The equation of state of the neutron stars with the d*(2380) degree of freedom in a hadronic molecular picture

Daris Samart
Khon Kaen University

In collaboration with
Tanech Klangburam and
Prin Sawasdipol

Outline

- Introduction to d*(2380)
- Molecular state of the d*(2380) and compositeness condition
- d*(2380) in neutron stars
- Relativistic Mean Field (RMF) with d*(2380) in molecular picture
- Results
- Conclusions and comments

Introduction

- Dibaryon has been long history in hadron spectroscopy.

- Dibaryon was predicted by Dyson and Xuong in 1964.

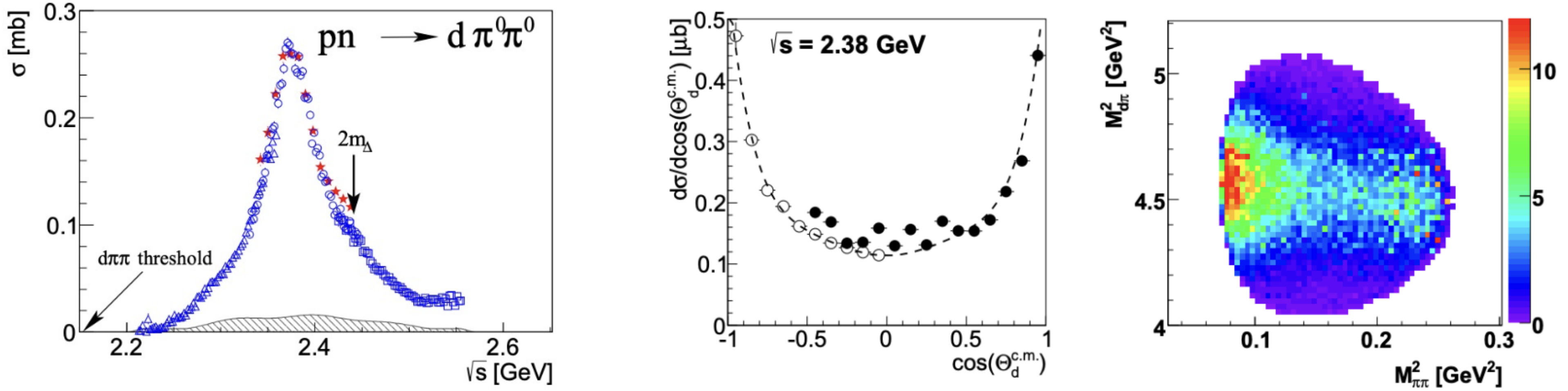
notation	I	J	asymptotic baryon-baryon configuration	mass (formula)	mass (value) (MeV)
D_{01}	0	1	deuteron	A	1876
D_{10}	1	0	$^1S_0$ NN virtual state	A	1876
D_{12}	1	2	ΔN	A + 6B	2160
D_{21}	2	1	ΔN	A + 6B	2160
D_{03}	0	3	$\Delta\Delta$	A + 10B	2350
D_{30}	3	0	$\Delta\Delta$	A + 10B	2350

Clement (2016)

- The deuteron is the first example of the dibaryon.

The d*(2380): very brief introduction

- The discovery of the six-quark state called d*(2380) is confirmed in the WASA@COSY laboratory: Adlarson et al.(2011).

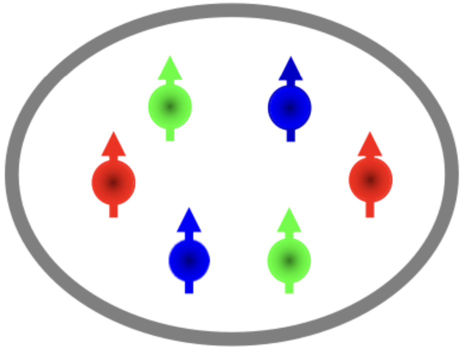


- Mass of d*(2389) ≈ 2380 MeV with binding energy ≈ 84 MeV.
- Total width ≈ 70 MeV: $\Gamma(d^* \to \Delta\Delta) \approx 64$ MeV, $\Gamma(d^* \to \mathrm{pn}) \approx 6$ MeV.
- The quantum number $(I)\,J^P = (0)\,3^+$.

Candidates of the d*(2380)

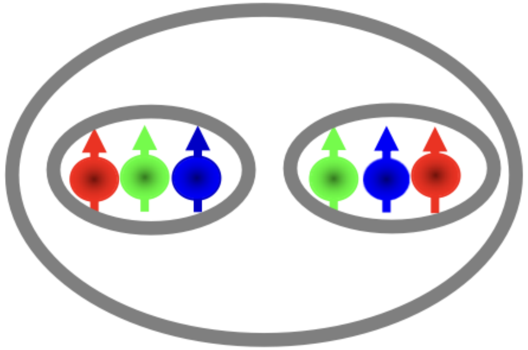
- Compact hexa-quark:

  Quark models by Thomas, Oka, Yamazaki, Goldman, Wang, Yuan, Zhang, Yu, Shen, Dong and others



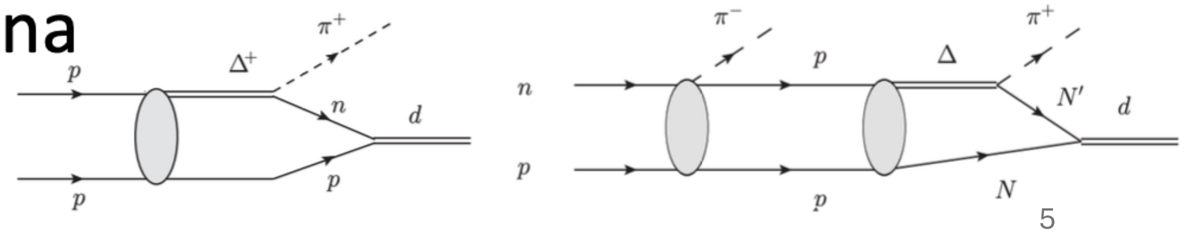
- Molecular state:

  Huang, Ping, Wang, Dong, Shen and others



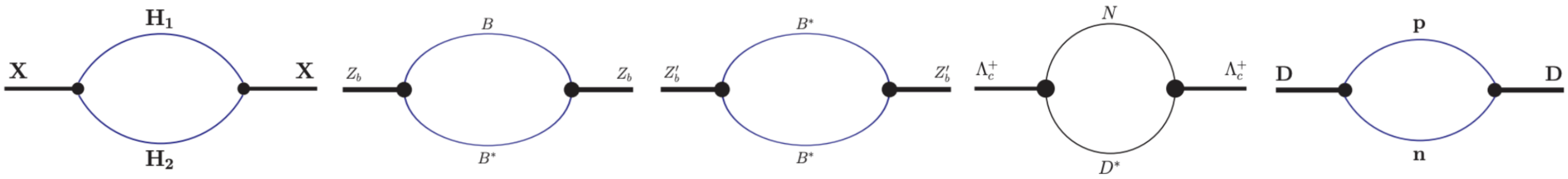
- Kinematic triangle singularity effect:

  Bar-nir, et.al and Oset, Ikeno and Molina



Phenomelogical Lagrangian approach to molecular states of exotic hadrons

- The (non-local) phenomenological Lagrangian is proposed by the Tubingen-Beijing group to study the molecular-like exotic hadron states, e.g., X(3872), Z_b(10610), Z_b(10650), Λ_c(2940) and deuteron: see review: Dong, Faessler, Lyubovitskij (2017).



- We assume that the d*(2380) is a hadronic molecular state of ΔΔ pair.

Phenomenological Lagrangian of the d*ΔΔ

- The phenomenological Lagrangian of the d*(2380) as ΔΔ molecular state is given by Dong and Shen (2022)

$$\mathcal{L}_{d^*\Delta\Delta}(x) = g_{d^*\Delta\Delta} \int d^4y\Phi\left(y^2\right) \bar{\Delta}_\alpha(x+y/2)\Gamma^{\alpha,(\mu_1\mu_2\mu_3),\beta} \Delta^C_\beta(x-y/2) d^*_{\mu_1\mu_2\mu_3}(x;\lambda) + \text{h.c.}$$

$$\Gamma^{\alpha,(\mu_1\mu_2\mu_3),\beta} = \frac{1}{6}\left[\gamma^{\mu_1}\left(g^{\mu_2\alpha}g^{\mu_3\beta} + g^{\mu_2\beta}g^{\mu_3\alpha}\right) + \gamma^{\mu_2}\left(g^{\mu_3\alpha}g^{\mu_1\beta} + g^{\mu_1\beta}g^{\mu_3\alpha}\right) + \gamma^{\mu_3}\left(g^{\mu_1\alpha}g^{\mu_2\beta} + g^{\mu_1\beta}g^{\mu_2\alpha}\right)\right]$$

- The correlation function $\Phi(y^2)$ describes the finite length of the d$*$ size as a bound state of ΔΔ baryons depending on the Jacobi coordinate y

$$\Phi\left(y^2\right) = \int \frac{d^4p}{(2\pi)^4} e^{-ipy}\tilde{\Phi}\left(-p^2\right), \qquad \tilde{\Phi}\left(-p^2\right) = \exp\left(p^2/\Lambda^2\right)$$

The Weinberg-compositeness condition

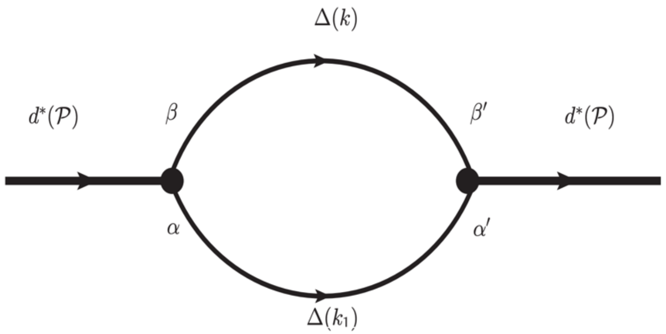
- The deuteron has been proved theoretically by Weinberg that it is bound state.
- The renormalization constant, Z can also be interpreted as the matrix element between the physical and the corresponding bare state.
- For Z= 0, the physical state does not contain the bare one and is solely described as a bound state.
- The Weinberg compositeness condition (Z=0 for bound state) use to evaluate the coupling constant.

Weinberg compositeness condition

$$Z = 1 - \left.\frac{d\,\Pi_{(\perp)}\left(q^2\right)}{d\,q^2}\right|_{q^2=m_{d*}^2}$$

Estimation of the d*ΔΔ coupling constant

- The mass operator can be computed via the diagram



$$
\begin{aligned}
\Sigma_{\Delta}^{(\mu_1'\mu_2'\mu_3'),(\mu_1\mu_2\mu_3)}(\mathcal{P}) \quad = \quad & |g_{d^*\Delta\Delta}(\Lambda)|^2 \int \frac{d^4k}{(2\pi)^4 i} \exp\left(-\frac{2\left(k-\frac{\mathcal{P}}{2}\right)_E^2}{\Lambda^2}\right) \\
\times \quad & \mathrm{Tr}\Bigg[\Gamma_{\alpha'}^{(\mu_1'\mu_2'\mu_3')}{}_{\beta'} \frac{\not{k}+M_\Delta}{k^2-M_\Delta^2}\left(-g^{\beta\beta'}+\frac{\gamma^\beta\gamma^{\beta'}}{3}+\frac{2k^\beta k^{\beta'}}{3M_\Delta^2}+\frac{\gamma_\beta k_{\beta'}-\gamma^{\beta'}k^\beta}{3M_\Delta}\right) \\
& \times \Gamma_{\beta}^{(\mu_1\mu_2\mu_3)}{}_{\alpha} \frac{\not{k}_1-M_\Delta}{k_1^2-M_\Delta^2}\left(-g^{\alpha'\alpha}+\frac{\gamma^{\alpha'}\gamma^\alpha}{3}+\frac{2k_1^{\alpha'}k_1^\alpha}{3M_\Delta^2}-\frac{\gamma^{\alpha'}k_1^\alpha-\gamma^\alpha k_1^\alpha}{3M_\Delta}\right)\Bigg]_{k_1=\mathcal{P}-k}
\end{aligned}
$$

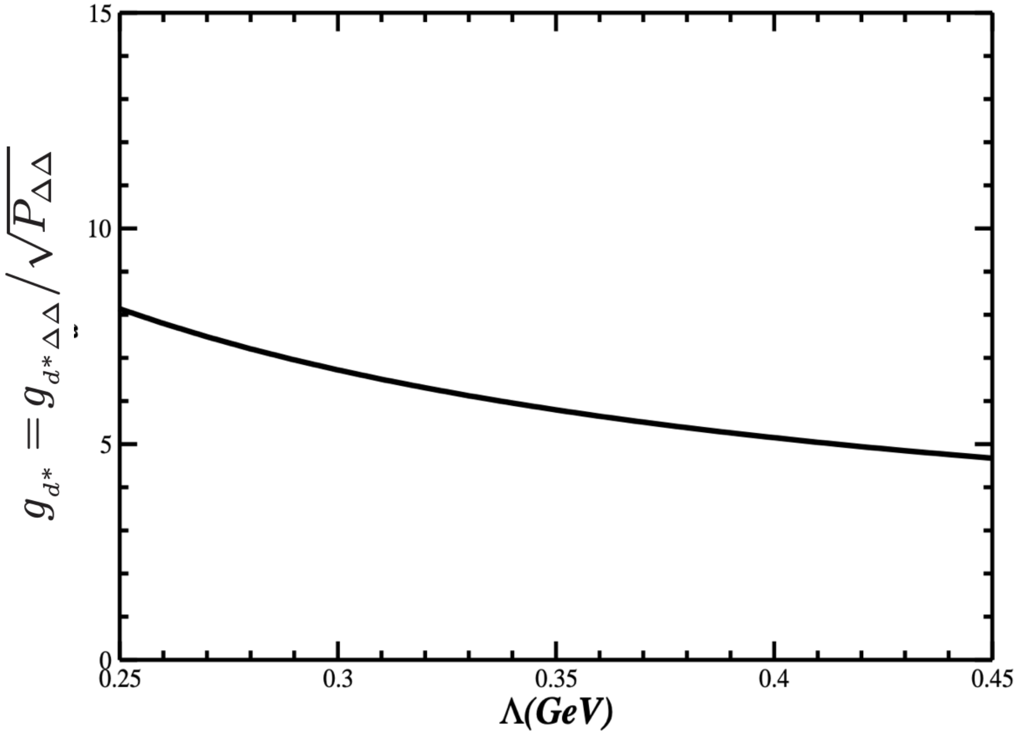
Molecular component in d*(2380)

- It is well known that the d*(2380) has two components.

$$|d^* > \sim \sqrt{\frac{1}{3}}|\Delta\Delta> + \sqrt{\frac{2}{3}}|CC>$$

- Then, the Weinberg condition is

$$Z_{d^*,(\Delta\Delta)} = \frac{1}{3} - \frac{\partial \Sigma^{(1)}_{(\Delta\Delta)}(\mathcal{P}^2)}{\partial \mathcal{P}^2}\Big|_{\mathcal{P}^2 = M^2_{d^*}} = 0$$



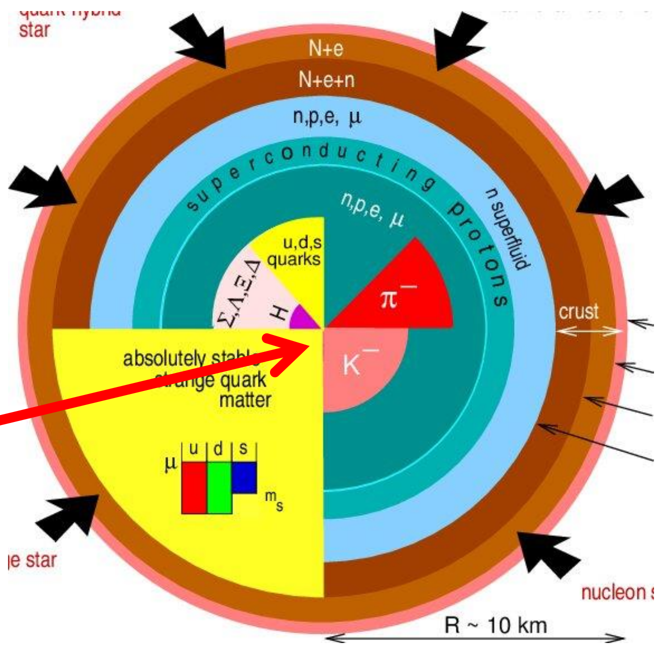
- We use $\Lambda^2 \sim 2/b^2$ for b~ 0.8 fm

- We find $g_{d^*\Delta\Delta} \sim 3.35$

Neutron stars with d*(2380) d.o.f.

- A neutron star is a compact object in astrophysics that provides many high-energy physics phenomena under extreme conditions.
- The core of neutron stars has very high density, a new d.o.f. is expected to appear in addition to nucleons, such as pion and kaon condensates, Δ isobars, deconfined quarks, and even di-baryonic matter.

d*(2380)???



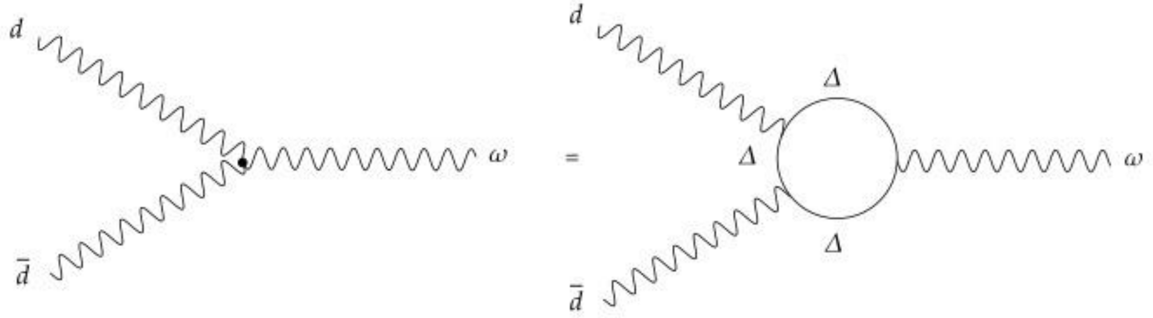
Relativistic Mean Field theory in neutron stars

- RMF theory is a framework used to describe the properties of nuclear matter and dense astrophysical objects like neutron stars. It is based on the principles of quantum field theory, where nucleons interact relativistically through the exchange of mesons.

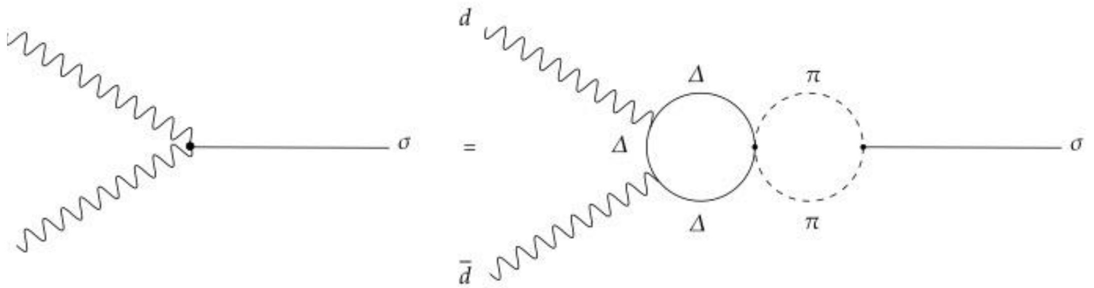
- The York group (Vidana et al. 2018 and Mantziris et al. 2020) has studied the d*(2380) in neutron stars using RMF and the results are compatible with the astrophysical constraints.

RMF with d*(2380) in molecular picture

- However, they consider the d*(2380) as scalar field , and the coupling constants of the d*(2380) with ω and σ mesons are not determine.
- We will compute the coupling constants of the d*(2380) with ω and σ mesons by mean of d*(2380) as the ΔΔ bound state.



$g_{\omega d^*} \propto g^2_{d^*\Delta\Delta} g_{\omega\Delta}$



$g_{\sigma d^*} \propto g^2_{d^*\Delta\Delta} g_{\sigma\pi\pi}/f_\pi$

RMF Langrangians (york group)

$$
\begin{aligned}
\mathcal{L}_N &= \bar{N}\left[i\gamma_\mu\partial^\mu - m_N + g_{\sigma N}\sigma - g_{\omega N}\gamma_\mu\omega^\mu - g_{\rho N}\gamma_\mu\frac{\vec{\tau}_N\cdot\vec{\rho}^{\,\mu}}{2}\right]N \\
\mathcal{L}_\Delta &= \bar{\Delta}_\nu\left[i\gamma_\mu\partial^\mu - m_\Delta + g_{\sigma\Delta}\sigma - g_{\omega\Delta}\gamma_\mu\omega^\mu - g_{\rho\Delta}\gamma_\mu\vec{I}_\Delta\cdot\vec{\rho}^{\,\mu}\right]\Delta^\nu \\
\mathcal{L}_l &= \bar{\Psi}_l\left[i\gamma_\mu\partial^\mu - m_l\right]\Psi_l \\
\mathcal{L}_M &= \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}bm_N g_{\sigma N}^3\sigma^3 - \frac{1}{4}cg_{\sigma N}^4\sigma^4 \\
&- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu \\
\mathcal{L}_{d^*} &= \left(\partial_\mu - ig_{\omega d^*}\omega_\mu\right)\phi_{d^*}^*\left(\partial^\mu + ig_{\omega d^*}\omega^\mu\right)\phi_{d^*} - \left(m_{d^*} - g_{\sigma d^*}\sigma\right)^2\phi_{d^*}^*\phi_{d^*}
\end{aligned}
$$

RMF Langrangians (present work)

$$\mathcal{L}_N = \bar{N}\left[i\gamma_\mu\partial^\mu - m_N + g_{\sigma N}\sigma - g_{\omega N}\gamma_\mu\omega^\mu - g_{\rho N}\gamma_\mu\frac{\vec{\tau}_N\cdot\vec{\rho}^\mu}{2}\right]N$$

$$\mathcal{L}_\Delta = \bar{\Delta}_\nu\left[i\gamma_\mu\partial^\mu - m_\Delta + g_{\sigma\Delta}\sigma - g_{\omega\Delta}\gamma_\mu\omega^\mu - g_{\rho\Delta}\gamma_\mu\vec{I}_\Delta\cdot\vec{\rho}^\mu\right]\Delta^\nu$$

$$\mathcal{L}_l = \bar{\Psi}_l\left[i\gamma_\mu\partial^\mu - m_l\right]\Psi_l$$

$$\mathcal{L}_M = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}bm_N g_{\sigma N}^3\sigma^3 - \frac{1}{4}cg_{\sigma N}^4\sigma^4 - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu$$

$$\mathcal{L}_{d^*} = \left(\partial_\mu - ig_{\omega d^*}\omega_\mu\right)\bar{d}_{\alpha\beta\gamma}\left(\partial^\mu + ig_{\omega d^*}\omega^\mu\right)d^{\alpha\beta\gamma} - \left(m_{d^*} - g_{\sigma d^*}\sigma\right)^2\bar{d}_{\alpha\beta\gamma}d^{\alpha\beta\gamma}$$

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RMF equations

$$
\begin{aligned}
m_\sigma^2\bar{\sigma} &= \sum_{B=N,\Delta} g_{\sigma B}\left(\frac{2J_B+1}{2\pi^2}\int_0^{k_{F_B}}\frac{m_B^*}{\sqrt{k^2+m_B^{*2}}}k^2\,\mathrm{d}k\right) - bm_N g_{\sigma N}\left(g_{\sigma N}\bar{\sigma}\right)^2 - cg_{\sigma N}\left(g_{\sigma N}\bar{\sigma}\right)^3 + g_{\sigma d^*}\rho_{d^*} \\
m_\omega^2\bar{\omega}_0 &= \sum_{B=N,\Delta} g_{\omega B}\frac{2J_B+1}{6\pi^2}k_{F_B}^3 - g_{\omega d^*}\cdot\rho_{d^*} \\
m_\rho^2\bar{\rho}_0^{(3)} &= \sum_{B=N,\Delta} g_{\rho B}\frac{2J_B+1}{6\pi^2}k_{F_B}^3 I_{3B}
\end{aligned}
$$

$$
\begin{aligned}
\varepsilon &= \sum_{B=N,\Delta}\frac{(2J_B+1)}{2\pi^2}\int_0^{k_{F_B}}\sqrt{k^2+m_B^{*2}}k^2\,\mathrm{d}k + \sum_{l=e^-,\mu^-}\frac{1}{\pi^2}\int_0^{k_F^l}\sqrt{k^2+m_l^2}k^2\,\mathrm{d}k + \frac{1}{2}m_\sigma^2\bar{\sigma}^2 \\
&+ \frac{1}{3}bm_N\left(g_\sigma\bar{\sigma}\right)^3 + \frac{1}{4}c\left(g_\sigma\bar{\sigma}\right)^4 + \frac{1}{2}m_\omega^2\bar{\omega}_0^2 + \frac{1}{2}m_\rho^2\left(\bar{\rho}_0^{(3)}\right)^2 + m_{d^*}^*\rho_{d^*} \\
P &= \sum_{B=N,\Delta}\frac{(2J_B+1)}{6\pi^2}\int_0^{k_{F_B}}\frac{k^4\,\mathrm{d}k}{\sqrt{k^2+m_B^{*2}}} + \sum_{l=e^-,\mu^-}\frac{1}{3\pi^2}\int_0^{k_F^l}\frac{k^4\,\mathrm{d}k}{\sqrt{k^2+m_l^2}} \\
&- \frac{1}{2}m_\sigma^2\bar{\sigma}^2 - \frac{1}{3}bm_N\left(g_\sigma\bar{\sigma}\right)^3 - \frac{1}{4}c\left(g_\sigma\bar{\sigma}\right)^4 + \frac{1}{2}m_\omega^2\bar{\omega}_0^2 + \frac{1}{2}m_\rho^2\left(\bar{\rho}_0^{(3)}\right)^2
\end{aligned}
$$

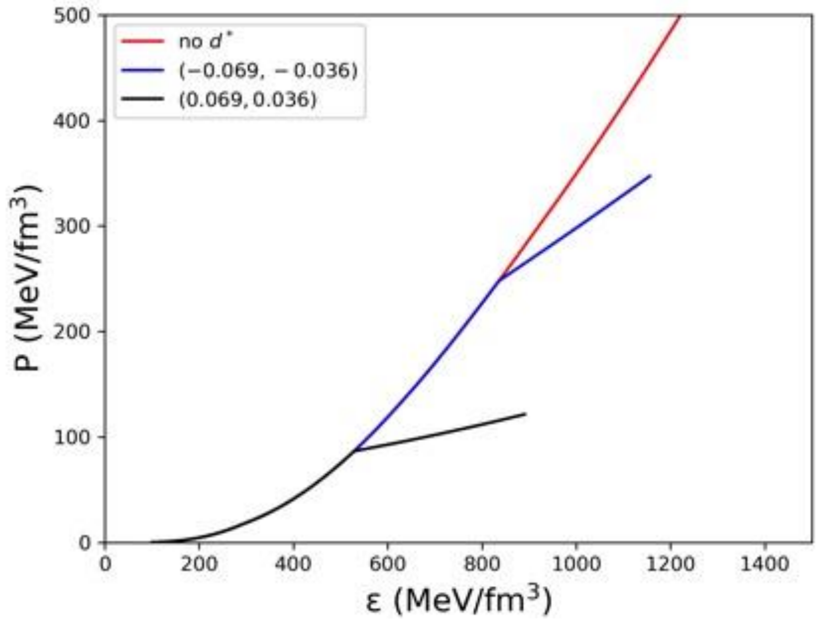
→ $P = P(\varepsilon)$

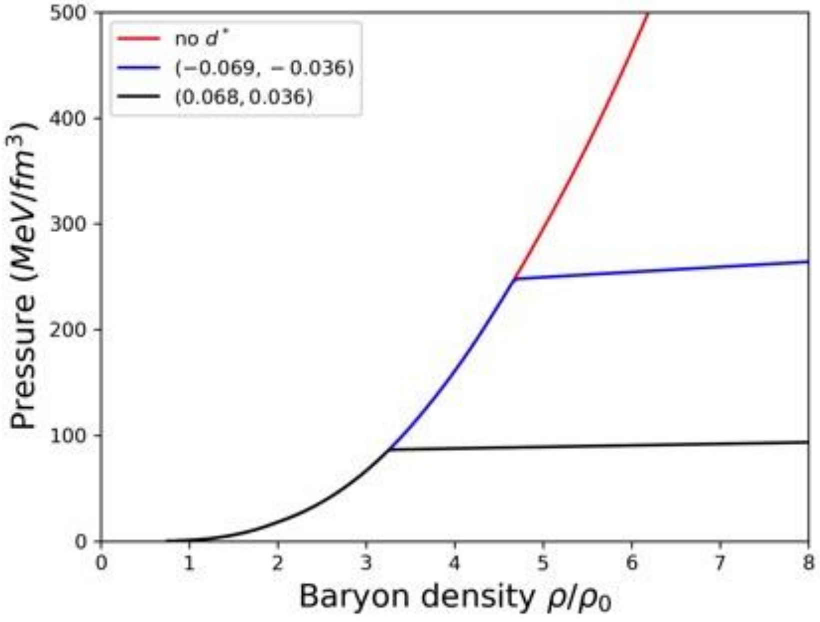
Equation of State (EoS)

$$
\begin{aligned}
\mu_B &= \sqrt{k_{F_B}^2+m_B^{*2}} + g_{\omega B}\bar{\omega}_0 + g_{\rho B}I_{3B}\bar{\rho}_0^{(3)}, \quad \left(B=n,p,\Delta^-,\Delta^0,\Delta^+,\Delta^{++}\right), \\
\mu_{d^*} &= m_{d^*} - g_{\sigma d^*}\bar{\sigma} - g_{\omega d^*}\bar{\omega}_0, \\
\mu_l &= \sqrt{k_{F_l}^2+m_l^2}, \quad \left(l=e^-,\mu^-\right)
\end{aligned}
$$

Results (preliminary)

We define $x_{\omega d^*} \equiv g_{\omega d^*}/g_{\omega N}$ and $x_{\sigma d^*} \equiv g_{\sigma d^*}/g_{\sigma N}$

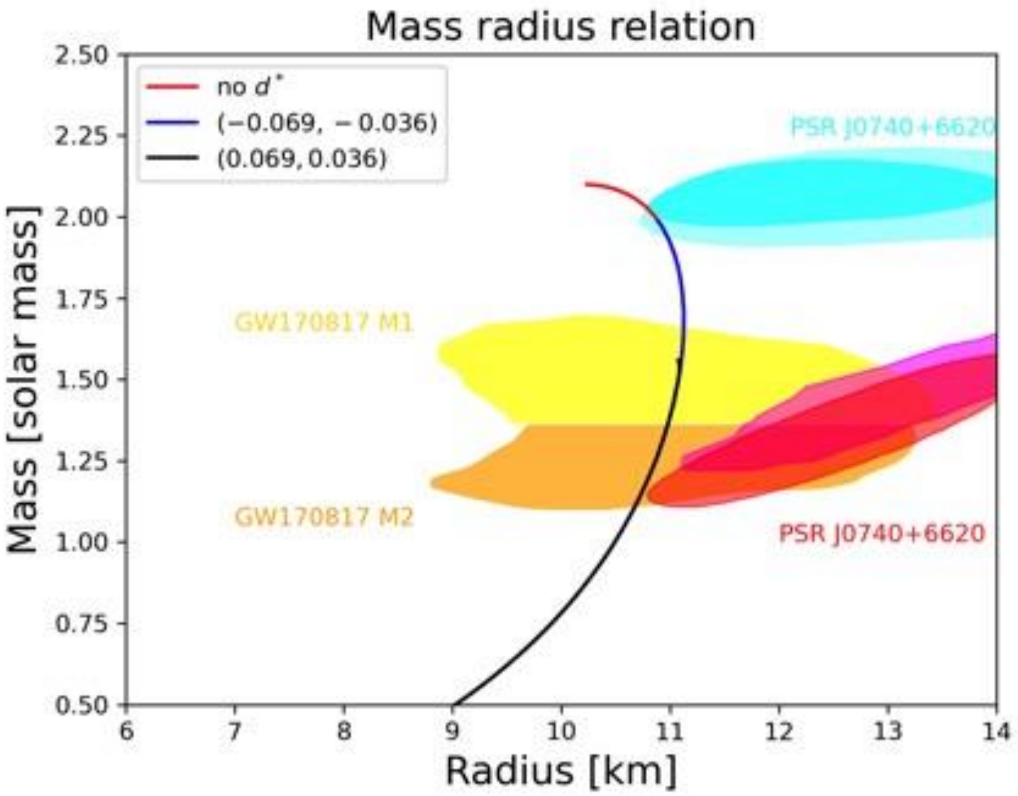




$$(x_{\omega d^*}, x_{\sigma d^*}) = (\pm 0.069, \pm 0.036)$$

TOV equation and Mass-radius relation

$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r) \qquad \frac{dP(r)}{dr} = -\frac{[\varepsilon(r) + P(r)][m(r) + 4\pi r^3 P(r)]}{r\big(r - 2Gm(r)\big)}$$



All relevant parameters are obtained from Glendenning–Moszkowski model (1991)

Conclusions

- We compute the $g_{\omega d^*} = 5.16$ and $g_{\sigma d^*} = 2.90$ in molecular picture of the d*(2380) which has been estimated yet in the literature.

- The effects of d*(2380) soften the EoS.

- Our results are compatible with astrophysical constraints.

Comments

- Our results are not compatible with the PSR J0740+6620.

- No d*(2380) in the neutron star pressure.

- Include the d*(2380) interaction term to nucleon might be needed.

Thank you for your attention