Light and hidden-charm pentaquark states in molecular and pentaquark pictures

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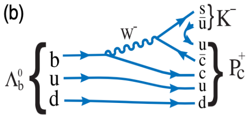
- Introduction
- Group theory approach

  Construction of the wave-function of multi-quark system
- Constituent quark models
- Light pentaquark states
- Hidden-charm pentaquark states
- Summary and outlook

Hidden-charm pentaquark states P_c

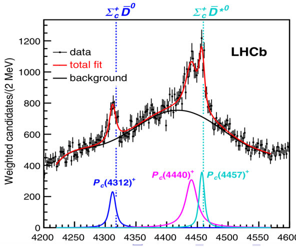
- LHCb first observation [PRL **115** (2015) 072001] two N^* from $\Lambda_b^0 \to J/\Psi p K^-$ decay: $P_c(4380)^+$ and $P_c(4450)^+$.



- LHCb new observations [PRL **122** (2019) 222001] three narrow pentaquarklike-states: $P_c(4312)^+$, $P_c(4440)^+$, and $P_c(4457)^+$ ($P_c(4440)^+$, and $P_c(4457)^+$ instead of $P_c(4450)^+$)

- LHCb [PRL **128** (2022) 062001]: $B_S^0 \to J/\psi p\bar{p}$ decay: P_c (4337).

States	M [MeV]	Γ [MeV]
$P_c(4380)^+$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$
$P_c(4337)^+$	$4337^{+7}_{-4}{}^{+2}_{-2}$	$29^{+26}_{-12}{}^{+14}_{-14}$



Construction of Pentaquark wave functions

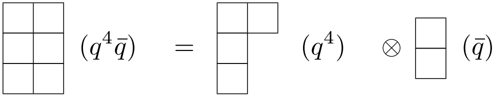
- Pentaquark wave functions contain contributions of the spatial degrees of freedom and the internal degrees of freedom of color, flavor and spin.
- The quark transforms under $SU(n)$, whereas the antiquark transforms under the conjugate representation of $SU(n)$, with $n = 2, 3, 3, 6$ for the spin, flavor, color and spin-flavor degree of freedom, respectively.
- The corresponding algebraic structure:

$$SU_{\mathrm{sf}}(6) \otimes SU_{\mathrm{c}}(3), \quad SU_{\mathrm{sf}}(6) = SU_{\mathrm{f}}(3) \otimes SU_{\mathrm{s}}(2) \tag{1}$$

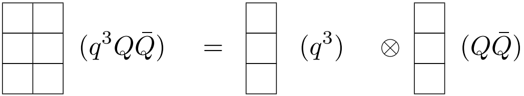
- The construction of pentaquark states is guided by
  - The pentaquark wave function should be a color singlet;
  - The pentaquark wave function should be antisymmetric under any permutation of the identical quark configuration.

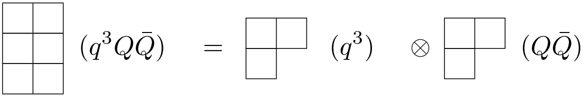
$q^4\,\overline{q}$ and $q^3 Q\,\overline{Q}$ Systems

- That the pentaquark wave function should be a color singlet demands that the color part of the pentaquark wave function must be a $[222]_1$ singlet.
- $q^4\bar{q}$ color configuration



- $q^3Q\bar{Q}$ ($q^3c\bar{c}$,$q^3b\bar{b}$) color configuration





$q^4\,\overline{q}$ and $q^3Q\,\overline{Q}$ Systems

- Total wave function of the q^4 configuration may be written in the general form

$$\psi = \sum_{i,j=\lambda,\rho,\eta} a_{ij}\psi^{c}_{[211]_i}\psi^{osf}_{[31]_j} \tag{2}$$

The coefficients can be determined by operating the permutations of S_4 on the general form, using the $[31]$ and $[211]$ representation matrices.

$$\psi_A = \frac{1}{\sqrt{3}}\left(\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\rho} - \psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\lambda} + \psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\eta}\right) \tag{3}$$

- Pentaquark wave function in $q^3Q\bar{Q}$ configuration:

$$\psi_A = \frac{1}{\sqrt{2}}\left(\psi^{c}_{[21]_\lambda}\psi^{osf}_{[21]_\rho} - \psi^{c}_{[21]_\rho}\psi^{osf}_{[21]_\lambda}\right) \tag{4}$$

The total color wave function for $q^3Q\bar{Q}$ pentaquark state takes the form,

$$\Psi^{c}_{[21]_{j=\rho,\lambda}} = \frac{1}{\sqrt{8}}\sum_{i}^{8}\psi^{c}_{[21]^{i}_{j}}(q^3)\psi^{c}_{[21]^{i}_{j}}(Q\bar{Q}) \tag{5}$$

where the ρ and λ stand for the types of $[21]_8$ color octet configurations.

Spatial-Spin-Flavor Configurations of q^4

- The spatial-flavor-spin wave function and flavor-spin wave function of the q^4 cluster take respectively the general forms,

$$\begin{aligned}\Psi^{osf}_{[31]} &= \sum_{i,j=S,A,\lambda,\rho,\eta} b_{ij}\Psi^{o}_{[X]_i}\Psi^{sf}_{[Y]_j} \\ \Psi^{sf}_{[Z]} &= \sum_{i,j=S,A,\lambda,\rho,\eta} c_{ij}\Phi^{f}_{[X]_i}\chi^{s}_{[Y]_j}\end{aligned} \quad (6)$$

- The explicit configurations can be decomposed according to the S_4 permutation group by applying characters of irreducible representations of S_4 .

$[31]_{OSF}$	
$[4]_O$	$[31]_{SF}$
$[1111]_O$	$[211]_{SF}$
$[22]_O$	$[31]_{SF}, [211]_{SF}$
$[211]_O$	$[31]_{SF}, [211]_{SF}, [22]_{SF}$
$[31]_O$	$[4]_{SF}, [31]_{SF}, [211]_{SF}, [22]_{SF}$

The coefficients can be determined by operating at least three permutations of S_4 on the general forms.

q^4 Spin-Flavor Configurations

	$[4]_{FS}$		
$[4]_{FS}[22]_F[22]_S$	$[4]_{FS}[31]_F[31]_S$	$[4]_{FS}[4]_F[4]_S$	
	$[31]_{FS}$		
$[31]_{FS}[31]_F[22]_S$	$[31]_{FS}[31]_F[31]_S$	$[31]_{FS}[31]_F[4]_S$	$[31]_{FS}[211]_F[22]_S$
$[31]_{FS}[211]_F[31]_S$	$[31]_{FS}[22]_F[31]_S$	$[31]_{FS}[4]_F[31]_S$	
	$[22]_{FS}$		
$[22]_{FS}[22]_F[22]_S$	$[22]_{FS}[22]_F[4]_S$	$[22]_{FS}[4]_F[22]_S$	$[22]_{FS}[211]_F[31]_S$
$[22]_{FS}[31]_F[31]_S$			
	$[211]_{FS}$		
$[211]_{FS}[211]_F[22]_S$	$[211]_{FS}[211]_F[31]_S$	$[211]_{FS}[211]_F[4]_S$	$[211]_{FS}[22]_F[31]_S$
$[211]_{FS}[31]_F[22]_S$	$[211]_{FS}[31]_F[31]_S$		

Spatial wave functions of pentaquark states

- In $q^4\bar{q}$ configuration:

$$\begin{aligned}\vec{\rho} &= \frac{1}{\sqrt{2}}(\vec{r_1}-\vec{r_2}), \vec{\lambda}=\frac{1}{\sqrt{6}}(\vec{r_1}+\vec{r_2}-2\vec{r_3}), \vec{\eta}=\frac{1}{\sqrt{12}}(\vec{r_1}+\vec{r_2}+\vec{r_3}-3\vec{r_4}),\\ \vec{\xi} &= \frac{1}{\sqrt{20}}(\vec{r_1}+\vec{r_2}+\vec{r_3}+\vec{r_4}-4\vec{r_5}), \vec{R}=\frac{1}{5}(\vec{r_1}+\vec{r_2}+\vec{r_3}+\vec{r_4}+\vec{r_5})\end{aligned}$$

$$\Psi^{[5]_S}_{NLM} = \psi^{q^4[4]_S}_{N'L'M'} \otimes \psi_{n_\xi,l_\xi}(\vec{\xi}) \quad (7)$$

  - $(\vec{\rho},\vec{\lambda},\vec{\eta})$ transform as $([31]_\rho,\ [31]_\lambda,\ [31]_\eta)$ of S_4
- In $q^3Q\bar{Q}$ configuration:

$$\begin{aligned}\vec{\rho} &= \frac{1}{\sqrt{2}}(\vec{r_1}-\vec{r_2}), \vec{\lambda}=\frac{1}{\sqrt{6}}(\vec{r_1}+\vec{r_2}-2\vec{r_3}), \vec{\sigma}=\frac{1}{\sqrt{2}}(\vec{r_4}-\vec{r_5}),\\ \vec{\chi} &= \frac{1}{\sqrt{30}}(2(\vec{r_1}+\vec{r_2}+\vec{r_3})-3(\vec{r_4}+\vec{r_5})), \vec{R}=\frac{3m(\vec{r_1}+\vec{r_2}+\vec{r_3})+2M(\vec{r_4}+\vec{r_5})}{3m+2M}\end{aligned}$$

$$\Psi^{[5]_S}_{NLM} = \psi^{q^3[3]_S}_{N'L'M'} \otimes \psi_{n_\sigma,l_\sigma}(\vec{\sigma}) \otimes \psi_{n_\chi,l_\chi}(\vec{\chi}) \quad (8)$$

$q^4\bar{q}$ Spatial Wave Functions

- Spatial wave functions of $q^4\bar{q}$ in the harmonic oscillator interaction take the general form,

$$
\begin{aligned}
\Psi^{\mathrm{O}}_{NLM} &= A(n_\lambda, n_\rho, n_\eta, n_\xi, l_\lambda, l_\rho, l_\eta, l_\xi) \\
&\cdot[\psi_{n_\lambda l_\lambda m_\lambda}(\vec{\lambda}) \otimes \psi_{n_\rho l_\rho m_\rho}(\vec{\rho}) \otimes \psi_{n_\eta l_\eta m_\eta}(\vec{\eta}) \otimes \psi_{n_\xi l_\xi m_\xi}(\vec{\xi})]_{NLM} \\
&= A(n_\lambda, n_\rho, n_\eta, n_\xi, l_\lambda, l_\rho, l_\eta, l_\xi) \\
&\cdot\psi_{n_\lambda l_\lambda m_\lambda}(\vec{\lambda})\psi_{n_\rho l_\rho m_\rho}(\vec{\rho})\psi_{n_\eta l_\eta m_\eta}(\vec{\eta})\psi_{n_\xi l_\xi m_\xi}(\vec{\xi}) \\
&\cdot C(l_\lambda, l_\rho, m_\lambda, m_\rho, l_{\lambda\rho}, m_{\lambda\rho}) \\
&\cdot C(l_{\lambda\rho}, l_\eta, m_{\lambda\rho}, m_\eta, l_{\lambda\rho\eta}, m_{\lambda\rho\eta}) \\
&\cdot C(l_{\lambda\rho\eta}, l_\xi, m_{\lambda\rho\eta}, m_\xi, LM)
\end{aligned} \tag{9}
$$

  with $N = 2(n_\lambda + n_\rho + n_\eta + n_\xi) + l_\lambda + l_\rho + l_\eta + l_\xi$

  where $\Psi_{n_r l_r m_r}(r) = R_{n_r l_r}(r) Y_{l_r m_r}(\hat{r})$, $R_{nl}(r) = L_n^{l+1/2}(r^2)e^{-\alpha^2 r^2}$.
- The coefficients A can be determined by applying the Yamanouchi basis representations of the S_4 group.
- Various types of spatial wave functions with the $[4], [31], [22], [211]$ and $[1111]$ symmetries are worked out.

q^4 sub-group spatial wave function

$q^4(N = 2n, L = M = 0)$ Spatial wave functions of the different symmetry

$$\psi^{[4]_S}_{N=4,L=M=0}(\vec{\rho},\vec{\lambda},\vec{\eta}) = \sum_{\{n_i,l_i\}} C_{n_\rho,l_\rho,n_\lambda,l_\lambda,n_\eta,l_\eta}\,(n_\rho,l_\rho,n_\lambda,l_\lambda,n_\eta,l_\eta)$$

$$= \sqrt{\tfrac{5}{33}}(2,0,0,0,0,0) + \sqrt{\tfrac{5}{33}}(0,0,2,0,0,0) + \sqrt{\tfrac{5}{33}}(0,0,0,0,2,0)$$

$$+\sqrt{\tfrac{2}{11}}(1,0,1,0,0,0), \sqrt{\tfrac{2}{11}}(1,0,0,0,1,0), \sqrt{\tfrac{2}{11}}(0,0,1,0,1,0)$$

$$\psi^{[31]_\rho}_{N=4,L=M=0}(\vec{\rho},\vec{\lambda},\vec{\eta}) = \sqrt{\tfrac{5}{39}}(0,1,0,0,1,1) + \sqrt{\tfrac{2}{13}}(0,1,0,1,1,0) + \tfrac{1}{\sqrt{13}}(0,1,1,0,0,1)$$

$$+\sqrt{\tfrac{10}{39}}(0,1,1,1,0,0) + \sqrt{\tfrac{5}{39}}(1,1,0,0,0,1) + \sqrt{\tfrac{10}{39}}(1,1,0,1,0,0)$$

$$\psi^{[31]_\lambda}_{N=4,L=M=0}(\vec{\rho},\vec{\lambda},\vec{\eta}) = \sqrt{\tfrac{5}{39}}(0,0,0,1,1,1) - \tfrac{1}{\sqrt{13}}(0,0,1,0,1,0) + \sqrt{\tfrac{5}{39}}(0,0,1,1,0,1)$$

$$-\sqrt{\tfrac{10}{39}}(0,0,2,0,0,0) + \tfrac{1}{\sqrt{13}}(1,0,0,0,1,0) + \tfrac{1}{\sqrt{13}}(1,0,0,1,0,1) + \sqrt{\tfrac{10}{39}}(2,0,0,0,0,0)$$

$$\psi^{[31]_\eta}_{N=4,L=M=0}(\vec{\rho},\vec{\lambda},\vec{\eta}) = -\sqrt{\tfrac{20}{39}}(0,0,0,0,2,0) - \tfrac{1}{\sqrt{26}}(0,0,1,0,1,0) + \sqrt{\tfrac{5}{39}}(0,0,2,0,0,0)$$

$$-\tfrac{1}{\sqrt{26}}(1,0,0,0,1,0) + \sqrt{\tfrac{2}{13}}(1,0,1,0,0,0) + \sqrt{\tfrac{5}{39}}(2,0,0,0,0,0) \qquad (1$$

Constituent quark model with a Cornell-like potential

- We apply, as complete bases, the full wave functions of pentaquarks worked out in previous sections to study the pentaquark system described by the Hamiltonian, [PRC **100**, 065207 (2019)],[PRD **101**, 076025 (2020)].

$$
\begin{aligned}
H &= H_0 + H_{hyp}^{OGE}, \\
H_0 &= \sum_{k=1}^{N} (m_k + \frac{p_k^2}{2m_k}) + \sum_{i<j}^{N} (-\frac{3}{8}\lambda_i^C \cdot \lambda_j^C)(A_{ij} r_{ij} - \frac{B_{ij}}{r_{ij}}), \\
H_{hyp}^{OGE} &= -C_{OGE} \sum_{i<j} \frac{\lambda_i^C \cdot \lambda_j^C}{m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j
\end{aligned} \tag{11}
$$

$$
A_{ij} = a\sqrt{\frac{m_{ij}}{m_u}}, \;\; B_{ij} = b\sqrt{\frac{m_u}{m_{ij}}} \tag{12}
$$

- 3 model coupling constants and 4 constituent quark masses,

$$
m_u = m_d = 327 \text{ MeV}\,, \quad m_s = 498 \text{ MeV}\,,
$$
$$
m_c = 1642 \text{ MeV}\,, \quad m_b = 4960 \text{ MeV}\,,
$$
$$
C_m = 18.3 \text{ MeV}, \quad a = 49500 \text{ MeV}^2, \quad b = 0.75
$$

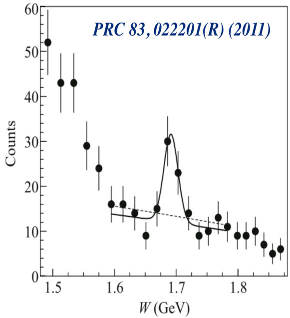
Mass spectrum of ground state $q^4\bar{q}$ pentaquarks.

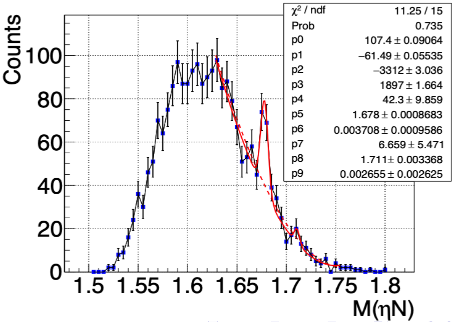
J^P	$q^4\bar{q}$ configurations	$(S^{q^4}, S^{\bar{q}}, S)$	$M^{EV}(q^4\bar{q})$
$\frac{5}{2}^-$	$\Psi^{sf}_{[31]_F[4]_S}(q^4\bar{q})$	(2,1/2,5/2)	2269
$\frac{3}{2}^-$	$\Psi^{sf}_{[4]_F[31]_S}(q^4\bar{q})$	(1,1/2,3/2)	2269
	$\begin{pmatrix}\Psi^{sf}_{[31]_F[4]_S}(q^4\bar{q})\\ \Psi^{sf}_{[31]_F[31]_S}(q^4\bar{q})\end{pmatrix}$	(2,1/2,3/2) (1,1/2,3/2)	$\begin{pmatrix}1805\\2269\end{pmatrix}$
	$\Psi^{sf}_{[22]_F[31]_S}(q^4\bar{q})$	(1,1/2,3/2)	2049
$\frac{1}{2}^-$	$\Psi^{sf}_{[4]_F[31]_S}(q^4\bar{q})$	(1,1/2,1/2)	2562
	$\begin{pmatrix}\Psi^{sf}_{[31]_F[31]_S}(q^4\bar{q})\\ \Psi^{sf}_{[31]_F[22]_S}(q^4\bar{q})\end{pmatrix}$	(1,1/2,1/2) (0,1/2,1/2)	$\begin{pmatrix}1986\\2162\end{pmatrix}$
	$\Psi^{sf}_{[22]_F[31]_S}(q^4\bar{q})$	(1,1/2,1/2)	1683

- An isospin $1/2$ narrow resonance $N^+(1685)$ ($\Gamma \leq 30$ MeV) firstly reported in GRAAL [PLB **647**: 23-29 (2007)]. Confirmed in A2@Mainz, CBELSA/TAPS, and LNS-Sendai.
- $N^+(1685)$ cannot be accommodated in the q^3 picture.
- Could be the lowest compact pentaquark state??

The experimental situation of N(1685)

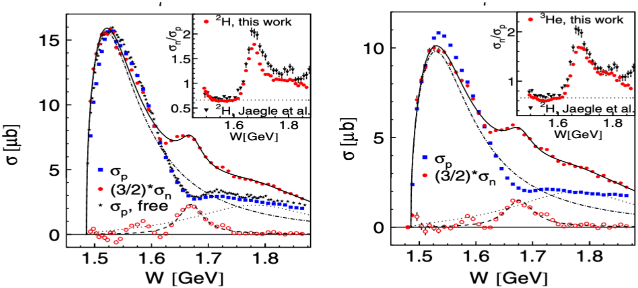
- $N(1685)$ was firstly reported in the photoproduction of η meson off the quasi-free neutron. [Phys. Lett. B **647**: 23-29 (2007)].
- It was also observed in quasifree Compton scattering on the neutron in the energy range of $E_\gamma = 0.75 - 1.5$ GeV. [Phys. Rev. C **83**, 022201(R) (2011)]
- Recently, the invariant mass spectra of ηN in the $\gamma N \to \pi\eta N$ reactions from GRAAL reveal the N(1685) resonance again.[JETP Letters **106**: 693-699(2017)]





N(1685)

- A2 experiment at Mainz MAMI accelerator, the measurements of η photoproduction with deuterium and also ^3He target also establish this narrow structure. [PRL 111, 232001 (2013), PRL 117, 132502 (2016)]

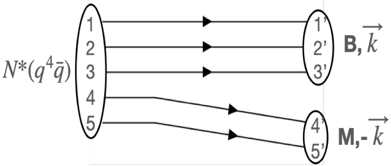


- In experiment, this isospin $1/2$ narrow peak-like structure ($\Gamma \leq 30$ MeV) was not (very poorly) seen in the $\gamma p \to \eta p$, but the existence should be beyond any doubts. (CBELSA/TAPS and LNS-Sendai collaborations all confirmed the observation.)

Partial decay widths of $N(1685)$

- Direct quark rearrangement diagram

$$\hat{O} = \delta^3(\vec{p}_1 - \vec{p}_1')\delta^3(\vec{p}_2 - \vec{p}_2')\delta^3(\vec{p}_3 - \vec{p}_3')\delta^3(\vec{p}_4 - \vec{p}_4')\delta^3(\vec{p}_5 - \vec{p}_5')$$



- Decay widths in the non-relativistic approximation,

$$\Gamma_{N(q^4\bar{q})\to BM} = \frac{2\pi E_1 E_2}{M}\frac{k}{2S_i+1}\sum_{m_i,m_j}|T(k)|^2,$$
$$T(k) = T^{CSF}\langle\psi_{(BM)}|\hat{O}|\psi_{N(q^4\bar{q})}\rangle \quad (14)$$

- Partial decay widths of these two channels are comparable,

$$T^{CSF}(N(1685)\to N\pi)/T^{CSF}(N(1685)\to N\eta) = \sqrt{3}:1$$

- Preliminary results: ratio of two decay widths.

$$\Gamma(N(1685)\to N\pi)/\Gamma(N(1685)\to N\eta) = 1:1.9$$

P_c states in the compact pentaquark and molecular pictures

- Coupling of the S-wave molecular states, $\Sigma_c^*\bar{D}^*$, $\Sigma_c\bar{D}^*$, $\Sigma_c\bar{D}$ with the ground hidden-charm pentaquarks of the same quantum numbers.

- The general mixing mass matrices of P_c states,

$$
\begin{aligned}
H &= \begin{pmatrix} M_{Mole} & \Delta_{hyp} \\ \Delta_{hyp} & M_{Penta} \end{pmatrix} \\
&= \begin{pmatrix} M_B + M_M + E_B & \langle \psi_{BM}^{CSF} | H_{hyp}^{OGE} | \psi_{P_c}^{CSF} \rangle \\ \langle \psi_{BM}^{CSF} | H_{hyp}^{OGE} | \psi_{P_c}^{CSF} \rangle & M_{Penta} \end{pmatrix}
\end{aligned}
$$

- Method: Solve the coupled Schrödinger equations.

- The $\Lambda_c^+\bar{D}^{(*)0}$ interactions are expected to be repulsive. [PRC **84** (2011) 015203; PRD **95**, 013010 (2017)].

Masses of the hidden-charm pentaquark states in the mixing pictures

- I=1/2, J=1/2 and J=3/2 mixing pentaquark states [PRD **109**, 036019 (2024)]
  $|A_i|^2$: Eigenvectors squared - contributions of compact pentaquark and molecule.

J^P	Mixing Configurations	$\vert A_i\vert^2$				M (MeV)
$\frac{1}{2}^-$	$\Sigma_c^*\bar{D}^*(4526)$	0.50	0.18	0.04	0.28	4535
	$\Psi^{csf}_{[21]_C[21]_F[21]_S}(q^3c\bar{c})$	0.47	0.33		0.21	4517
	$\Psi^{csf}_{[21]_C[21]_F[21]_S}(q^3c\bar{c})$	0.03	0.18	0.71	0.08	4455
	$\Psi^{csf}_{[21]_C[21]_F[3]_S}(q^3c\bar{c})$		0.31	0.26	0.43	4433
$\frac{1}{2}^-$	$\Sigma_c\bar{D}^*(4462)$		0.48	0.02	0.50	4526
	$\Psi^{csf}_{[21]_C[21]_F[21]_S}(q^3c\bar{c})$	0.55	0.04	0.42		4479
	$\Psi^{csf}_{[21]_C[21]_F[21]_S}(q^3c\bar{c})$	0.22	0.38	0.10	0.30	4444
	$\Psi^{csf}_{[21]_C[21]_F[3]_S}(q^3c\bar{c})$	0.24	0.10	0.47	0.20	4426
$\frac{1}{2}^-$	$\Sigma_c\bar{D}(4322)$		0.49	0.02	0.49	4526
	$\Psi^{csf}_{[21]_C[21]_F[21]_S}(q^3c\bar{c})$	0.03	0.38	0.34	0.25	4458
	$\Psi^{csf}_{[21]_C[21]_F[21]_S}(q^3c\bar{c})$	0.09	0.09	0.61	0.21	4451
	$\Psi^{csf}_{[21]_C[21]_F[3]_S}(q^3c\bar{c})$	0.88	0.05	0.02	0.06	4298

Masses of the hidden-charm pentaquark states in the mixing pictures

J^P	Mixing Configurations	$\|A_i\|^2$				M^{EV} (MeV)
$\frac{3}{2}^-$	$\Sigma_c^*\overline{D}^*(4526)$	0.20	0.12	0.64	0.04	4586
	$\Psi^{csf}_{[21]_C[21]_F[21]_S}(q^3c\bar{c})$		0.08	0.11	0.81	4532
	$\Psi^{csf}_{[21]_C[21]_F[3]_S}(q^3c\bar{c})$	0.77	0.10	0.13		4509
	$\Psi^{csf}_{[21]_C[21]_F[3]_S}(q^3c\bar{c})$	0.02	0.70	0.13	0.16	4473
$\frac{3}{2}^-$	$\Sigma_c^*\overline{D}(4386)$		0.18	0.77	0.05	4571
	$\Psi^{csf}_{[21]_C[21]_F[21]_S}(q^3c\bar{c})$		0.08	0.13	0.79	4532
	$\Psi^{csf}_{[21]_C[21]_F[3]_S}(q^3c\bar{c})$	0.05	0.69	0.11	0.16	4479
	$\Psi^{csf}_{[21]_C[21]_F[3]_S}(q^3c\bar{c})$	0.95	0.05			4376
$\frac{3}{2}^-$	$\Sigma_c\overline{D}^*(4462)$		0.17	0.78	0.04	4570
	$\Psi^{csf}_{[21]_C[21]_F[21]_S}(q^3c\bar{c})$	0.01	0.08	0.11	0.80	4533
	$\Psi^{csf}_{[21]_C[21]_F[3]_S}(q^3c\bar{c})$	0.03	0.73	0.11	0.13	4474
	$\Psi^{csf}_{[21]_C[21]_F[3]_S}(q^3c\bar{c})$	0.95	0.02		0.02	4457

- Six mass eigenstates of isospin 1/2 below the mass threshold.
  -Name: $(J^P = 1/2^-)$ $X(4298)$, $X(4426)$, $X(4444)$,
  $(J^P = 3/2^-)$ $X(4457)$, $X(4378)$ and $X(4509)$.

Stability of $I = \frac{1}{2}$ mixing pentaquark states

- Masses of the compact pentaquark states and the hadronic molecules are individually varied to check the stability of X states.

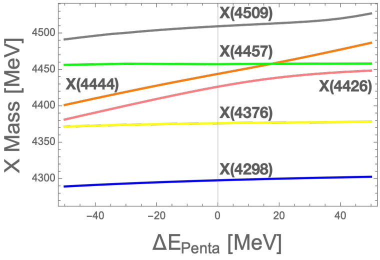


Figure: X mass dependence on the mass of compact pentaquark states generally varied in the range of -50 to 50 MeV.

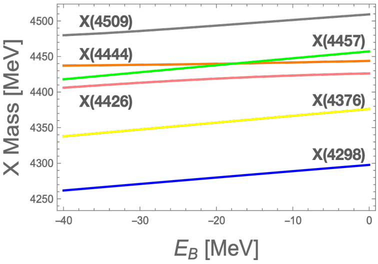


Figure: X mass dependence on the binding energy E_B of molecular states varied from -40 to -1 MeV.

- X(4298), X(4457), X(4378) and X(4509), the masses are very stable with the change of ΔE_{Penta}, but change according to the E_B.
- X(4444) and X(4426) change oppositely because of large pentaquark components.

Partial decay widths of $I=\frac{1}{2}$ mixing pentaquark states

- Partial decay width ratios of I= 1/2 mixing pentaquark states, normalized to 4457 MeV state.

J	Threshold	Mass	Eigenvector^2	Total	$p\eta_c$	pJ/ψ	$\Sigma_c^*\bar{D}$	$\Sigma_c\bar{D}$	$\Lambda_c^+\bar{D}$	$\Sigma_c^*\bar{D}^*$	$\Sigma_c\bar{D}^*$	$\Lambda_c^+\bar{D}^*$
	$\Sigma_c\bar{D}(4322)$	4298	(0.88,0.05,0.02,0.06)	0.57	0.21	0.11						0.25
$\frac{1}{2}$	$\Sigma_c\bar{D}^*(4462)$	4444	(0.22,0.38,0.10,0.30)	14.71	0.01	0.13		9.99	2.74			1.85
	$\Sigma_c\bar{D}^*(4462)$	4426	(0.24,0.10,0.47,0.20)	17.53	0.01	0.15		10.61	1.63			5.13
	$\Sigma_c^*\bar{D}^*(4526)$	4509	(0.77,0.10,0.13,0)	1.87		0.28	0.08				0.43	1.08
$\frac{3}{2}$	$\Sigma_c^*\bar{D}(4386)$	4376	(0.95,0.05,0,0)	1.06		0.35						0.71
	$\Sigma_c\bar{D}^*(4462)$	4457	(0.95,0.02,0.01,0.02)	1.00		0.09	0.61					0.31

- X(4298), X(4457), X(4378) and X(4509) are dominantly hadronic molecules while X(4426) has considerable both the molecular and compact pentaquark components.

- $P_c(4312)$, $P_c(4457)$ and $P_c(4380)$ resonances might be mainly $\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*$ and $\Sigma_c^*\bar{D}$ hadronic molecules respectively, and $P_c(4440)$ might include sizable pentaquark components.

Summary

- Mass spectra of light (compact pentaquark picture) and hidden-charm pentaquark states (mixing picture)

- N(1685) could be the lowest compact pentaquark state: need further study!!!

- $P_c(4312)$, $P_c(4457)$ and $P_c(4380)$ might mainly hadronic molecules, $P_c(4440)$ might include sizable pentaquark components.

Thank You Very Much For Your Attentions!

Representations of S_4

$$
\begin{aligned}
D^{[211]}(13) &= D^{[21]}(13) \oplus D^{[111]}(13) \\
&= \begin{pmatrix} -1/2 & -\sqrt{3}/2 & 0 \\ -\sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & -1 \end{pmatrix} \\
D^{[211]}(12) &= D^{[21]}(12) \oplus D^{[111]}(12) \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \\
D^{[211]}(23) &= D^{[21]}(23) \oplus D^{[111]}(23) \\
&= \begin{pmatrix} -1/2 & \sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad (15)
\end{aligned}
$$

Representations of S_4

For the element (34) of S_4, we have

$$\begin{aligned}
(34)|[211](3211)\rangle &= -|[211](3211)\rangle \\
(34)|[211](3121)\rangle &= \sigma_{31}|[211](3121)\rangle + \sqrt{1-\sigma_{31}^2}|[211](1321)\rangle \\
(34)|[211](1321)\rangle &= \sigma_{13}|[211](1321)\rangle + \sqrt{1-\sigma_{13}^2}|[211](3121)\rangle \qquad (16)
\end{aligned}$$

with

$$\sigma_{31} = \frac{1}{(\lambda_3 - 3) - (\lambda_1 - 1)} = -\frac{1}{3} = -\sigma_{13} \qquad (17)$$

Thus in the basis of ϕ_1, ϕ_2 and ϕ_3, the $[211]$ matrix of the element (34) is

$$D^{[211]}(34) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1/3 & 2\sqrt{2}/3 \\ 0 & 2\sqrt{2}/3 & 1/3 \end{pmatrix} \qquad (18)$$

The representation matrices for other elements of S_4 can be derived from the matrices above.

Method of characters of S_4 irreducible representations

- $\boldsymbol{Definition}$ **:** Let $\Gamma = \{D(g)\}$ be the representation of the group G of order n, the traces of the $nD(g)$ form the characters of the representation Γ
- The orthogonal theorem in group theory leads to the following property for the characters of a group,

$$\chi(g) \;=\; \sum_{\beta=1}^{h} m_\beta \chi^{(\beta)}(g),\; m_\alpha = \frac{1}{n}\sum_{g}\chi^{(\alpha)*}(g)\,\chi(g) \tag{19}$$

where g are group elements, $\chi(g)$ are the characters of a product (reducible) representation of the group, and $\chi^{(\beta)}(g)$ are the characters of the irreducible representation labeled by β.

- From the above equation and the properties of characters, one gets

$$m_{[31]_{OSF}} \;=\; \frac{1}{n}\sum_{g}\chi^{[31]_{OSF*}}(g)\,(\chi^{[X]_O}(g)\chi^{[Y]_{SF}}(g)) \tag{20}$$

- By applying Eq. (20), one gets all the spatial-spin-flavor configurations and spin-flavor configurations of the q^4 cluster of pentaquarks,

Character tables of conjugacy classes of S_4

C_i	ρ_i	$\chi^{[4]}$	$\chi^{[31]}$	$\chi^{[22]}$	$\chi^{[211]}$	$\chi^{[1111]}$
(e)	1	1	3	2	3	1
(ij)	6	1	1	0	-1	-1
$(ij)(kl)$	3	1	-1	2	-1	1
(ijk)	8	1	0	-1	0	1
$(ijkl)$	6	1	-1	0	1	-1

$q^4\,\overline{q}$ Systems

- The total states of q^4 is antisymmetric implies that the orbital-spin-flavor part must be a [31] state

$$\psi^{osf}_{[31]}(q^4) =$$  (21)

which is obtained from the Young tabloid of the color part by interchanging rows and columns.

- Total wave function of the q^4 configuration may be written in the general form

$$\psi = \sum_{i,j=\lambda,\rho,\eta} a_{ij}\psi^{c}_{[211]_i}\psi^{osf}_{[31]_j} \tag{22}$$

The coefficients can be determined by operating the permutations of S_4 on the general form, using the $[31]$ and $[211]$ representation matrices.

- For example, applying the permutation (12) first by using

$$D^{[31]}(12) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad D^{[211]}(12) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{23}$$

$q^4\,\overline{q}$ Systems

One gets

$$\begin{aligned}(12)\psi \;=\; & +a_{\lambda\lambda}\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\lambda} - a_{\lambda\rho}\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\rho} + a_{\lambda\eta}\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\eta}\\ & -a_{\rho\lambda}\Psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\lambda} + a_{\rho\rho}\psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\rho} - a_{\rho\eta}\psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\eta}\\ & -a_{\eta\lambda}\Psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\lambda} + a_{\eta\rho}\psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\rho} - a_{\eta\eta}\psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\eta}\end{aligned}$$

An antisymmetric ψ requires $a_{\lambda\lambda} = a_{\lambda\eta} = a_{\rho\rho} = a_{\eta\rho} = 0$. Therefore, we have

$$\begin{aligned}\psi \;=\; & a_{\lambda\rho}\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\rho} + a_{\rho\lambda}\psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\lambda} + a_{\rho\eta}\psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\eta}\\ & +a_{\eta\lambda}\psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\lambda} + a_{\eta\eta}\psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\eta}\end{aligned}$$

The action of the permutation (13) of S_4 on the above equation and the application of the antisymmetric restriction, $(13)\psi = -\psi$ lead to $a_{\eta\lambda} = a_{\rho\eta} = 0$ and $a_{\rho\lambda} = -a_{\lambda\rho}$, and hence

$$\psi \;=\; a_{\lambda\rho}\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\rho} - a_{\lambda\rho}\psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\lambda} + a_{\eta\eta}\psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\eta}$$

$q^4\,\overline{q}$ Systems

Applying the permutation (34) of S_4 to the above equation, we have

$$\begin{aligned}(34)\psi &= -a_{\lambda\rho}\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\rho} \\ &+ a_{\rho\lambda}\left(-\frac{1}{3}\psi^{c}_{[211]_\rho}+\frac{2\sqrt{2}}{3}\psi^{c}_{[211]_\eta}\right)\left(\frac{1}{3}\psi^{osf}_{[31]_\lambda}+\frac{2\sqrt{2}}{3}\psi^{osf}_{[31]_\eta}\right) \\ &+ a_{\eta\eta}\left(\frac{2\sqrt{2}}{3}\psi^{c}_{[211]_\rho}+\frac{1}{3}\psi^{c}_{[211]_\eta}\right)\left(\frac{2\sqrt{2}}{3}\psi^{osf}_{[31]_\lambda}-\frac{1}{3}\psi^{osf}_{[31]_\eta}\right).\end{aligned} \tag{24}$$

Here we have used the $[31]$ and $[211]$ representation matrices for the permutation (34),

$$D^{[31]}(34)=\begin{pmatrix}1/3 & 0 & 2\sqrt{2}/3\\ 0 & 1 & 0\\ 2\sqrt{2}/3 & 0 & -1/3\end{pmatrix},\quad D^{[211]}(34)=\begin{pmatrix}-1 & 0 & 0\\ 0 & -1/3 & 2\sqrt{2}/3\\ 0 & 2\sqrt{2}/3 & 1/3\end{pmatrix} \tag{25}$$

An antisymmetric ψ demands $a_{\lambda\rho}=a_{\eta\eta}$. Finally, we derive a fully antisymmetric wave function for the q^4 configuration

$$\psi = \frac{1}{\sqrt{3}}\left(\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\rho}-\psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\lambda}+\psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\eta}\right) \tag{26}$$

q^4 Spin-Flavor Wave Functions

For the pentaquark states with isospin $I = 0$ and strangeness $S = 1$ (compared to), the q^4 flavor-spin wave function of must be as follows:

$$\begin{array}{ccccc} [31] & = & [22] & \otimes & [31] \\ SU_{sf}(6) & & SU_f(3) & & SU_s(2) \end{array} \tag{27}$$

Again, the spin-flavor wave functions of various permutation symmetries take the general form,

$$\psi^{\rm sf} = \sum_{i=\lambda,\rho} \sum_{j=\lambda,\rho,\eta} a_{ij}\, \phi_{[22]_i} \chi_{[31]_j} \tag{28}$$

a_{ij} can be determined by acting the permutations of S_4 on the general form. The spin-flavor wave functions for the q^4 cluster are derived as,

$$\begin{aligned} \psi^{\rm sf}_{[31]_\rho} &= -\frac{1}{2}\phi_{[22]_\rho}\chi_{[31]_\lambda} - \frac{1}{2}\phi_{[22]_\lambda}\chi_{[31]_\rho} + \frac{1}{\sqrt{2}}\phi_{[22]_\rho}\chi_{[31]_\eta} \\ \psi^{\rm sf}_{[31]_\lambda} &= -\frac{1}{2}\phi_{[22]_\rho}\chi_{[31]_\rho} + \frac{1}{2}\phi_{[22]_\lambda}\chi_{[31]_\lambda} + \frac{1}{\sqrt{2}}\phi_{[22]_\lambda}\chi_{[31]_\eta} \\ \psi^{\rm sf}_{[31]_\eta} &= \frac{1}{\sqrt{2}}\phi_{[22]_\rho}\chi_{[31]_\rho} + \frac{1}{\sqrt{2}}\phi_{[22]_\lambda}\chi_{[31]_\lambda} \end{aligned} \tag{29}$$

q^4 Spin Wave Functions

The spin wave functions of the four-quark subsystem with the [31] symmetry can be derived by operating $P_{[31]_{\lambda,\rho,\eta}}$ on any q^4 spin state, for example, the state $\uparrow\uparrow\uparrow\downarrow$,

$$\left| \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \uparrow & \uparrow & \uparrow \\ \hline \downarrow \\ \cline{1-1}\end{array} \right\rangle = P_{[31]_\eta}(\uparrow\uparrow\uparrow\downarrow)$$

$$\Longrightarrow \chi_{[31]_\eta}(s_{q^4}=1, m_{q^4}=1) = \frac{1}{2\sqrt{3}} \mid 3 \uparrow\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow\uparrow - \uparrow\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow\uparrow \rangle$$

$$\left| \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \uparrow & \uparrow & \uparrow \\ \hline \downarrow \\ \cline{1-1}\end{array} \right\rangle = P_{[31]_\rho}(\uparrow\downarrow\uparrow\uparrow)$$

$$\Longrightarrow \chi_{[31]_\rho}(s_{q^4}=1, m_{q^4}=1) = \frac{1}{\sqrt{2}} \mid \uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow\uparrow \rangle$$

$$\left| \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \uparrow & \uparrow & \uparrow \\ \hline \downarrow \\ \cline{1-1}\end{array} \right\rangle = P_{[31]_\lambda}(\uparrow\uparrow\downarrow\uparrow)$$

$$\Longrightarrow \chi_{[31]_\lambda}(s_{q^4}=1, m_{q^4}=1) = \frac{1}{\sqrt{6}} \mid 2 \uparrow\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\uparrow - \uparrow\downarrow\uparrow\uparrow \rangle \tag{30}$$

q^4 Flavor Wave Functions

The flavor wave functions of the four-quark subsystem with the [22] symmetry can be derived by operating $P_{[22]_{\lambda,\rho}}$ on any q^4 state. For example,

$$\left| \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}, \begin{array}{|c|c|}\hline u & u \\ \hline d & d \\ \hline \end{array} \right\rangle = P_{[22]_\rho}(udud)$$
$$\Longrightarrow \phi_{[22]_\rho} = \frac{1}{2}(dudu - duud + udud - uddu) \tag{31}$$

$$\left| \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}, \begin{array}{|c|c|}\hline u & u \\ \hline d & d \\ \hline \end{array} \right\rangle = P_{[22]_\lambda}(uudd)$$
$$\Longrightarrow \phi_{[22]_\lambda} = \frac{1}{2\sqrt{3}}(2uudd + 2dduu - duud - udud - uddu - dudu) \tag{32}$$

The flavor wave functions for the $I = I_3 = 0$, $S = 1$ pentaquark are given by

$$\begin{aligned} \Phi_{[22]_\rho} &= \phi_{[22]_\rho}\bar{q} \\ \Phi_{[22]_\lambda} &= \phi_{[22]_\lambda}\bar{q} \ . \end{aligned} \tag{33}$$

The flavor states with other values of the isospin I, its projection I_3 and hypercharge Y can be derived the same way.

q^4 Color Wave Functions

The color state of the antiquark in pentaquarks is a $[11]$ antitriplet, thus the color wave function of the four-quark configuration must be a $[211]_3$ triplet,

$$\psi^c_{[211]_\lambda}(q^4) = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array} \quad \psi^c_{[211]_\rho}(q^4) = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array} \quad \psi^c_{[211]_\eta}(q^4) = \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1}\end{array} \tag{34}$$

The q^4 color wave functions can be derived by applying the [211] λ-type, ρ-type and η-type projection operators of the permutation group S_4 onto

$$\begin{aligned} P_{[211]_\lambda}(RRGB) &\Longrightarrow \psi^c_{[211]_\lambda}(R) \\ P_{[211]_\rho}(RGRB) &\Longrightarrow \psi^c_{[211]_\rho}(R) \\ P_{[211]_\eta}(RGBR) &\Longrightarrow \psi^c_{[211]_\eta}(R) \end{aligned} \tag{35}$$

Thus, the corresponding singlet color wave function of the pentaquark at color symmetry pattern $j = \lambda, \rho, \eta$ is given by

$$\Psi^c_{[211]_j}(q^4\bar{q}) = \frac{1}{\sqrt{3}}\left[\psi^c_{[211]_j}(R)\,\bar{R} + \psi^c_{[211]_j}(G)\,\bar{G} + \psi^c_{[211]_j}(B)\,\bar{B}\right]. \tag{36}$$

$q^4\bar{q}$ Color Wave Functions

The explicit forms q^4 color wave functions are

$$\begin{aligned}
\chi^{c}_{[211]_\lambda}(R) &= \frac{1}{\sqrt{16}}(2|RRGB\rangle - 2|RRBG\rangle \\
&\quad -|GRRB\rangle - |RGRB\rangle - |BRGR\rangle - |RBGR\rangle \\
&\quad +|BRRG\rangle + |GRBR\rangle + |RBRG\rangle + |RGBR\rangle), \\
\chi^{c}_{[211]_\rho}(R) &= \frac{1}{\sqrt{48}}(3|RGRB\rangle - 3|GRRB\rangle \\
&\quad +3|BRRG\rangle - 3|RBRG\rangle + 2|GBRR\rangle - 2|BGRR\rangle \\
&\quad -|BRGR\rangle + |RBGR\rangle + |GRBR\rangle - |RGBR\rangle), \\
\chi^{c}_{[211]_\eta}(R) &= \frac{1}{\sqrt{6}}(|BRGR\rangle + |RGBR\rangle + |GBRR\rangle \\
&\quad -|RBGR\rangle - |GRBR\rangle - |BGRR\rangle).
\end{aligned} \tag{37}$$

q^4 Flavor Wave Functions

The λ-type and ρ-type projection operators for the representation $[22]$ are derived as

$$
\begin{aligned}
P_{[22]_\lambda} &= \sum_{i=1}^{24} \langle [22](2211)|R_i|[22](2211)\rangle R_i \\
&= 2 + 2(12) - (13) - (14) - (23) - (24) + 2(34) \\
&\quad +2(12)(34) + 2(14)(23) + 2(13)(24) \\
&\quad -(123) - (124) - (132) - (134) - (142) - (143) - (234) - (243) \\
&\quad -(1234) - (1243) + 2(1324) - (1342) + 2(1423) - (1432) \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
P_{[22]_\rho} &= \sum_{i=1}^{24} \langle [22](2121)|R_i|[22](2121)\rangle R_i \\
&= 2 - 2(12) + (13) + (14) + (23) + (24) - 2(34) \\
&\quad +2(12)(34) + 2(14)(23) + 2(13)(24) \\
&\quad -(123) - (124) - (132) - (134) - (142) - (143) - (234) - (243) \\
&\quad +(1234) + (1243) - 2(1324) + (1342) - 2(1423) + (1432) \qquad (39)
\end{aligned}
$$

q^4 Spin Wave Functions

The λ-type, ρ-type and η-type projection operators for the representation $[31]$ are derived as

$$\begin{aligned} P_{[31]_\lambda} &= 6+6(12)-3(13)+5(14)-3(23)+5(24)+2(34) \\ &+2(12)(34)-4(14)(23)-4(13)(24) \\ &-3(123)+5(124)-3(132)-(134)+5(142)-(143)-(234)-(243) \\ &-(1234)-(1243)-4(1324)-(1342)-4(1423)-(1432) \end{aligned} \tag{40}$$

$$\begin{aligned} P_{[31]_\rho} &= 2-2(12)+(13)+(14)+(23)+(24)+2(34) \\ &-2(12)(34) \\ &-(123)-(124)-(132)+(134)-(142)+(143)+(234)+(243) \\ &-(1234)-(1243)-(1342)-(1432) \end{aligned} \tag{41}$$

$$\begin{aligned} P_{[31]_\eta} &= 3+3(12)+3(13)-(14)+3(23)-(24)-(34) \\ &-(12)(34)-(14)(23)-(13)(24) \\ &+3(123)-(124)+3(132)-(134)-(142)-(143)-(234)-(243) \\ &-(1234)-(1243)-(1324)-(1342)-(1423)-(1432) \end{aligned} \tag{42}$$

$q^4\bar{q}$ Spin-Flavor Wave Functions

- The total spin wave function of the pentaquark states with $s = 1/2$ and [31] symmetry is the combination of the spin wave function of the four-quark subsystem with $[31]$ symmetry (with $s = 1$) and that of the antiquark with $s = 1/2$, that is

$$\chi(q^4\bar{s})_{[31]_\alpha} = \sqrt{\frac{2}{3}}\chi_{[31]_\alpha}(m_{q^4} = 1)\chi_{\bar{s}}(-\frac{1}{2}) - \sqrt{\frac{1}{3}}\chi_{[31]_\alpha}(m_{q^4} = 0)\chi_{\bar{s}}(\frac{1}{2}) \quad (43)$$

with $\alpha = \rho,\ \lambda,\ \eta$.

- Combining the flavor wave functions in eq. (33) and the spin wave functions in eq. (43), we derive the total spin-flavor wave function of the pentaquark state with isospin $I = 0$, strangemess $S = 1$ and spin $s = 1/2$,

$$\begin{aligned}
\Psi^{\rm sf}_{[31]_\rho} &= -\frac{1}{2}\Phi_{[22]_\rho}\chi(q^4\bar{s})_{[31]_\lambda} - \frac{1}{2}\Phi_{[22]_\lambda}\chi(q^4\bar{s})_{[31]_\rho} + \frac{1}{\sqrt{2}}\Phi_{[22]_\rho}\chi(q^4\bar{s})_{[31]_\eta} \\
\Psi^{\rm sf}_{[31]_\lambda} &= -\frac{1}{2}\Phi_{[22]_\rho}\chi(q^4\bar{s})_{[31]_\rho} + \frac{1}{2}\Phi_{[22]_\lambda}\chi(q^4\bar{s})_{[31]_\lambda} + \frac{1}{\sqrt{2}}\Phi_{[22]_\lambda}\chi(q^4\bar{s})_{[31]_\eta} \\
\Psi^{\rm sf}_{[31]_\eta} &= \frac{1}{\sqrt{2}}\Phi_{[22]_\rho}\chi(q^4\bar{s})_{[31]_\rho} + \frac{1}{\sqrt{2}}\Phi_{[22]_\lambda}\chi(q^4\bar{s})_{[31]_\lambda} \quad (44)
\end{aligned}$$

q^3 SU(6) supermultiplets until N≤2

N	$SU(6)_{SF}$ Representations	l^P O(3)	$SU(6)_{SF}\times O(3)$ wave functions: $SU(3)_F$ octet	$SU(3)_F$ decuplet
0	56	0^+	$J^P=\frac{1}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\phi^0_{00s}(\Phi_\lambda\chi_\lambda+\Phi_\rho\chi_\rho)$	$J^P=\frac{3}{2}^+$ $\psi^c_{[111]}\phi^0_{00S}\Phi_S\chi_S$
1	70	1^-	$J^P=\frac{1}{2}^-,\ \frac{3}{2}^-$ $\frac{1}{2}\psi^c_{[111]}[\phi^1_{1m\rho}(\Phi_\lambda\chi_\rho+\Phi_\rho\chi_\lambda)+\phi^1_{1m\lambda}(\Phi_\rho\chi_\rho-\Phi_\lambda\chi_\lambda)]$ $J^P=\frac{1}{2}^-,\ \frac{3}{2}^-,\ \frac{5}{2}^-$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\chi_S(\phi^1_{1m\lambda}\Phi_\lambda+\phi^1_{1m\rho}\Phi_\rho)$	$J^P=\frac{1}{2}^-,\ \frac{3}{2}^-$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\Phi_S(\phi^1_{1m\lambda}\chi_\lambda+\phi^1_{1m\rho}\chi_\rho)$
2	56	0^+	$J^P=\frac{1}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\phi^2_{00s}(\Phi_\lambda\chi_\lambda+\Phi_\rho\chi_\rho)$	$J^P=\frac{3}{2}^+$ $\psi^c_{[111]}\Phi_S\phi^2_{00S}\chi_S$
	70	0^+	$J^P=\frac{1}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}[\phi^2_{00\rho}(\Phi_\lambda\chi_\rho+\Phi_\rho\chi_\lambda)+\phi^2_{00\lambda}(\Phi_\rho\chi_\rho-\Phi_\lambda\chi_\lambda)]$ $J^P=\frac{3}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\chi_S(\phi^2_{00\lambda}\Phi_\lambda+\phi^2_{00\rho}\Phi_\rho)$	$J^P=\frac{1}{2}^+$ $\frac{1}{2}\psi^c_{[111]}\Phi_S(\phi^2_{00\lambda}\chi_\lambda+\phi^2_{00\rho}\chi_\rho)$
2	20	1^+	$J^P=\frac{1}{2}^+,\ \frac{3}{2}^+$ $\psi^c_{[111]}\phi^2_{1mA}(\Phi_\rho\chi_\rho-\Phi_\lambda\chi_\lambda)$	
2	56	2^+	$J^P=\frac{3}{2}^+,\ \frac{5}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\phi^2_{2mS}(\Phi_\rho\chi_\rho+\Phi_\lambda\chi_\lambda)$	$J^P=\frac{1}{2}^+,\ \frac{3}{2}^+,\ \frac{5}{2}^+,\ \frac{7}{2}^+$ $\psi^c_{[111]}\phi^2_{2mS}\Phi_S\chi_S$
	70	2^+	$J^P=\frac{3}{2}^+,\ \frac{5}{2}^+$ $\frac{1}{2}\psi^c_{[111]}[\phi^2_{2m\rho}(\Phi_\lambda\chi_\rho+\Phi_\rho\chi_\lambda)+\phi^2_{2m\lambda}(\Phi_\rho\chi_\rho-\Phi_\lambda\chi_\lambda)]$ $J^P=\frac{1}{2}^+,\ \frac{3}{2}^+,\ \frac{5}{2}^+,\ \frac{7}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\chi_S(\phi^2_{2m\lambda}\Phi_\lambda+\phi^2_{2m\rho}\Phi_\rho)$	$J^P=\frac{3}{2}^+,\ \frac{5}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\Phi_S(\phi^2_{2m\lambda}\chi_\lambda+\phi^2_{2m\rho}\chi_\rho)$

Resonances of negative-parity applied to fit the model parameters.

$(\Gamma, {}^{2s+1}D, N, L^P)$	Status	J^P	$M^{exp}(\mathrm{MeV})$	$M^{cal}(\mathrm{MeV})$
$N(70, {}^2 10, 1, 1^-)$	****	$\frac{3}{2}^-$	N(1520)	1380
$N(70, {}^2 10, 1, 1^-)$	****	$\frac{1}{2}^-$	N(1535)	1380
$N(70, {}^4 10, 1, 1^-)$	****	$\frac{1}{2}^-$	N(1650)	1672
$N(70, {}^4 10, 1, 1^-)$	****	$\frac{5}{2}^-$	N(1675)	1672
$N(70, {}^4 10, 1, 1^-)$	***	$\frac{3}{2}^-$	N(1700)	1672
$\Delta(70, {}^2 10, 1, 1^-)$	****	$\frac{1}{2}^-$	Δ(1620)	1380
$\Delta(70, {}^2 10, 1, 1^-)$	****	$\frac{3}{2}^-$	Δ(1700)	1380

- We can't locate $N(1685)$ in the q^3 negative party spectrum.

Mass of ground state pentaquark $q^3s\bar{s}$ considering the mixtures between pentaquark states

J^P	$q^3s\bar{s}$ configurations	$(S^{q^3s}, S^{\bar{s}}, S)$	$M^{EV}(q^3s\bar{s})$
$\frac{5}{2}^-$	$\Psi^{sf}_{[31]_F[4]_S}(q^3s\bar{s})$	(2,1/2,5/2)	2546
$\frac{3}{2}^-$	$\Psi^{sf}_{[4]_F[31]_S}(q^3s\bar{s})$	(1,1/2,3/2)	2586
	$\begin{pmatrix}\Psi^{sf}_{[31]_F[4]_S}(q^3s\bar{s})\\ \Psi^{sf}_{[31]_F[31]_S}(q^3s\bar{s})\end{pmatrix}^{mix1}$	(2,1/2,3/2) (1,1/2,3/2)	$\begin{pmatrix}2289\\2545\end{pmatrix}$
	$\Psi^{sf}_{[211]_F[31]_S}(q^3s\bar{s})$	(1,1/2,3/2)	2243
	$\Psi^{sf}_{[22]_F[31]_S}(q^3s\bar{s})$	(1,1/2,3/2)	2354
$\frac{1}{2}^-$	$\Psi^{sf}_{[4]_F[31]_S}(q^3s\bar{s})$	(1,1/2,1/2)	2762
	$\begin{pmatrix}\Psi^{sf}_{[31]_F[31]_S}(q^3s\bar{s})\\ \Psi^{sf}_{[31]_F[22]_S}(q^3s\bar{s})\end{pmatrix}^{mix2}$	(1,1/2,1/2) (0,1/2,1/2)	$\begin{pmatrix}2370\\2471\end{pmatrix}$
	$\begin{pmatrix}\Psi^{sf}_{[211]_F[31]_S}(q^3s\bar{s})\\ \Psi^{sf}_{[211]_F[22]_S}(q^3s\bar{s})\end{pmatrix}^{mix3}$	(1,1/2,1/2) (0,1/2,1/2)	$\begin{pmatrix}1997\\2200\end{pmatrix}$
	$\Psi^{sf}_{[22]_F[31]_S}(q^3s\bar{q})$	(1,1/2,1/2)	2135

- The lower mixing state of $[31]_{FS}[211]_F[31]_S$ and $[31]_{FS}[211]_F[22]_S$ $q^3s\bar{s}$ pentaquark configurations with quantum number $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ has the lowest mass, 1997 MeV.

Mass of ground state pentaquark $q^3 s\bar{q}$ considering the mixtures between pentaquark states

J^P	$q^3 s\bar{q}$ configurations	$(S^{q^3 s}, S^{\bar{q}}, S)$	$M^{EV}(q^3 s\bar{q})$
$\frac{5}{2}^-$	$\Psi^{sf}_{[31]_F[4]_S}(q^3 s\bar{q})$	(2,1/2,5/2)	2408
$\frac{3}{2}^-$	$\Psi^{sf}_{[4]_F[31]_S}(q^3 s\bar{q})$	(1,1/2,3/2)	2392
	$\begin{pmatrix} \Psi^{sf}_{[31]_F[4]_S}(q^3 s\bar{q}) \\ \Psi^{sf}_{[31]_F[31]_S}(q^3 s\bar{q}) \end{pmatrix}^{mix1}$	(2,1/2,3/2) (1,1/2,3/2)	$\begin{pmatrix} 1966 \\ 2407 \end{pmatrix}$
	$\Psi^{sf}_{[211]_F[31]_S}(q^3 s\bar{q})$	(1,1/2,3/2)	2116
	$\Psi^{sf}_{[22]_F[31]_S}(q^3 s\bar{q})$	(1,1/2,3/2)	2229
$\frac{1}{2}^-$	$\Psi^{sf}_{[4]_F[31]_S}(q^3 s\bar{q})$	(1,1/2,1/2)	2659
	$\begin{pmatrix} \Psi^{sf}_{[31]_F[31]_S}(q^3 s\bar{q}) \\ \Psi^{sf}_{[31]_F[22]_S}(q^3 s\bar{q}) \end{pmatrix}^{mix2}$	(1,1/2,1/2) (0,1/2,1/2)	$\begin{pmatrix} 2162 \\ 2314 \end{pmatrix}$
	$\begin{pmatrix} \Psi^{sf}_{[211]_F[31]_S}(q^3 s\bar{q}) \\ \Psi^{sf}_{[211]_F[22]_S}(q^3 s\bar{q}) \end{pmatrix}^{mix3}$	(1,1/2,1/2) (0,1/2,1/2)	$\begin{pmatrix} 1742 \\ 2052 \end{pmatrix}$
	$\Psi^{sf}_{[22]_F[31]_S}(q^3 s\bar{q})$	(1,1/2,1/2)	1894

- Here all $q^3 s\bar{q}$ ground state pentaquarks have isospin $I = 0$.

Mass of ground state pentaquark $q^3c\bar{c}$

- Ground hidden-charm pentaquark $q^3c\bar{c}$ mass spectrum, where the q^3 and $Q\bar{Q}$ components are in the color octet states.

$q^3Q\bar{Q}$ configurations	J^P	$S^{c\bar{c}}$	$M(q^3c\bar{c})$(MeV)
$\Psi^{csf}_{[21]_C[21]_{FS}[21]_F[21]_S}(q^3c\bar{c})$	$\frac{1}{2}^-$	0	4447
	$\frac{1}{2}^-, \frac{3}{2}^-$	1	4416, 4459
$\Psi^{csf}_{[21]_C[21]_{FS}[3]_F[21]_S}(q^3c\bar{c})$	$\frac{1}{2}^-$	0	4666
	$\frac{1}{2}^-, \frac{3}{2}^-$	1	4665, 4665
$\Psi^{csf}_{[21]_C[21]_{FS}[21]_F[3]_S}(q^3c\bar{c})$	$\frac{3}{2}^-$	0	4520
	$\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$	1	4445, 4489, 4562

P_c decay in quark rearrangement diagram

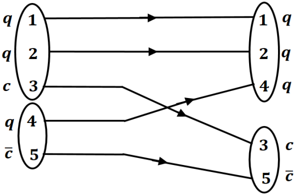


Figure: hidden charm diagram

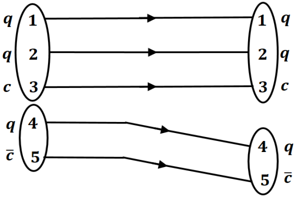


Figure: open charm diagram

- Direct quark rearrangement diagrams, with the wave function of baryon, meson and pentaquark states, the transition amplitude is

$$T = T^{CSF}\langle \psi_f|\hat{O}|P_c\rangle \tag{45}$$

with $\hat{O}$ taking the form,

$$\hat{O}_d = \delta^3(\vec{p}_1 - \vec{p_1'})\delta^3(\vec{p}_2 - \vec{p_2'})\delta^3(\vec{p}_3 - \vec{p_3'})\delta^3(\vec{p}_4 - \vec{p_4'}) \delta^3(\vec{p}_5 - \vec{p_5'}), \tag{46}$$

$$\hat{O}_c = \delta^3(\vec{p}_1 - \vec{p_1'})\delta^3(\vec{p}_2 - \vec{p_2'})\delta^3(\vec{p}_3 - \vec{p_4'})\delta^3(\vec{p}_4 - \vec{p_3'}) \delta^3(\vec{p}_5 - \vec{p_5'}). \tag{47}$$

Partial decay widths of $I = \frac{1}{2}$ mixing pentaquark states

- Partial decay width ratios of I= 1/2 mixing pentaquark states, normalized to 4457 MeV state.

J	Threshold	E_B	Mass	Eigenvector^2	Total	$p\eta_c$	pJ/ψ	$\Sigma_c^*\bar{D}$	$\Sigma_c\bar{D}$	$\Lambda_c^+\bar{D}$	$\Sigma_c^*\bar{D}^*$	$\Sigma_c\bar{D}^*$	$\Lambda_c^+\bar{D}^*$
$\frac{1}{2}$	$\Sigma_c\bar{D}(4322)$	0	4298	(0.88,0.05,0.02,0.06)	0.57	0.21	0.11						0.25
		−5	4293	(0.88,0.04,0.02,0.05)	0.33	0.22	0.11						
		−10	4289	(0.89,0.04,0.02,0.05)	0.34	0.22	0.11						
	$\Sigma_c\bar{D}^*(4462)$	0	4444	(0.22,0.38,0.10,0.30)	14.71	0.01	0.13		9.99	2.74			1.85
		−5	4443	(0.22,0.39,0.05,0.33)	16.11	0.01	0.13		11.42	2.19			2.36
		−10	4442	(0.21,0.40,0.02,0.36)	17.67	0.01	0.13		12.92	1.66			2.95
	$\Sigma_c\bar{D}^*(4462)$	0	4426	(0.24,0.10,0.47,0.20)	17.53	0.01	0.15		10.61	1.63			5.13
		−5	4425	(0.29,0.08,0.46,0.17)	15.72	0.01	0.19		9.18	1.74			4.60
		−10	4423	(0.36,0.06,0.44,0.14)	13.77	0.02	0.23		7.68	1.82			4.02
$\frac{3}{2}$	$\Sigma_c^*\bar{D}^*(4526)$	0	4509	(0.77,0.10,0.13,0)	1.87		0.28	0.08				0.43	1.08
		−5	4505	(0.79,0.11,0.11,0)	1.82		0.29	0.03				0.34	1.16
		−10	4501	(0.79,0.12,0.09,0)	1.86		0.30					0.27	1.28
	$\Sigma_c^*\bar{D}(4386)$	0	4376	(0.95,0.05,0,0)	1.06		0.35						0.71
		−5	4371	(0.95,0.05,0,0)	1.01		0.36						0.65
		−10	4367	(0.95,0.04,0,0)	0.96		0.36						0.60
	$\Sigma_c\bar{D}^*(4462)$	0	4457	(0.95,0.02,0.01,0.02)	1.00		0.09	0.61					0.31
		−5	4452	(0.97,0.01,0,0.02)	0.65		0.09	0.37					0.19
		−10	4448	(0.97,0.01,0,0.01)	0.47		0.09	0.25					0.13

Spatial and spin-flavor wave functions

- Spatial wave functions are in symmetric type for ground state pentaquarks.
- States in the harmonic oscillator interaction served as complete bases.

- All possible spin-flavor [31] configurations of q^4 cluster

	$[31]_{FS}$		
$[31]_{FS}[31]_F[22]_S$	$[31]_{FS}[31]_F[31]_S$	$[31]_{FS}[31]_F[4]_S$	$[31]_{FS}[211]_F[22]_S$
$[31]_{FS}[211]_F[31]_S$	$[31]_{FS}[22]_F[31]_S$	$[31]_{FS}[4]_F[31]_S$	

- All possible spin-flavor [21] configurations of q^3 cluster

	$[21]_{FS}$	
$[21]_{FS}[21]_F[3]_S$	$[21]_{FS}[3]_F[21]_S$	$[21]_{FS}[21]_F[21]_S$