Overview of Black Hole Superradiance

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2024/09/07 Weihai

Outline

\diamond Introduction

\diamond The fast-growing cloud

♦ Evolution of BH-cloud system

\diamond Observables

\diamond Summary

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Kerr Metric

• Describing a rotating black hole

$$ds^{2} = \left(1 - \frac{2r_{g}r}{\Sigma}\right)dt^{2} + \frac{4 a r_{g}r}{\Sigma}\sin^{2}\theta \, dt \, d\varphi - \frac{\Sigma}{\Delta}dr^{2}$$
$$- \Sigma \, d\theta^{2} - \left[(r^{2} + a^{2})\sin^{2}\theta + 2\frac{r_{g}r}{\Sigma}a^{2}\sin^{4}\theta\right]d\varphi^{2}.$$
with $a = J/M, r_{g} = GM$
$$\Delta = r^{2} - 2r_{g}r + a^{2}$$
$$\Sigma = r^{2} + a^{2}\cos^{2}\theta$$

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Horizons: $\Delta = 0 \implies r_{\pm} = r_{g} \pm \sqrt{r_{g}^{2} - a^{2}}$ Ergo-sphere: $1 - \frac{2r_{g}r}{\Sigma} = 0 \implies r_{\mathrm{E},\pm} = r_{g} \pm \sqrt{r_{g}^{2} - a^{2} \cos^{2} \theta^{2}}$

Ergo-region

• Region between outer horizon and outer ergo-sphere



- In the ergoregion, energy can be negative.
- Penrose process: extracting energy and angular mom. From BH

Superradiance

• A wave shining on a Kerr BH could be enhanced in magnitude.



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Superradiance

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• With a BH wrapped with mirrors, the wave grows exponentially with time, until the mirror breaks.



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Letter | Published: 28 July 1972

Floating Orbits, Superradiant Scattering and the Black-hole Bomb

WILLIAM H. PRESS & SAUL A. TEUKOLSKY

<u>Nature</u> 238, 211–212 (1972) Cite this article

Superradiance with a Bound State

- If the wave can from a **bound state** around the BH, the wave cannot propagate to infinity.
 - ➤ A natural "mirror".

$$M_{\odot} \sim 10^{-10} \text{ eV}$$

- Requires de Brogile wavelength ~ BH horizon
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- Physics at the outer horizon r_+

Use tortoise coordinate, $r_* = r - \frac{r_+ + r_-}{r_+ - r_-} \left[r_- \log \left(\frac{r - r_-}{2M} \right) - r_+ \log \left(\frac{r - r_+}{2M} \right) \right]$

The wave close to the outer horizon is $\propto e^{-i\omega t + i(\omega - m\Omega_H)r_*}$

angular velocity of the outer horizon: $\Omega_H = a/2r_ar_+$

The group velocity and phase velocity are in different direction if $\omega < m\Omega_H$ **Superradiance condition!**

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Non-interacting Real Scalar Field

- Simplest example: non-interacting massive real scalar
- Assume small energy density **—** can still use Kerr metric
- Klein-Gordon equation in Kerr metric

 $(\nabla^{\nu}\nabla_{\nu} + \mu^2)\Phi = 0$ μ : scalar mass

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• The bound state eigen-energy is complex $\omega_{n\ell m} = \omega_{n\ell m}^{(R)} + i\omega_{n\ell m}^{(I)}$

Three indices: $(n \ge 0, l, m)$, similar to hydrogen atom

 The condensate mass and angular momentum grow exponentially

$$M_s \propto \exp(2\omega_{nlm}^{(I)}t)$$

If $\omega_{nlm}^{(l)} > 0$, called superradiance rate



Superradiance Rate

 $M_s \propto \exp(2\omega_{nlm}^{(I)} t)$

• Analytic approximation at LO of $\alpha = M\mu$

M: BH mass μ: axion mass

KLEIN-GORDON EQUATION AND ROTATING BLACK HOLES							
Steven L. Detweiler (Yale U.) (1980)							
Published in: <i>Phys.Rev.D</i> 22 (1980) 2323-2326							
& DOI	[∃ cite	🔁 claim	বি reference search				

• Complicated numerical solution

Instability of the massive Klein-Gordon field on the Kerr spacetime #4									
Sam R. Dolan (University Coll., Dublin) (May, 2007)									
Published in: Phys.Rev.D 76 (2007) 084001 • e-Print: 0705.2880 [gr-qc]									
🔎 pdf	∂ DOI	[→ cite	🗟 claim		a reference search				

NLO Calculation

- We confirm a mistake in the LO calculation of $\omega_{nlm'}^{(I)}$ and calculate the NLO contribution for the first time.
- Leading order

At small α with a = 0.99,

Err. of original result $\sim 150\%$

Err. of corrected result $\sim 30\%$

Analytic and numerical results do not converge at $\alpha \rightarrow 0$!



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NLO Sol. of KNBH

• NLO solution greatly improves the precision

BH mass is normalized to 1, BH charge Q = 0.02



• In the rest of this part, I focus only on Kerr BH.

Important Modes

- Three indices $(n \ge 0, l, m)$
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- Three indices $(n \ge 0, l, m)$
- (n, l, l) are the most important, modes with $m \neq l$ can be ignored
- (*n*, *l*, *l*) modes with different *l* have very different rates
 - Very different time scales
 Can consider a single *l* in each time range
- In each *l*-stage, (0, *l*, *l*) is the fastest, followed by (1, *l*, *l*)



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- In each *l*-stage, (0, *l*, *l*) is the fastest, followed by (1, l, l)



Superradiance condition $\mu \sim \omega_R < m \Omega_H$ gives the max. of $M\mu$ $\Omega_H = a/2r_+$ is the angular velocity of the BH outer horizon. 21

Fields with Nonzero Spin

 $M_s \propto \exp(2\omega_{nlm}^{(I)} t)$

- Complex scalar: same eigenvalue as the real scalar. Strongest mode $\omega^{(I)} \sim \mu (M\mu)^9$
- Fermion: free Dirac field in Kerr metric always have $\omega_{nlm}^{(I)} < 0$

lyer, Kumar, Phys. Rev. 1978

- > No superradiance when cloud density is small!
- > Not clear with the cloud's contribution to the metric.
- > Not clear with self-interaction.
- Vector: strongest mode has $\omega^{(I)} \sim \mu (M\mu)^7$

Baumann, Chia, Stout, ter Haar, JCAP 2019

• Rank-2 Tensor: strongest mode has $\omega^{(I)} \sim \mu (M\mu)^3$

Brito, Cardoso, Pani, PRD 2013

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Isolated BH-cloud Evolution

- Consider non-interacting real scalar field in a Kerr metric
- Evolution equation of real scalar field



M: BH mass

J: BH angular mom.

 $M_s^{(nlm)}$: mass of mode (nlm) $J_s^{(nlm)}$: angular mom. of mode (nlm)

$$\begin{split} \dot{M} &= -\sum_{nlm} 2M_s^{(nlm)} \omega_I^{(nlm)}, \\ \dot{J} &= -\sum_{nlm} 2m M_s^{(nlm)} \omega_I^{(nlm)} / \omega_R^{(nlm)} & \text{Energy and angular mom.} \\ \dot{M}_s^{(nlm)} &= 2M_s^{(nlm)} \omega_I^{(nlm)} - \dot{E}_{\rm GW}^{(nlm)} \\ \dot{J}_s^{(nlm)} &= 2m M_s^{(nlm)} \omega_I^{(nlm)} / \omega_R^{(nlm)} - m \dot{E}_{\rm GW}^{(nlm)} / \omega_R^{(nlm)} \\ \end{split}$$

Isolated BH-condesate Evolution

- Keep four (n, l, m = l) modes: (0,1,1), (1,1,1), (0,2,2), (1,2,2)
- Evolution can be split into l = 1 and l = 2 stages



Guo, Bao, and **HZ**, PRD (2023)

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BH Evolution

 When transiting between stages, the BH spin drops a lot (~50%) and the BH mass decreases slightly (a few percent)



BH Evolution

- When transiting between stages, the BH spin drops a lot (~50%) and the BH mass decreases slightly (a few percent)
- In each *l* stage, the BH spin and mass are almost constants
 If BH spin increases a little, *E* and *J* flow from BH to condensate
 If BH spin drops a little, *E* and *J* flow from condensate to BH

The condensate is a "angular momentum reservoir" Attractor!



- A Kerr BH is characterized by its angular momentum *J* and mass *M*
- Regge plot: a plot of BH spin $a_* \equiv J/M^2$ vs. BH mass



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High-spin BHs are very rare with superradiance

Condensate Evolution

- At the beginning of each *l* stage, the modes with the same *l* grow exponentially
- The modes lose energy via GW emission \implies GW signal!



Condensate Evolution

- At the beginning of each *l* stage, the modes with the same *l* grow exponentially
- The modes lose energy via GW emission \implies GW signal!
- The coexistence of (0, l, l) and (1, l, l) modes produces an unique GW beat signal

$$\cos[(\omega + \Delta \omega)t] + \cos[(\omega - \Delta \omega)t] = 2\cos(\Delta \omega t)\cos(\omega t)$$



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High-spin BHs are Rare

Guo, Bao and HZ, PRD 2023



- Angular mom. transfers from BH to the cloud.
- BHs prefer to reside on Regge trajectories.

BH Spin Measurement

- LVK can measure the individual BH spin, with huge error.
- The error can be reduced to $\sim 30\%$ in the future

LIGO, Virgo, PRX 2016



Survive with the Current Data

• Consider 3 scenarios: high, flat, low to estimate the effect of the initial BH spin.



- Assume the Lifetime of BHs distributes log-uniformly between $10^6 \mbox{ to } 10^{10} \mbox{ years}$

Constrain Scalar Mass

- Include all BBHs in three phases of GTWC data reported by LVK collaboration, only excluding the events with neutron.
- scalar mass prior is log-uniform between $10^{-13.5}$ to 10^{-11} eV.



Two slightly favored ranges are identified, but evidence is weak.

• Previous calculation only consider the (n = 0, l = 1, m = 1) mode



Monochromatic, constant energy flux, Cannot distinguish from neutron stars

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Monochromatic, constant energy flux, Cannot distinguish from neutron stars

• Different modes have slightly different angular speeds

$$\phi(t, \vec{r}) = \sum_{l,m} \int d\omega \left[e^{i(m\varphi - \omega t)} R_{lm}(r) S_{lm}(\theta) + \text{c.c.} \right] \quad \omega_R^{nlm} \approx \mu \left[1 - \frac{\alpha^2}{2(n+l+1)^2} \right] + O(\alpha^4)$$

e.g $\cos[(\omega + \Delta \omega)t] + \cos[(\omega - \Delta \omega)t] = 2\cos(\Delta \omega t)\cos(\omega t)$

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Modulation of amp. and energy flux. Beat!

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• Strength of the beat signal.

Two (0,1,1) axions \implies graviton: Amp. $\propto N_{011}$, freq. = $2\omega^{011}$ (0,1,1) + (1,1,1) \implies graviton: Amp. $\propto \sqrt{N_{011}N_{111}}$, freq. = $\omega^{011} + \omega^{111}$ Energy flux $\propto Amp^2$, so beat Amp. $\propto \sqrt{\frac{N_{111}}{N_{011}}}$, with freq. $\omega^{111} - \omega^{011}_{44}$



 $(0,1,1) + (1,1,1) \implies \text{graviton: Amp.} \propto \sqrt{N_{011}N_{111}}, \text{ freq.} = \omega^{011} + \omega^{111}$ Energy flux $\propto Amp^2$, so beat Amp. $\propto \sqrt{\frac{N_{111}}{N_{011}}}$, with freq. $\omega^{111} - \omega^{011}_{_{45}}$

Μοι

Can



Different modes have slightly dif

 $\phi(t, \vec{r}) = \sum_{l} \int d\omega \left[e^{i(m\varphi - \omega t)} R_{lm}(r) S_{lm} \right]$

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GW Beat: Observation



- Parameters: $M\mu = 0.17$ (so $a_C = 0.6$), $M_s/M = 0.1$, $N_{111}/N_{011} = 0.1$
- The red shift ranges from 0.001 to 10
- The current and future GW telescope can cover a large range of scalar mass.

Fields with Other Spins

• Real vector



Guo, Jia, Bao, HZ, Zhang, Accepted by PRD

- Real tensor field: unknown
- Complex field does not radiate GW. Then BHs reside on the first Regge trajectory for very long time.

Other Gravitational Observables

• Since real vector and tensor fields grow very fast, one could search the GW right after BH merger events.

Arvanitaki, Baryakhtar, Dimopoulos, Dubovsky, and Lasenby, PRD 2017 . Baryakhtar, Lasenby, and Teo, PRD 2017

• With perturbation (e.g. self-interaction or a companion star), the transitions between different modes radiate GW.

Scalar: Yoshino and Kodama, Class Quant. Grav. 2015

Vector: Siemonsen and East, PRD 2020

Stochastic GW background

Brito, Ghosh, Barausse, Berti, Cardoso, Dvorkin, Klein, and Pani, PRL 2017

GW burst in bosenova

Yoshino and Kodama, Prog. Theor. Phys. 2012 Arvanitaki, Baryakhtar, and Huang, PRD 2015

• Floating orbit of companion stars

Kavic, Liebling, Lippert, and Simonetti, JCAP 2020

Optical Observables

• BH shadow from lensing

Cunha, Herdeiro, Radu, and Runarsson, PRL 2015 Cunha, Herdeiro, Radu, and Runarsson, Int. J. Mod. Phys. 2016 Vincent, Gourgoulhon, Herdeiro, and Radu, PRD 2016

• If assuming coupling between the boson field and photon

birefringence causes time-dependent polarization

Plascencia and Urbano, JCAP 2018

Chen, Shu, Xue, Yuan, and Zhao, PRL 2020

> BH laser: oscillating charged field generates photon field

Ikeda, Brito, and Cardoso, PRL 2019

Boskovic, Brito, Cardoso, Ikeda, and Witek, PRD 2019

Summary

- If boson de Brogile wave length ~ BH horizon size, the boson field could grow exponentially around Kerr BHs.
- With the same parameters, the tensor field has the largest superradiance rate, while the scalar has the smallest rate.
- The superradiance can be detected without assuming the coupling between the boson field and the SM.



Model-independent search of new boson field.

• The superradiance can also test the strong field region of GR.

Jia, Guo, Liang, Mai, Zhang, arXiv:2309.05108

• Very rich phenomenology if assuming coupling between the boson field and the SM.

Sorry if I have Missed Your Work...

• New ideas emerge quickly.

Number of papers with "superradiance" in the title

Good reviews and lecture notes:

- Brito, Cardoso and Pani, Lect.Notes Phys. 906 (2015)
- Brack, Cardoso, Nissanke, Sotiriou, and Askar, Class.Quant.Grav. 36 (2019) 14, 143001
- Cardoso and Pani, Living Rev. Rel. 22 (2019) 1



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