The East Asian Workshop on Exotic Hadrons 2024

Correlation function and the inverse problem in the two-body interactions

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arXiv: 2409.05787 (Just accepted by PRD)

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Outline

1. Introduction
2. Two-body interaction
3. Inverse problem
4. Summary

§1. Introduction

A $\Sigma^*(1/2-)$ state with mass 1430 MeV near $\bar{K}N$ threshod was **predicted**:

N. Kaiser, P. B. Siegel, and W. Weise, Nucl. Phys. A 594, 325 (1995)

E. Oset and A. Ramos, Nucl. Phys. A 635, 99 (1998)

Isospin I = 1

J. A. Oller and U.-G. Meißner, Phys. Lett. B 500, 263 (2001)

D. Jido, J. A. Oller, E. Oset, A. Ramos and U.-G. Meißner, Nucl. Phys. A 725, 181 (2003)

two-poles structure of Λ(1405) was found

But, still NOT found yet........

A recent review on the $\Sigma^*(1/2-)$ state:

E. Wang, L.-S. Geng, J.-J. Wu, J.-J. Xie, and B.-S. Zou, arXiv:2406.07839

There are some proposals to search for this **predicted** state:

Y.-H. Lyu, H. Zhang, N.-C. Wei, B.-C. Ke, E. Wang, and J.-J. Xie, Chin. Phys. C 47, 053108 (2023)

$$\gamma n \to K^+ \Sigma^{*-}_{1/2^-}$$

X.-L. Ren, E. Oset, L. Alvarez-Ruso, and M. J. Vicente Vacas, Phys. Rev. C 91, 045201 (2015)
J.-J. Wu and B.-S. Zou, Few Body Syst. 56, 165 (2015)

$$\bar{\nu}_l p \to l^+ \Phi B$$

E. Wang, J.-J. Xie, and E. Oset, Phys. Lett. B 753, 526 (2016)

$$\chi_{c0}(1P) \to \bar{\Sigma}\Sigma\pi$$

L.-J. Liu, E. Wang, J.-J. Xie, K.-L. Song, and J.-Y. Zhu, Phys. Rev. D 98, 114017 (2018)

$$\chi_{c0} \to \bar{\Lambda}\Sigma\pi$$

J.-J. Xie and E. Oset, Phys. Lett. B 792, 450 (2019)

$$\Lambda_c^+ \to \pi^+\pi^0\pi^-\Sigma^+ \qquad \Lambda_c^+ \to \pi^+\pi^+\pi^-\Lambda$$

A recent **evidence** of the Σ∗(1/2−) state:

Y. Ma et al. (Belle), Phys. Rev. Lett. 130, 151903 (2023)

$$\Lambda_c^+ \to \Lambda\pi^+\pi^+\pi^-$$

But, from their analysis, they can NOT discriminate from the peak being due to a resonance or to a cusp in the $\bar{K}N$ threshold.

$\Sigma^*(1430)$?

§2. Two-body interaction

(1) Coupled channel interaction from the chiral unitary approach

$\bar{K}^0 p, \pi^+\Sigma^0, \pi^0\Sigma^+, \pi^+\Lambda$, and $\eta\Sigma^+$

Without the Coulomb interaction

$$|\pi^+\Sigma^0\rangle = -\frac{1}{\sqrt{2}}\left(|\pi\Sigma, I=2, I_3=1\rangle + |\pi\Sigma, I=1, I_3=1\rangle\right)$$

$$|\pi^0\Sigma^+\rangle = -\frac{1}{\sqrt{2}}\left(|\pi\Sigma, I=2, I_3=1\rangle - |\pi\Sigma, I=1, I_3=1\rangle\right)$$

$$|\bar{K}^0 p\rangle = |\bar{K}N, I=1, I_3=1\rangle,$$

$$|\pi^+\Lambda\rangle = -|\pi\Lambda, I=1, I_3=1\rangle$$

$$|\eta\Sigma^+\rangle = -|\eta\Sigma, I=1, I_3=1\rangle$$

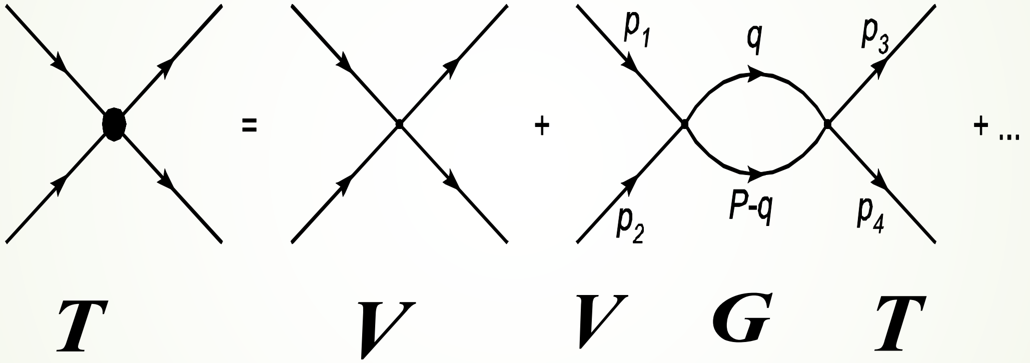
$$V_{ij} = -\frac{1}{4f^2}C_{ij}(k_i^0 + k_j^0)$$

$f = 93$ MeV

C_{ij}	$\bar{K}^0 p$	$\pi^+\Sigma^0$	$\pi^0\Sigma^+$	$\pi^+\Lambda$	$\eta\Sigma^+$
$\bar{K}^0 p$	1	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{2}}$	$\sqrt{\frac{3}{2}}$
$\pi^+\Sigma^0$		0	-2	0	0
$\pi^0\Sigma^+$			0	0	0
$\pi^+\Lambda$				0	0
$\eta\Sigma^+$					0

- **Coupled Channel Unitary Approach**： solving Bethe-Salpeter equations, which take on-shell approximation for the loops.

$$T = V + V\,G\,T,\; T = [1 - V\,G]^{-1}\,V$$



where V matrix (potentials) can be evaluated from the interaction Lagrangians.

J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438
E. Oset and A. Ramos, Nucl. Phys. A 635 (1998) 99
J. A. Oller and U. G. Meißner, Phys. Lett. B 500 (2001) 263

G is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{2M_l}{(P-q)^2 - m_{l1}^2 + i\varepsilon} \frac{1}{q^2 - m_{l2}^2 + i\varepsilon}$$

The coupled channel scattering amplitudes **T matrix satisfy the unitary** ：

$$\mathrm{Im}\, T_{ij} = T_{in}\, \sigma_{nn}\, T^*_{nj}$$

$$\sigma_{nn} \equiv \mathrm{Im}\, G_{nn} = -\frac{q_{cm}}{8\pi\sqrt{s}} \theta(s - (m_1 + m_2)^2))$$

To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G^{II}_{ll}(s) = G^{I}_{ll}(s) + i \boxed{\frac{i}{2\pi} \frac{M_l q_{cml}(s)}{\sqrt{s}}}$$

(2) Correlation functions

$$\mathcal{C}_{\bar{K}^0 p}(p_{\bar{K}^0}) = 1 + 4\pi\theta(q_{\max} - p_{\bar{K}^0}) \int \mathrm{d}r r^2 S_{12}(r) \times \Big\{ \left| j_0(p_{\bar{K}^0} r) + T_{11}(E)\tilde{G}_1(r,E) \right|^2 + \left| T_{21}(E)\tilde{G}_2(r,E) \right|^2 + \left| T_{31}(E)\tilde{G}_3(r,E) \right|^2 + \left| T_{41}(E)\tilde{G}_4(r,E) \right|^2 + \left| T_{51}(E)\tilde{G}_5(r,E) \right|^2 - j_0^2(p_{\bar{K}^0} r) \Big\}, \quad (7)$$

$$\mathcal{C}_{\pi^+ \Sigma^0}(p_{\pi^+}) = 1 + 4\pi\theta(q_{\max} - p_{\pi^+}) \int \mathrm{d}r r^2 S_{12}(r) \times \Big\{ \left| j_0(p_{\pi^+} r) + T_{22}(E)\tilde{G}_2(r,E) \right|^2 + \left| T_{12}(E)\tilde{G}_1(r,E) \right|^2 + \left| T_{32}(E)\tilde{G}_3(r,E) \right|^2 + \left| T_{42}(E)\tilde{G}_4(r,E) \right|^2 + \left| T_{52}(E)\tilde{G}_5(r,E) \right|^2 - j_0^2(p_{\pi^+} r) \Big\}, \quad (8)$$

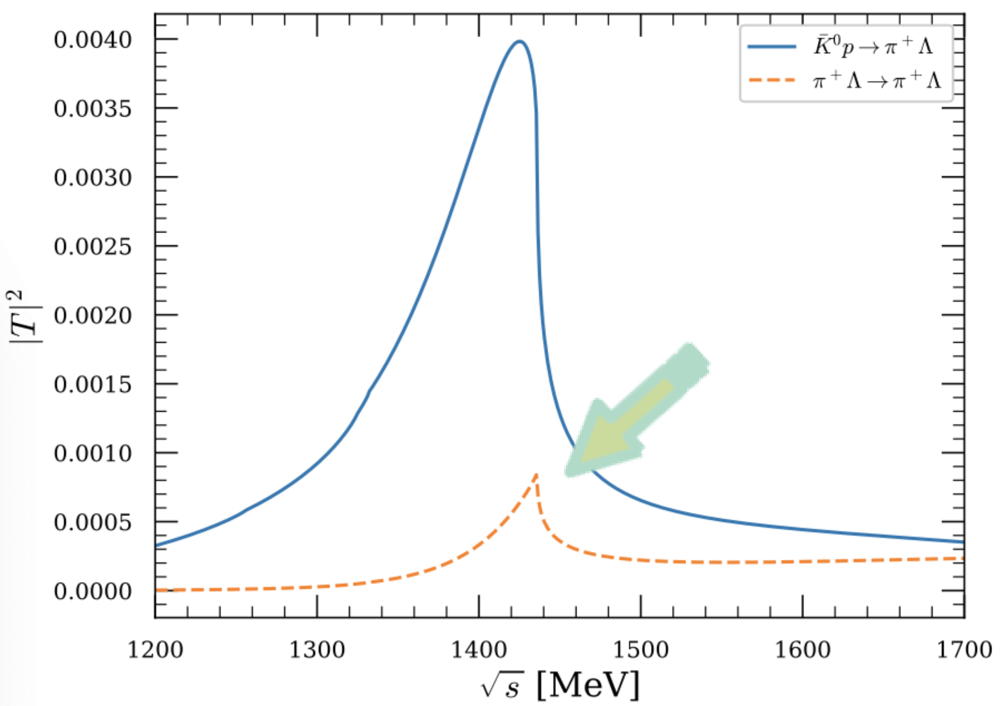
source function

$$S_{12}(r) = \frac{1}{(\sqrt{4\pi}R)^3} \exp\left(-\frac{r^2}{4R^2}\right)$$

$$\tilde{G}_i = 2M_i \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{w_1(\vec{q}) + w_2(\vec{q})}{2\, w_1(\vec{q})\, w_2(\vec{q})} \times \frac{j_0(|\vec{q}|r)}{s - [w_1(\vec{q}) + w_2(\vec{q})]^2 + i\epsilon}$$

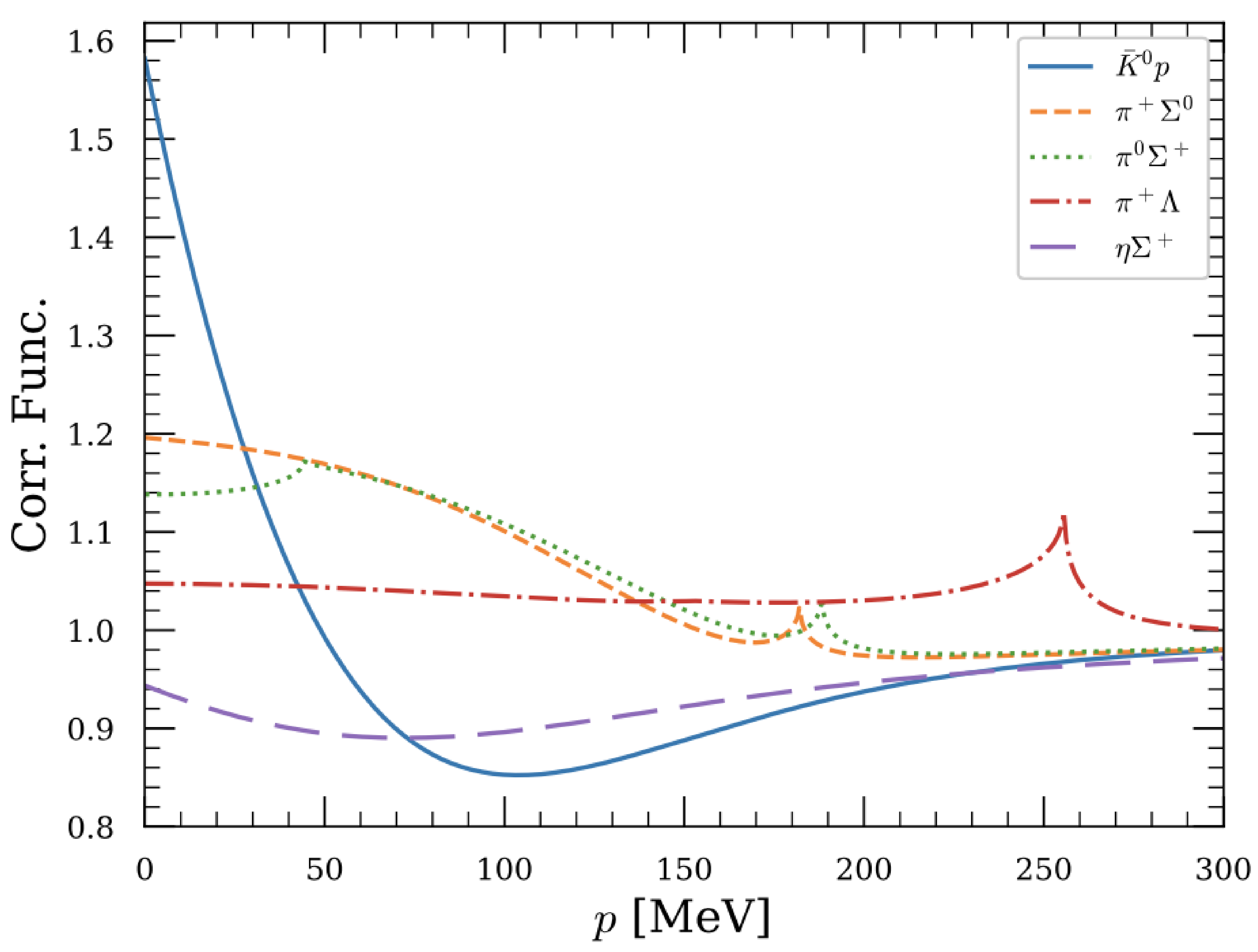
The zeroth order spherical Bessel function

(3) results



$\sqrt{s_0}$ (MeV)	g_1	g_2	g_3	g_4	g_5
(1431.83, 104.75)	(3.03, 2.71)	(1.98, 1.45)	(1.98, 1.47)	(0.19, 1.21)	(0.21, 1.27)
Probabilities	P_1	P_2	P_3	P_4	P_5
	(−0.60, 0.19)	(0.24, −0.41)	(0.25, −0.41)	(0.09, 0.05)	(−0.02, −0.00)
Scattering	a_1	a_2	a_3	a_4	a_5
lengths (fm)	(0.45, −1.13)	(−0.15, −0.03)	(−0.12, −0.00)	(−0.05, −0.00)	(0.08, −0.15)
Effective	r_1	r_2	r_3	r_4	r_5
ranges (fm)	(0.04, −0.45)	(−35.24, −16.61)	(−67.00, 0.39)	(−66.12, 0.00)	(0.31, 0.32)

Correlation functions



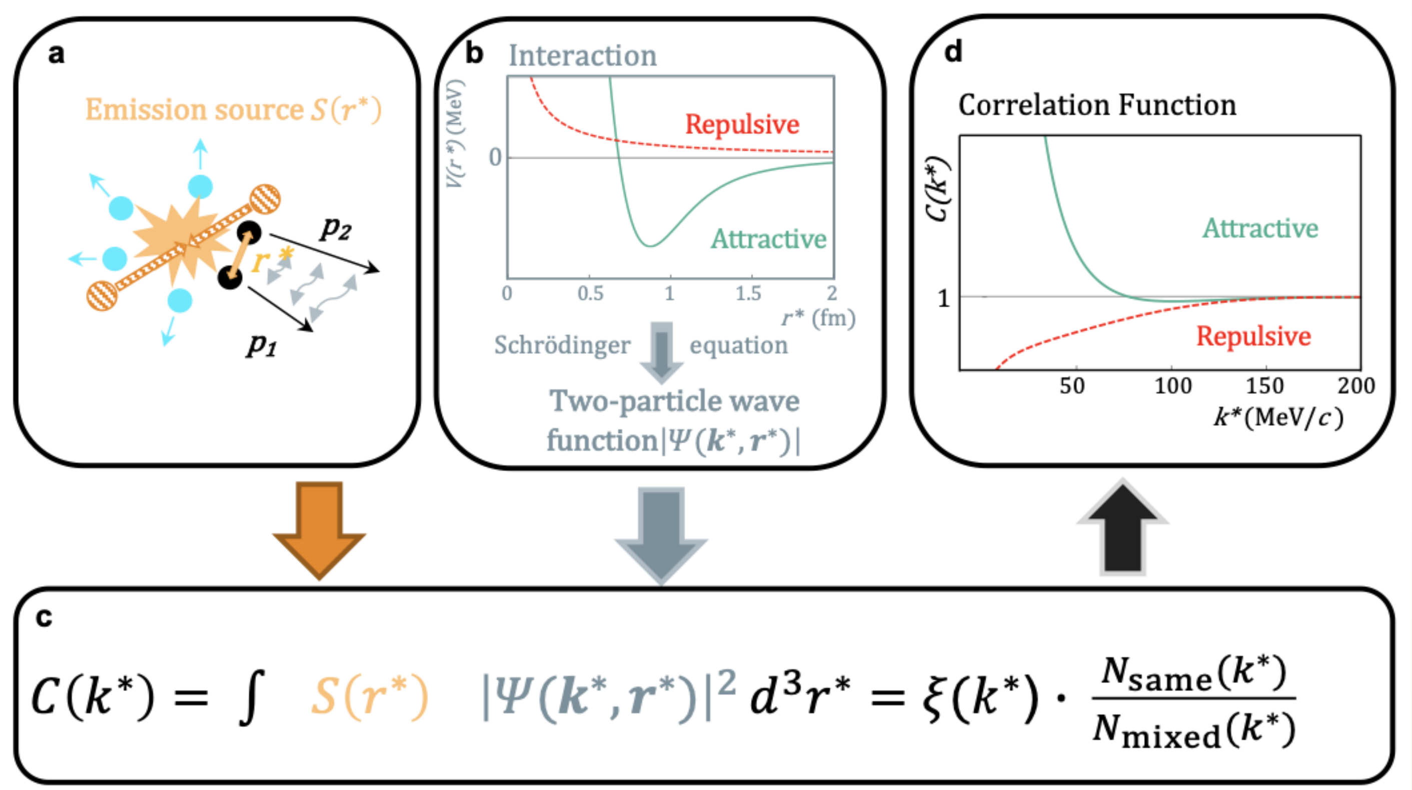
Generating pseudo data

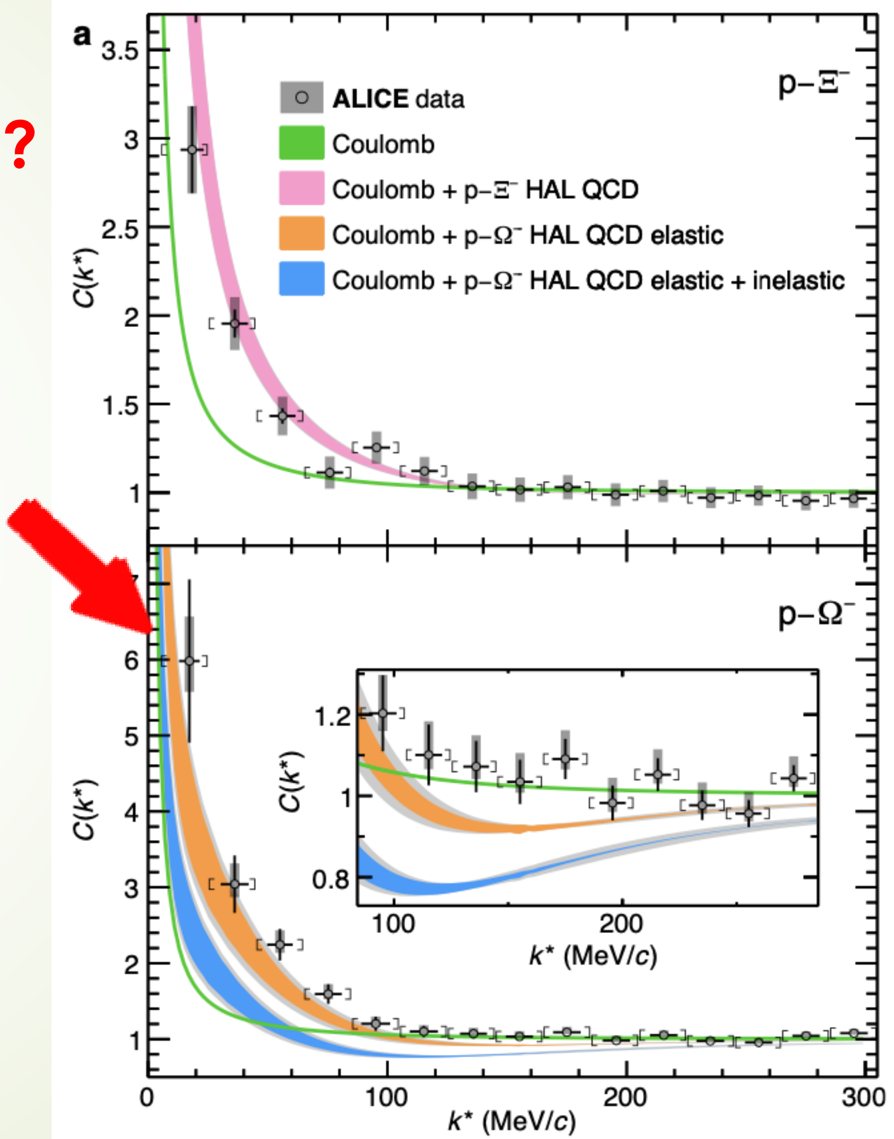
Assuming ± 0.02 error

§3. Inverse problem

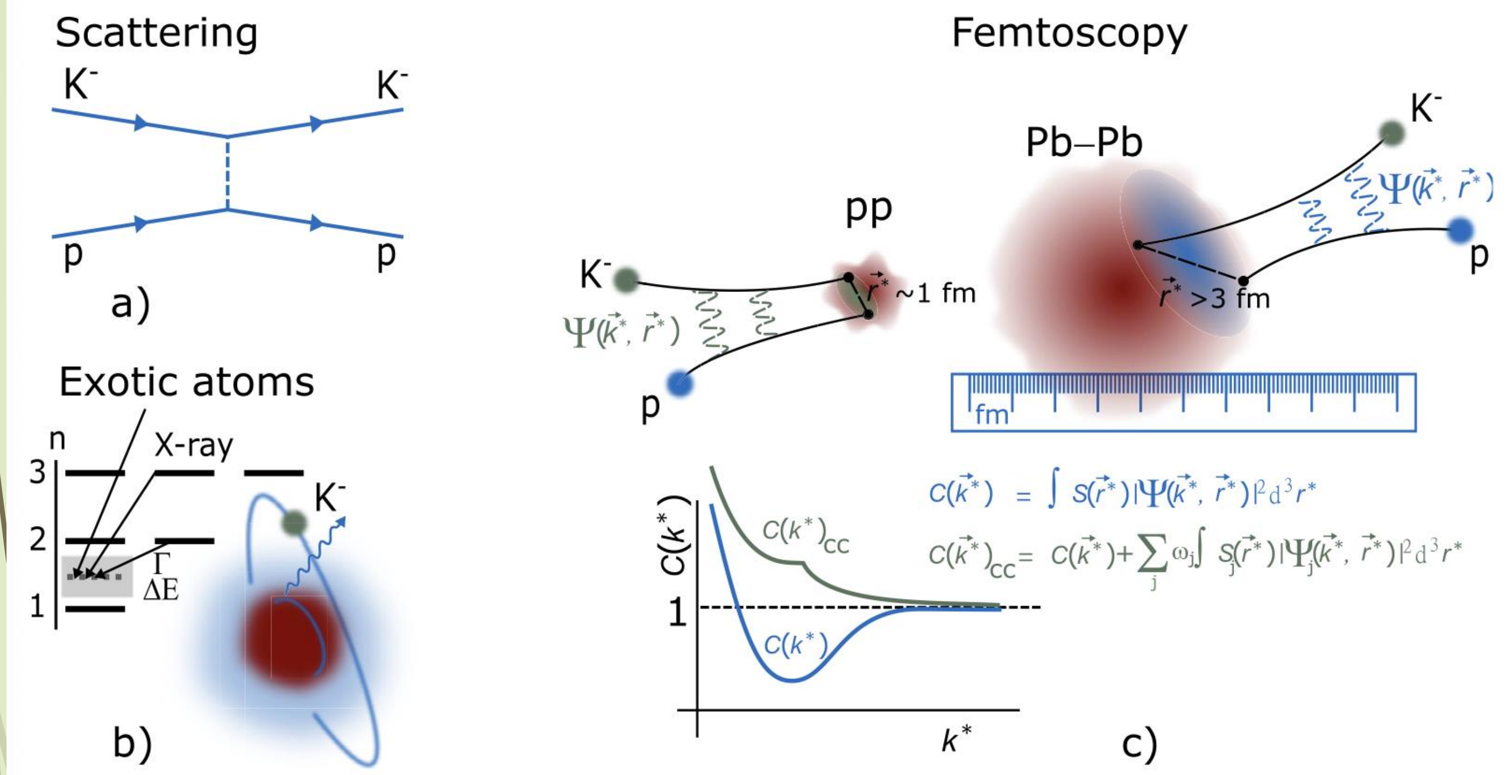
Why, we do the inverse problem?

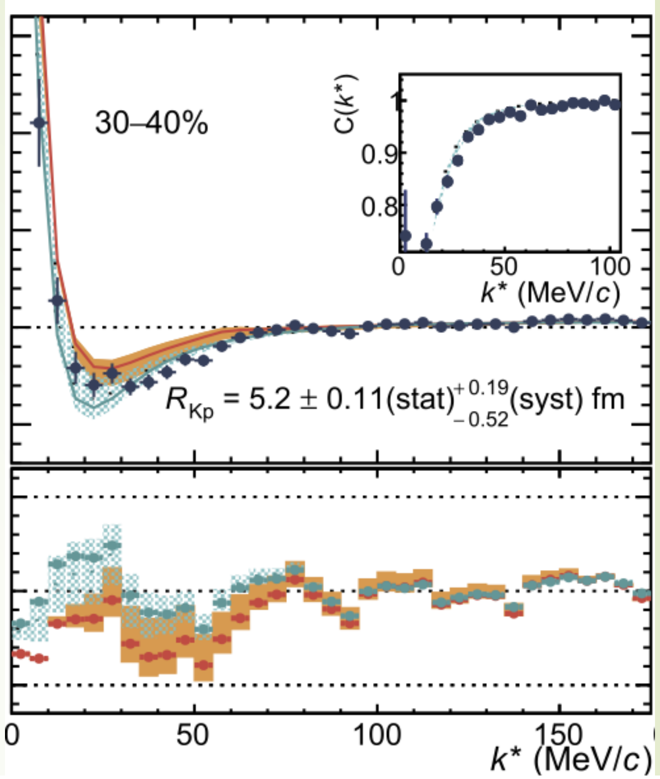
(1) What can we learn from the correlation functions?



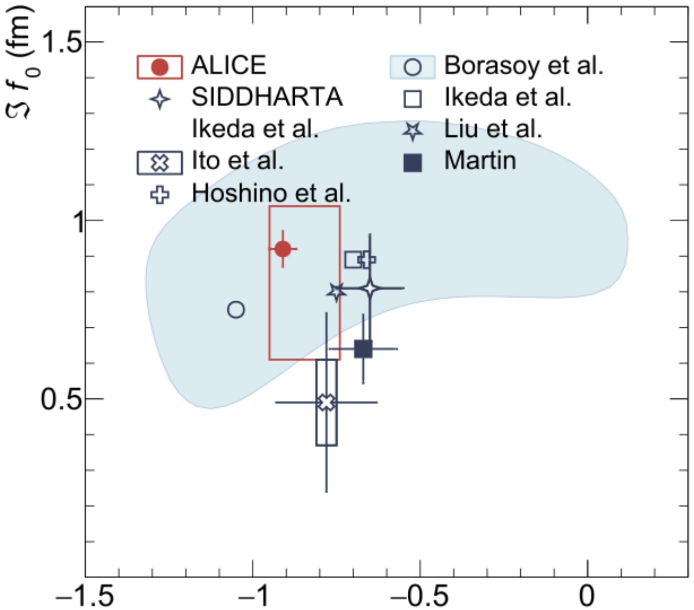


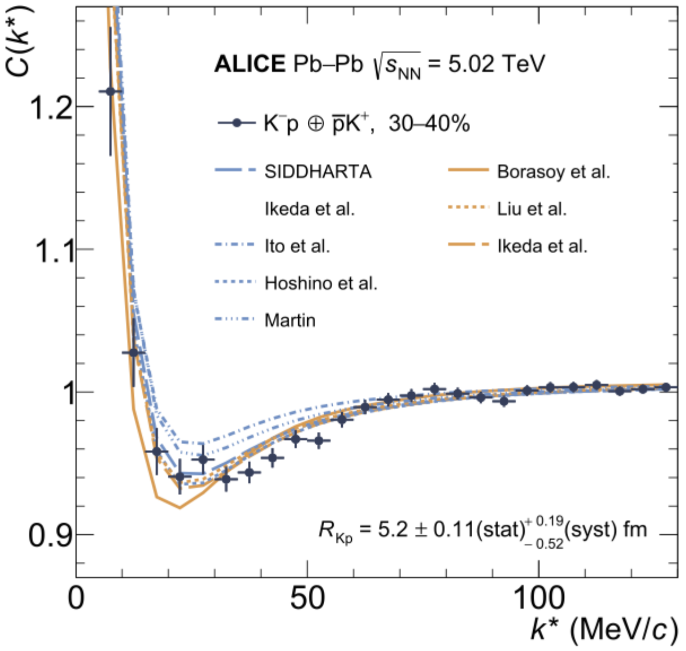
A. Collaboration et al. (ALICE), Nature 588, 232 (2020)





Femtoscopic correlation functions are emerging as a relevant experimental tool, showing the potential to provide information on the interaction of hadrons, scattering parameters and the nature of hadronic resonances.

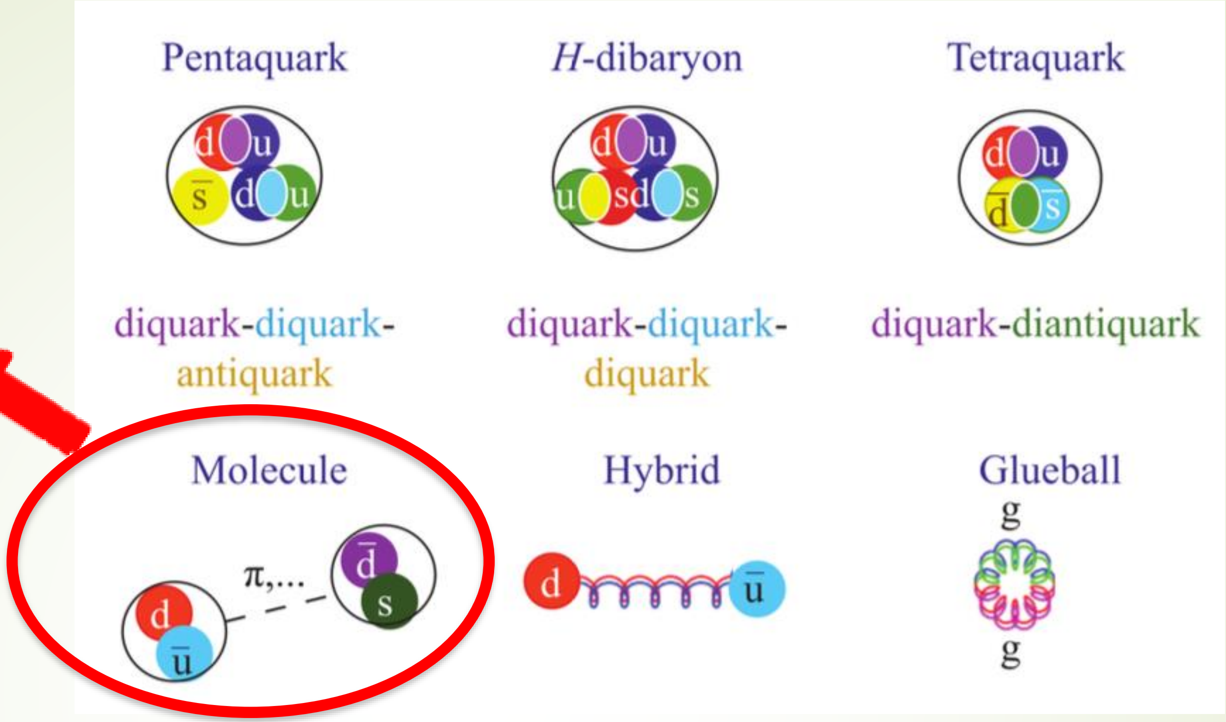




S. Acharya et al. (ALICE), Phys. Lett. B 822, 136708 (2021)

In the molecular picture

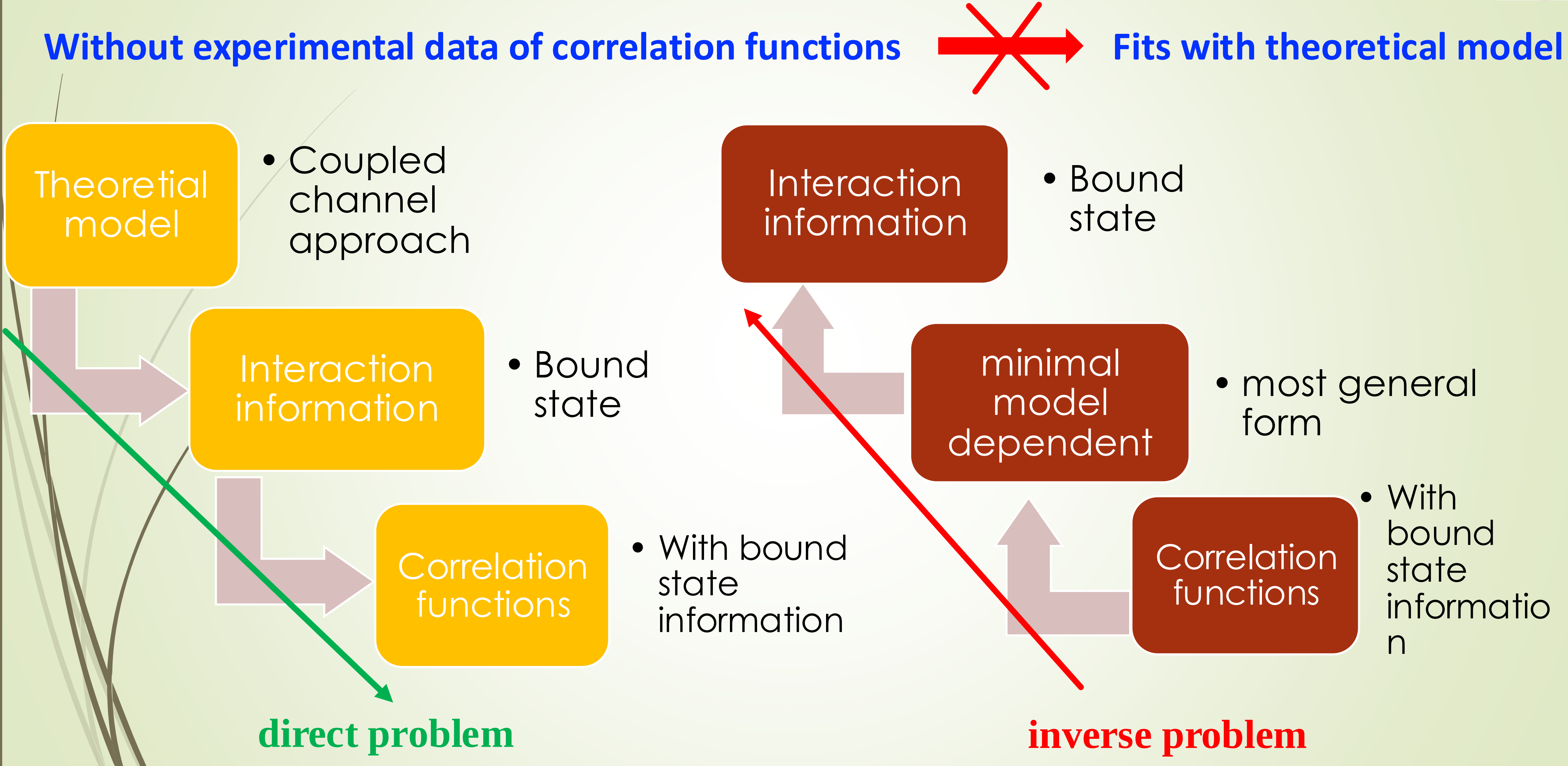
Correlation functions in coupled channels



One can obtain the information on the possible bound states contained in the correlation functions of these coupled channel components.

The information will be different, if the state is three quark states, or compact multiquark state, and so on.

(2) What is the inverse problem?



(3) How to do the inverse problem?

Assume an energy dependence interaction potential

$$V_{ij} = -\frac{1}{4f^2}\tilde{C}_{ij}(k_i^0 + k_j^0)$$

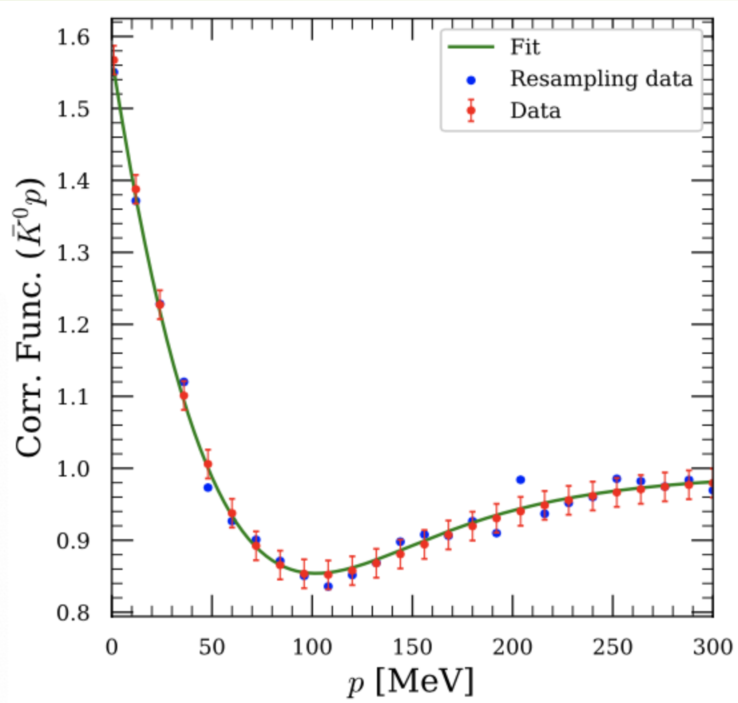
Under the isospin constrain

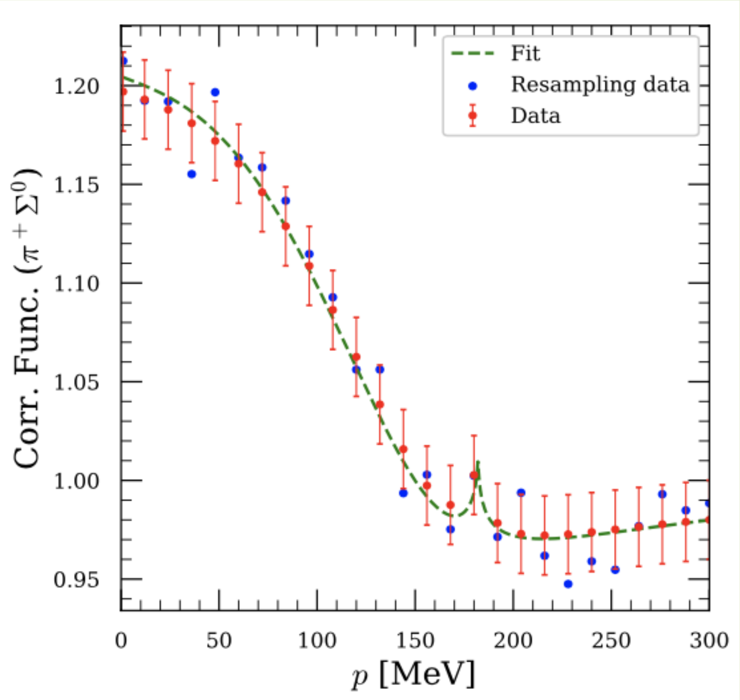
$\tilde{C}_{ij}$	$\bar{K}^0 p$	$\pi^+\Sigma^0$	$\pi^0\Sigma^+$	$\pi^+\Lambda$	$\eta\Sigma^+$
$\bar{K}^0 p$	$\tilde{C}_{11}$	$-\frac{1}{\sqrt{2}}\tilde{C}_{12}$	$\frac{1}{\sqrt{2}}\tilde{C}_{12}$	$-\tilde{C}_{14}$	$-\tilde{C}_{15}$
$\pi^+\Sigma^0$		$\frac{1}{2}(\tilde{C}_{22} + \tilde{C}'_{22})$	$\frac{1}{2}(-\tilde{C}_{22} + \tilde{C}'_{22})$	$\frac{1}{\sqrt{2}}\tilde{C}_{24}$	$\frac{1}{\sqrt{2}}\tilde{C}_{25}$
$\pi^0\Sigma^+$			$\frac{1}{2}(\tilde{C}_{22} + \tilde{C}'_{22})$	$-\frac{1}{\sqrt{2}}\tilde{C}_{24}$	$-\frac{1}{\sqrt{2}}\tilde{C}_{25}$
$\pi^+\Lambda$				$\tilde{C}_{44}$	$\tilde{C}_{45}$
$\eta\Sigma^+$					$\tilde{C}_{55}$

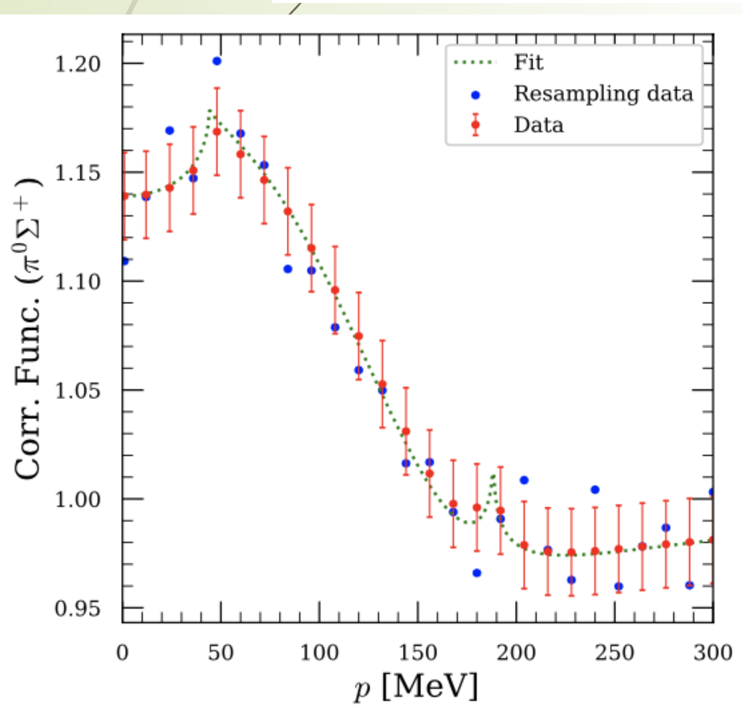
11 free parameters + R + q_{max} : 13 totally

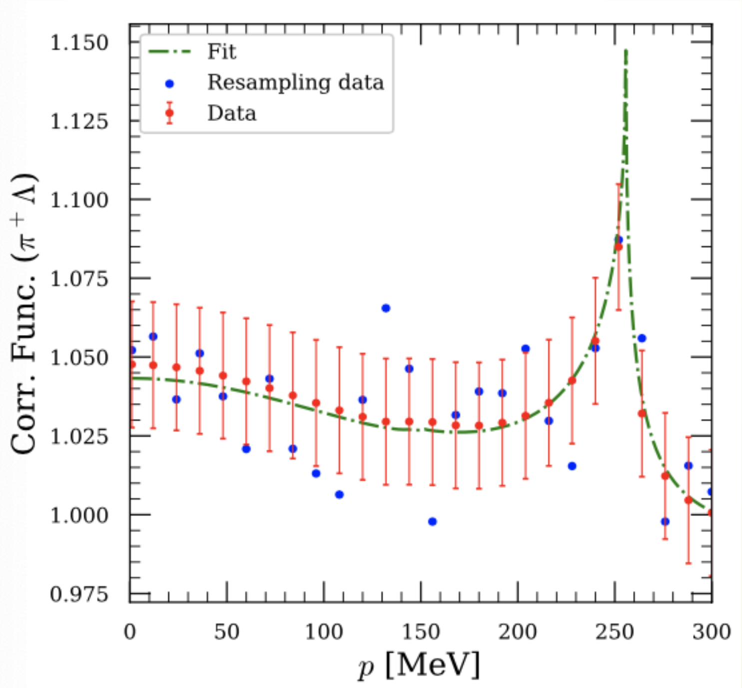
Using the bootstrap or resampling method → **Doing 50 fits**

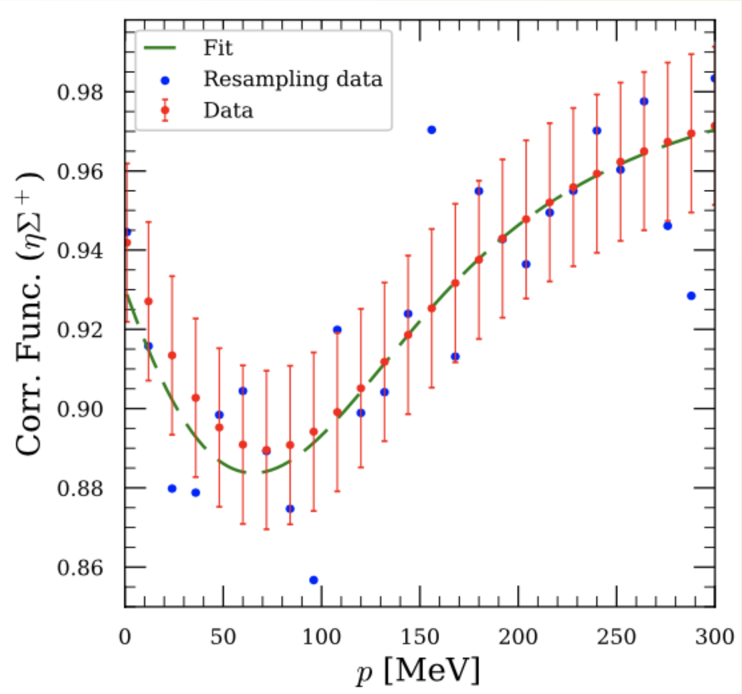
Generating random centroids of the data


Fit
Resampling data
Data
Corr. Func. $(\bar{K}^0 p)$
p [MeV]


Fit
Resampling data
Data
Corr. Func. $(\pi^+ \Sigma^0)$
p [MeV]


Fit
Resampling data
Data
Corr. Func. $(\pi^0 \Sigma^+)$
p [MeV]


Fit
Resampling data
Data
Corr. Func. $(\pi^+ \Lambda)$
p [MeV]


Fit
Resampling data
Data
Corr. Func. $(\eta \Sigma^+)$
p [MeV]

Average of the fitted parameters

$\tilde{C}_{11}$	$\tilde{C}_{12}$	$\tilde{C}_{14}$	$\tilde{C}_{15}$	$\tilde{C}_{22}$
1.036 ± 0.261	-0.985 ± 0.138	-1.204 ± 0.220	-0.829 ± 0.406	1.924 ± 0.147
$\tilde{C}'_{22}$	$\tilde{C}_{24}$	$\tilde{C}_{25}$	$\tilde{C}_{44}$	$\tilde{C}_{45}$
-2.136 ± 0.465	-0.057 ± 0.342	-0.028 ± 0.571	-0.053 ± 0.141	-0.066 ± 0.706
$\tilde{C}_{55}$	$q_{\max}(\text{MeV})$	$R(\text{fm})$		
0.043 ± 0.447	653.468 ± 63.802	0.995 ± 0.029		

Observables： the scattering length and effective range

$$\frac{1}{a_i} = \frac{8\pi\sqrt{s}}{2M_i}(T_{ii})^{-1}\Big|_{\sqrt{s}_{\text{th},i}}$$

$$r_i = \frac{1}{\mu_i}\frac{\partial}{\partial\sqrt{s}}\left[\frac{-8\pi\sqrt{s}}{2M_i}(T_{ii})^{-1} + ik_i\right]_{\sqrt{s}_{\text{th},i}}$$

$$\sqrt{s}_p = (1420 \pm 10) - i(101 \pm 19)\ \text{MeV}$$

J. A. Oller and U.-G. Meißner, Phys. Lett. B 500, 263 (2001)
D. Jido, J. A. Oller, E. Oset, A. Ramos and U.-G. Meißner, Nucl. Phys. A 725, 181 (2003)

Average of the scattering lengths

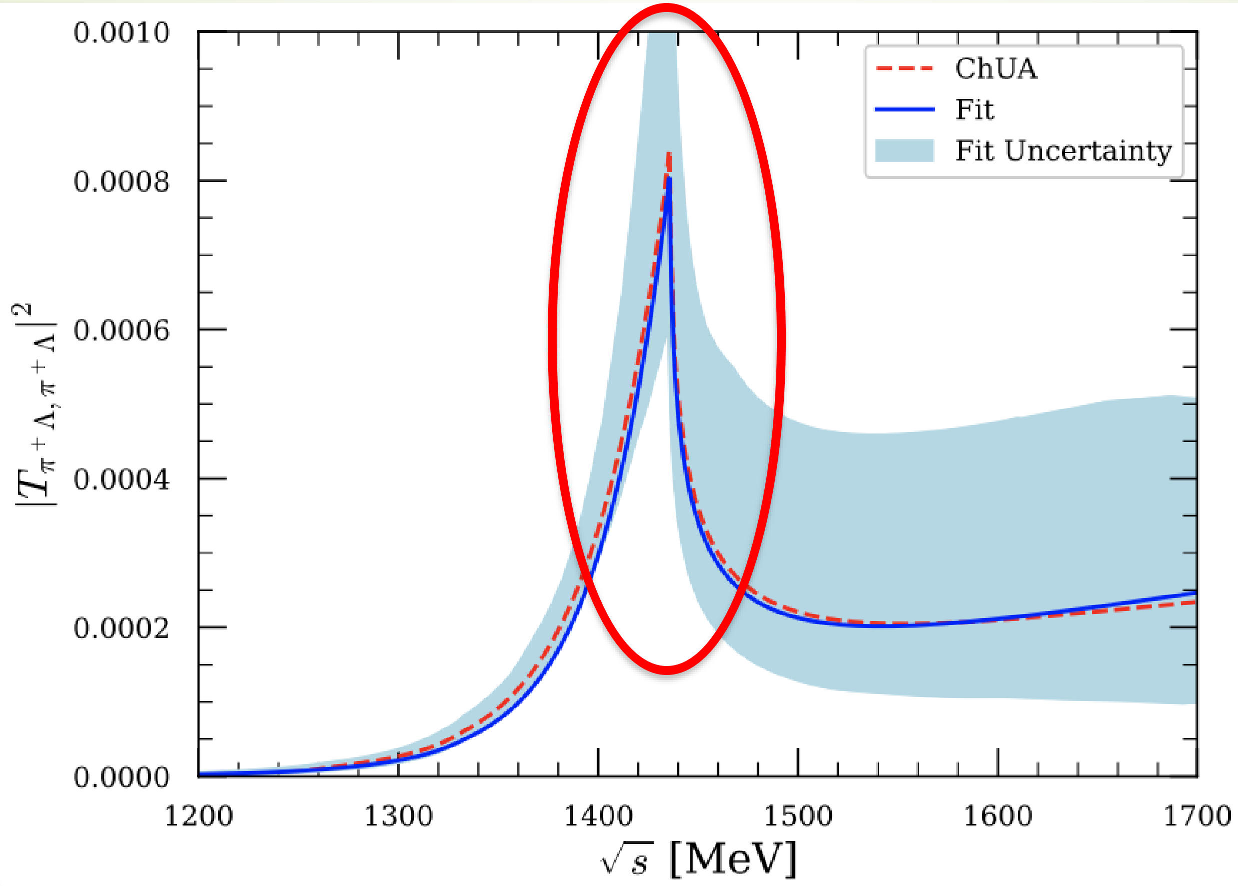
a_1	a_2	a_3
$(0.468 \pm 0.088) - i(1.130 \pm 0.041)$	$-(0.148 \pm 0.010) - i(0.030 \pm 0.004)$	$-(0.113 \pm 0.010) - i(0.004 \pm 0.003)$
a_4	a_5	
$-(0.045 \pm 0.008)$	$(0.083 \pm 0.010) - i(0.161 \pm 0.026)$	

Average of the effective ranges

r_1	r_2	r_3
$(0.025 \pm 0.150) - i(0.452 \pm 0.089)$	$-(38.019 \pm 6.345) - i(16.534 \pm 1.932)$	$-(75.053 \pm 17.150) + i(1.143 \pm 1.456)$
r_4	r_5	
$-(75.035 \pm 19.508)$	$(0.334 \pm 0.761) + i(0.380 \pm 0.947)$	

Consistent with the theoretical results before

A cusp- like structure



§4. Summary

- We use the chiral unitary approach to dynamically generate the state $\Sigma^*(1430)$
- Taking the pseudo data from theory, we use the resampling method for the inverse problem in the fitting of the correlation functions.
- The existing of this resonance can be tested by the information from the correlation functions.

Hope future experiments bring more clarifications on these issues…….

Thanks for your attention!

感谢大家的聆听！