Properties of X(3872) from hadronic potentials coupled to quarks

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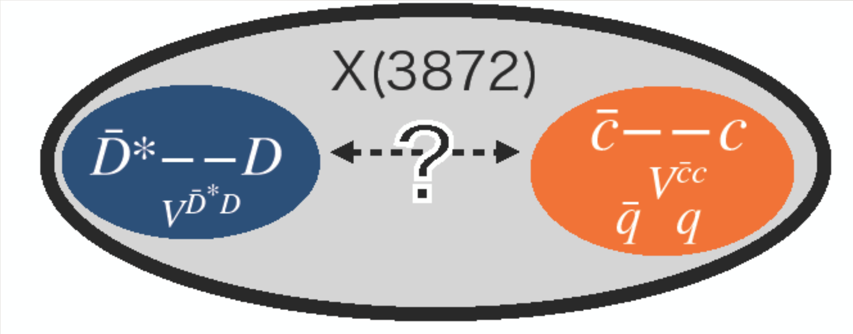
[I. Terashima and T. Hyodo, PhysRevC.108.035204 (2023)]

Exotic hadron $X(3872)$

- There is no restriction by QCD which prohibits the mixing with each d.o.f
  - States with same quantum numbers mix by definition
- Structure of $X(3872)$ [A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, PTEP **2016** (2016)]
  - **Mixing with quark and hadron** degrees of freedom
  - Not enough experimental data and lattice QCD results

> - How about a channel coupling between **quark** and **hadron** degrees of freedom like $\mathbf{X(3872)}$?
> - Revealing the internal structure of exotic hadrons by compositeness



1 : Molecule ⟺ Compositeness ⟺ 0 : Elementary

Channel coupling

✓ Formulation according to Feshbach method [H. Feshbach, Ann. Phys. 5, 357 (1958); ibid., 19, 287 (1962)]

- Hamiltonian H with channel between quark potential V^q and hadron V^h

$$H = \begin{pmatrix} T^q & 0 \\ 0 & T^h + \Delta \end{pmatrix} + \begin{pmatrix} V^q & V^t \\ V^t & V^h \end{pmatrix}$$

T^q, T^h:Kinetic energy

Δ:Threshold energy

V^t:Transition potential

- Schrödinger equation with wave functions of quark and hadron channels $|q\rangle, |h\rangle$

$$H \begin{pmatrix} |q\rangle \\ |h\rangle \end{pmatrix} = E \begin{pmatrix} |q\rangle \\ |h\rangle \end{pmatrix}$$

$$\langle \boldsymbol{r}'_h \mid V^h_{\text{eff}}(E) \mid \boldsymbol{r}_h \rangle = \langle \boldsymbol{r}'_h \mid V^h \mid \boldsymbol{r}_h \rangle + \sum_n \frac{\langle \boldsymbol{r}'_h \mid V^t \mid \phi_n \rangle \langle \phi_n \mid V^t \mid \boldsymbol{r}_h \rangle}{E - E_n}$$

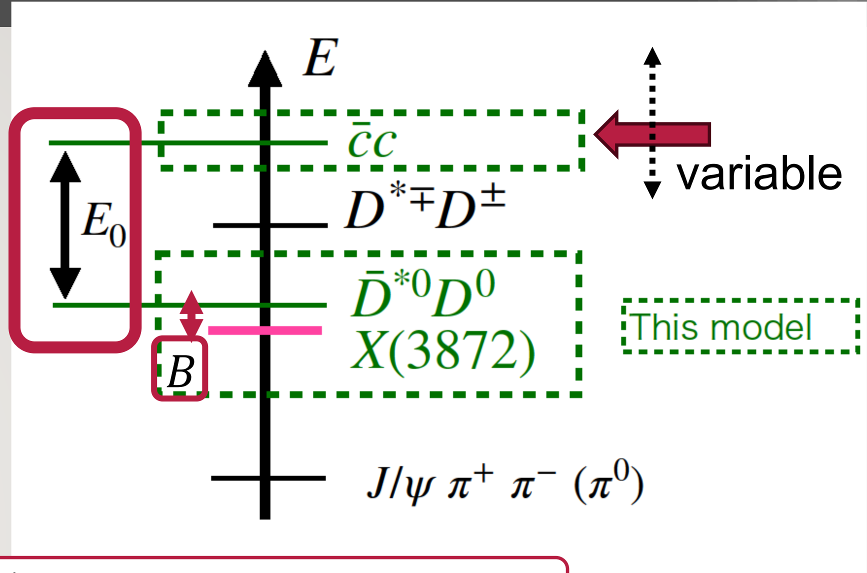
➢ Quark channel contribution. Sum of discrete eigenstates E_n

Formulation of $X(3872)$

◆ Quark channel : $\bar{c}c$

$$H = \begin{pmatrix} T^q & 0 \\ 0 & T^h + \Delta \end{pmatrix} + \begin{pmatrix} V^q & V^t \\ V^t & V^h \end{pmatrix}$$

◆ Hadron channel : $D^0\,\bar{D}^{*0}$

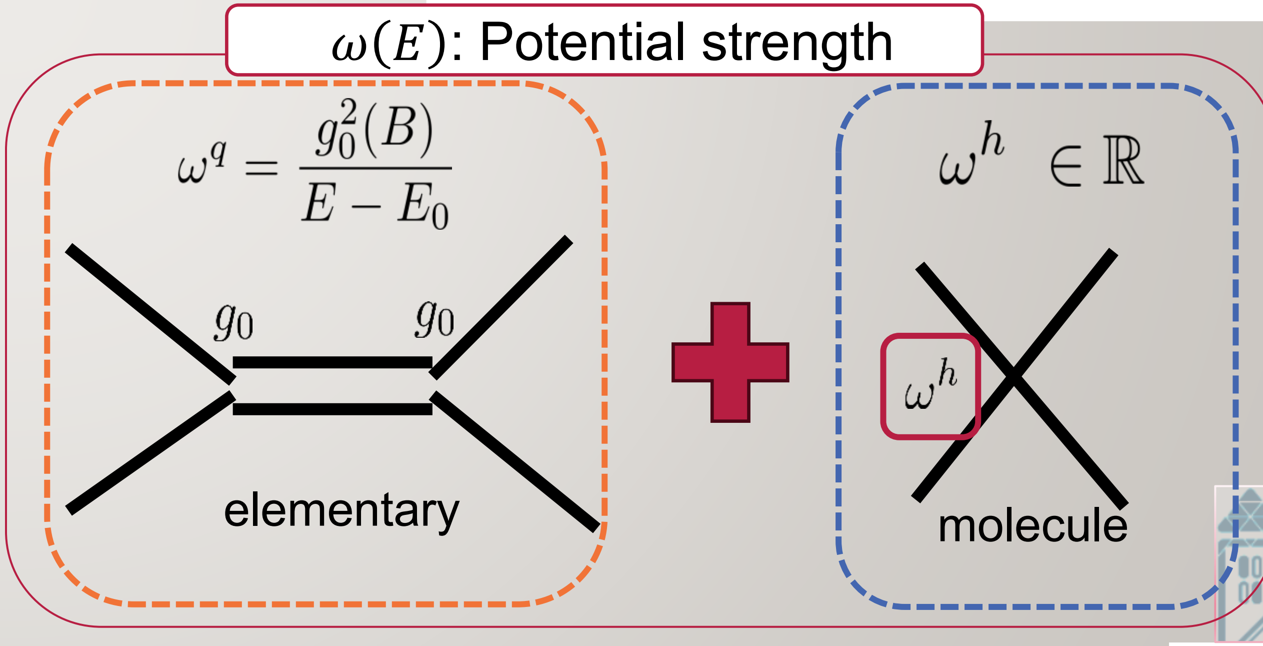


$\langle \boldsymbol{r}'_h \mid V^t \mid \boldsymbol{r}_h \rangle$ ✓ Separable
✓ Yukawa

$$V_{\mathrm{eff}}^{\bar{D}^* D}(\boldsymbol{r}, \boldsymbol{r}', E) = \left[\omega^q(E) + \omega^h\right] V(\boldsymbol{r}) V(\boldsymbol{r}')$$

$$= \omega(E) \frac{e^{-\mu r}}{r} \frac{e^{-\mu r'}}{r'}$$

μ: cut-off



$g_0(B)$: coupling constant

➢ Determine to reproduce mass of $X(3872)$

$$g_0^2(B) = (B + E_0) \cdot \left(-1/G(E = -B) + \omega^h\right)$$

$G(E)$ is a loop function

Formulation: Compositeness

- Bound state wave function is normalized as:

[Kenta Miyahara and Tetsuo Hyodo. Phys. Rev. C, 93(1):015201, 2016.]

$$1 = \int \mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{r}' \Psi_E^*(\boldsymbol{r}')(\delta(\boldsymbol{r}-\boldsymbol{r}') - \frac{\partial}{\partial E} V(\boldsymbol{r},\boldsymbol{r}',E))\Psi_E(\boldsymbol{r})$$

- Definition of compositeness $1 = X_1 + Z_1 = X_2 + Z_2$

$$X_1 = \int \mathrm{d}\boldsymbol{r} |\Psi_{E=-B}(\boldsymbol{r})|^2 \quad Z_1 = -\int \mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{r}' \Psi_E^*(\boldsymbol{r}') \frac{\partial}{\partial E} V(\boldsymbol{r},\boldsymbol{r}',E)\Psi_E(\boldsymbol{r})$$

$$X_1 = [1 + \frac{g_0^2 \kappa\mu(\kappa+\mu)^3}{8\pi m^2 (g_0^2 + (E-E_0)\omega^h)^2}]^{-1} = X_2 = [1 + 2\pi \frac{g_0^2}{(B+E_0)^2} \frac{\kappa}{\mu(\mu+\kappa)}]^{-1}$$

X_2 from L-S e.q.

- Scattering wave function ψ^s

$$\psi_k^s(r) = \frac{\sin[kr + \delta(k)] - \sin\delta(k) e^{-\mu r}}{kr}$$

$$k\cot\delta(k) = -\frac{\mu[4\pi m\omega(E) + \mu^3]}{8\pi m\omega(E)} + \frac{1}{2\mu}\left[1 - \frac{2\mu^3}{4\pi m\omega(E)}\right] k^2 - \frac{1}{8\pi m\omega(E)} k^4$$

- Bound state wave function ψ^b

$$\psi_{k=i\kappa}^b(r) = \mathcal{N}_b \left(-\frac{\kappa e^{-\kappa r}}{r} + \frac{\kappa e^{-\mu r}}{r}\right)$$

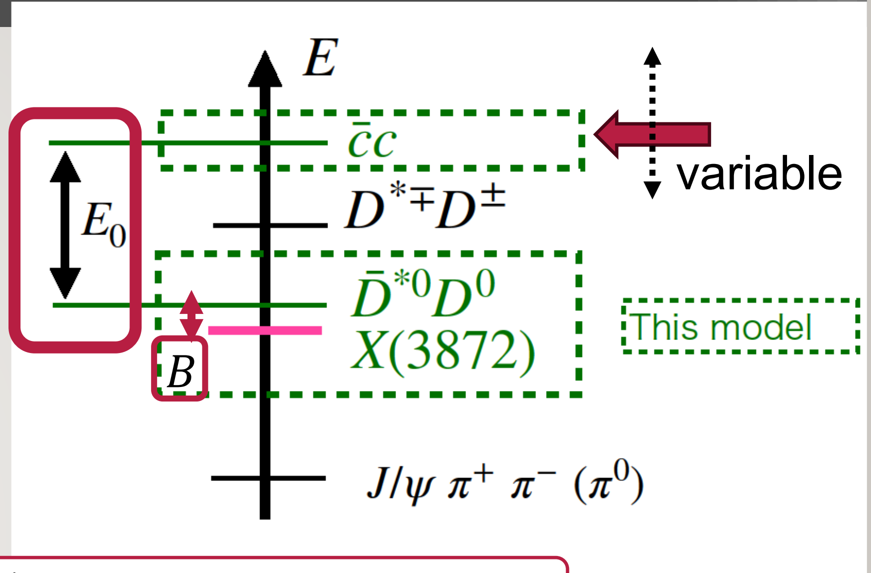
N_b: normalization constant

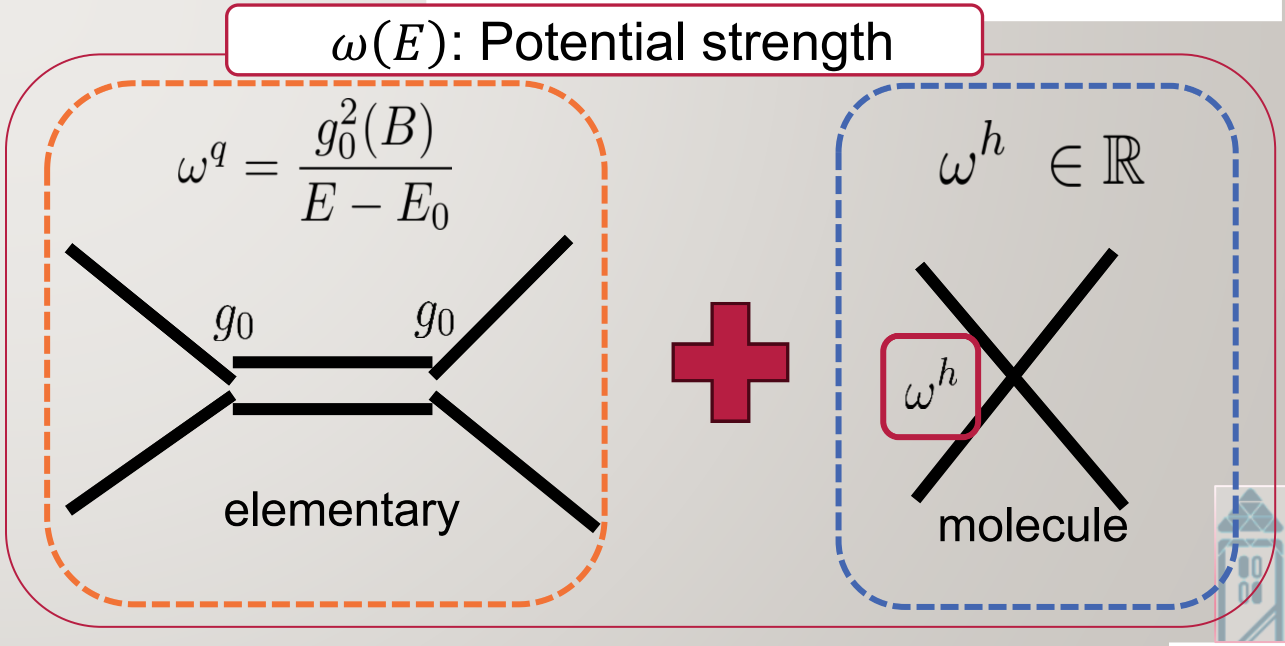
Parameters

- Parameters in this model

Physical observable	Typical value
E_0	0.0078 [GeV] ($\chi_{C1}(2P)$)
B	4×10^{-5} [GeV]
μ	0.14 [GeV]
ω^h	0 [dim'less]

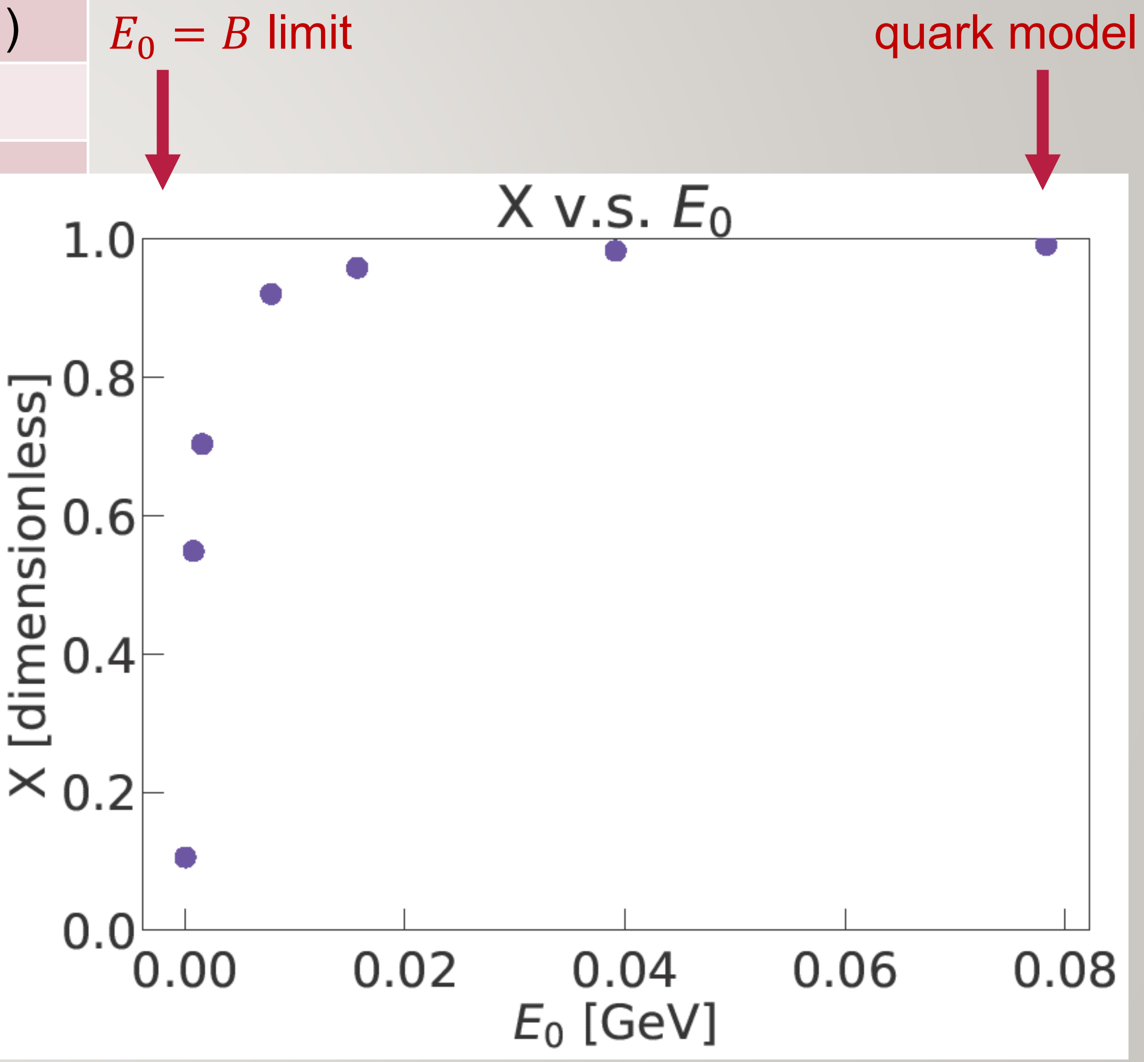
$$V_{\rm eff}^{\bar{D}^* D}(\boldsymbol{r}, \boldsymbol{r}', E) = \omega(E) \frac{e^{-\mu r}}{r} \frac{e^{-\mu r'}}{r'}$$





Result : E_0 dependence

Physical observable	Fixed quantity	Typical value
$\boldsymbol{E_0}$	-	0.0078 [GeV] ($\chi_{c1}(2P)$)
B	4×10^{-5} [GeV]	4×10^{-5} [GeV]
μ	0.14 [GeV]	0.14 [GeV]
ω^h	0 [dim'less]	0 [dim'less]



- E_0 Increase → X Increase

Reason:

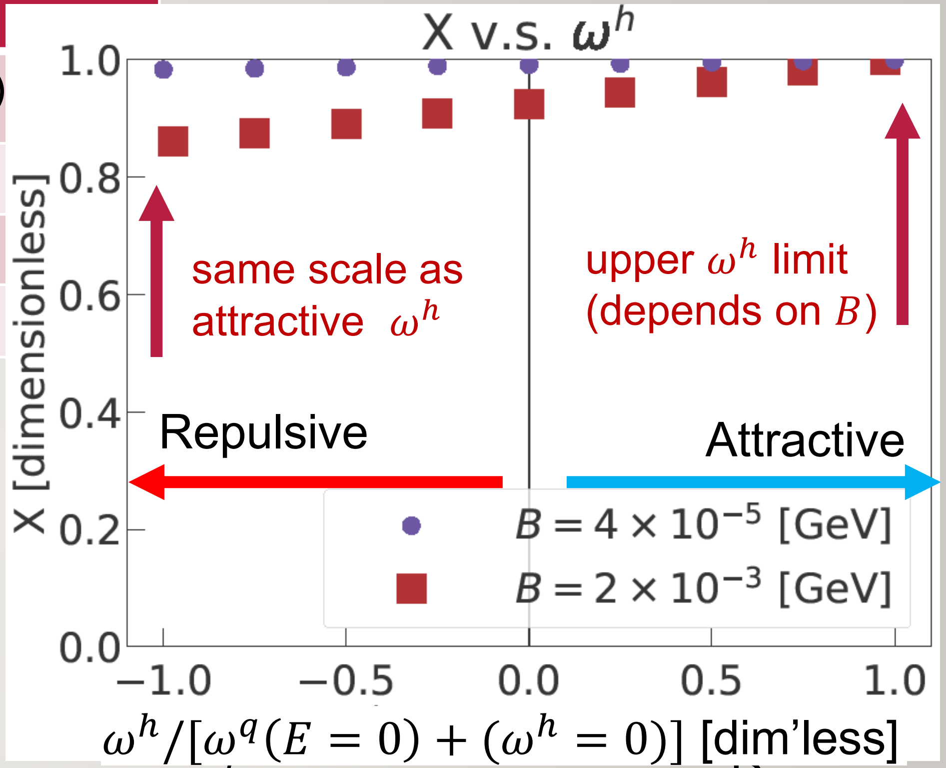
- Self energy increases as quark channel energy E_0 is far from the bare mass

Memo:

✓Changes are **huge** in $X(3872)$

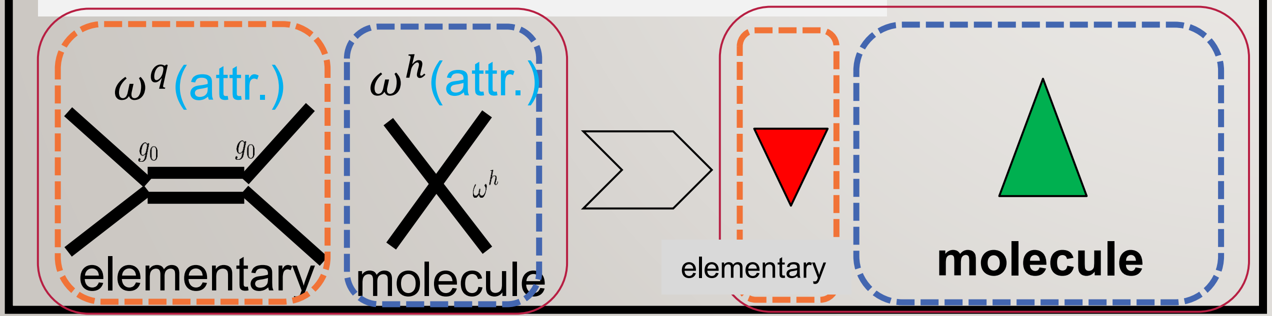
Result : ω^h dependence

Physical observable	Fixed quantity	Typical value
E_0	0.0078 [GeV] ($\chi_{c1}(2P)$)	0.0078 [GeV] ($\chi_{c1}(2P)$)
B	4×10^{-5} [GeV]	4×10^{-5} [GeV]
μ	0.14 [GeV]	0.14 [GeV]
$\boldsymbol{\omega^h}$	-	0 [dim'less]



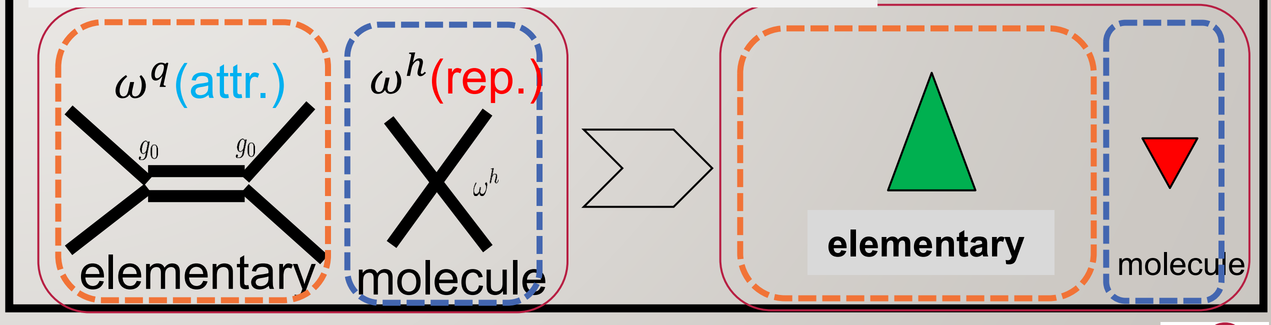
- ω^h(attractive) Strength ⟹ X Increase
- ω^h(repulsive) Strength ⟹ X Decrease

Reason for attractive V^h



Decrease

Reason for repulsive V^h



local approximation of $V^h_{\text{eff}}(\boldsymbol{r}, \boldsymbol{r}', E)$

- To visualize the effective potential, we need to,

$$V^h_{\text{eff}}(\boldsymbol{r}, \boldsymbol{r}', E) \;\Rightarrow\; V^h_{\text{eff}}(\boldsymbol{r}, E_{k_i, k_j, \cdots})$$

Energy independent

HAL QCD method

- Schrödinger equation with **non-local potential** at $n+1$ points of k_i $(i = 0, 1, \cdots, n)$

order of derivative

$$-\frac{1}{2m}\nabla^2 \psi_{k_i}(\boldsymbol{r}) + \int d^3\boldsymbol{r}' \, V_n(\boldsymbol{r}, \boldsymbol{r}', E)\psi_{k_i}(\boldsymbol{r}') = E_{k_i}\psi_{k_i}(\boldsymbol{r})$$

Obtain wavefunctions $\psi_{k_i}(\boldsymbol{r})$

Unknown: $\psi_{k_i}(\boldsymbol{r})$

Assume

- Wave functions $\psi_{k_i}(\boldsymbol{r})$ satisfy the Schrödinger equation with local potentials

$$\left(-\frac{1}{2m}\boldsymbol{\nabla}^2 + V_n(\boldsymbol{r}, \boldsymbol{\nabla})\right)\psi_{k_i}(\boldsymbol{r}) = E_{k_i}\psi_{k_i}(\boldsymbol{r}),$$

Unknown: $V_n(\boldsymbol{r}, \boldsymbol{\nabla})$

HAL QCD method

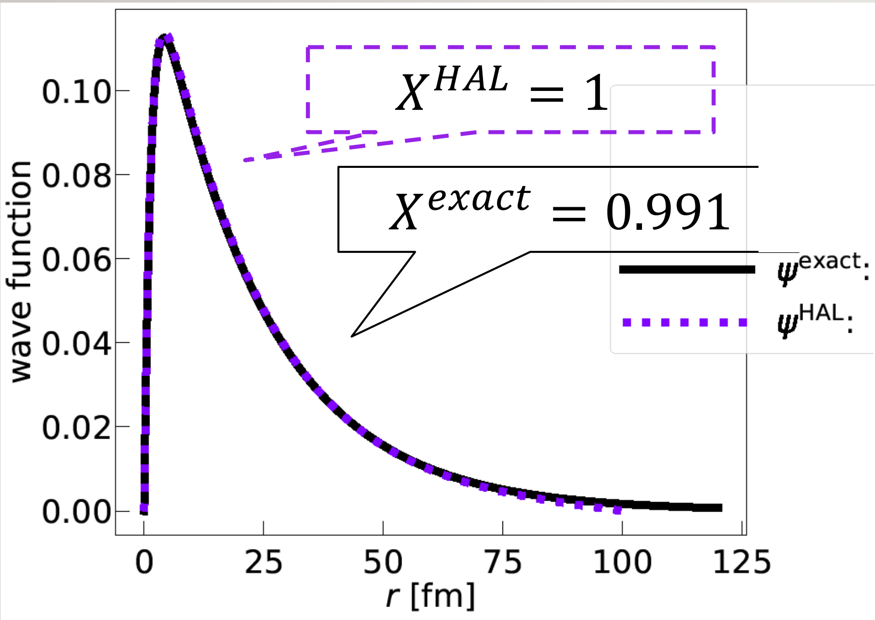
- Obtain local potential $V_n(\boldsymbol{r}, \boldsymbol{\nabla})$ by solving above equation for the potential inversely

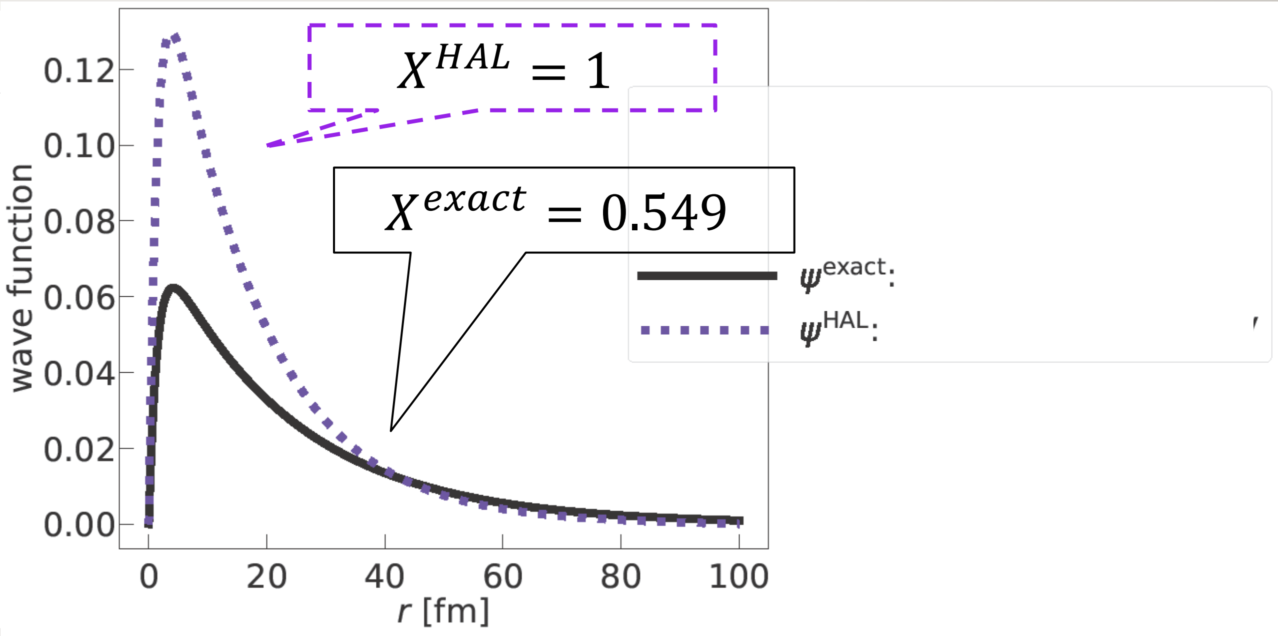
[S.Aoki and K.Yazaki, PTEP 2022, no.3, 033B04 (2022)]

Result: Bound state wavefunctions

- HAL potentials: Energy independent potential so that $\psi^{exact} \leq \psi^{HAL}$

$$1 = \int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'\Psi_E^*(\boldsymbol{r}')\left(\delta(\boldsymbol{r}-\boldsymbol{r}')-\frac{\partial}{\partial E}V(\boldsymbol{r},\boldsymbol{r}',E)\right)\Psi_E(\boldsymbol{r}). \qquad X_1 = \int \mathrm{d}\boldsymbol{r}|\Psi_{E=-B}(\boldsymbol{r})|^2$$



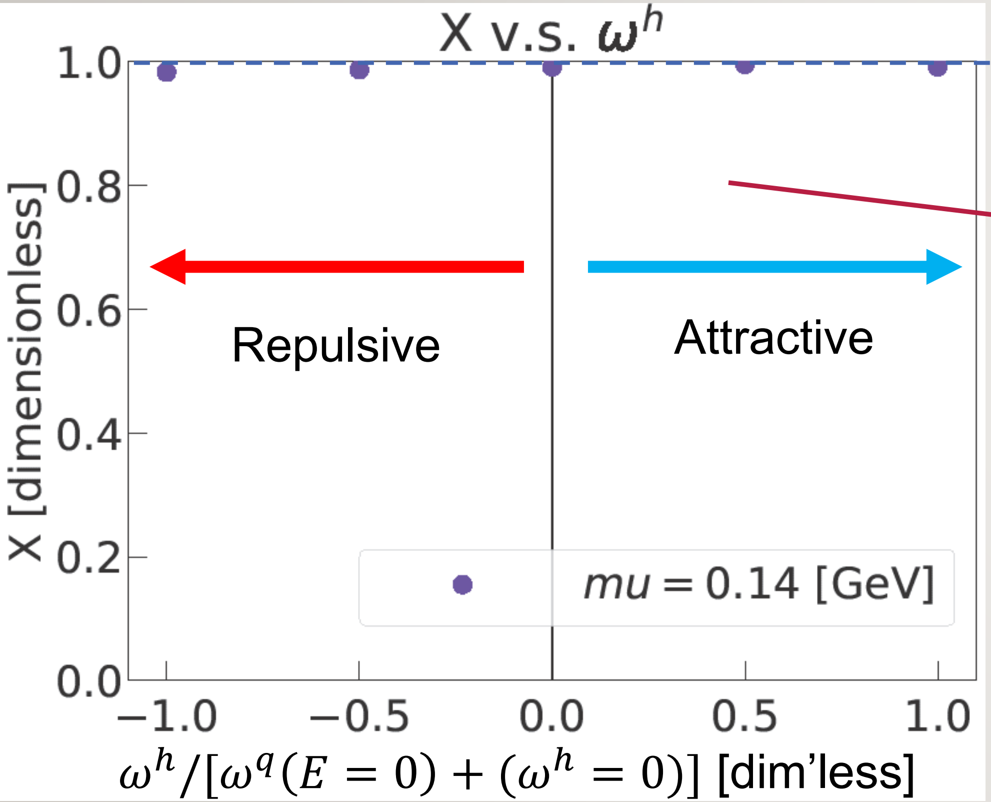


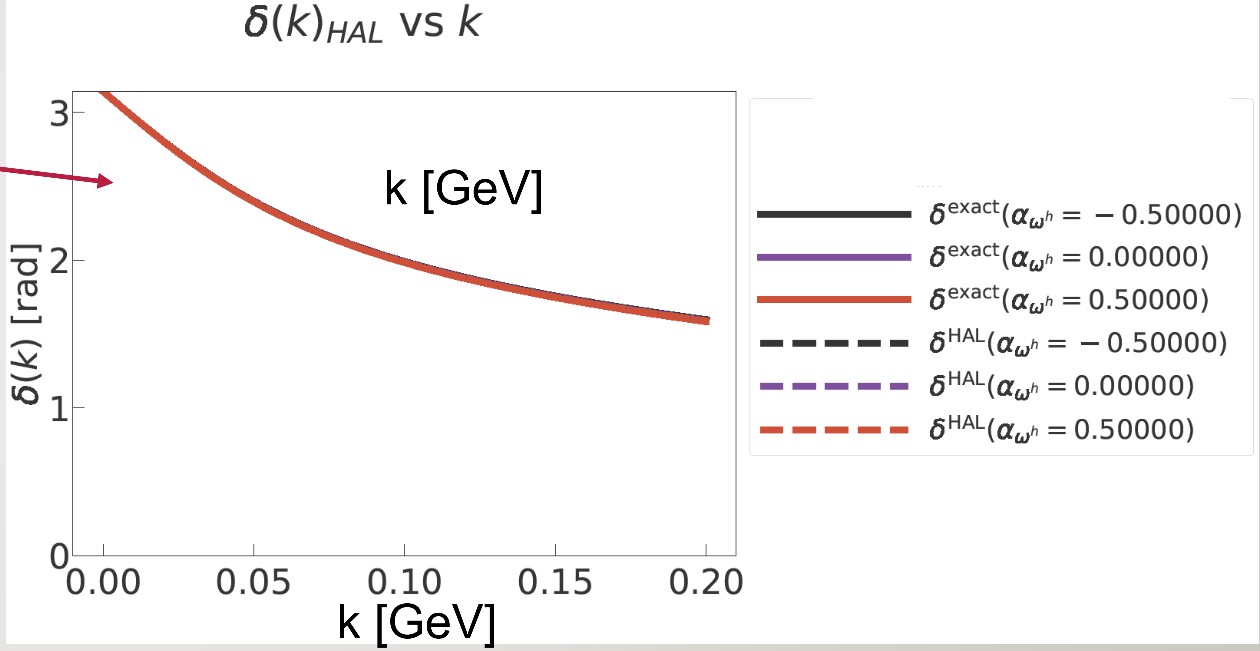
Result: Compare δ^{exact} and δ^{HAL}

- V^h dependences of X and δ

Case: exact $X \simeq 1$

$X_{HAL} = 1$



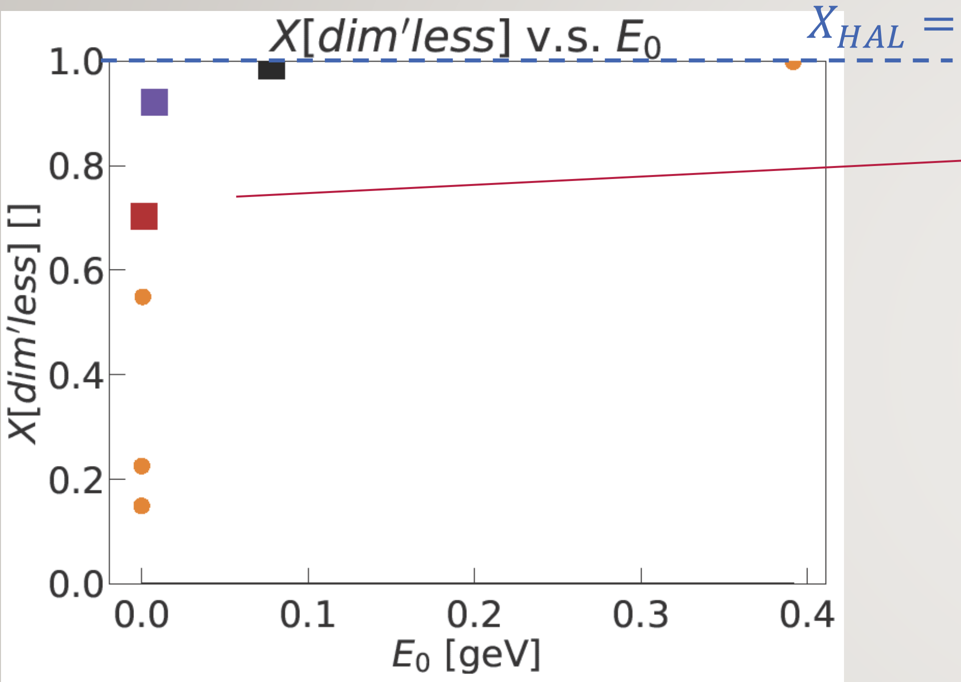


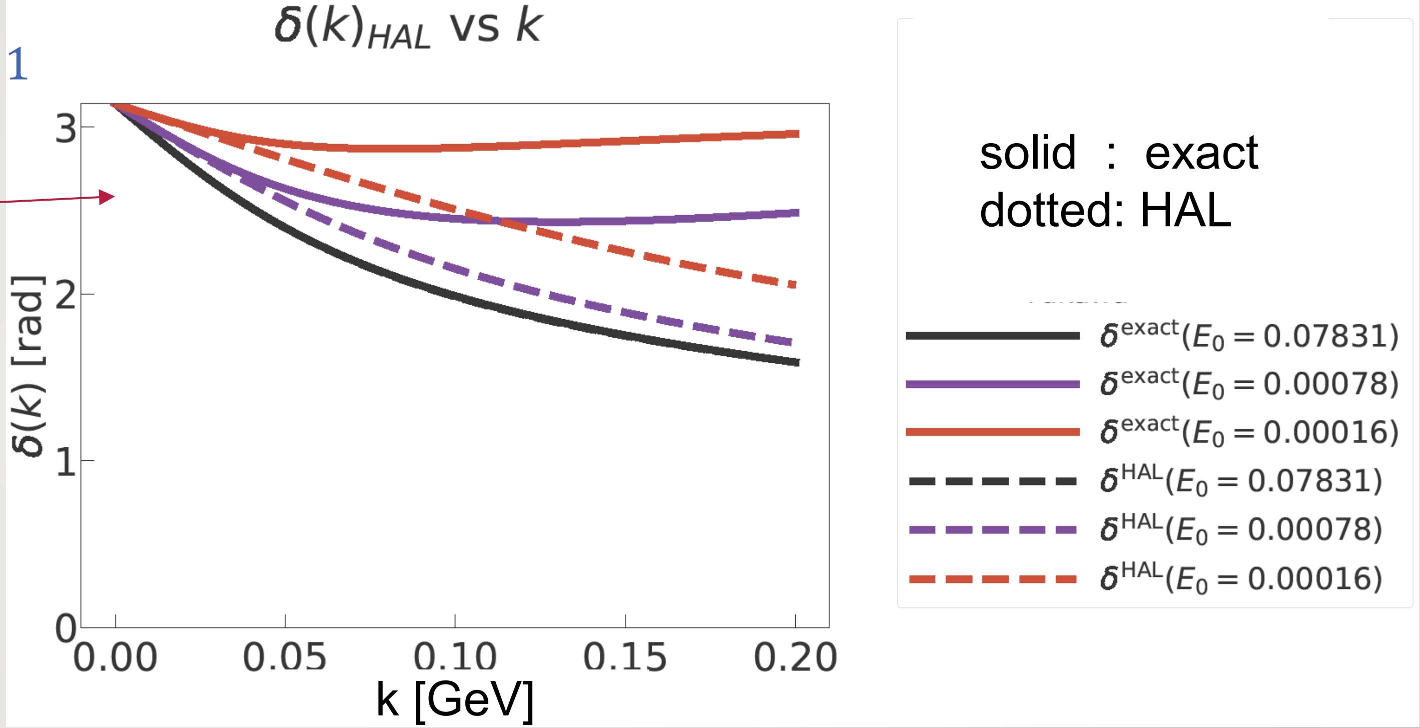
- δ^{exact} and δ^{HAL} are not so different especially for low energy region

Result: Compare δ^{exact} and δ^{HAL}

Case: exact $X < 1$

- E_0 dependences of X and δ

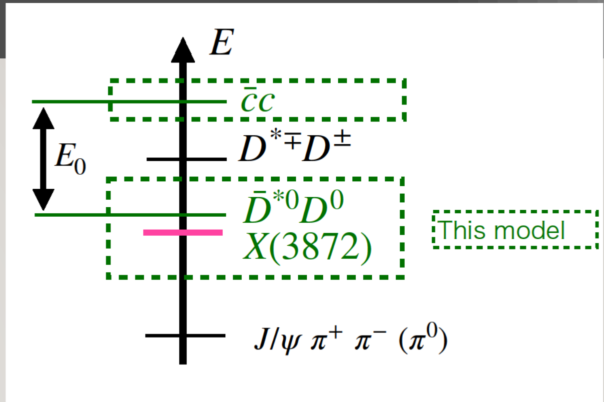




- δ^{exact} and δ^{HAL} are different even for low energy region
  - $X = 1$ for universality from small B, but E_0 fine-tuned to be $X \ll 1$
  - HAL potentials dislike of odd parameters

Summary

[I. Terashima and T. Hyodo, PhysRevC.108.035204 (2023)]

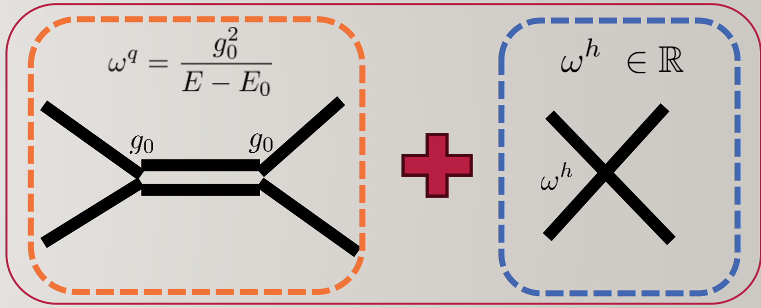


- Channel coupling between $c\bar{c}$ and $D\bar{D}^*$ in $X(3872)$

$$H = \begin{pmatrix} T^{c\bar{c}} & 0 \\ 0 & T^{\bar{D}^*D} + \Delta \end{pmatrix} + \begin{pmatrix} V^{c\bar{c}} & V^t \\ V^t & V^{\bar{D}^*D} \end{pmatrix}$$

- Effective potential with explicit V^q and V^h

$$V_{\text{eff}}^{\bar{D}^*D}(\boldsymbol{r}, \boldsymbol{r}', E) = [\omega^q(E) + \omega^h(E)] V(\boldsymbol{r}) V(\boldsymbol{r}')$$

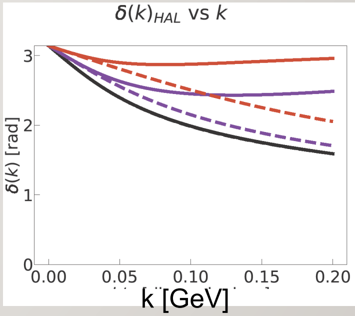


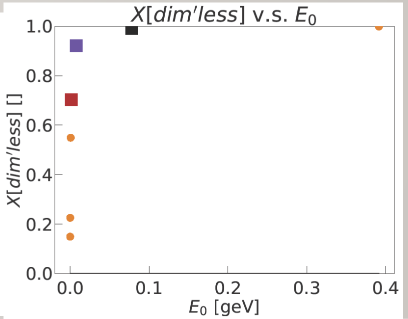
- Compositeness X in analytical form

$$X = \left[1 + \frac{g_0^2 \kappa \mu (\kappa + \mu)^3}{8\pi m^2 (g_0^2 + (E - E_0)\omega^h)^2}\right]^{-1} = \left[1 + 2\pi \frac{g_0^2}{(B + E_0)^2} \frac{\kappa}{\mu(\mu + \kappa)}\right]^{-1}$$

- Parameter dependences for compositeness

Physical observable	Correlation to compositeness
E_0 (quark channel energy)	**Positive (large)**
$\omega^h_{attr.}$ (attractive hadron-ch. potential)	Positive (small)
$\omega^h_{rep.}$ (repulsive hadron-ch. potential)	Negative (small)





- Compare $\delta(k)$ from the non-local exact potential and from the local HAL potential
  - ➢ δ^{HAL} get disclosed from δ^{HAL} in the specialized parameters