Dynamical generation of Exotic Heavy Mesons in the heavy meson scattering

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Outline

- *Introduction*
- *Coupled-channel formalism*
- *Effective Lagrangian*
- *Heavy and light meson scattering with* $C=1$ *and* $S=1$
- *Heavy meson scattering in hidden-charm channels*
- *Heavy meson scattering in doubly charmed-channels*
- *Summary*

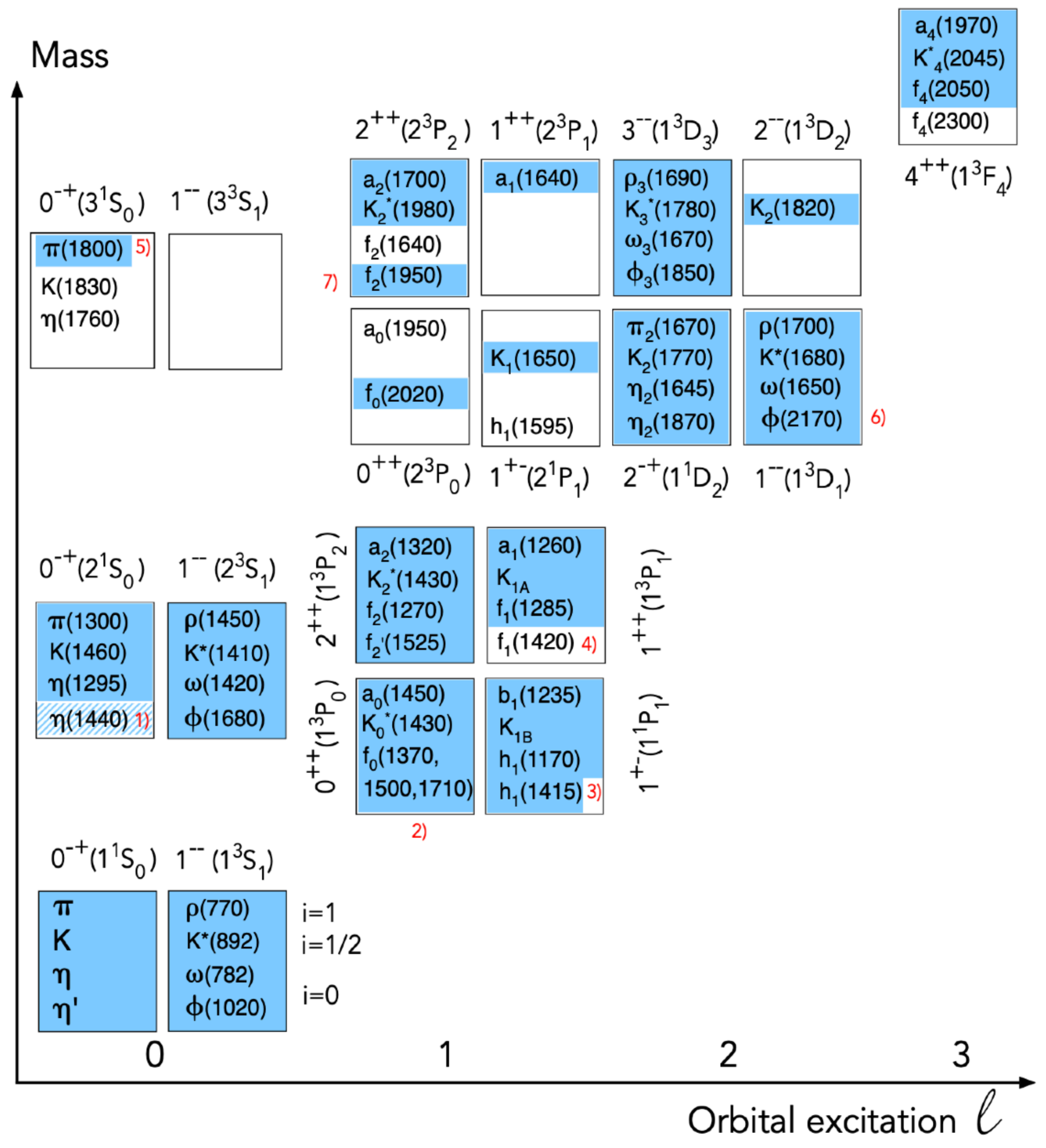
Introduction

Hadron spectroscopy

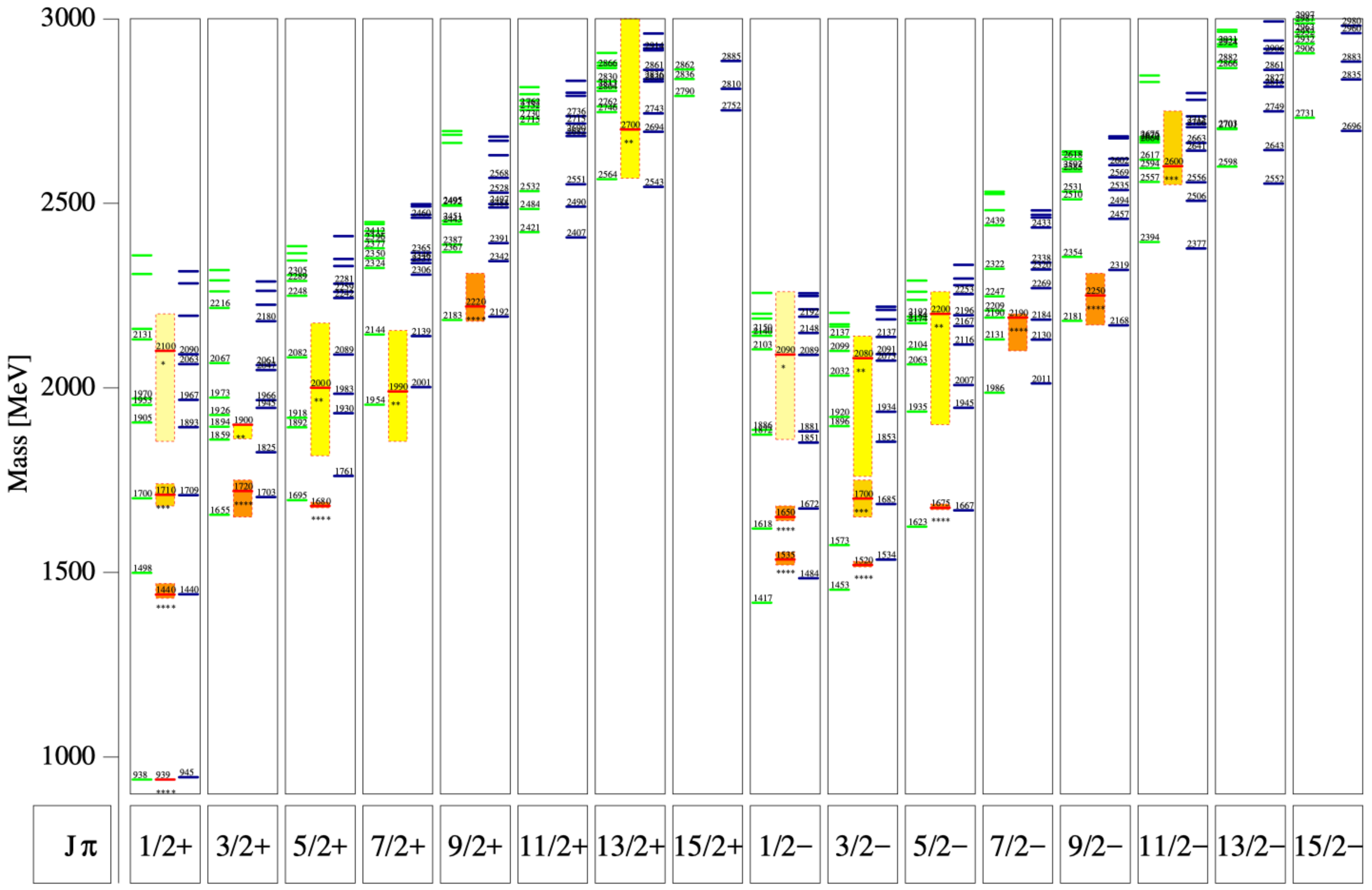
Conventional Quark Model

- The spectrum of strongly interacting particles consists of a tower of many states.

Conventional hadrons : Mesons($q\bar{q}$) and Baryons(qqq)



R.L. Workman *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01.



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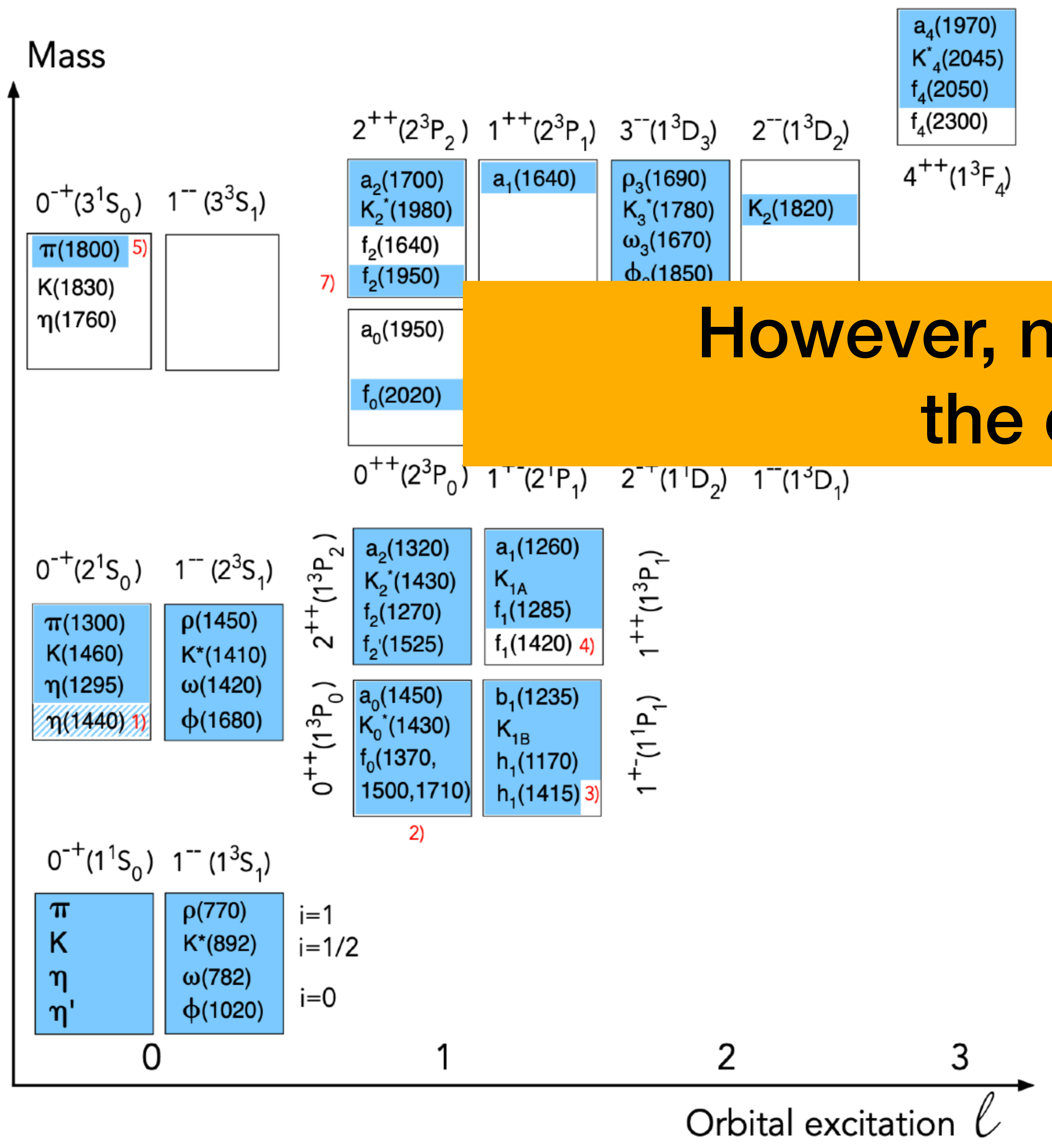

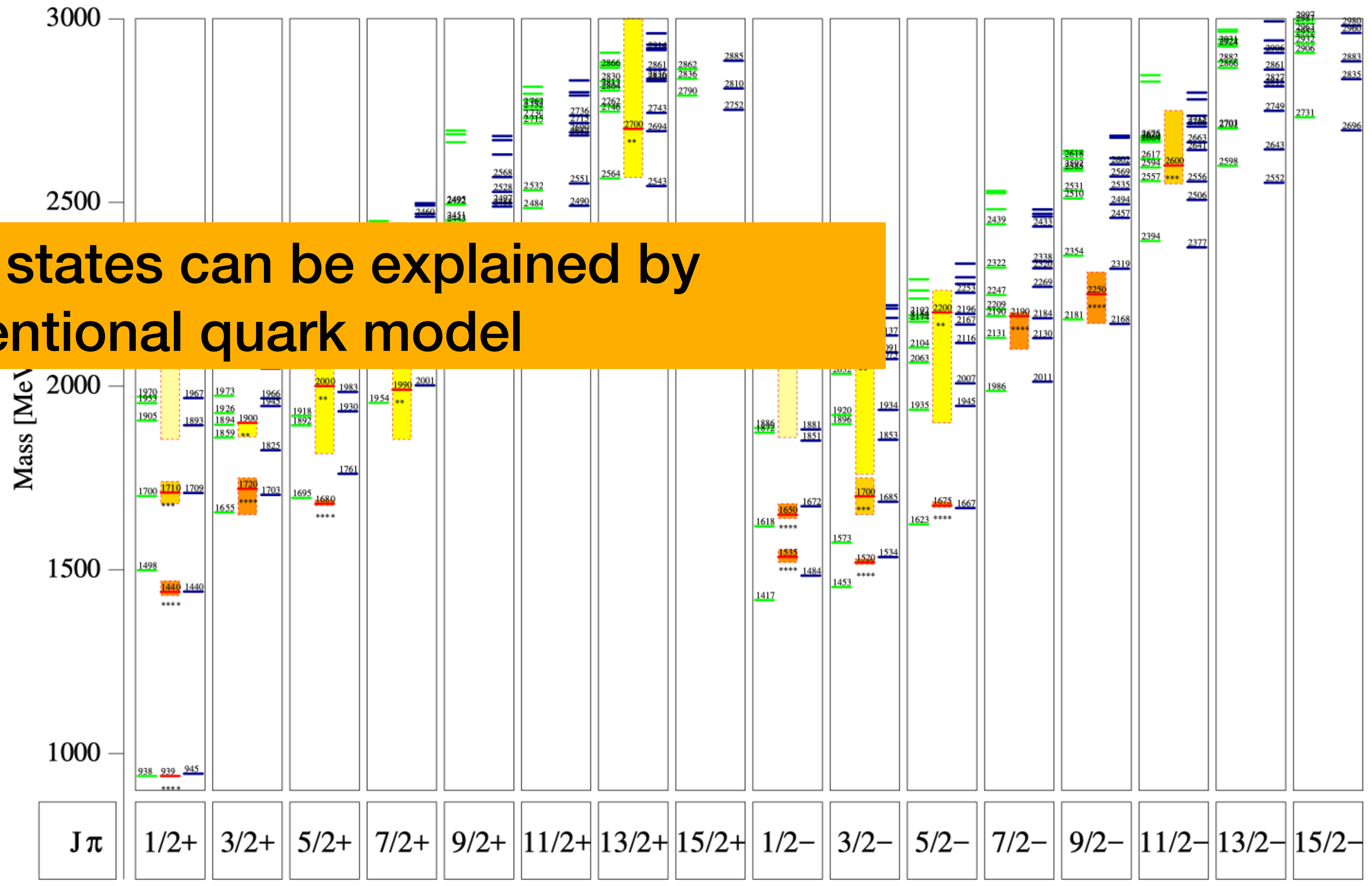

However, not all states can be explained by the conventional quark model

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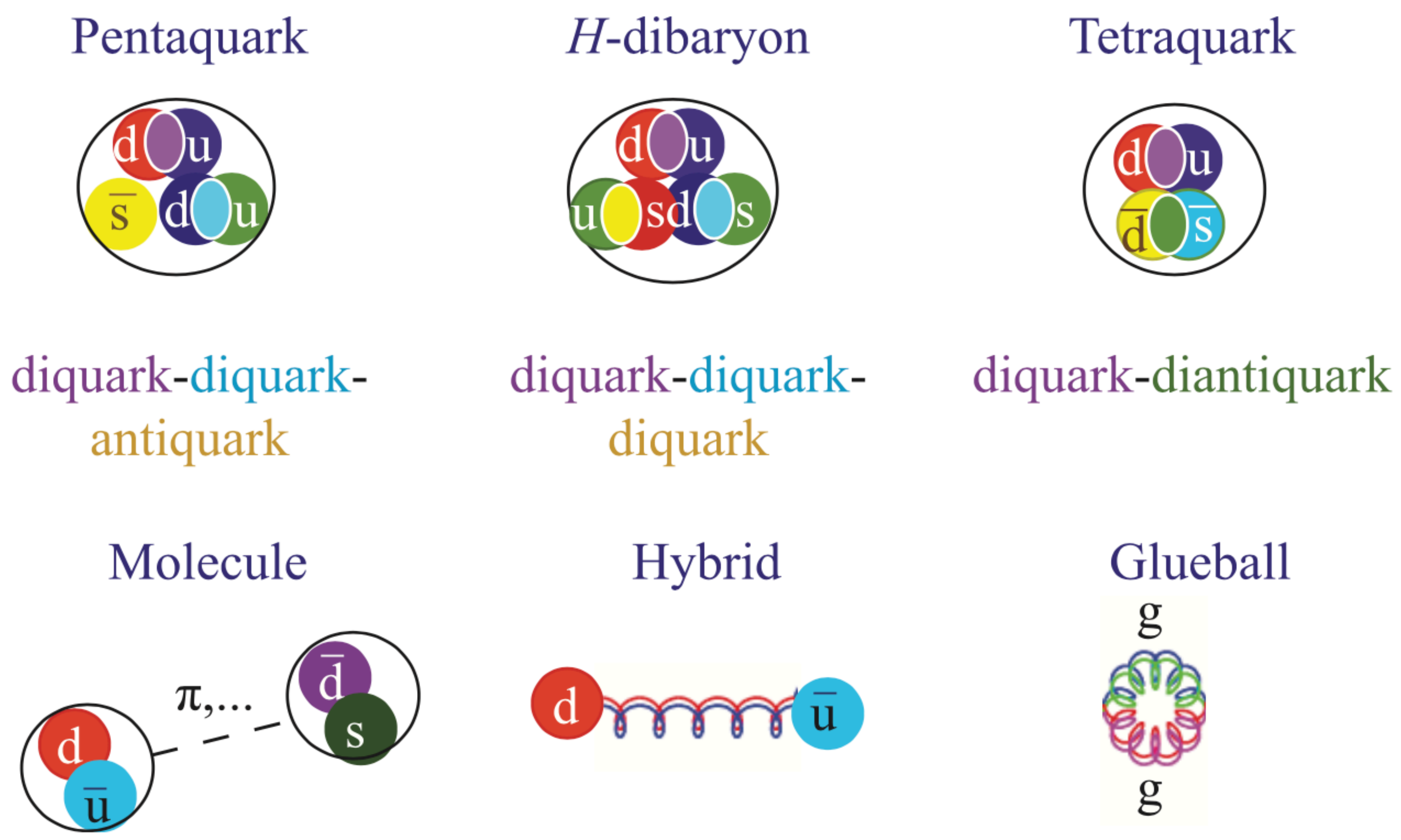
Beyond Quark Model

QCD allows many different types of color-neutral objects

Meson : $q\bar{q}$ $qq\bar{q}\bar{q}$(tetraquark), $q\bar{q}g$(hybrids), glueballs, …

Baryon : qqq $qqqq\bar{q}$(pentaquark), $qqqqqq$…



S.L.Olsen Front.Phys.(Beijing) 10 (2015) 2, 121-154

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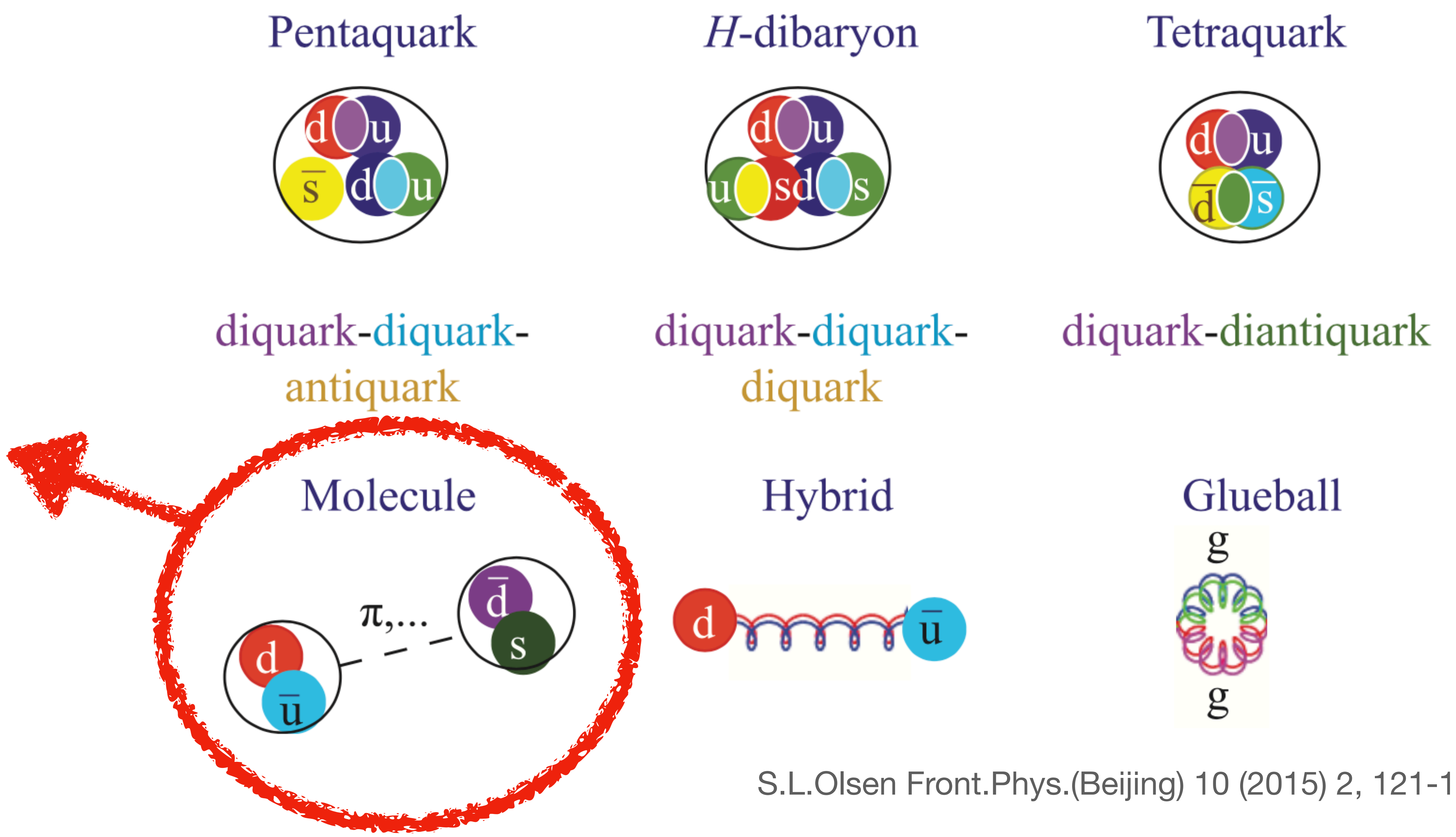
Baryon : qqq $\quad qqqq\bar{q}$(pentaquark), $qqqqqq$…

Hadronic molecule

- Bound states of color-neutral states via meson-exchanges
- Near certain two-particle thresholds
- Dominantly decay into the two-particle channels



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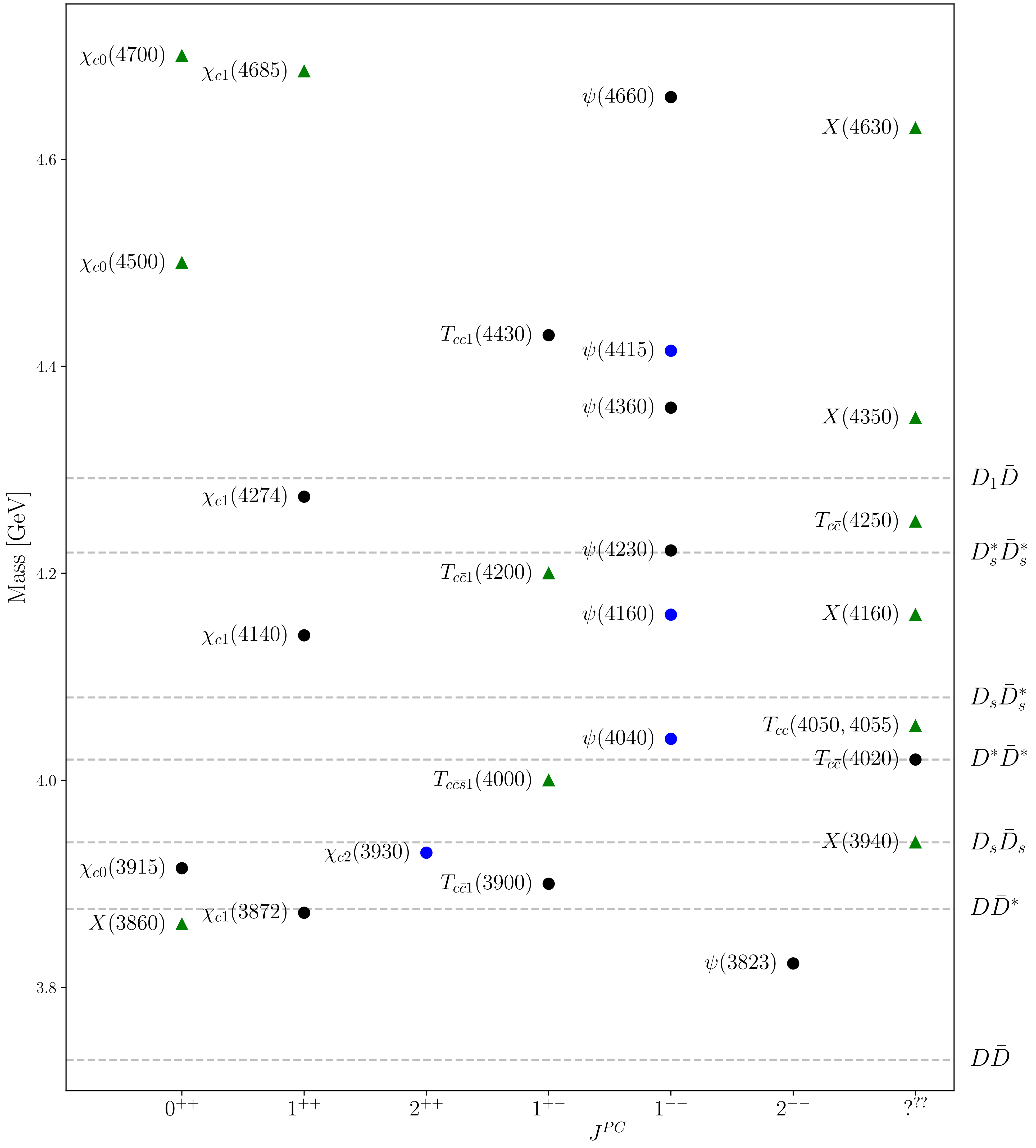
Exotic states



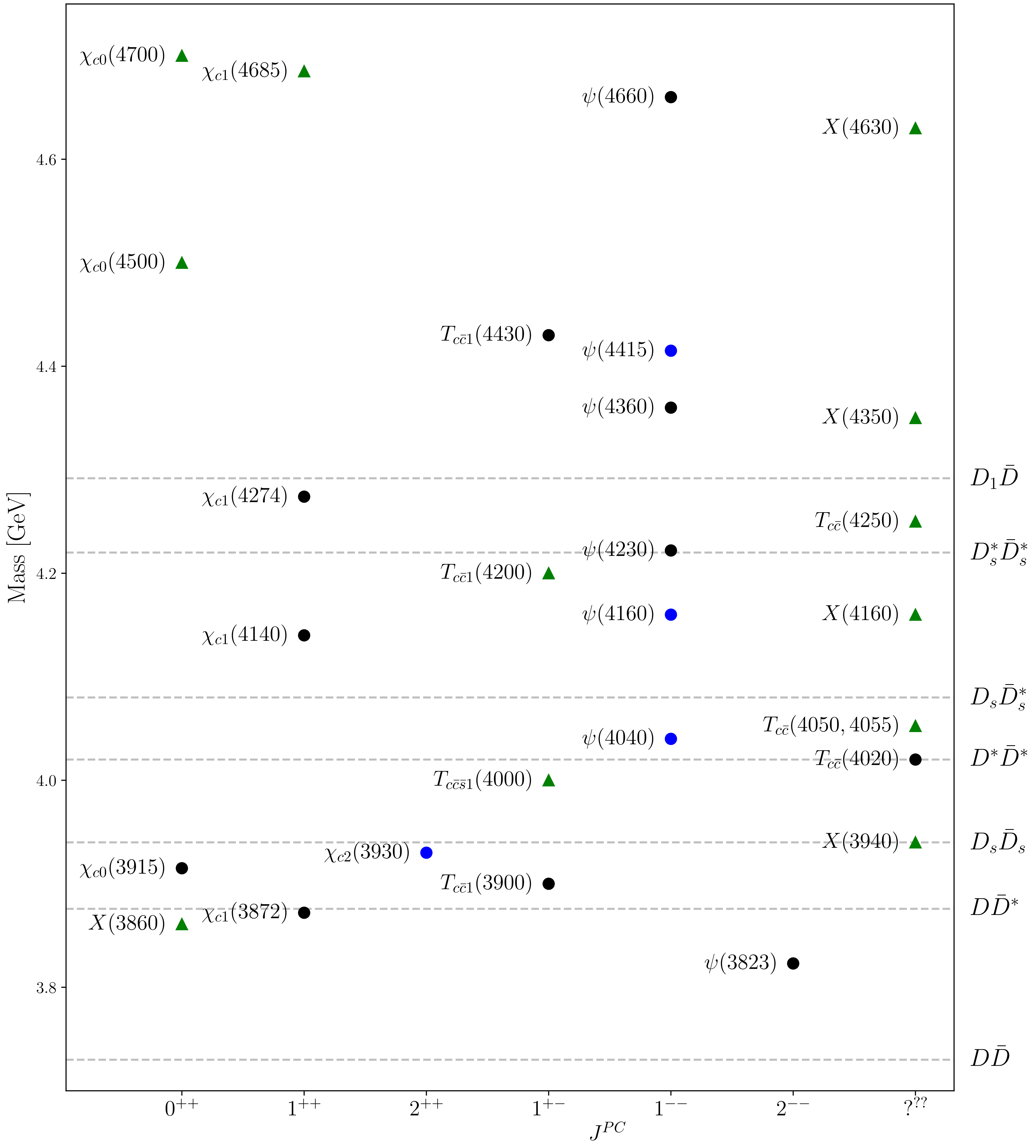
The representative hadronic-molecule candidate is $\chi_{c1}(3872)$

$$I^G(J^{PC}) = 0^+(1^{++})$$

QM candidates: $M(2^3P_1\ c\bar{c}) \approx 3950\,\mathrm{MeV}$

$$M_{\chi_{c1}} = 3871.84\,\mathrm{MeV}$$

Exotic states



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$M_{\chi_{c1}} = 3871.84\,\text{MeV}$

Its mass is very close to the $D^0\bar{D}^{*0}$ threshold:

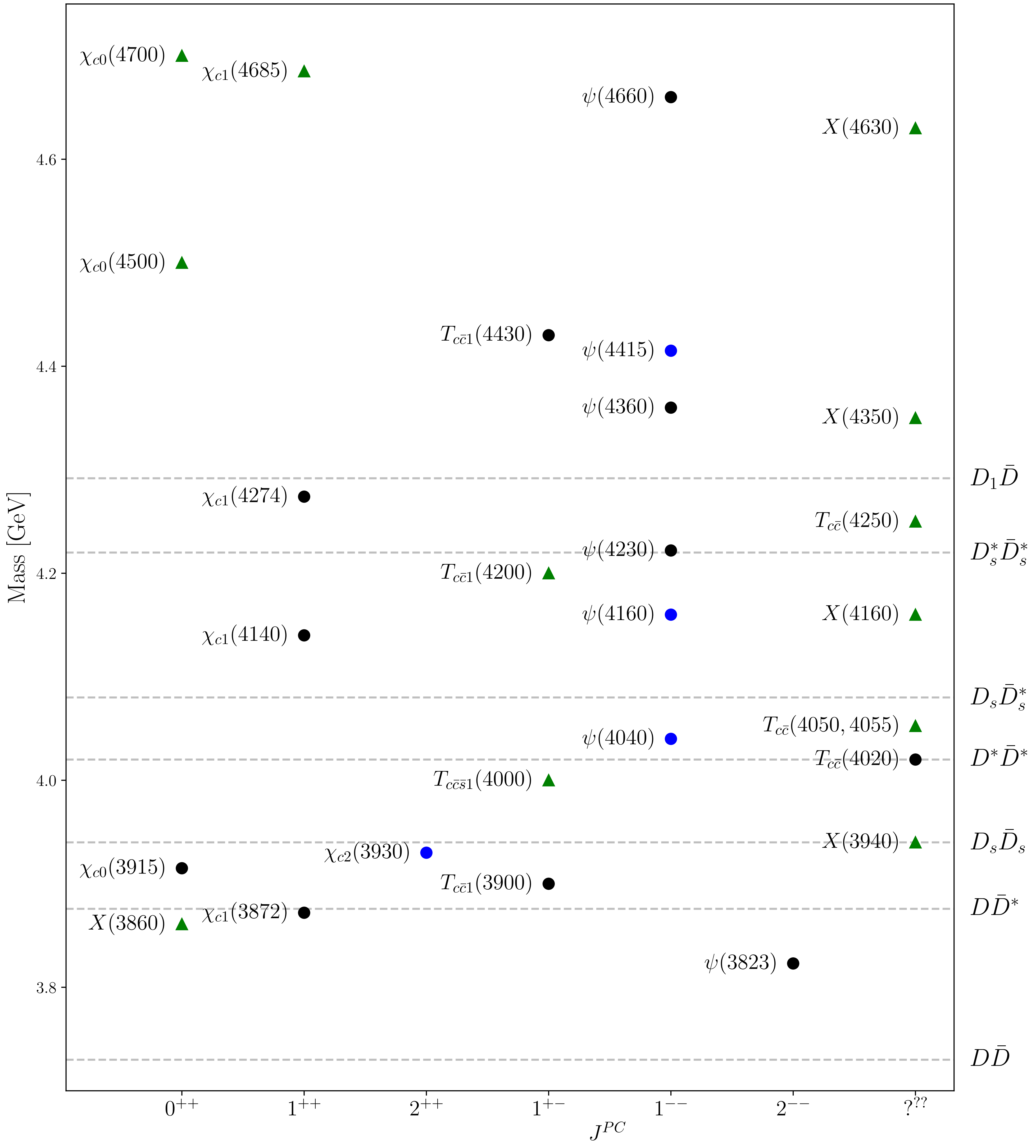
$$M_{\chi_{c1}} - (m_{D^0} + m_{\bar{D}^{*0}}) = -0.09 \pm 0.28\,\text{MeV}$$

Dominantly decay into this channel:

$$\mathscr{B}(\chi_{c1}(3872) \to D^0\bar{D}^{0*}) > 34\,\%$$

Consistent with the characteristics of hadronic molecules

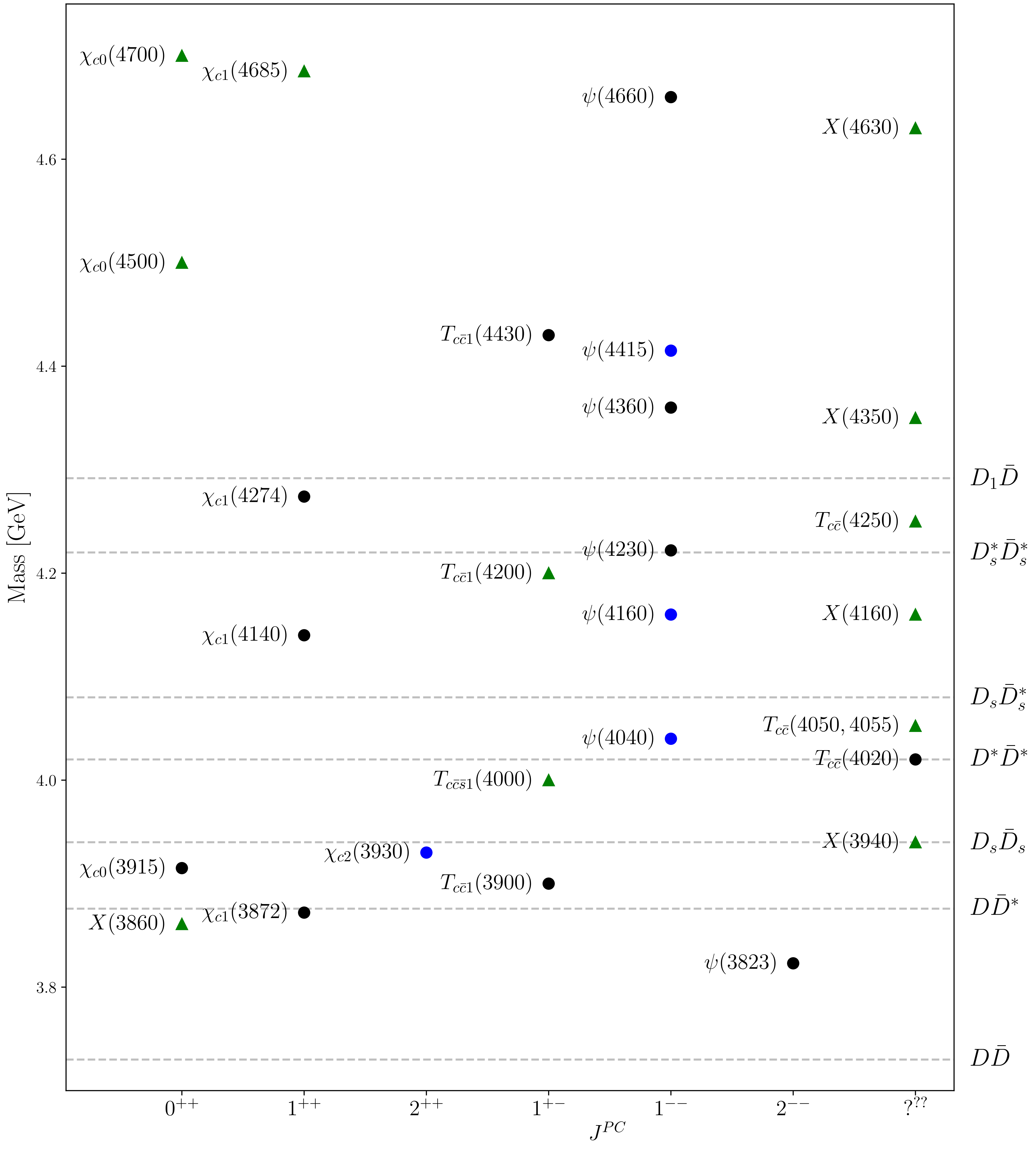
Exotic heavy mesons



Observed multiquark candidates listed in PDG:

- Low-lying scalar mesons : $a_0/f_0(980), f_0(500), a_1(1260), b_1(1235)...$
- Exotic states with an heavy flavor : $D_{s0}^*(2317)$, $D^*(2400)$, ...
- Exotic $c\bar{c}$ or $b\bar{b}$ states : $\chi_{c1}(3872)$, $T_{c\bar{c}1}(3900)$, $T_{b\bar{b}1}(10610)$, ...
- Open heavy-flavored state : T_{cc}^+
- Fully heavy tetraquark : $T_{cc\bar{c}\bar{c}}(6900)$

Exotic heavy mesons



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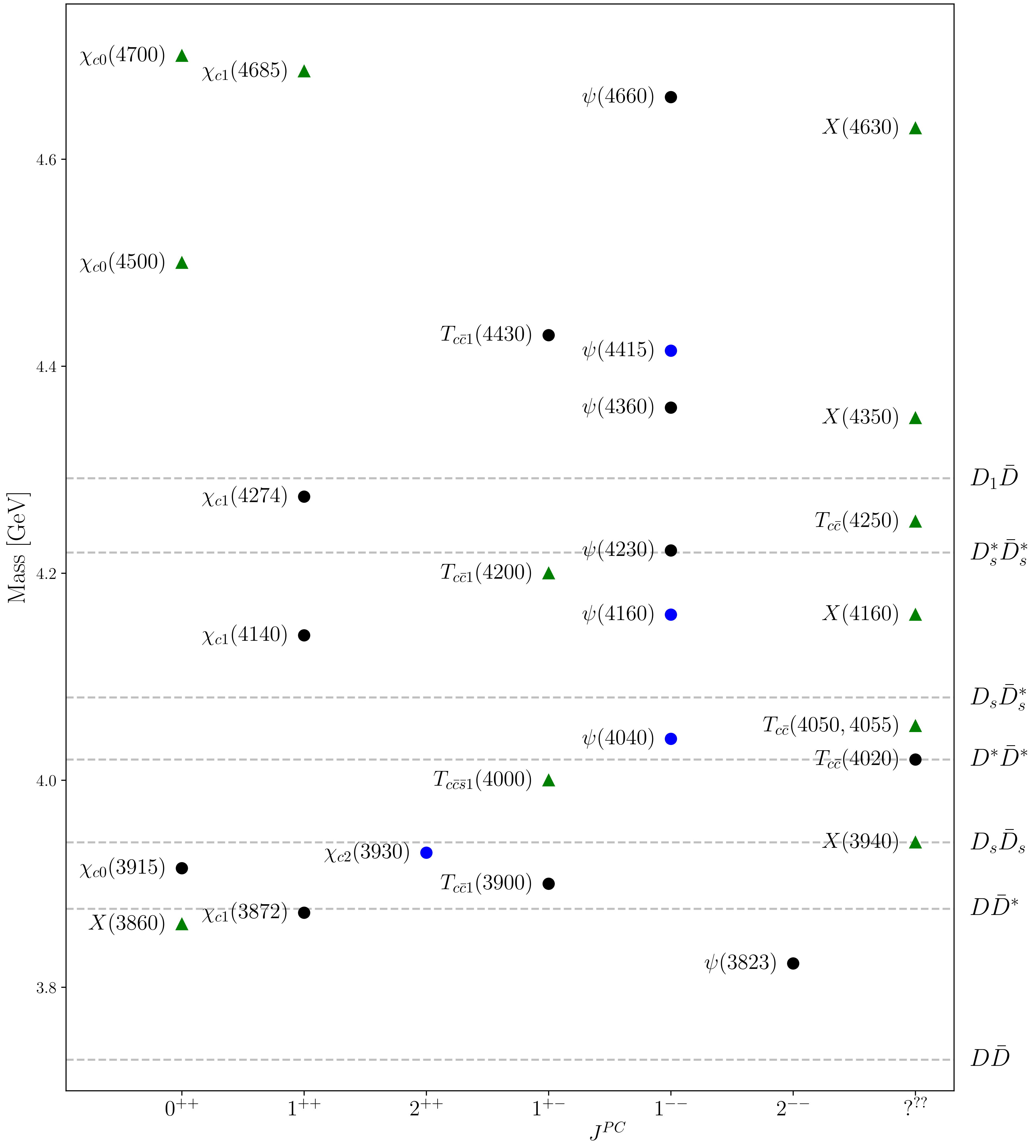

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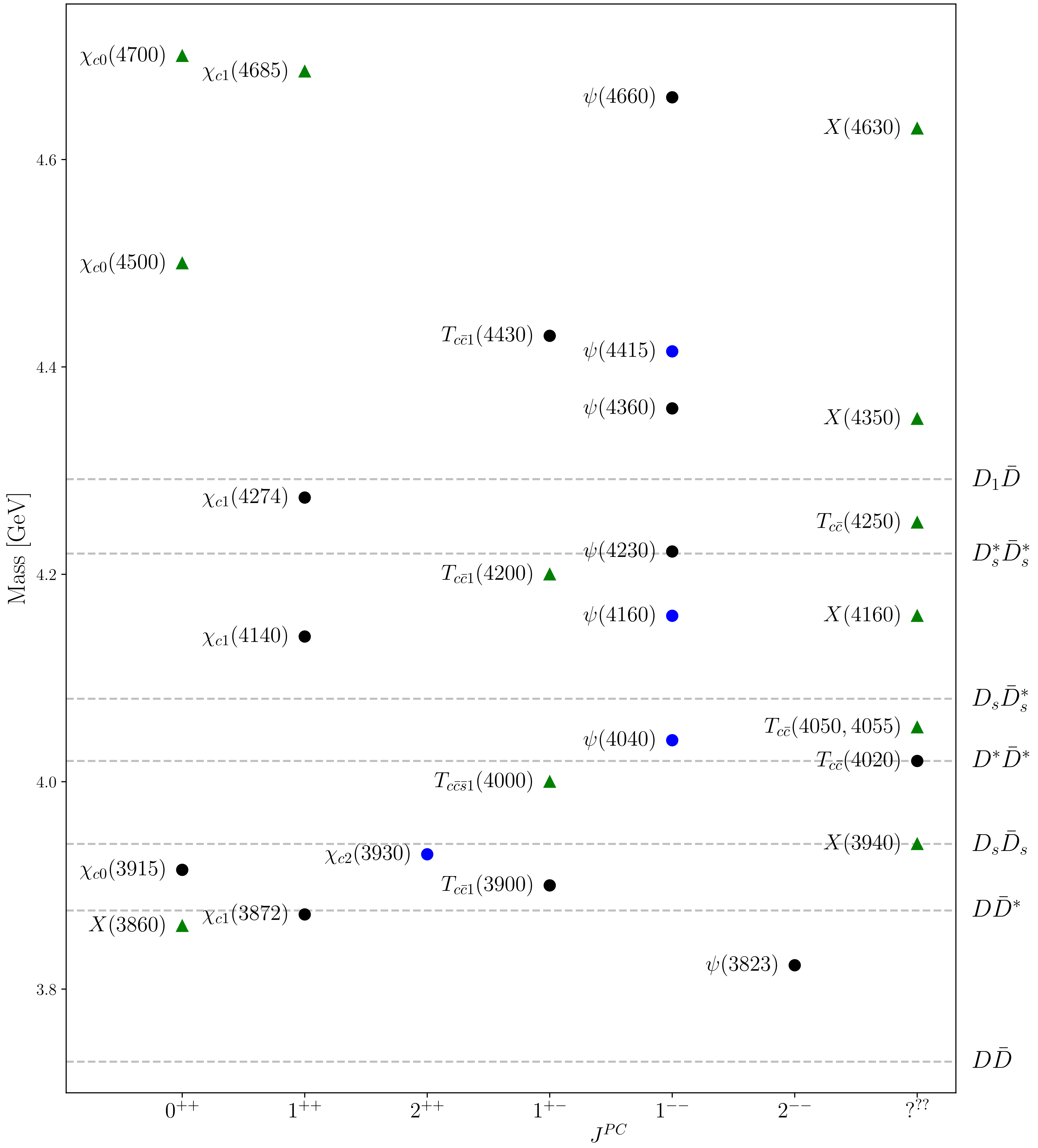

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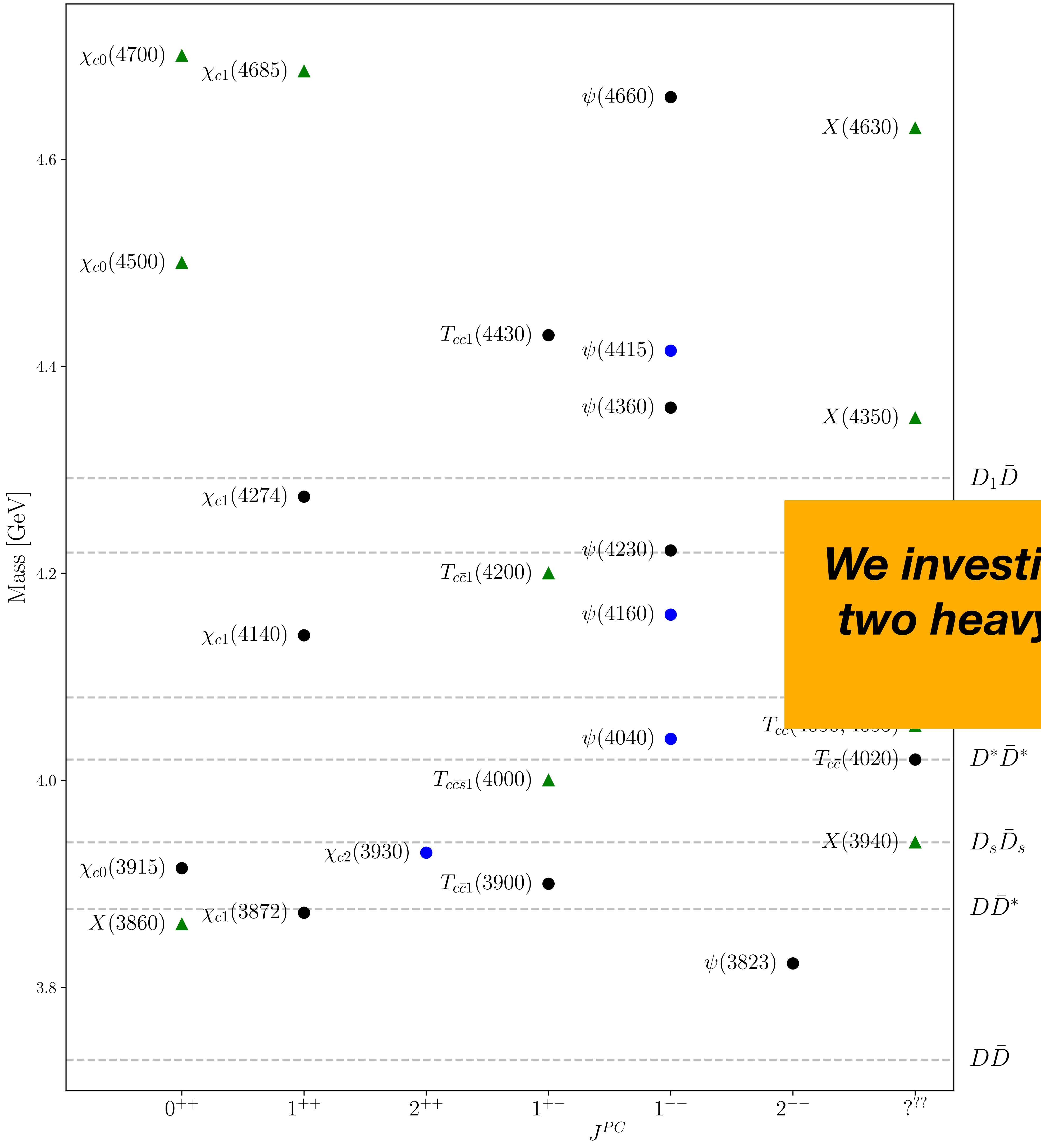

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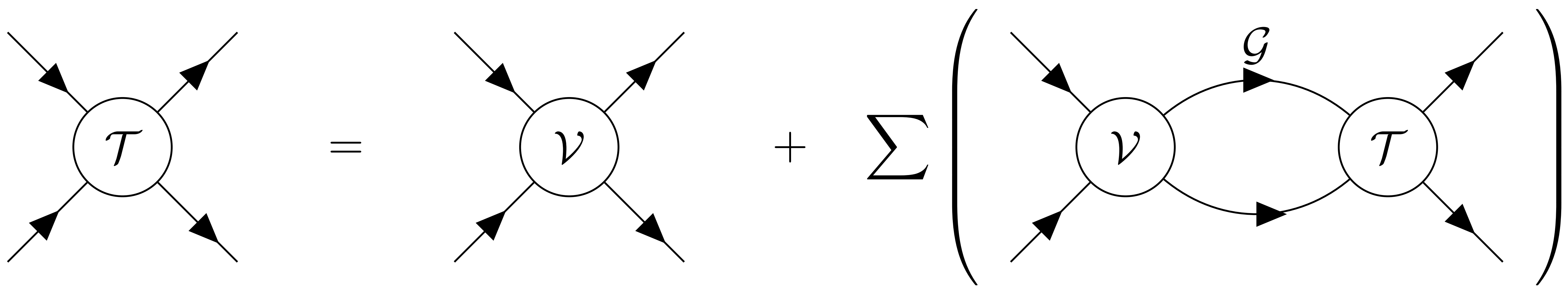
We investigate the hadron molecular features of the exotic states containing two heavy quarks using the fully off-mass-shell coupled-channel formalism within the meson-exchange framework.

Coupled-channel formalism

Two-body scattering equation

- **Blankenbecler-Sugar equation**



$$T = V + VGT$$

- The two-body T-matrix are obtained by solving the Bethe-Salpeter equation:

$$T_{fi} = V_{fi} + \sum_n \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{V_{fn}T_{ni}}{(k_1^2 - m_1^2 + i\epsilon)(k_2^2 - m_2^2 + i\epsilon)}$$

- The three-dimensional reduction via the Blankenbecler-Sugar scheme preserves unitarity and off-shellness:

$$G_k(q) = \frac{\pi}{\omega_1^k \omega_2^k} \delta\left(q^0 - \frac{\omega_1^k - \omega_2^k}{2}\right) \frac{\omega_1^k + \omega_2^k}{s - (\omega_1^k + \omega_2^k)^2 + i\epsilon}$$

$$T_{fi}(\boldsymbol{p}, \boldsymbol{p}') = V_{fi}(\boldsymbol{p}, \boldsymbol{p}') + \sum_n \int \frac{d^3q}{(2\pi)^3} \frac{\omega_1^n + \omega_2^n}{s - (\omega_1^n + \omega_2^n)^2 - i\varepsilon} V_{ni}(\boldsymbol{p}, \boldsymbol{q}) T_{fn}(\boldsymbol{q}, \boldsymbol{p}')$$

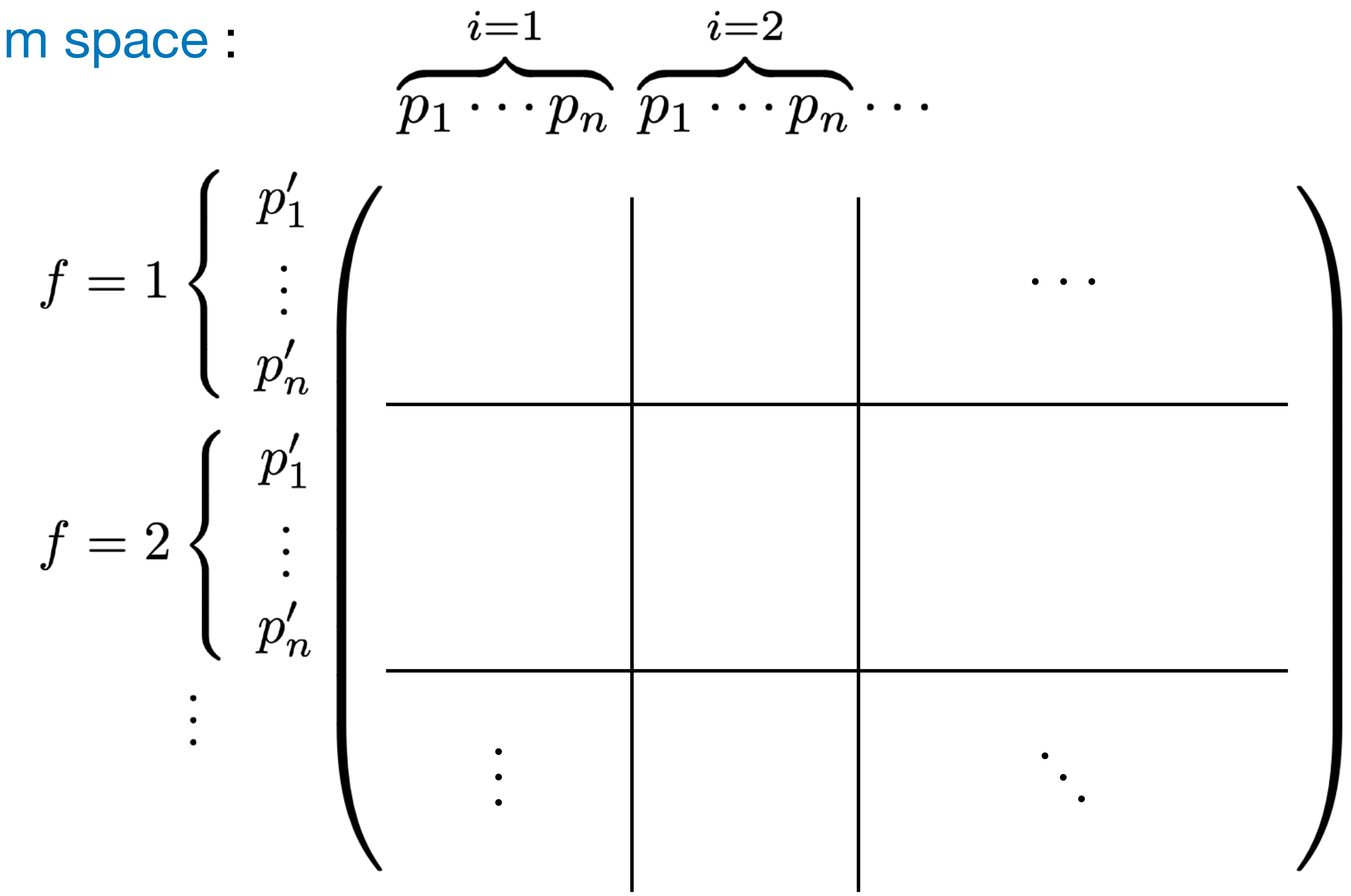
Two-body scattering equation

- **Blankenbecler-Sugar equation**
  - Total angular momentum projection

$$T^J_{fi}(p,p') = V^J_{fi}(p,p') + \frac{1}{(2\pi)^3}\int_0^\infty dq\, \frac{\omega_1+\omega_2}{2\omega_1\omega_2}\frac{q^2 V^J_{fk}(p,q)T^J_{ki}(q,p')}{s-(\omega_1+\omega_2)^2 - i\varepsilon}$$

Matrix inversion method: $T^J = V^J + V^J G T^J \implies T^J = (1 - V^J G)^{-1} V^J$

We obtain the *off-mass-shell* T matrix in the full-channel momentum space :



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Regularization of the two-body propagator:

- The two-body propagator is singular at the on-mass-shell momentum point, $\tilde{q} = \frac{\sqrt{\left(s-(m_1^2+m_2^2)^2\right)\left(s-(m_1^2-m_2^2)^2\right)}}{2\sqrt{s}}$
- Change of the variable

$$\omega = \omega_1+\omega_2, \quad d\omega = \frac{\omega_1+\omega_2}{\omega_1\omega_2} q\, dq \quad \Longrightarrow \quad \int_{m_1+m_2}^\infty d\omega \frac{f(q)}{\omega^2 - s}, \quad \text{where } f(q) = \frac{1}{2} qV(q)T(q)$$

- Decompose into regular part and singular part.

$$\int_{m_1+m_2}^\infty d\omega \frac{f(q)-f(\tilde{q})}{\omega^2-s} + \int_{m_1+m_2}^\infty d\omega \frac{f(\tilde{q})}{\omega^2-s}$$

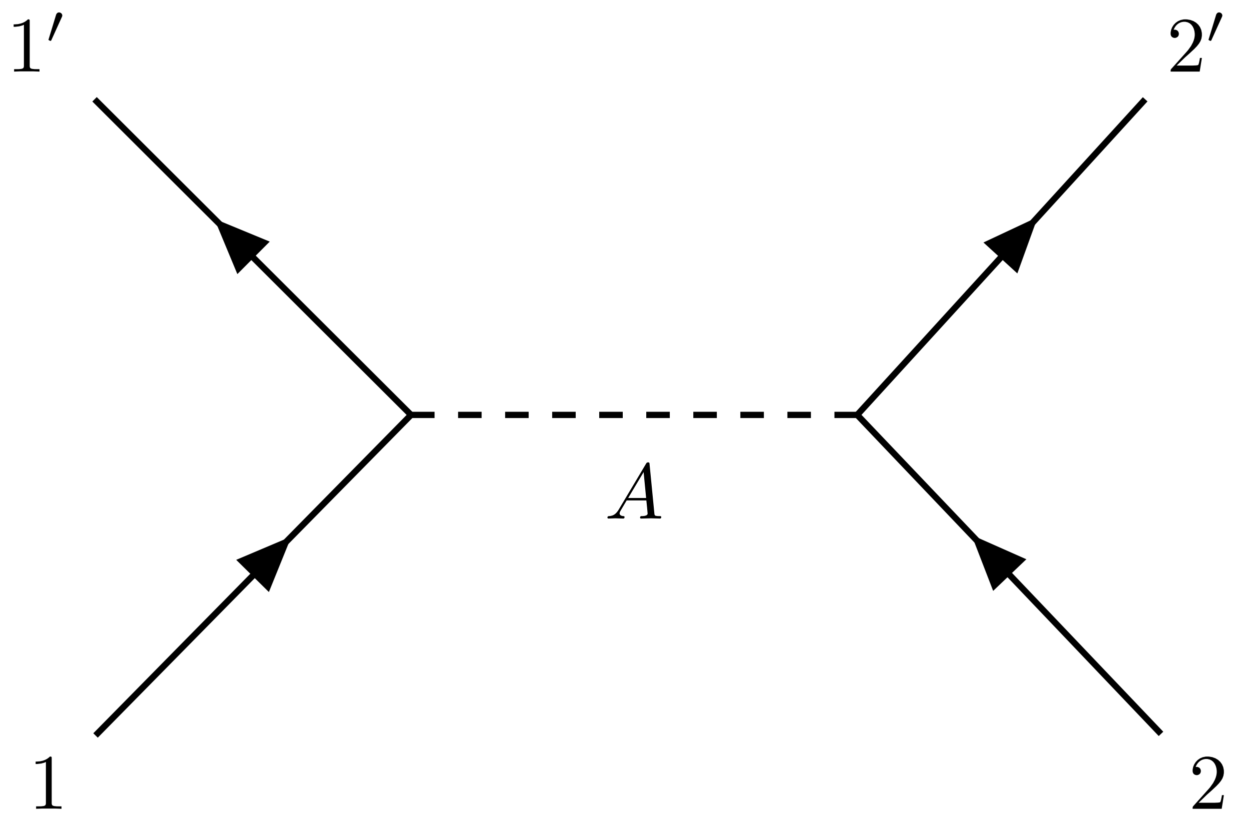
- One can regularize the singular part:

$$\int_{m_1+m_2}^\infty d\omega \frac{f(\tilde{q})}{\omega^2-s} = P\int_{m_1+m_2}^\infty d\omega \frac{f(\tilde{q})}{\omega^2-s} + \int_C d\omega \frac{f(\tilde{q})}{\omega^2-s} = \frac{f(\tilde{q})}{2\sqrt{s}}\left(i\pi - \log\left|\frac{\sqrt{s}-m_1-m_2}{\sqrt{s}+m_1+m_2}\right|\right)$$

Kernel amplitudes

- **Scattering amplitudes**

$$\mathcal{V}_{12\to1'2'} = \sum_A \mathcal{M}^A_{12\to1'2'}$$



$$\mathcal{M}^A_{12\to1'2'} = \mathrm{IS} F_A^2 \Gamma^A_{12} P^A \Gamma^A_{1'2'}$$

Since the hadron has a finite size, form factor is need to be considered at each vertex: $F(q^2) = \left(\frac{n\Lambda^2 - m_{\rm ex}^2}{n\Lambda^2 - q^2}\right)^n$

For minimal uncertainty from the cutoff parameters, we strictly fixed the values about $\Lambda = m_{\rm ex} + 600$ MeV.

IS is the isospin symmetric factor from the isospin projection (for definite isospin channels)

Effective Lagrangian

Effective Lagrangian

- **Heavy chiral Lagrangian**

The coupling constants between heavy and light mesons are determined by the interaction Lagrangian based on the *Heavy Quark Effective Field Theory*(HQEFT).

$$\mathcal{L}_{\text{heavy}} = ig\text{Tr}[H_b\gamma_\mu\gamma_5\mathcal{A}^\mu_{ba}\bar{H}_a] + i\beta\text{Tr}[H_b v^\mu(\mathcal{V}_\mu - \rho_\mu)_{ba}\bar{H}_a] + i\lambda\text{Tr}[H_b\sigma_{\mu\nu}F^{\mu\nu}_{ba}(\rho)\bar{H}_a] + g_\sigma\bar{H}_a H_a\sigma$$

A heavy-light meson is made up by a heavy quark Q and a light antiquark $\bar{q}$.

→ heavy quark spin symmetry(HQSS), heavy quark flavor symmetry(HQFS) + chiral symmetry

- *Heavy superfield:* HQSS, HQFS, Lorentz invariance, Parity invariance

$$H^a = \frac{1+\not{v}}{2}(P^{*a}_\mu\gamma^\mu - P^a\gamma_5), \quad \bar{H} = \gamma_0 H^\dagger\gamma_0 = (P^{*\dagger a}_\mu\gamma^\mu + P^{\dagger a}\gamma_5)\frac{1+\not{v}}{2}$$

Pseudoscalar heavy field: $P^a = \{D^+, D^0, D^+_s\}$ or $\{B^-, \bar{B}^0, \bar{B}^0_s\}$

Vector heavy field: $P^{*a}_\mu = \{D^{*+}_\mu, D^{*0}_\mu, D^{*+}_{s\mu}\}$ or $\{B^{*-}_\mu, \bar{B}^{*0}_\mu, \bar{B}^{*0}_{s\mu}\}$

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- *Light pseudoscalar mesons:* chiral symmetry spontaneous break down

$$\mathcal{A}^\mu = \frac{i}{f_\pi}\partial^\mu\mathcal{M} + \cdots \qquad \mathcal{M} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

- *Light vector mesons:* dynamical gauge boson of the hidden local symmetry

$$\rho^\mu = i\frac{g_V}{\sqrt{2}}V^\mu, \quad V^\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega \end{pmatrix}^\mu$$

Effective Lagrangian

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- **Effective Lagrangian for charmonium interactions**

$$\mathcal{L}_J = g_\psi\text{Tr}[J\bar{H}^{\bar{Q}}_a\gamma_\mu\partial^\mu\bar{H}^Q_a]$$

Charminum superfield: $J = \frac{1+\not{v}}{2}\left[\psi^\mu\gamma_\mu - \eta_c\gamma_5\right]\frac{1-\not{v}}{2}$

- **SU(3) symmetric Lagrangians for the light flavors**

$$\mathcal{L}_{\mathcal{P}\mathcal{P}V} = -\frac{i}{2}g_{\mathcal{P}\mathcal{P}V}\,\text{Tr}([\mathcal{M},\partial_\mu\mathcal{M}]V_\mu),$$

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\sigma} = 2g_{\mathcal{P}\mathcal{P}\sigma}m_{\mathcal{P}}\mathcal{M}\mathcal{M}\sigma$$

Heavy and light meson scattering with $C = S = 1$

Kernel matrix

- **Decay to isospin violated channel $D_{s0}^*(2317) \to D_s\pi^0$**

  Isospin violation decay implies the mixing of the isoscalar and isovector channels → π^0-η mixing

$$|D_s^+\pi^0\rangle = |D_s^+\tilde{\pi}^0\rangle\cos\epsilon - |D_s^+\tilde{\eta}\rangle\sin\epsilon$$
$$|D_s^+\eta\rangle = |D_s^+\tilde{\pi}^0\rangle\sin\epsilon + |D_s^+\tilde{\eta}\rangle\cos\epsilon$$

- **Kernel matrix**

  Consider four $C = S = 1$ channels : $|1\rangle = |D_s^+\pi^0\rangle,\ |2\rangle = |D^0K^+\rangle,\ |3\rangle = |D^+K^0\rangle,\ |4\rangle = |D_s^+\eta\rangle$

  Kernel matrix element: $\mathcal{V}_{ba} \equiv \langle b|\mathcal{V}|a\rangle$

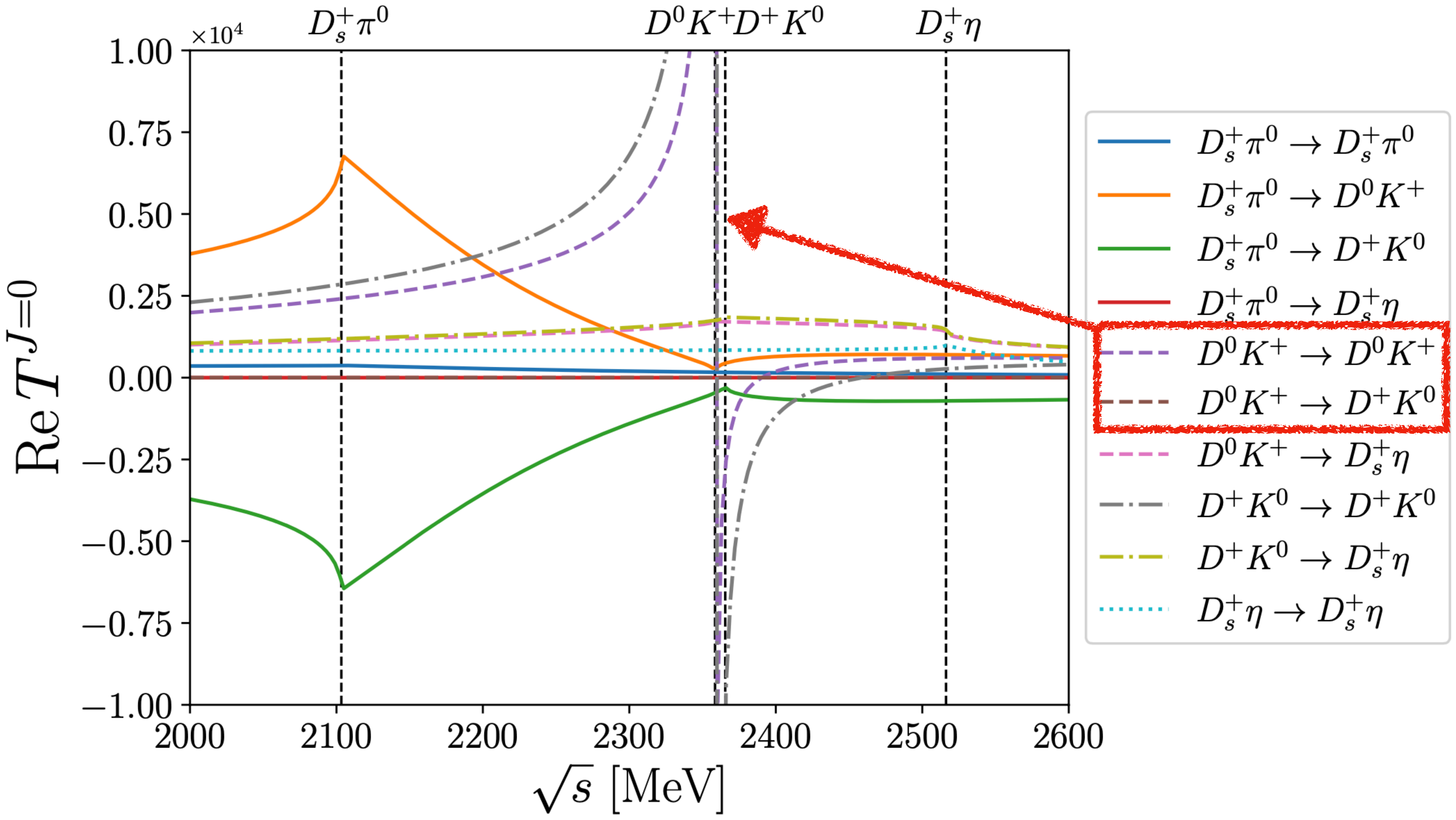
$$\hat{\mathcal{V}} = \begin{pmatrix} \mathcal{V}_{D_s^+\pi^0\to D_s^+\pi^0} & \mathcal{V}_{D_s^+\pi^0\to D^0K^+} & \mathcal{V}_{D_s^+\pi^0\to D^+K^0} & \mathcal{V}_{D_s^+\pi^0\to D_s^+\eta} \\ \mathcal{V}_{D^0K^+\to D_s^+\pi^0} & \mathcal{V}_{D^0K^+\to D^0K^+} & \mathcal{V}_{D^0K^+\to D^+K^0} & \mathcal{V}_{D^0K^+\to D_s^+\eta} \\ \mathcal{V}_{D^+K^0\to D_s^+\pi^0} & \mathcal{V}_{D^+K^0\to D^0K^+} & \mathcal{V}_{D^+K^0\to D^+K^0} & \mathcal{V}_{D^+K^0\to D_s^+\eta} \\ \mathcal{V}_{D_s^+\eta\to D_s^+\pi^0} & \mathcal{V}_{D_s^+\eta\to D^0K^+} & \mathcal{V}_{D_s^+\eta\to D^+K^0} & \mathcal{V}_{D_s^+\eta\to D_s^+\eta} \end{pmatrix}$$

  We construct the *off-mass-shell* kernel matrix in the full-channel momentum space.

Dynamical generation of the poles

Single channel T matrix elements

$$\text{ex)}\, T_{11} = (1 - V_{11}G_1)^{-1}V_{11}$$

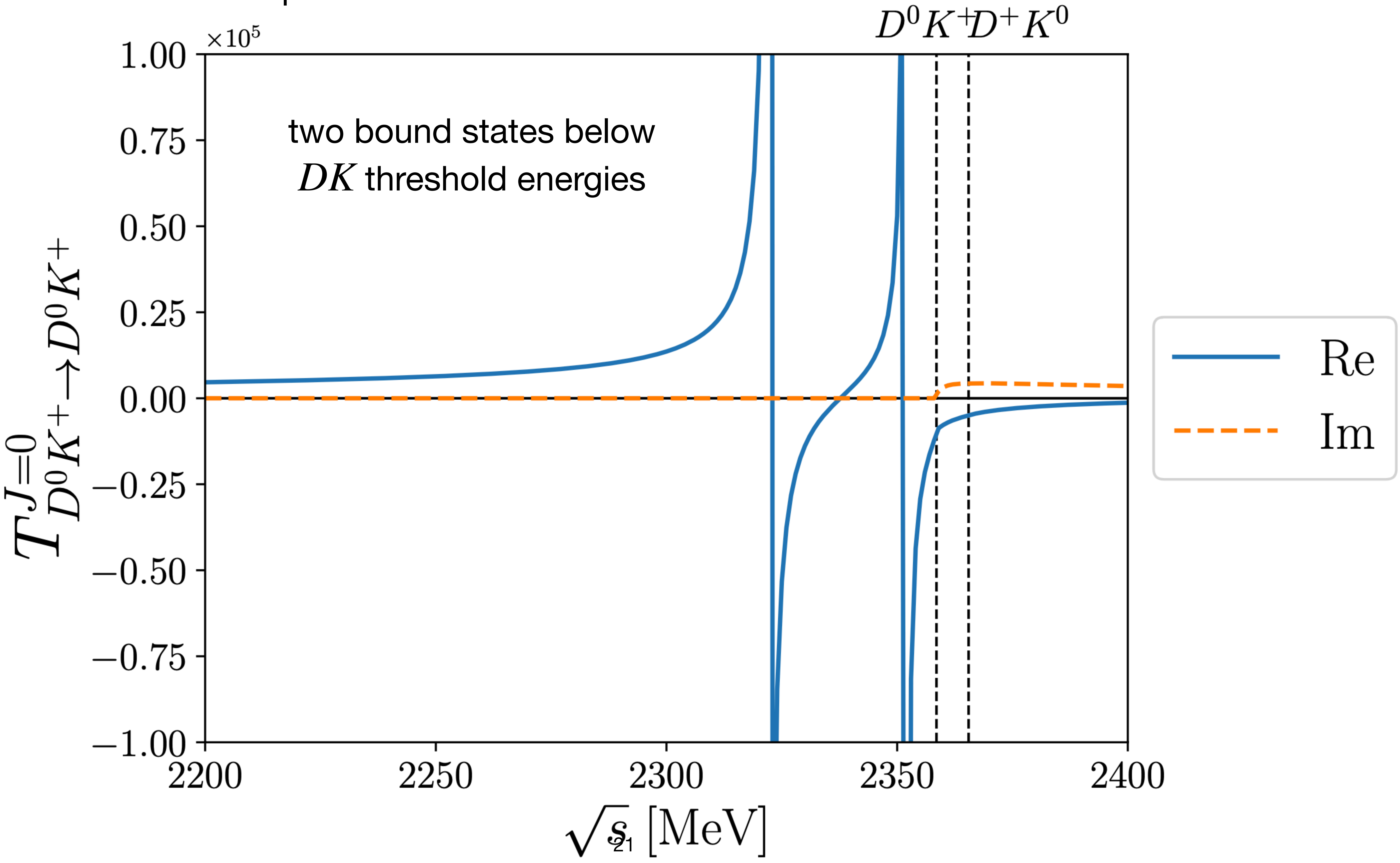


The u-channel processes in DK scattering generate two poles through strong attractive interactions

Dynamical generation of the poles

Coupled-channel dynamics

When the D^0K^+ channel is coupled to the D^+K^0 :



Dynamical generation of the poles

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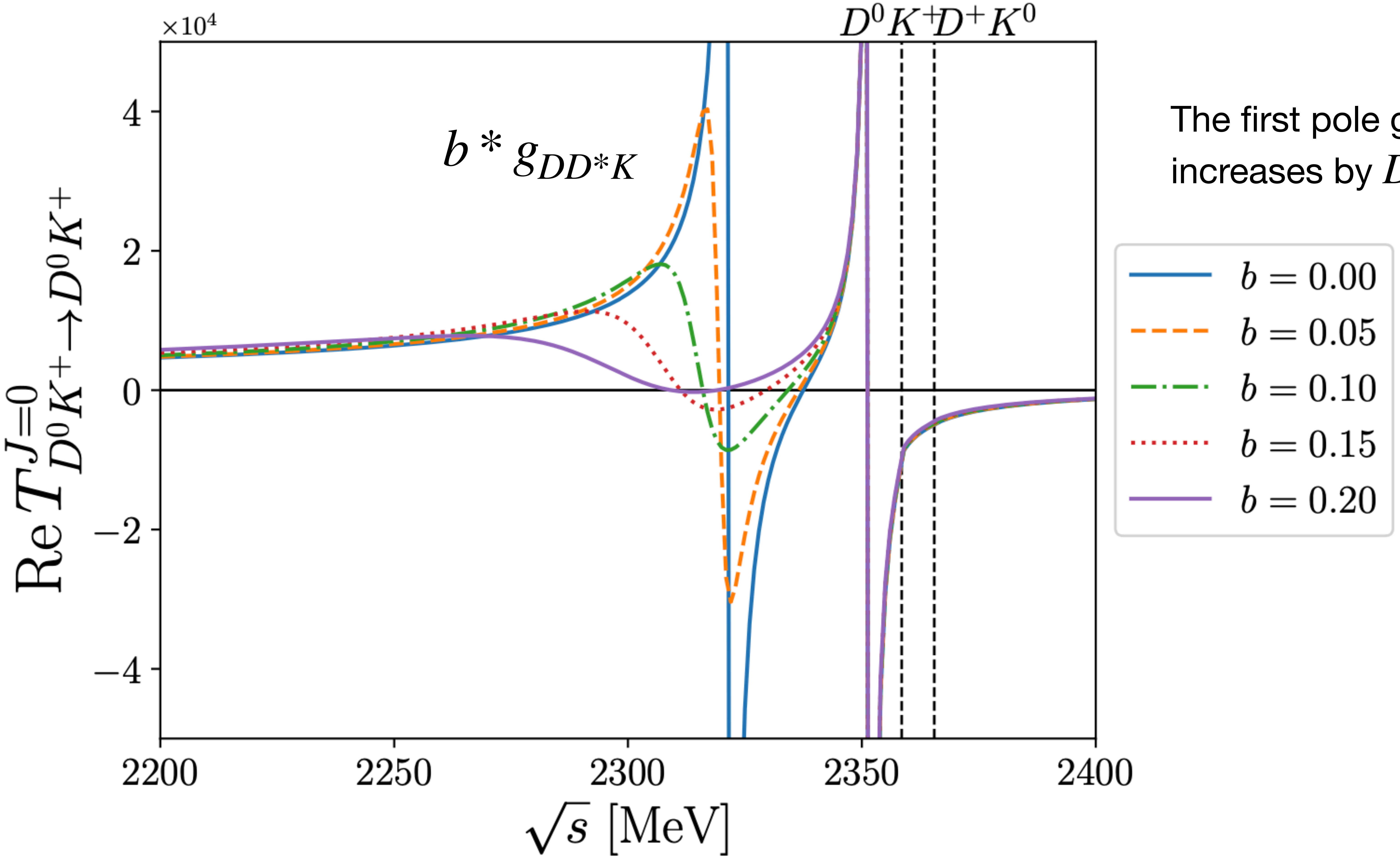
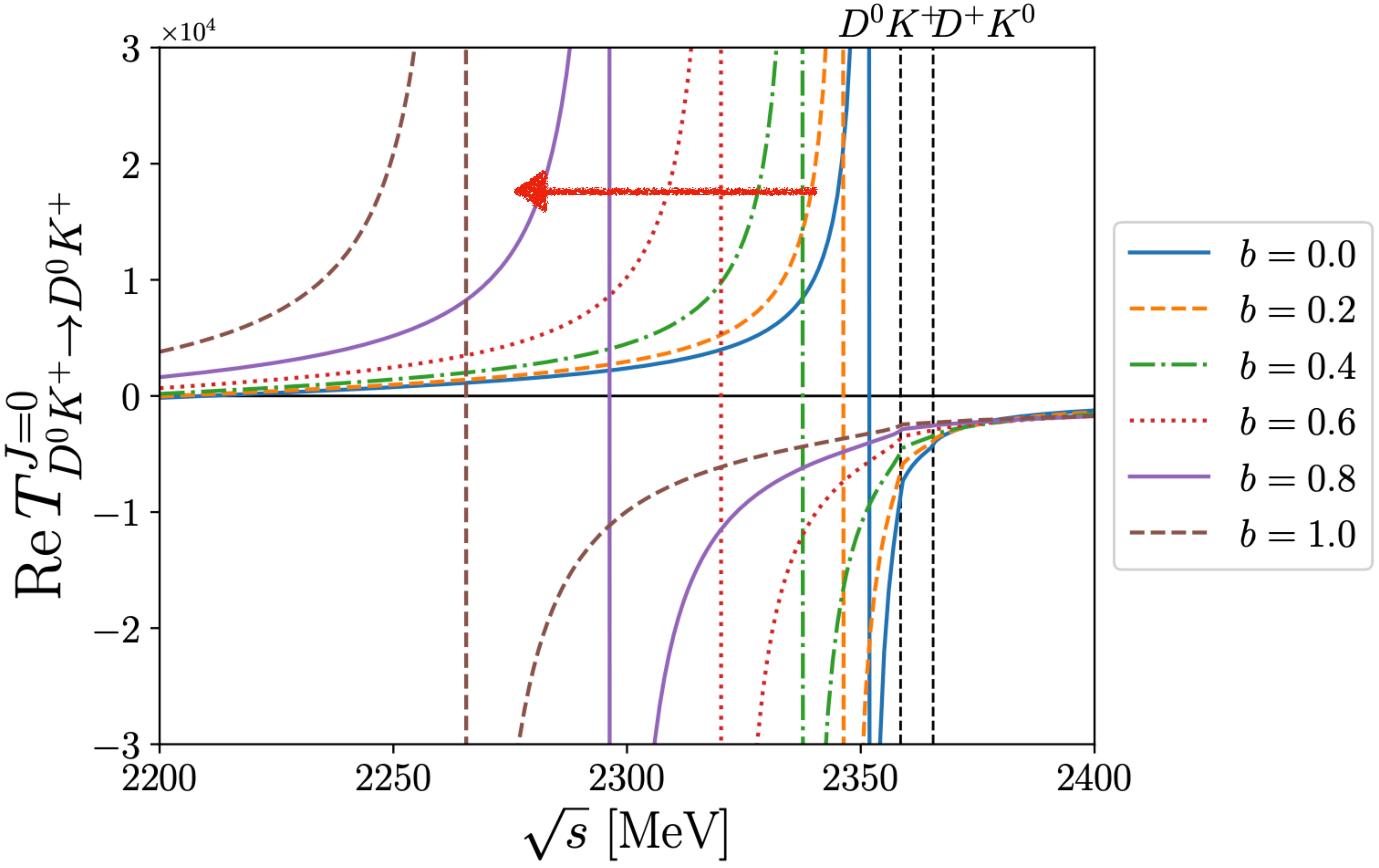


The first pole gains width rapidly as b increases by $D_s^+\pi^0$ channel.

Dynamical generation of the poles

Coupled-channel dynamics

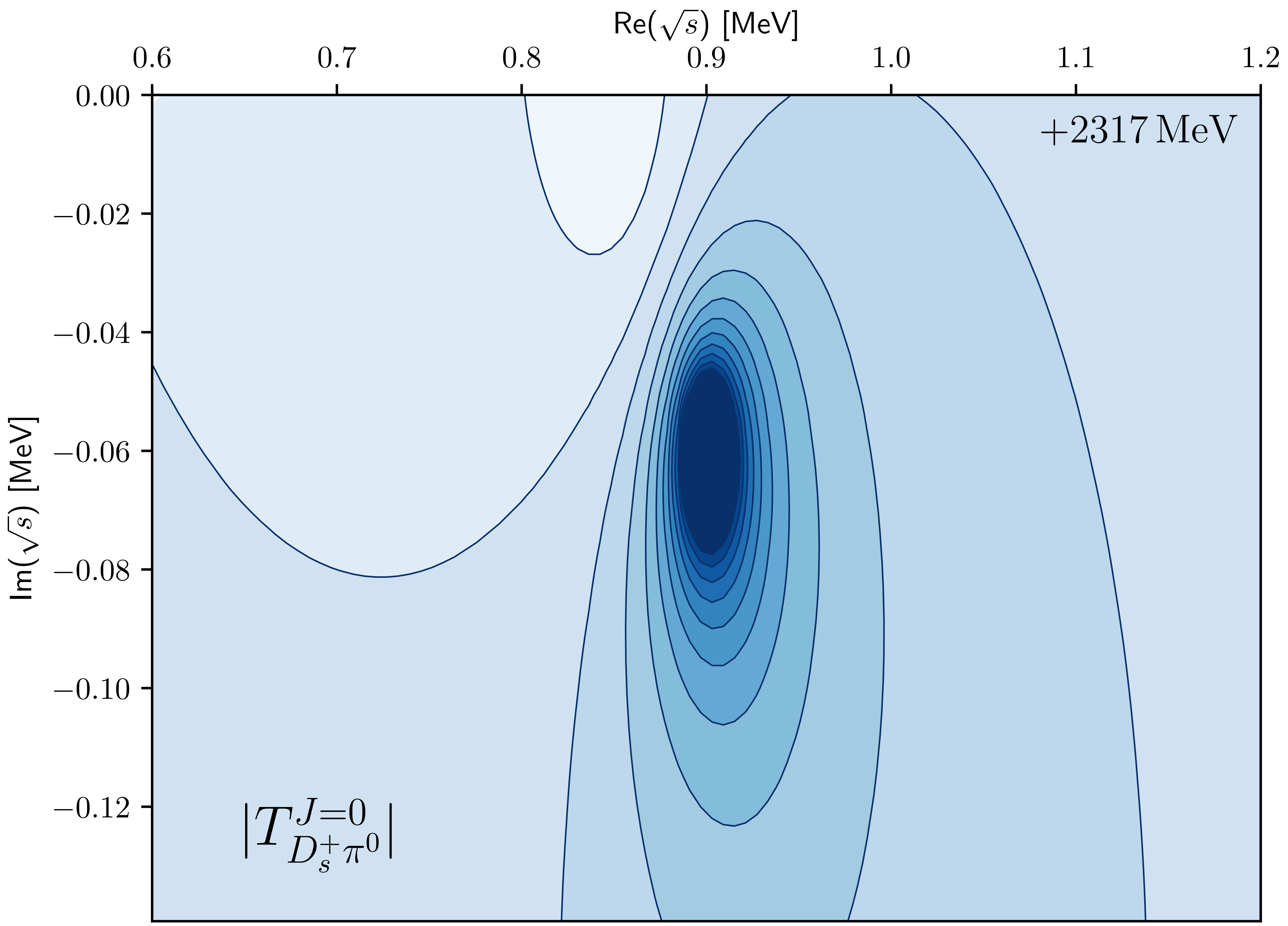
In the case of fully coupled-channel, the second pole moves to the lower energy



Dynamical generation of the poles

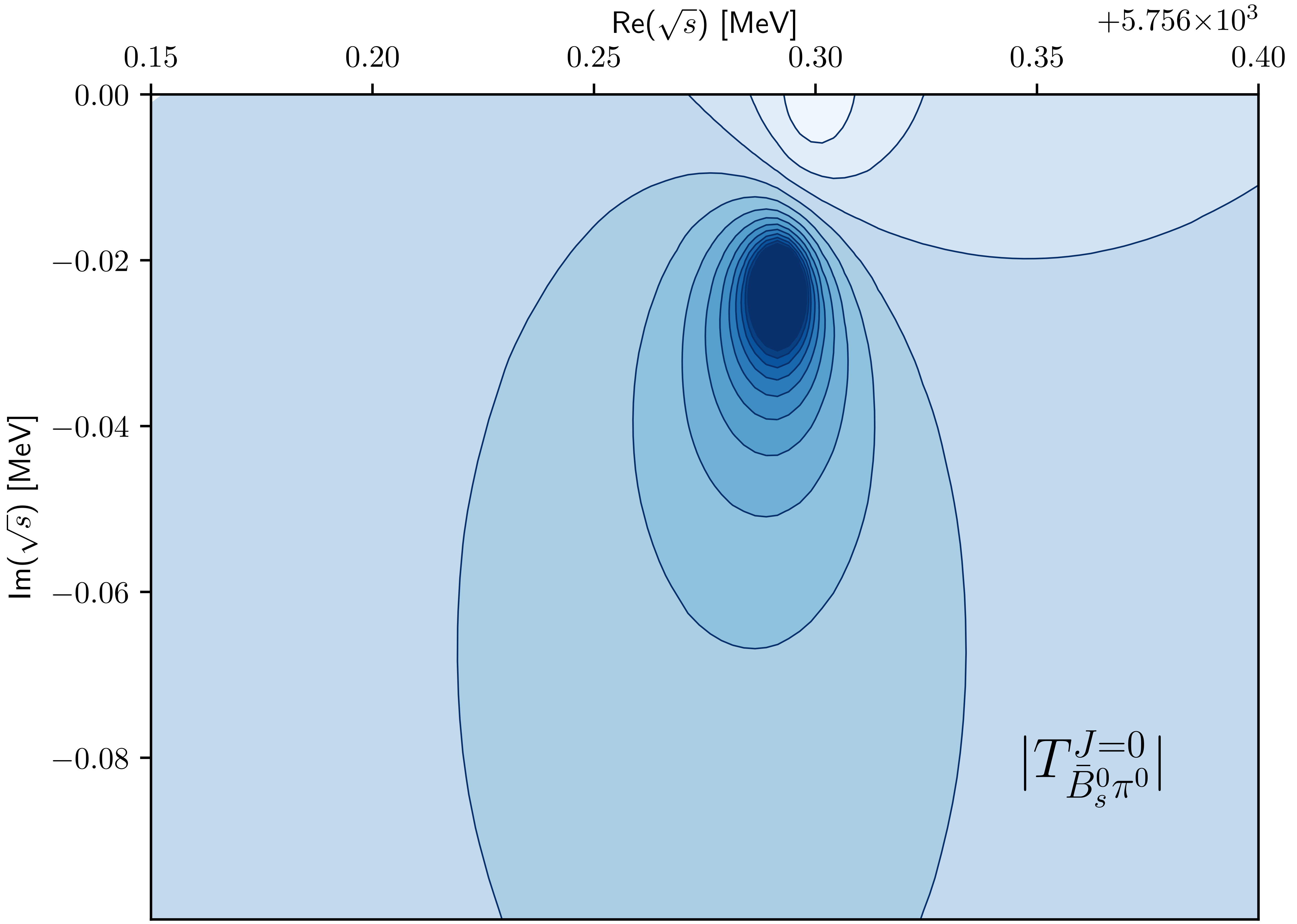
Coupled-channel dynamics

We find the pole at $\sqrt{s}_R = (2317.90 - i0.0593)$ MeV in the complex plane.



Bottom sector: B^*_{s0}

- **Model prediction for the scalar bottom-strange state:** $\sqrt{s}_R = 5756.32 - i0.0228\,\mathrm{MeV}$



Heavy meson scattering in the hidden charm channel

Feynman amplitudes

- **Hadron channels in hidden-charm sector**

Possible two-hadron states with $c\bar{c}q\bar{q}'$:

- $D\bar{D},\ D\bar{D}^*,\ D^*\bar{D}^*\ (I=0,1)$
- $D_s\bar{D}_s,\ D_s\bar{D}_s^*,\ D_s^*\bar{D}_s^*,\ \eta_c\omega,\ J/\psi\omega,\ \eta_c\phi,\ J/\psi\phi\ (I=0)$
- $\eta_c\pi,\ J/\psi\pi,\ \eta_c\rho,\ J/\psi\rho\ (I=1).$

Four sets of coupled channels: mesons can be classified by quantum numbers, $I^G(J^{PC})$

- $0^\pm(0^{+\pm},2^{+\pm})$: $D\bar{D},\ J/\psi\omega,\ D_s\bar{D}_s,\ D^*\bar{D}^*,\ J/\psi\phi,\ D_s^*\bar{D}_s^*$
- $0^\pm(1^{+\pm})$: $\eta_c\omega,\ D\bar{D}^*,\ \eta_c\phi,\ D^*\bar{D}^*,\ D_s\bar{D}_s^*,\ D_s^*\bar{D}_s^*$
- $1^\pm(0^{+\mp})$: $D\bar{D},\ J/\psi\rho,\ D^*\bar{D}^*$
- $1^\pm(1^{+\mp})$: $\eta_c\rho,\ D\bar{D}^*,\ D^*\bar{D}^*$

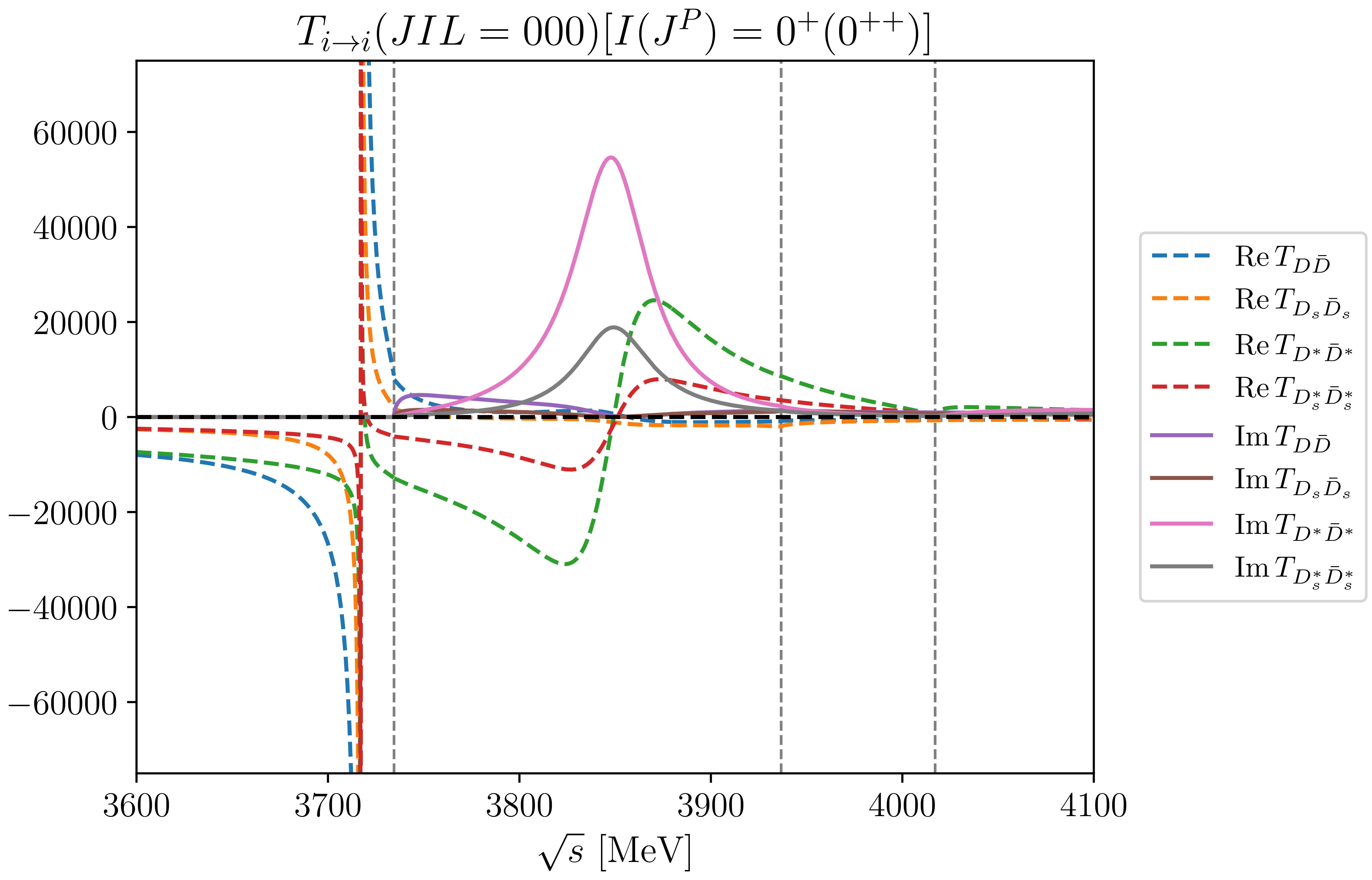
- $0^+(0^{++})$: $\chi_{c0}(3860)$, $\chi_{c0}(3915)$
- $0^+(1^{++})$: $\chi_{c1}(3872)$, $\chi_{c1}(4140)$, $\chi_{c1}(4274)$
- $0^+(2^{++})$: $\chi_{c2}(3930)$
- $1^+(1^{+-})$: $T_{c\bar{c}1}(3900)$, $T_{c\bar{c}1}(4200)$, $T_{c\bar{c}1}(4430)$

Isoscalar channels ($I = 0$)

Dynamical generation of the poles

Fully-coupled T-matrices

Scalar-isoscalar(J=0, I=0) channel



- A bound state below $D\bar{D}$ threshold

  $\sqrt{s_B} = 3720$ MeV

  lower charmonium channels($\eta_c\eta, J/\psi\eta, \ldots$) are additionally coupled, leading to resonance

- A resonance between $D\bar{D}$ and $D_s\bar{D}_s$

  $\sqrt{s_R} = 3861.34 - i22.76$ MeV

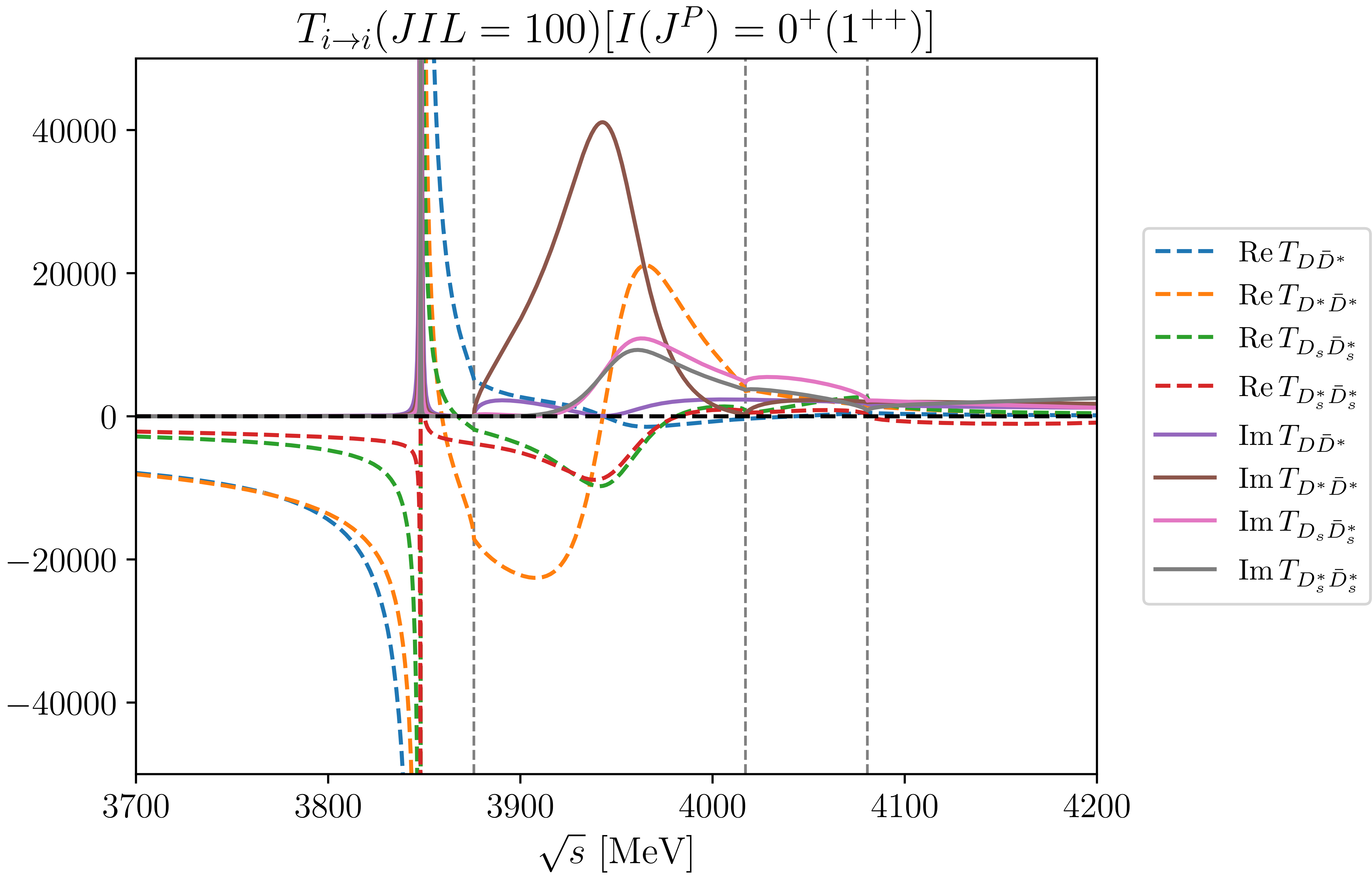
  very close to the mass of X(3860).
  ($M_{X(3860)} = 3862^{+50}_{35}$ MeV)

  but much narrower width than X(3860).
  ($\Gamma_{X(3860)} \simeq 200^{+180}_{-110}$ MeV)

Dynamical generation of the poles

Fully-coupled T-matrices

Vector-isoscalar(J=1, I=0) channel



- Nearly bound state below $D\bar{D}^*$ threshold

  $\sqrt{s_B} = 3848.11$ MeV

  Within our cutoff scheme,
  larger binding energy than $\chi_{c1}(3872)$

  $M_{\chi_{c1}} = 3871.84\,\mathrm{MeV}$

- A resonance between $D\bar{D}^*$ and $D^*\bar{D}^*$

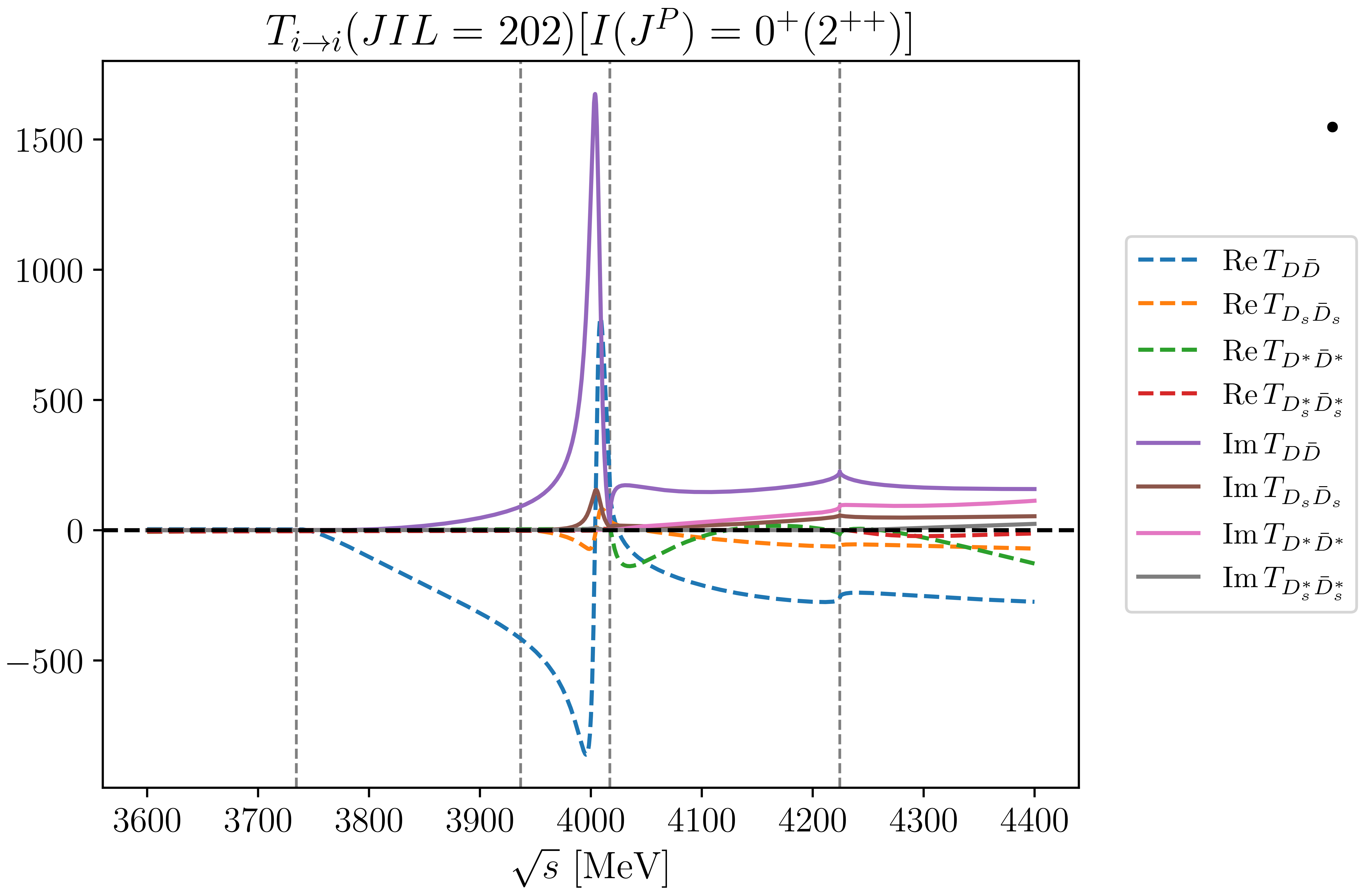
  $\sqrt{s_R} = 3948.622 - i26.98$ MeV

  nice candidate for $X(3940)$:
  $m_{X(3940)} = 3942 \pm 9$ MeV
  $\Gamma_{X(3940)} = 37^{+27}_{-17}$ MeV

Dynamical generation of the poles

Fully-coupled T-matrices

Tensor-isoscalar(J=2, I=0) channel



- A very narrow resonance near $D^*\bar{D}^*$ threshold

$$\sqrt{s_R} = 4005.26 - i5.95 \text{ MeV}$$

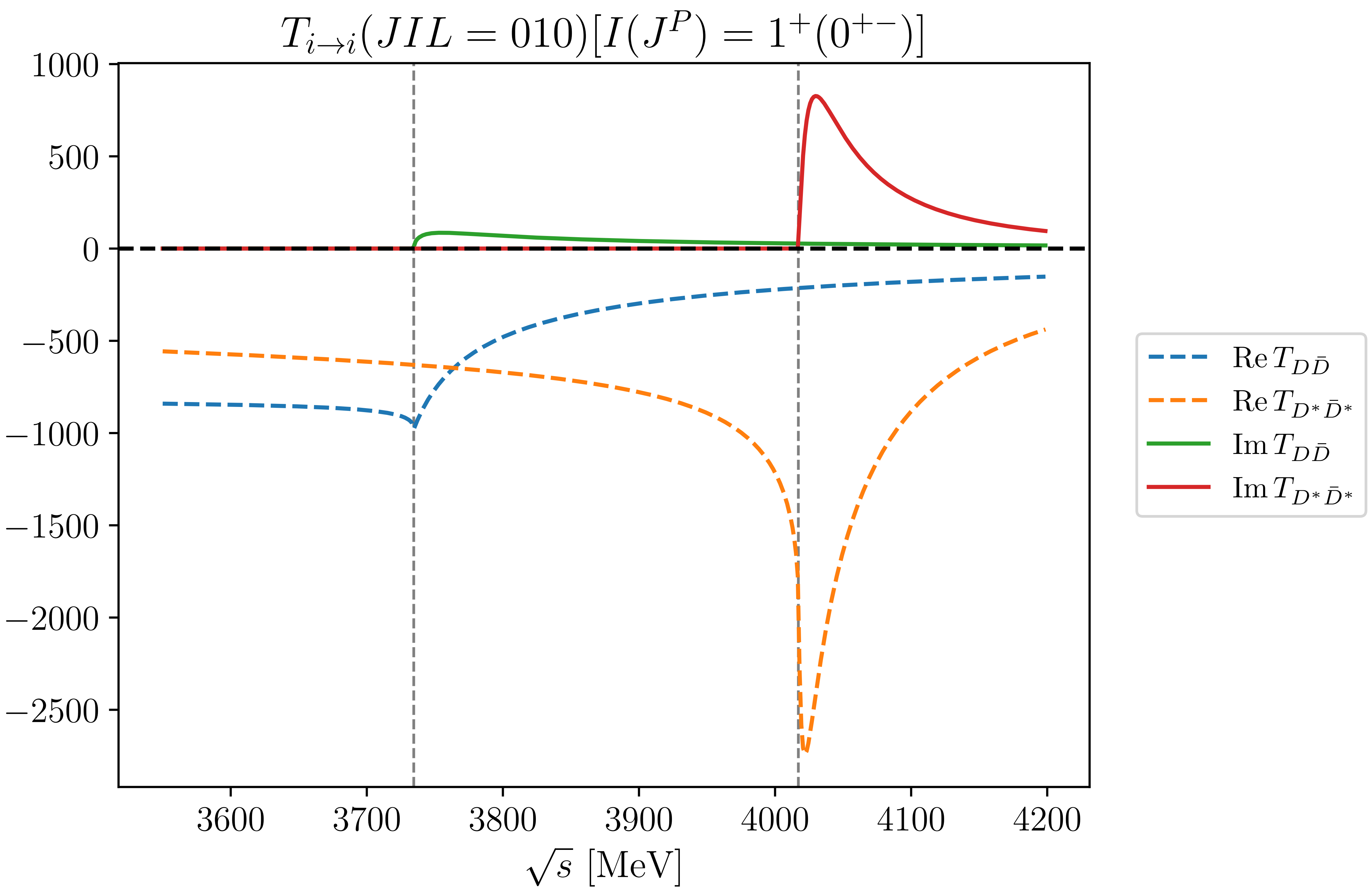
positioned near $D^*\bar{D}^*$ threshold but dominantly coupled to the $D\bar{D}$ like $\chi_{c2}(3930)$.

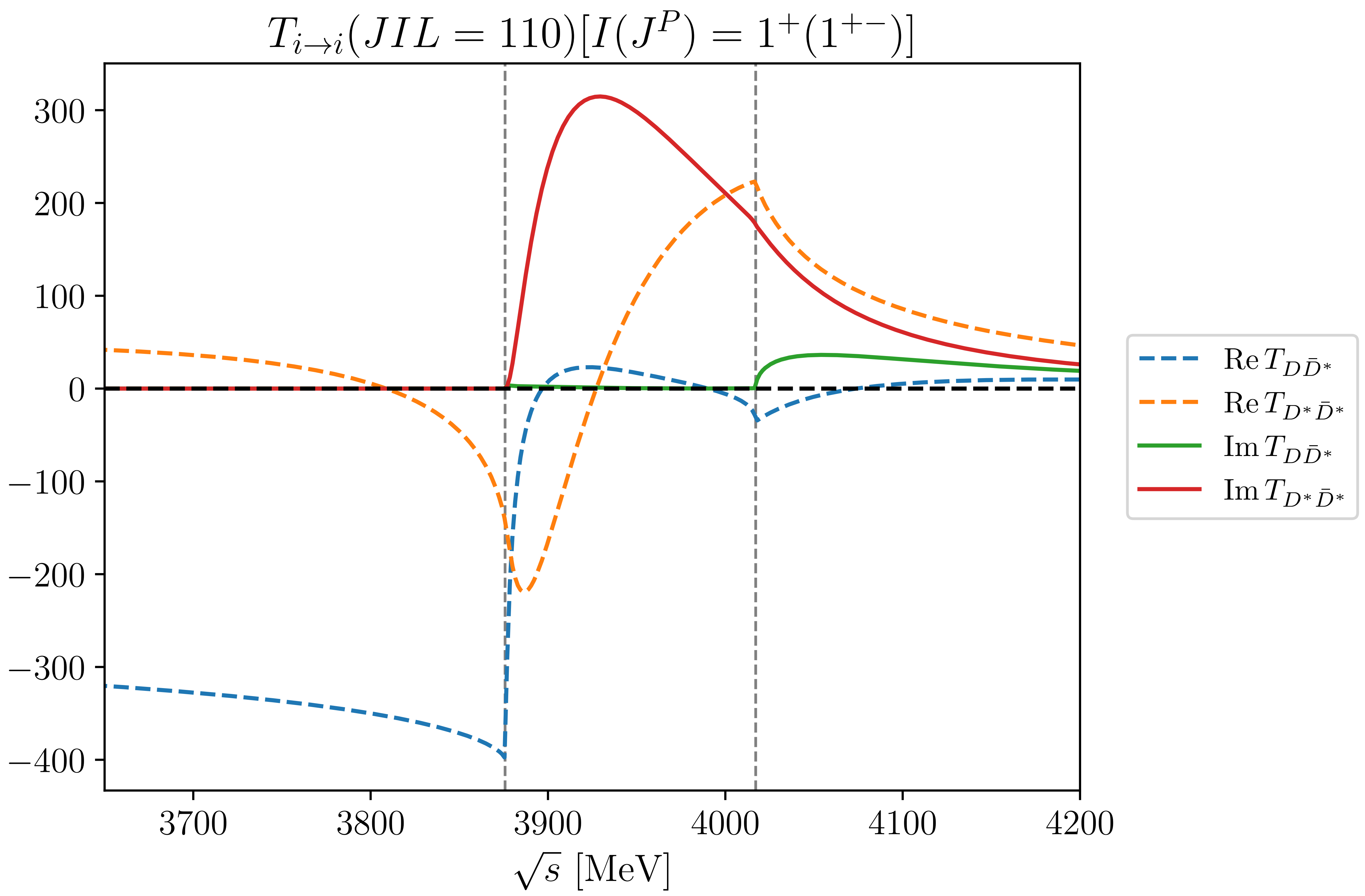
about 80 MeV heavier than $\chi_{c2}(3930)$...

Isovector channels ($I = 1$)

Dynamical generation of the poles

Fully-coupled T-matrices





- No resonant shape in scalar channels
- A pronounce cusp at the $D\bar{D}^*$ mass threshold

- A peak appears between two mass thresholds
- This structure is found to be a virtual state

Heavy meson scattering in the doubly charm channel

Feynman amplitudes

- **Hadron channels in doubly-charm sector**

Possible two-hadron states with $cc\bar{q}\bar{q}'$:

- $DD,\ DD^{*},\ D^{*}D^{*}\ (I=0,1)$

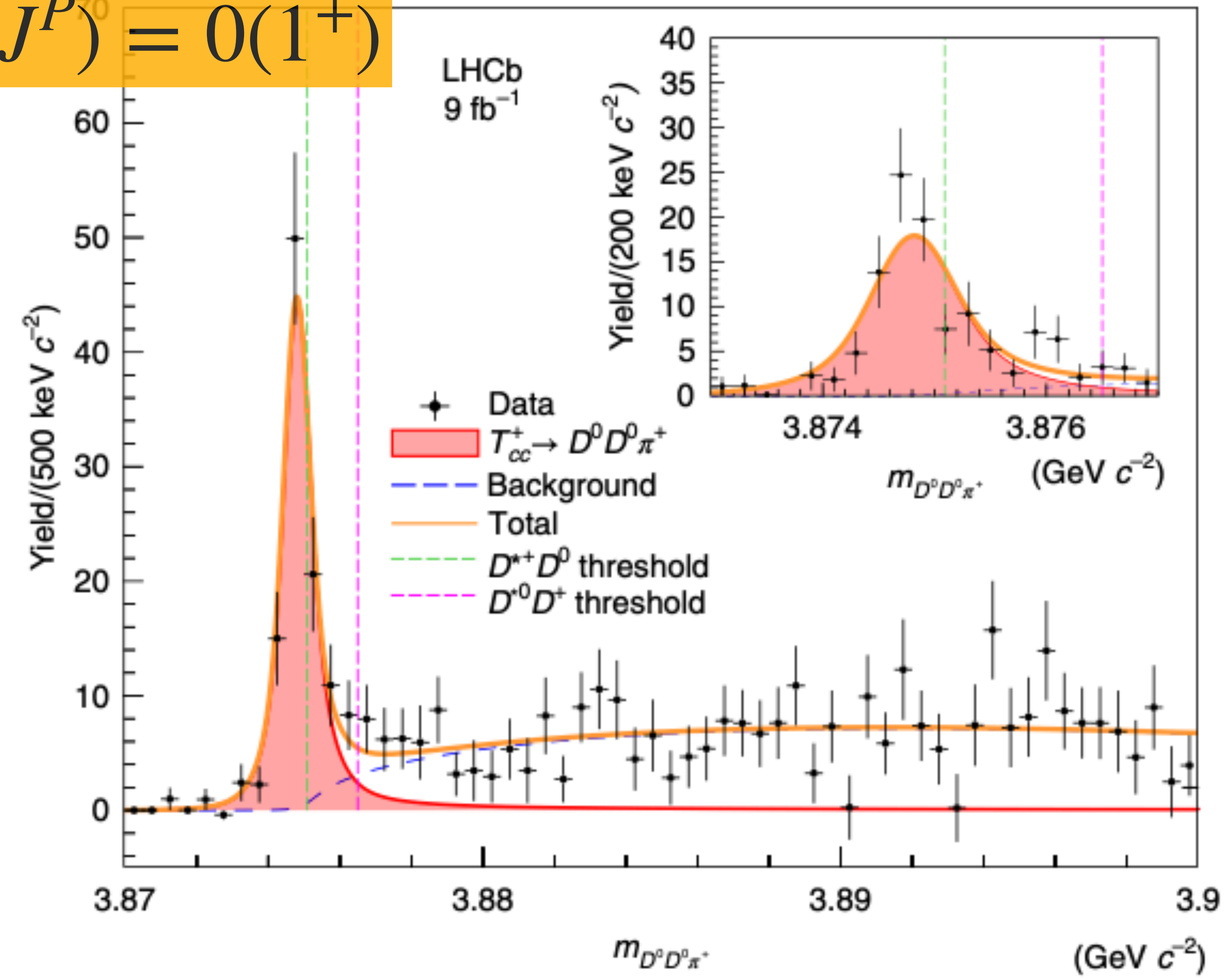
Four sets of coupled channels: mesons can be classified by quantum numbers, $I^{G}(J^{PC})$

- $0^{\pm}(0^{+\pm},2^{+\pm})$: $DD,\ D^{*}D^{*}$
- $0^{\pm}(1^{+\pm})$: $DD^{*},\ D^{*}D^{*}$
- $1^{\pm}(0^{+\mp})$: $DD,\ D^{*}D^{*}$
- $1^{\pm}(1^{+\mp})$: $DD^{*},\ D^{*}D^{*}$

Kernel matrix element:

$$\mathcal{V}=\begin{pmatrix} \mathcal{V}_{DD\to DD} & \mathcal{V}_{DD\to DD^{*}} & \mathcal{V}_{DD\to D^{*}D^{*}} \\ \mathcal{V}_{DD^{*}\to DD} & \mathcal{V}_{DD^{*}\to DD^{*}} & \mathcal{V}_{D^{*}D^{*}\to D^{*}D^{*}} \\ \mathcal{V}_{D^{*}D^{*}\to DD} & \mathcal{V}_{D^{*}D^{*}\to DD^{*}} & \mathcal{V}_{D^{*}D^{*}\to D^{*}D^{*}} \end{pmatrix}$$

$I(J^{P})=0(1^{+})$

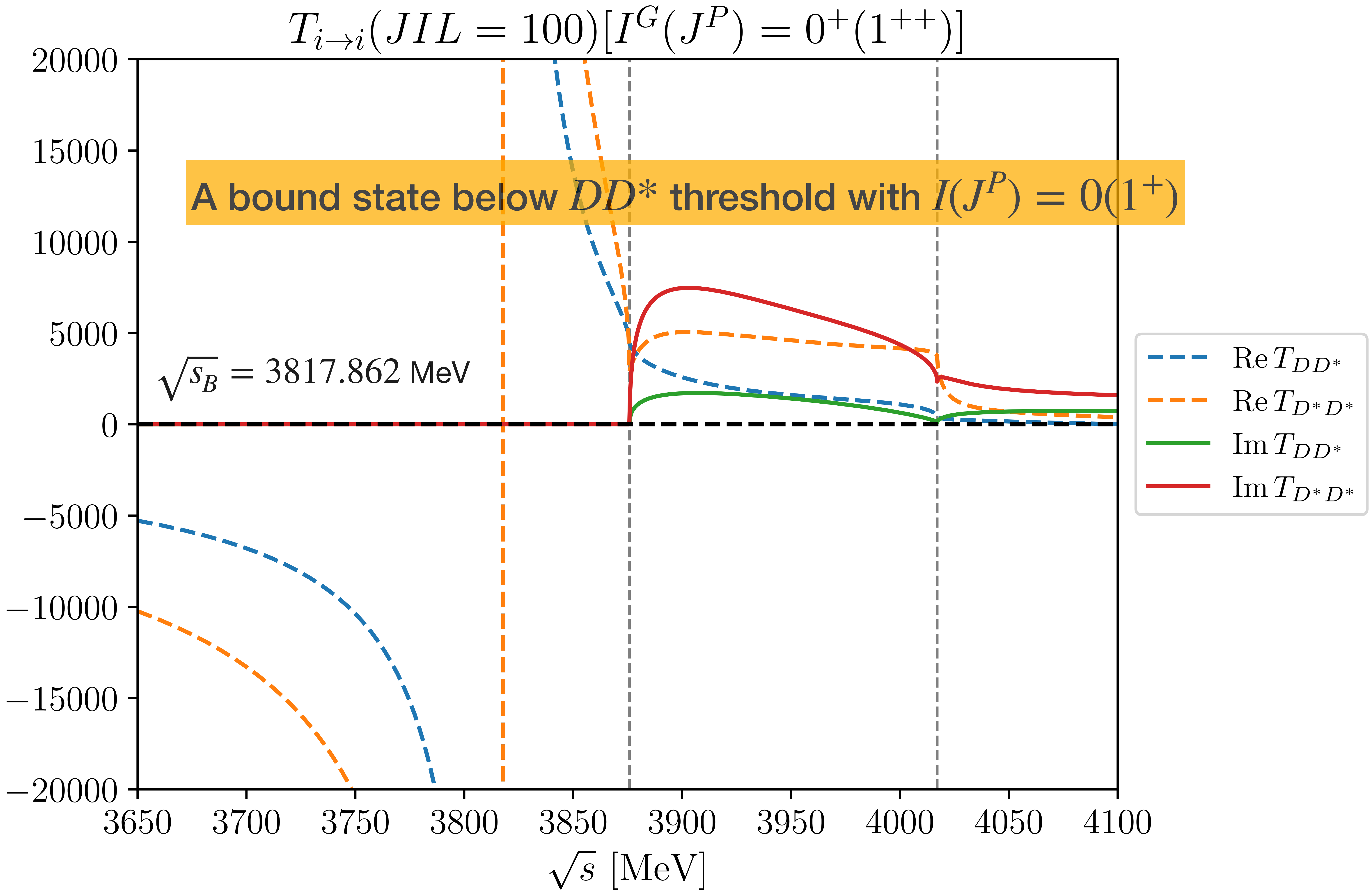


R. Aaij et.al.(LHCb Collabration) Nature Physics. 18 (2022) 7, 751-754

Dynamical generation of the poles

Fully-coupled T-matrices

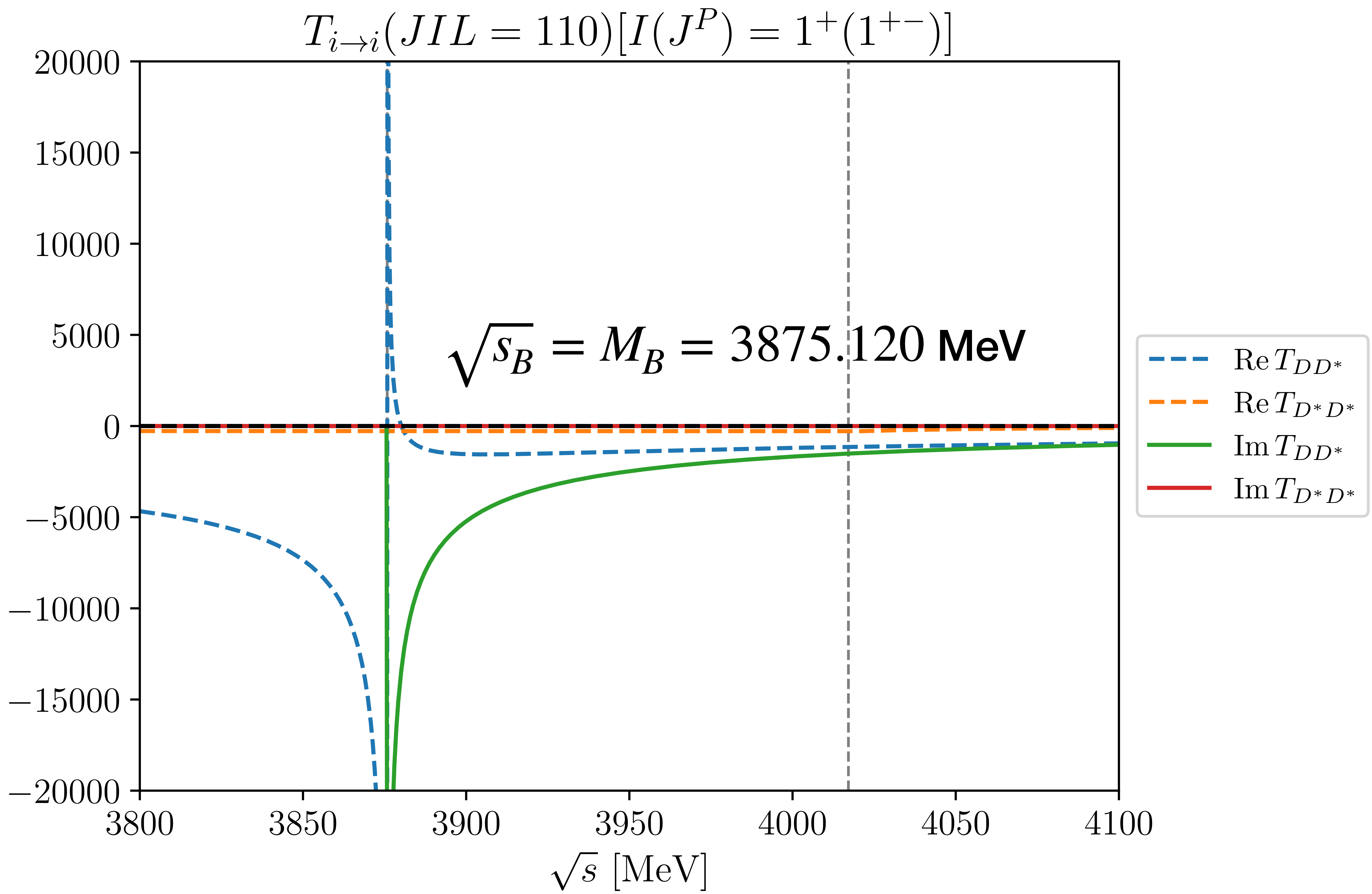
Vector-isoscalar channel (I=0,J=1)



Dynamical generation of the poles

Fully-coupled T-matrices

Vector-isovector channel (I=1,J=1)



Summary

Summary

- We investigated the production of exotic mesons containing one or more heavy quarks using coupled-channel dynamics within meson-molecule framwork.
- The tree-level interactions between heavy mesons and light unflavored mesons are described through the effective Lagrangian approach based on HQEFT.
- Through coupled-channel dynamics, we demonstrated the dynamical generation of the $D_{s0}^*(2317)$ and predicted its bottom-sector partner B_{s0}^* from heavy quark flavor symmetry.
- Our investigation in the hidden-charm sector revealed three states: a new scalar bound state below the $D\bar{D}$ threshold, along with scalar, vector and tensor resonances as $D^*\bar{D}^*$, $D\bar{D}^*$ and $D\bar{D}$ molecular states, respectively: $X(3860)$, $\chi_{c1}(3872)$, $X(3940)$, $\chi_{c2}(3930)$.
- In the doubly charm sector, we searched two vector bound states. The isoscalar one may be nice candidate for observed doubly charmed meson by LHCb: $T_{cc}^+(3875)$.
- This study enhances the understanding of production mechanism of molecule-like exotic mesons across three distinct sectors.

Thank you

Back up

Feynman amplitudes

- **Kernel matrix**

Consider four $C = S = +1$ channels : $|1\rangle = |D_s^+\pi^0\rangle$, $|2\rangle = |D^0K^+\rangle$, $|3\rangle = |D^+K^0\rangle$, $|4\rangle = |D_s^+\eta\rangle$

Kernel matrix element: $\mathcal{V}_{ba} \equiv \langle b|\mathcal{V}|a\rangle$

$$\hat{\mathcal{V}} = \begin{pmatrix} \mathcal{V}_{D_s^+\pi^0\to D_s^+\pi^0} & \mathcal{V}_{D_s^+\pi^0\to D^0K^+} & \mathcal{V}_{D_s^+\pi^0\to D^+K^0} & \mathcal{V}_{D_s^+\pi^0\to D_s^+\eta} \\ \mathcal{V}_{D^0K^+\to D_s^+\pi^0} & \mathcal{V}_{D^0K^+\to D^0K^+} & \mathcal{V}_{D^0K^+\to D^+K^0} & \mathcal{V}_{D^0K^+\to D_s^+\eta} \\ \mathcal{V}_{D^+K^0\to D_s^+\pi^0} & \mathcal{V}_{D^+K^0\to D^0K^+} & \mathcal{V}_{D^+K^0\to D^+K^0} & \mathcal{V}_{D^+K^0\to D_s^+\eta} \\ \mathcal{V}_{D_s^+\eta\to D_s^+\pi^0} & \mathcal{V}_{D_s^+\eta\to D^0K^+} & \mathcal{V}_{D_s^+\eta\to D^+K^0} & \mathcal{V}_{D_s^+\eta\to D_s^+\eta} \end{pmatrix}$$

We construct the *off-mass-shell* kernel matrix in the full-channel momentum space.

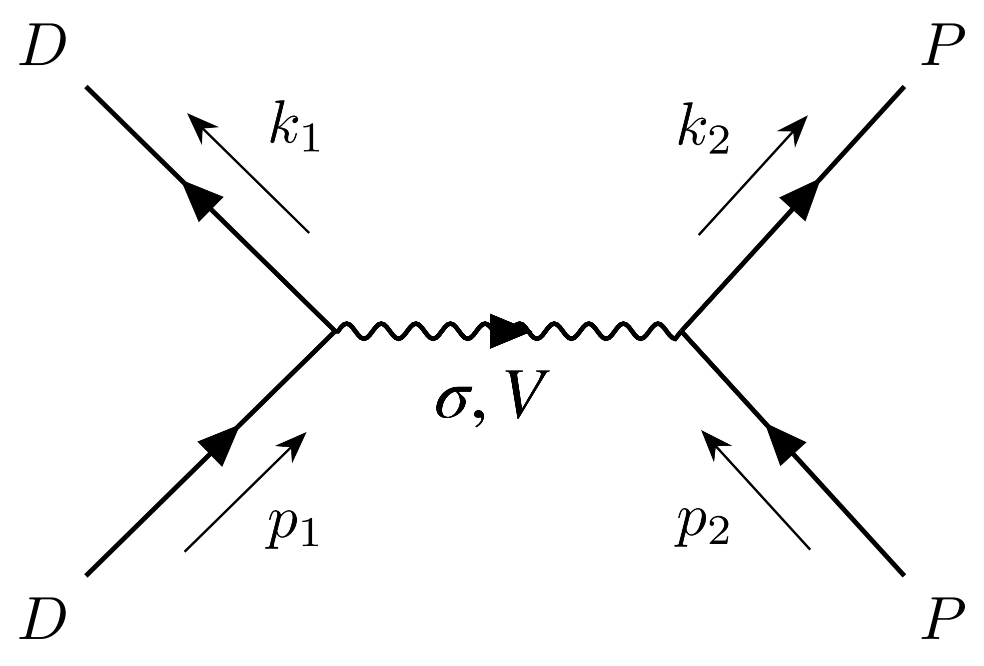
Feynman amplitudes

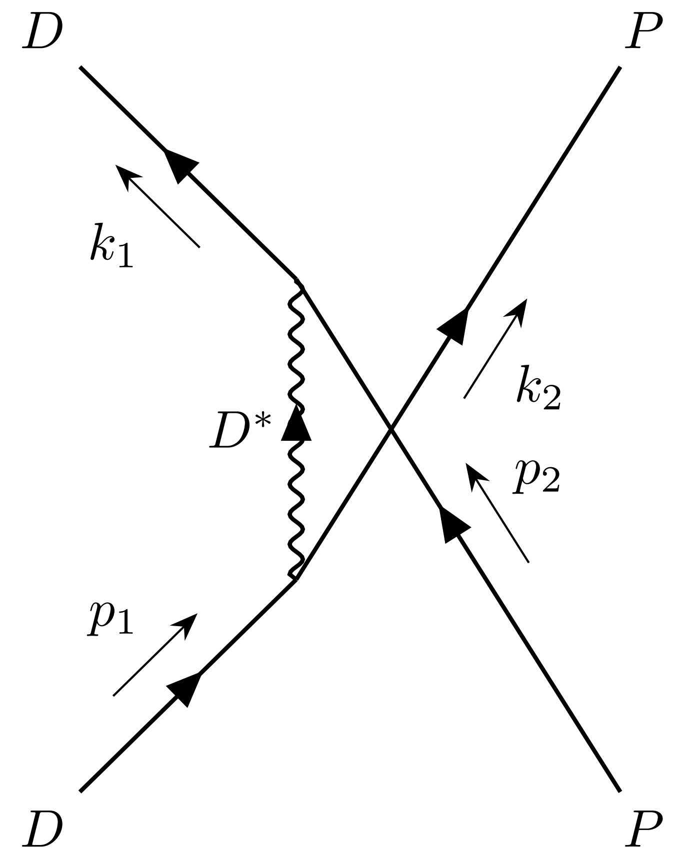
- **Scattering amplitudes**

$$\mathcal{V}_\sigma^t(t) = 4F^2(t) g_{DD\sigma} g_{PP\sigma} m_D m_P \frac{1}{t - m_\sigma^2}$$

$$\mathcal{V}_V^t(t) = -F^2(t) g_{DDV} g_{PPV} \frac{(p_1 + k_1)\cdot(p_2 + k_2)}{t - m_V^2}$$

$$\mathcal{V}_{D^*}^u(u) = F^2(u) g_{DD^*P}^2 \frac{(p_1 + k_2)\cdot(p_2 + k_1)}{u - m_{D^*}^2},$$





- **Coupling constants**

$g_{\pi^0\pi^0\sigma} = 8.7$, $g_{\eta\eta\sigma} = 5.0$, $g_{PPV} = 5.137$, $g_{DD^*P} = 17.9$, $g_{DDV} = 1.65$[2], $g_{DD\sigma} = 1.5$[2]

$$g_{D_s^+K^+D^{*0}} = g_{D_s^+K^0D^{*+}} = g_{DD^*P},$$

$$g_{\pi^0D^0D^{*0}} = -g_{\pi^0D^+D^{*+}} = g_{DD^*P}/\sqrt{2},$$

$$g_{\eta D^0D^*} = g_{\eta D^+D^{*+}} = g_{DD^*P}/\sqrt{6},$$

$$g_{D_s^+D^0K^{*+}} = g_{D_s^+D^+K^{*0}} = g_{DDV},$$

$$g_{D^0D^0\rho^0} = g_{D^0D^0\omega} = g_{DDV}/\sqrt{2}$$

$$g_{\pi^0K^0K^{*0}} = -g_{\pi^0K^+K^{*+}} = g_{PPV},$$

$$g_{\eta K^0K^{*0}} = g_{\eta K^+K^{*+}} = -\sqrt{3}g_{PPV},$$

$$g_{K^+K^+\rho^0} = g_{K^0K^0\rho^0} = g_{K^+K^0\rho^-}/\sqrt{2} = g_{PPV},$$

$$g_{K^+K^+\omega} = -g_{K^0K^0\omega} = g_{PPV}$$

[2] H-J. Kim and H.Ch. Kim Phys.Rev.D102 014026(2020)

Model calculations

	Present work	LO χ-BS(3) [18]	NLO χ-BS(3) [20]	SU(4) [19]
m_R [MeV]	2317.90	2317	2317.6	2317.25
$\Gamma_{D^*_{s0}}$ [keV]	13.86	8.69	140	-
$g_{D_s^+\pi^0}$ [GeV]	5.381×10^{-3}	-	-	-
$g_{D^0K^+}$ [GeV]	77.59	10.203	7.579	9.08
$g_{D^+K^0}$ [GeV]	80.17	10.203	7.579	9.08
$g_{D_s^+\eta}$ [GeV]	85.25	5.876	5.795	5.25

- small strong mode partial width due to the isospin violation
- large coupling constants compared to the on-shell approximated models
- significant coupling strength with $D_s^+\eta$ channel
- dominantly coupled with DK

Feynman amplitudes

- **Kernel matrix**

Kernel matricies:

$$\mathcal{V}^{J=0(2),I=0} = \begin{pmatrix} \mathcal{V}_{D\bar{D}\to D\bar{D}} & \mathcal{V}_{D\bar{D}\to J/\psi\omega} & \mathcal{V}_{D\bar{D}\to D_s\bar{D}_s} & \mathcal{V}_{D\bar{D}\to D^*\bar{D}^*} & \mathcal{V}_{D\bar{D}\to J/\psi\phi} & \mathcal{V}_{D\bar{D}\to D_s^*\bar{D}_s^*} \\ \mathcal{V}_{J/\psi\omega\to D\bar{D}} & \mathcal{V}_{J/\psi\omega\to J/\psi\omega} & \mathcal{V}_{J/\psi\omega\to D_s\bar{D}_s} & \mathcal{V}_{J/\psi\omega\to D^*\bar{D}^*} & \mathcal{V}_{J/\psi\omega\to J/\psi\phi} & \mathcal{V}_{J/\psi\omega\to D_s^*\bar{D}_s^*} \\ \mathcal{V}_{D_s\bar{D}_s\to D\bar{D}} & \mathcal{V}_{D_s\bar{D}_s\to J/\psi\omega} & \mathcal{V}_{D_s\bar{D}_s\to D_s\bar{D}_s} & \mathcal{V}_{D_s\bar{D}_s\to D^*\bar{D}^*} & \mathcal{V}_{D_s\bar{D}_s\to J/\psi\phi} & \mathcal{V}_{D_s\bar{D}_s\to D_s^*\bar{D}_s^*} \\ \mathcal{V}_{D^*\bar{D}^*\to D\bar{D}} & \mathcal{V}_{D^*\bar{D}^*\to J/\psi\omega} & \mathcal{V}_{D^*\bar{D}^*\to D_s\bar{D}_s} & \mathcal{V}_{D^*\bar{D}^*\to D^*\bar{D}^*} & \mathcal{V}_{D^*\bar{D}^*\to J/\psi\phi} & \mathcal{V}_{D^*\bar{D}^*\to D_s^*\bar{D}_s^*} \\ \mathcal{V}_{J/\psi\phi\to D\bar{D}} & \mathcal{V}_{J/\psi\phi\to J/\psi\omega} & \mathcal{V}_{J/\psi\phi\to D_s\bar{D}_s} & \mathcal{V}_{J/\psi\phi\to D^*\bar{D}^*} & \mathcal{V}_{J/\psi\phi\to J/\psi\phi} & \mathcal{V}_{J/\psi\phi\to D_s^*\bar{D}_s^*} \\ \mathcal{V}_{D_s^*\bar{D}_s^*\to D\bar{D}} & \mathcal{V}_{D_s^*\bar{D}_s^*\to J/\psi\omega} & \mathcal{V}_{D_s^*\bar{D}_s^*\to D_s\bar{D}_s} & \mathcal{V}_{D_s^*\bar{D}_s^*\to D^*\bar{D}^*} & \mathcal{V}_{D_s^*\bar{D}_s^*\to J/\psi\phi} & \mathcal{V}_{D_s^*\bar{D}_s^*\to D_s^*\bar{D}_s^*} \end{pmatrix}$$

$$\mathcal{V}^{J=1,I=0} = \begin{pmatrix} \mathcal{V}_{\eta_c\omega\to\eta_c\omega} & \mathcal{V}_{\eta_c\omega\to D\bar{D}^*} & \mathcal{V}_{\eta_c\omega\to\eta_c\phi} & \mathcal{V}_{\eta_c\omega\to D\bar{D}^*} & \mathcal{V}_{\eta_c\omega\to D_s\bar{D}_s^*} & \mathcal{V}_{\eta_c\omega\to D_s^*\bar{D}_s^*} \\ \mathcal{V}_{D\bar{D}^*\to\eta_c\omega} & \mathcal{V}_{D\bar{D}^*\to D\bar{D}^*} & \mathcal{V}_{D\bar{D}^*\to\eta_c\phi} & \mathcal{V}_{D\bar{D}^*\to D\bar{D}^*} & \mathcal{V}_{D\bar{D}^*\to D_s\bar{D}_s^*} & \mathcal{V}_{D\bar{D}^*\to D_s^*\bar{D}_s^*} \\ \mathcal{V}_{\eta_c\phi\to\eta_c\omega} & \mathcal{V}_{\eta_c\phi\to D\bar{D}^*} & \mathcal{V}_{\eta_c\phi\to\eta_c\phi} & \mathcal{V}_{\eta_c\phi\to D\bar{D}^*} & \mathcal{V}_{\eta_c\phi\to D_s\bar{D}_s^*} & \mathcal{V}_{\eta_c\phi\to D_s^*\bar{D}_s^*} \\ \mathcal{V}_{D\bar{D}^*\to\eta_c\omega} & \mathcal{V}_{D\bar{D}^*\to D\bar{D}^*} & \mathcal{V}_{D\bar{D}^*\to\eta_c\phi} & \mathcal{V}_{D\bar{D}^*\to D\bar{D}^*} & \mathcal{V}_{D\bar{D}^*\to D_s\bar{D}_s^*} & \mathcal{V}_{D\bar{D}^*\to D_s^*\bar{D}_s^*} \\ \mathcal{V}_{D_s\bar{D}_s^*\to\eta_c\omega} & \mathcal{V}_{D_s\bar{D}_s^*\to D\bar{D}^*} & \mathcal{V}_{D_s\bar{D}_s^*\to\eta_c\phi} & \mathcal{V}_{D_s\bar{D}_s^*\to D\bar{D}^*} & \mathcal{V}_{D_s\bar{D}_s^*\to D_s\bar{D}_s^*} & \mathcal{V}_{D_s\bar{D}_s^*\to D_s^*\bar{D}_s^*} \\ \mathcal{V}_{D_s^*\bar{D}_s^*\to\eta_c\omega} & \mathcal{V}_{D_s^*\bar{D}_s^*\to D\bar{D}^*} & \mathcal{V}_{D_s^*\bar{D}_s^*\to\eta_c\phi} & \mathcal{V}_{D_s^*\bar{D}_s^*\to D\bar{D}^*} & \mathcal{V}_{D_s^*\bar{D}_s^*\to D_s\bar{D}_s^*} & \mathcal{V}_{D_s^*\bar{D}_s^*\to D_s^*\bar{D}_s^*} \end{pmatrix}$$

Each kernel matrix element is the sum of all possible Feynman amplitudes allowed.

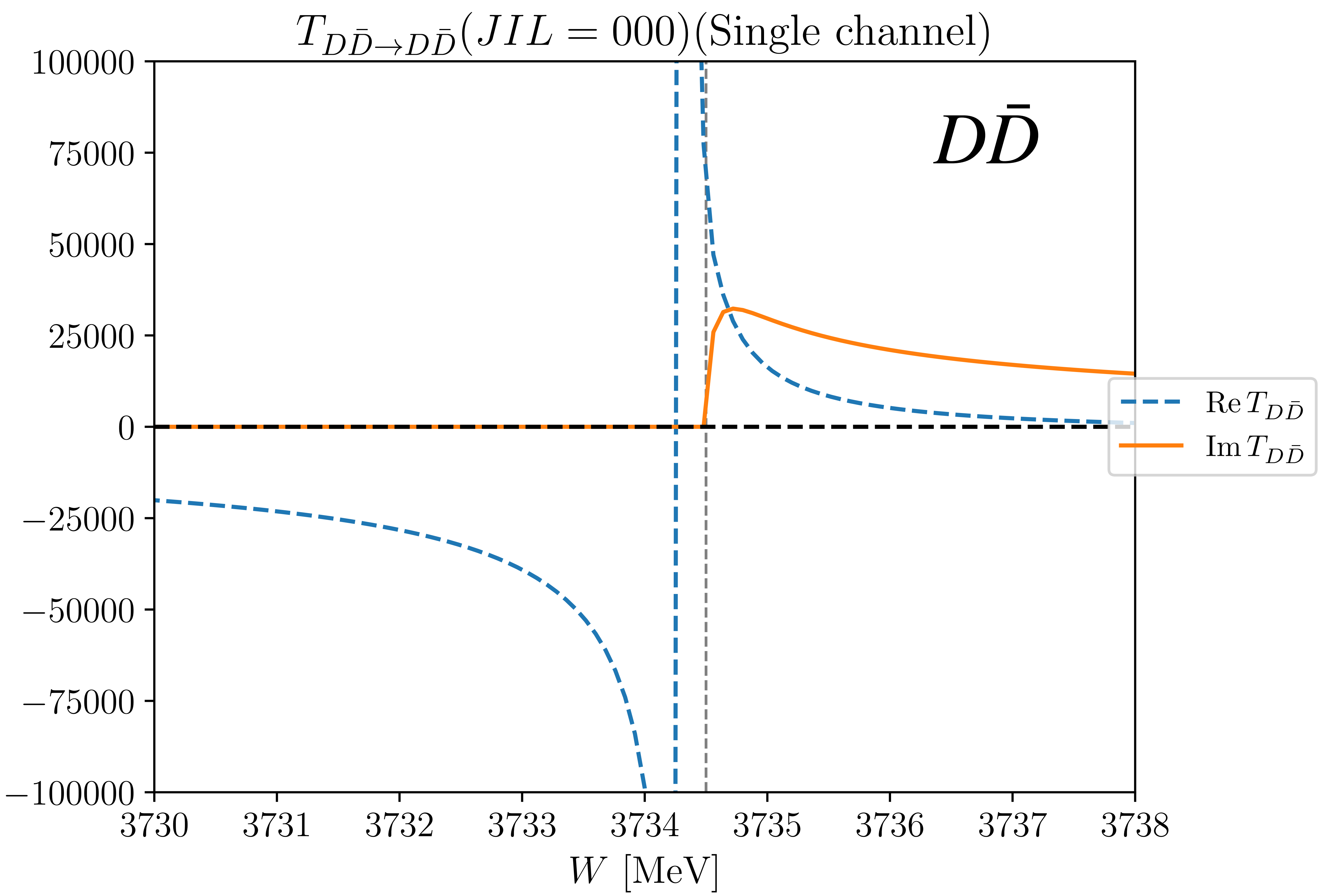
Every elements are spanned in the momentum space for initial and final states.

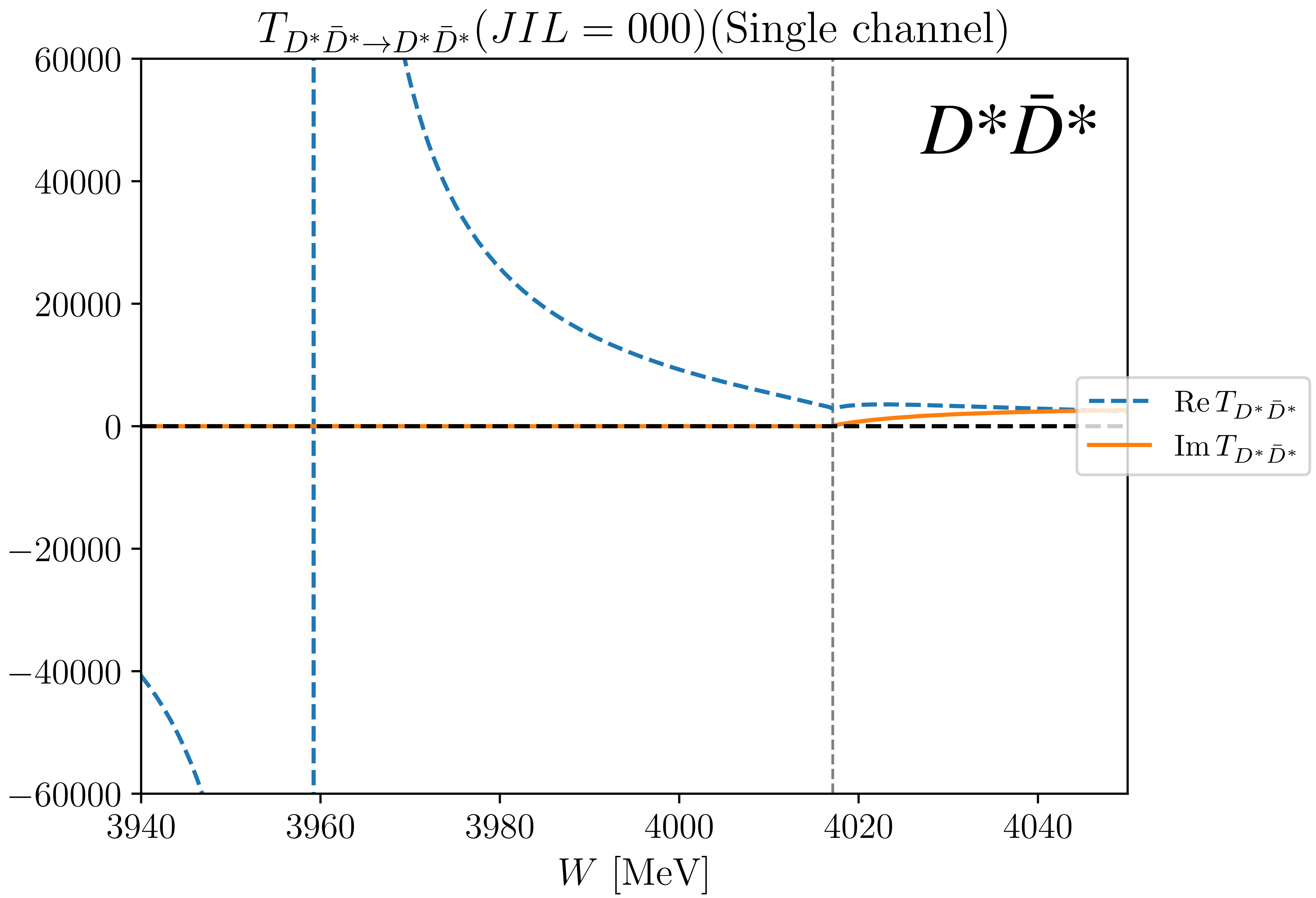
We thus construct the *off-mass-shell* kernel matrix in the full-channel momentum space.

Dynamical generation of the poles

Single channel T matrix elements   ex) $T_{11} = (1 - V_{11}G_1)^{-1}V_{11}$

scalar-isoscalar(J=I=0) channel



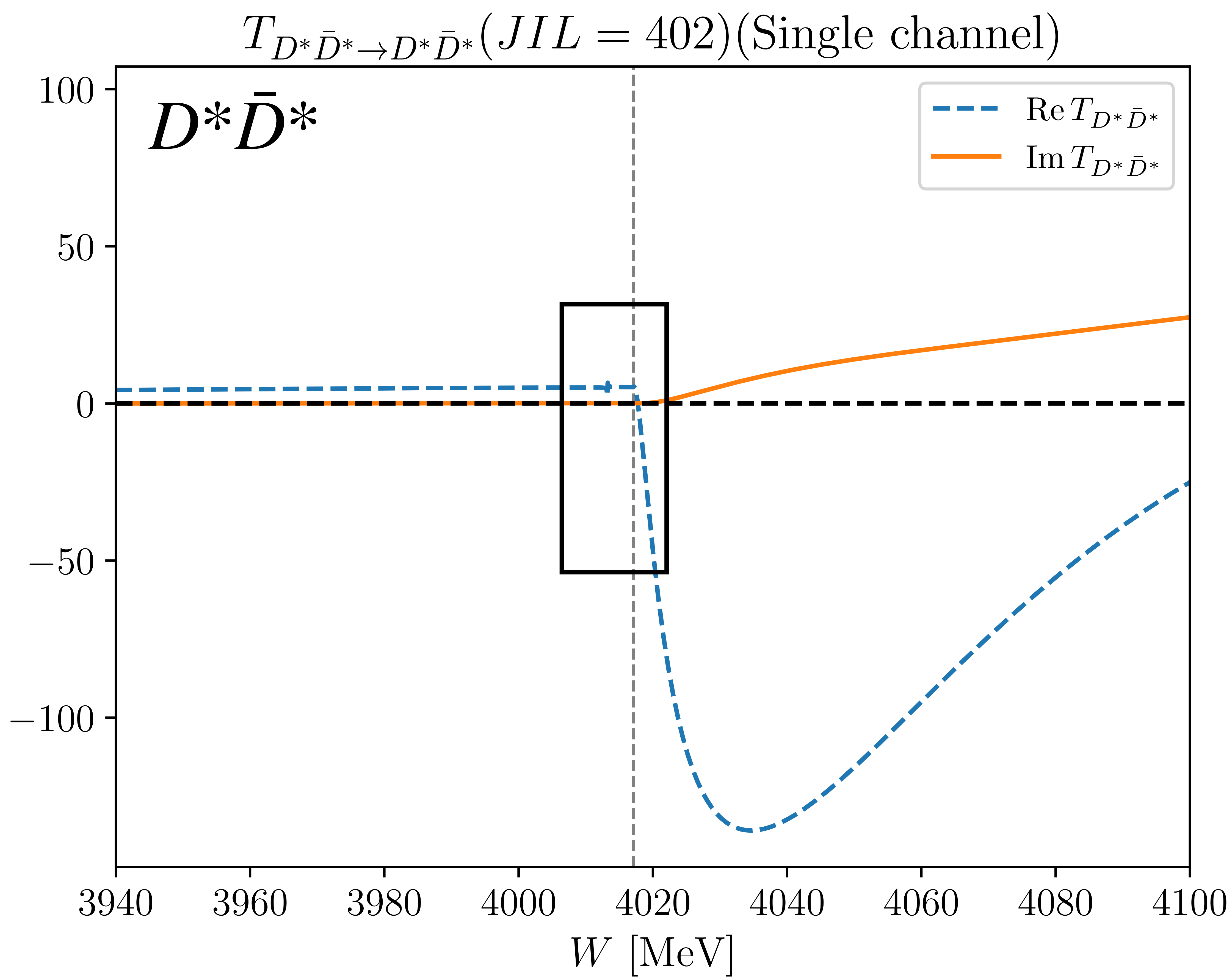


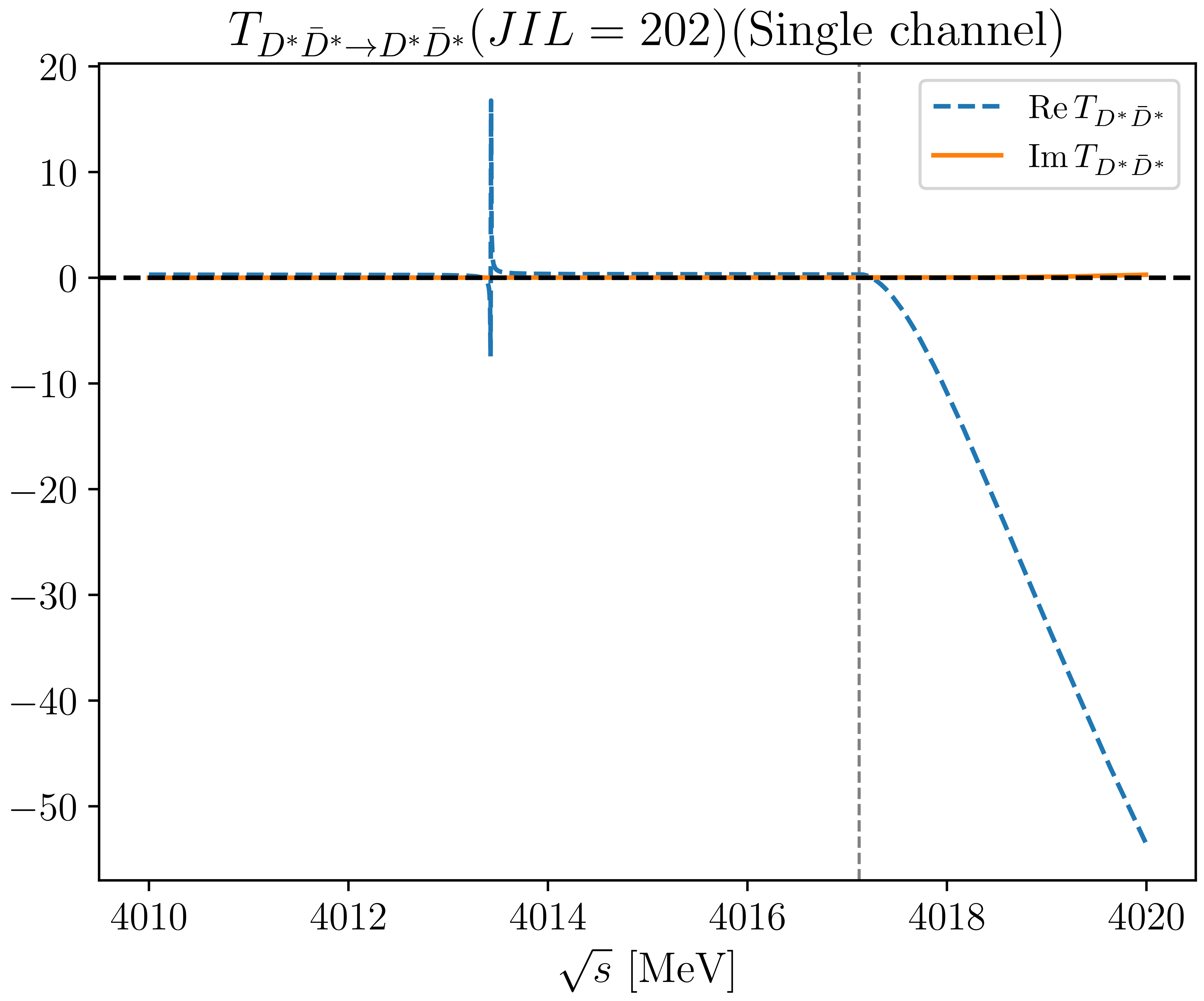
generated solely by $D\bar{D}$ and $D^*\bar{D}^*$ interactions, respectively

Dynamical generation of the poles

Single channel T matrix elements

tesor-isoscalar(J=2,I=0) channel

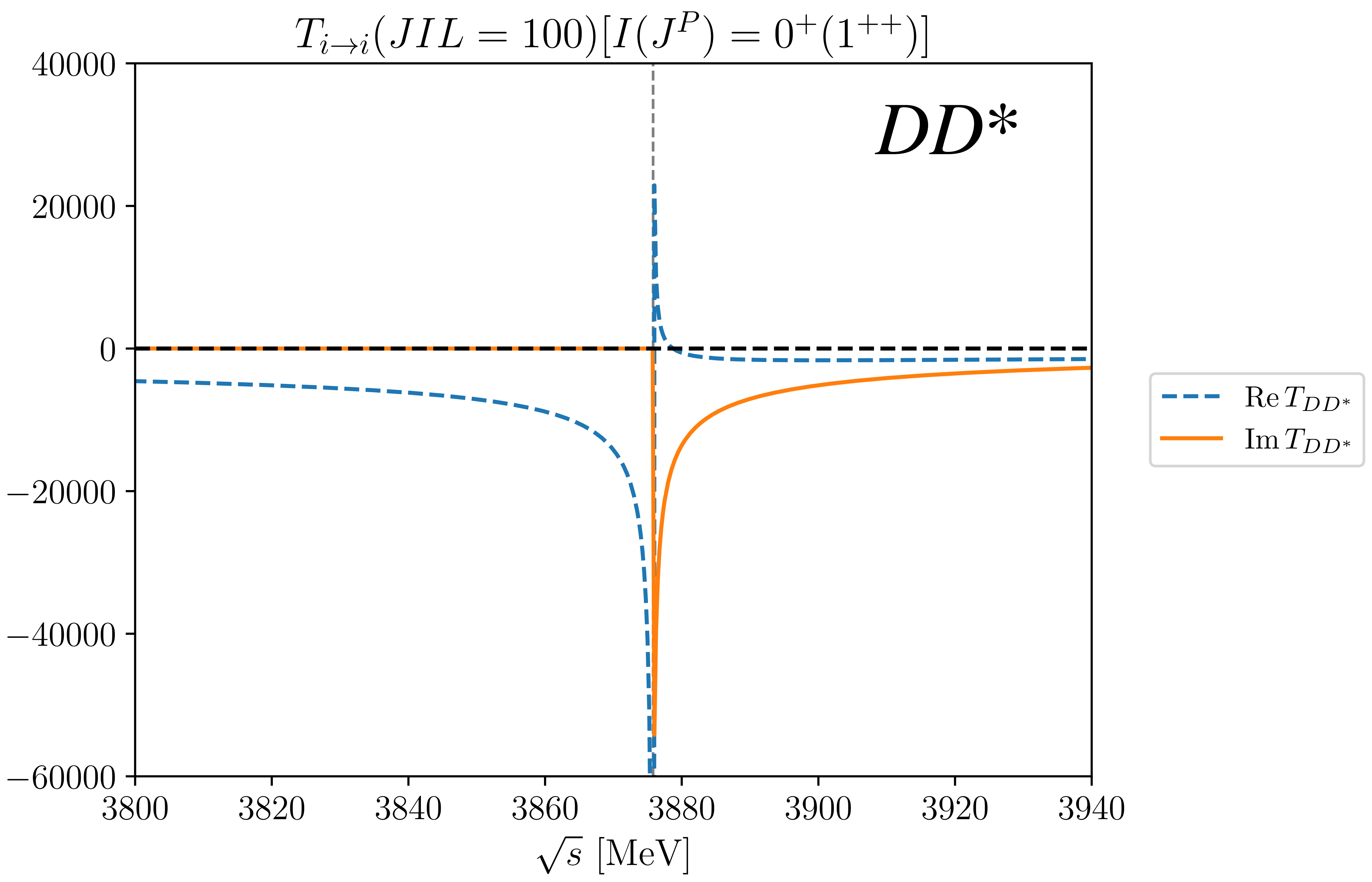


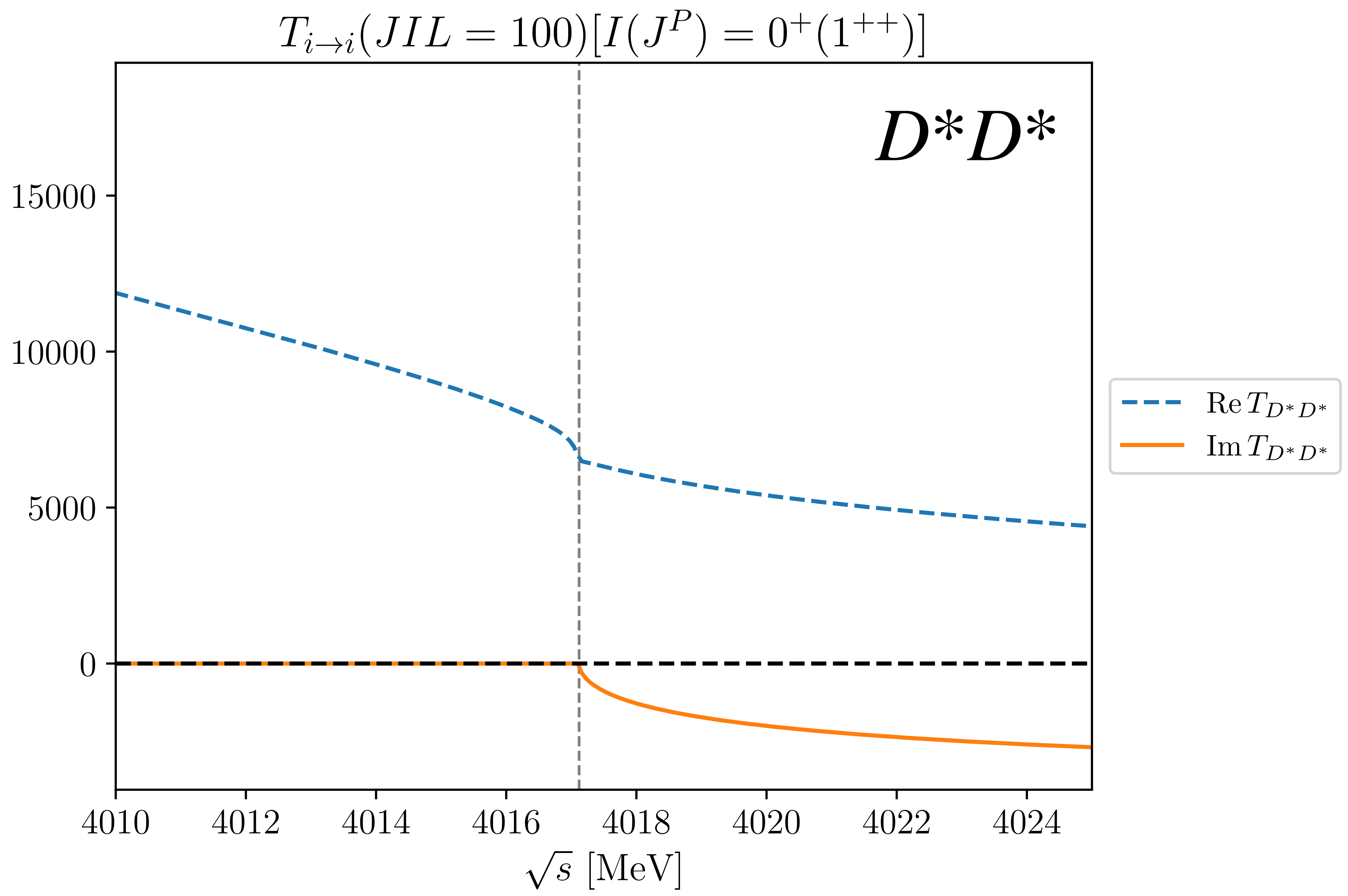


Dynamical generation of the poles

Single channel T matrix elements

Vector-isoscalar channel (J=1, I=0)





The attraction in DD^* kernel is nearly sufficient to generate a pole at the threshold.

Feynman amplitudes

- **Kernel matrix**

Kernel matrix element:

$$\mathcal{V} = \begin{pmatrix} \mathcal{V}_{D\bar{D}\to D\bar{D}} & \mathcal{V}_{D\bar{D}\to D\bar{D}^*} & \mathcal{V}_{D\bar{D}\to D^*\bar{D}^*} \\ \mathcal{V}_{D\bar{D}^*\to D\bar{D}} & \mathcal{V}_{D\bar{D}^*\to D\bar{D}^*} & \mathcal{V}_{D\bar{D}^*\to D^*\bar{D}^*} \\ \mathcal{V}_{D^*\bar{D}^*\to D\bar{D}} & \mathcal{V}_{D^*\bar{D}^*\to D\bar{D}^*} & \mathcal{V}_{D^*\bar{D}^*\to D^*\bar{D}^*} \end{pmatrix}$$

We neglect the charmonium channel due to their marginal coupling to heavy-meson pairs.

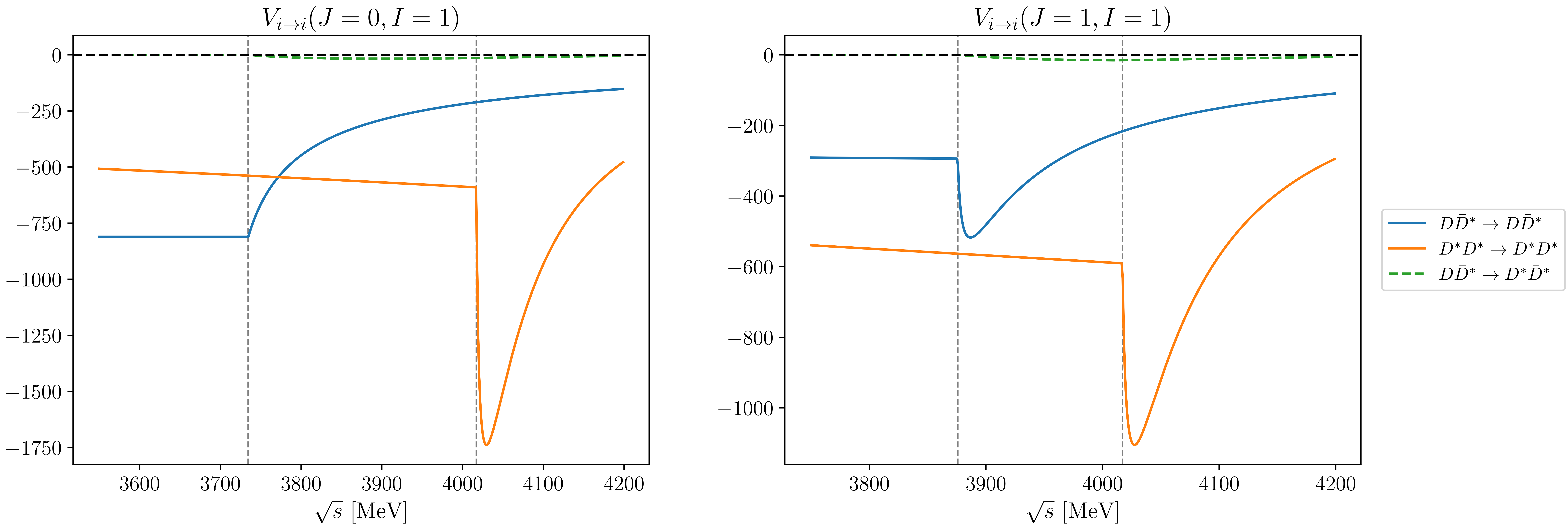
Each kernel matrix element is the sum of all possible Feynman amplitudes allowed.

Every elements are spanned in the momentum space for initial and final states.

We thus construct the *off-mass-shell* kernel matrix in the full-channel momentum space.

Dynamical generation of the poles

Kernel amplitudes

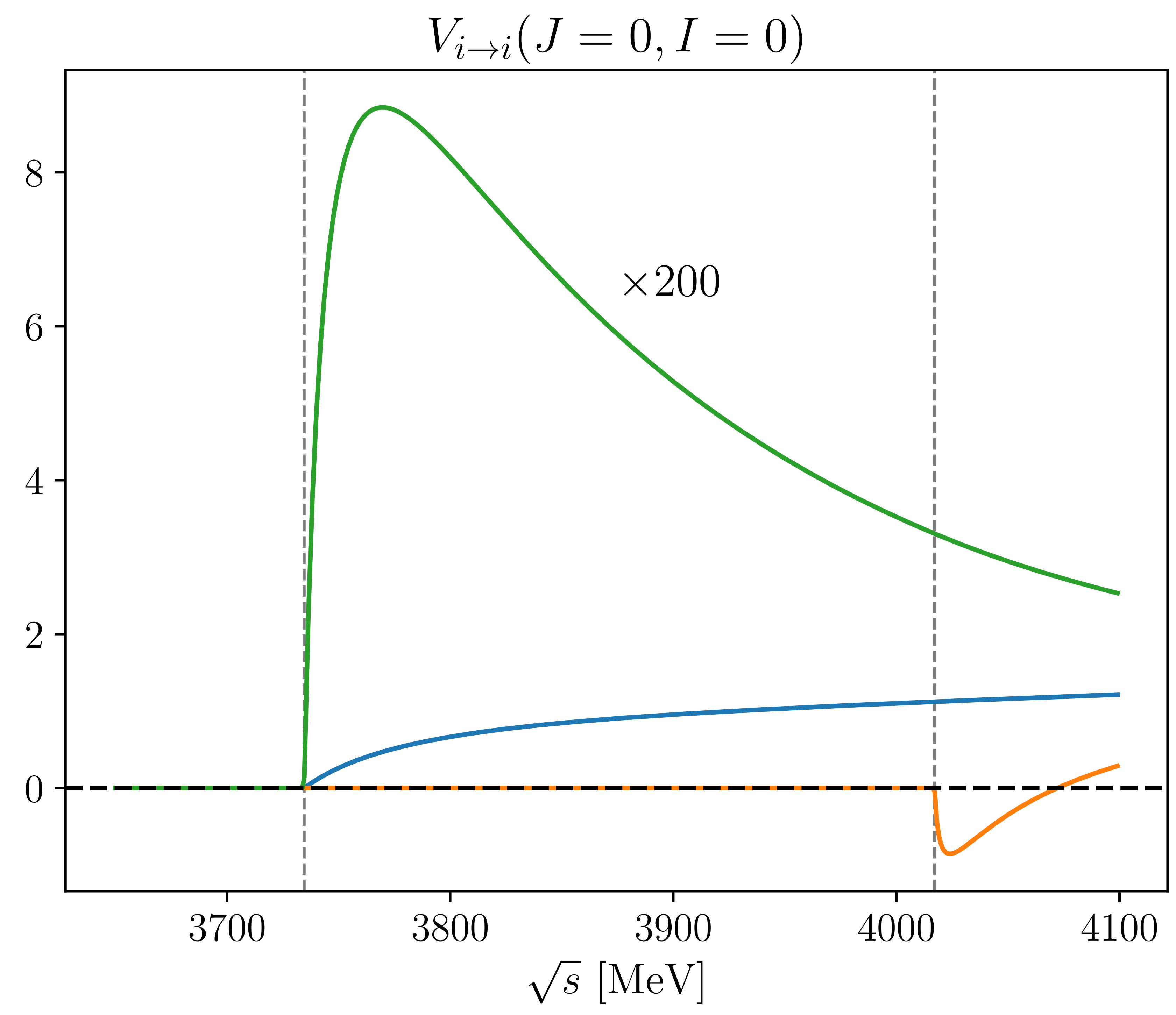


The attractive interactions appears in diagonal elements, while the $D\bar{D}^* \to D^*\bar{D}^*$ transition are absent in both scalar and vector channels.

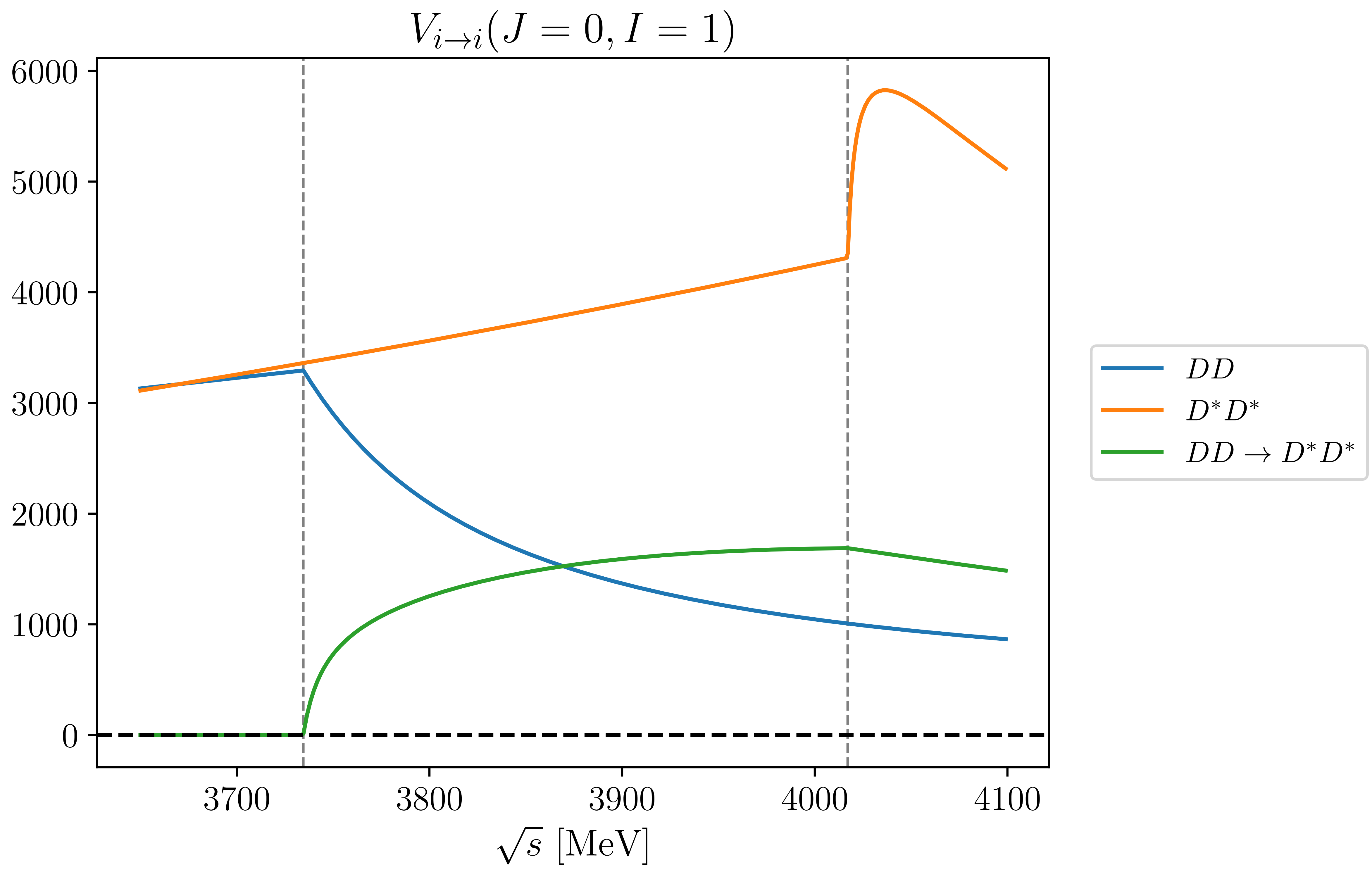
Dynamical generation of the poles

Kernel amplitudes

Scalar channels (J=0)



destructive interference between kernel amplitudes in the diagonal elements due to the IS factors.

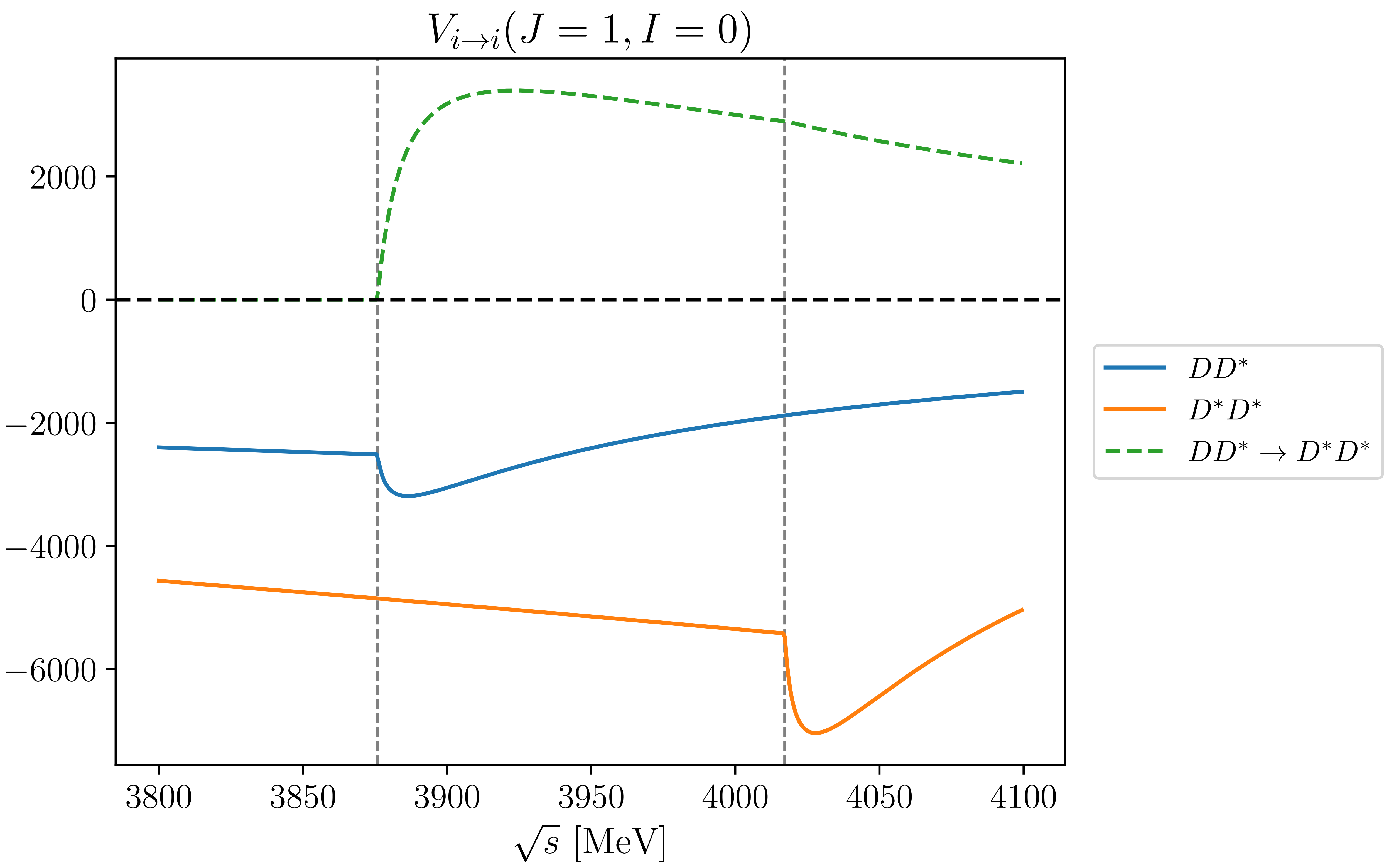


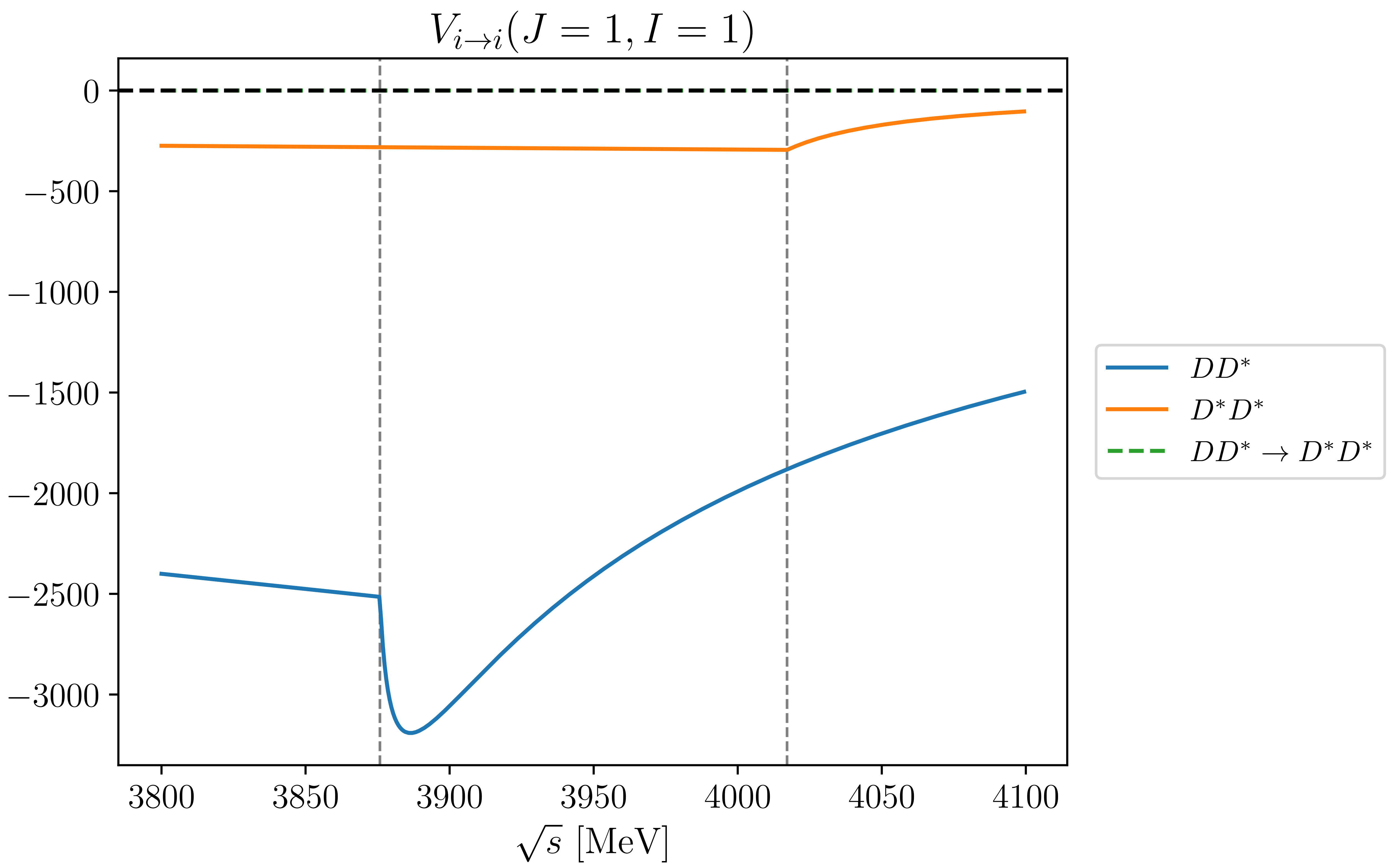
all interactions are strongly repulsive.

Dynamical generation of the poles

Kernel amplitudes

Vector channels (J=1)





Strong attractions for both diagonal elements

→ Pole is expected to generate in the coupled-channel solution

Strong attraction for $DD^* \to DD^*$

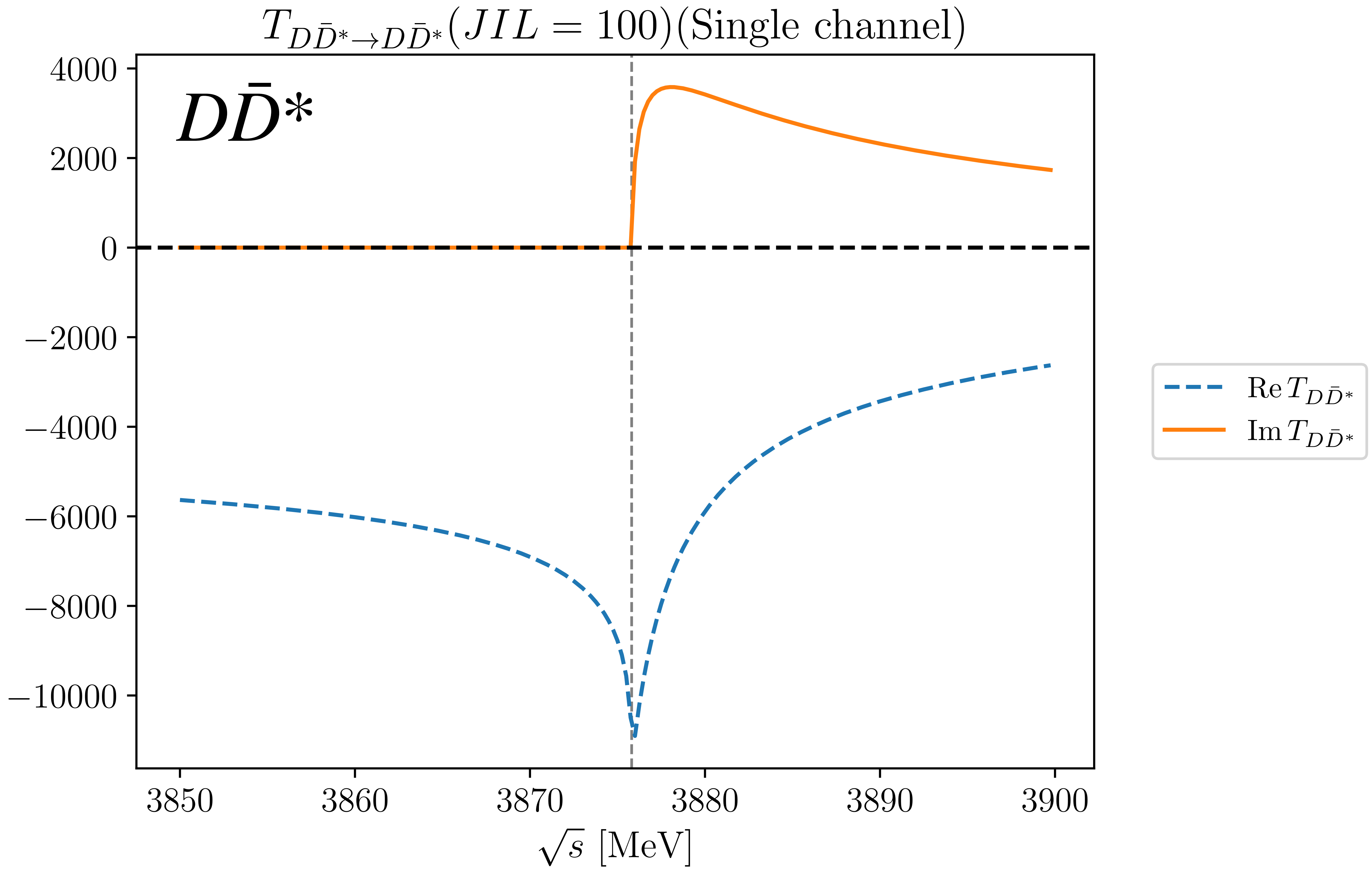
No transition appears in the $DD^* \to D^*D^*$ process.

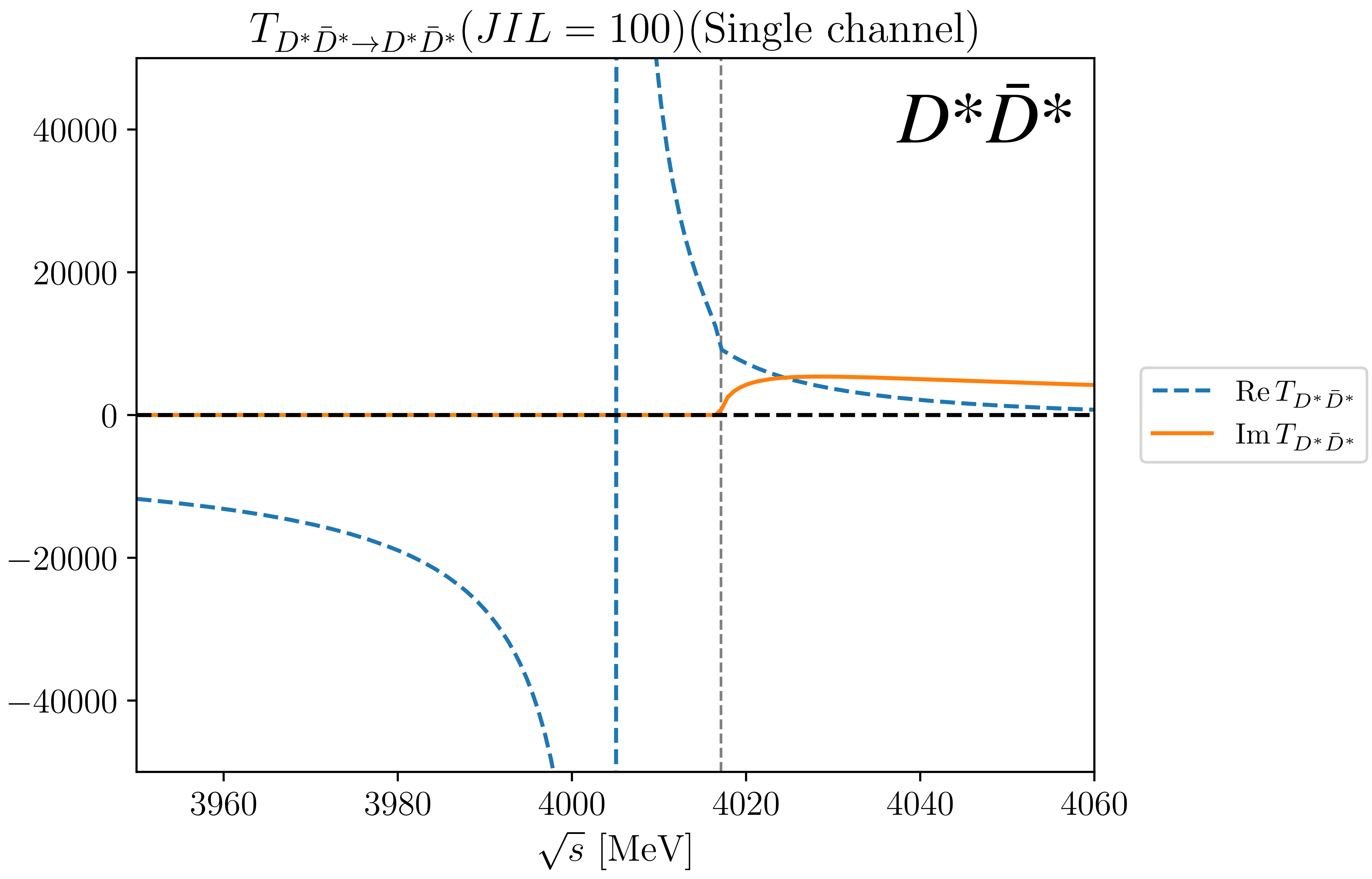
Dynamical generation of the poles

Single channel T matrix elements

ex) $T_{11} = (1 - V_{11}G_1)^{-1}V_{11}$

vector-isoscalar(J=1, I=0) channel



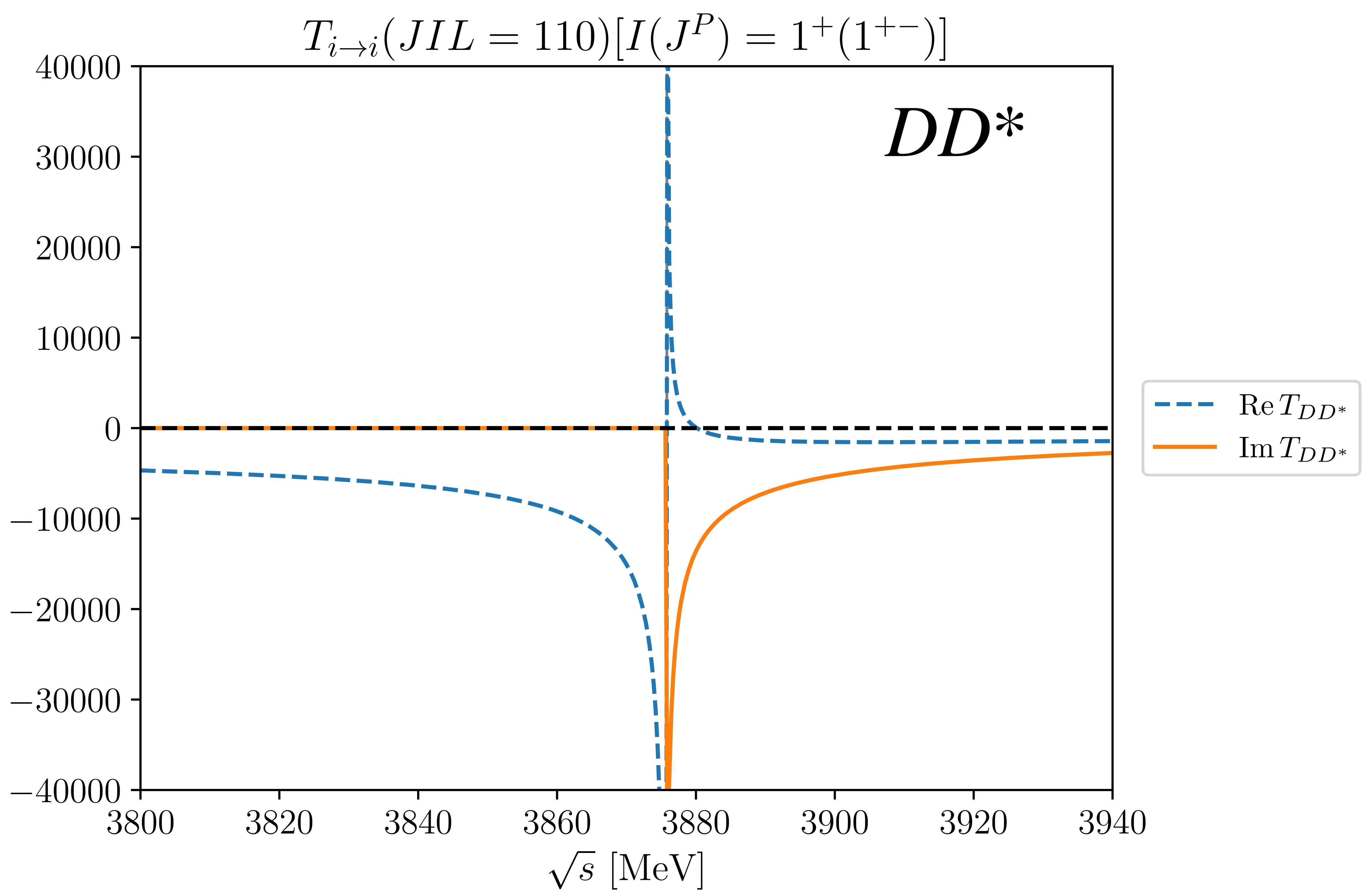


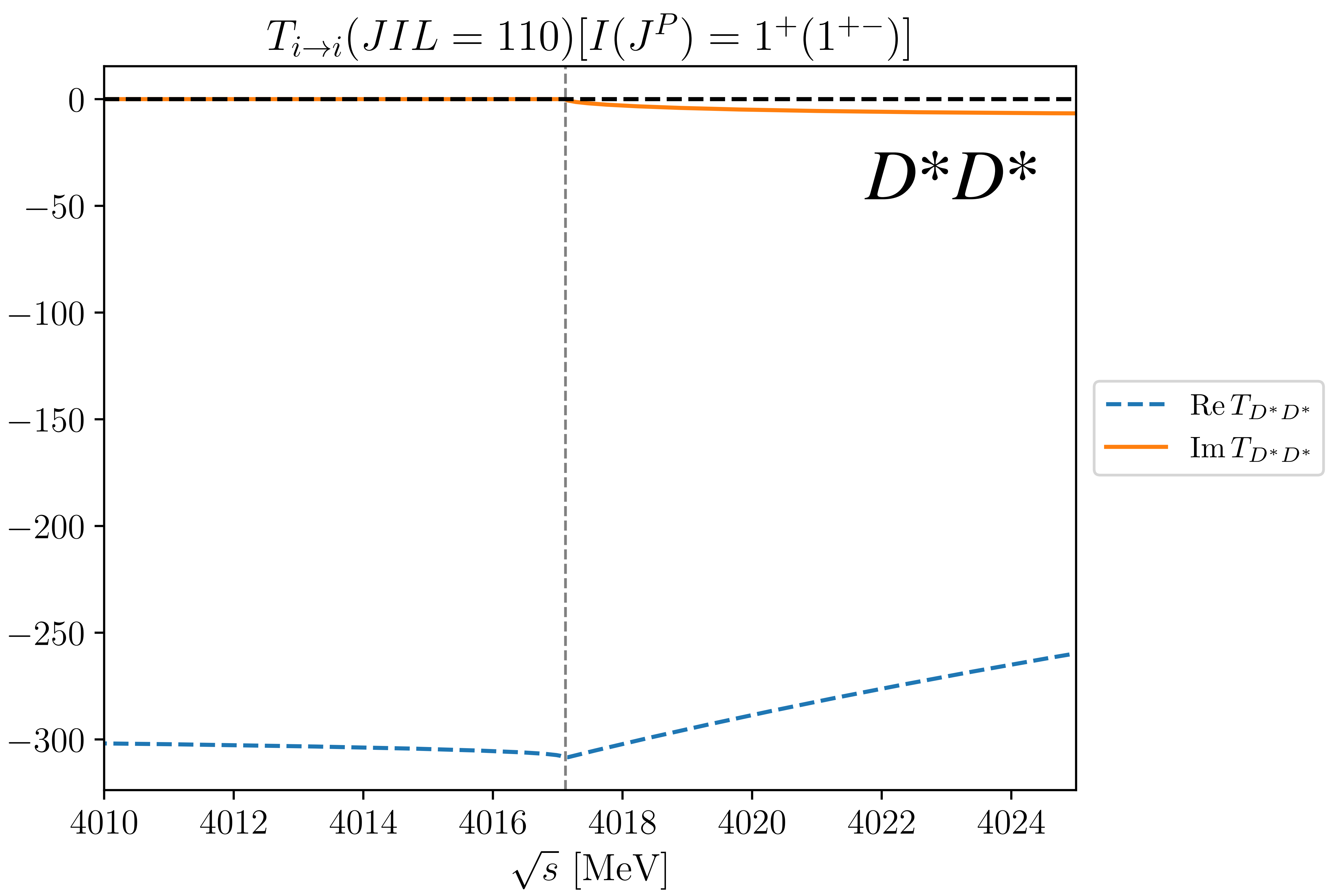
generated solely by $D\bar{D}$ and $D^*\bar{D}^*$ interactions, respectively

Dynamical generation of the poles

Single channel T matrix elements

Vector-isovector channel (J=1)





The pole is about to emerge at DD^* mass threshold.

Decay width and channel coupling strengths

- $D_{s0}^{*} \to D_s \pi^0$ **decay width**

Partial decay width of a resonance R to a channel a : $\Gamma_{R\to a} = |g_a|^2 \frac{p_{\rm cm}}{4\pi M_R^2}$

Strong decay mode of the $D_{s0}^{*}(2317)$

$$\Gamma_{D_{s0}^{*}\to D_s^+\pi^0} = \frac{g_1^2}{m_{D_{s0}^{*}}} \rho_{D_s^+\pi^0}(m_{D_{s0}^{*}}^2) = \frac{g_1^2}{m_{D_{s0}^{*}}^2} \frac{p_{\rm cm}}{4\pi} = 13.86\,\text{keV} < 3.8\,\text{MeV}$$

- **Residue of the transition amplitude**

$$\mathcal{R}_{ab} = \lim_{s\to s_R} (s_R - s) T_{ab}/(4\pi)$$

One can introduce channel couplings which characterize the coupling strength of the resonance with a certain channel a: $g_a = \sqrt{\mathcal{R}_{aa}}$

$$g_1 = |g_{D_s^+\pi^0}| = 5.381 \times 10^{-2}\,\text{GeV},$$
$$g_2 = |g_{D^0K^+}| = 77.59\,\text{GeV},$$
$$g_3 = |g_{D^+K^0}| = 80.17\,\text{GeV},$$
$$g_4 = |g_{D_s^+\eta}| = 85.25\,\text{GeV}.$$

Channel coupling strengths

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J^{PC}	0^{++}		2^{++}
$\sqrt{s_R}$	3720.535	$3861.34 - i22.76$	$4005.264 - i5.950$
$g_{D\bar{D}}$	8.140	$1.747 + i5.350$	$2.487 + i0.650$
$g_{\omega J/\psi}$	0.155	$0.472 + i0.184$	$8.25 \times 10^{-3} - i3.34 \times 10^{-3}$
$g_{D_s\bar{D}_s}$	4.304	$6.70 \times 10^{-2} - i3.818$	$0.775 + i8.13 \times 10^{-2}$
$g_{D^*\bar{D}^*}$	1.328	$27.83 + i0.860$	$1.76 \times 10^{-2} + i1.26 \times 10^{-2}$
$g_{\phi J/\psi}$	0.100	$0.327 + i9.90 \times 10^{-2}$	$2.39 \times 10^{-4} + i4.91 \times 10^{-5}$
$g_{D_s^*\bar{D}_s^*}$	1.182	$16.44 - i0.514$	$0.172 + i4.58 \times 10^{-2}$

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$1^{++}\sqrt{s_R}$	$3848.111 - i0.0637$	$3948.622 - i26.98$
$g_{\eta_c \omega}$	$0.376 + i2.653 \times 10^{-4}$	$6.538 \times 10^{-3} + i0.275$
$g_{D\bar{D}^*}$	$15.323 + i1.934 \times 10^{-3}$	$1.561 + i0.867$
$g_{\eta_c \phi}$	$0.128 + i4.821 \times 10^{-5}$	$0.273 - i0.223$
$g_{D^*\bar{D}^*}$	$10.132 + i9.984 \times 10^{-4}$	$26.850 + i5.962$
$g_{D_s\bar{D}_s^*}$	$6.955 + i1.110 \times 10^{-3}$	$12.122 - i7.193$
$g_{D_s^*\bar{D}_s^*}$	$1.964 + i3.921 \times 10^{-3}$	$11.411 - i5.695$

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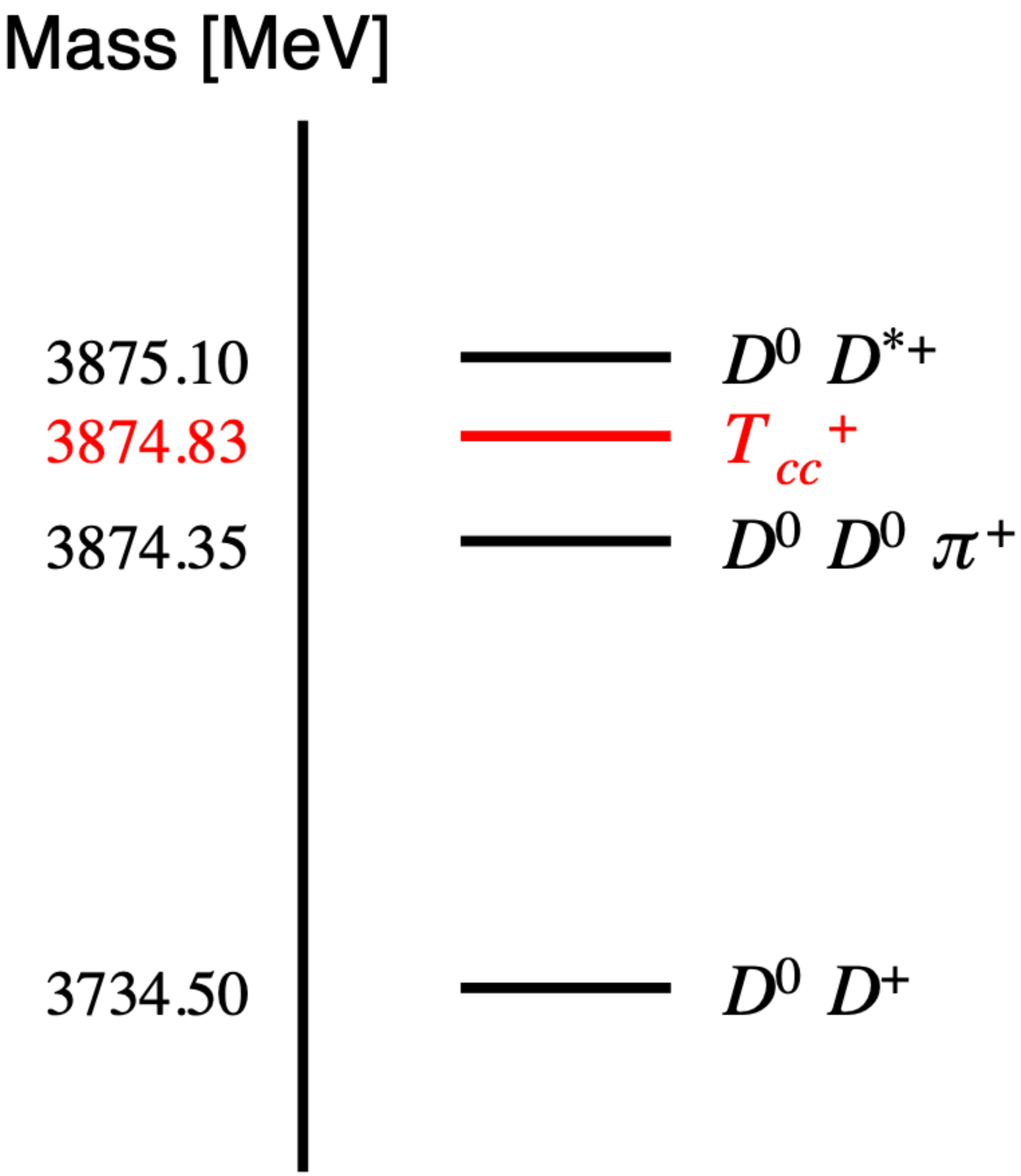
$\sqrt{s_R}$	$3817.862[0^+(1^{++})]$	$3875.720[1^+(1^{+-})]$
g_{DD^*}	18.36	2.242
$g_{D^*D^*}$	25.48	2.97×10^{-4}


Mass [MeV]
3875.10
$D^0\ D^{*+}$
3874.83
$T_{cc}{}^+$
3874.35
$D^0\ D^0\ \pi^+$
3734.50
$D^0\ D^+$