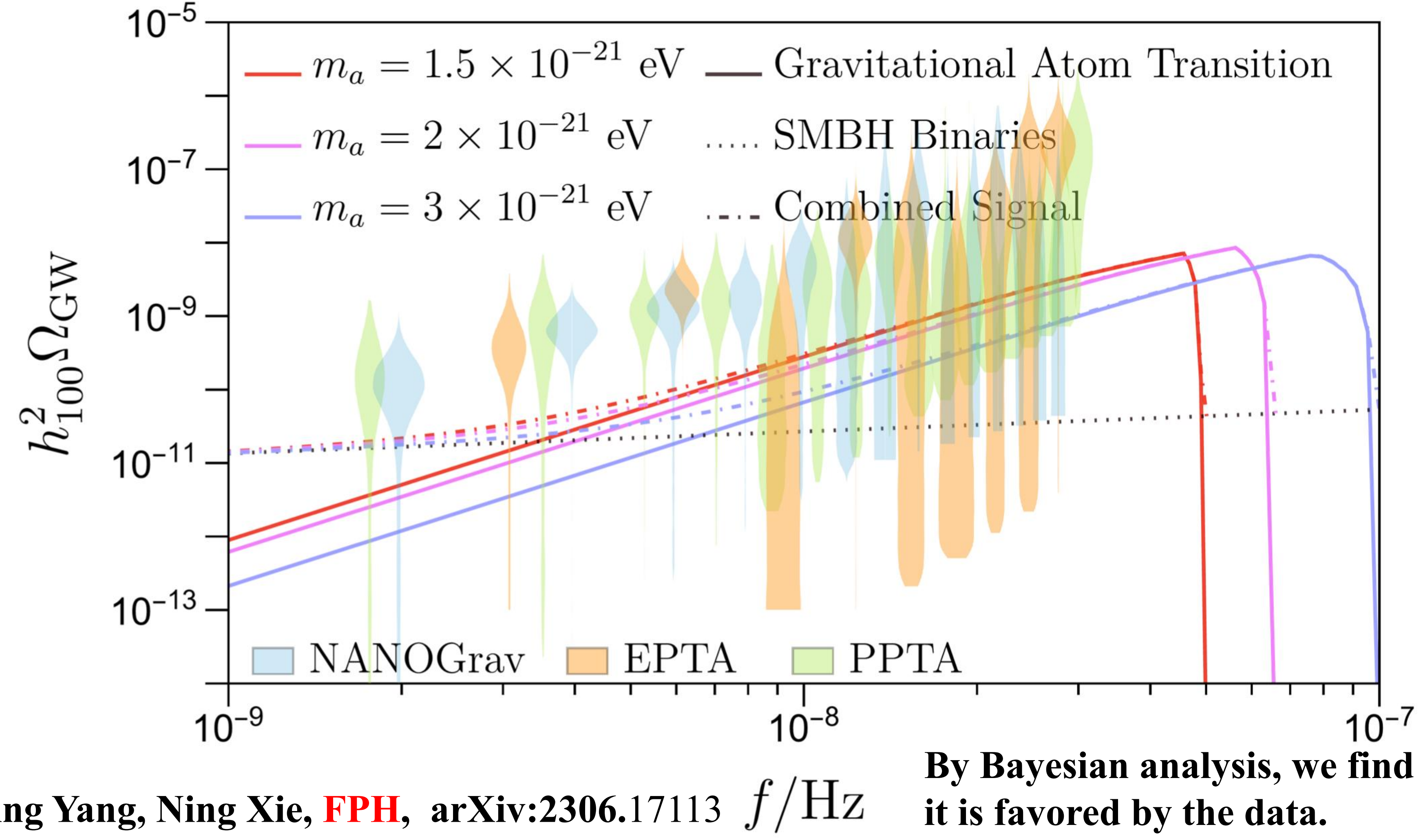


Jing Yang, Ning Xie, FPH, arXiv:2306.17113

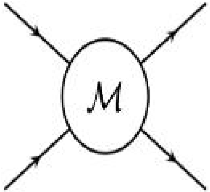
By Bayesian analysis, we find it is favored by the data.

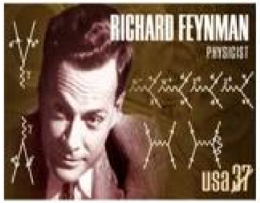
State of the art: GW precise calculation

Scattering amplitude method in GW precise calculations.

See Zvi Bern's recent works

Modern tools from collider physics!





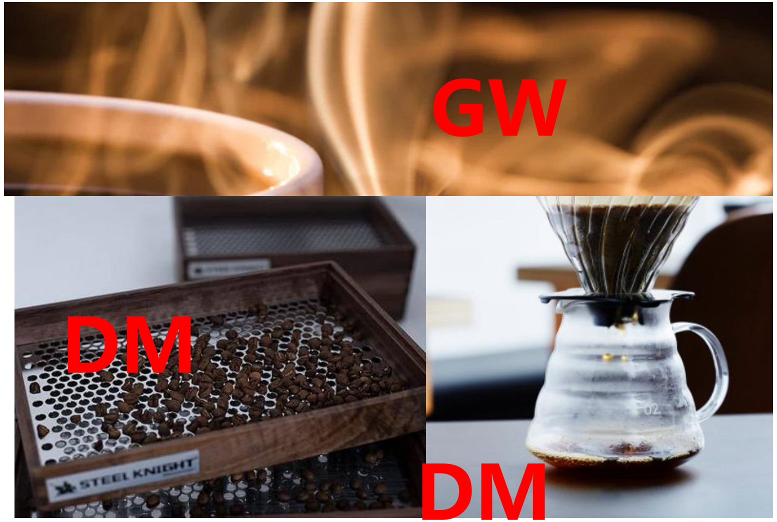
Towards accurate calculations of GW power spectrum from new physics models: non-linear evolution (turbulence, shocks) not well understood; P.Auclair, C.Caprini, D.Cutting, M.Hindmarsh, K.Rummukainen, D.A.Steer and D.J.Weir, [arXiv:2205.02588]
J.Dahl, M.Hindmarsh, K.Rummukainen and D.J.Weir, [arXiv:2112.12013].

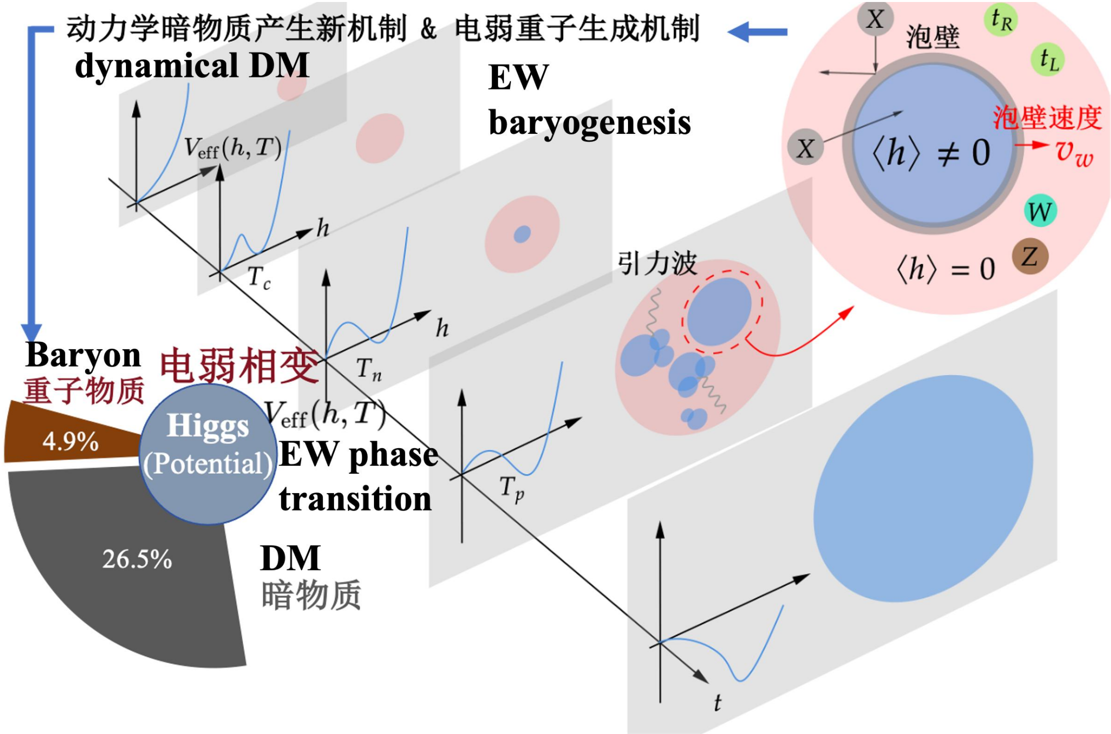
EW baryogenesis with high bubble wall velocity

See James Cline and Hindmarsh's recent works

Summary and outlook

- Bubble wall can be a natural filter to produce DM.
- GW provides new approaches to explore the nature of DM.





Thanks!　Comments and collaborations are welcome!

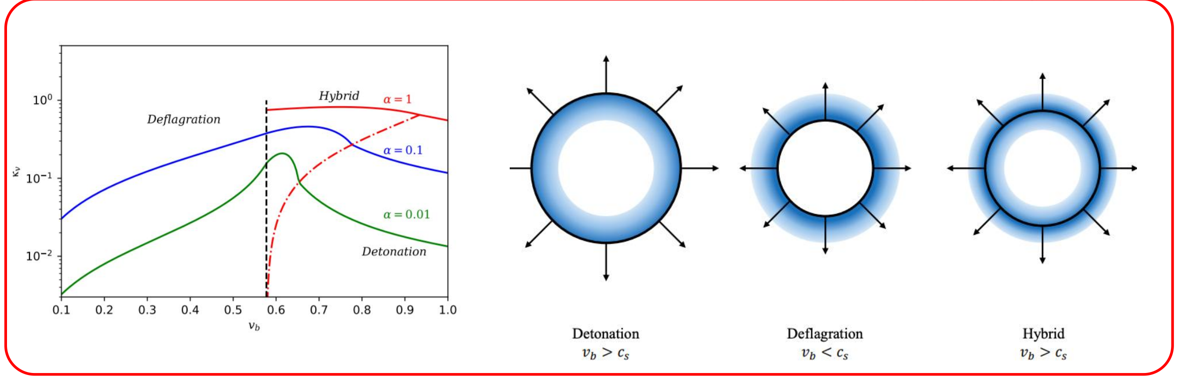
Email:huangfp8@sysu.edu.cn

Backup slides

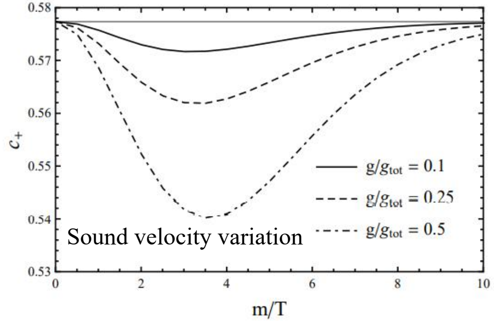
Energy budget (to measure the efficiency of the energy released by a SFOPT converting to the kinetic energy of sounding plasma)

LISA Cosmology working group *JCAP* 03 (2020) 024

$$h^2\Omega_{\rm sw}(f) \simeq 1.64\times10^{-6}(H_*\tau_{\rm sw})(H_*R_*)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}(f/f_{\rm sw})^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$



Xiao Wang, **FPH** and Xinmin Zhang, JCAP 05, 045 (2020)



L. Leitao and A. Megevand, Nucl. Phys. B **891**, 159-199 (2015)

Most of current studies of the efficiency parameter are based on bag EoS, which assume the sound velocity is $1/\sqrt{3}$ in both phases. But for a realistic SFOPT, particle can obtain the mass, hence, the sound velocity can deviate from pure radiation phase. The efficiency parameter (based on the bag model), the energy budget/efficiency parameter beyond the bag model are study in our recent work:

Xiao Wang,**FPH** and Xinmin Zhang, PRD 103 (2021) 10, 103520

Xiao Wang, **FPH** and Xinmin Zhang, JCAP 05, 045 (2020)

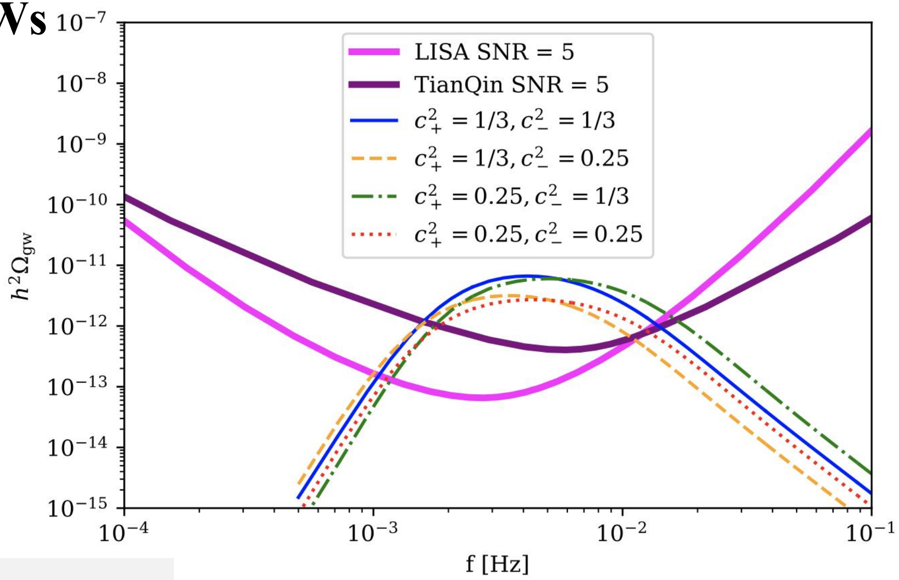
Xiao Wang, Chi Tian, **FPH**, arXiv: 2301.12328

Phase transition Dynamics

The effect of sound velocity on the GWs

reheating effect is also important

重点是计算宇宙早期的高温粒子汤中的形成的声波的速度分布和能量分布，进而可以得到剪切应力张量。



$$P_{\dot h} \sim U_\Pi \sim \langle \tau\tau \rangle \sim \langle \tilde v \tilde v \tilde v \tilde v \rangle = \sum \langle \tilde v \tilde v \rangle \langle \tilde v \tilde v \rangle \sim \sum P_v P_v$$

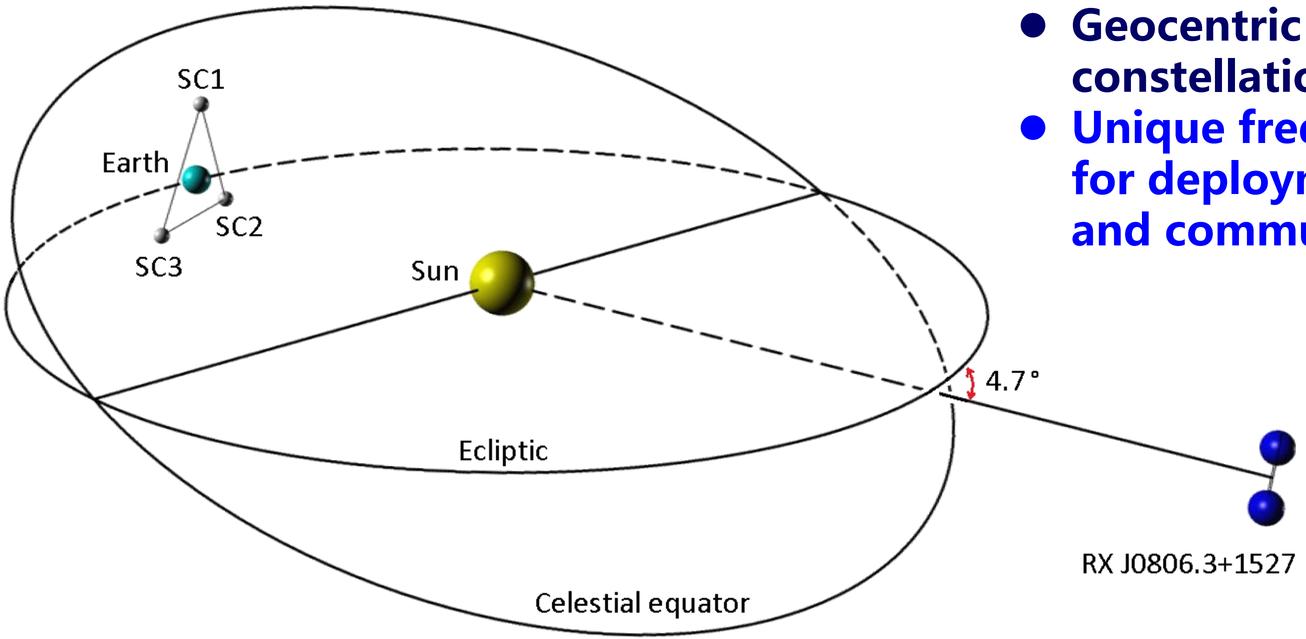
Xiao Wang, **FPH**, Yongping Li, Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model, Phys.Rev.D 105 (2022) 103513

Xiao Wang, **FPH,** Xinmin Zhang, Energy budget and the gravitational wave spectra beyond the bag model ,Phys.Rev.D 103 (2021) 10, 103520

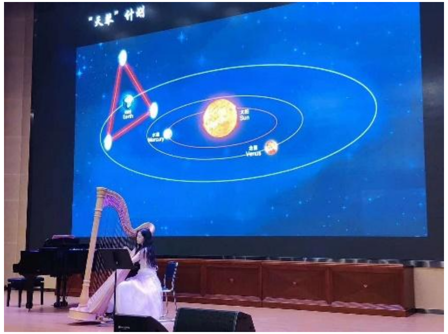
Xiao Wang, Chi Tian, **FPH**, arXiv: 2301.12328

Backup slides

- **Expected in 2035**
- **Geocentric orbit, normal triangle constellation, radius ~10^5km**
- **Unique frequency band, easier for deployment, tracking, control, and communication**



J. Luo et al. TianQin: a space-borne gravitational wave detector, Class. Quant. Grav. 33 (2016) no.3, 035010.



"天琴"
"Harpe in space"

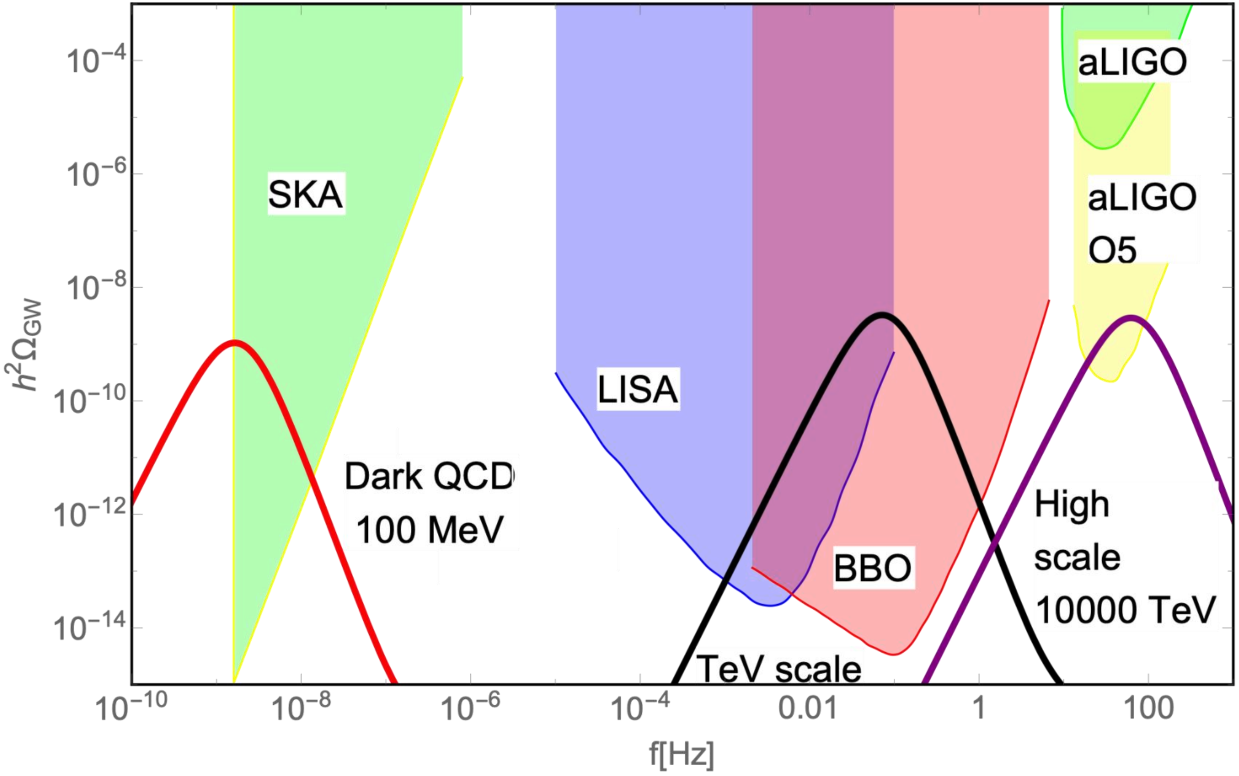
SFOPT and new DM mechanism/signal

Phase transition GW can provide a unique way to probe many important physics processes: inflation, PQ-symmetry breaking, neutrino physics, axion physics, extra dimension, primordial magnetic field, cosmic defects...

Wei-Chih Huang, et. al, arXiv: 2012.11614
Mark Hindmarsh, et. al. arXiv: 2011.12878
Bhupal Dev, et. al. arXiv: 1905.00891
Yiyang Zhang, et. al. arXiv:1902.02751
Yang Bai, et. al. arXiv:1810.04360
Andrew Long, et. al. arXiv:1703.04902
Graciela Gelmini, et. al. arXiv:2009.01903
Stephen King, et.al. arXiv:2005.13549
Bhupal Dev, et.al. arXiv:1602.04203
Astrid Eichhorn et.al. arXiv:2010.00017
Yuefeng Di, et. al., arXiv: 2012.15625
Haipeng An,et.al. arXiv:2009.12381
FPH, Xinmin Zhang, Phys.Lett. B788 (2019) 288-29,
Jia Liu, et.al. arXiv:2104.06421
Zhao Zhang, et. al. arXiv:2102.01588
Wei Liu， et.al. arXiv:2101.10469
Cheng-wei Chiang, et.al. arXiv:2012.14071
Ke-Pan Xie, et.al. arXiv:2011.04821
Ligong Bian， et.al. arxiv:1907.13589
Zhaofeng Kang,et.al. arXiv:2101.03795
Zhaofeng Kang, et. al. arXiv:2003.02465
a lot of new and nice works unmentioned here

SFOPT and new physics/early universe

Generally



FPH, Xinmin Zhang, Phys.Lett. B788 (2019) 288-29,

Phase transition dynamics

Energy budget for phase transition GW

Bag EoS

$$p_+ = \frac{1}{3}a_+T_+^4 - \epsilon_+, \quad e_+ = a_+T_+^4 + \epsilon_+ \,,$$

$$p_- = \frac{1}{3}a_-T_-^4 - \epsilon_-, \quad e_- = a_-T_-^4 + \epsilon_- \,.$$

$$\alpha_\theta = \frac{4}{3}\frac{\Delta\epsilon}{w_+}, \quad \epsilon_\pm = \frac{1}{4}(e_\pm - 3p_\pm)$$

Strength parameter

$$\partial p/\partial e = c_s^2 = \text{constant}$$

EoS with different sound velocity (DSVM)

$$p_+ = c_+^2 a_+T_+^4 - \epsilon_+, \quad e_+ = a_+T_+^4 + \epsilon_+ \,,$$

$$p_- = c_-^2 a_-T_-^4 - \epsilon_-, \quad e_- = a_-T_-^4 + \epsilon_- \,,$$

$$\alpha_{\bar\theta} = \frac{\Delta\bar\theta}{3w_+}, \quad \bar\theta = e - p/c_-^2$$

Strength parameter

F. Giese, T. Konstandin, and J. van de Vis, JCAP. 07 (2020) 057.

Phase transition dynamics

Energy budget for phase transition GW

- Matching condition

$$w_- v_-^2 \gamma_-^2 + p_- = w_+ v_+^2 \gamma_+^2 + p_+ ,$$
$$w_- v_- \gamma_-^2 = w_+ v_+ \gamma_+^2 .$$

$$\longrightarrow \quad v_+ v_- = \frac{p_+ - p_-}{e_+ - e_-}, \quad \frac{v_+}{v_-} = \frac{e_- + p_+}{e_+ + p_-} .$$

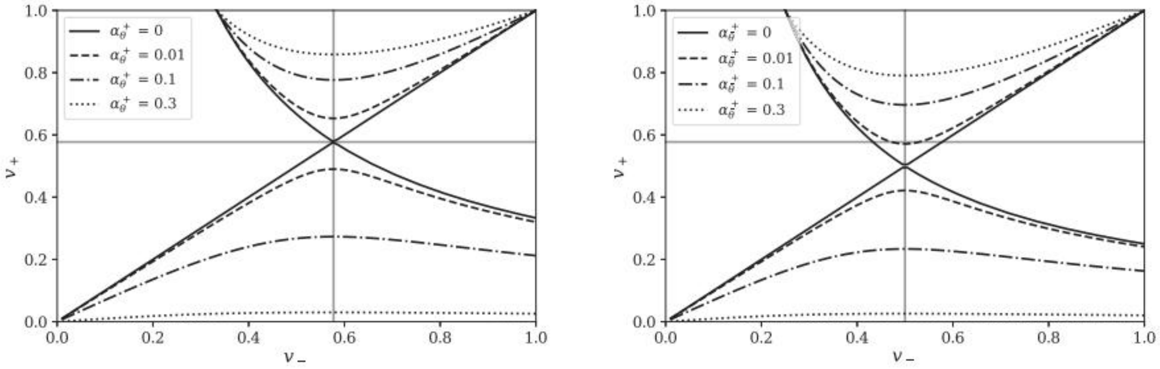


FIG. 1. The fluid velocities v_+ and v_- in the reference frame of bubble wall for different definitions and values of phase transition strength parameter. The horizontal and vertical gray lines indicate the sound velocities of symmetric and broken phase. Left panel: the bag model. Right panel: the DSVM with $c_+^2 = 1/3$ and $c_-^2 = 0.25$.

X. Wang, F. P. Huang and X. Zhang, PRD 103 (2021) 10, 103520

Phase transition dynamics

Energy budget for phase transition GW

- Energy momentum conservation derive fluid equation:

$$(\xi - v)\frac{\partial_\xi e}{w} = 2\frac{v}{\xi} + \gamma^2(1 - v\xi)\partial_\xi v \,,$$

$$(1 - v\xi)\frac{\partial_\xi p}{w} = \gamma^2(\xi - v)\partial_\xi v \,.$$

$$2\frac{v}{\xi} = \gamma^2(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v$$

Velocity profile

$$\frac{\partial_\xi w}{w} = \left(1 + \frac{1}{c_s^2}\right)\mu\gamma^2\partial_\xi v$$

Enthalpy profile

$$\frac{\partial_\xi T}{T} = \gamma^2\mu\partial_\xi v$$

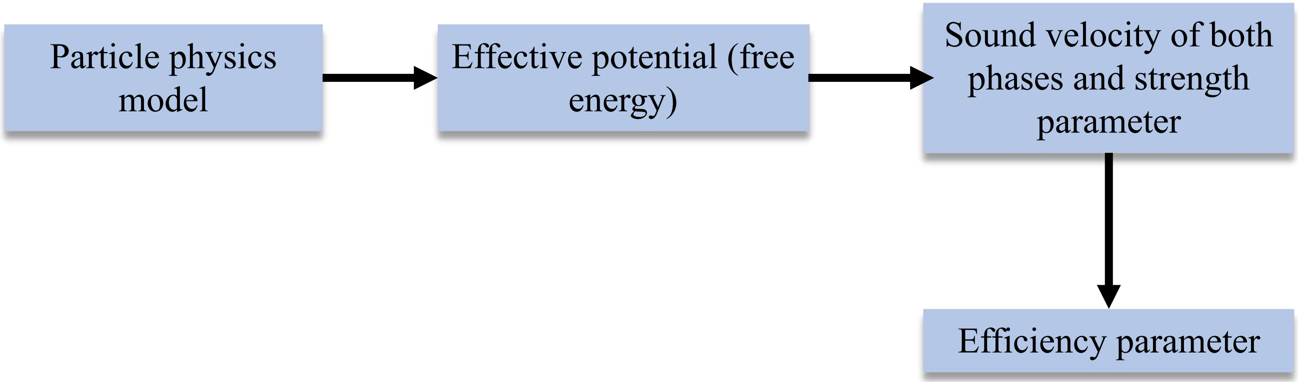
Temperature profile

Different boundary conditions give different hydrodynamical modes.

Phase transition dynamics

Energy budget for phase transition GW

- The method to map a particle physics model on the DSVM to get efficiency parameter

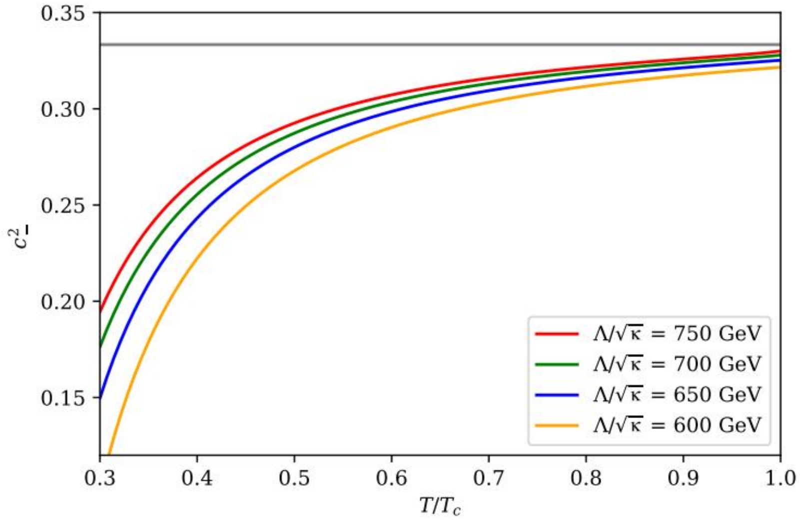


Phase transition dynamics

Energy budget for phase transition GW

The evolution of sound velocity of broken phase and symmetric phase in Dim-6 effective model:

$$\mathcal{F}(\phi,T) \approx -\frac{a_\pm}{3}T^4 + \frac{\mu^2 + cT^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\kappa}{8\Lambda^2}\phi^6$$



X. Wang, F. P. Huang and X. Zhang, PRD 103 (2021) 10, 103520

Phase transition dynamics

Energy budget for phase transition GW

GW spectrum and SNR for different EoS with different parameter combination:

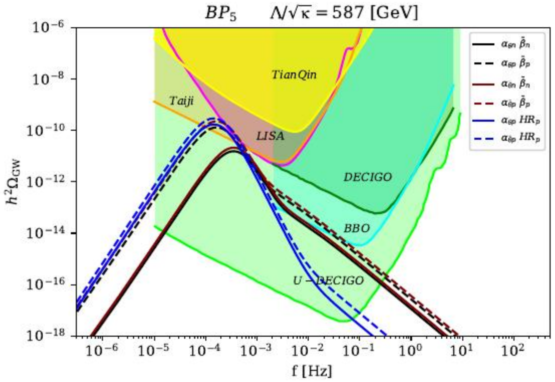


TABLE II. The SNR of BP_5 for different experiment configurations with different combinations of phase transition parameters and models of EOS.

	$\alpha_{\theta n}\,\tilde{\beta}_n$	$\alpha_{\theta p}\,\tilde{\beta}_p$	$\alpha_{\bar{\theta} n}\,\tilde{\beta}_n$	$\alpha_{\bar{\theta} p}\,\tilde{\beta}_p$	$\alpha_{\theta p}\,HR_p$	$\alpha_{\bar{\theta} p}\,HR_p$
$\mathrm{SNR}_{(\mathrm{LISA})}$	7.949	16.930	10.913	28.836	16.009	27.468
$\mathrm{SNR}_{(\mathrm{Taiji})}$	14.760	58.607	20.271	100.343	66.216	113.609
$\mathrm{SNR}_{(\mathrm{TianQin})}$	0.452	1.506	0.620	2.576	1.629	2.794

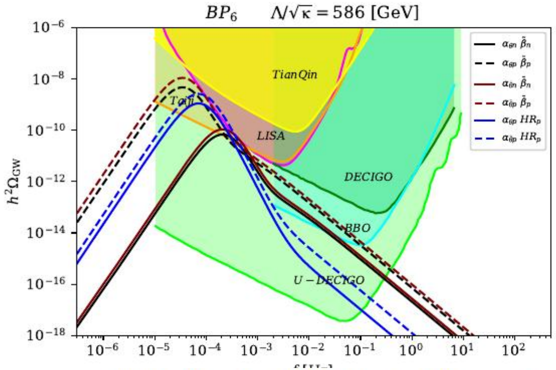


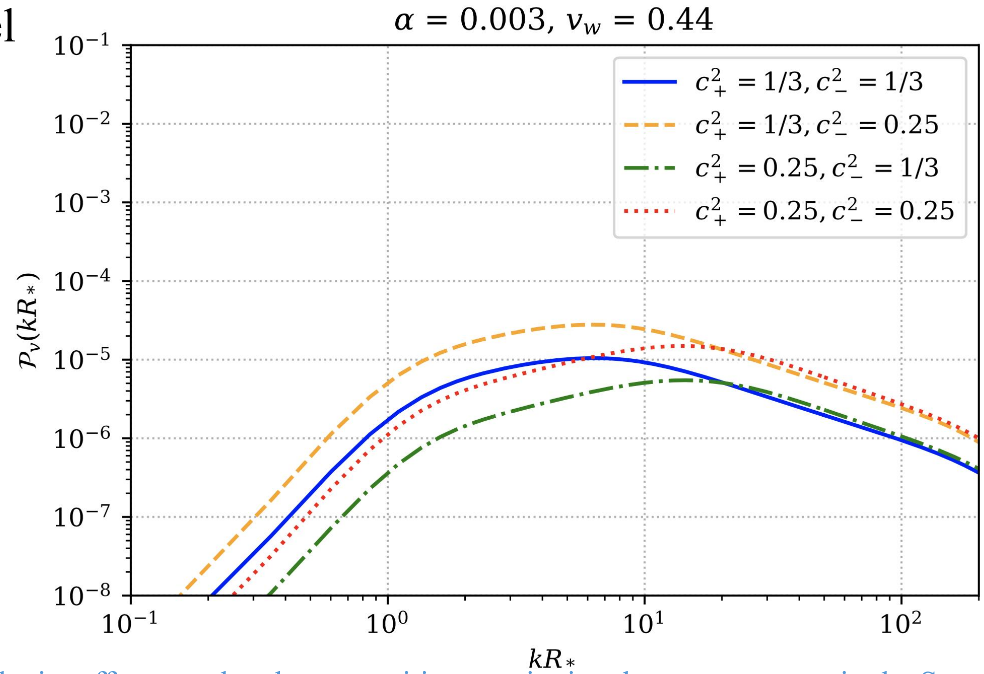
TABLE III. The SNR of BP_6 for different experiment configurations with different combinations of phase transition parameters and models of EOS.

	$\alpha_{\theta n}\,\tilde{\beta}_n$	$\alpha_{\theta p}\,\tilde{\beta}_p$	$\alpha_{\bar{\theta} n}\,\tilde{\beta}_n$	$\alpha_{\bar{\theta} p}\,\tilde{\beta}_p$	$\alpha_{\theta p}\,HR_p$	$\alpha_{\bar{\theta} p}\,HR_p$
$\mathrm{SNR}_{(\mathrm{LISA})}$	14.230	15.368	22.470	26.382	17.367	40.816
$\mathrm{SNR}_{(\mathrm{Taiji})}$	38.666	427.813	61.208	1000.501	213.123	500.668
$\mathrm{SNR}_{(\mathrm{TianQin})}$	1.060	5.569	1.678	12.934	3.973	9.333

X. Wang, F. P. Huang and X. Zhang, PRD 103 (2021) 10, 103520

Phase Transition dynamics

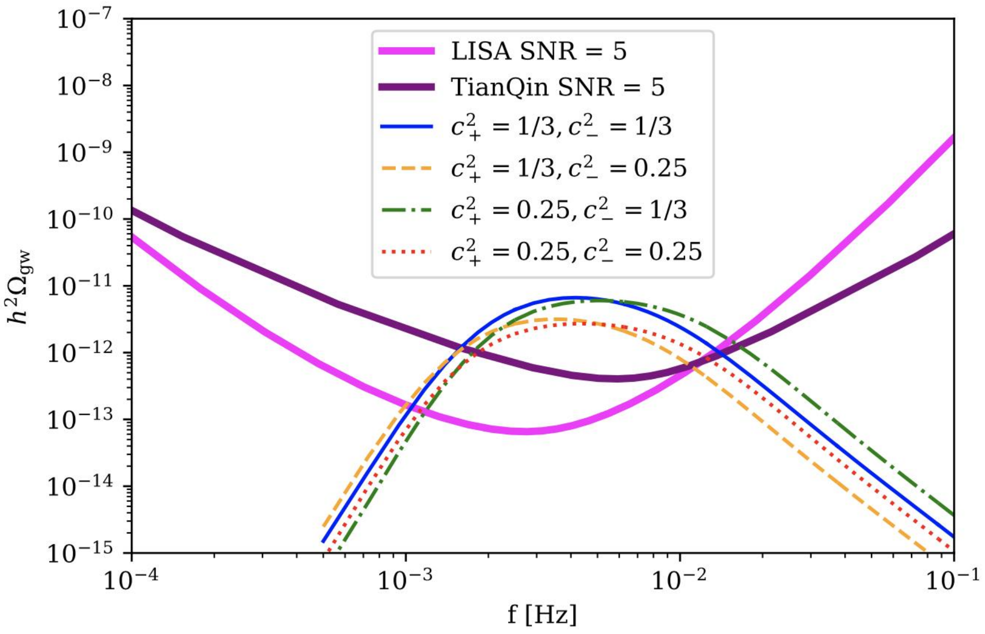
Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model



Xiao Wang, **FPH**, Yongping Li, Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model, arXiv:2112.14650

Xiao Wang, **FPH,** Xinmin Zhang, Energy budget and the gravitational wave spectra beyond the bag model ,Phys.Rev.D 103 (2021) 10, 103520

Phase transition dynamics



Xiao Wang, **FPH**, Yongping Li, Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model, arXiv:2112.14650

Xiao Wang, **FPH,** Xinmin Zhang, Energy budget and the gravitational wave spectra beyond the bag model ,Phys.Rev.D 103 (2021) 10, 103520

Anisotropy and primordial seeds

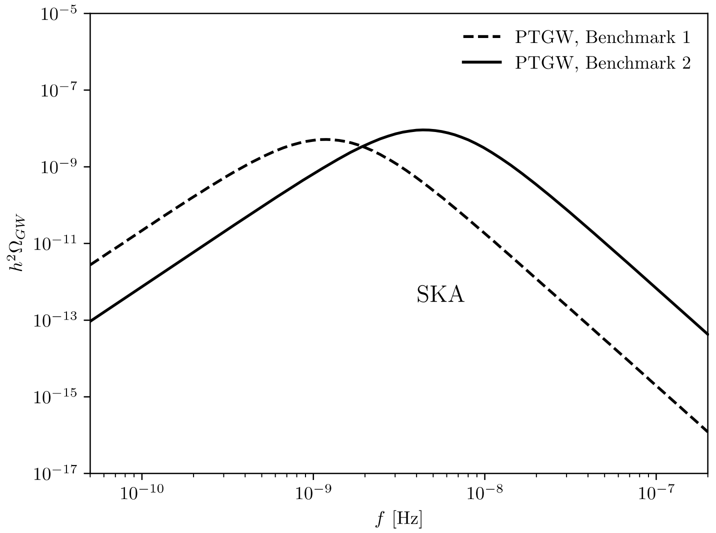
$$\boxed{H_*^2 = \rho/3M_{\rm pl}^2}$$

$$h^2\Omega_{\rm GW}(f) \simeq 1.64\times 10^{-6}\left(\frac{4}{3}\right)^{\frac{1}{2}} \boxed{(H_*R_*)^2} \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^{\frac{3}{2}} \times \left(\frac{100}{g_*}\right)^{\frac{1}{3}} (f/f_{\rm sw})^3 \left(\frac{7}{4+3\,(f/f_{\rm sw})^2}\right)^{\frac{7}{2}}$$

$$f_{\rm sw} \simeq 2.6\times 10^{-5}\ {\rm Hz}\frac{1}{H_*R_*}\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$ Peak frequency

QCD-like DM model

	Benchmark 1	Benchmark 2
phase transition strength	$\alpha = 0.5$	
D.O.F	$g_* = 10$	
efficiency factor	$\kappa_v \approx 0.44$	
bubble wall velociy	$v_b = 0.95$	
characteristic temperature	$T_* = 1$ MeV	$T_* = 5$ MeV
mean bubble separation	$H_*R_* = 0.15$	$H_*R_* = 0.2$

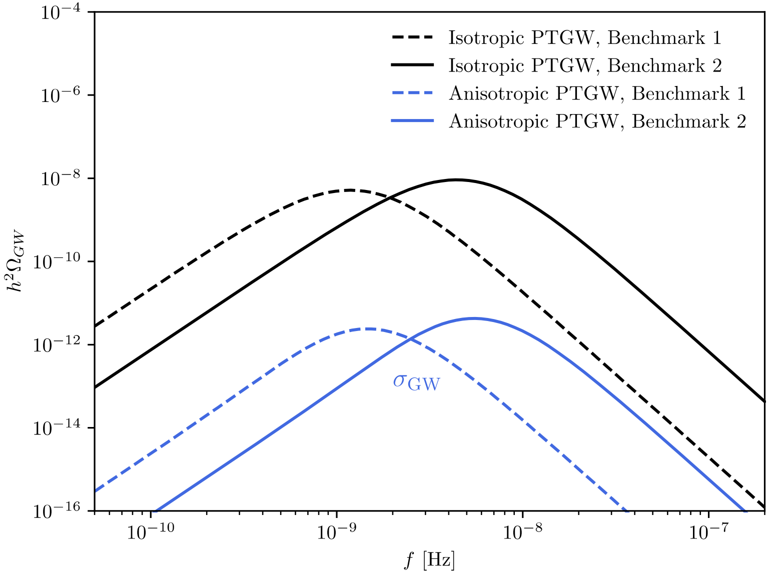


Anisotropy and primordial seeds

$$\mathrm{Var}^{\mathcal{G}} = \frac{1}{4\pi}\sum_{\ell}(2\ell+1)C_{\ell}^{\mathcal{G}}$$

$$\sigma_{\mathrm{GW}}(p) \equiv h^2\Omega_{\mathrm{GW}}(p)\sqrt{\mathrm{Var}^{\delta_{\mathrm{GW}}}(p)}$$

CMB TT anisotropy	4×10^{-5}
PTGW anisotropy	1×10^{-4}
PTGW energy spectra anisotropy	8×10^{-4} $(> f_{\mathrm{sw}})$



Anisotropy and primordial seeds

$$\rho_{\rm GW}(\eta,\boldsymbol{x}) = \int d^3\boldsymbol{p}\, p f(\eta,\boldsymbol{x},\boldsymbol{p}) = \int dp d\hat{\boldsymbol{p}} p^3 f(\eta,\boldsymbol{x},p,\hat{\boldsymbol{p}})$$

$$\Omega_{\rm GW}(\eta,\boldsymbol{x},p) = \int \frac{d\hat{\boldsymbol{p}}}{4\pi}\bar{\Omega}_{\rm GW}(\eta,p)\left[1+\delta_{\rm GW}(\eta,\boldsymbol{x},p,\hat{\boldsymbol{p}})\right]$$

$$= \int d\hat{\boldsymbol{p}}\frac{p^4}{\rho_c}\left[\bar{f}(\eta,p) - p\frac{\partial\bar{f}(\eta,p)}{\partial p}\mathcal{G}(\eta,\boldsymbol{x},\hat{\boldsymbol{p}})\right]$$

anisotropy of GW energy spectra $\delta_{\rm GW} = \frac{\delta\Omega_{\rm GW}(\eta,\boldsymbol{x},p,\hat{p})}{\bar{\Omega}_{\rm GW}(\eta,p)}$

$$\delta_{\rm GW}(\eta,\boldsymbol{x},p,\hat{p}) = \left[4 - \frac{\partial\ln\bar{\Omega}_{\rm GW}(\eta,p)}{\partial\ln p}\right]\mathcal{G}(\eta,\boldsymbol{x},\hat{p})$$

$$C_l^{\delta_{\rm GW}}(p) = g^2(p)C_l^{\mathcal{G}}$$