

New insights into the OZI suppression and the $X_{0,1}(4140)$ and $X_1(4685)$ as hadronic molecules

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1 Motivation

2 $X_{0,1}(4140)$ in $J/\psi\phi$ scattering

- $D_s\bar{D}_s - J/\psi\phi - D_s^*\bar{D}_s^*$ scattering in $J^{PC} = 0^{++}$ sector
- $D_s\bar{D}_s^* - J/\psi\phi$ scattering in $J^{PC} = 1^{++}$ sector

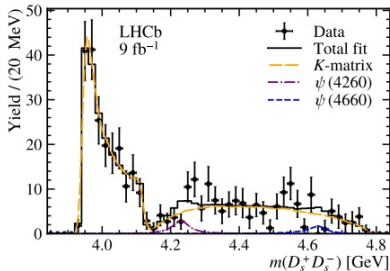
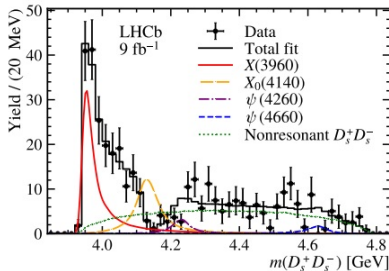
3 $X_1(4685)$ in $J/\psi\phi - \psi(2S)\phi$ scattering

4 Summary

5 Backup

Motivation

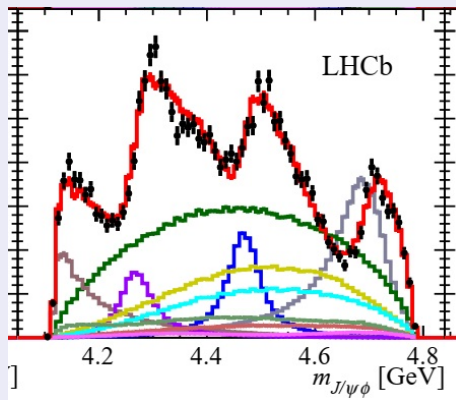
Observation and explanations on $X_0(4140)$ with $J^{PC} = 0^{++}$



- $X_0(4140)$ is a BW (4.1σ) or dip (3.7σ)
- BW: $(M, \Gamma) = 4133 \pm 6 \pm 6, 67 \pm 17 \pm 7 \text{ MeV}$
- K-matrix: $M_R = 3957 \pm 14 \text{ MeV}$ ($X(3960)$) , [LHCb, PRL.131,2210.15153](#)

Motivation

Observation and explanation on $X_1(4140)$ with $J^{PC} = 1^{++}$



- BW: $X_1(4140)$ (13σ), $(M, \Gamma) = 4118 \pm 11^{+19}_{-36}, 162 \pm 27^{+24}_{-49} \text{ MeV}$,

LHCb, PRL127, 2103.01803

$D_s \bar{D}_s - J/\psi \phi - D_s^* \bar{D}_s^*$ scattering in $J^{PC} = 0^{++}$ sector

EFT and RGE

- $\{\Lambda_{QCD}, m_\nu\}$ and $\frac{d}{d\Lambda} \langle \Psi | C(\Lambda) | \Psi \rangle = 0$, $\Lambda_0 = m_\phi$ is adapted.

$D_s^{(*)} \bar{D}_s^{(*)}$ scattering

- HQSS, $H = \frac{1}{\sqrt{2}} [D_s + \vec{\sigma} \cdot \vec{D}_s^*]$, $\mathcal{C}|D_s\rangle = |\bar{D}_s\rangle$ and $\mathcal{C}|D_s^*\rangle = |\bar{D}_s^*\rangle$
- $\mathcal{L}_c = C_0 \text{Tr} [H^\dagger H] \text{Tr} [\bar{H}^\dagger \bar{H}] + C_1 \text{Tr} [H^\dagger \sigma_k H] \text{Tr} [\bar{H}^\dagger \sigma_k \bar{H}]$
- $|\vec{q}| = \sqrt{m_{D_s^*} \Delta} \simeq 478 \text{ MeV}$ with $\Delta = 2m_{D_s^*} - m_{J/\psi} - m_\phi$, $\frac{\vec{q}^2}{m_\phi^2} \simeq 0.22$,
 $V_{D_s \bar{D}_s, D_s \bar{D}_s} = V_{D_s^* \bar{D}_s^*, D_s^* \bar{D}_s^*}$

$J/\psi \phi$ scattering

- $\mathcal{L}_d = D_0 (J/\psi \cdot J/\psi^\dagger) (\phi \cdot \phi^\dagger) + D_1 (J/\psi \vec{S}_1 J/\psi^\dagger) (\phi \vec{S}_2 \phi^\dagger)$
- $V_{C_d}(p) = D_0 + D_1 (\vec{S}_1 \cdot \vec{p}) (\vec{S}_2 \cdot \vec{p})$, the D_1 term is switched off.

$D_s \bar{D}_s - J/\psi \phi - D_s^* \bar{D}_s^*$ scattering in $J^{PC} = 0^{++}$ sector

Inelastic scattering with HQSS

- $\mathcal{L}_e = E_0 \bar{H} \epsilon_k J/\psi \phi \epsilon^k H, + E_1 \eta_c \eta^{(\prime)} \bar{H} H$, with a vanished E_1 corresponding to the excluded $\eta_c \eta^{(\prime)}$ channel in present study
- Spin decomposition:

$$\begin{pmatrix} |D_s \bar{D}_s\rangle \\ |D_s^* \bar{D}_s^*\rangle \end{pmatrix}_{J=0} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} |0_l \otimes 0_Q\rangle \\ |1_l \otimes 1_Q\rangle \end{pmatrix}_{J=0},$$

with subscripts l and Q standing for the light and heavy degrees of freedom in the scattering.

- $V_{D_s \bar{D}_s, J/\psi \phi} = \sqrt{3} E_0, V_{D_s^* \bar{D}_s^*, J/\psi \phi} = -E_0.$
- $D_s \bar{D}_s - D^* \bar{D}^*$ is counted into $V_{ij} (a_{ij})$ after fitting.

T-matrix

- $T_{ij} = [(1 - VG)^{-1} V]_{ij},$
 $G_i^\Lambda = \int_0^\Lambda \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{2\mu_i}{q^2 - k_i^2 - i\epsilon} = i \frac{\mu_i}{2\pi} k_i + \frac{\mu_i}{\pi^2} \Lambda \left[1 + O\left(\frac{k_i^2}{\Lambda^2}\right) \right]$

$D_s \bar{D}_s - J/\psi \phi - D_s^* \bar{D}_s^*$ scattering in $J^{PC} = 0^{++}$ sector

IAM

$$T^{-1} = V^{-1} - G = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{12} & V_{22} & V_{23} \\ V_{13} & V_{23} & V_{33} \end{pmatrix}^{-1} - \begin{pmatrix} G_{11} & 0 & 0 \\ 0 & G_{22} & 0 \\ 0 & 0 & G_{33} \end{pmatrix}$$

$V_{13} = 0$, $V_{11} = V_{33}$, $V_{im} V_{mj}$ attributed to the subleading terms

- $(T^{-1})_{11} = \frac{1}{V_{11}} - G_{11}$
- $(T^{-1})_{22} = \frac{1}{V_{22}} - G_{22}$
- $(T^{-1})_{33} = \frac{1}{V_{33}} - G_{33}$
- $(T^{-1})_{12} = \frac{V_{12}}{V_{12}^2 + V_{23}^2 - V_{11} V_{22}} \sim -\frac{V_{12}}{V_{11} V_{22}}$
- $(T^{-1})_{23} = \frac{V_{23}}{V_{12}^2 + V_{23}^2 - V_{11} V_{22}} \sim -\frac{V_{23}}{V_{11} V_{22}}$

$D_s \bar{D}_s - J/\psi \phi - D_s^* \bar{D}_s^*$ scattering in $J^{PC} = 0^{++}$ sector

T-Matrix in ERE in three coupled-channel scattering

$$\bullet \quad \frac{T}{8\pi th_2} = \begin{pmatrix} \frac{1}{a_{11}} - ik_1 & \frac{1}{a_{12}} & \frac{1}{a_{13}} \\ \frac{1}{a_{12}} & \frac{1}{a_{22}} - ik_2 & \frac{1}{a_{23}} \\ \frac{1}{a_{13}} & \frac{1}{a_{23}} & \frac{1}{a_{33}} - ik_3 \end{pmatrix}^{-1},$$

where $i = 1, 2$ and 3 denote the scattering channels, respectively, μ_i denotes the reduced mass, and the p_i is the NR momentum in c.m. frame. S. Sakai et al, PLB808, 2004.09824

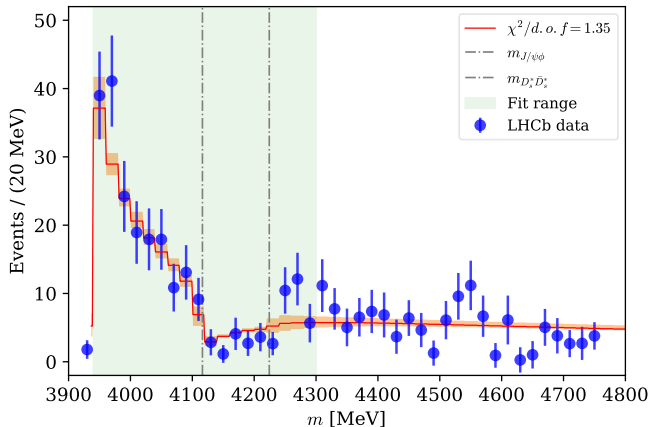
Production amplitude

$$\begin{aligned} \mathcal{M}_a &= P_1^\Lambda + P_1^\Lambda G_1^\Lambda T_{11} + P_1^\Lambda G_2^\Lambda T_{21} + P_3^\Lambda G_3^\Lambda T_{31} \\ &= \textcolor{violet}{P}_1 T_{11}(w) + P_2 T_{21}(w) + \textcolor{violet}{P}_3 T_{31}(w). \end{aligned} \quad \textcolor{violet}{X. K. Dong et al, PRL126, 2011.14517}$$

Lineshape in $D_s \bar{D}_s$ invariant mass distributions

$$\frac{dN}{dm} = |\mathcal{M}_a|^2 \frac{|\vec{k}_1|}{8\pi m}$$

Fitting results



a_{11} [fm]	a_{22} [fm]	a_{23} [fm]	P_1	P_3
1.48 ± 0.17	1.11 ± 0.65	0.87 ± 0.10	-0.29 ± 0.02	0.04 ± 0.03

Fitting results

Scattering length

- a_{11} consistent with $a_{11}^{Ji} = 1.87_{-0.25}^{+0.30}$ fm (1.0 GeV) T. Ji et al, Sci.Bull. 68 (2023)
- Effective $a_{J/\psi\phi} : 0.12_{-0.10}^{+0.20} + i0.78_{-0.40}^{+0.20}$ fm
(0.24 ± 0.04 fm) $_{latt.}$ ($X_1(4140)$) S. Ozaki and S. Sasaki, PRD87(2013),1211.5512
- The $D^{(*)}\bar{D}^{(*)}$ interactions are not included in the analysis directly, and partly counted into δa_{22} ,

$$\delta a_{22}^{D^*\bar{D}^*} = \frac{|\langle X(3960) | D^* \bar{D}^* \rangle|^2}{|\langle X(3960) | D_s \bar{D}_s \rangle|^2} a_{11} = \frac{|g_{X(3960), D^* \bar{D}^*}|^2}{|g_{X(3960), D_s \bar{D}_s}|^2} a_{11} = 0.30 \pm 0.03 \text{ fm.}$$

Poles

$(-++)$	$(-++)$	$(--+)$
$3904.4_{-14.7}^{+5.2}$	$4106.0_{-58.3}^{+10.4} + i24.6_{+29.1}^{-6.7}$	$4223.5_{-4.3}^{+1.9} + i4.3_{+14.5}^{-0.7}$

The 2nd pole is a virtual state ($\theta_E = 2.63_{+0.16}^{-0.13} \pi$), T. Nishibuchi et al, PRC109(2024)
where $J/\psi\phi - D_s^* \bar{D}_s^*$ plays a crucial role. H.J. Lipkin and B.S. Zou, PRD53(1996)

$D_s \bar{D}_s^* - J/\psi \phi$ scattering in $J^{PC} = 1^{++}$ sector

HQSS

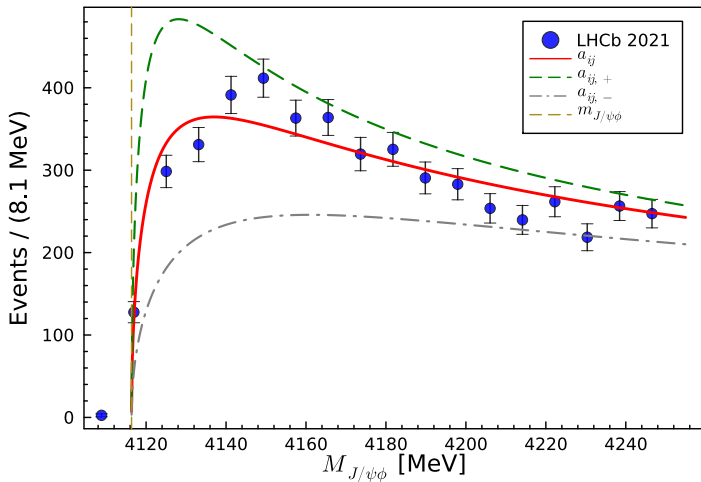
- $X(1^{++}) = \frac{1}{\sqrt{2}} (|D_s^* \bar{D}_s\rangle + |D_s \bar{D}_s^*\rangle)$,

$$\left(\begin{array}{c} |D_s^* \bar{D}_s\rangle \\ |D_s \bar{D}_s^*\rangle \\ |D_s^* \bar{D}_s^*\rangle \end{array} \right)_{J=1} = \left(\begin{array}{ccc} -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{array} \right) \left(\begin{array}{c} |0_I \otimes 1_Q\rangle \\ |1_I \otimes 0_Q\rangle \\ |1_I \otimes 1_Q\rangle \end{array} \right)_{J=1},$$
- $V_{J/\psi \phi, D_s^* \bar{D}_s^*} = 0$
- $D_s \bar{D}_s^* - J/\psi \phi$ couples to $J^{PC} = 1^{++}$ sector

Production amplitude

- $\mathcal{M}_b = P_{1^{++}} \frac{th_2}{8\pi} \frac{1}{1/a^{eff} - ik_2}$, $P_{1^{++}}$ is unknown.
- a virtual pole $4106.7_{+15.5}^{-4.0} + i 8.3_{-6.2}^{+2.6}$ MeV near $J/\psi \phi$ threshold.
- $\frac{dN}{dM_{J/\psi \phi}} = |\mathcal{M}_b|^2 \frac{|\vec{k}_2|}{8\pi M_{J/\psi \phi}}$

$D_s \bar{D}_s^* - J/\psi \phi$ scattering in $J^{PC} = 1^{++}$ sector



$$P_{1^{++}} = 1.16 \text{ MeV}^{-1/2}.$$

$X_1(4685)$ in $J/\psi\phi - \psi(2S)\phi$ scattering

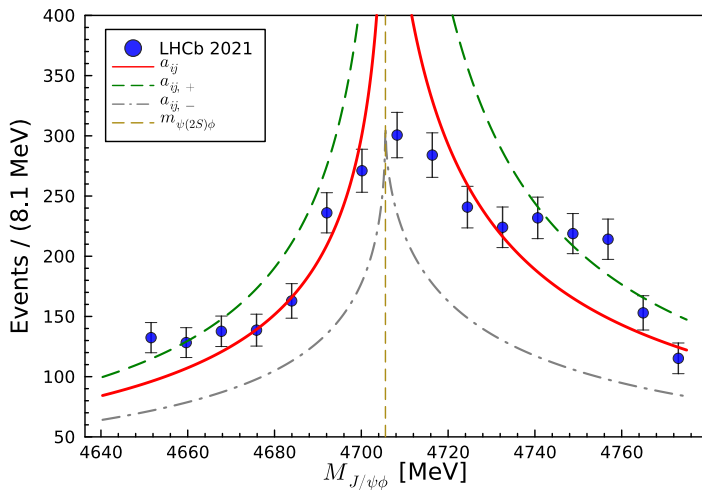
HQSS

- $\mathcal{L}_{J/\psi\phi_i, \psi(2S)\phi_f} = D_0 \frac{E_{\phi_i}}{m_{\phi_f}} \vec{\epsilon}_{J/\psi} \cdot \vec{\epsilon}_{\psi(2S)} \vec{\epsilon}_{\phi_i} \cdot \vec{\epsilon}_{\phi_f},$
- $D_s^{(*)} \bar{D}_s^{(*)}$ are far away from $\psi(2S)\phi$ threshold, where the transitions are kinematically suppressed.
- $\{J/\psi\phi, \psi(2S)\phi\}$ is considered around $\psi(2S)\phi$ threshold.

Production amplitude

- $\mathcal{M}_c = P^{J/\psi\phi} t_{11} + P^{\psi(2S)\phi} t_{21}$
- a virtual pole $4690.9_{+5.1}^{-5.7} + i7.0_{-7.4}^{+13.5}$ MeV near $\psi(2S)\phi$ threshold.
- $\frac{dN}{dM_{J/\psi\phi}} = |\mathcal{M}_c|^2 \frac{|\vec{k}_2|}{8\pi M_{J/\psi\phi}}$

$X_1(4685)$ in $J/\psi\phi - \psi(2S)\phi$ scattering



$$P_{1^{++}}^{J/\psi\phi} = 0 \text{ and } P_{1^{++}}^{\psi(2S)\phi} = 0.48 \text{ MeV}^{-1/2}.$$

Summary

- ① HQSS inspired EFT is introduced to study $D_s^{(*)} \bar{D}_s^{(*)} - J/\psi\phi$ scattering in the LO approach.
- ② The dip around $J/\psi\phi$ threshold in $D_s \bar{D}_s$ invariant mass distributions provides a hint about $J/\psi\phi$ interaction, where a virtual state appears in elastic $J/\psi\phi$ scattering and is a counterpart of X(6200) in di- J/ψ scattering.
- ③ The $X_1(4140)$ may correspond to a $J/\psi\phi$ virtual state.
- ④ The X(4685) may be a cusp effect, driven by a virtual pole, at $\psi(2S)\phi$ threshold.
- ⑤ The 2^{++} pole in $J/\psi\phi$ scattering might be observed in 2γ final states.
- ⑥ Coupled channel effect and OZI interaction work together in low energy scattering.

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Thanks !

Backup: K-matrix parameterization

2 coupled-channel scattering

$$\mathcal{K}_{ba}(m) = \sum_R \frac{g_b^R g_a^R}{M_R^2 - m^2} + f_{ba}, \quad \mathcal{P}_b(m) = \sum_R \frac{\beta_R g_b^R}{M_R^2 - m^2} + \beta_b, \text{ cplx. valued } \beta_i.$$
$$\mathcal{M}_a = \sum_b (I - i\rho\mathcal{K})_{ab}^{-1} \mathcal{P}_b,$$

Table S3: Main results found from the K -matrix fit. Uncertainties are statistical only.

Contribution	J^{PC}	M_R (MeV)	g_1^R (MeV)	Γ_0 (MeV)	$\mathcal{F}$ (%)
$ \mathcal{M}_1 ^2$	0^{++}	3957 ± 14	1350 ± 344		94.7 ± 0.4
$\psi(4260)$	1^{--}	4230 [59]		55 [59]	3.2 ± 0.5
$\psi(4660)$	1^{--}	4633 [31]		64 [31]	2.1 ± 0.2
β_R		$(1, 0i)$	β_1	$(-1.2, 2.5i) \pm (4.5, 3.1i)$	
β_2	$(-137.2, -1.5i) \pm (2.7, 218.6i)$		f_{11}	0.8 ± 1.2	
$f_{12} = f_{21}$		0.1 ± 0.1	f_{22}	8.0 ± 5.1	

Backup: $X_1(4685)$

Coupled channel scattering

$\{J/\psi\phi, \psi(2S)\phi\}$ coupled channel scattering involves a large mass difference between thresholds that is a new scale

$\Lambda_{CC} = \sqrt{2\mu (m_{\psi(2S)} - m_{J/\psi})} = 951.50 \text{ MeV} \simeq 0$, which implies the coupled channel effect is suppressed by $\mathcal{O}(\vec{k}_2^2/\Lambda_{CC}^2)$.

Parameterization of cusp

$$\Gamma_{cusp} = \frac{1}{\mu} \left(\frac{4}{|a_0|^2} - \sum_x x \sqrt{\frac{3}{|a_0|^2} + x^2} \right), = 60.90_{-24.24}^{+62.54} \text{ MeV}.$$

$$(M, \Gamma_{BW}) = (4684 \pm 7_{-16}^{+13}, 126 \pm 15_{-41}^{+37}) \text{ MeV},$$

with the $\psi(2S)J/\psi$ reduced mass μ and the effective scattering length $a_0 = 1.03_{+0.38}^{-0.39} + i 0.40_{-0.01}^{+0.01} \text{ fm}$, where the sum runs over $x = \text{Im}(1/a_0)$ and $\text{Re}(1/a_0)$.