Classification of eigenstates in coupled-channel scattering amplitude with the chiral unitary method

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T.Nisihibuchi and T.Hyodo, Phys. Rev. C **109**, no1, 015203 (2024)

Recent results for $\Xi(1620)$

Recently new results of $\Xi(1620)$ are obtained

Belle experiment of $\Xi_c \to \pi\pi\Xi$ (2019)[1]

Ξ excited states are observed in $\pi^+\Xi^-$ spectrum.

The mass M_R and width Γ_R of $\Xi(1620)$

$$M_R = 1610.4 \pm 6.0(\mathrm{stat.})^{+6.1}_{-4.2}(\mathrm{syst.})\ \mathrm{MeV}$$

$$\Gamma_R = 59.9 \pm 4.8(\mathrm{stat.})^{+2.8}_{-7.1}(\mathrm{syst.})\ \mathrm{MeV}$$

ALICE experiment(2021)[2]

The scattering length f_0 of $K^-\Lambda$ was determined with femtoscopy in Pb-Pb collisions

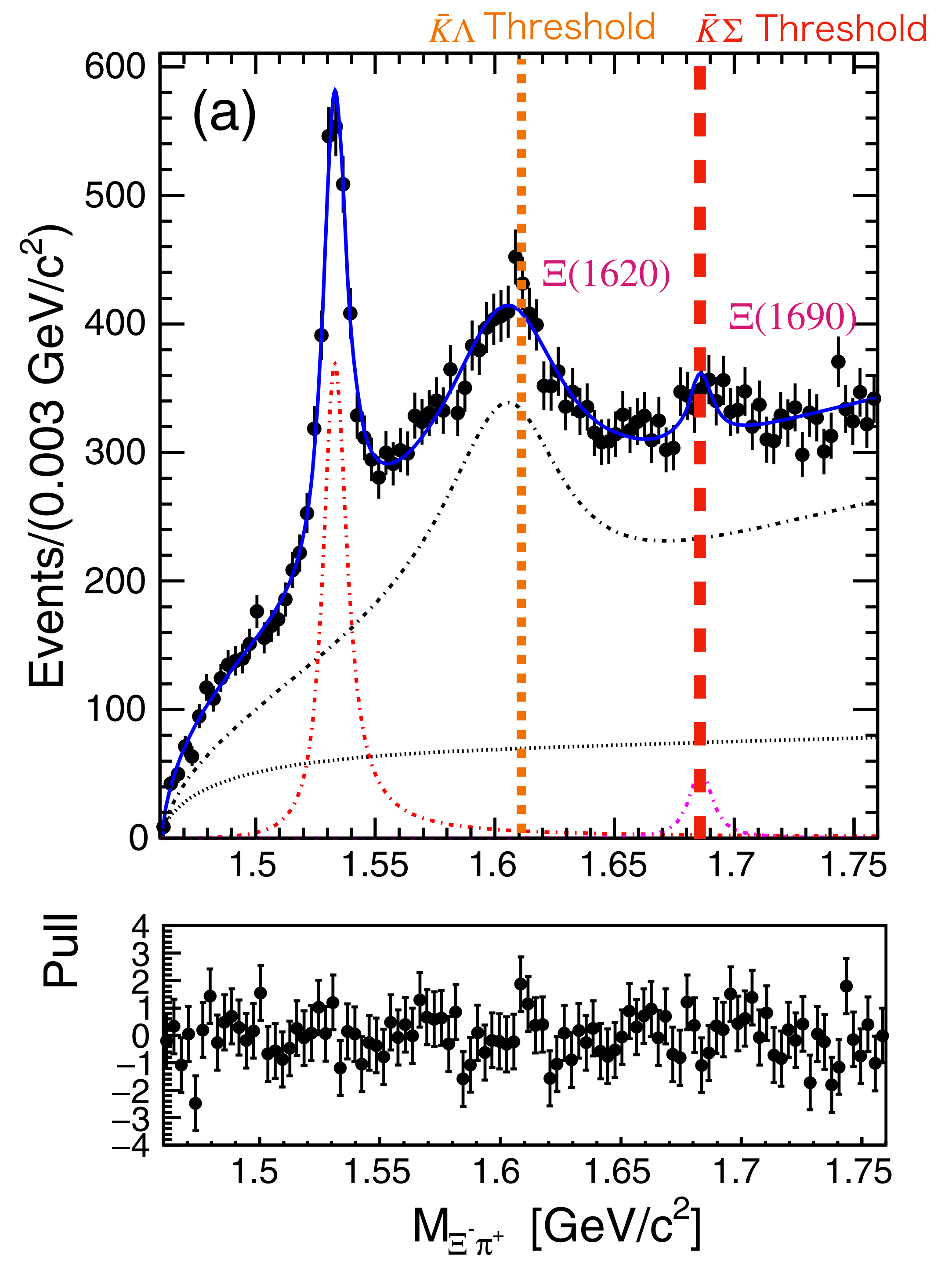


fig1.$\pi^+\Xi^-$ spectrum in $\Xi_c \to \pi\pi\Xi$ decay[1].

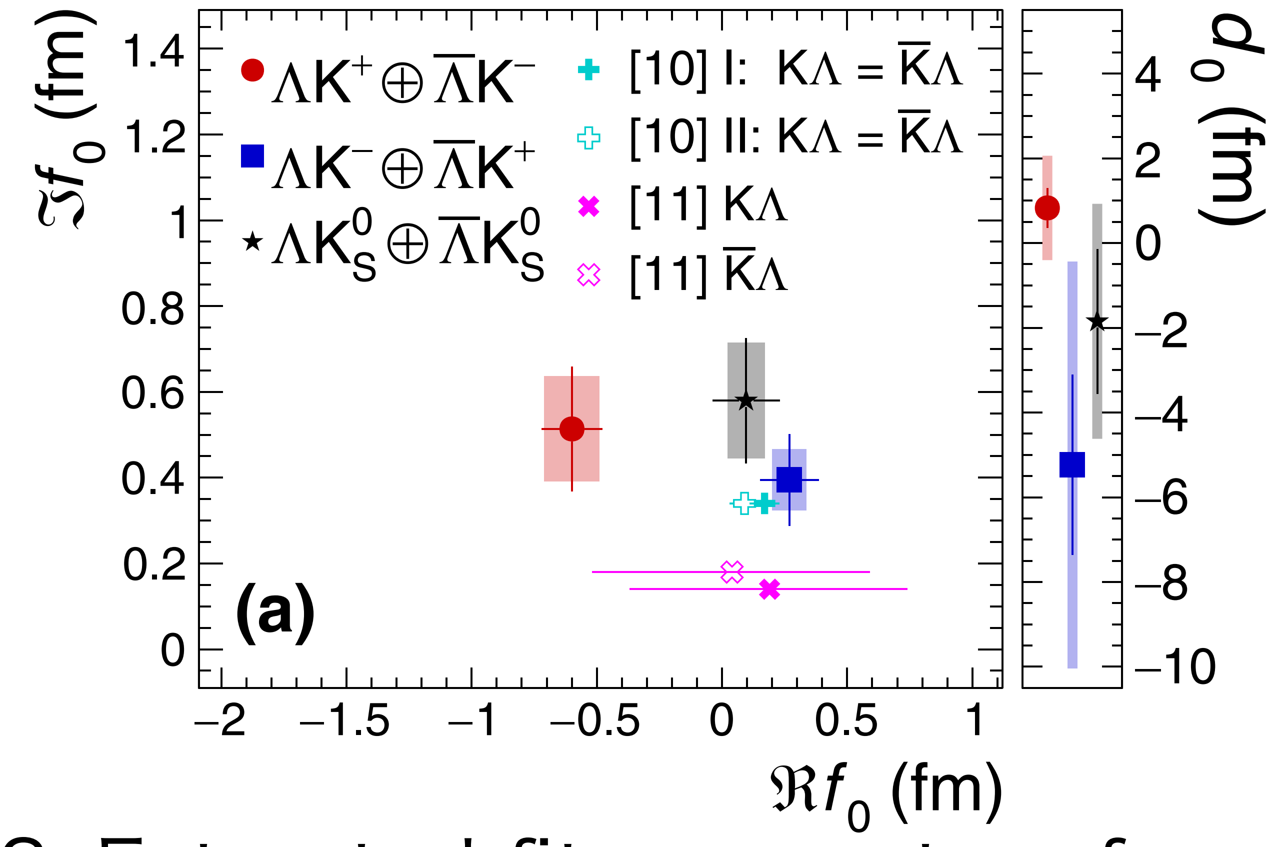


fig2. Extracted fit parameters for all of the $\bar{K}\Lambda$ system [2].

[1]Belle collaboration, M.Sumihama et al., Phys. Rev. Lett. **122**, 072501 (2019).

[2]S. Acharya et al. (ALICE Collaboration)Phys. Rev. C **103**, 055201(2021).

Outline

- **Construction of models(Model 1/Model 2)**[3]

We expected Model 1 as QB and Model 2 as QV. (eigenstates with decay widths, respectively)

- **The Change of $B \to V$ with decay width**

Before apply to Model 1 and Model 2, we confirm the general change

- **Model extrapolation(Model 1/Model 2)**

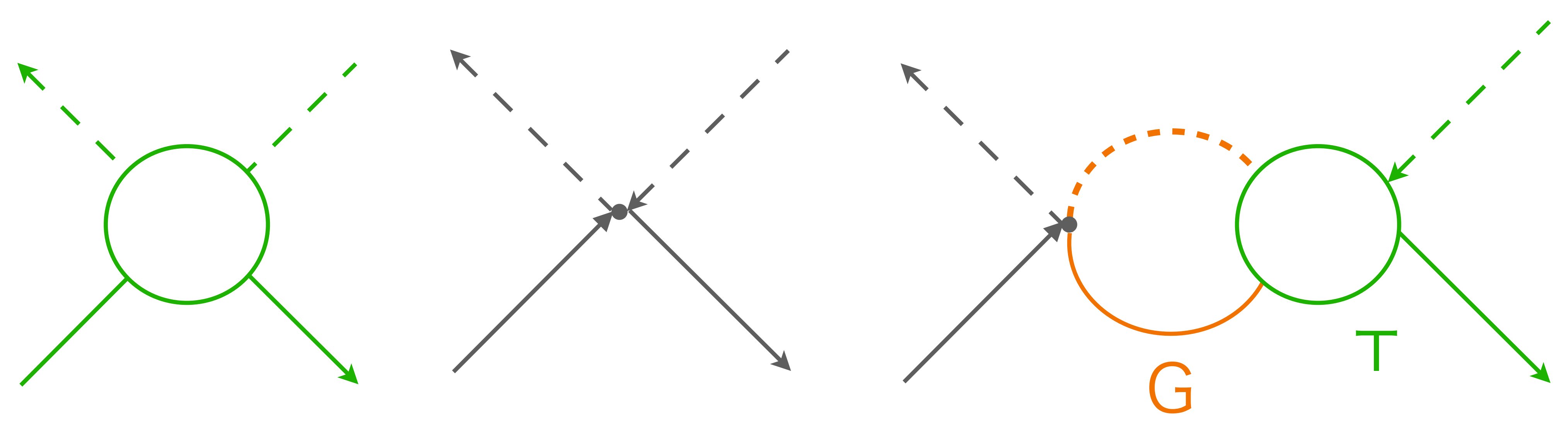
[3]T.Nisihibuchi and T.Hyodo, Phys. Rev. C **109**, no1, 015203 (2024)

Formulation of the scattering model

The scattering length $T_{ij}(W)$

satisfies the scattering equation.

$$T_{ij}(W) = V_{ij}(W) + V_{ik}(W)G_k(W, a_i)T_{kj}(W)$$



Interaction kernel $V_{ij}(W)$:Weinberg-Tomozawa interaction

$$V_{ij}(W) = -\frac{C_{ij}}{4f_if_j}N_iN_j(2W - M_i - M_j)$$

f_i:Meson decay constant, N_i: Kinematical coefficient,

C_{ij}:Group theoretical coefficient, M_i:Baryon Mass

Loop function $G_i(W)$ (Removed divergence by dimensional regularization)

$$G_i(W) \to G_i(W, a_i)$$

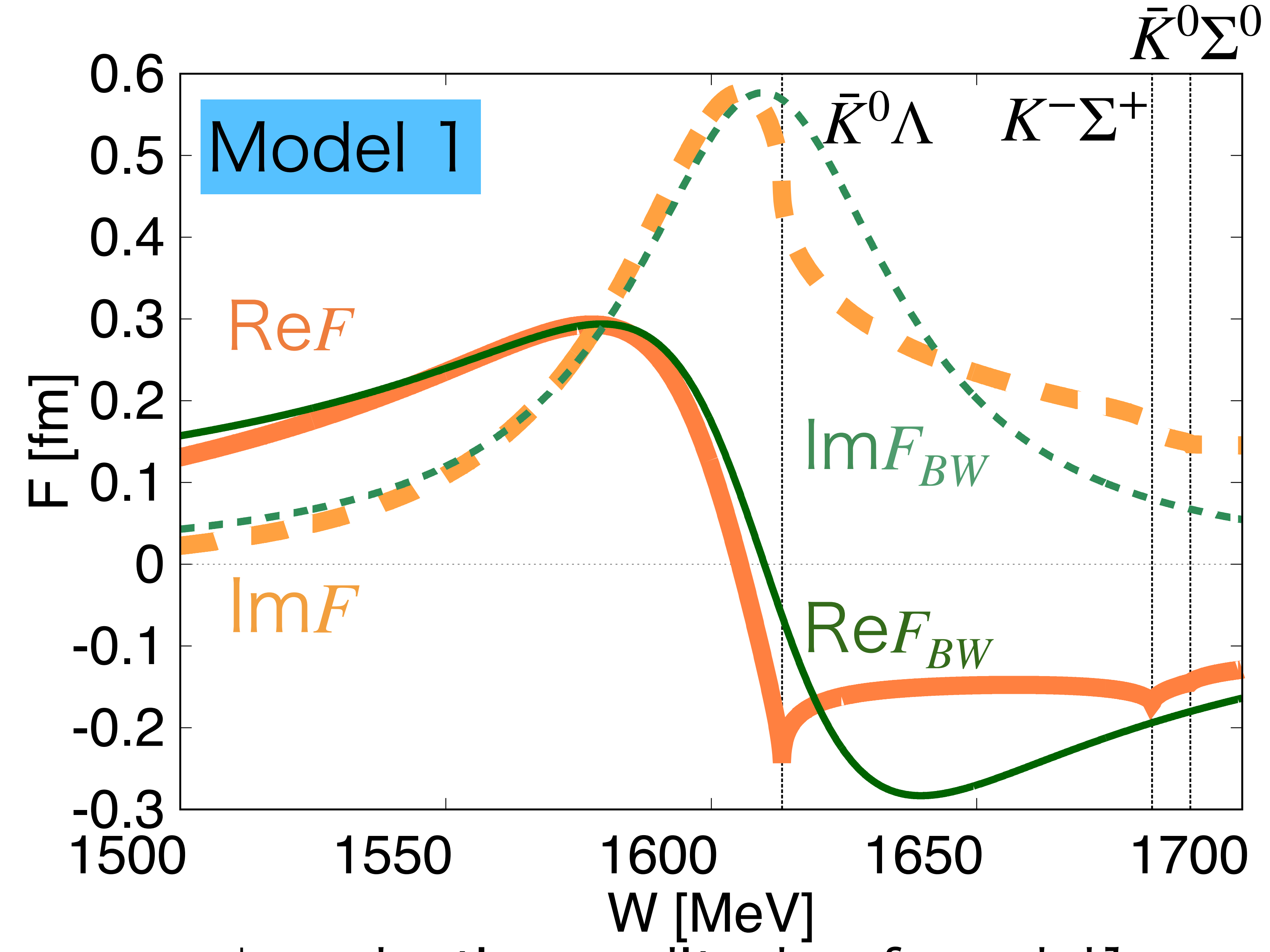
W:Total energy, a_i:subtraction constant

Construction theoretical models

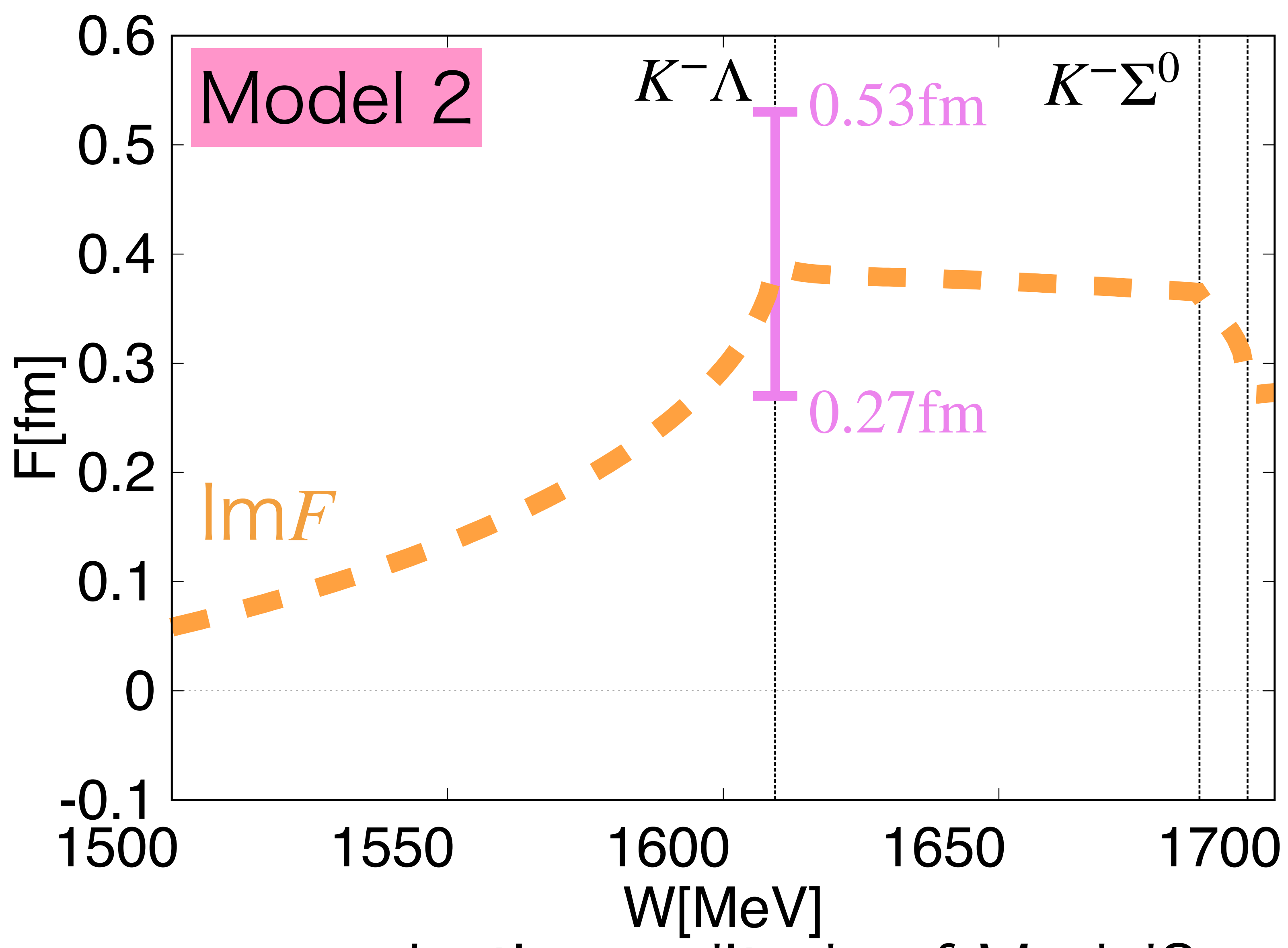
Construct the models which based on Belle and ALICE respectively

Model 1 Assume the pole position as $z_{\rm ex} = [1610 - 30i]$ MeV, and construct the model with the pole at $z_{\rm ex}$.

Model 2 Reproduce the $K^-\Lambda$ scattering length of ALICE.



$\pi^+\Xi^-$ elastic amplitude of model1 with BW amplitude



$K^-\Lambda$ elastic amplitude of Model2

Contracted by adjusting $a_{\pi\Xi}$ and $a_{\bar{K}\Lambda}$.

They have poles at different position each other

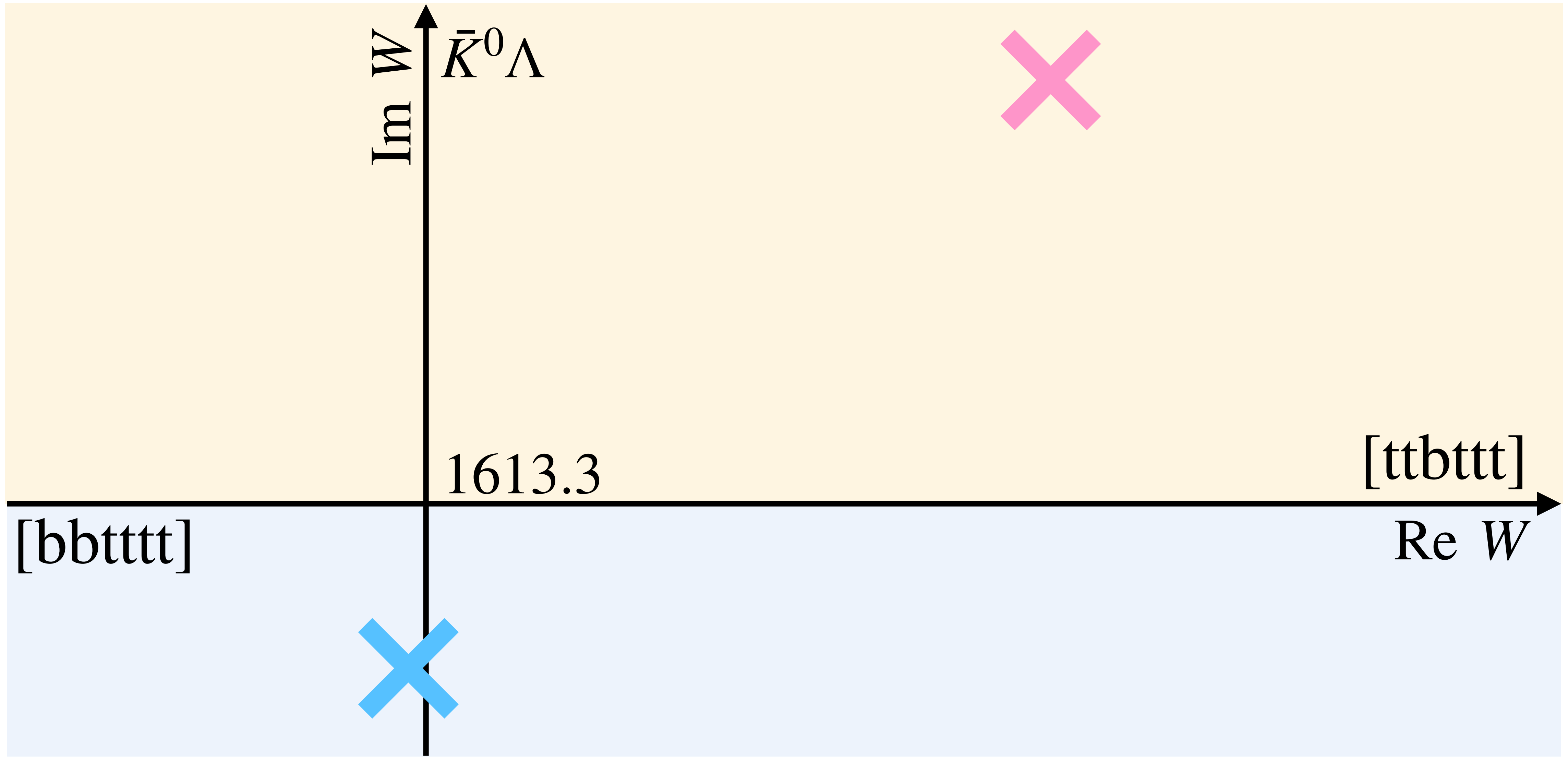
There are no cusps near $\bar{K}\Sigma$ threshold

[3]T.Nisihibuchi and T.Hyodo, Phys. Rev. C **109**, no1, 015203 (2024)

Poles of $\Xi(1620)$ in theoretical models

Pole position of each models as follows

	Pole of $\Xi(1620)$
Model 1	$z = 1610 - 30i$ MeV [bbtttt]
Model 2	$z = 1726 + 80i$ MeV [ttbttt]



→We consider that Model 1 pole of $\Xi(1620)$ as QB, Model 2 pole as QV.

We summarized pole classification(QB/QV) in latter slides

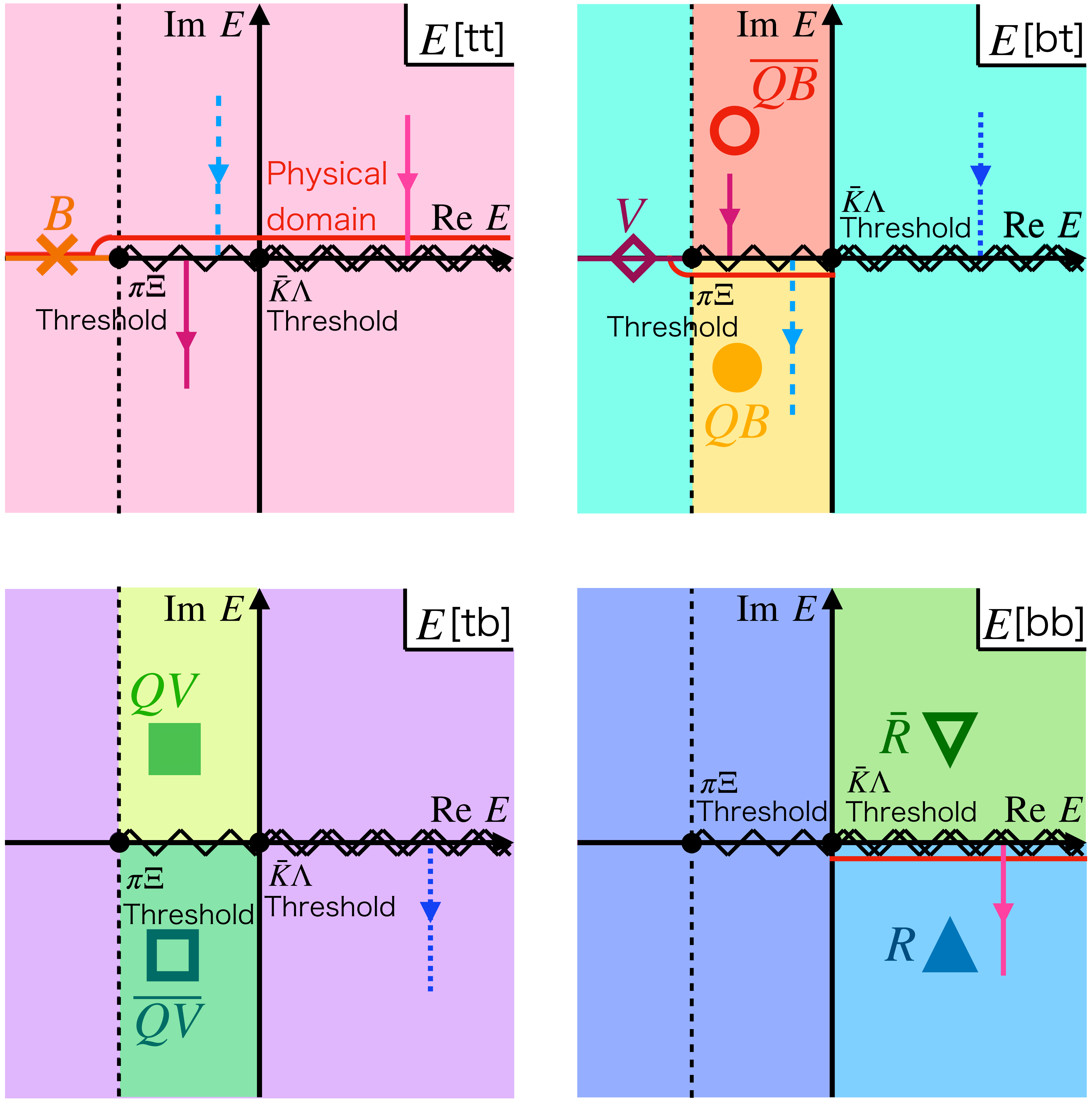
Eigenstates

- Riemann sheets at complex energy plane. The case of 2ch, then we have 4 Riemann sheets.

Classification of eigenstates

× B	Bound state	**Same as 1channel scattering**
◇ V	Virtual state	
▲ R	Resonance	

● QB	Quasi-Bound	**Bound and Virtual with decay width**
■ QV	Quasi-Virtual	



Riemann sheets of complex E plane([tt],[tb],[bt],[bb])

Pole trajectory in simplified system

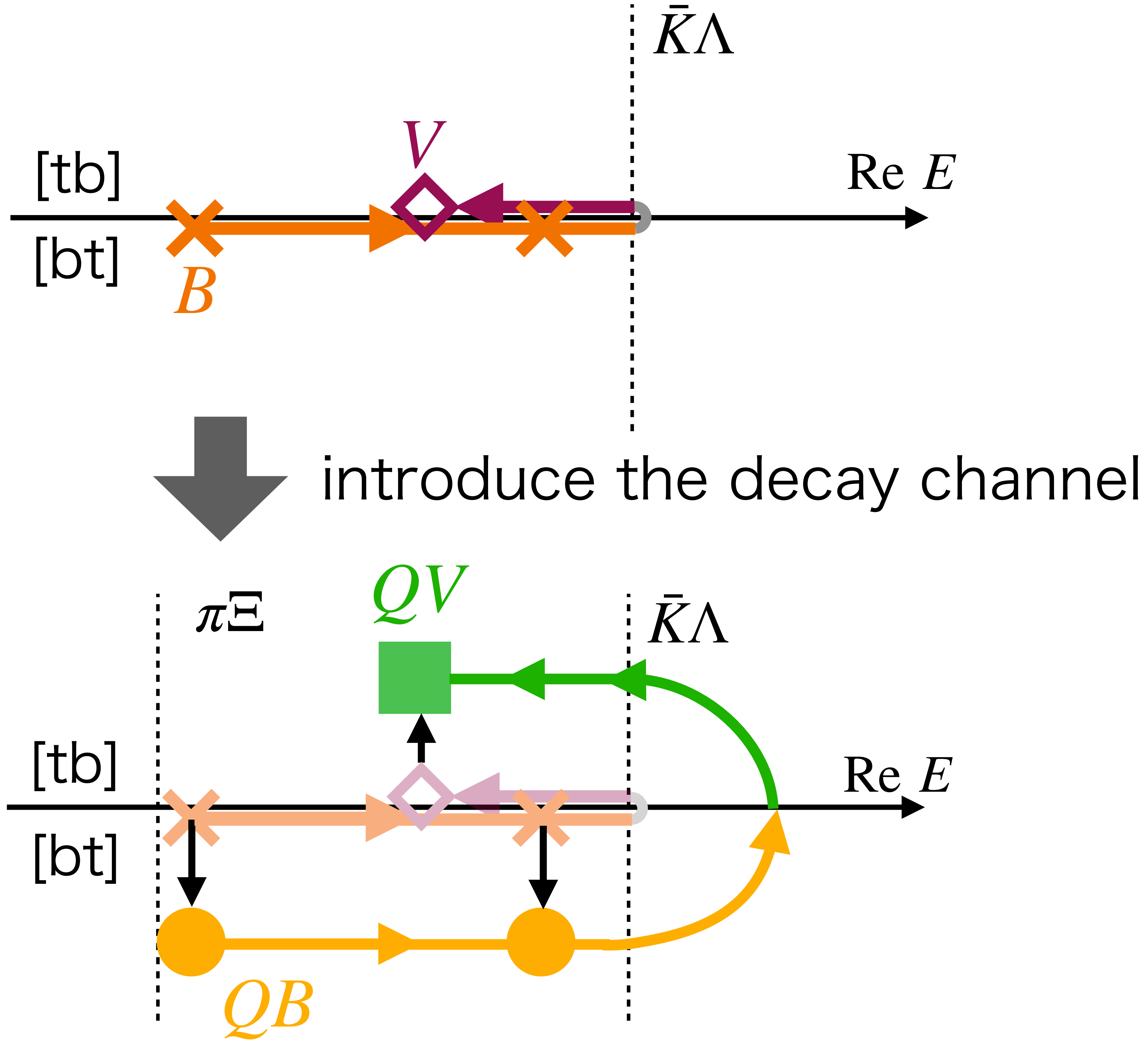
In 1channel scattering, pole trajectory on B→V is well known.

Now we introduce the decay channel to consider the pole trajectory QB→QV.

- We consider the 2 channel system with in mind the $\Xi(1620)$ resonance
- Changing a_i is corresponding to the changing interaction.

When we changing the $a_{\bar{K}\Lambda}$ continuously,

pole moving on real axis



B and V get the decay width

Pole trajectory in simplified system

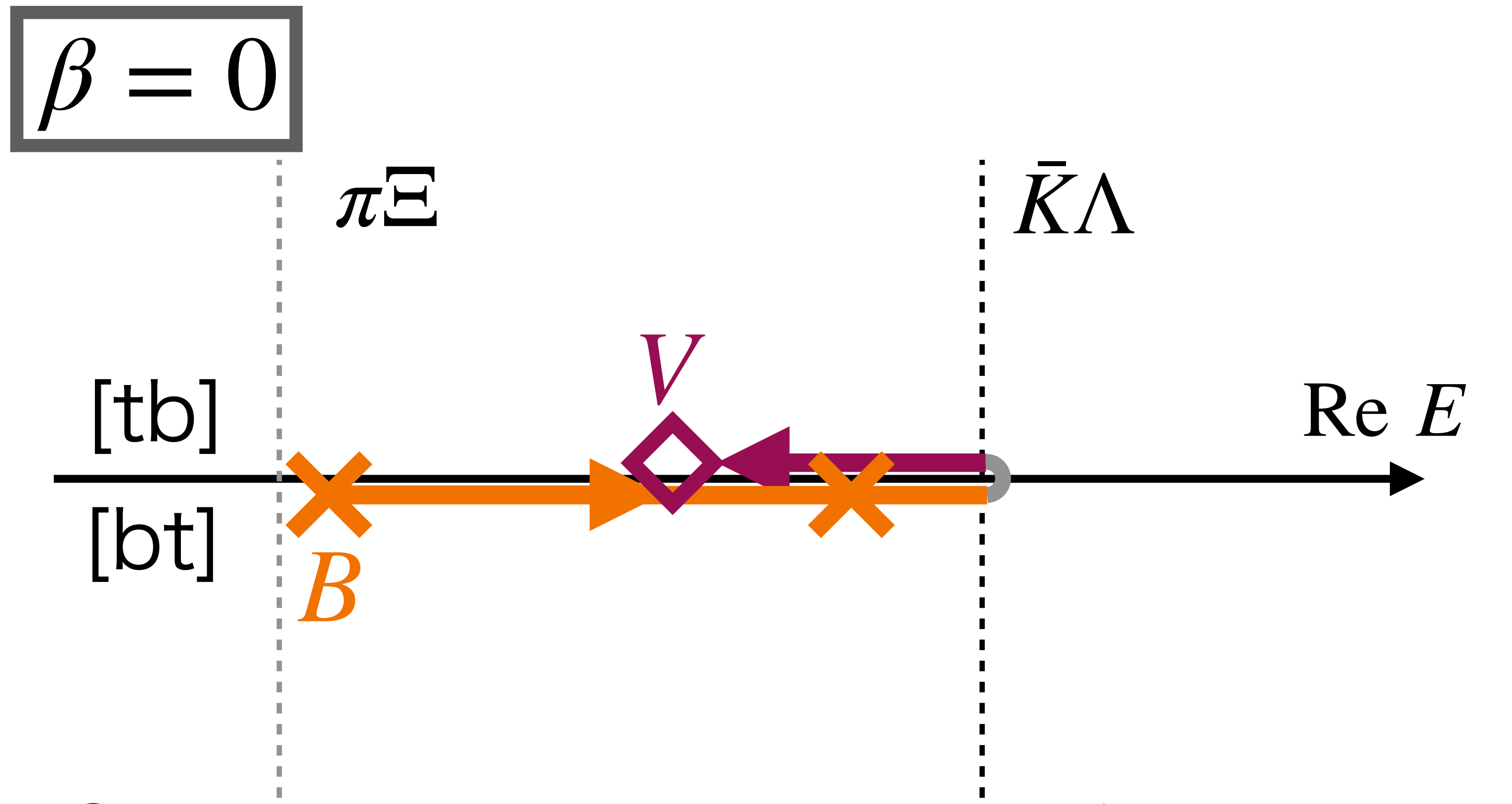
To introduce channel coupling, we rewrite interaction kernel V_{ij}.

- Rewrite the C_{ij} which is included of V_{ij} as shown in follows.

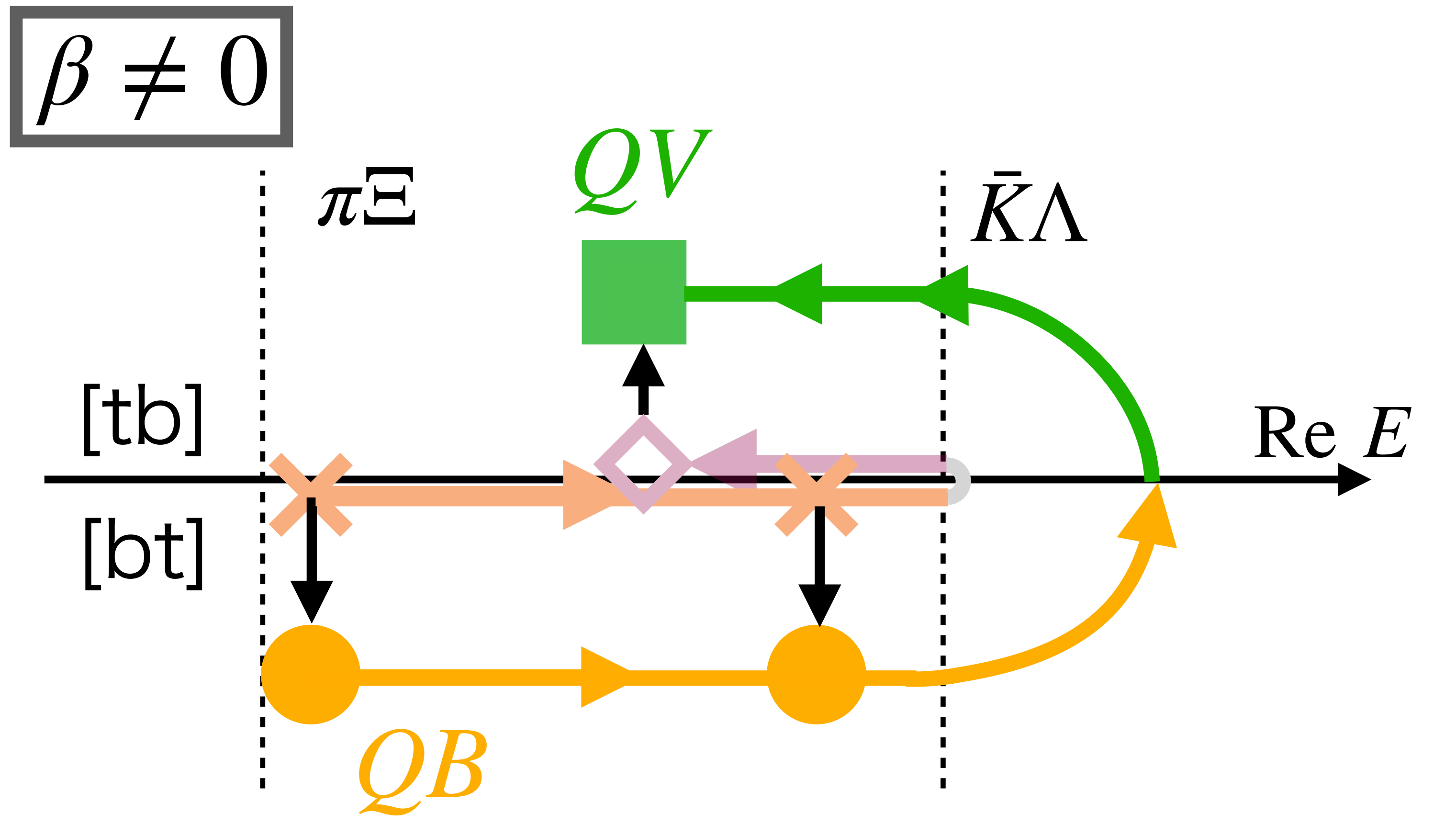
$$C_{ij} = \begin{pmatrix} 2 & \beta \\ \beta & 4 \end{pmatrix}$$

The strength of channel coupling is variable by adjusting β,

(When $\beta = 0$, there are no coupling channels.)



Same as 1ch scattering (no coupling)



Non-zero β represents the 2 channel scattering.

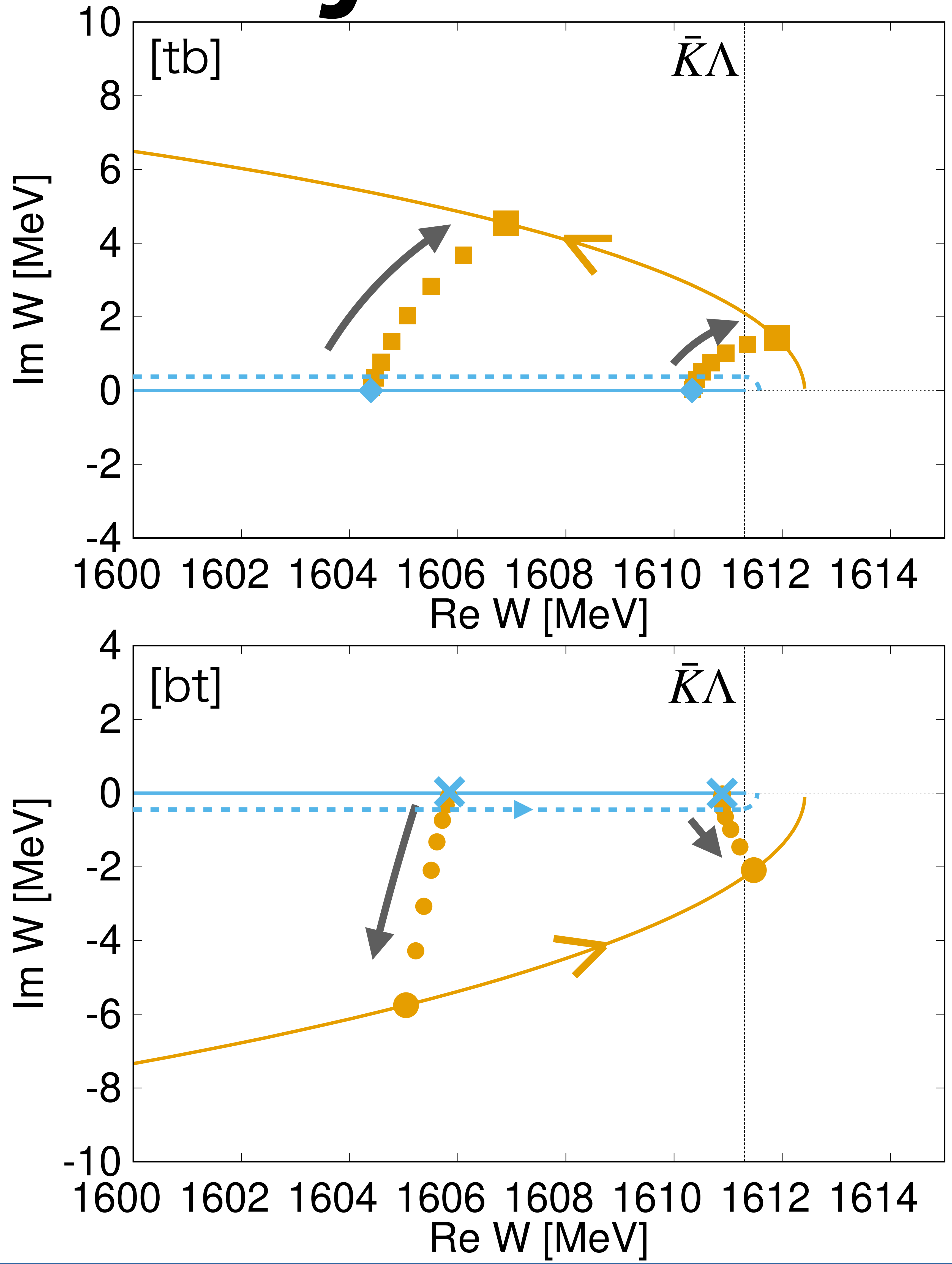
Pole trajectory in simplified system

Pole trajectories with $\beta = 0$ and $\beta = 0.5$

When $\beta = 0$, trajectory is same as 1ch.

When $\beta = 0.8$, pole acquire imaginary part as expected.

We can confirm the transition from QB to QV as expected.

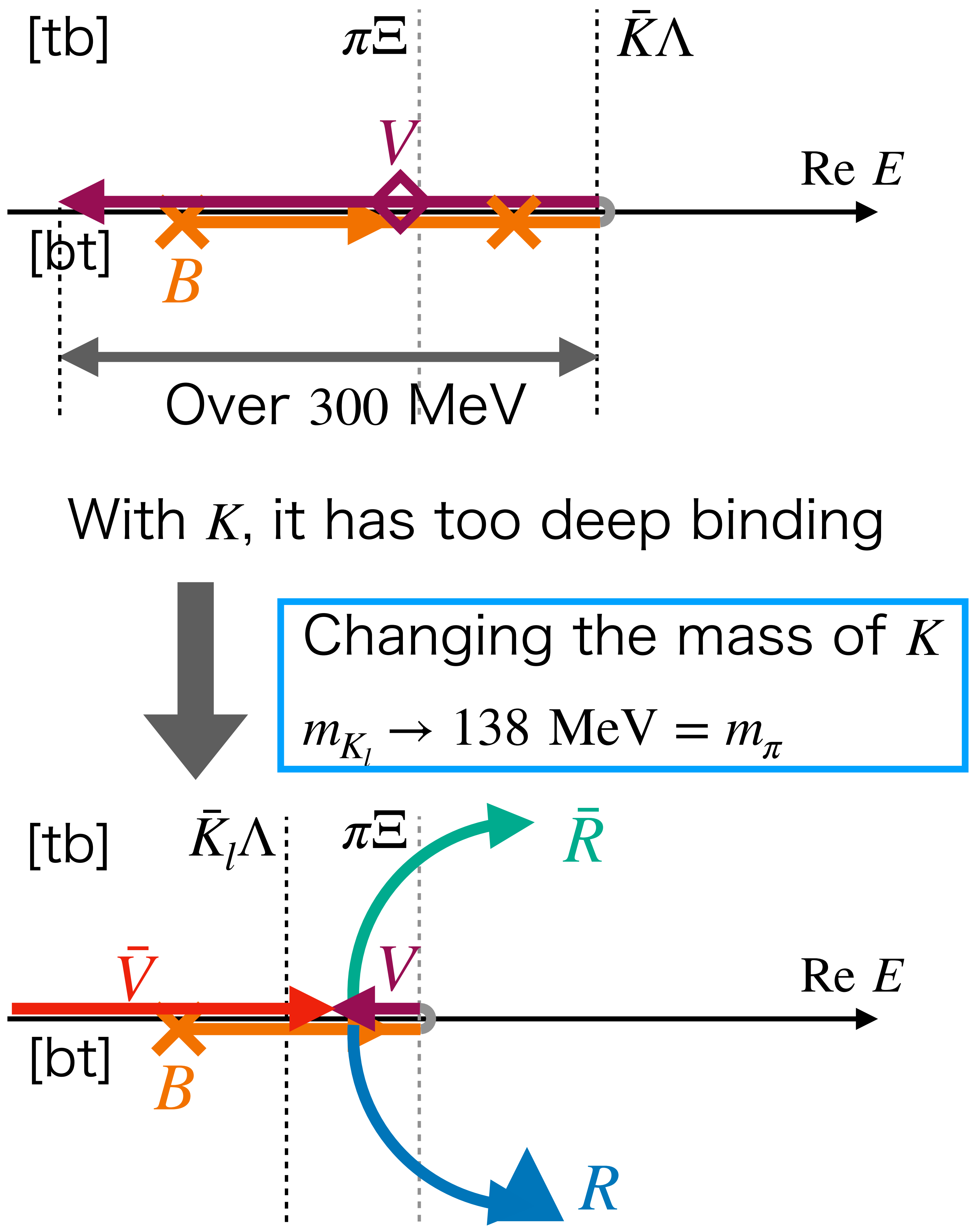


Pole trajectory in simplified system v2

Now, we consider extended pole trajectory B→V→R and the one with decay width.

But K has too deep binding,
so it is difficult to see the trajectory to R

- To see pole trajectory easily, making the $m_{\bar{K}}$ lighter (138 MeV $= m_\pi$)
- Introduce channel coupling (adjusting β)



Pole trajectory can be seen easily.

Pole trajectory in simplified system v2

Pole trajectories with $\beta = 0$, $\beta = 0.3$ and $\beta = 0.5$ in complex energy plane.

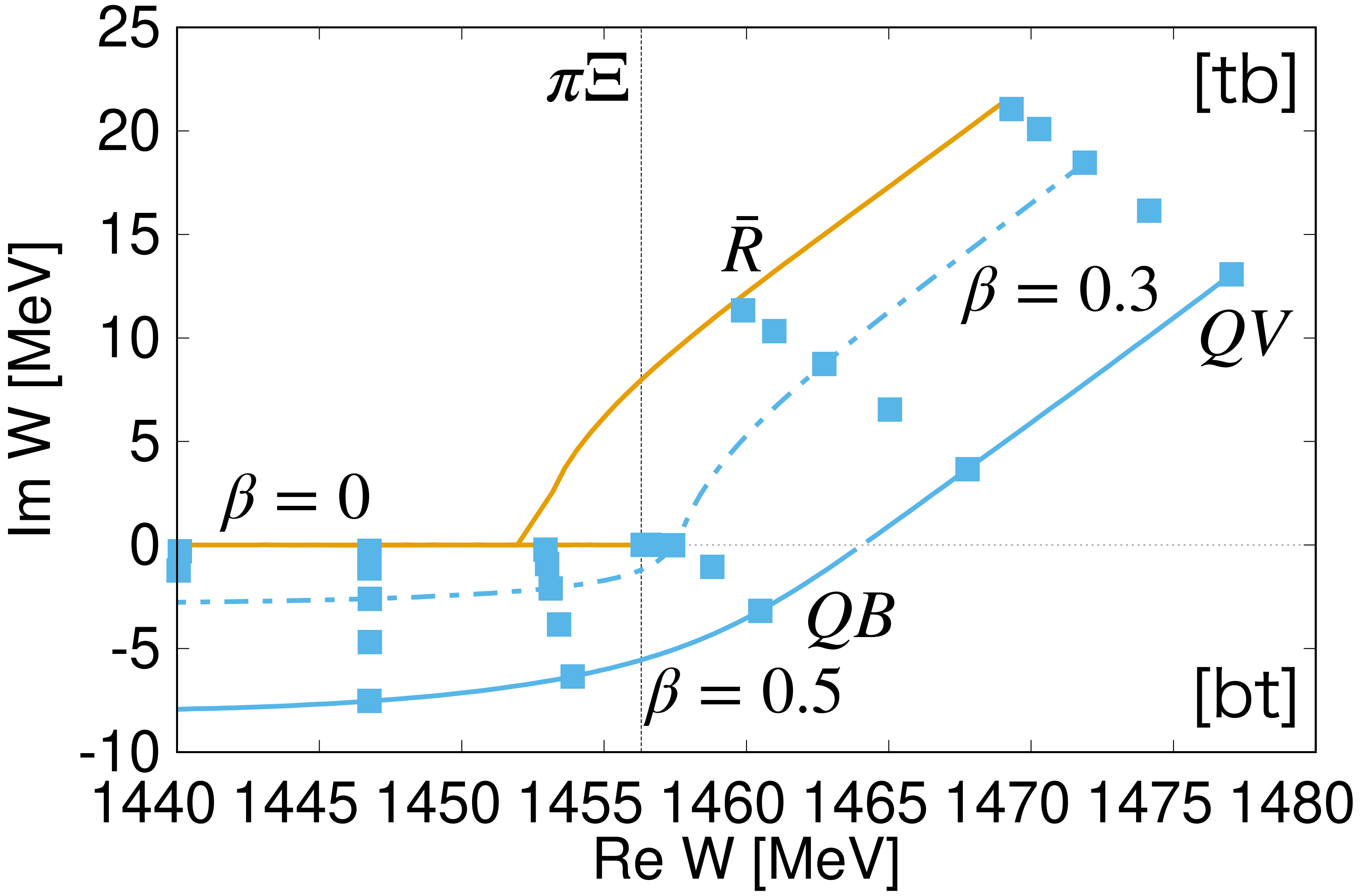
When $\beta = 0$,

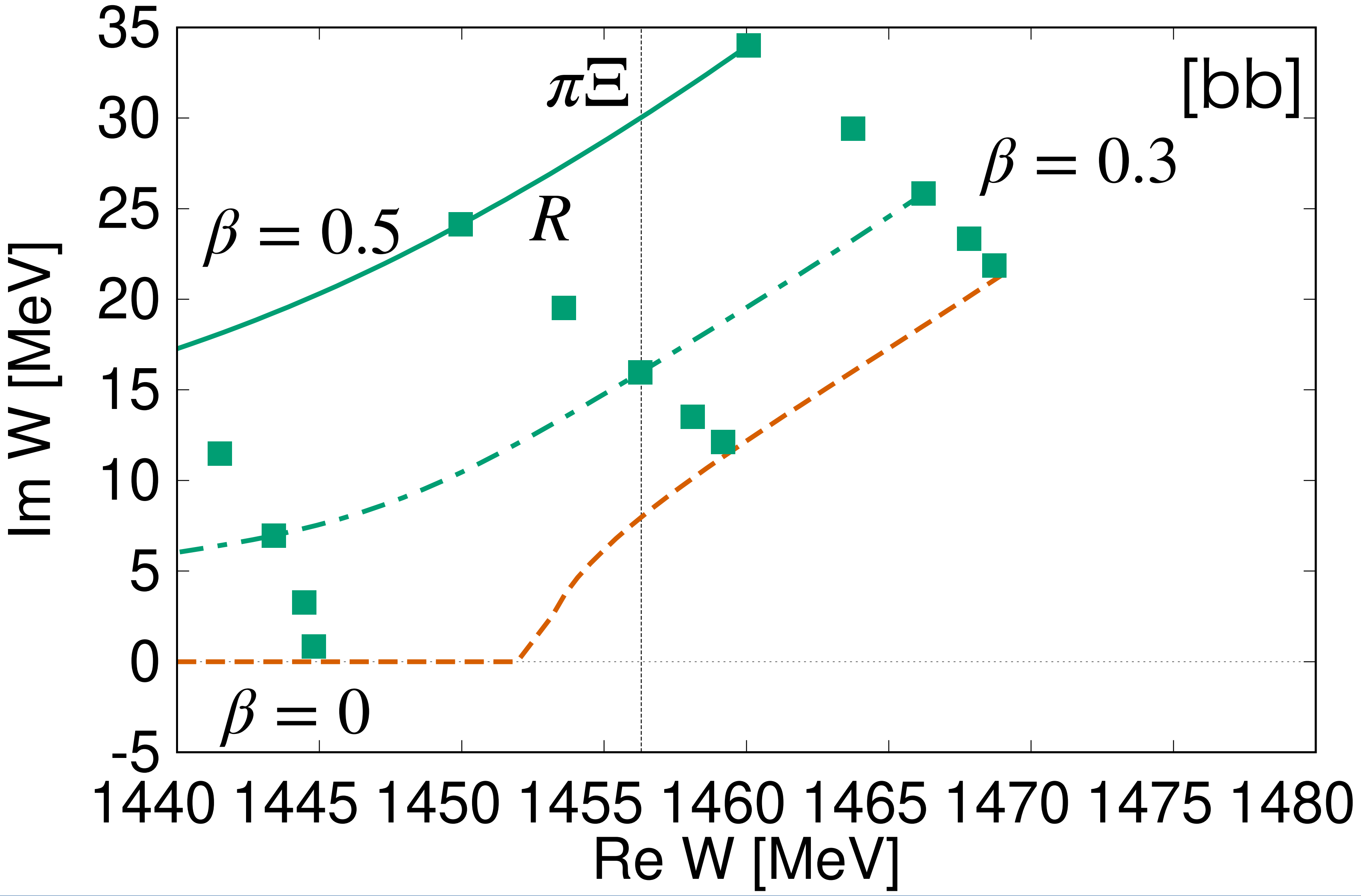
trajectory is same as 1channel scattering.

When $\beta = 0.3$ and $\beta = 0.5$

[tb]sheet : It shows the trajectory QB to QV

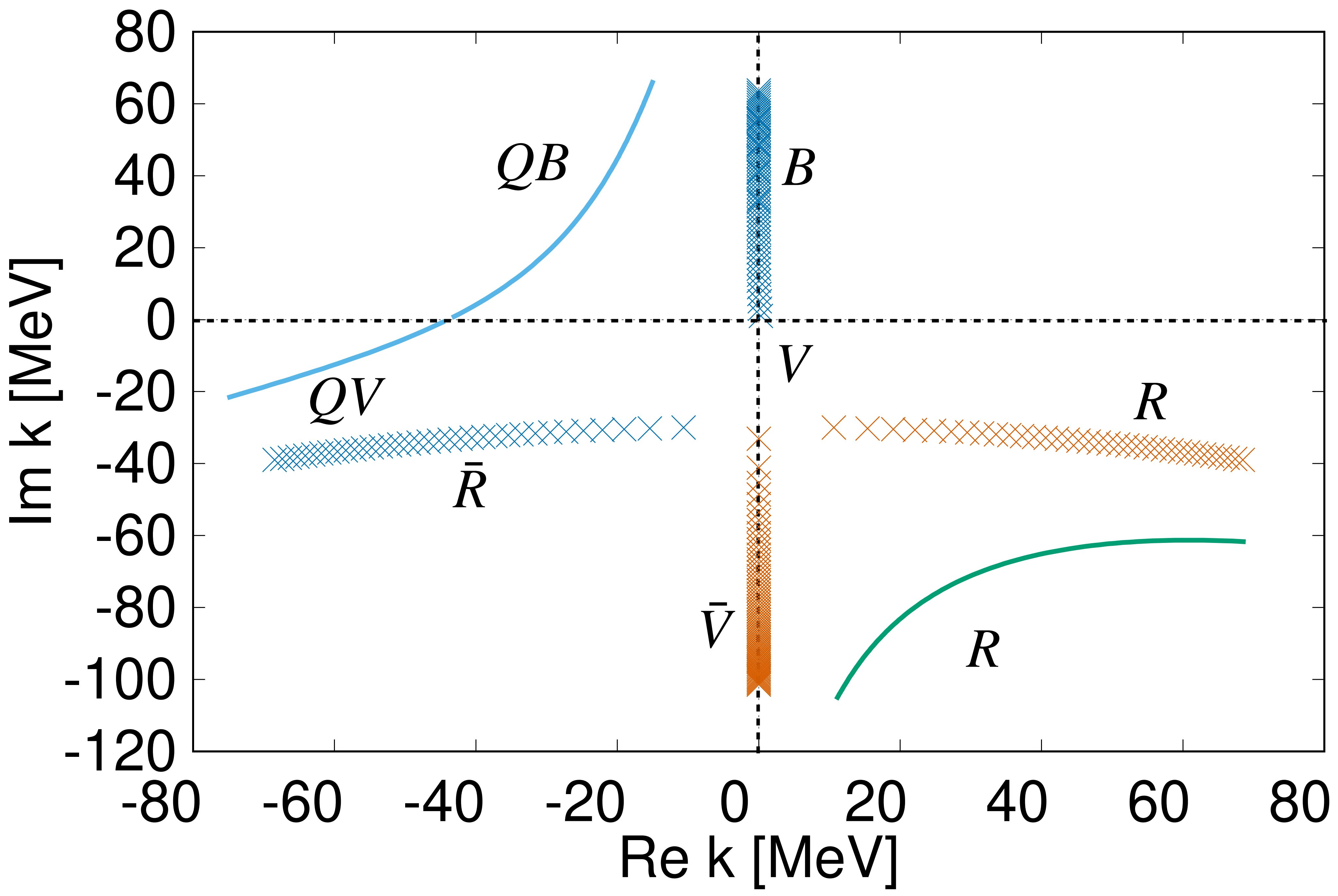
[bb]sheet : R pole exists





Pole trajectory in simplified system v2

To make easy to follow pole trajectory, we write pole trajectories with $\beta = 0$ and $\beta = 0.5$ with momentum.



Focus on $\beta = 0.5$

[tb]/[bt]sheet : It shows the trajectory QB to QV

[bb]sheet :R pole exists

There is no continuous between R and QV/QB.

Pole trajectory in actual models

From the previous result…

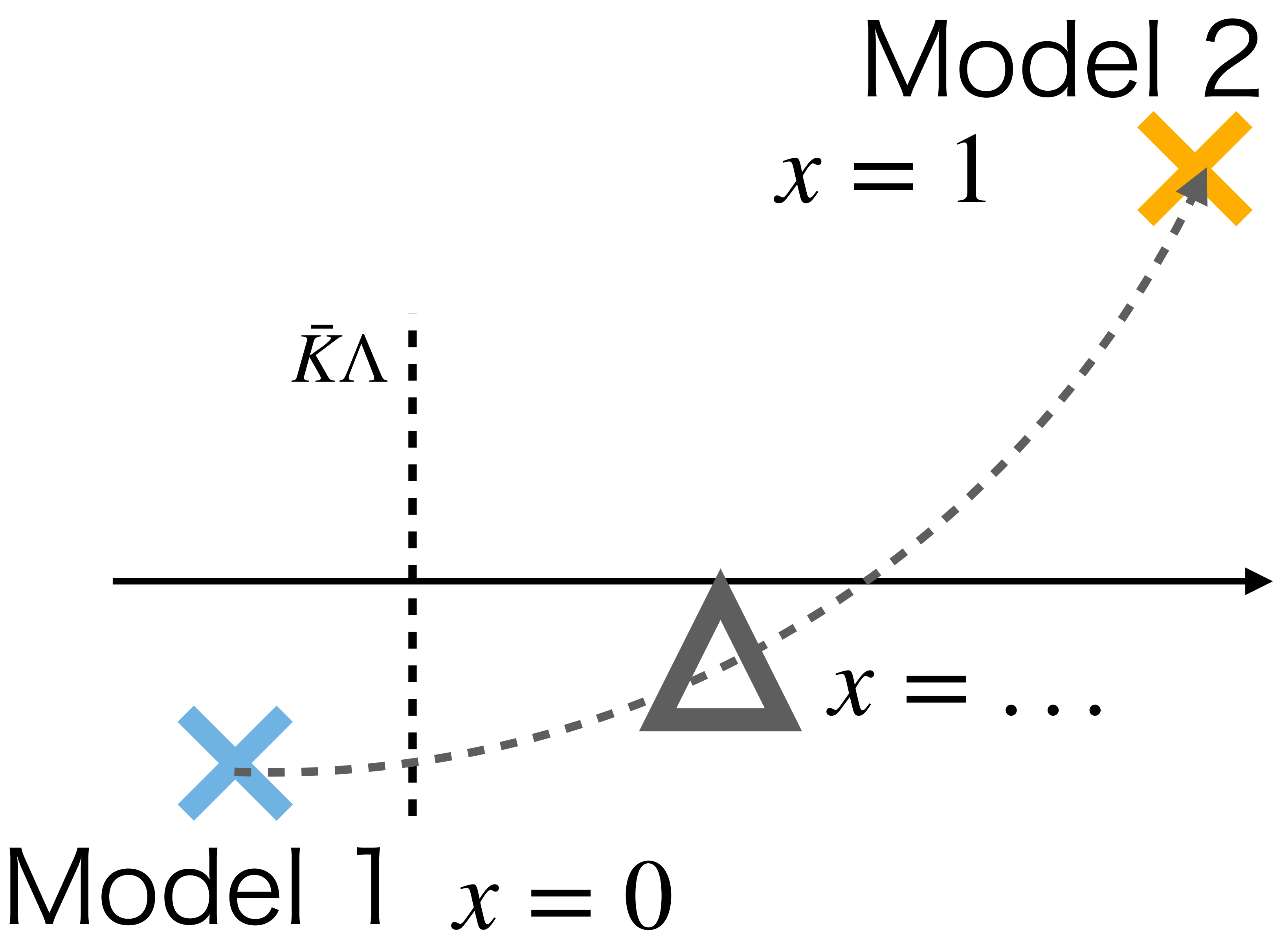
It expects Model 1 and Model 2 to be continuously connected, but how does the actual pole transition?

Model extrapolation by changing a_i

$$a_i(x) = x a_i'' + (1-x) a_i' \qquad (0 \le x \le 1)$$

a_i' ・・・subtraction constant of Model 1

a_i'' ・・・subtraction constant of Model 2



Extrapolation can be done by calculating the poles at each point and connecting them consecutively.

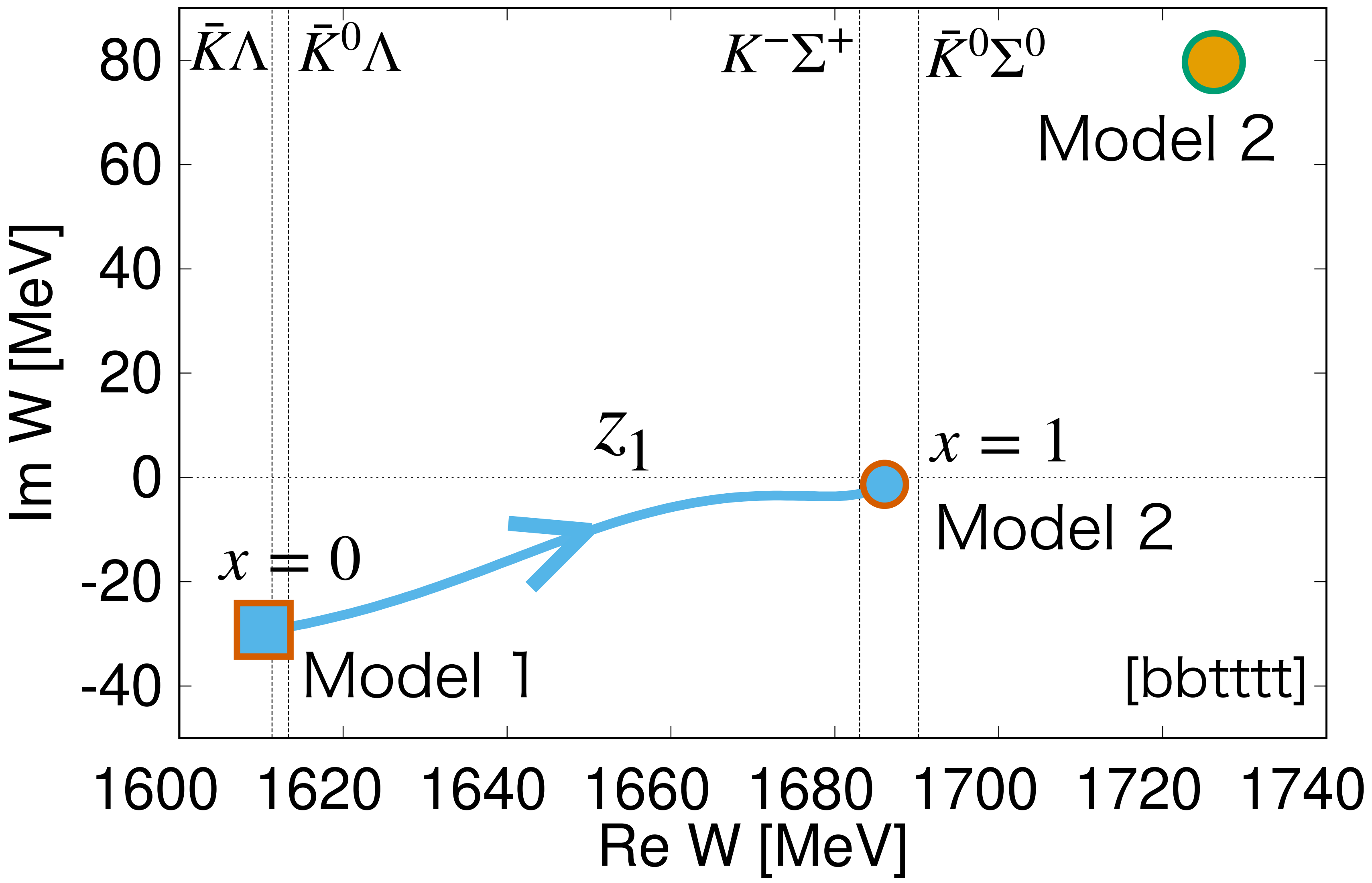
Pole trajectory in actual models

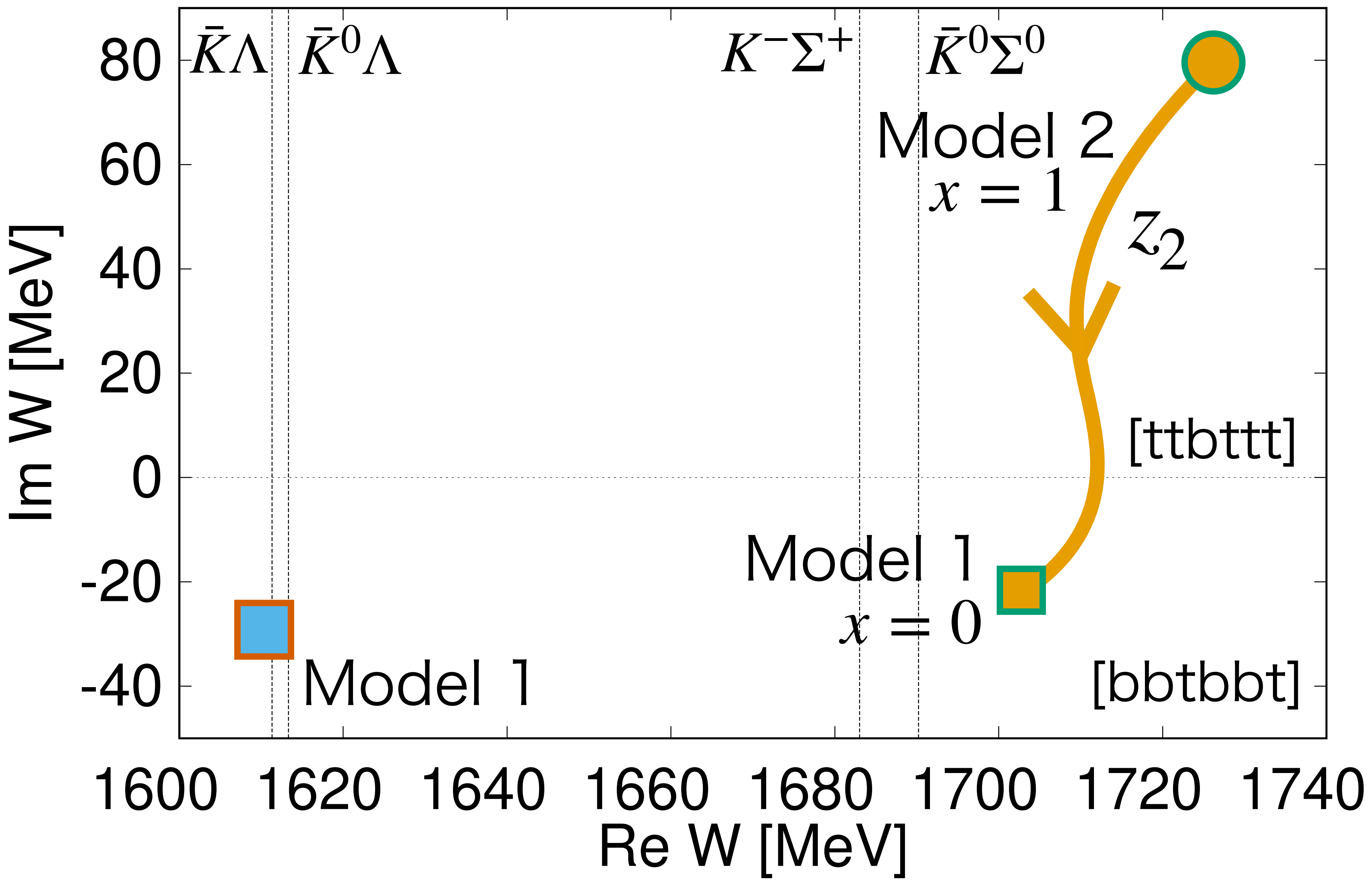
Two poles that are supposed $\Xi(1620)$ are not continuously connected.

z_1 : $\Xi(1620)$ pole of Model 1 [bbtttt]

z_2 : $\Xi(1620)$ pole of Model 2 [ttbttt]

This means that Model 1 and Model 2 poles have different physical origins.





Summary

In recent years, experimental data about $\Xi(1620)$ have been reported, and theoretical studies have also been conducted actively.

- We construct the models based on Belle and ALICE(Model 1/Model 2)
- Confirm the pole trajectory QB→QV in simplified system(K/π)
- We found that the QB pole on Model 1 and QV pole on Model 2 are different states.

Future work

- Investigate spectrum change on the pole trajectory in simplified system.

Formulation of the scattering model

Coupled-channel meson-baryon scattering amplitude $T_{ij}(W)$ at total energy W.

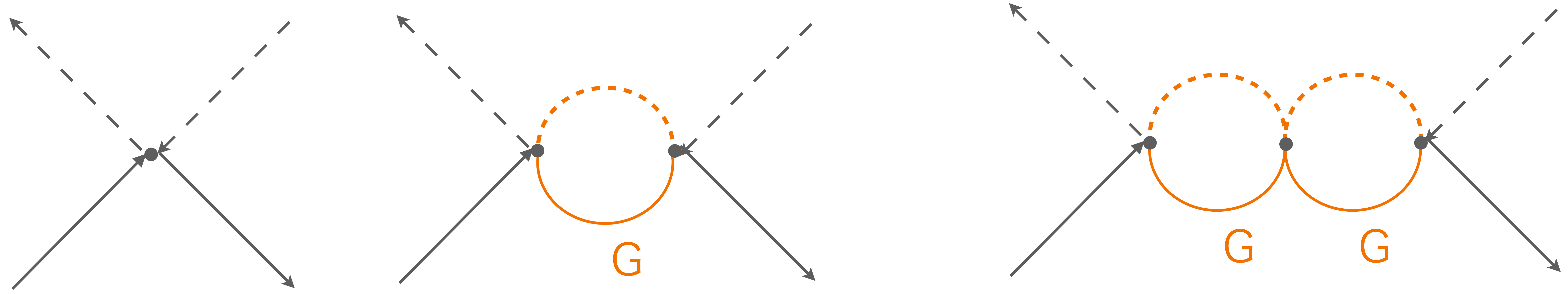
Scattering equation

$$T_{ij}(W) = V_{ij}(W) + V_{ik}(W)G_k(W)T_{kj}(W)$$

$V_{ij}(W)\cdots$Interaction kernel

$G_i(W)\cdots$Loop function

$$T_{ij}(W) = V_{ij}(W) + V_{ik}(W)G_k(W)V_{kj}(W) + V_{ik}(W)G_k(W)V_{kl}(W)G_l(W)V_{lj}(W) + \cdots$$



Meson-baryon multiple scattering

The solution of the equation is obtained as

$$T_{ij}(W) = [[V(W)]^{-1} - G(W)]^{-1}_{ij}$$

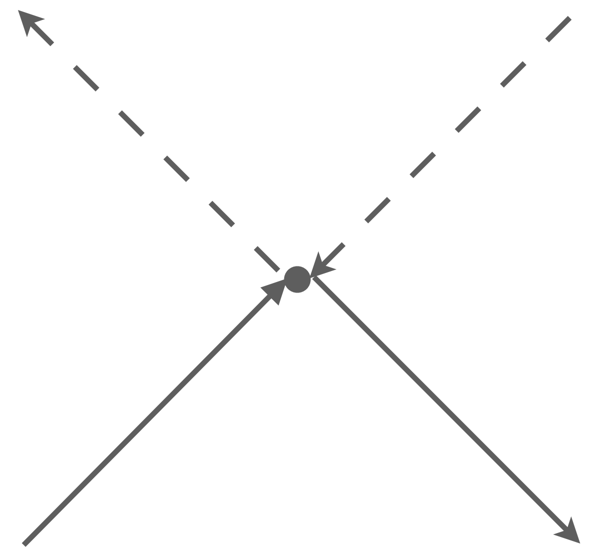
Formulation of the scattering model

$V_{ij}(W)\cdots$Interaction kernel (Weinberg-Tomozawa term)
s-wave interaction satisfying chiral low energy theorem.

$$V_{ij}(W) = -\frac{C_{ij}}{4f_i f_j} N_i N_j (2W - M_i - M_j)$$

f_i : Meson decay constant, C_{ij} : Group theoretical coefficient,

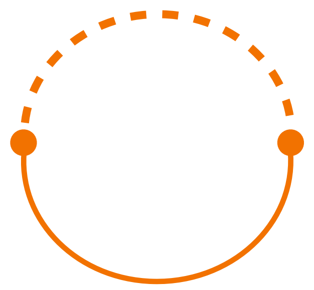
M_i : Baryon Mass, N_i : Kinematical coefficient



$G_i(W, a_i)\cdots$Loop function
(Divergence renormalized by dimensional regularization)

$$G_i(W) \rightarrow G_i(W, a_i)$$

W : Total energy, a_i : Subtraction constant

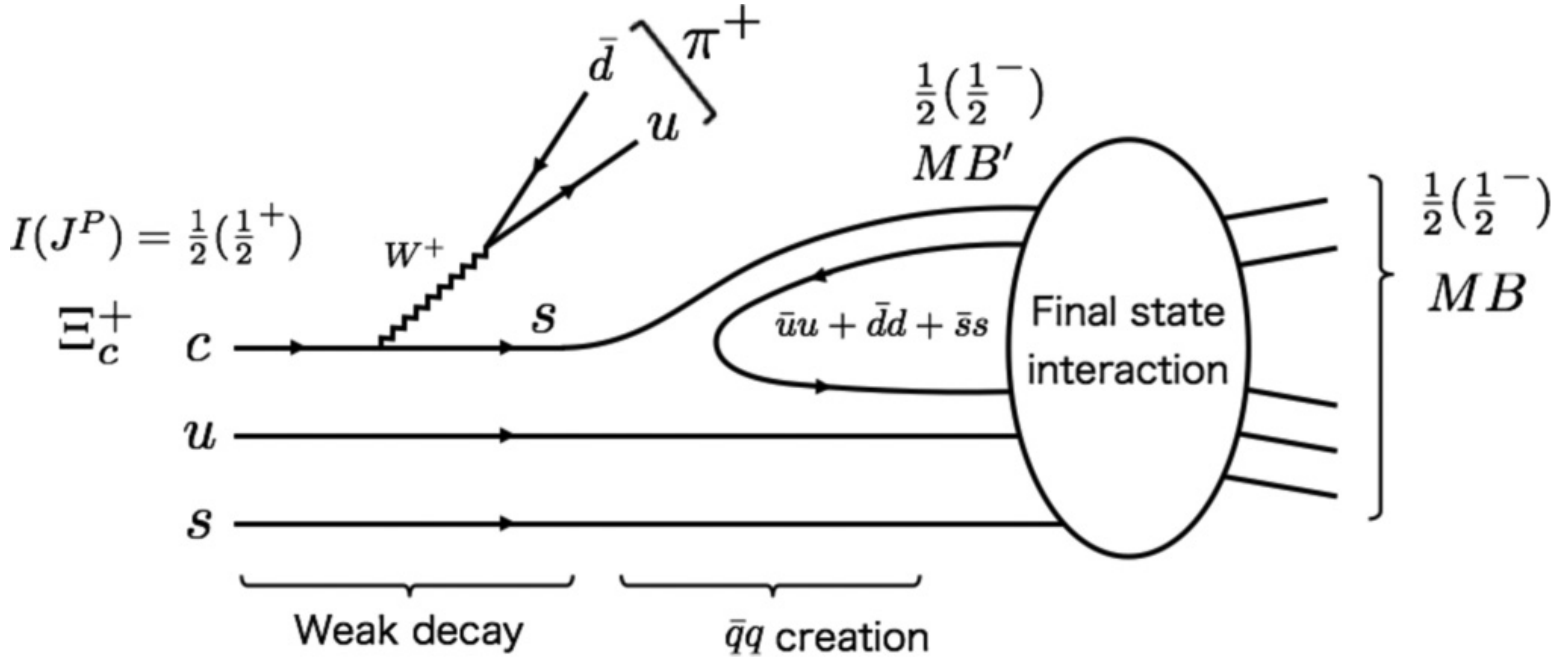


Formulation of $\mathscr{M}$ in 3 body decay

3 body decay($\Xi_c^+ \to \pi^+ \mathrm{MB}$)

The decay amplitude to
the final meson baryon state

$$\mathscr{M}_j = V_P \left(h_j + \sum_i h_i G_i(M_{\mathrm{inv}}) T_{ij}(M_{\mathrm{inv}}) \right)$$



The diagram for $\Xi_c^+ \to \pi^+ MB$ decay. [6]

V_P : the constant includes all dynamics before FSI.

h_i : the weight coefficient of intermediate state, M_{inv} : Invariant Mass,

T_{ij} : Meson baryon scattering amplitude, G_i : Meson baryon loop function

[6]K.Miyahara, T.Hyodo, M.oka, J.Nieves and E.Oset Phys.Rev.C **95** (2017) 3, 035212

Formulation of $\mathscr{M}$ in 3 body decay

3 body decay($\Xi_c^+ \to \pi^+ MB$)

The invariant mass distribution reduced to

$$\frac{d\Gamma_j}{dM_{\text{inv}}} = \frac{1}{(2\pi)^3}\frac{p_{\pi^+}\tilde{p}_j M_j}{M_{\Xi_c^+}}|\mathscr{M}_j|^2$$

$$p_{\pi^+} = \frac{\lambda^{1/2}(M_{\Xi_c^+}^2, m_{\pi^+}^2, M_{\text{inv}}^2)}{2M_{\Xi_c^+}}$$

$$\tilde{p}_j = \frac{\lambda^{1/2}(M_{\text{inv}}^2, M_j^2, m_j^2)}{2M_{\text{inv}}}$$

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$$

$M_{\Xi_c^+}$: mass of Ξ_c^+, m_{π^+} : mass of π^+

p_{π^+} : three-momentum of the π^+ which emitted in weak decay (Ξ_c^+ rest flame)

$\tilde{p}_j$: three-momentum of meson baryon emitted in weak decay (MB rest flame)

Back up New studies for Ξ excited states

LHCb Collaboration(2021)[6]

- $\Xi^-(1690)$ and $\Xi^-(1820)$ are observed in $\Xi_b^- \to J/\psi \Lambda K^-$ decay.
- Mass M_R and width Γ_R of $\Xi^-(1690)$ are reported as follows.

$$M_R = 1692.0 \pm 1.3(\text{stat.})^{+1.2}_{-0.4}(\text{syst.})\ \text{MeV}$$
$$\Gamma_R = 25.9 \pm 9.5(\text{stat.})^{+14.0}_{-13.5}(\text{syst.})\ \text{MeV}$$

New theoretical analysis of $\Xi(1620)$ and $\Xi(1690)$ (2023)[7]

The study based on chiral unitary approach which is added the Born and NLO terms.

$\Xi(1620)$	$M_R = 1599.95$ MeV, $\Gamma_R = 158.88$ MeV.	$M_R = 1608.51$ MeV, $\Gamma_R = 170.00$ MeV.
$\Xi(1690)$	$M_R = 1683.04$ MeV, $\Gamma_R = 11.51$ MeV.	$M_R = 1686.17$ MeV, $\Gamma_R = 29.72$ MeV.

[6]R. Aaij, et al., Sci. Bull. 66 (2021) 1278–1287. [7]Feijoo, A. and Valcarce, V. and Magas, V. K., arXiv:2303.01323 [hep-ph].

Back up Definition of scattering length

- In this study, we define the scattering length f_0 as follows.

  (It is the value of scattering amplitude at threshold energy.)

$$f(k) = \frac{1}{\frac{1}{f_0} + \frac{d_0}{2}k^2 + \cdots - ik}$$

$f(k)$: Scattering amplitude

k: Complex momentum

r_0: effective range

- But in general, scattering length a_0 is defined as follow.

  (It is reverse sign of f_0.)

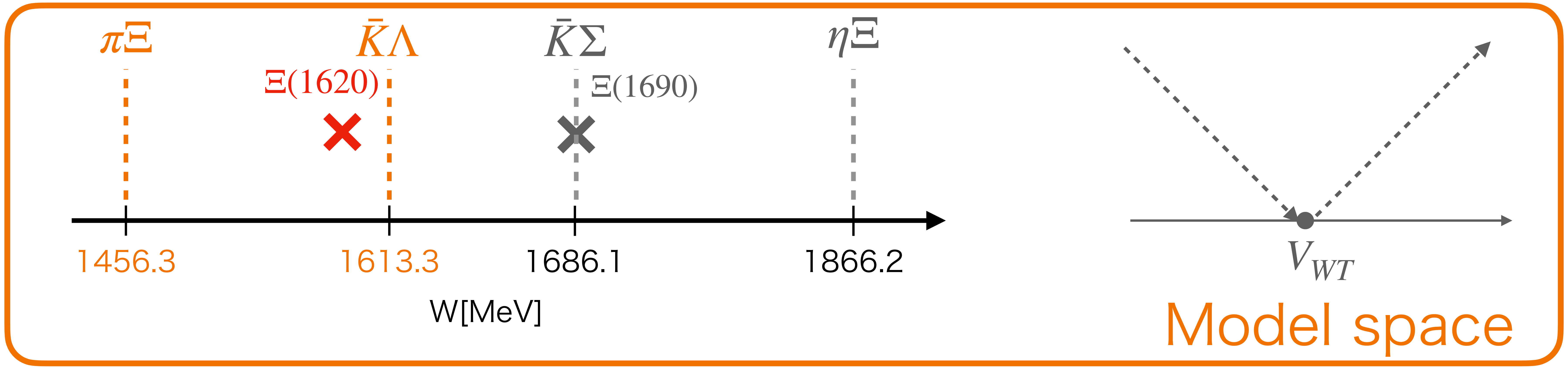
$$f(k) = \frac{1}{-\frac{1}{a_0} + \frac{r_0}{2}k^2 + \cdots - ik}$$

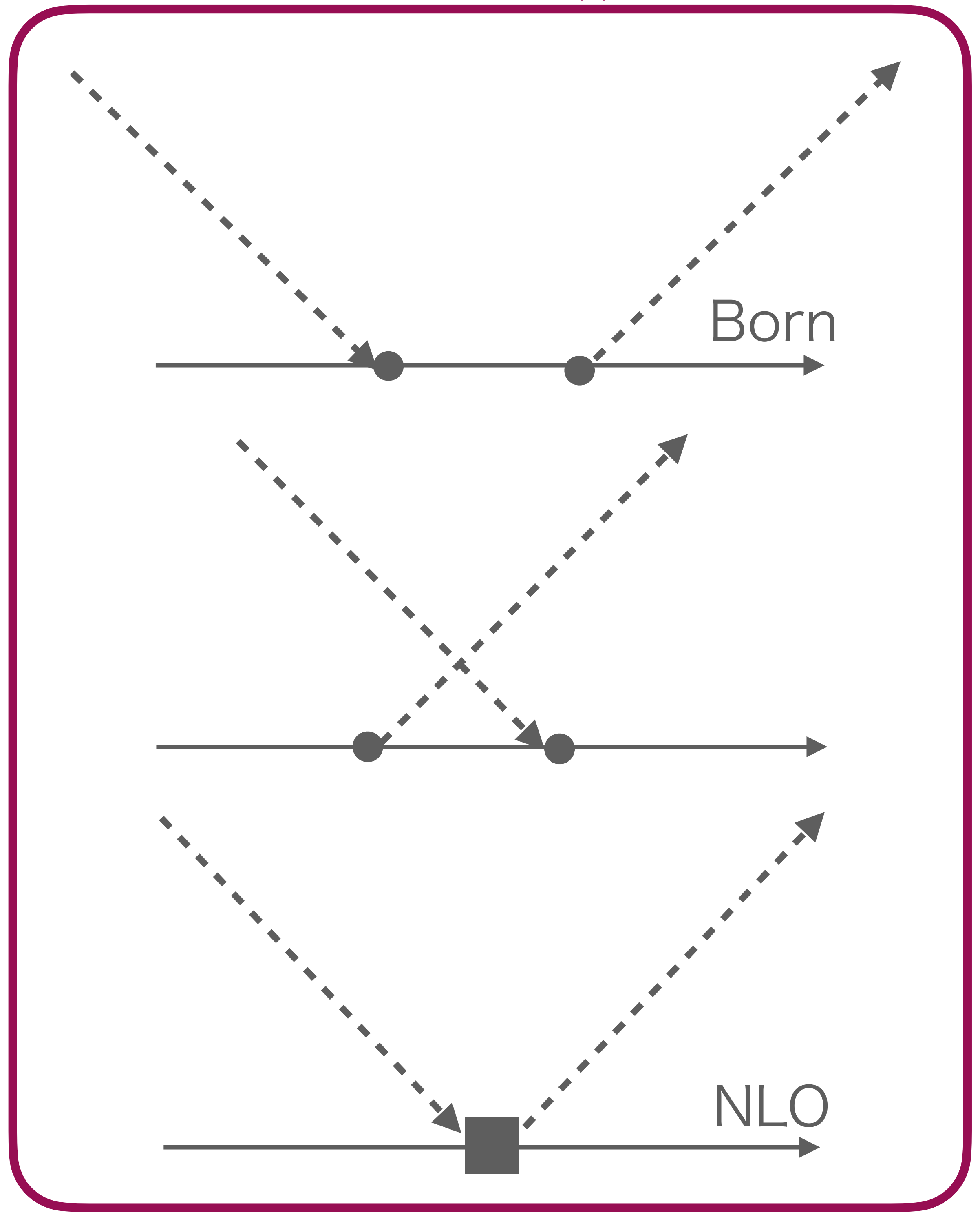
Back up The roles of subtraction constants

- By changing subtraction constants, the effects from outside of model space can be absorbed.

➡ When the subtraction constants closer to natural value, outside effect become smaller.

Effects from except V_{WT} .





Effects from other channels(Ξ^*_{uss},$\bar{K}^*\Lambda$,$\bar{K}^*\Sigma$,$\pi\bar{K}\Lambda$,$\cdots$).

[8] T.Hyodo, D.Jido and A.Hosaka Phys. Rev. C 78.025203 (2008)

Back up Detail of loop function

Loop function $G_i(W)$

$$G_i(W) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - m_i^2 + i0^+}\frac{1}{(P-q)^2 - M_i^2 + i0^+}$$

Loop function $G_i(W, a_i)$ (Removed divergence by dimensional regularization)

$$G_i(W,a_i) = \frac{1}{16\pi^2}\Bigg[a_i(\mu_{reg}) + \ln\frac{mM}{\mu_{reg}^2} + \frac{M^2 - m^2}{2W^2}\ln\frac{M^2}{m^2} + \frac{\lambda^{1/2}}{2W^2}\left\{\ln(W^2 - m^2 + M^2 + \lambda^{1/2})\right.$$

$$\left. + \ln(W^2 + m^2 - M^2 + \lambda^{1/2}) - \ln(-W^2 + m^2 - M^2 + \lambda^{1/2}) - \ln(-W^2 - m^2 + M^2 + \lambda^{1/2})\right\}\Bigg]$$

$$\lambda^{1/2} = \sqrt{W^4 + m_k^4 + M_k^4 - 2W^2m_k^2 - 2m_k^2M_k^2 - 2M_k^2W^2}$$

Back up Lednický and Lyuboshitz model

When Coulomb interaction is not at work, the correlation function can be described analytically with the Lednický and Lyuboshitz model.

$$f^s(k^*) = \left(\frac{1}{f_0^s} + \frac{1}{2} d_0^s k^{*2} - ik^*\right)^{-1}$$

$f_0^s(k)$: complex s-wave scattering length

d_0^s : Effective range

F_1, F_2 : Analytic functions

ρ_S :Weight factor (the normalized emission probability for a state of total spin S)

$$C(k^*)_{\text{Lednický}} = 1 + \sum_S \rho_S \left[\frac{1}{2}\left|\frac{f^s(k^*)}{R_{\text{inv}}}\right|^2 \left(1 - \frac{d_0^s}{2\sqrt{\pi}R_{\text{inv}}}\right) + \frac{2\text{Re}\, f^s(k^*)}{\sqrt{\pi}R_{\text{inv}}} F_1(2k^*R_{\text{inv}}) + \frac{\text{Im}\, f^s(k^*)}{R_{\text{inv}}} F_2(2k^*R_{\text{inv}})\right]$$

$$\rho_S = \frac{(2S+1)}{[(2j_1+1)(2j_2+1)]}$$