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A model study of tetraquark and pentaquark states

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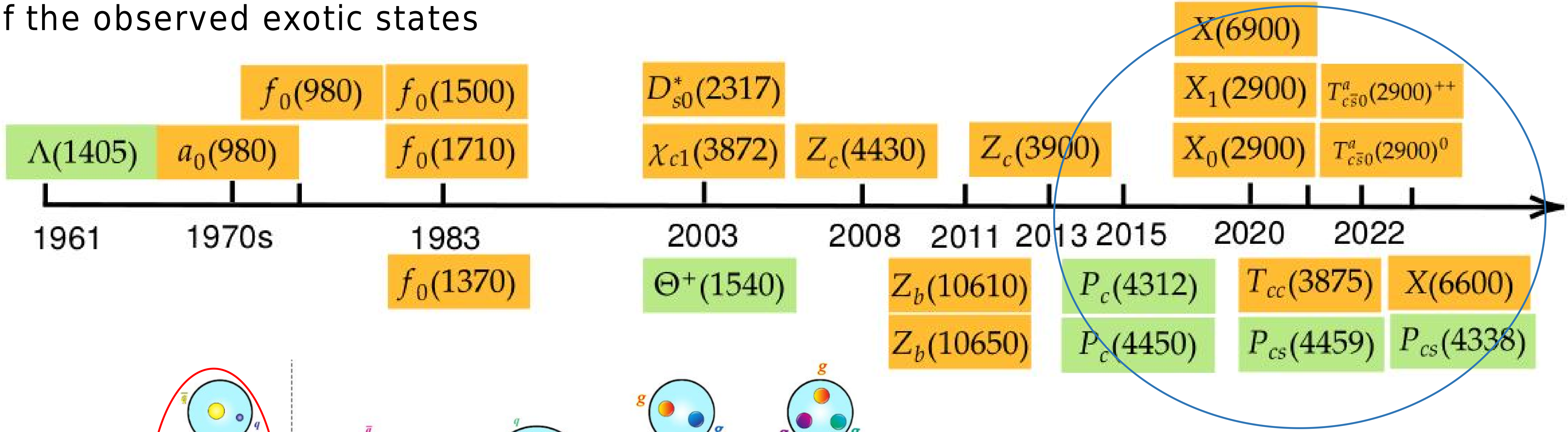
Content

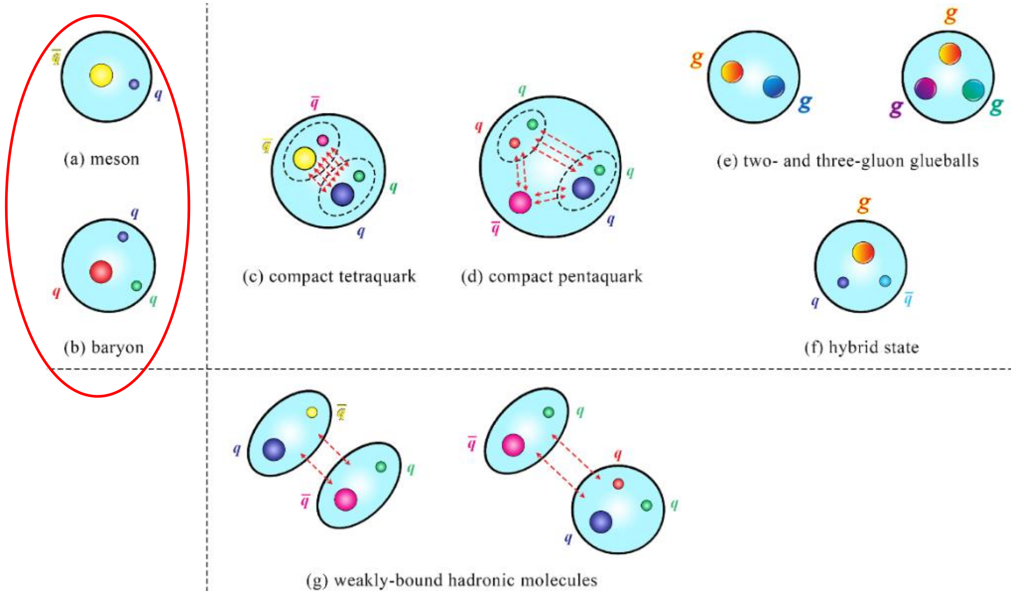
Related papers:

Y.R. Liu, X. Liu, S.L. Zhu, Phys. Rev. D 93, 074023 (2016)
J. Wu, Y.R. Liu, K.Chen, X.Liu, S.L. Zhu, Phys. Rev. D 94, 094031 (2016)
S.Q. Luo, K. Chen, X. Liu, Y.R. Liu, S.L. Zhu, Eur. Phys. J. C 77, 709 (2017)
J. Wu, Y.R. Liu, K. Chen, X. Liu, S.L. Zhu, Phys. Rev. D 95, 034002 (2017)
J. Wu, Y.R. Liu, K.Chen, X.Liu, S.L. Zhu, Phys. Rev. D 97, 094015 (2018)
J. Wu, X. Liu, Y.R. Liu, S.L. Zhu, Phys. Rev. D 99, 014037 (2019)
J.B. Cheng, Y.R. Liu, Phys. Rev. D 100, 054002 (2019)
J.B. Cheng, S.Y. Li, Y.R. Liu, Y.N. Liu, Z.G. Si, T. Yao, Phys. Phys. D 101, 114017 (2020)
J.B. Cheng, S.Y. Li, Y.R. Liu, Z.G. Si, T. Yao, Chin. Phys. C 45, 043102 (2021)
S.Y. Li, Y.R. Liu, Z.L. Man, Z.G. Si, J. Wu, Phys. Rev. D 108, 056015 (2023)
S.Y. Li, Y.R. Liu, Z.L. Man, Z.G. Si, J. Wu, Chin. Phys. C 48, 063109 (2024)
S.Y. Li, Y.R. Liu, Z.L. Man, Z.G. Si, J. Wu, Phys. Rev. D 110, 094044 (2024)

Some of the observed exotic states



Conventional hadrons

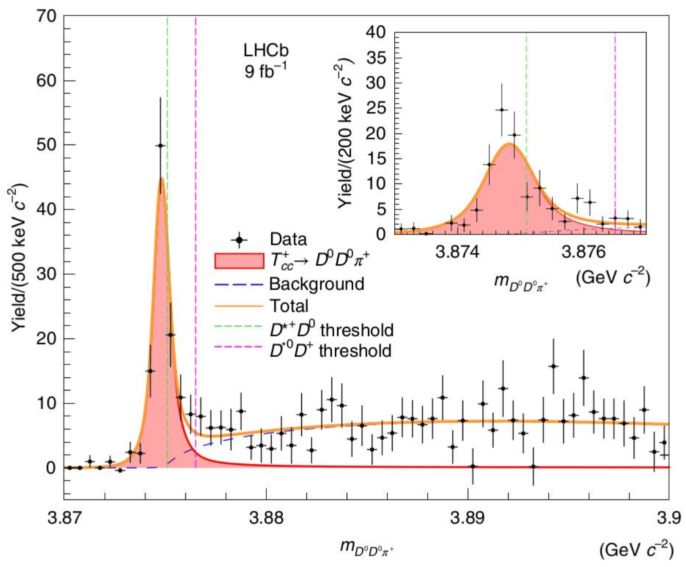


H.X. Chen, W. Chen, X. Liu, Y.R. Liu, S.L. Zhu, Rep. Prog. Phys. 86, 026201 (2023)

$(n = u, d)$

	State	Mass(MeV)	Γ (MeV)	Observed channels
	$\chi_{c1}(4140)$[CDF:2009jgo]	$4143.0 \pm 2.9 \pm 1.2$	$11.7^{+8.3}_{-5.0} \pm 3.7$	$B^+ \to J/\psi\phi K^+$
	$X(4350)$[Belle:2009rkh]	$4350^{+4.6}_{-5.1} \pm 0.7$	$13^{+18}_{-9} \pm 4$	$\gamma\gamma \to J/\psi\phi$
	$\chi_{c1}(4274)$[CDF:2011pep]	$4274^{+8.4}_{-6.7} \pm 1.9$	$32.3^{+21.9}_{-15.3} \pm 7.6$	$B^+ \to J/\psi\phi K^+$
	$\chi_{c0}(4500)$[LHCb:2016axx]	$4506 \pm 11^{+12}_{-15}$	$92 \pm 21^{+21}_{-20}$	$B^+ \to J/\psi\phi K^+$
	$\chi_{c0}(4700)$[LHCb:2016axx]	$4704 \pm 10^{+14}_{-24}$	$120 \pm 31^{+42}_{-33}$	$B^+ \to J/\psi\phi K^+$
	$X(4630)$[LHCb:2021uow]	$4626 \pm 16^{+18}_{-110}$	$174 \pm 27^{+134}_{-73}$	$B^+ \to J/\psi\phi K^+$
	$X(4685)$[LHCb:2021uow]	$4684 \pm 7^{+13}_{-16}$	$126 \pm 15^{+37}_{-41}$	$B^+ \to J/\psi\phi K^+$
$cs\overline{cs}$	$X(3960)$[LHCb:2022aki]	$3956 \pm 5 \pm 10$	$43 \pm 13 \pm 8$	$B^+ \to D_s^+ D_s^- K^+$
	$X_0(4140)$[LHCb:2022aki]	$4133 \pm 6 \pm 6$	$67 \pm 17 \pm 7$	$B^+ \to D_s^+ D_s^- K^+$

$T_{cc}(3875)$



Collaboration	State	(M, Γ)		Observation Channel
LHCb [1]	X(6900)	$(6905 \pm 11 \pm 7,\ 80 \pm 19 \pm 33)$		$J/\psi J/\psi$
CMS[3]		Interference model	No-interference model	$J/\psi J/\psi$
	X(6600)	$(6638^{+43+16}_{-38-31},\ 440^{+230+110}_{-200-240})$	$(6552 \pm 10 \pm 12,\ 124^{+32}_{-26} \pm 33)$	
	X(6900)	$(6847^{+44+48}_{-28-20},\ 191^{+66+52}_{-49-17})$	$(6927 \pm 9 \pm 4,\ 122^{+24}_{-21} \pm 18)$	
	X(7200)	$(7134^{+48+41}_{-25-15},\ 97^{+40+29}_{-29-26})$	$(7287^{+20}_{-18} \pm 5,\ 95^{+59}_{-40} \pm 19)$	
ATLAS[4]		Model A	Model B	$J/\psi J/\psi$
	X(6400)	$(6410 \pm 80^{+80}_{-30},\ 590 \pm 350^{+120}_{-200})$	$(6650 \pm 20^{+30}_{-20},\ 440 \pm 50^{+60}_{-50})$	
	X(6600)	$(6630 \pm 50^{+80}_{-10},\ 350 \pm 110^{+110}_{-40})$		
	X(6900)	$(6860 \pm 30^{+10}_{-20},\ 110 \pm 50^{+20}_{-10})$	$(6910 \pm 10 \pm 10,\ 150 \pm 30 \pm 10)$	
		Model α	Model β	$J/\psi\psi(2S)$
	X(7200)	$(7200 \pm 30^{+10}_{-40},\ 90 \pm 60^{+60}_{-50})$	$(6960 \pm 50 \pm 30,\ 510 \pm 170^{+110}_{-100})$	

$cc\overline{cc}$

$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{BW} = 273 \pm 61 \pm 5^{+11}_{-14}\ \text{keV}$$

$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38}\ \text{keV}$$

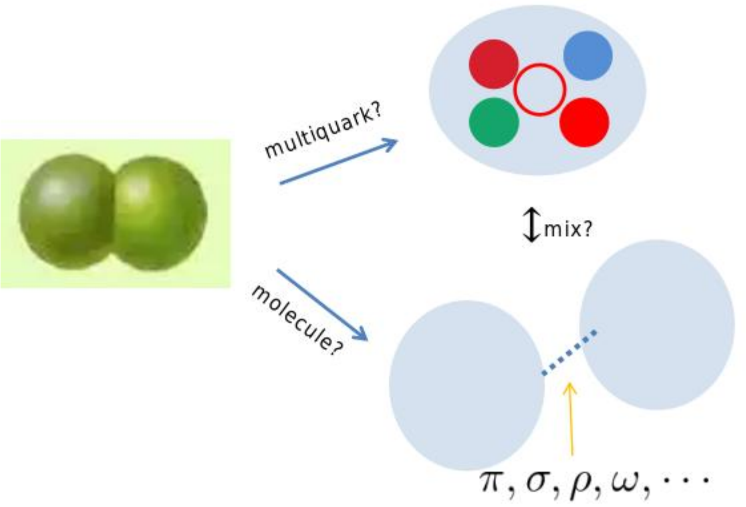
$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0}\ \text{keV}$$

$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14}\ \text{keV}$$

	State	Mass(MeV)	Γ (MeV)	Observed channels
$cs\overline{nn}$	$T_{cs0}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+ D^- K^+$
	$T_{cs1}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$B^+ \to D^+ D^- K^+$
$cn\overline{sn}$	$T^a_{c\bar{s}0}(2900)^0$[LHCb:2022sfr,LHCb:2022lzp]	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to \bar{D}^0 D_s^+ \pi^-$
	$T^a_{c\bar{s}0}(2900)^{++}$[LHCb:2022sfr,LHCb:2022lzp]	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^- D_s^+ \pi^+$

Hidden-charm pentaquark-like baryons ($n = u, d$):

	State	Mass (MeV)	Γ (MeV)	observable channels
$uudc\bar{c}$	$P_c(4380)^+$[LHCb:2015yax]	$4380 \pm 8 \pm 29$	$215 \pm 18 \pm 86$	$\Lambda_b^0 \to J/\psi p K^-$
	$P_c(4312)^+$[LHCb:2019kea]	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	$\Lambda_b^0 \to J/\psi p K^-$
	$P_c(4440)^+$[LHCb:2019kea]	$4440 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.2}$	$\Lambda_b^0 \to J/\psi p K^-$
	$P_c(4457)^+$[LHCb:2019kea]	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	$\Lambda_b^0 \to J/\psi p K^-$
$udsc\bar{c}$	$P_{cs}(4459)^0$[LHCb:2020jpq]	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$	$\Xi_b^- \to J/\psi \Lambda K^-$
	$P_c(4337)^+$[LHCb:2021chn]	$4337^{+7}_{-4}{}^{+2}_{-2}$	$29^{+26}_{-12}{}^{+14}_{-14}$	$B_s^0 \to J/\psi p \bar{p}$
	$P_{cs}(4338)^0$ [LHCb:2022jad]	$4338.2 \pm 0.7 \pm 0.4$	$7.0 \pm 1.2 \pm 1.3$	$B^- \to J/\psi \Lambda \bar{p}$



Can we understand exotic mesons/baryons in the compact multiquark picture?

Where are other tetraquark/pentaquark states?

Here: S-wave spectra and rearrangement decays in a model based on color-magnetic interaction (CMI) [symmetry analysis]

Hadron	CMI	Hadron	CMI	Parameter(MeV)
N	$-8C_{nn}$	Δ	$8C_{nn}$	$C_{nn} = 18.4$
Σ	$\frac{8}{3}C_{nn} - \frac{32}{3}C_{ns}$	Σ^*	$\frac{8}{3}C_{nn} + \frac{16}{3}C_{ns}$	$C_{ns} = 12.4$
Ξ^0	$\frac{8}{3}(C_{ss} - 4C_{ns})$	Ξ^{*0}	$\frac{8}{3}(C_{ss} + C_{ns})$	
Ω	$8C_{ss}$			$C_{ss} = 6.5$
Λ	$-8C_{nn}$			
D	$-16C_{c\bar{n}}$	D^*	$\frac{16}{3}C_{c\bar{n}}$	$C_{c\bar{n}} = 6.7$
D_s	$-16C_{c\bar{s}}$	D_s^*	$\frac{16}{3}C_{c\bar{s}}$	$C_{c\bar{s}} = 6.7$
B	$-16C_{b\bar{n}}$	B^*	$\frac{16}{3}C_{b\bar{n}}$	$C_{b\bar{n}} = 2.1$
B_s	$-16C_{b\bar{s}}$	B^*	$\frac{16}{3}C_{b\bar{s}}$	$C_{b\bar{s}} = 2.3$
η_c	$-16C_{c\bar{c}}$	J/ψ	$\frac{16}{3}C_{c\bar{c}}$	$C_{c\bar{c}} = 5.3$
η_b	$-16C_{b\bar{b}}$	Υ	$\frac{16}{3}C_{b\bar{b}}$	$C_{b\bar{b}} = 2.9$
Σ_c	$\frac{8}{3}C_{nn} - \frac{32}{3}C_{cn}$	Σ_c^*	$\frac{8}{3}C_{nn} + \frac{16}{3}C_{cn}$	$C_{cn} = 4.0$
Ξ'_c	$\frac{8}{3}C_{ns} - \frac{16}{3}C_{cn} - \frac{16}{3}C_{cs}$	Ξ_c^*	$\frac{8}{3}C_{ns} + \frac{8}{3}C_{cn} + \frac{8}{3}C_{cs}$	$C_{cs} = 4.8$
Σ_b	$\frac{8}{3}C_{nn} - \frac{32}{3}C_{bn}$	Σ_b^*	$\frac{8}{3}C_{nn} + \frac{16}{3}C_{bn}$	$C_{bn} = 1.3$
Ξ'_b	$\frac{8}{3}C_{ns} - \frac{16}{3}C_{bn} - \frac{16}{3}C_{bs}$	Ξ_b^*	$\frac{8}{3}C_{ns} + \frac{8}{3}C_{bn} + \frac{8}{3}C_{bs}$	$C_{bs} = 1.2$

Original CMI:

$$H = \sum_i m_i - \sum_{i<j} C_{ij}\lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j,$$

$$\Rightarrow \quad M = \sum_i m_i + E_{\text{CMI}}$$

$$\begin{aligned} m_n &= 361.8\ \text{MeV}, \quad (n = u, d) \\ m_s &= 540.4\ \text{MeV}, \\ m_c &= 1724.8\ \text{MeV}, \\ m_b &= 5052.9\ \text{MeV}. \end{aligned}$$

TABLE III. Comparison for hadron masses between experimental data and theoretical estimation. All the values are in units of MeV.

Hadron	Theory	Experiment	Deviation	Hadron	Theory	Experiment	Deviation
D	1975.9	1864.8	111.1	D^*	2121.0	2007.0	114.0
D_s	2154.5	1968.3	186.2	D_s^*	2299.5	2112.1	187.4
η_c	3361.0	2983.6	377.4	J/ψ	3474.1	3096.9	377.2
Σ_c	2452.9	2454.0	1.1	Σ_c^*	2516.9	2518.4	−1.5
Ω_c	2796.2	2695.2	101.0	Ω_c^*	2845.3	2765.9	79.4
Ξ_c	2525.9	2471.0	54.9	Ξ'_c	2612.3	2577.9	34.4
Ξ_c^*	2680.6	2645.9	34.7				

Bad theoretical results! mainly due to *quark masses*

- Alternative schemes to study multiquark spectrum:

  (1) Reference scale → hadron-hadron threshold

  $M = [M_{ref} - (E_{CMI})_{ref}] + E_{CMI}$

  (same quark content for *ref* and multiquark)

  Studies for $cs\overline{cs}$, $QQ\overline{QQ}$, $qq\overline{QQ}$, $Qq\overline{Qq}$, $QQ\overline{Qq}$ $\to M_{low}$

  [1605.01134, 1608.07900, 1609.06117, 1707.01180, 1810.06886, 2001.05287, 2008.00737]

  (2) Tetraquark: reference scale $\to M_{X(4140)}$ $\to M_{reasonable}$

  Assumption: $X(4140)$ observed in $J/\psi\phi$ as the ground 1^{++} $cs\overline{cs}$ tetraquark

  Pentaquark: reference scale $\to M_{Pc(4312)}$ $\to M_{reasonable}$

  Assumption: $Pc(4312)$ observed in $J/\psi p$ as a $3/2^-$ $c\bar{c}uud$ pentaquark [consistency requires]

Formalism: mass splitting model for tetraquarks

$$M = M_{X(4140)} - (E_{CMI})_{X(4140)} + \sum_{ij} n_{ij}\Delta_{ij} + E_{CMI}$$

where $\Delta_{ij} = m_i - m_j$ denotes the effective quark mass gap between quark i and quark j.

Wu, Liu, Liu, Zhu, PRD 99, 014037 (2019);
Cheng, Li, Liu, Liu, Si, Yao, PRD 101, 114017 (2020)

Δ_{bc}=3340.2MeV,
Δ_{cn}=1280.7MeV,
Δ_{sn}=90.6MeV,
Δ_{cs}=1180.6MeV,
Δ_{bs}=4520.2MeV.

C_{ij}	n	s	c	b	$C_{i\bar{j}}$	$\bar{n}$	$\bar{s}$	$\bar{c}$	$\bar{b}$
n	18.3	12.1	4.0	1.3	n	29.8	18.7	6.6	2.1
s		6.5	4.3	1.3	s		9.8	6.7	2.3
c			3.5	2.0	c			5.3	3.3
b				1.9	b				2.9

Consistent with Buccella et al., EPJC 49, 743 (2007).

Approximate relations:

$$\frac{C_{cc}}{C_{c\bar{c}}} = \frac{C_{bb}}{C_{b\bar{b}}} = \frac{C_{bc}}{C_{b\bar{c}}} = \frac{C_{nn}}{C_{n\bar{n}}} \approx \frac{2}{3}$$

$$\Delta_{cn} \approx \Delta_{cs} + \Delta_{sn},$$
$$\Delta_{bs} \approx \Delta_{bc} + \Delta_{cs}.$$

$(n = u, d)$

Godfrey-Isgur model: $m_{B_c^*} - m_{B_c} = 70\text{ MeV}$

Why is X(4140) selected?

1. It is a $J/\psi\phi$ resonance confirmed by different experiments. $J^{PC}=1^{++}$ determined; suppressed mixing with $c\bar{c}$ states;
2. $J^{PC}=1^{++}$ partner states X(4274) and X(4140) can be consistently interpreted as compact $cs\overline{cs}$ tetraquark states;

   [Stancu, J.Phys.G 37, 075017 (2010); Wu et. al., PRD 94, 094031 (2016)]
3. X(4140) as the reference state can provide more reasonable explanations for other observed $cs\overline{cs}$ states.

   [Li et. al., Chin.Phys.C 48, 063109 (2024)]

Formalism: mass fomulas for compact *tetraquark* states

In original CMI (v1):

$$M = \sum_i m_i + E_{\mathrm{CMI}}$$

additional attraction needed

In threshold scheme (v2):

$$M = [M_{ref} - (E_{\mathrm{CMI}})_{ref}] + E_{\mathrm{CMI}}$$

included superfluous attraction

where *ref=hadron-hadron state* and mesured M_{ref} is its threshold.

In the scheme we use (v3):

$$M = M_{X(4140)} - (E_{CMI})_{X(4140)} + \sum_{ij} n_{ij}\Delta_{ij} + E_{CMI}$$

Usually, $M_{v2} < M_{v3} < M_{v1}$

(size: hadron-hadron state > compact tetraquark > conventional hadron)

- **Combine information from spectrum and decay to analyze multiquark properties**

- A simple decay scheme with assumptions:

  1. decay Hamiltonian is a constant: $\boldsymbol{H_{decay}} = \mathcal{C}$ ← system-dependent $\mathcal{C}$

  2. measured width ≈sum of two-body rearrangement decay widths: $\boldsymbol{\Gamma_{exp}} \approx \boldsymbol{\Gamma_{sum}}$

$$\mathcal{M} = \langle initial|H_{decay}|final\rangle = \mathcal{C}\sum_{ij} x_i y_j$$

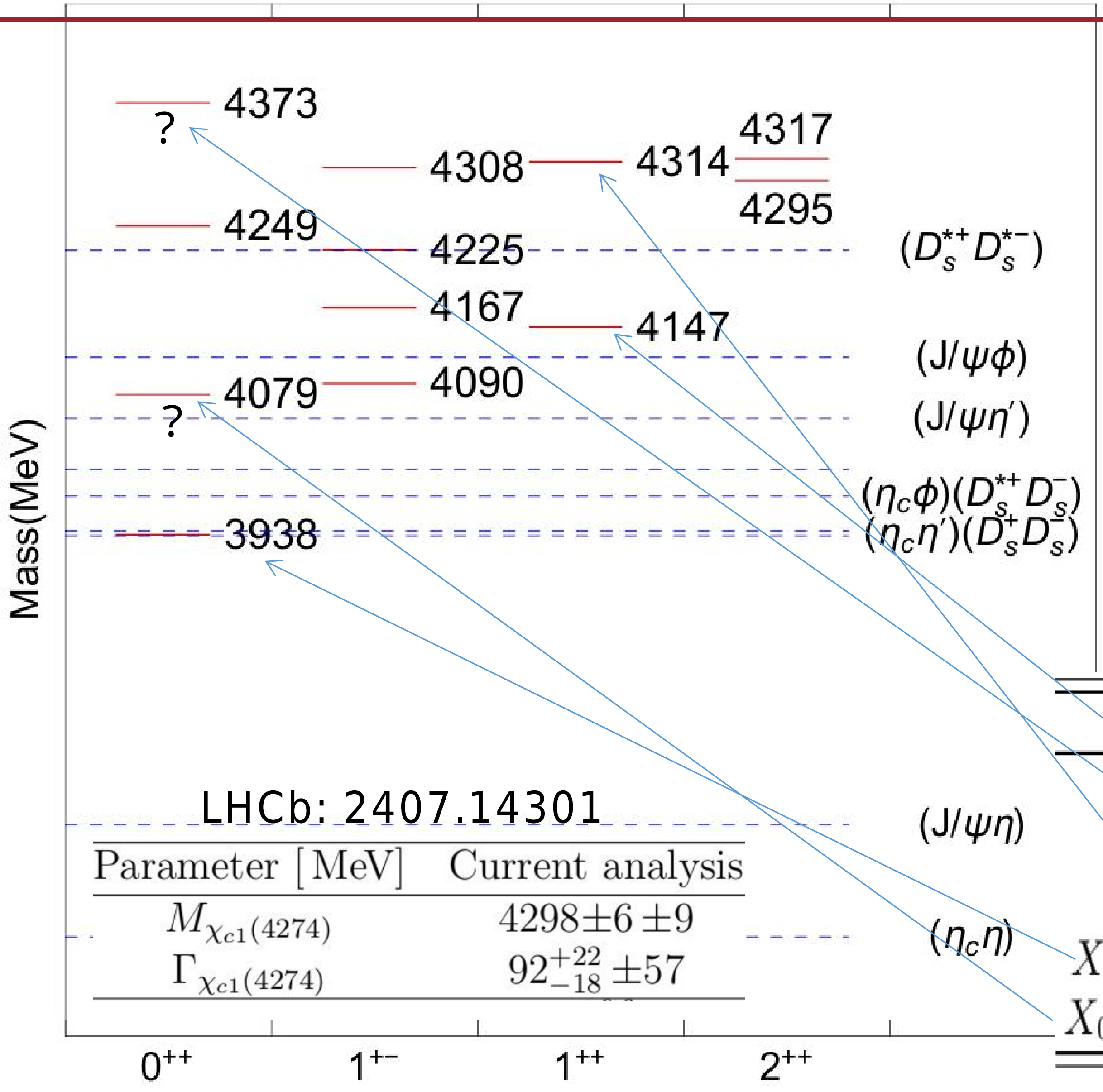
$$\Psi_{initial} = \sum_i x_i(q_1 q_2 \bar{q}_3 \bar{q}_4),$$

$$\Psi_{final} = \sum_i y_i(q_1 q_2 \bar{q}_3 \bar{q}_4).$$

$$\Gamma = |\mathcal{M}|^2 \frac{|\mathbf{P}|}{8\pi M_{initial}^2}$$

(1) $cs\bar{c}\bar{s}$ states

J. Wu, Y.R. Liu, K.Chen, X.Liu, S.L. Zhu, PRD 94, 094031 (2016)
S.Y. Li, Y.R. Liu, Z.L. Man, Z.G. Si, J. Wu, CPC 48, 063109 (2024)



LHCb: 2407.14301

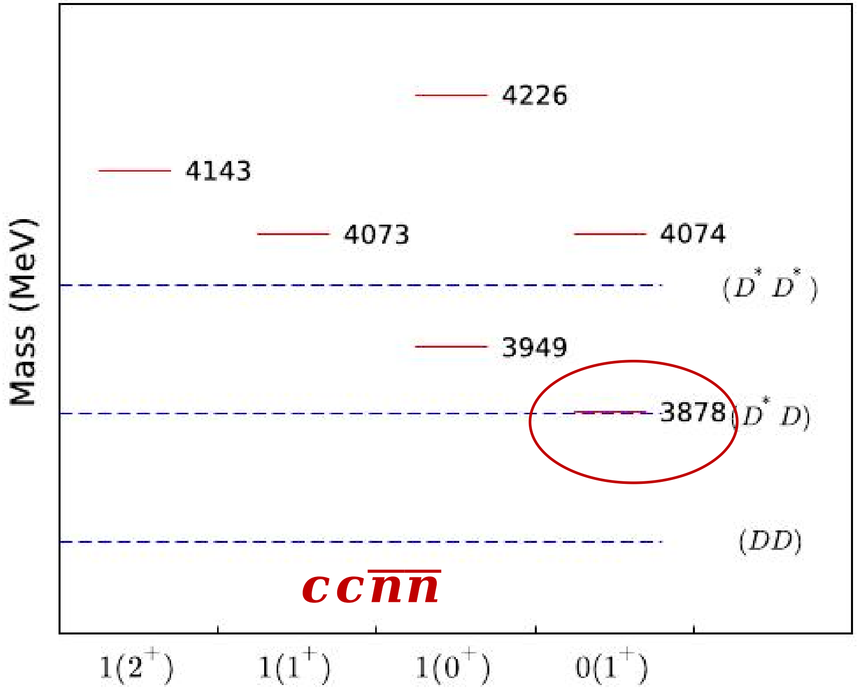
Parameter [MeV]	Current analysis
$M_{\chi_{c1}(4274)}$	$4298 \pm 6 \pm 9$
$\Gamma_{\chi_{c1}(4274)}$	$92^{+22}_{-18} \pm 57$

$M_{X(4140)} = 4146.5$ MeV $\mathcal{C} = 7282.15$ MeV

X(3960) is a good candidate of the lowest 0^{++} $cs\bar{c}\bar{s}$ tetraquark state

State	Mass_{PDG}(MeV)	Γ_{PDG} (MeV)	Γ_{sum}
$\chi_{c1}(4140)$	4146.5 ± 3.0	19^{+7}_{-5}	83.0 MeV
$X(4350)$	$4350^{+4.6}_{-5.1} \pm 0.7$	$13^{+18}_{-9} \pm 4$	74.0 MeV
$\chi_{c1}(4274)$	4286^{+8}_{-9}	51 ± 7	76.9 MeV
$X(3960)$[LHCb:2022aki]	$3956 \pm 5 \pm 10$	$43 \pm 13 \pm 8$	31.8 MeV
$X_0(4140)$[LHCb:2022aki]	$4133 \pm 6 \pm 6$	$67 \pm 17 \pm 7$	60.2 MeV

(2) Lowest $I(J^P) = 0(1^+)\, cc\bar{n}\bar{n}$ tetraquark state: $T_{cc} = cc\bar{u}\bar{d}$



$\mathcal{C}$=7282.15 MeV from X(4140)

S.Y. Li, Y.R. Liu, Z.L. Man, Z.G. Si, J. Wu, 2401.00115
S.Q. Luo, K. Chen, X. Liu, Y.R. Liu, S.L. Zhu, EPJC 77, 709 (2017)
J.B. Cheng, S.Y. Li, Y.R. Liu, Z.G. Si, T. Yao, CPC 45, 043102 (2021)

The lowest $0(1^+)$ $cc\bar{u}\bar{d}$ tetraquark state can be used to understand the LHCb T_{cc} state.

Width sensitive to mass for near-threshold states.
If M → 3878 MeV, Γ=7.2 MeV;
If M → 3876 MeV, Γ=3.0 MeV;
If M → 3880 MeV, Γ=9.7 MeV.

$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38}\ \text{keV}$$

$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14}\ \text{keV}$$

With measured mass $M_{Tcc} = M_{D^{*+}} + M_D - 273$ keV, quasi-two-body decay width [Capstick, Roberts, PRD 49, 4570 (1994)]:

$$k_{max} = \frac{\sqrt{M^2_{T^+_{cc}} - (2M_{D^0} + M_\pi)^2}\sqrt{M^2_{T^+_{cc}} - M^2_\pi}}{2M_{T^+_{cc}}}$$

$$\Gamma = \int_0^{k_{max}} dk \frac{\Gamma_{D^{*+}\to D^0\pi^+}}{(M_{T^+_{cc}} - E_{D^{*+}}(k) - E_{D^0}(k))^2 + \frac{1}{4}\Gamma_{D^{*+}}} \frac{k^2|\mathcal{M}|^2}{(2\pi)^2 M_{T^+_{cc}} E_{D^{*+}}(k) E_{D^0}(k)}$$

~105 keV

(2) Lowest $I(J^P) = 0(1^+)\, cc\bar{n}\bar{n}$ tetraquark state: $T_{cc} = cc\bar{u}\bar{d}$

PHYSICAL REVIEW D **104,** 114009 (2021)

Color and baryon number fluctuation of preconfinement system in production process and T_{cc} structure

Yi Jin,[1] Shi-Yuan Li,[2] Yan-Rui Liu,[2] Qin Qin,[3] Zong-Guo Si,[2] and Fu-Sheng Yu[4,5,6]

IV. CONCLUSION

The consistency between the theoretical analysis on the T_{cc} production by Qin, Shen and Yu [37] and the data [8,9] strongly favors that the newly discovered resonance T_{cc} is produced as a real four-quark state. We in this paper clarify

lation. The cross section $pp \to T_{DD^*} + X$ is around $3 \times 10^2\, pb$, which is one order lower than that of the production rate of the four-quark state [37].

Chinese Physics C Vol. 45, No. 10 (2021) 103106

Discovery potentials of double-charm tetraquarks*

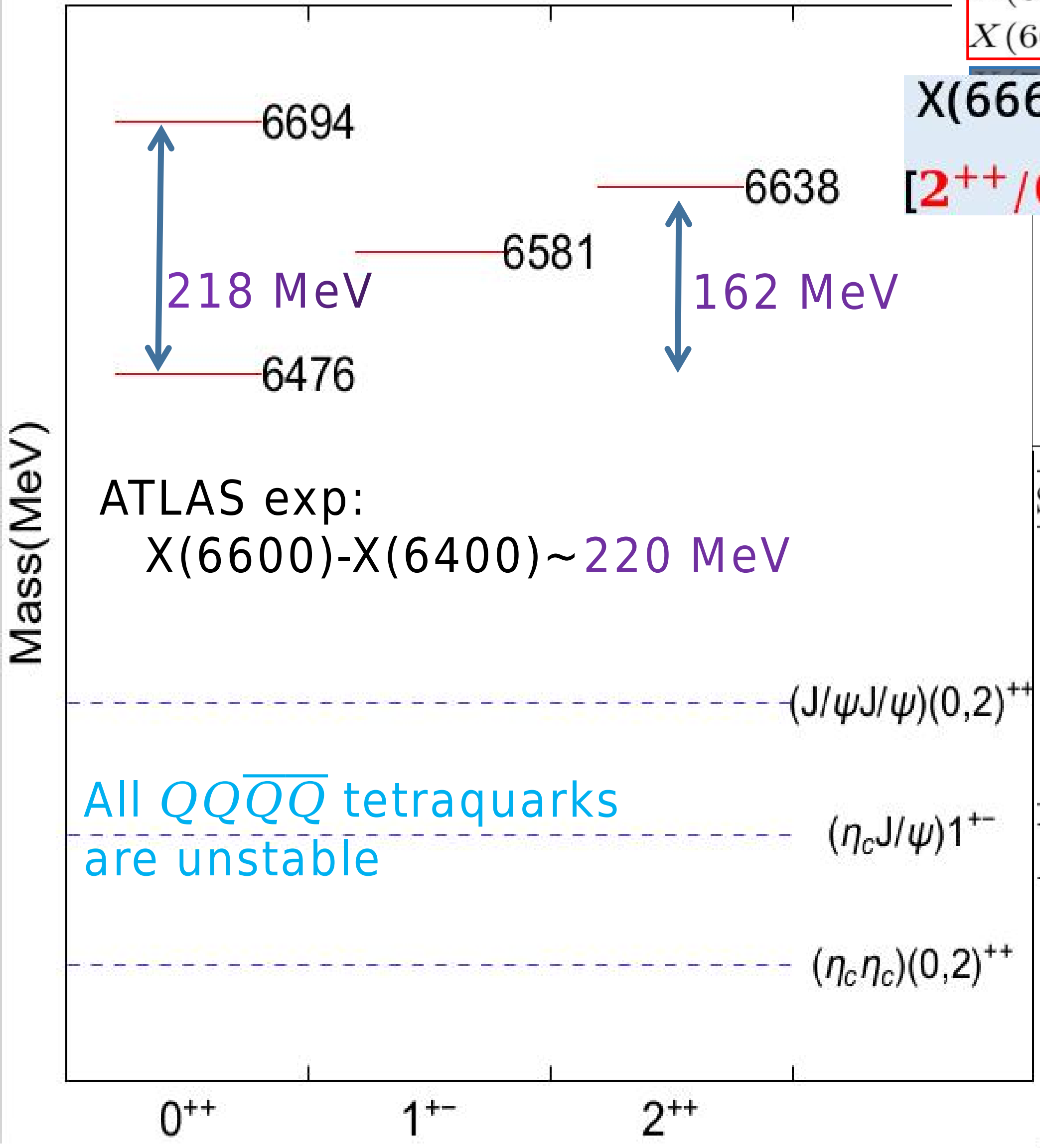
Qin Qin(秦溱)[1†] Yin-Fa Shen(沈胤发)[1] Fu-Sheng Yu(于福升)[2,3,4‡]

From mass, width, and production properties, it is possible to assign the LHCb T_{cc} as the lowest $I(J^P) = 0(1^+)\ cc\bar{u}\bar{d}$ tetraquark state.

(3) $cc\overline{cc}$ states

State	Mass (MeV)	Γ(MeV)	Observed channels
[redacted]			
$X(6600)$[CMS:2023owd]	$6552 \pm 10 \pm 12$ MeV	$124^{+32}_{-26} \pm 33$ MeV	diJ/ψ
[redacted]			
$X(6400)$[ATLAS:2023bft]	$6.41 \pm 0.08^{+0.08}_{-0.03}$ GeV	$0.59 \pm 0.35^{+0.12}_{-0.2}$GeV	di-J/ψ
$X(6600)$[ATLAS:2023bft]	$6.63 \pm 0.05^{+0.08}_{-0.01}$ GeV	$0.35 \pm 0.11^{+0.11}_{-0.04}$GeV	di-J/ψ

X(6660)/X(6400) consistent with $0^{++}/0^{++}$ $cc\overline{cc}$ tetraquark states [$2^{++}/0^{++}$?].



System	$J^{P(C)}$	Mass	$J/\psi J/\psi$	$J/\psi\eta_c$	$\eta_c\eta_c$	Γ
$cc\bar{c}\bar{c}$	2^{++}	6637.5	(33.3, 80.2)			80.2
	1^{+-}	6581.0		(16.7, 172.1)		172.1
	0^{++}	6694.3	(54.9, 138.2)		(0.1, 0.3)	138.5
		6476.4	(3.5, 6.9)		(41.6, 110.7)	117.7

If CMS X(6600) → 138.5

System	$J^{P(C)}$	Mass	Channels			Γ
			$J/\psi J/\psi$	$J/\psi\eta_c$	$\eta_c\eta_c$	
$cc\bar{c}\bar{c}$	2^{++}	6637.5	(33.3, 213.5)			213.5
	1^{+-}	6581.0		(16.7, 458.0)		458.0
	0^{++}	6694.1	(54.9, 367.7)		(0.1, 0.9)	368.6
		6476.4	(3.5, 18.5)		(41.6, 294.8)	313.2

If ATLAS X(6600) → 368.6

Narrower ATLAS X(6400) → 313.2

System	$J^{P(C)}$	Mass	Channels			Γ
			$J/\psi J/\psi$	$J/\psi\eta_c$	$\eta_c\eta_c$	
$cc\bar{c}\bar{c}$	2^{++}	6637.5	(33.3, 352.3)			352.3
	1^{+-}	6581.0		(16.7, 755.8)		755.8
	0^{++}	6694.1	(54.9, 606.8)		(0.1, 1.4)	608.2
		6476.4	(3.5, 30.5)		(41.6, 486.4)	516.9

If ATLAS X(6600) → 352.3

Broader ATLAS X(6400) → 516.9

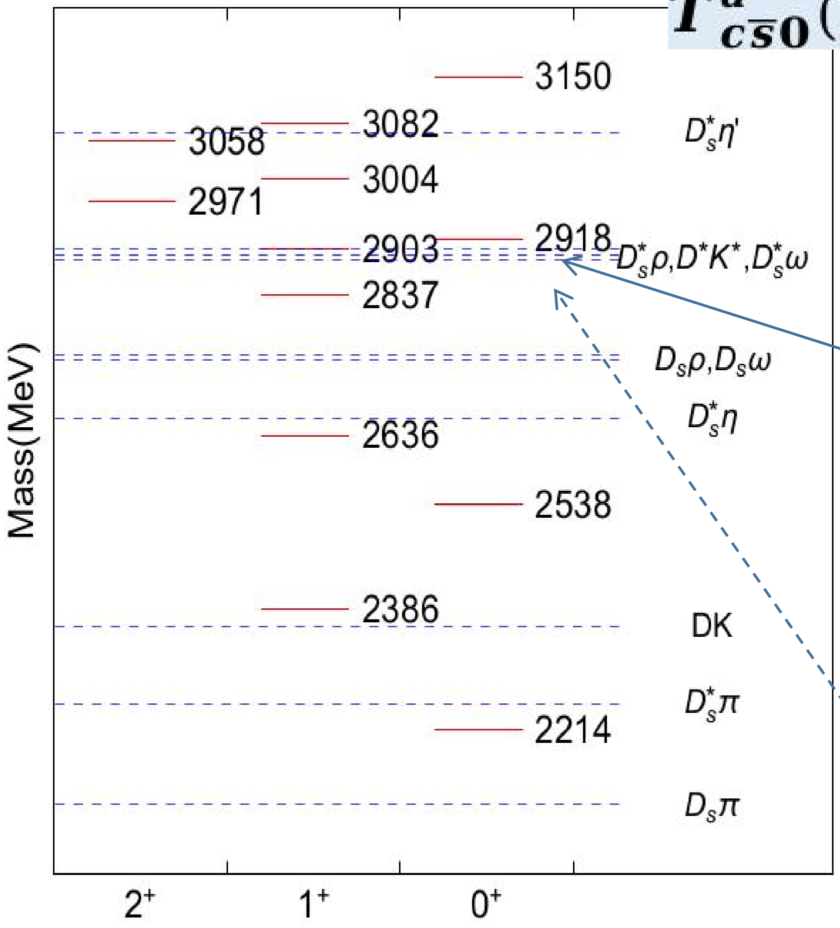
(4) $Qq\overline{qq}$ states

State	Mass(MeV)	Γ (MeV)	Observed channels
$T_{cs0}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+D^-K^+$
$T_{cs1}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$B^+ \to D^+D^-K^+$
$T^a_{c\bar{s}0}(2900)^0$[LHCb:2022sfr,LHCb:2022lzp]	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to \bar{D}^0 D_s^+ \pi^-$
$T^a_{c\bar{s}0}(2900)^{++}$[LHCb:2022sfr,LHCb:2022lzp]	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^- D_s^+ \pi^+$

P=- →

$\mathcal{C}$=13.577 GeV as input to get widths

$T^a_{c\bar{s}0}(2900)$ as the second highest I=1 $cn\bar{s}\bar{n}$ tetraquark state



$I(J^P)$	Mass							Γ
1(2⁺)	3058.0 2971.3	(95.8, 340.4) (4.2, 10.9)	(26.9, 94.8) (73.1, 179.3)					435.2 190.1
		$D_s^*\rho$	$D_s^*\pi$	$D_s\rho$	D^*K^*	D^*K	DK^*	
1(1⁺)	3082.1 3004.4 2902.7 2837.3 2636.4 2385.9	(71.1, 266.6) (26.8, 80.9) (0.0, 0.0) (1.3, –) (0.8, –) (0.0, –)	(0.0, 0.2) (0.3, 2.0) (0.7, 4.2) (0.0, 0.0) (10.7, 51.5) (88.2, 253.0)	(4.5, 22.7) (22.7, 103.6) (44.1, 166.0) (26.8, 80.2) (1.8, –) (0.0, –)	(22.5, 84.1) (61.8, 181.3) (12.6, 2.7) (0.1, –) (1.5, –) (1.4, –)	(0.8, 4.6) (1.3, 7.5) (0.0, 0.2) (3.2, 16.5) (72.7, 258.8) (22.1, –)	(9.3, 46.7) (1.3, 6.1) (12.4, 45.5) (70.9, 198.4) (4.4, –) (1.7, –)	424.9 381.4 218.5 295.1 310.3 253.0
		$D_s^*\rho$	$D_s\pi$	D^*K^*	DK			
1(0⁺)	3149.7 2917.9 2537.5 2214.3	(63.3, 266.6) (34.1, 54.9) (2.5, –) (0.1, –)	(0.0, 0.3) (0.7, 5.1) (17.3, 96.4) (81.9, 235.6)	(46.3, 196.1) (47.4, 56.9) (4.1, –) (2.1, –)	(0.2, 1.7) (3.1, 20.1) (67.0, 297.1) (29.6, –)			464.6 137.0 393.5 235.6

$I(J^P)$	Mass	Channels							Γ
		$D_s^*\omega$	D^*K^*						
0(2⁺)	3058.0 2971.3	(95.8, 333.6) (4.2, 10.4)	(26.9, 94.8) (73.1, 179.3)						428.4 189.7
		$D_s^*\omega$	$D_s^*\eta$	$D_s^*\eta'$	$D_s\omega$	D^*K^*	D^*K	DK^*	
0(1⁺)	3082.1 3004.4 2902.7 2837.3 2636.4 2385.9	(71.1, 262.0) (26.8, 78.5) (0.0, 0.0) (1.3, –) (0.8, –) (0.0, –)	(0.0, 0.1) (0.2, 0.8) (0.3, 1.4) (0.3, 0.0) (5.0, –) (41.5, –)	(0.0, 0.0) (0.2, –) (0.4, –) (0.0, –) (5.7, –) (46.7, –)	(4.5, 22.5) (22.7, 102.3) (44.1, 162.4) (26.8, 77.1) (1.8, –) (0.0, –)	(22.5, 84.1) (61.8, 181.3) (12.6, 2.7) (0.1, –) (1.5, –) (1.4, –)	(0.8, 4.6) (1.3, 7.5) (0.0, 0.2) (3.2, 16.5) (72.7, 258.8) (22.1, –)	(9.3, 46.7) (1.3, 6.1) (12.4, 45.5) (70.9, 198.4) (4.4, –) (1.7, –)	420.0 376.4 212.1 292.0 258.8 –
		$D_s^*\omega$	$D_s\eta$	$D_s\eta'$	D^*K^*	DK			
0(0⁺)	3149.7 2917.9 2537.5 2214.3	(63.3, 263.3) (34.1, 47.8) (2.5, –) (0.1, –)	(0.0, 0.1) (0.4, 2.0) (8.1, 12.6) (38.6, –)	(0.0, 0.1) (0.4, 0.1) (9.1, –) (43.3, –)	(46.3, 196.1) (47.4, 56.9) (4.1, –) (2.1, –)	(0.2, 1.7) (3.1, 20.1) (67.0, 297.1) (29.6, –)			461.3 126.9 309.7 –

$cn\bar{s}\bar{n}$: I=0 & I=1 degenerate

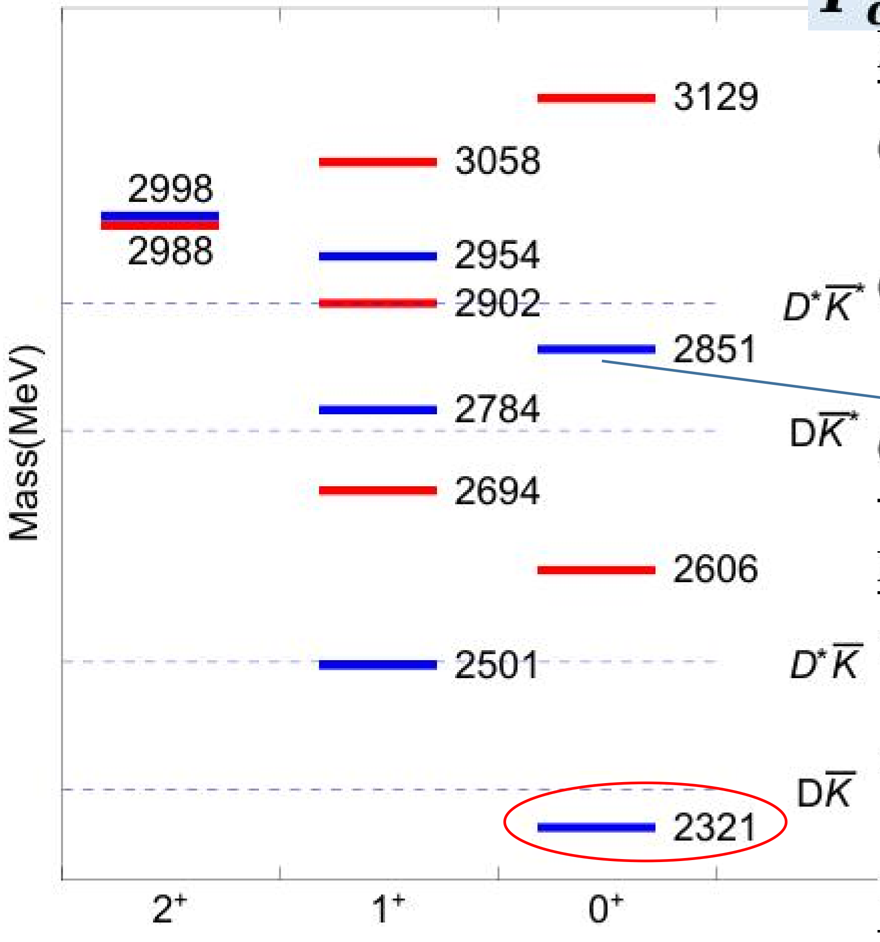
(4) $Qq\overline{q}\overline{q}$ states

State	Mass(MeV)	Γ (MeV)	Observed channels
$T_{cs0}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+ D^- K^+$
$T_{cs1}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$B^+ \to D^+ D^- K^+$
$T^a_{c\bar{s}0}(2900)^0$[LHCb:2022sfr,LHCb:2022lzp]	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to \bar{D}^0 D_s^+ \pi^-$
$T^a_{c\bar{s}0}(2900)^{++}$[LHCb:2022sfr,LHCb:2022lzp]	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^- D_s^+ \pi^+$

P=- →

$\mathcal{C}$=13.577 GeV as input to get widths

$T_{cs0}(2900)$ as the higher I=0 $cs\overline{u}\overline{d}$ tetraquark state



(e) $I = 0/1$ $cs\bar{n}\bar{n}$ states

$I(J^P)$	Mass	Channels: D^*K^* / $D^*\bar{K}^*$	$D^*\bar{K}$	$D\bar{K}^*$	$D\bar{K}$	Γ
$0(2^+)$	2998.3	(66.7, 380.2)				380.2
$0(1^+)$	2953.5	(47.6, 203.0)	(1.3, 14.5)	(10.6, 88.5)		306.1
	2784.3	(2.4, −)	(1.8, 17.8)	(47.6, 151.9)		169.7
	2500.6	(0.1, −)	(55.2, −)	(0.1, −)		0
$0(0^+)$	2850.5	(41.4, −)			(2.7, 34.4)	34.4
	2320.7	(0.3, −)			(55.6, −)	0

$I(J^P)$	Mass	Channels: D^*K^* / $D^*\bar{K}^*$	$D^*\bar{K}$	$D\bar{K}^*$	$D\bar{K}$	Γ
$1(2^+)$	2987.5	(33.3, 180.1)				180.1
$1(1^+)$	3058.0	(46.9, 329.8)	(0.4, 4.5)	(7.1, 69.0)		403.3
	2902.3	(2.2, 0.3)	(1.1, 12.1)	(30.7, 225.3)		237.7
	2693.7	(0.9, −)	(40.2, 333.5)	(3.9, −)		333.5
$1(0^+)$	3128.9	(55.1, 451.1)			(0.1, 2.1)	453.2
	2606.4	(3.2, −)			(41.5, 419.3)	419.3

$$M = M_{X(4140)} - (E_{CMI})_{X(4140)} + \sum_{ij} n_{ij}\Delta_{ij} + E_{CMI}$$

With one mass formulae and a simple decay scheme:

- X(3960) is a good candidate of the lowest 0^{++} $cs\bar{c}\bar{s}$ tetraquark state.

  Wu, Liu, Chen, Liu, Zhu, PRD 94, 094031 (2016); Li, Liu, Man, Si, Wu, CPC 48, 063109 (2024)

- The lowest $0(1^{+})$ $cc\bar{u}\bar{d}$ tetraquark state can be used to understand the LHCb T_{cc} state. [mass, width, production]

  Li, Liu, Man, Si, Wu, 2401.00115 (to appear in PRD); Luo, Chen, Liu, Liu, Zhu, EPJC 77, 709 (2017)

  J.B. Cheng, S.Y. Li, Y.R. Liu, Z.G. Si, T. Yao, CPC 45, 043102 (2021)

- X(6660)/X(6400) consistent with $0^{++}/0^{++}$ $cc\bar{c}\bar{c}$ tetraquark states [$2^{++}/0^{++}$?].

- $T^{a}_{c\bar{s}0}(2900)$ as the second highest I=1 $cn\bar{s}\bar{n}$ tetraquark state;

  $T_{cs0}(2900)$ as the higher I=0 $cs\bar{u}\bar{d}$ tetraquark state.

Studies of $Q\overline{Q}qqq/QQqq\bar{q}$ states in the literature

Compact states:

- Zhou, Chen, Liu, Liu, Zhu, PRC 98, 045204 (2018)
- Wang, EPJC 78, 826 (2018)
- Park, Cho, Lee, PRD 99, 094023 (2019)
- Giannuzzi, PRD 99, 094006 (2019)
- Xing, Niu, EPJC 81, 978 (2021)
- Özdem, EPJPlus 137, 936 (2022)
- Park, Noh, PRD 108, 014026 (2023)

For those of $Q\overline{Q}qqq$:

Phys. Rept.639,1 (2016)
Prog.Part.Nucl.Phys.,93, 143 (2017)
Prog.Part.Nucl.Phys.,97, 123 (2017)
Rev.Mod. Phys.,90, 015003 (2018)
Rev.Mod. Phys.,90, 015004 (2018)
Prog.Part.Nucl.Phys.,107, 237 (2019)
Symmetry 15, 1298 (2023)
Phys. Rept.1019,1 (2023)
Rept.Prog.Phys.,86,026201 (2023)

Molecules:

- Xu, Liu, Jin, PRD 86, 114032 (2012)
- Shimizu, Harada, PRD 96, 094012 (2017)
- Chen, Hosaka, Liu, PRD 96, 116012 (2017)
- Z.H. Guo, PRD 96, 074004 (2017)
- Dias, Debastiani, Xie, Oset, PRD 98, 094017 (2018)
- Yang, Ping, Segovia, PRD 101, 074030 (2020)
- Chen, Li, Sun, Liu, Zhu, PLB 822, 136693 (2021)
- Chen, Wang, Zhu, 103, 116017 (2021)
- Dong, Guo, Zou, CTP 73, 125201 (2021)
- Wang, Feijoo, Song, Oset, PRD 106, 116004 (2022)
- Zhou, Wang, Liu, Liu, PRD 106, 034034 (2022)
- Yalikun, Dong, Zou, CPC 47, 123101 (2023)
- Liu et al, 2312.04390
- Wang, Wang, IJMPA 39, 2450067 (2024)
- Duan, Wang, Yang, Chen, Chen, PRD 109, 094018 (2024)
- Wang, Xiao, Sun, Liu, PRD 109, 034038 (2024)
- Wang, Liu, PRD 108, 074022 (2023); PRD 109, 014043 (2024)
- Wang, Chen, Meng, Zhu, PRD 109, 074035 (2024)
- U. Özdem, 2405.07273

Formalism: mass fomulas for hidden/double-charm *pentaquark* states

$$M = [M_{P_c(4312)} - (E_{CMI})_{P_c(4312)}] + E_{CMI} + \sum_{ij} n_{ij}\Delta_{ij}$$
$$= \tilde{m}_{penta} + E_{CMI} + \sum_{ij} n_{ij}\Delta_{ij}.$$

$$M = \sum_i m_i + E_{\rm CMI}$$

$$M = [M_{X(4140)} - (E_{\rm CMI})_{X(4140)}] + E_{\rm CMI} + \sum_{ij} n_{ij}\Delta_{ij}$$
$$= \tilde{m}_{tetra} + E_{\rm CMI} + \sum_{ij} n_{ij}\Delta_{ij}.$$

$$M_{c\bar{c}nnn} = \tilde{m}_{penta} + E_{CMI},$$
$$M_{c\bar{c}nns} = \tilde{m}_{penta} + E_{CMI} + \Delta_{sn},$$
$$M_{c\bar{c}nss} = \tilde{m}_{penta} + E_{CMI} + 2\Delta_{sn},$$
$$M_{c\bar{c}sss} = \tilde{m}_{penta} + E_{CMI} + 3\Delta_{sn}.$$

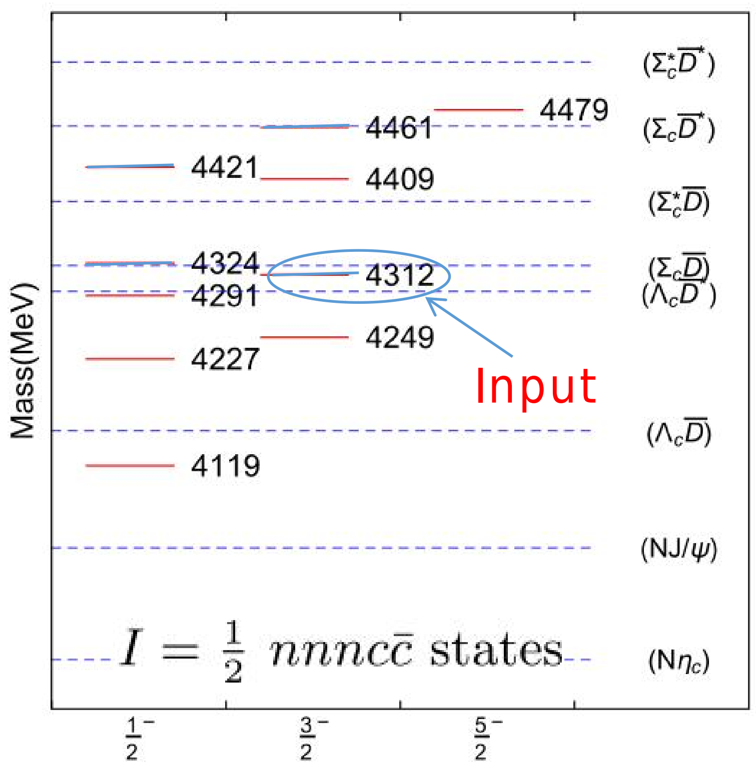
$$M_{ccnn\bar{n}} = \tilde{m}_{penta} + E_{CMI},$$
$$M_{ccnn\bar{s}} = \tilde{m}_{penta} + E_{CMI} + \Delta_{sn},$$
$$M_{ccns\bar{n}} = \tilde{m}_{penta} + E_{CMI} + \Delta_{sn},$$
$$M_{ccns\bar{s}} = \tilde{m}_{penta} + E_{CMI} + 2\Delta_{sn},$$
$$M_{ccss\bar{n}} = \tilde{m}_{penta} + E_{CMI} + 2\Delta_{sn},$$
$$M_{ccss\bar{s}} = \tilde{m}_{penta} + E_{CMI} + 3\Delta_{sn}.$$

Assumption: $P_c(4312)$ observed in $J/\psi p$ as a $3/2^-$ $c\bar{c}uud$ pentaquark [consistency requires]

$\widetilde{m}_{penta}$ = 4382.6 MeV, Δ_{sn} = 90.6 MeV
$\widetilde{m}_{tetra}$ = 4231.1 MeV
$\widetilde{m}_{penta}$-$\widetilde{m}_{tetra}$=$\widetilde{m}_n$-2Δ_{sn}=151.5 MeV ⇒ $\widetilde{m}_n$=332.7 MeV

Pc states $(n = u, d)$

$C = 4647.9$ MeV $(nnn)_{8_c}(c\bar{c})_{8_c}$-$(nnn)_{1_c}(c\bar{c})_{1_c}$



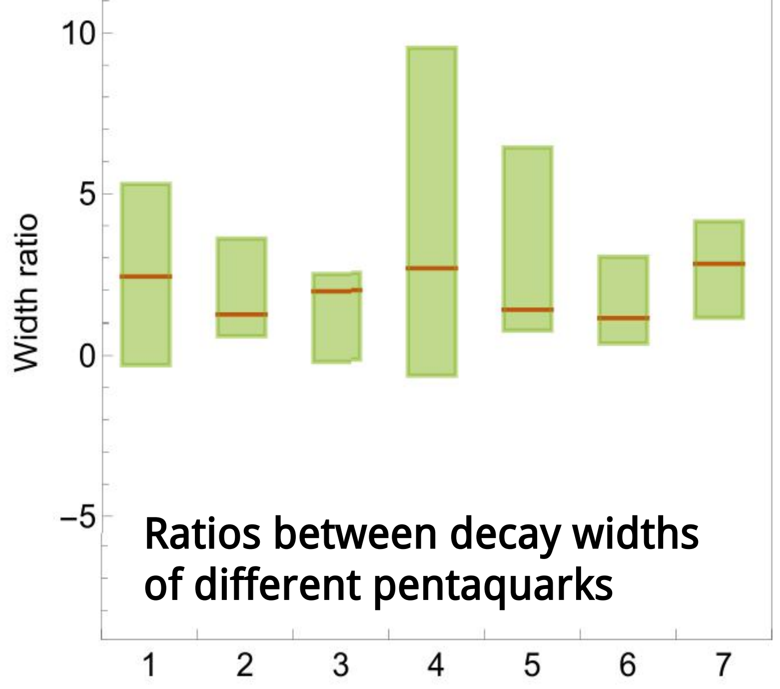
Exp.

$$\Gamma(P_c(4440)^+):\Gamma(P_c(4457)^+) = 3.2^{+2.1}_{-3.5},$$
$$\Gamma(P_c(4440)^+):\Gamma(P_c(4312)^+) = 2.1^{+1.5}_{-1.5},$$
$$\Gamma(P_c(4312)^+):\Gamma(P_c(4457)^+) = 1.5^{+1.0}_{-1.7},$$
$$\Gamma(P_c(4337)^+):\Gamma(P_c(4457)^+) = 4.5^{+5.0}_{-5.2},$$
$$\Gamma(P_c(4337)^+):\Gamma(P_c(4312)^+) = 3.0^{+3.4}_{-2.3},$$
$$\Gamma(P_c(4337)^+):\Gamma(P_c(4440)^+) = 1.4^{+1.6}_{-1.1}.$$

Th.

$$\Gamma(\tilde{P}_c(4421)^+):\Gamma(\tilde{P}_c(4461)^+) = 2.42,$$
$$\Gamma(\tilde{P}_c(4421)^+):\Gamma(\tilde{P}_c(4312)^+) = 1.24,$$
$$\Gamma(\tilde{P}_c(4312)^+):\Gamma(\tilde{P}_c(4461)^+) = 1.96,$$
$$\Gamma(\tilde{P}_c(4324)^+):\Gamma(\tilde{P}_c(4461)^+) = 2.64,$$
$$\Gamma(\tilde{P}_c(4324)^+):\Gamma(\tilde{P}_c(4312)^+) = 1.35,$$
$$\Gamma(\tilde{P}_c(4324)^+):\Gamma(\tilde{P}_c(4421)^+) = 1.09.$$

J.B. Cheng, Y.R. Liu, PRD100, 054002(2019);
Li, Liu, Man, Si, Wu, PRD 108, 056015 (2023)



$P_c(4457)^+$, $P_c(4440)^+$, $P_c(4337)^+$ can be assigned as the 3/2, 1/2, and 1/2 pentaquark states, respectively.

For $P_c(4457)^+$ $[\tilde{P}_c(4461)^+]$ $\Gamma(\Sigma_c^*\bar{D}):\Gamma(\Lambda_c\bar{D}):\Gamma(NJ/\psi) = 2.3:4.0:1.0$

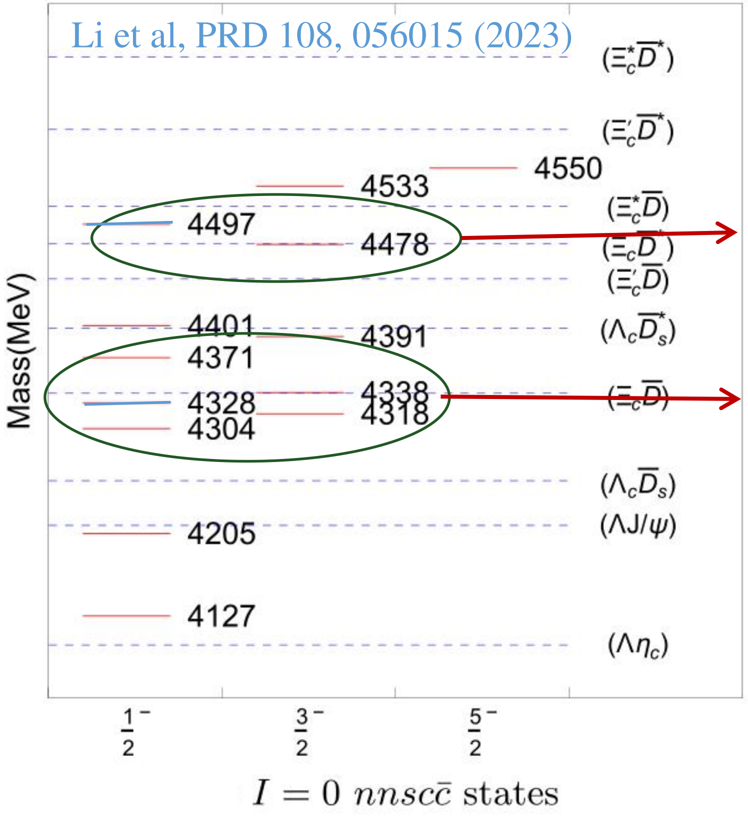
For $P_c(4440)^+$ $[\tilde{P}_c(4421)^+]$ $\Gamma(\Lambda_c\bar{D}^*):\Gamma(\Sigma_c\bar{D}):\Gamma(\Lambda_c\bar{D}):\Gamma(NJ/\psi):\Gamma(N\eta_c) = 45.5:3.0:3.0:7.5:1.0$

For $P_c(4312)^+$ $\Gamma(NJ/\psi):\Gamma(\Lambda_c\bar{D}^*) = 1.1$

For $P_c(4337)^+$ $[\tilde{P}_c(4324)^+]$ $\Gamma(\Lambda_c\bar{D}):\Gamma(NJ/\psi) = 1.3$

} **Predictions**

Pcs states $(n = u, d)$ $(nns)_{8_c}(c\bar{c})_{8_c}$-$(nns)_{1_c}(c\bar{c})_{1_c}$



If we assign the $P_{cs}(4459)^0$, $P_{cs}(4338)^0$ to be J=3/2 pentaquark states $\tilde{P}_{cs}(4478)$, $\tilde{P}_{cs}(4338)$, respectively, $\Gamma(\tilde{P}_{cs}(4478)) : \Gamma(\tilde{P}_{cs}(4338)) \sim 0.12$ which is contradicted with the experimantal value.

Other possible assignments:

$$\begin{aligned}
\Gamma(\tilde{P}_{cs}(4478)^0) : \Gamma(\tilde{P}_{cs}(4371)^0) &= 0.15,\\
\Gamma(\tilde{P}_{cs}(4478)^0) : \Gamma(\tilde{P}_{cs}(4328)^0) &= 0.56,\\
\Gamma(\tilde{P}_{cs}(4478)^0) : \Gamma(\tilde{P}_{cs}(4318)^0) &= 2.57,\\
\Gamma(\tilde{P}_{cs}(4478)^0) : \Gamma(\tilde{P}_{cs}(4304)^0) &= 0.17,\\
\Gamma(\tilde{P}_{cs}(4497)^0) : \Gamma(\tilde{P}_{cs}(4371)^0) &= 0.72,\\
\Gamma(\tilde{P}_{cs}(4497)^0) : \Gamma(\tilde{P}_{cs}(4338)^0) &= 0.61,\\
\Gamma(\tilde{P}_{cs}(4497)^0) : \Gamma(\tilde{P}_{cs}(4328)^0) &= 2.78,\\
\Gamma(\tilde{P}_{cs}(4497)^0) : \Gamma(\tilde{P}_{cs}(4318)^0) &= 12.71,\\
\Gamma(\tilde{P}_{cs}(4497)^0) : \Gamma(\tilde{P}_{cs}(4304)^0) &= 0.83.
\end{aligned}$$

Theoretical widths are much smaller than the measured results.

Exp:

$$\Gamma(P_{cs}(4459)^0) : \Gamma(P_{cs}(4338)^0) = 2.5^{+1.6}_{-1.4}$$

Both $P_{cs}(4459)^0$ and $P_{cs}(4338)^0$ can be assigned as $\frac{1}{2}^-$ pentaquark states.

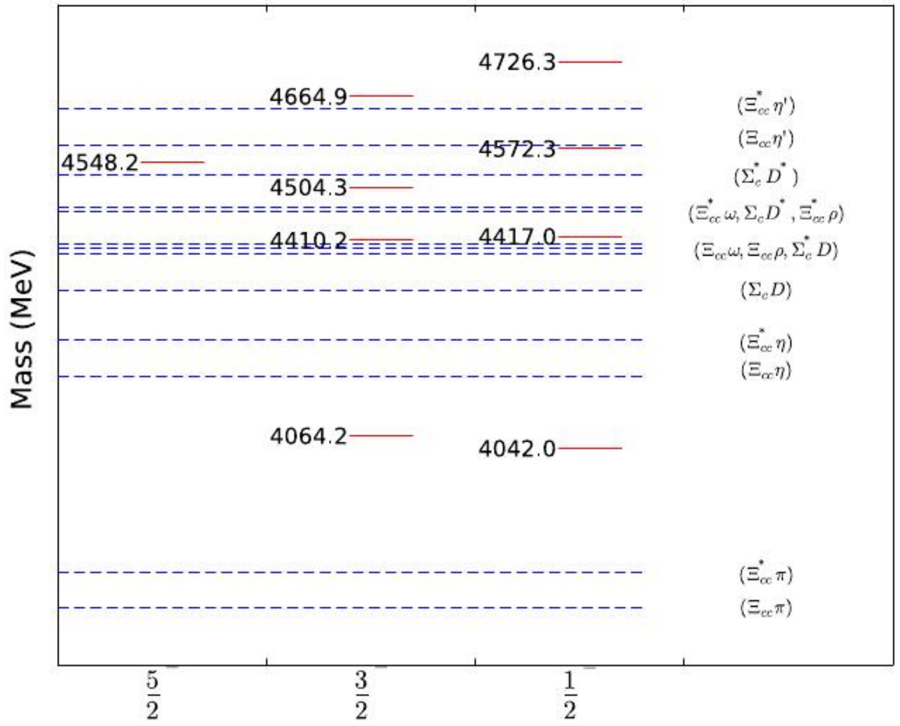
For $P_{cs}(4459)^0$ $[\tilde{P}_{cs}(4497)^0]$, $\Gamma(\Lambda_c\bar{D}_s^*) : \Gamma(\Lambda J/\psi) = 2.3 : 1.0$

For $P_{cs}(4338)^0$ $[\tilde{P}_{cs}(4328)^0]$, $\Gamma(\Lambda J/\Psi) : \Gamma(\Lambda_c\bar{D}_s) = 3.0$

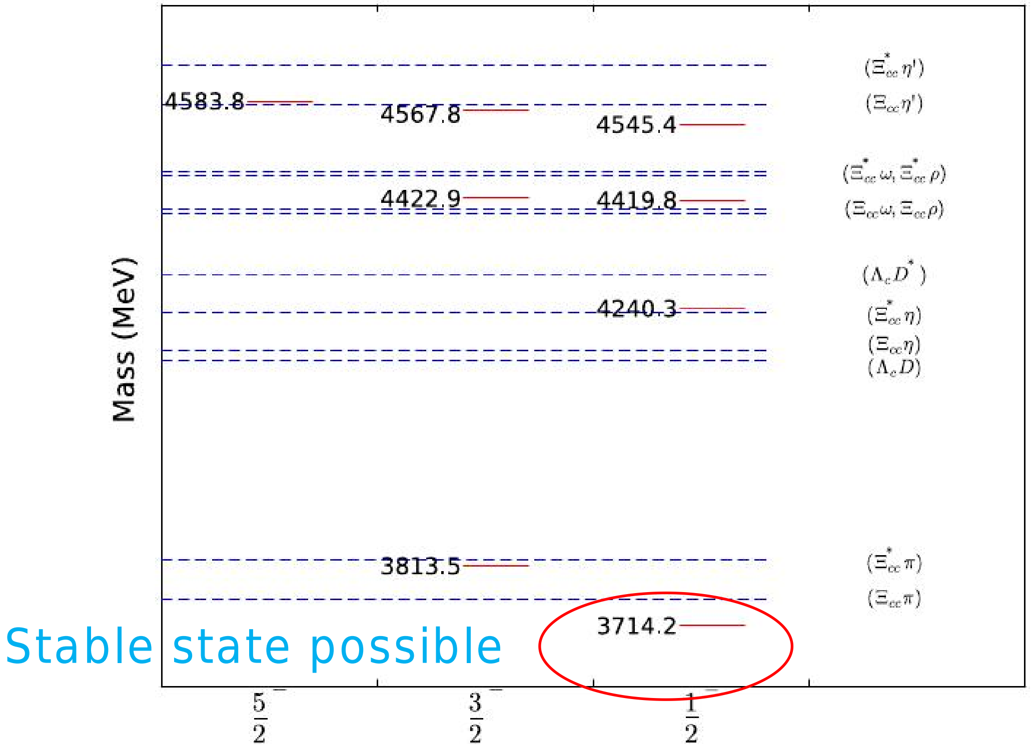
Predictions

The J=5/2 state，the lighest J=3/2 state, and the lighest J=1/2 state are narrow.

$ccnn\bar{n}$ states $(n = u, d)$



(a) $ccnn\bar{n}$ states with $I_{nn} = 1$ and $I = \frac{3}{2}, \frac{1}{2}$



(b) $ccnn\bar{n}$ states with $I_{nn} = 0$ and $I = \frac{1}{2}$

Lowest $ccud\bar{n}$ $(I_{nn} = 0,\ I = \frac{1}{2})$:

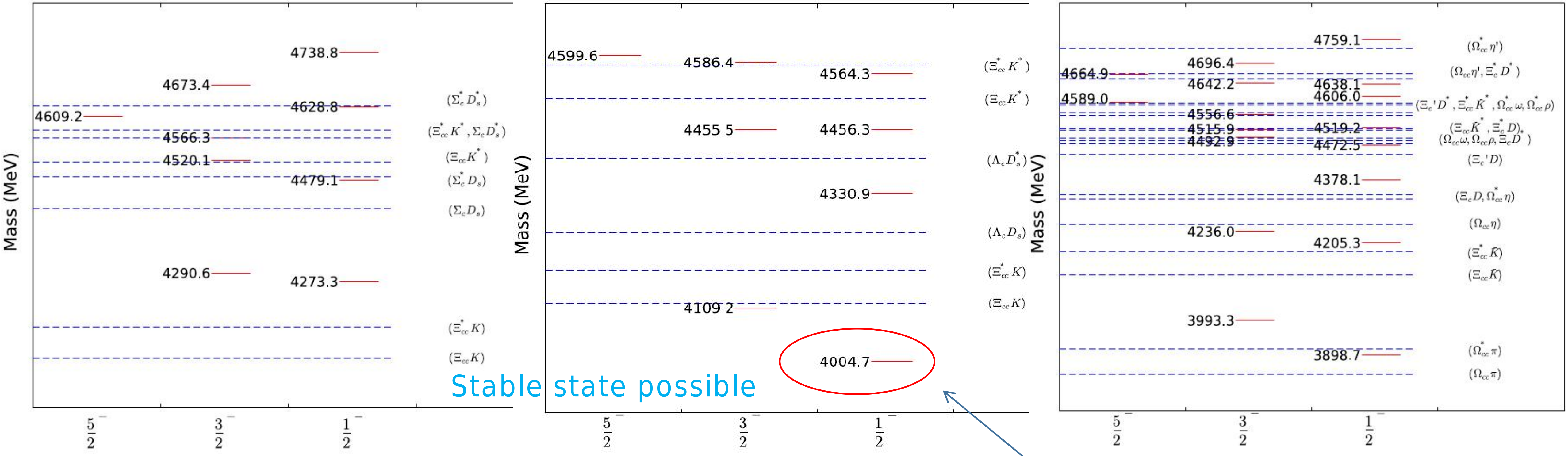
3755 MeV: Zhou, Chen, Liu, Liu, Zhu, PRC 98, 045204 (2018)

$4.21^{+0.10}_{-0.11}$ GeV: Wang, EPJC 78, 826 (2018) [QSR]

4.54 GeV: Giannuzzi, PRD 99, 094006 (2019) [di-di-anti]

3.841±0.290 GeV: Xing, Niu, EPJC 81, 978 (2021) [di-tri]

$ccnn\bar{s}$ / $ccsn\bar{n}$ states ($n = u, d$)



(c) $ccnn\bar{s}$ states with $I = 1$

(d) $ccnn\bar{s}$ states with $I = 0$

(e) $ccns\bar{n}$ states with $I = 1, 0$

Lowest $ccud\bar{s}$:

4060 MeV: Zhou, Chen, Liu, Liu, Zhu, PRC 98, 045204 (2018)

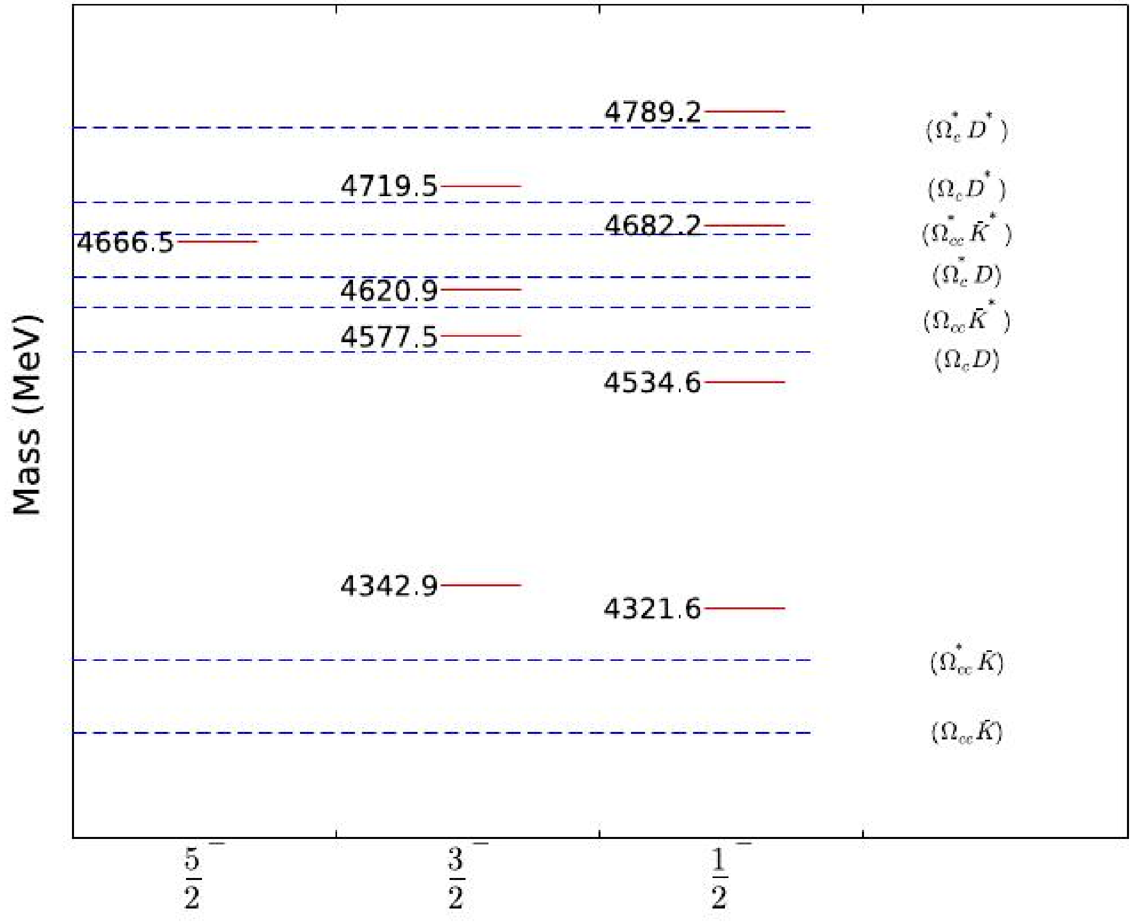
$\Xi_{cc}K$-135 MeV: Park, Cho, Lee, PRD 99, 094023 (2019)

4.092±0.298 GeV: Xing, Niu, EPJC 81, 978 (2021) [di-tri]

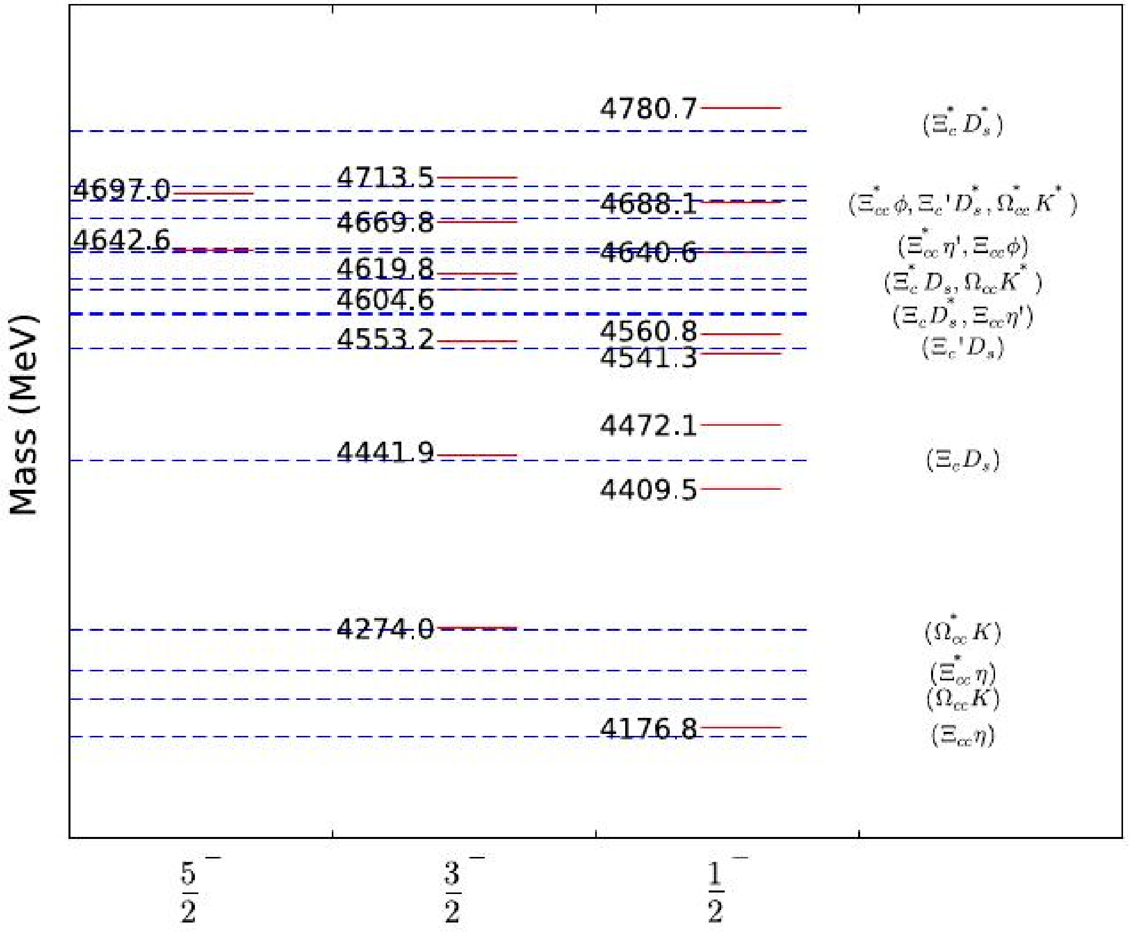
$\Xi_{cc}K$ +18.5 MeV: Park, Noh, PRD 108, 014026 (2023)

+100 MeV, still below $\Xi_{cc}K$ threshold

$ccss\bar{n}$ / $ccns\bar{s}$ states ($n = u, d$)



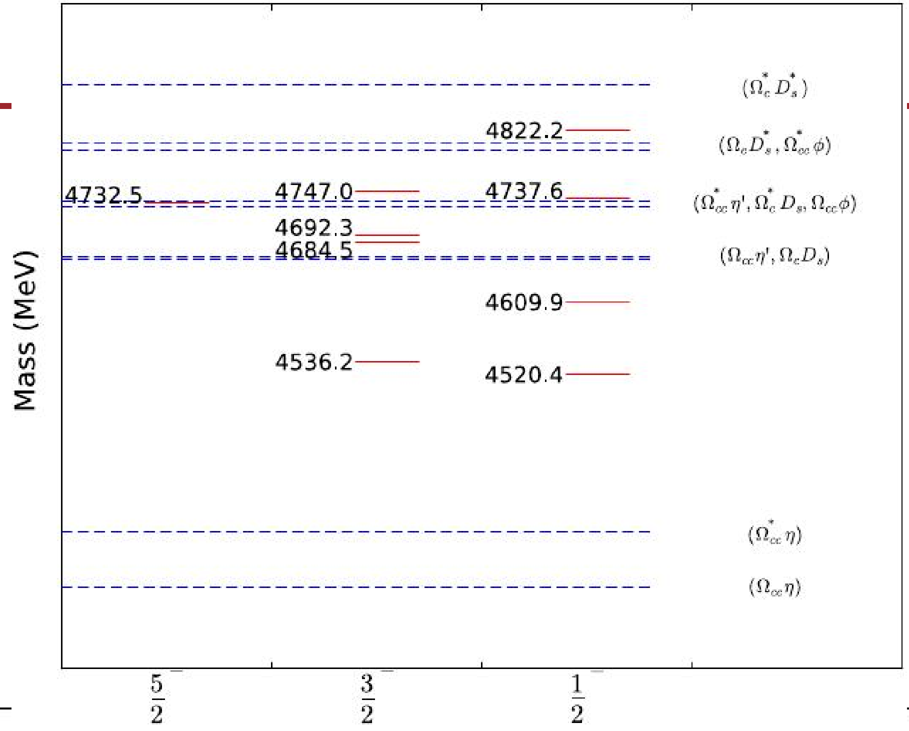
(g) $ccss\bar{n}$ states with $I = \frac{1}{2}$



(h) $ccns\bar{s}$ states with $I = \frac{1}{2}$

$ccss\bar{s}$ states

Unstable but may be narrow



(f) $ccss\bar{s}$ states

$ccss\bar{s}$ states

		$\Omega_c^* D_s^*$	$\Omega_{cc}^* \phi$						
$\frac{5}{2}^-$	4732.5	(−, −)	(−, −)						−
		$\Omega_c^* D_s^*$	$\Omega_c D_s^*$	$\Omega_c^* D_s$	$\Omega_{cc}^* \phi$	$\Omega_{cc} \phi$	$\Omega_{cc}^* \eta$	$\Omega_{cc}^* \eta'$	
$\frac{3}{2}^-$	4747.0	(−, −)	(−, −)	(2.3, 0.4)	(−, −)	(10.2, 1.4)	(0.0, 0.0)	(0.0, 0.0)	1.8
	4692.3	(−, −)	(−, −)	(−, −)	(−, −)	(−, −)	(4.2, 2.2)	(−, −)	2.2
	4684.5	(−, −)	(−, −)	(−, −)	(−, −)	(−, −)	(0.4, 0.2)	(−, −)	0.2
	4536.2	(−, −)	(−, −)	(−, −)	(−, −)	(−, −)	(15.2, 6.0)	(−, −)	6.0
		$\Omega_c^* D_s^*$	$\Omega_c D_s^*$	$\Omega_c D_s$	$\Omega_{cc}^* \phi$	$\Omega_{cc} \phi$	$\Omega_{cc} \eta$	$\Omega_{cc} \eta'$	
$\frac{1}{2}^-$	4822.2	(−, −)	(0.1, 0.0)	(0.0, 0.0)	(37.3, 5.6)	(0.1, 0.0)	(0.0, 0.0)	(0.0, 0.0)	5.6
	4737.6	(−, −)	(−, −)	(0.3, 0.0)	(−, −)	(18.8, 1.8)	(0.1, 0.0)	(0.1, 0.0)	1.8
	4609.9	(−, −)	(−, −)	(−, −)	(−, −)	(−, −)	(3.7, 2.0)	(−, −)	2.0
	4520.4	(−, −)	(−, −)	(−, −)	(−, −)	(−, −)	(15.9, 7.2)	(−, −)	7.2

$$M = [M_{P_c(4312)} - (E_{CMI})_{P_c(4312)}] + E_{CMI} + \sum_{ij} n_{ij}\Delta_{ij}$$

With one mass formula and a simple decay scheme using the same parameters:

- Consistent hidden-charm pentaquark assignments in the model:

  $P_c(4312)$, $P_c(4457)$: $\frac{3}{2}^-$ $c\bar{c}uud$;

  $P_c(4440)$, $P_c(4337)$: $\frac{1}{2}^-$ $c\bar{c}uud$;

  $P_{cs}(4459)$, $P_{cs}(4338)$: $\frac{1}{2}^-$ $c\bar{c}uds$.

- Stable double-charm pentaquarks:

  (1) lowest $ccnn\bar{n}$ with $I_{nn} = 0$, $IJ = \frac{1}{2}\frac{1}{2}$;

  (2) lowest $ccnn\bar{s}$ with $IJ = 0\frac{1}{2}$.

- Unstable states seem to have narrow widths

- It is possible to understand X(3960), LHCb T_{cc}, $T^{a}_{c\bar{s}0}(2900)$, $T_{cs0}(2900)$, and X(6660)/X(6400) in tetraquark picture.

- It is possible to understand $P_c(4457)$, $P_c(4440)$, $P_c(4337)$, $P_{cs}(4459)$, and $P_{cs}(4338)$ in pentaquark picture if $P_c(4312)$ is a $J^P = \frac{3}{2}^-$ state.

- More narrow/stable multiquark states are possible, e.g.

$I = 0\ cs\bar{u}\bar{d}$, $IJ = 0\frac{1}{2}\ ccud\bar{s}$

Thank you for your attention!