

# **The Coupled-channel framework for the exotic structures near thresholds**

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Based on [Phys.Rev.Lett. 128,112001\(2022\)](#); [JHEP01\(2023\)058](#); [Sci.Bull. 69,3036\(2024\)](#)

In collaboration with Guang-Juan Wang, Jia-Jun Wu, Makoto Oka, Shi-Lin Zhu

East Asian Workshop on Exotic Hadrons 2024  
Nanjing 2024/12/10

- ❖ Background

- ❖ Masses of exotic hadrons:

$D_{s0}(2317)$  and  $D_{s1}(2460)$

$X(3872)$

- ❖ Decay width of  $X(3872)$

- ❖ Summary



A SCHEMATIC MODEL OF BARYONS AND MESONS \*

M. GELL-MANN

California Institute of Technology, Pasadena, California

Received 4 January 1964

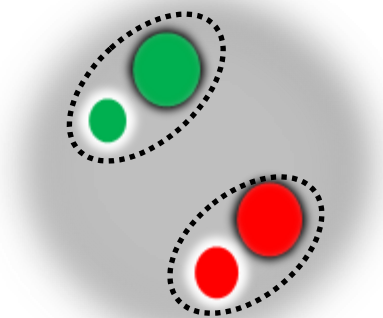
If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" <sup>1-3</sup>, we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone <sup>4</sup>. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

ber  $n_t - n_{\bar{t}}$  would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin  $\frac{1}{2}$  and  $z = -1$ , so that the four particles  $d^-$ ,  $s^-$ ,  $u^0$  and  $b^0$  exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon  $b$  if we assign to the triplet  $t$  the following properties: spin  $\frac{1}{2}$ ,  $z = -\frac{1}{3}$ , and baryon number  $\frac{1}{3}$ . We then refer to the members  $u^{\frac{2}{3}}$ ,  $d^{-\frac{1}{3}}$ , and  $s^{-\frac{1}{3}}$  of the triplet as "quarks" <sup>6</sup>  $q$  and the members of the anti-triplet as anti-quarks  $\bar{q}$ . Baryons can now be constructed from quarks by using the combinations  $(qqq)$ ,  $(qqq\bar{q})$ , etc., while mesons are made out of  $(q\bar{q})$ ,  $(q\bar{q}\bar{q})$ , etc. It is assuming that the lowest



Compact multiquark



Hadronic molecule

8419/TH.412  
21 February 1964

AN  $SU_3$  MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II \*)

G. Zweig  
CERN---Geneva

\*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

...

6) In general, we would expect that baryons are built not only from the product of three aces,  $AAA$ , but also from  $\bar{A}AAAA$ ,  $\bar{A}AAAAAA$ , etc., where  $\bar{A}$  denotes an anti-ace. Similarly, mesons could be formed from  $\bar{A}A$ ,  $\bar{A}AAA$  etc. For the low mass mesons and baryons we will assume the simplest possibilities,  $\bar{A}A$  and  $AAA$ , that is, "deuces and treys".



# GI quark model for $D_s$ meson



## Mesons in a Relativized Quark Model with Chromodynamics

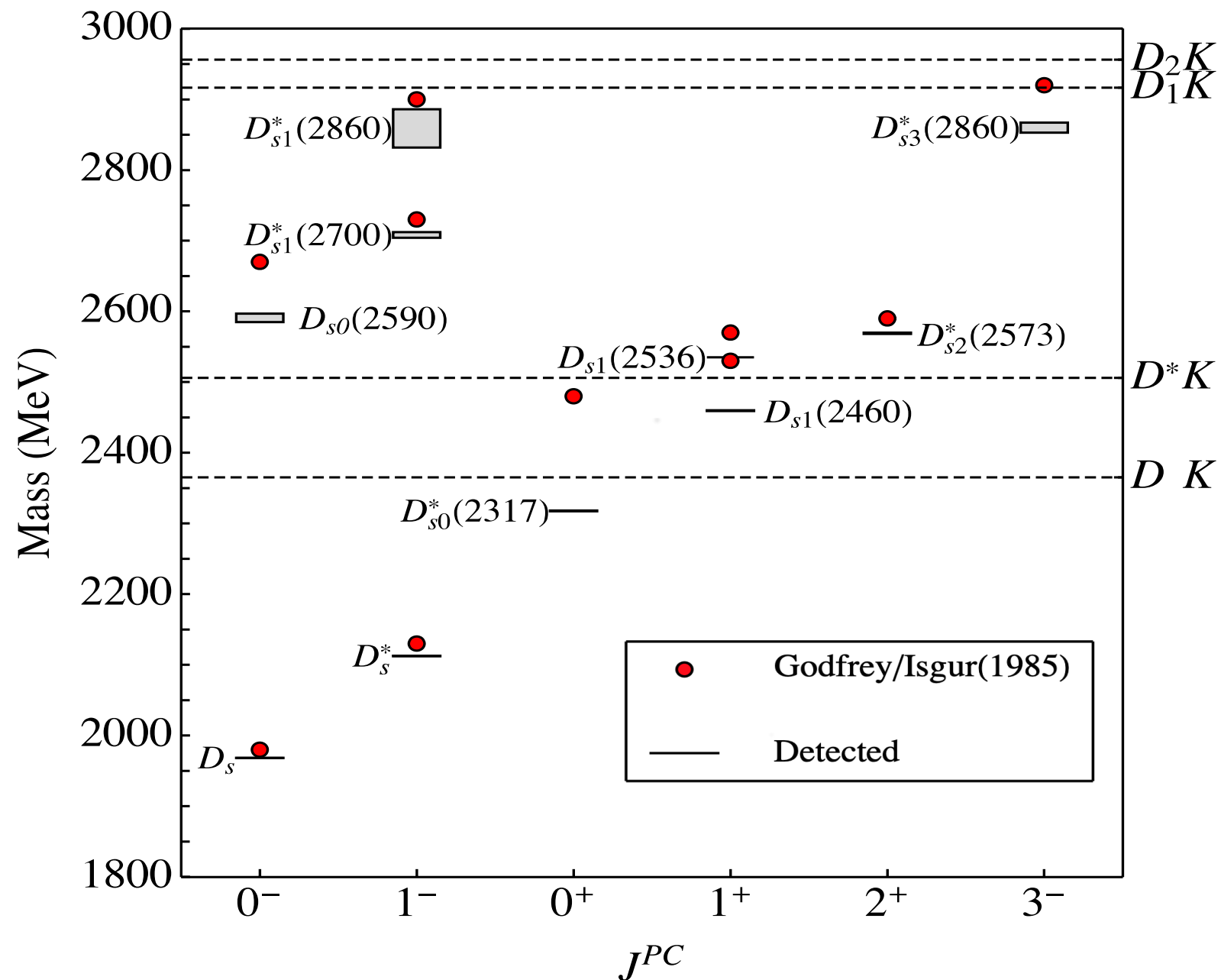
#1

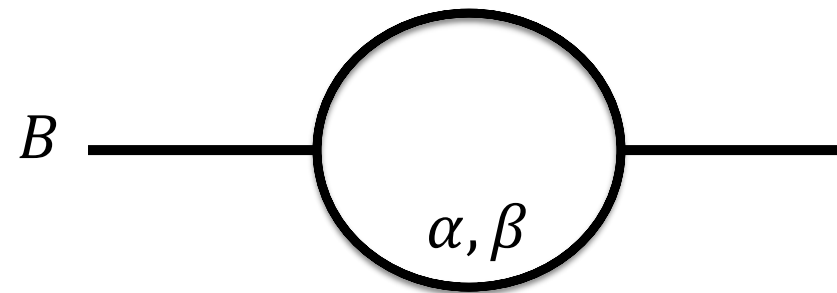
S. Godfrey (Toronto U.), Nathan Isgur (Toronto U.) (1985)

Published in: *Phys.Rev.D* 32 (1985) 189-231

[DOI](#) [cite](#) [claim](#)

[reference search](#) [3,134 citations](#)

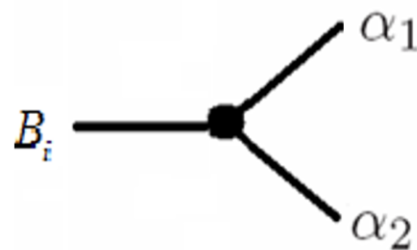




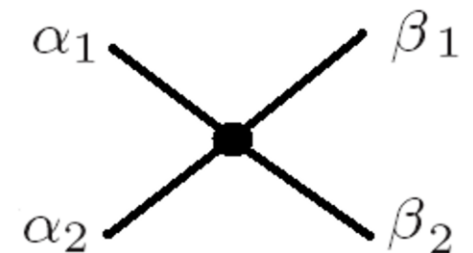
- Coupled-channel effect due to hadron loop could cause sizable mass shift on the state in quark model.

Yu. S. Kalashnikova, *Phys.Rev.D* 72, 034010 (2005);  
Z.-Y. Zhou and Z. Xiao, *Phys. Rev. D* 84, 034023 (2011)

bare state core  $\rightarrow$  channel:



channel  $\rightarrow$  channel:



*Phys.Rev.Lett.* 128,112001(2022)

$$g = \sum_{\alpha, B} \int d^3 \vec{k} \left\{ |\alpha(\vec{k})\rangle g_{\alpha B}(|\vec{k}|) \langle B| + h.c. \right\}$$

$$v = \sum_{\alpha, \beta} \int d^3 \vec{k} d^3 \vec{k}' |\alpha(\vec{k})\rangle V_{\alpha, \beta}^L(|\vec{k}|, |\vec{k}'|) \langle \beta(\vec{k}')|$$

Quark pair creation model (QPC):

$$g_{\alpha B}(|\vec{k}|) = \gamma I_{\alpha B}(|\vec{k}|) e^{-\frac{\vec{k}^2}{2\Lambda'^2}}$$



P. G. Ortega, et al,  
*Phys. Rev. D* 94, 074037 (2016)

truncate the hard vertices given  
by usual QPC

Effective Lagrangian: (exchanging mesons, e.g.  $\rho/\omega$ )

Form factor:  $\left( \frac{\Lambda^2}{\Lambda^2 + p_f^2} \right)^2 \left( \frac{\Lambda^2}{\Lambda^2 + p_i^2} \right)^2$

- For the T-matrix,

$$T(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) = \mathcal{V}(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) + \int d\vec{q} \frac{\mathcal{V}(\vec{k}_{D^*}, \vec{q}; E) T(\vec{q}, \vec{k}'_{D^*}; E)}{E - \sqrt{m_D^2 + q^2} - \sqrt{m_{D^*}^2 + q^2} + i\epsilon}$$

$$\mathcal{V}(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) = \frac{g(\vec{k}_{D^*})g(\vec{k}'_{D^*})}{E - m_B} + v(\vec{k}_{D^*}, \vec{k}'_{D^*})$$

- For the Hamiltonian

$$H = H_0 + H_I,$$

where the non-interacting one is

$$H_0 = \sum_B |B\rangle m_B \langle B| + \sum_\alpha \int d^3\vec{k} |\alpha(\vec{k})\rangle E_\alpha(\vec{k}) \langle \alpha(\vec{k})|.$$

And the interacting one includes two parts

$$H_I = g + v$$

❖  $D_{s0}(2317)$  and  $D_{s1}(2460)$

❖  $X(3872)$

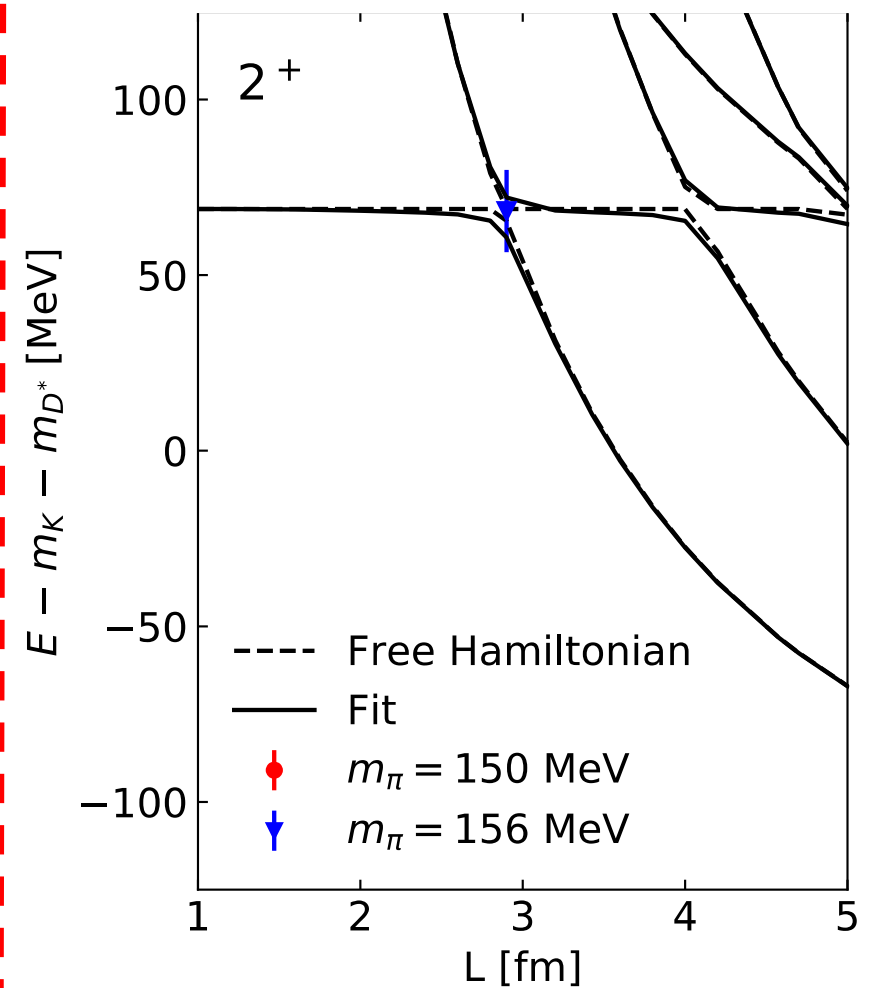
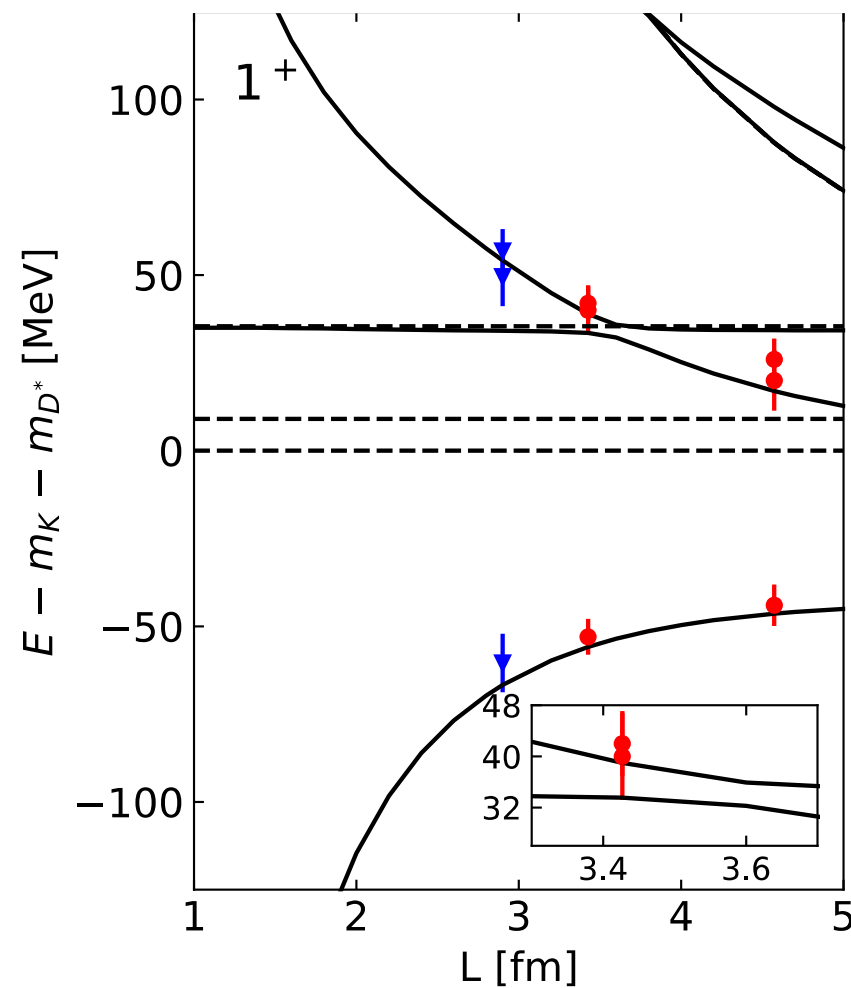
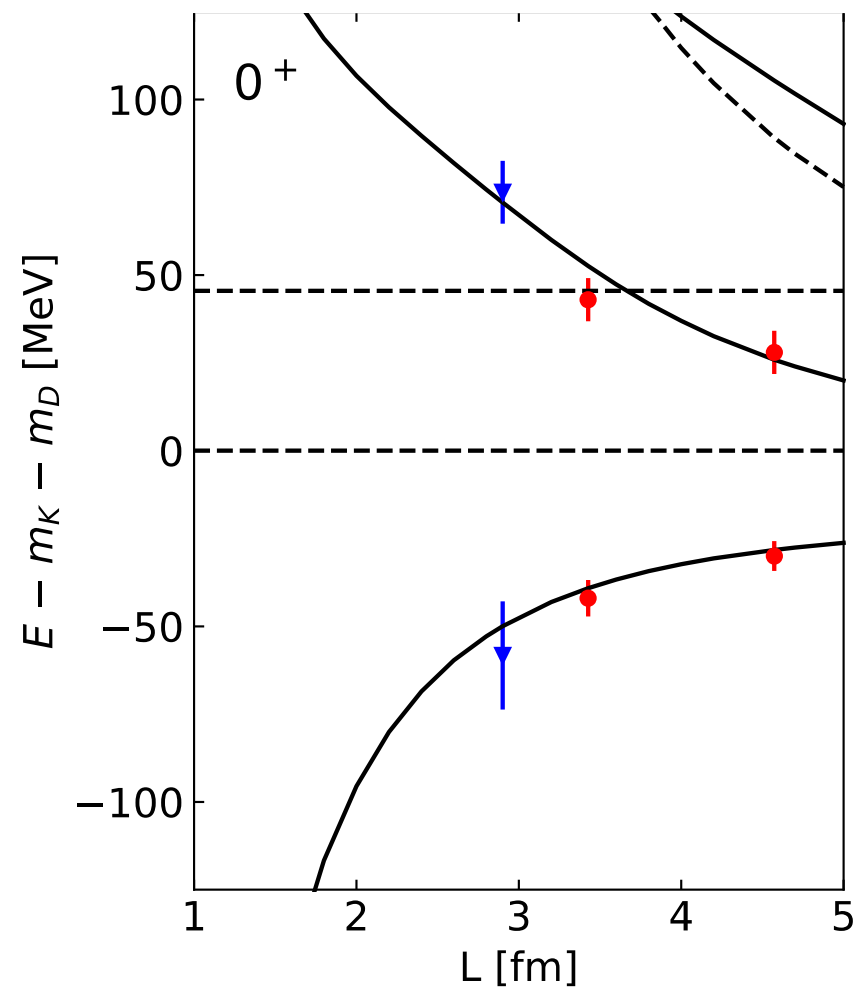
# Fit the lattice data : $D_s(2317,2460,2536)$



Lattice data from: C. B. Lang et al., [Phys. Rev. D 90, 034510 \(2014\)](#);  
G. S. Bali et al., [Phys. Rev. D 96, 074501 \(2017\)](#)

## Fit

## Postpredict



|                  | $P(c\bar{s})[\%]$    | ours                   | exp                |
|------------------|----------------------|------------------------|--------------------|
| $D_{s0}^*(2317)$ | $32.0^{+5.2}_{-3.9}$ | $2338.9^{+2.1}_{-2.7}$ | $2317.8 \pm 0.5$   |
| $D_{s1}^*(2460)$ | $52.4^{+5.1}_{-3.8}$ | $2459.4^{+2.9}_{-3.0}$ | $2459.5 \pm 0.6$   |
| $D_{s1}^*(2536)$ | $98.2^{+0.1}_{-0.2}$ | $2536.6^{+0.3}_{-0.5}$ | $2535.11 \pm 0.06$ |
| $D_{s2}^*(2573)$ | $95.9^{+1.0}_{-1.5}$ | $2570.2^{+0.4}_{-0.8}$ | $2569.1 \pm 0.8$   |

$D_{s0}(2317), D_{s1}(2460)$

- Bare  $c\bar{s}$  has strong coupling to S-wave  $D^{(*)}K$  channels, and significant mass shift.
- Both the bare  $c\bar{s}$  core and molecular components are significant and essential.

$D_{s1}(2536), D_{s2}(2573)$

- Coupling to D-wave  $D^{(*)}K$  channels can be neglected.
- Mainly pure  $c\bar{s}$ .

|                  | $P(c\bar{s})[\%]$    | ours                   | exp                |
|------------------|----------------------|------------------------|--------------------|
| $D_{s0}^*(2317)$ | $32.0^{+5.2}_{-3.9}$ | $2338.9^{+2.1}_{-2.7}$ | $2317.8 \pm 0.5$   |
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A. M. Torres, E. Oset, S. Prelovsek, and A. Ramos [JHEP 05, 153 \(2015\)](#)

$$P(KD) = 72 \pm 13 \pm 5 \%, \text{ for the } D_{s0}^*(2317)$$

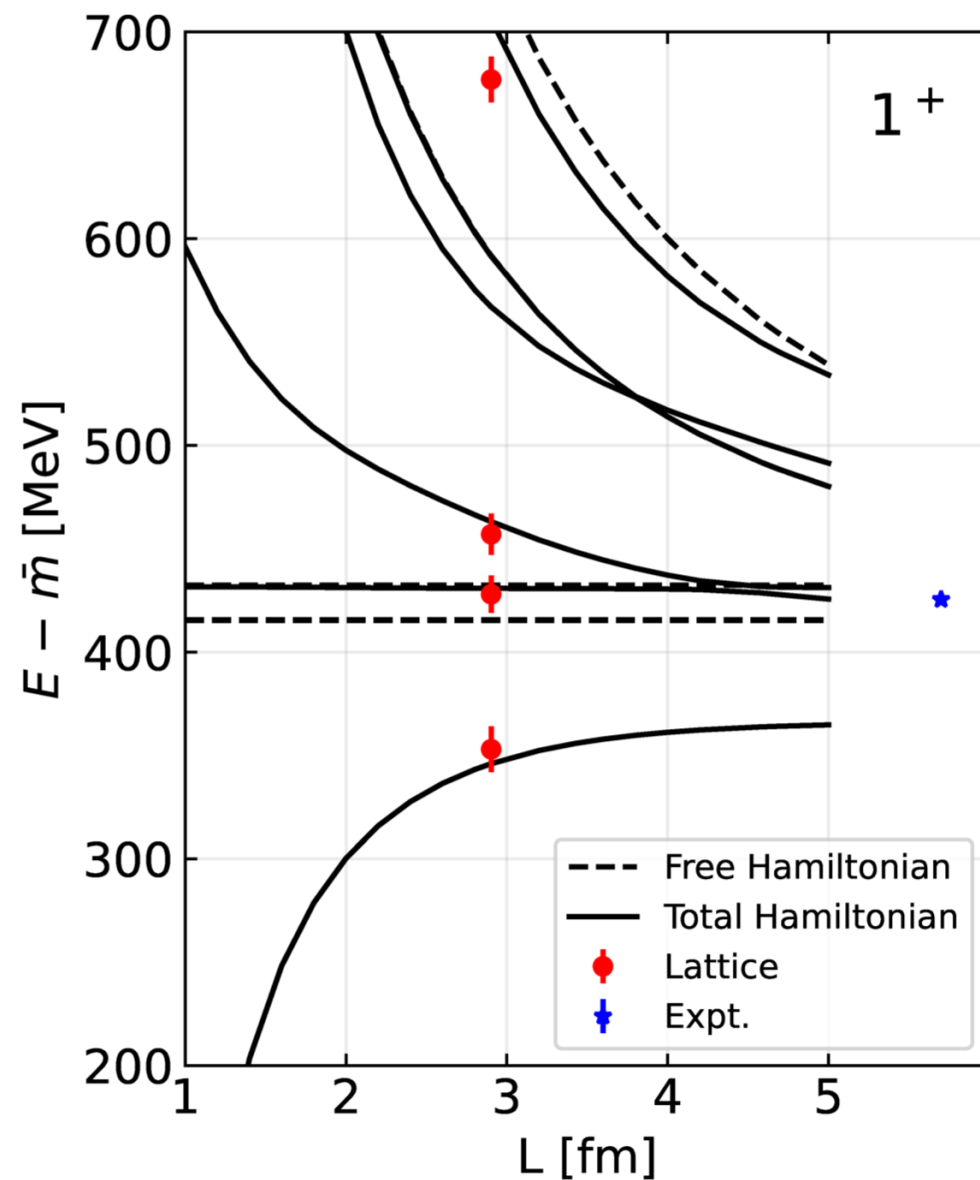
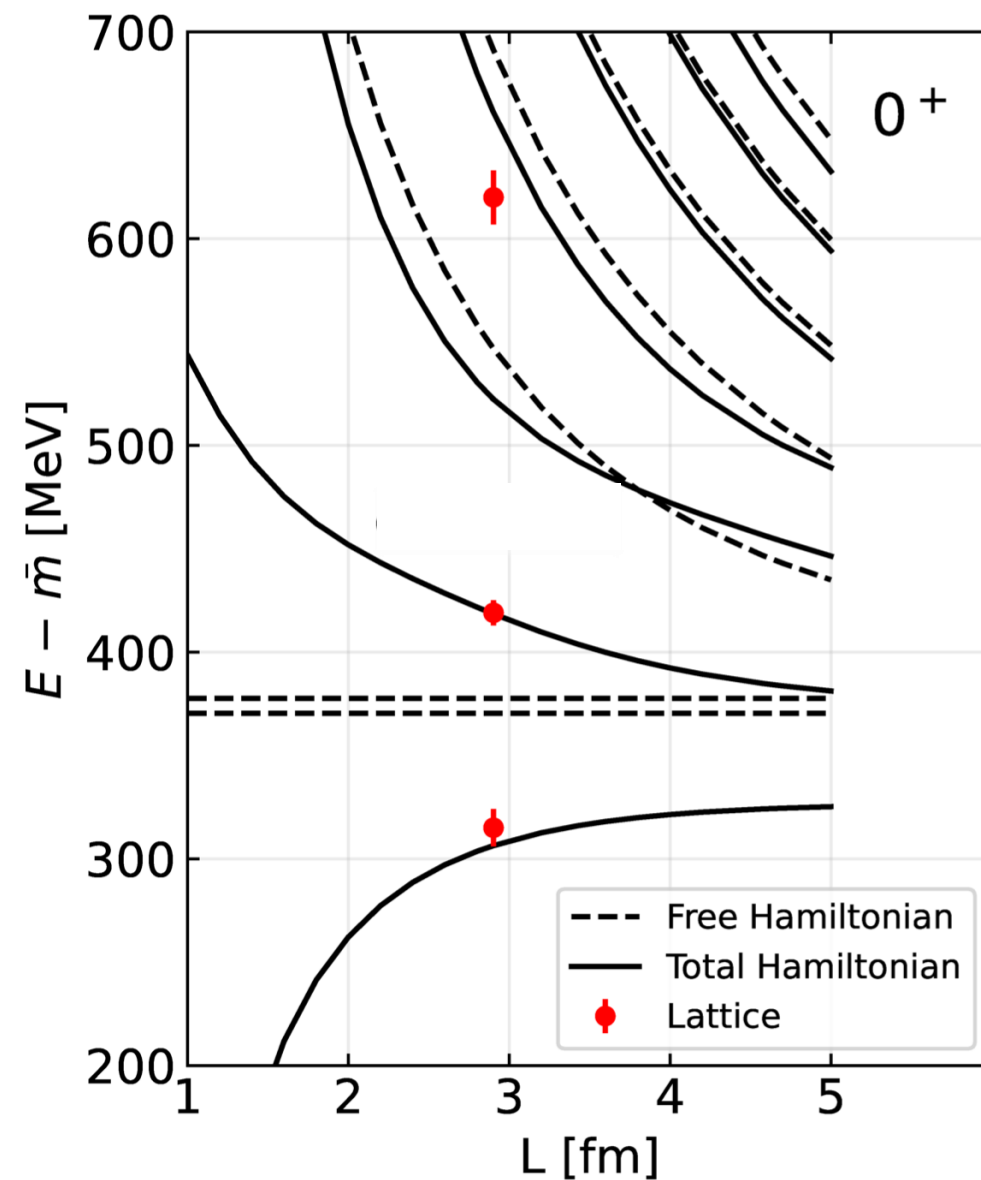
$$P(KD^*) = 57 \pm 21 \pm 6 \%, \text{ for the } D_{s1}(2460)$$

L.M. Liu, K. Orginos, F.-K. Guo, C. Hanhart, Ulf-G. Meissner [Phys.Rev.D 87 \(2013\) 1, 014508](#)

$$P(KD) = [0.68, 0.73], \text{ for the } D_{s0}^*(2317)$$

- The heavy quark symmetry seems to be a good symmetry here.
- Use the same parameters as  $D_s$ .

Postprediction, not a fit !



$$\bar{m} = \frac{1}{4}(m_{B_s} + 3m_{B_s^*})$$

Lattice data from: C. B. Lang et al., [Phys. Lett. B 750, 17 \(2015\)](#)

❖  $D_{s0}(2317)$  and  $D_{s1}(2460)$

❖  $X(3872)$

# GI quark model for charmonium



## Mesons in a Relativized Quark Model with Chromodynamics

#1

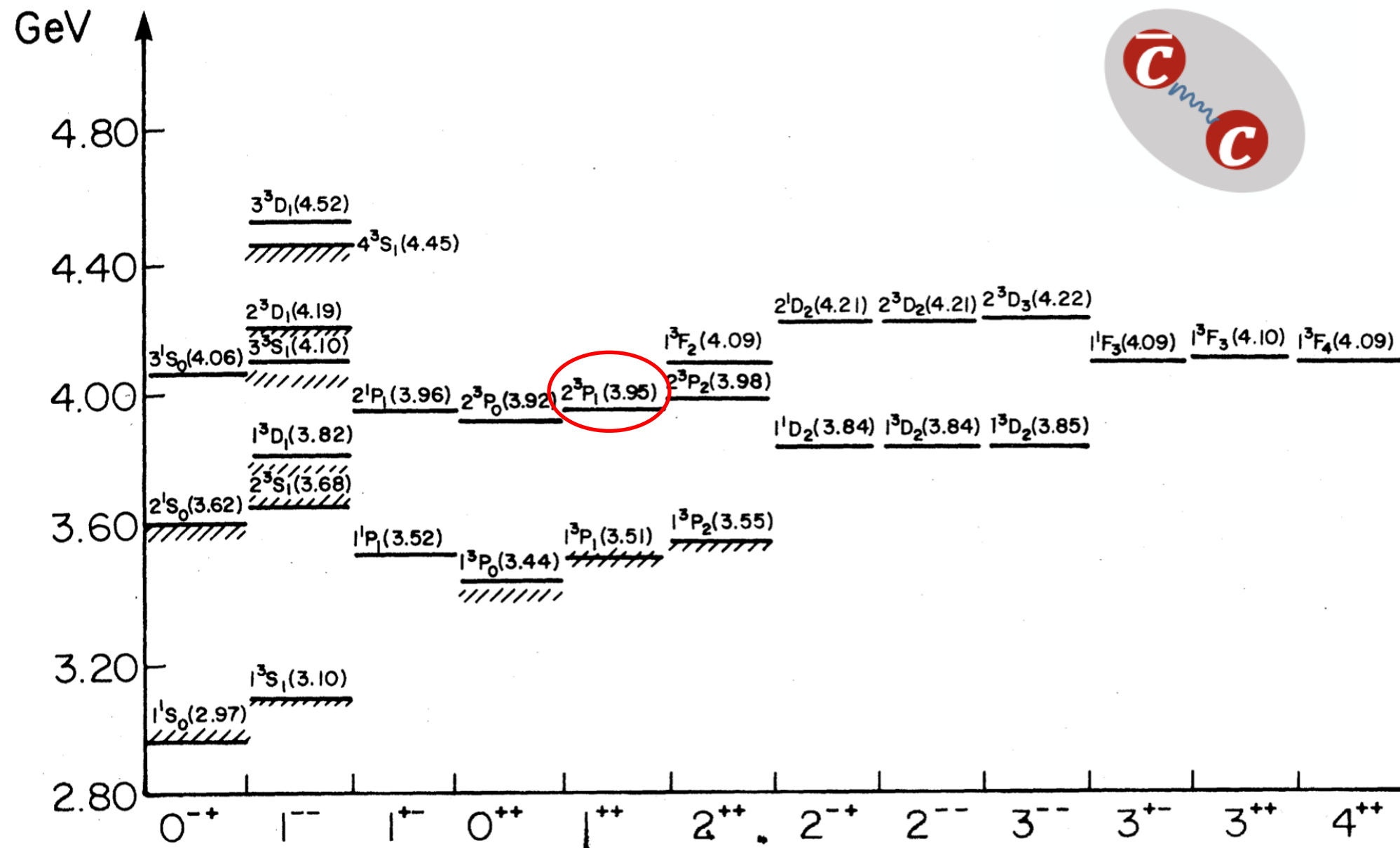
S. Godfrey (Toronto U.), Nathan Isgur (Toronto U.) (1985)

Published in: *Phys.Rev.D* 32 (1985) 189-231

[DOI](#) [cite](#) [claim](#)

[reference search](#)

[3,134 citations](#)



| Experiment  | Mass [MeV]                             | Width [MeV]                 |
|-------------|----------------------------------------|-----------------------------|
| Belle [63]  | $3872 \pm 0.6 \pm 0.5$                 | $< 2.3$                     |
| Belle [75]  | —                                      | —                           |
| Belle [76]  | $3875.4 \pm 0.7^{+0.4}_{-1.7} \pm 0.9$ | —                           |
| Belle [77]  | $3871.46 \pm 0.37 \pm 0.07$            | —                           |
| Belle [78]  | $3872.9^{+0.6+0.4}_{-0.4-0.5}$         | $3.9^{+2.8+0.2}_{-1.4-1.1}$ |
| Belle [79]  | —                                      | —                           |
| Belle [80]  | $3871.84 \pm 0.27 \pm 0.19$            | $< 1.2$                     |
| CDF [67]    | $3871.3 \pm 0.7 \pm 0.4$               | —                           |
| CDF [81]    | —                                      | —                           |
| CDF [82]    | —                                      | —                           |
| CDF [83]    | $3871.61 \pm 0.16 \pm 0.19$            | —                           |
| DØ [68]     | $3871.8 \pm 3.1 \pm 3.0$               | —                           |
| BaBar [84]  | $3873.4 \pm 1.4$                       | —                           |
| BaBar [85]  | $3871.3 \pm 0.6 \pm 0.1$               | $< 4.1$                     |
|             | $3868.6 \pm 1.2 \pm 0.2$               | —                           |
| BaBar [86]  | —                                      | —                           |
| BaBar [87]  | $3875.1^{+0.7}_{-0.5} \pm 0.5$         | $3.0^{+1.9}_{-1.4} \pm 0.9$ |
| BaBar [88]  | $3871.4 \pm 0.6 \pm 0.1$               | $< 3.3$                     |
|             | $3868.7 \pm 1.5 \pm 0.4$               | —                           |
| BaBar [89]  | —                                      | —                           |
| BaBar [90]  | $3873.0^{+1.8}_{-1.6} \pm 1.3$         | —                           |
| LHCb [91]   | $3871.95 \pm 0.48 \pm 0.12$            | —                           |
| LHCb [70]   | —                                      | —                           |
| LHCb [92]   | —                                      | —                           |
| CMS [73]    | —                                      | —                           |
| BESIII [93] | $3871.9 \pm 0.7 \pm 0.2$               | $< 2.4$                     |

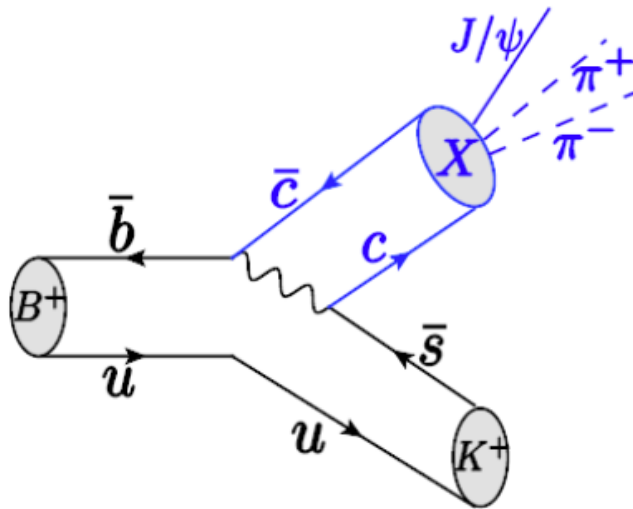
Observation of a narrow charmonium-like state in exclusive  $B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi$  decays

Belle Collaboration • S.K. Choi (Gyeongsang Natl. U.) et al. (Sep, 2003)

Published in: *Phys.Rev.Lett.* 91 (2003) 262001 • e-Print: [hep-ex/0309032](#) [hep-ex]

 pdf  links  DOI  cite  claim

 2,295 citations



- The  $D\bar{D}^*/D^*\bar{D}$  molecular state.

[Swanson, Wong, Guo, liu,....](#)

Close to  $D^0\bar{D}^{*0}/D^{*0}\bar{D}^0$  thresholds

$$\begin{aligned}\delta m &= m_{D^0\bar{D}^{*0}} - m_{X(3872)} \\ &= 0.00 \pm 0.18 \text{ MeV}\end{aligned}$$

PDG 22

## Where is the $\chi_{c1}(2P)$ in quark model?

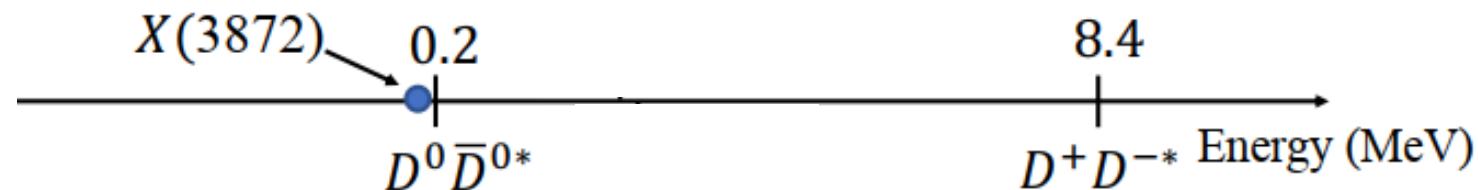
- The mixing of the  $\bar{c}c$  core with  $D\bar{D}^*/D^*\bar{D}$  component.  
Chao, H. Q. Zheng, Yu. S. Kalashnikova, P. G. Ortega...

Close to charmonium  $\chi_{c1}(2P)$ :  $m=3953.5$  MeV

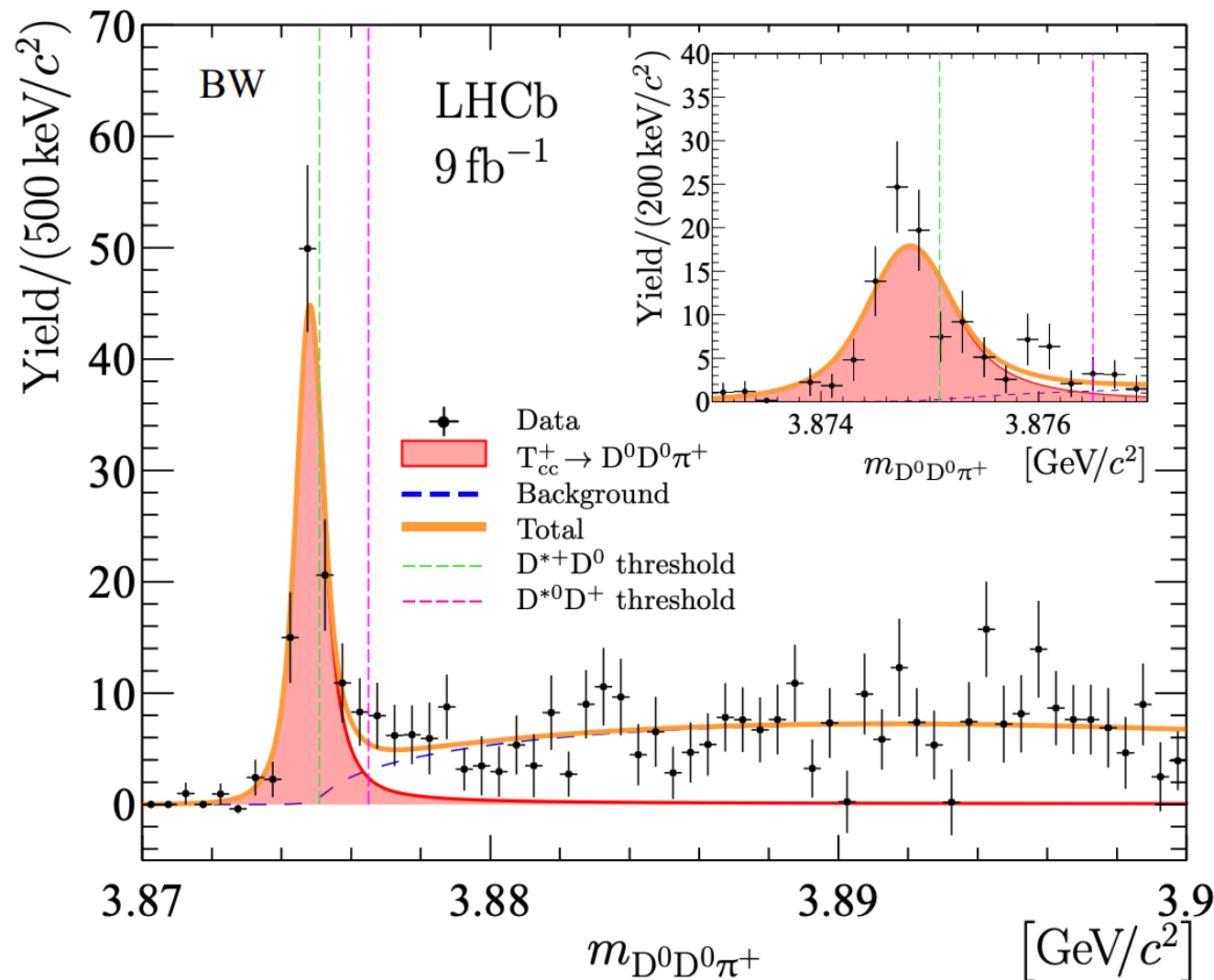
$$\delta m = m_{\chi_{c1}(2P)} - m_{X(3872)} = 81.35 \text{ MeV}$$

→ *Complicated coupled-channel effect:  $\bar{c}c$  &  $D\bar{D}^*/D^*\bar{D}$*

Phys. Rev. D 32, 189 (1985)



# How to determine the component in the X(3872): from Tcc



- $D^0 D^0 \pi^+$  channel
- Close to  $D^{*+} D^0$  thresholds:

Conventional Breit-Wigner: assumed  $J^P = 1^+$ .

$$\begin{aligned} \delta m_{BW} &= m_{T_{cc}} - m_{D^{*+} D^0} \\ &= -273 \pm 61 \text{ keV} \end{aligned}$$

$$\Gamma_{BW} = 410 \pm 165 \text{ keV}$$

EPS-HEP conference, Ivan Polyakov's talk, 29/07/2021; Nature Physics, 22'

Unitarized Breit-Wigner:

$$\begin{aligned} \delta m_U &= m_{T_{cc}} - m_{D^{*+} D^0} \\ &= -361 \pm 40 \text{ keV} \end{aligned}$$

$$\Gamma_U = 47.8 \pm 1.9 \text{ keV}$$

LHCb, Nature Commun. 13 (2022) 1, 3351

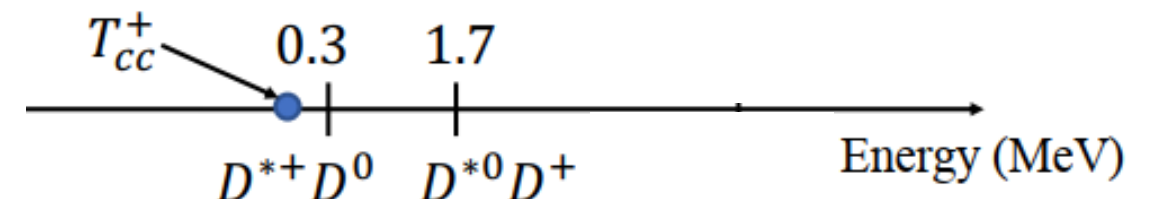
❖ Quark content:  $cc\bar{u}\bar{d}$

❖ *Only the  $D^* D$  coupled channel effect*



**C-parity**

$\bar{D}^* D / \bar{D} D^*$  interaction



$DD^*$

$$H_a^{(Q)} = \frac{1+\not{p}}{2} [P_a^{*\mu} \gamma_\mu - P_a \gamma_5]$$

$$\bar{H}_a^{(Q)} \equiv \gamma_0 H_a^{(Q)\dagger} \gamma_0 = [P_a^{*\dagger\mu} \gamma_\mu + P_a^\dagger \gamma_5] \frac{1+\not{p}}{2}$$

$$P = (D^0, D^+, D_s^+) \text{ \& } P^* = (D^{*0}, D^{*+}, D_s^{*+})$$

$$\begin{aligned} \mathcal{L}_{MH^{(Q)}H^{(Q)}} &= ig \text{Tr} \left[ H_b^{(Q)} \gamma_\mu \gamma_5 A_{ba}^\mu \bar{H}_a^{(Q)} \right] \\ \mathcal{L}_{VH^{(Q)}H^{(Q)}} &= i\beta \text{Tr} \left[ H_b^{(Q)} v_\mu (V_{ba}^\mu - \rho_{ba}^\mu) \bar{H}_a^{(Q)} \right] \\ &\quad + i\lambda \text{Tr} \left[ H_b^{(Q)} \sigma_{\mu\nu} F^{\mu\nu}(\rho)_{ba} \bar{H}_a^{(Q)} \right] \end{aligned}$$

$D\bar{D}^*$

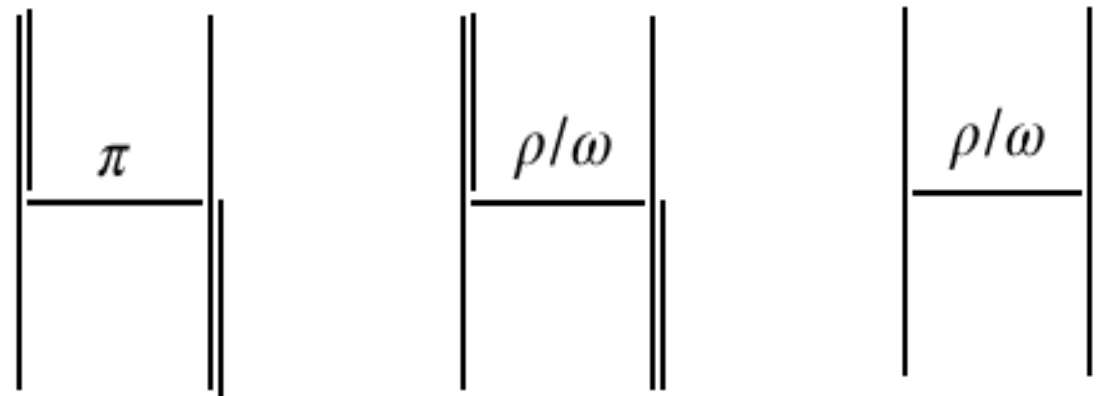
$$H_a^{(\bar{Q})} \equiv C \left( C H_a^{(Q)} C^{-1} \right)^T C^{-1} = [P_{a\mu}^{(\bar{Q})*} \gamma^\mu - P_a^{(\bar{Q})} \gamma_5] \frac{1-\not{p}}{2}$$

$$\bar{H}_a^{(\bar{Q})} \equiv \gamma_0 H_a^{(\bar{Q})\dagger} \gamma_0 = \frac{1-\not{p}}{2} [P_{a\mu}^{(\bar{Q})*} \gamma^\mu + P_a^{(\bar{Q})\dagger} \gamma_5]$$

$$\tilde{P} = (\bar{D}^0, D^-, D_s^-) \text{ \& } \tilde{P}^* = (\bar{D}^{*0}, D^{*-}, D_s^{*-})$$

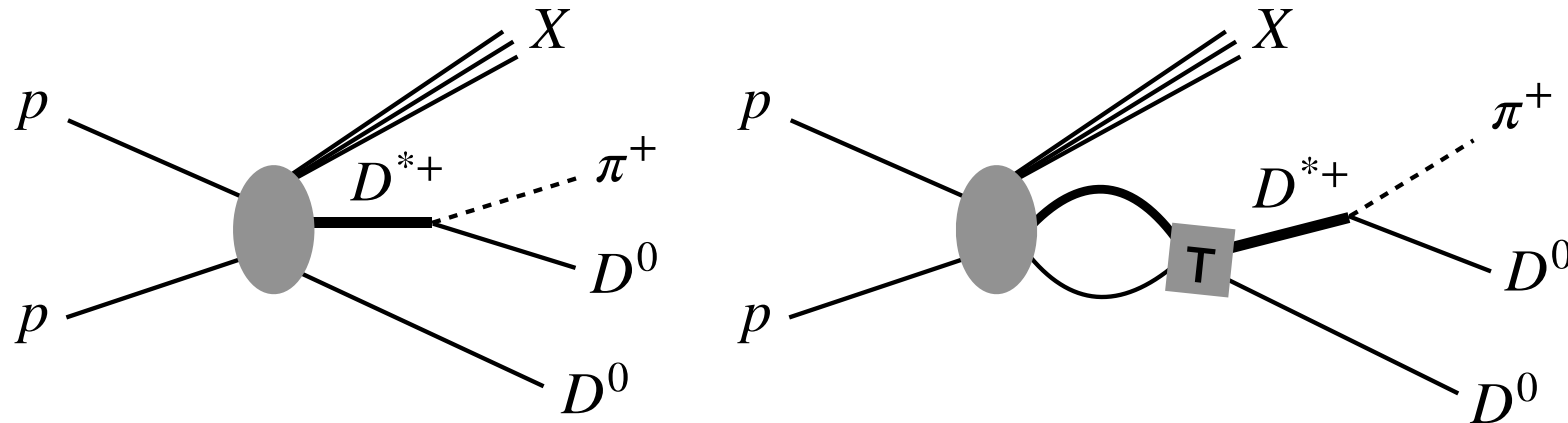
$$\begin{aligned} \mathcal{L}_{MH^{(\bar{Q})}H^{(\bar{Q})}} &= ig \text{Tr} \left[ \bar{H}_a^{(\bar{Q})} \gamma_\mu \gamma_5 A_{ab}^\mu H_b^{(\bar{Q})} \right] \\ \mathcal{L}_{VH^{(\bar{Q})}H^{(\bar{Q})}} &= -i\beta \text{Tr} \left[ \bar{H}_a^{(\bar{Q})} v_\mu (V_{ab}^\mu - \rho_{ab}^\mu) H_b^{(\bar{Q})} \right] \\ &\quad + i\lambda \text{Tr} \left[ \bar{H}_a^{(\bar{Q})} \sigma_{\mu\nu} F'^{\mu\nu}(\rho)_{ab} H_b^{(\bar{Q})} \right] \end{aligned}$$

- $g = 0.57$  is determined by the strong decays  $D^* \rightarrow D\pi$ .
- undetermined  $\lambda$  &  $\beta$ .



# The inclusive production of the $T_{cc}$

$pp \rightarrow D^0(p_{D_1})D^0(p_{D_2})\pi^+(p_\pi)X$ ,  $X$  denotes all the other produced particles



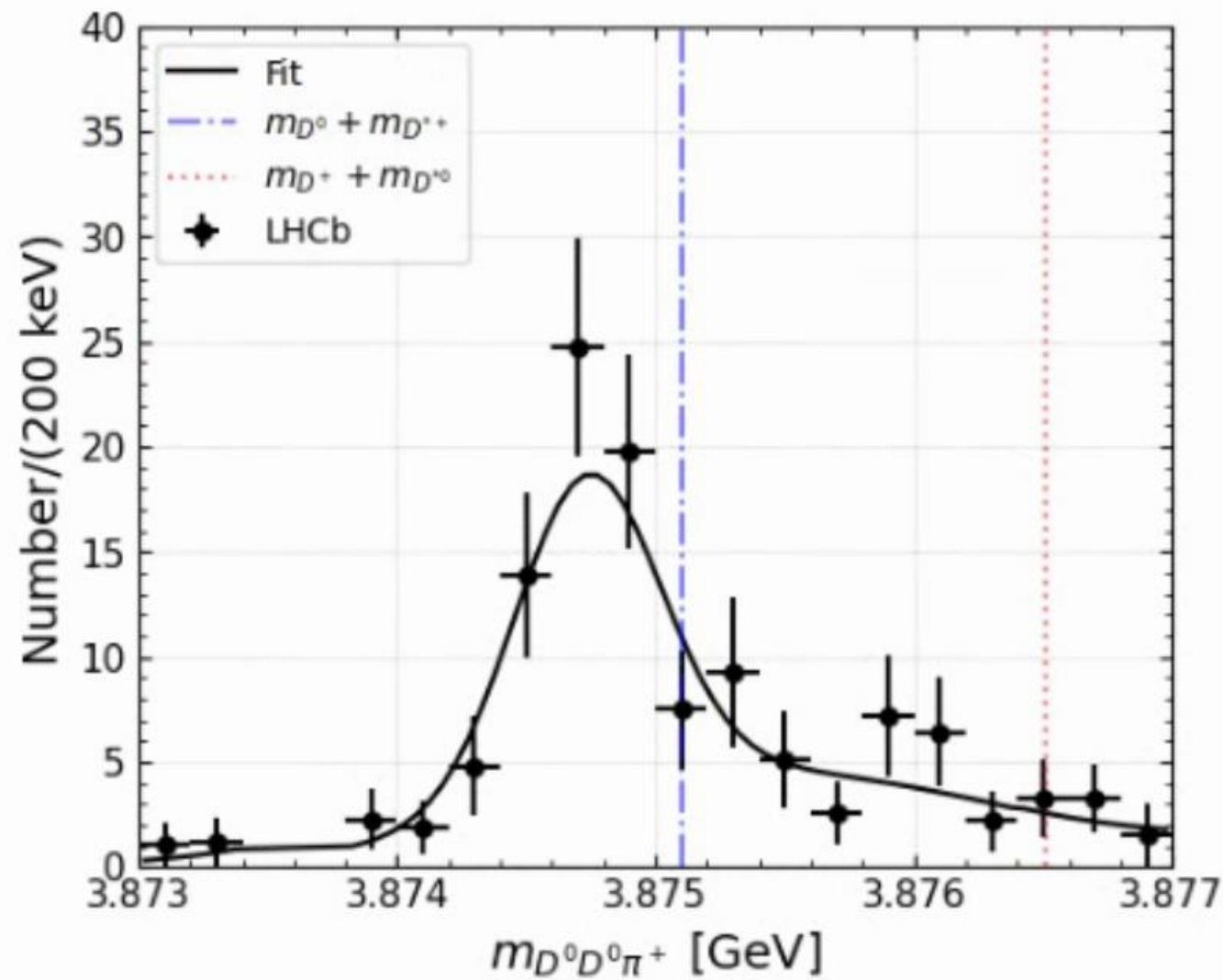
The T-matrix can be solved from the Lippmann-Schwinger equation

$$T(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) = \mathcal{V}(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) + \int d\vec{q} \frac{\mathcal{V}(\vec{k}_{D^*}, \vec{q}; E) T(\vec{q}, \vec{k}'_{D^*}; E)}{E - \sqrt{m_D^2 + q^2} - \sqrt{m_{D^*}^2 + q^2} + i\epsilon}$$

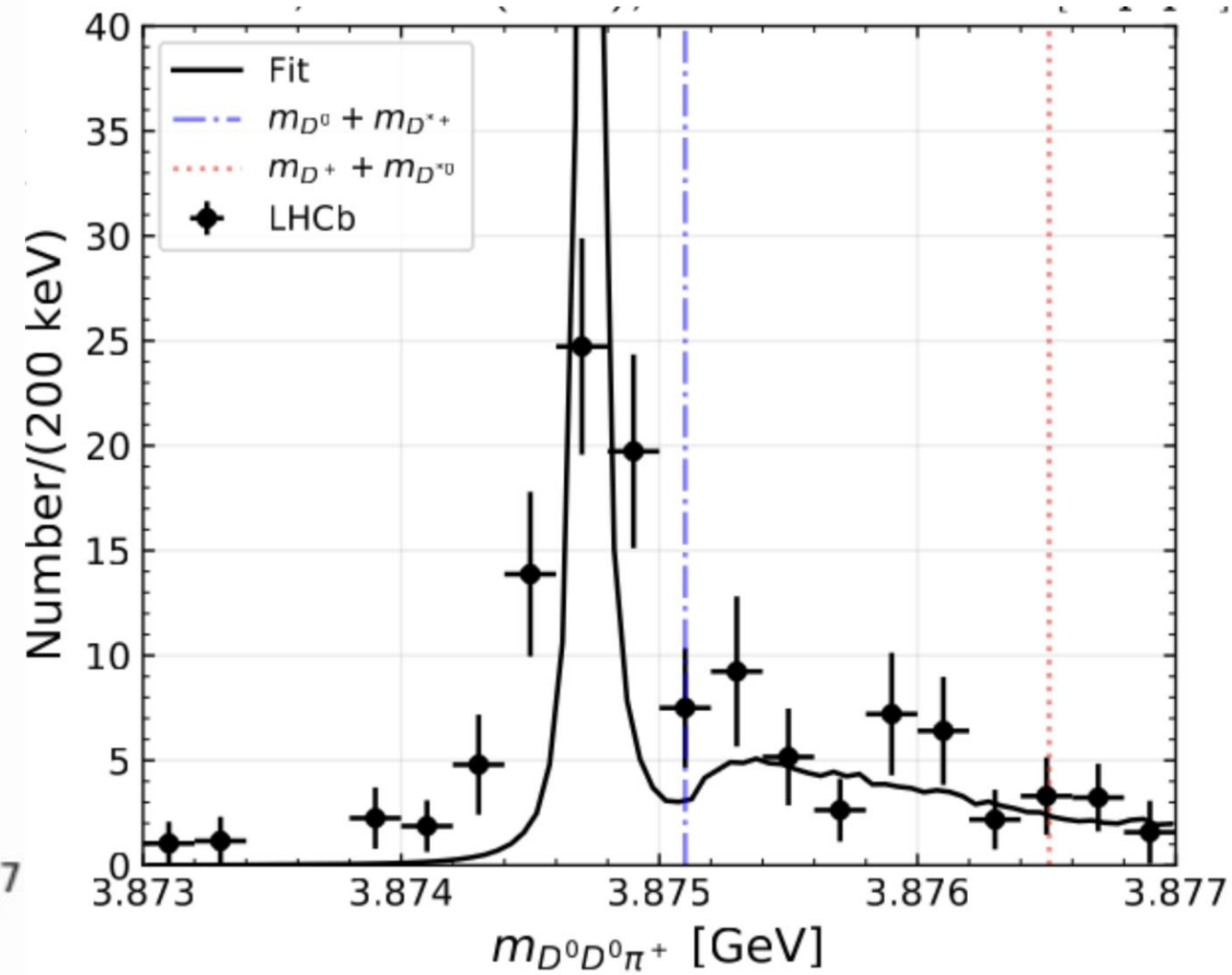
The effective potential is obtained with light-meson exchange potentials

$$\mathcal{V} = (V_\pi + V_{\rho/\omega}^t + V_{\rho/\omega}^u) \left( \frac{\Lambda^2}{\Lambda^2 + p_f^2} \right)^2 \left( \frac{\Lambda^2}{\Lambda^2 + p_i^2} \right)^2$$

$\Lambda = 0.8 \text{ GeV}$ ,  $\chi^2/dof = 0.76$



Without resolution function



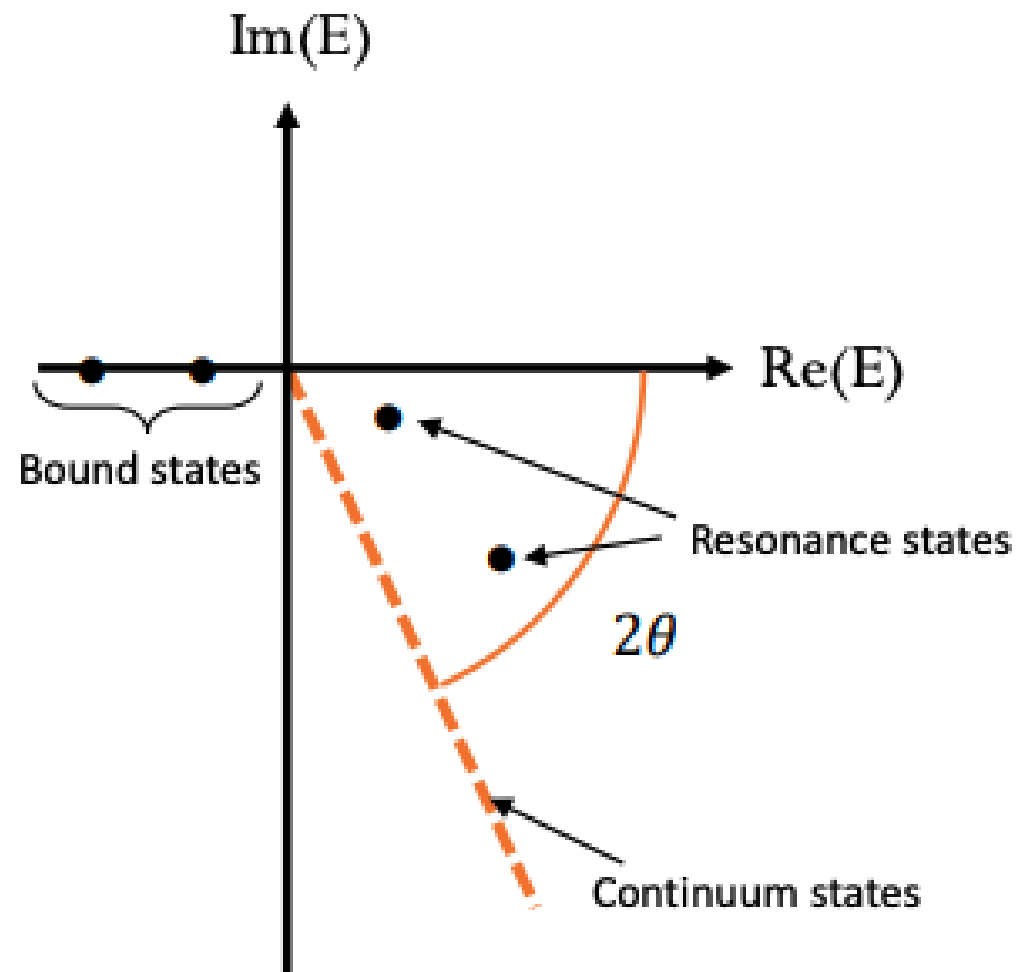
# Complex scaling method



The radius and momentum will rotate with an angle  $\theta$ :

$$\mathbf{r} \rightarrow \mathbf{r}e^{i\theta}, \quad \mathbf{q} \rightarrow \mathbf{q}e^{-i\theta}$$

$$H_\theta \Phi_\theta = E_\theta \Phi_\theta, \quad H_\theta = H(\mathbf{r}_\theta, \mathbf{q}_\theta) = \frac{q^2}{2u}e^{-2i\theta} + V(\mathbf{r}e^{i\theta}, \mathbf{q}e^{-i\theta})$$



S.Aoyama et al. PTP. 116, 1 (2006).  
T. Myo et al. PPNP. 79, 1 (2014)  
N. Moiseyev, Physics reports 302, 212 (1998)

With the varying  $\theta$ :

- the scattering states will rotate with  $2\theta$
- while the bound and resonant states will stay stable

# Results with $\Lambda = 0.8 \text{ GeV}$

- Only one pole appears—bound states

$$m_{T_{cc}} = 3874.7 \text{ MeV}, \Delta E = -387.7 \text{ keV}$$

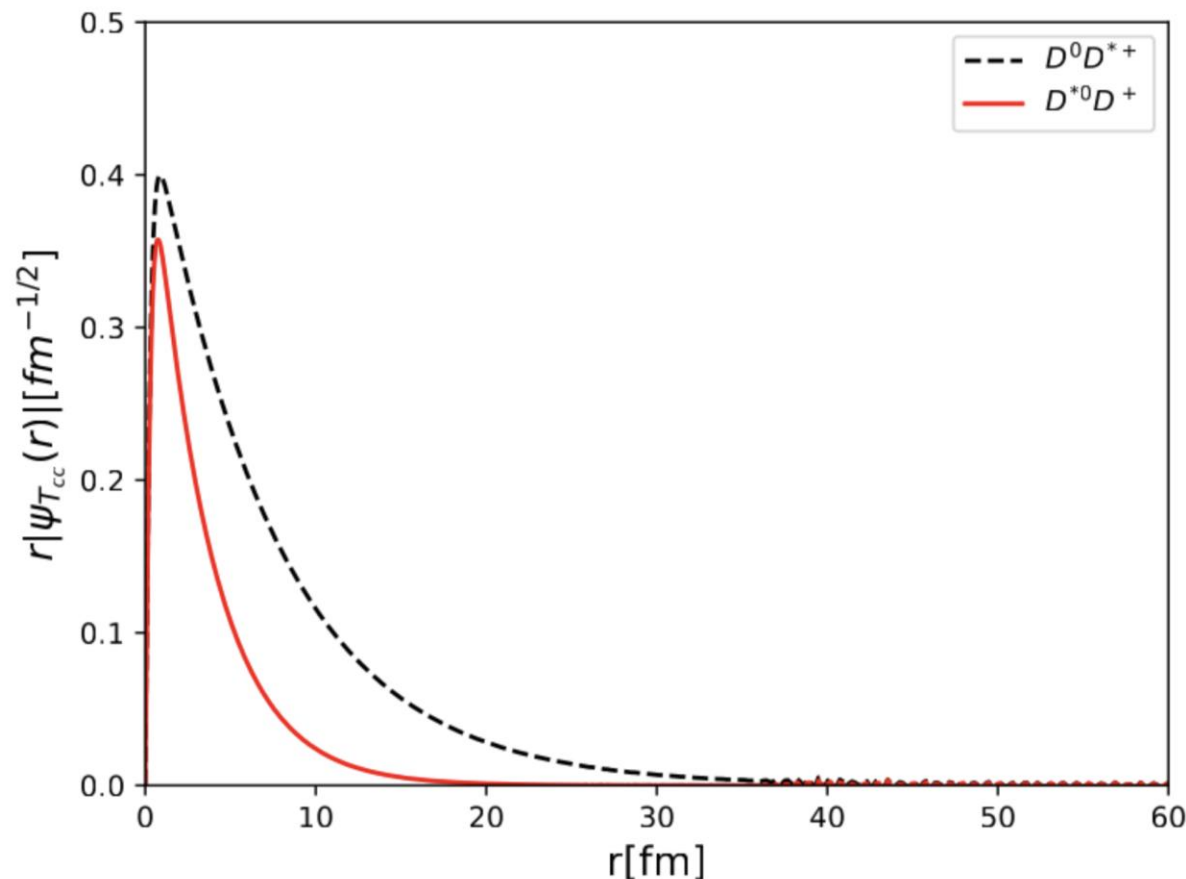
$$\Gamma_{T_{cc}} = 67.3 \text{ keV}$$

- $\sqrt{\langle r^2 \rangle} = 4.8 \text{ fm}$

- 70.1%  $D^{*+}D^0$ , 30%  $D^+D^{*0}$   $\longleftrightarrow$  95.8%  $DD^*(I=0)$   
4.2%  $DD^*(I=1)$

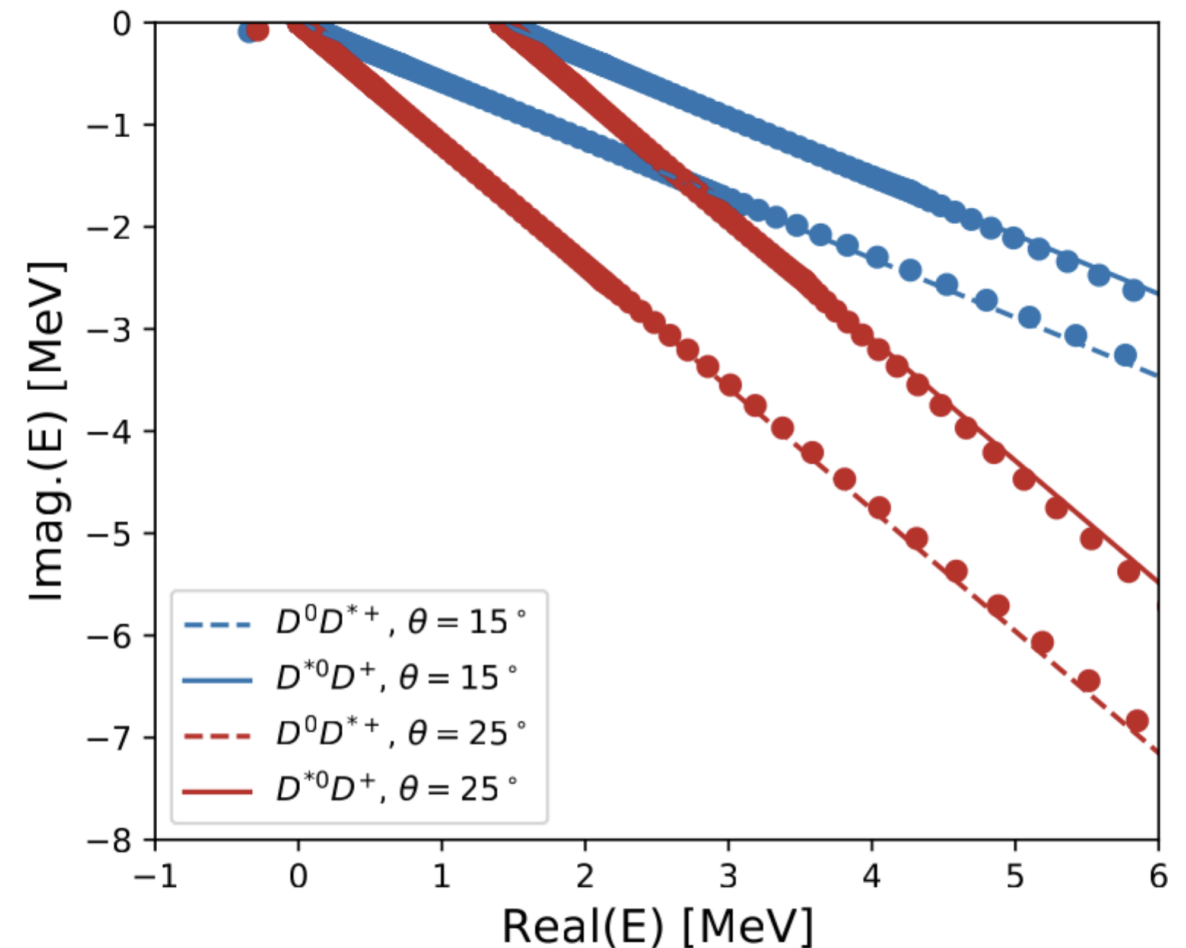


Mass differences of  $D^{*+}D^0$  and  $D^+D^{*0}$



$$[I=0] = \frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+)$$

$$[I=1] = \frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+)$$



# Results with three $\Lambda$



| $\Lambda$ (GeV) | BE (keV) | $\Gamma$ (keV) | $\sqrt{\langle r^2 \rangle}$ | $I = 0$ | $I = 1$ | $P(D^0 D^{*+})$ | $P(D^+ D^{*0})$ | $\frac{\text{Res}(D^0 D^{*+})}{\text{Res}(D^+ D^{*0})}$ |
|-----------------|----------|----------------|------------------------------|---------|---------|-----------------|-----------------|---------------------------------------------------------|
| 0.8             | -387.7   | 67.3           | 4.8 fm                       | 95.8%   | 4.2%    | 70.0%           | 30.0%           | $-1.063 + 0.001I$                                       |
| 1.0             | -393.0   | 70.4           | 4.7 fm                       | 95.8%   | 4.2%    | 70.0%           | 30.0%           | $-1.055 + 0.001I$                                       |
| 1.2             | -391.6   | 72.7           | 4.7 fm                       | 95.7%   | 4.3%    | 70.3%           | 29.7%           | $-1.052 + 0.001I$                                       |

- The conclusion remains the same using the three different cutoff values.
- The binding energy of the bound state is around  $\Delta E \sim -390\text{keV}$ , which is consistent

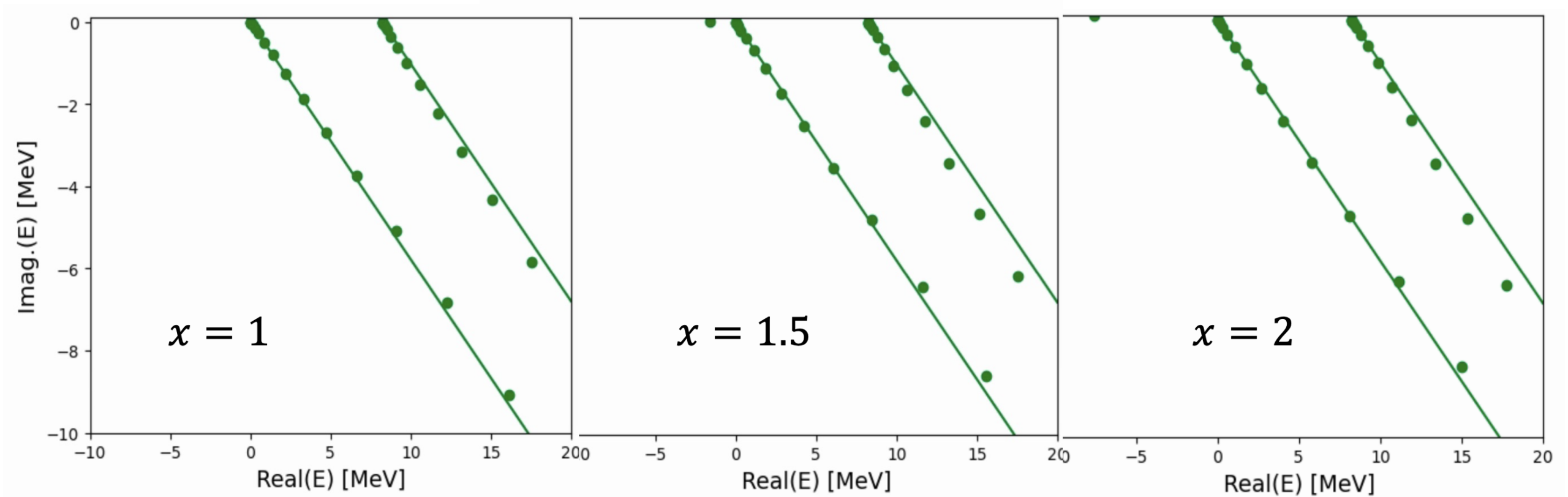
with that of the measurement ( $\Delta E_{\text{exp}} = -360(40)\text{keV}$ ).

LHCb, Nature Commun. 13 (2022) 1, 3351

# Direct application to $D\bar{D}^*: X(3872)$



- Without the  $c\bar{c}$  core, there are no bound states.
- $V'_{D\bar{D}^*} = x * V_{D\bar{D}^*}$



*$D\bar{D}^*$  interaction is attractive but not strong enough to form a bound state.*



*Inclusion of  $c\bar{c}$  core*

# $X(3872) : D\bar{D}^* + c\bar{c}$

- The  $D\bar{D}^*$  system with quantum number  $I(J^{PC}) = 0(1^{++})$  can couple with the  $\chi_{c1}(2P)$ .
- The coupled channel effect between them can be described by the quark-pair-creation model:

$$g_{D\bar{D}^*,c\bar{c}}(|\vec{k}_{D\bar{D}^*}|) = \gamma I_{D\bar{D}^*,c\bar{c}}(|\vec{k}_{D\bar{D}^*}|)$$

where  $\vec{k}_{D\bar{D}^*}$  is the relative momentum in the  $D\bar{D}^*$  channel.

$I_{D\bar{D}^*,c\bar{c}}(|\vec{k}_{D\bar{D}^*}|)$  is the overlap of the meson wave functions ← GI quark model

- $\gamma$  is determined to reproduce the  $\psi(3770)$ :

$$\gamma = 4.69$$

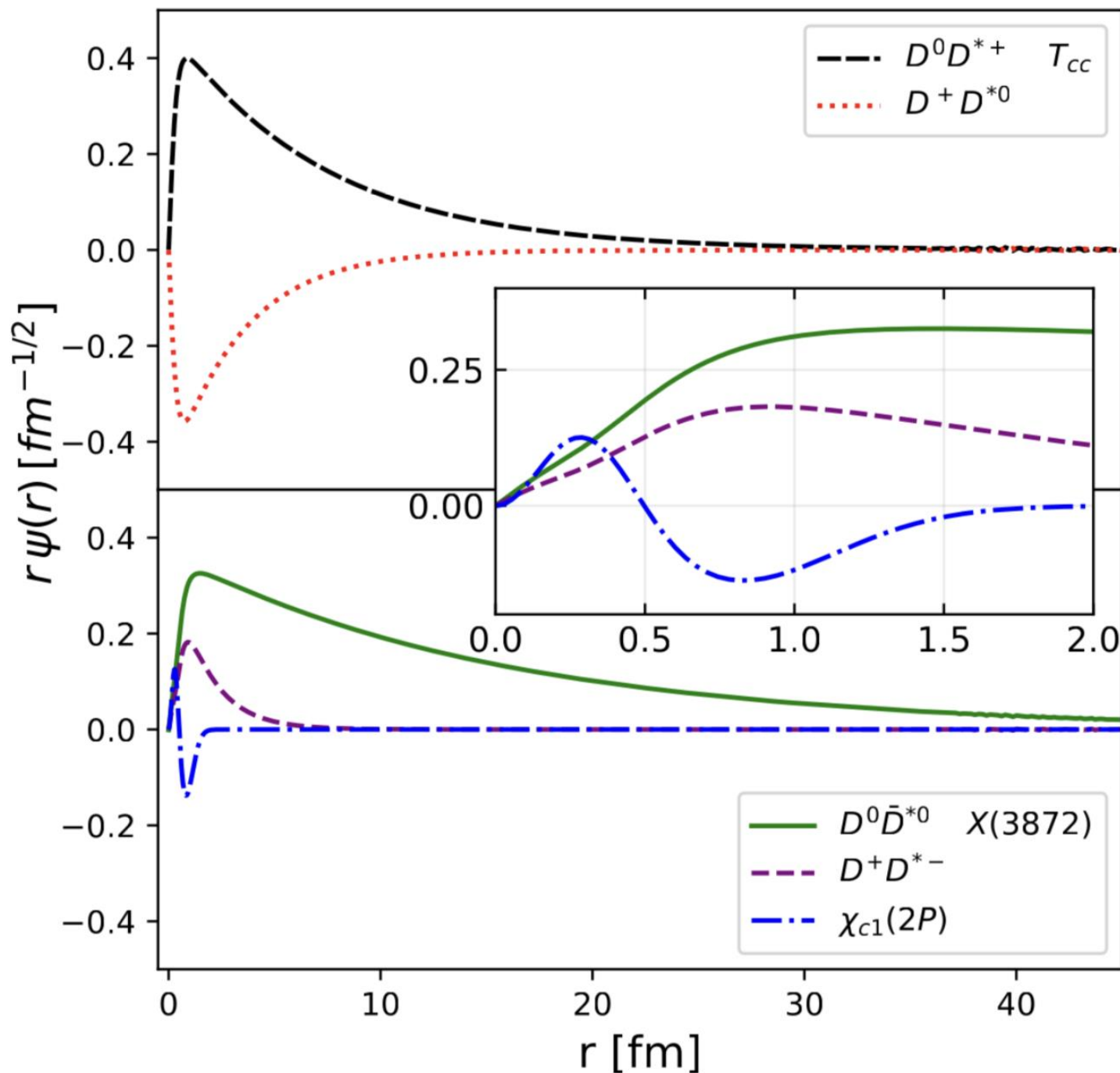
- The the X(3872) can be obtained:

| $X(3872)$ | BE (keV) | $\Gamma$ (keV) | $\sqrt{\langle r^2 \rangle}$ | $I = 0$ | $I = 1$ | $P(D^0\bar{D}^{*0})$ | $P(D^+D^{*-})$ | $P(c\bar{c})$ |
|-----------|----------|----------------|------------------------------|---------|---------|----------------------|----------------|---------------|
|           | -80.4    | 32.5           | 11.2 fm                      | 71.9%   | 28.1%   | 94.0%                | 4.8%           | 1.2%          |

# Direct application to $D\bar{D}^* : X(3872)$



Wave functions of  $T_{cc}$  and  $X(3872)$



- Long tails for the radius distribution.
- $X(3872)$  has a even longer tails than  $T_{cc}$
- ✓  $r < 2$  fm,  $c\bar{c} + \bar{D}D^*$  are important.
- ✓  $r < 0.5$  fm,  $c\bar{c}$  core dominates.
- ✓  $D\bar{D}^*$  plays the dominant role in the long-distance region, which contributes to  $\sqrt{\langle r^2 \rangle}$ .

Ours:  $\chi_{c1}(2P) \rightarrow M = 3957.9\text{MeV}$

Haozheng Li et al, arXiv: 2402.14541

|                        |         |         |         |         |
|------------------------|---------|---------|---------|---------|
| $m_\pi(\text{MeV})$    | 250(3)  | 307(2)  | 362(1)  | 417(1)  |
| $m_R(\text{MeV})$      | 3924(5) | 3926(6) | 3969(4) | 3995(4) |
| $\Gamma_R(\text{MeV})$ | 63(23)  | 57(18)  | 37(13)  | 57(10)  |

$X \approx 1$  and indicates a predominant  $D\bar{D}^*$  component. This state may correspond to  $X(3872)$ . On the other hand, our results of the finite volume energies also hint at the existence of a  $1^{++}$  resonance below 4.0 GeV with a width around 60 MeV.

# Compare with the experimental results



Ours: virtual state with  $1^{+-}$  and  $M = 3870.2 \text{ MeV}$

COMPASS:  $\tilde{X}(3872)$  with  $M = 3860.0 \pm 10.4 \text{ MeV}$  COMPASS, PLB783,334

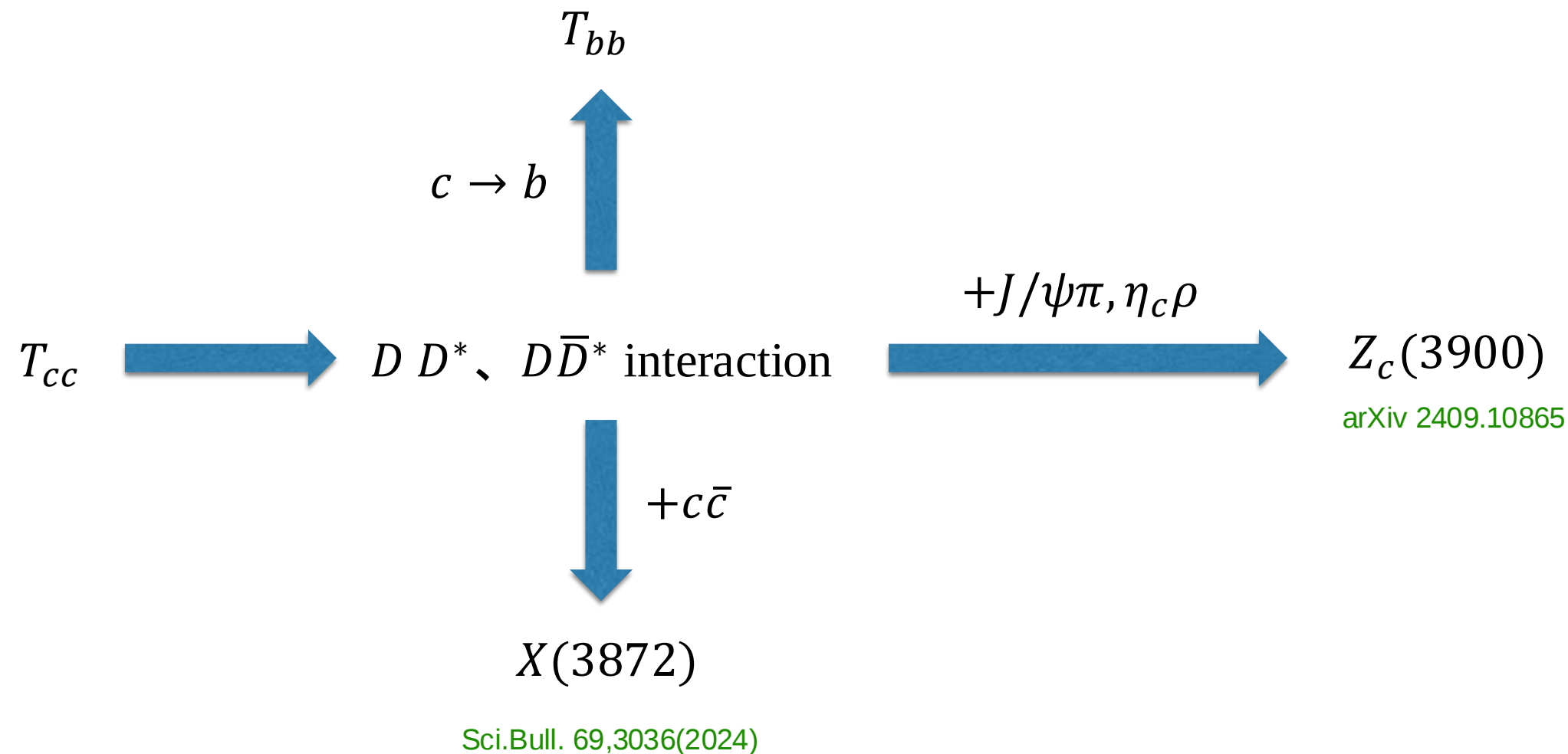
Ours:  $h_c(2P) \rightarrow M = 3961.3\text{MeV}$

$\chi_{c1}(2P) \rightarrow M = 3957.9\text{MeV}$

LHCb, arXiv:2406.03156

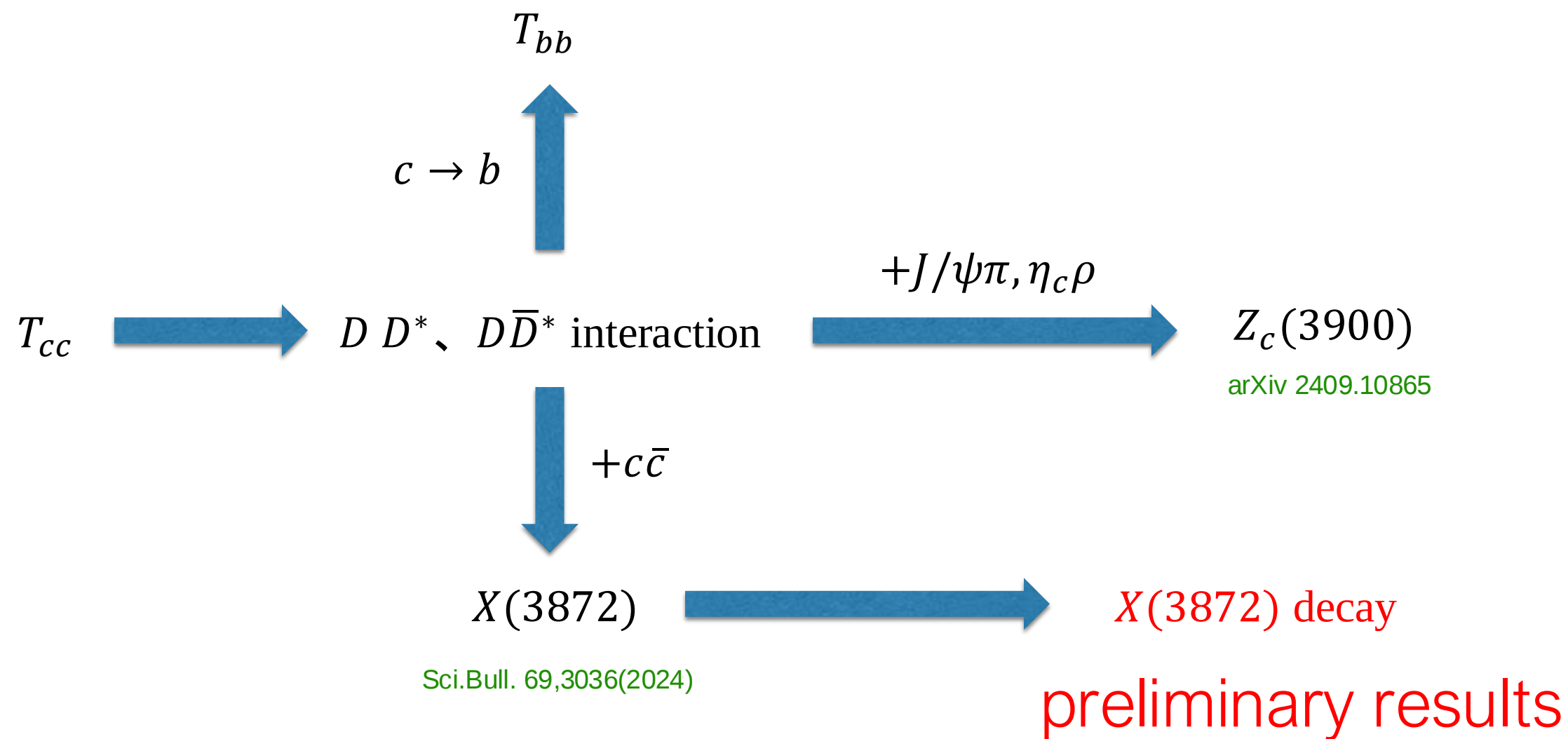
| This work                                     |                                                  | Known states [6]                     |                                   | $c\bar{c}$ prediction [34]        |
|-----------------------------------------------|--------------------------------------------------|--------------------------------------|-----------------------------------|-----------------------------------|
| $\eta_c(3945)$                                | $J^{PC} = 0^{-+}$                                | $X(3940)$ [9][10]                    | $J^{PC} = ?^{??}$                 | $\eta_c(3S)$ $J^{PC} = 0^{-+}$    |
| $m_0 = 3945^{+28}_{-17} {}^{+37}_{-28}$       | $\Gamma_0 = 130^{+92}_{-49} {}^{+101}_{-70}$     | $m_0 = 3942 \pm 9$                   | $\Gamma_0 = 37^{+27}_{-17}$       | $m_0 = 4064$ $\Gamma_0 = 80$      |
| $h_c(4000)$                                   | $J^{PC} = 1^{+-}$                                | $T_{c\bar{c}}(4020)^0$ [35]          | $J^{PC} = ?^{?-}$                 | $h_c(2P)$ $J^{PC} = 1^{+-}$       |
| $m_0 = 4000^{+17}_{-14} {}^{+29}_{-22}$       | $\Gamma_0 = 184^{+71}_{-45} {}^{+97}_{-61}$      | $m_0 = 4025.5^{+2.0}_{-4.7} \pm 3.1$ | $\Gamma_0 = 23.0 \pm 6.0 \pm 1.0$ | $m_0 = 3956$ $\Gamma_0 = 87$      |
| $\chi_{c1}(4010)$                             | $J^{PC} = 1^{++}$                                |                                      |                                   | $\chi_{c1}(2P)$ $J^{PC} = 1^{++}$ |
| $m_0 = 4012.5^{+3.6}_{-3.9} {}^{+4.1}_{-3.7}$ | $\Gamma_0 = 62.7^{+7.0}_{-6.4} {}^{+6.4}_{-6.6}$ |                                      |                                   | $m_0 = 3953$ $\Gamma_0 = 165$     |

PRD110, 074007(2024)



- $T_{cc}$  is used to fix the  $D \bar{D}^*$  interactions in  $X(3872)$ .
- Short-range interactions and structures of  $X(3872)$  should be studied by considering the  $c\bar{c}$  core.

PRD110, 074007(2024)



# Preliminary results of $X(3872)$ decay



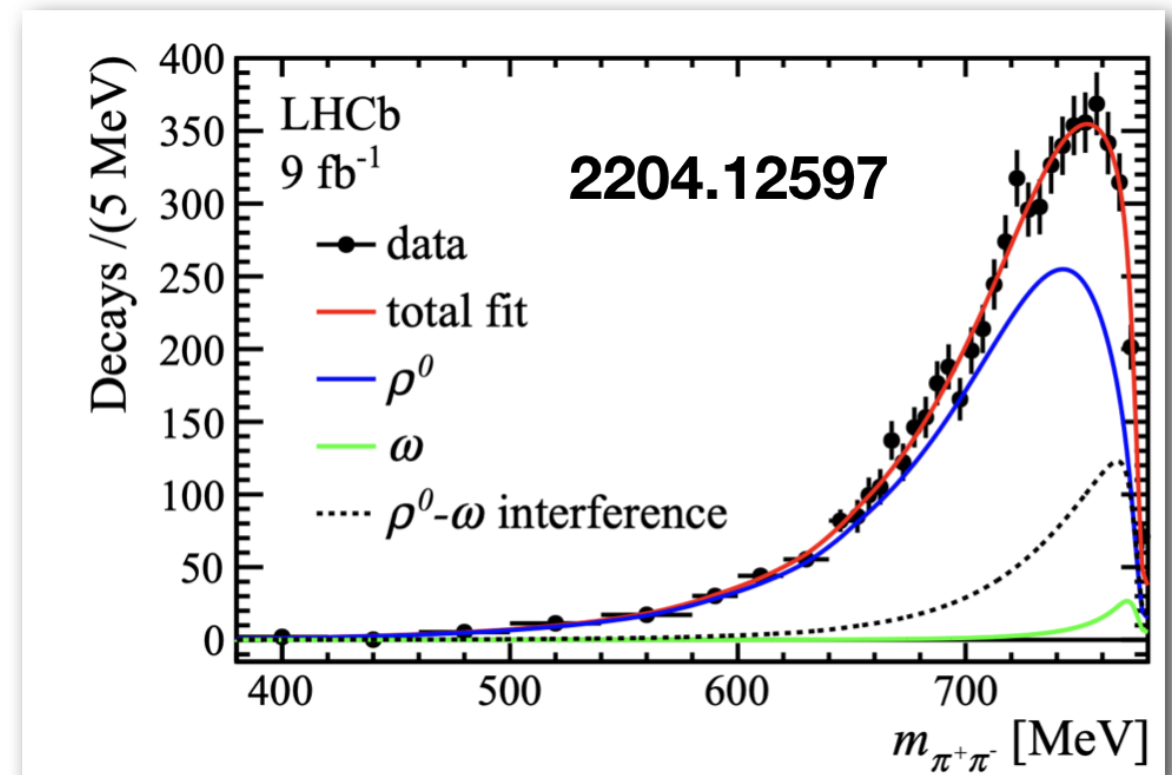
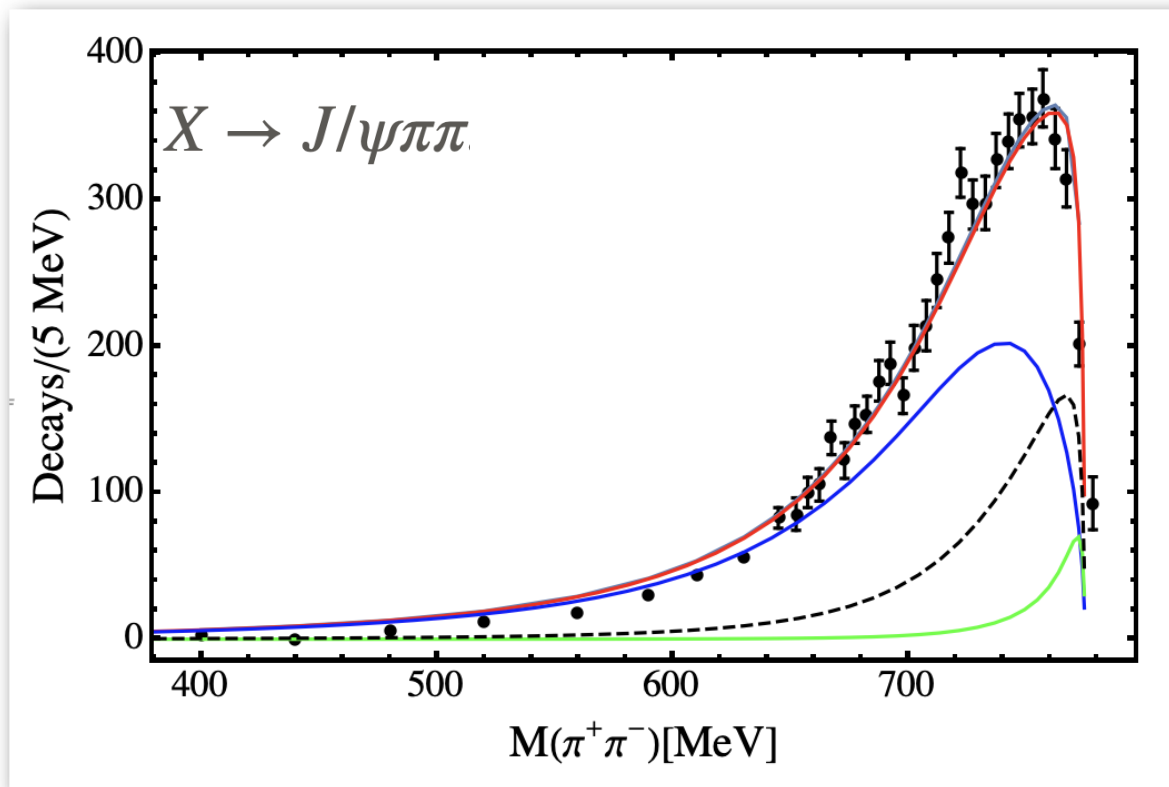
The Quark exchanging model

| $\Gamma_i/\Gamma_{\text{total}}$                            | PDG                            | Our result<br>with $\Gamma_{D^*} = 55.9\text{keV}$ | C.-H. Li, C.-Z. Yuan<br>[PRD100 094003] |
|-------------------------------------------------------------|--------------------------------|----------------------------------------------------|-----------------------------------------|
| $\Gamma(\chi_{c1}(3872) \rightarrow \pi^+\pi^-J/\psi(1S))$  | $0.035 \pm 0.009$              | 0.03                                               | $(4.1^{+1.9}_{-1.1})\%$                 |
| $\Gamma(\chi_{c1}(3872) \rightarrow \rho(770)^0J/\psi(1S))$ | $(2.8 \pm 0.7)\%$              | 3.7%                                               | -                                       |
| $\Gamma(\chi_{c1}(3872) \rightarrow \omega J/\psi(1S))$     | $(4.1 \pm 1.4)\%$              | 12.4%                                              | $(4.4^{+2.3}_{-1.3})\%$                 |
| $\Gamma(\chi_{c1}(3872) \rightarrow \pi\pi\pi J/\psi(1S))$  | not seen                       | 6.4%                                               | -                                       |
| $\Gamma(\chi_{c1}(3872) \rightarrow D^0\bar{D}^0\pi^0)$     | $(45 \pm 21)\%$                | 10.3%                                              | -                                       |
| $\Gamma(\chi_{c1}(3872) \rightarrow \bar{D}^{*0}D^0)$       | $(34 \pm 12)\%$                | 52.6%                                              | $(52.4^{+25.3}_{-14.3})\%$              |
| $\Gamma(\chi_{c1}(3872) \rightarrow \pi^0\chi_{c2})$        | $< 4\%$                        | 1.6%                                               | -                                       |
| $\Gamma(\chi_{c1}(3872) \rightarrow \pi^0\chi_{c1})$        | $(3.1^{+1.5}_{-1.3})\%$        | 2.0%                                               | $(3.6^{+2.2}_{-1.6})\%$                 |
| $\Gamma(\chi_{c1}(3872) \rightarrow \pi^0\chi_{c0})$        | $< 13\%$                       | 1.3%                                               | -                                       |
| $\Gamma(\chi_{c1}(3872) \rightarrow \gamma D^+D^-)$         | $< 3.5\%$                      | 0.1%                                               | -                                       |
| $\Gamma(\chi_{c1}(3872) \rightarrow \gamma\bar{D}^0D^0)$    | $< 6\%$                        | 5.8%                                               | -                                       |
| $\Gamma(\chi_{c1}(3872) \rightarrow \gamma J/\psi)$ [VMD]   | $(7.8 \pm 2.9) \times 10^{-3}$ | $0.9 \times 10^{-3}$                               | $(1.1^{+0.6}_{-0.3})\%$                 |
| $\Gamma(\chi_{c1}(3872) \rightarrow \gamma\psi(2S))$        | possibly seen                  | $(7.0) \times 10^{-3}$                             | $(2.4^{+1.3}_{-0.8})\%$                 |

# Preliminary results of $X(3872)$ decay



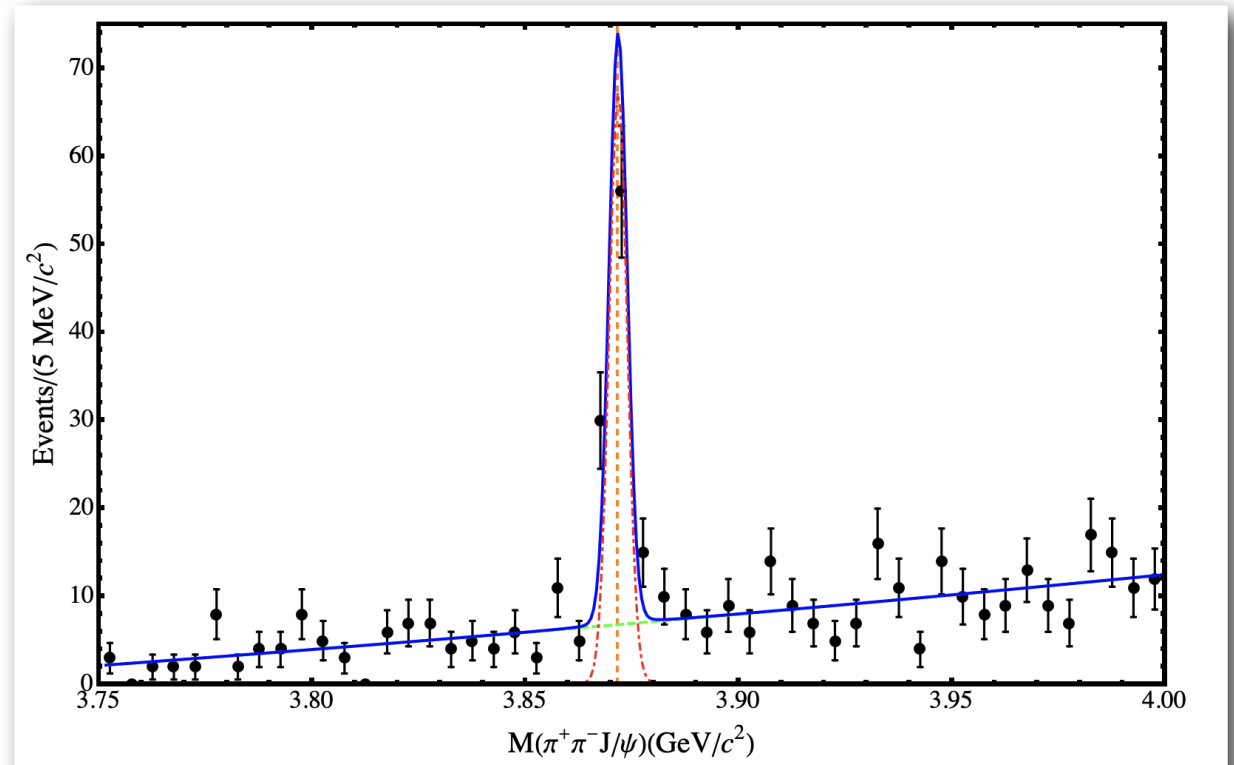
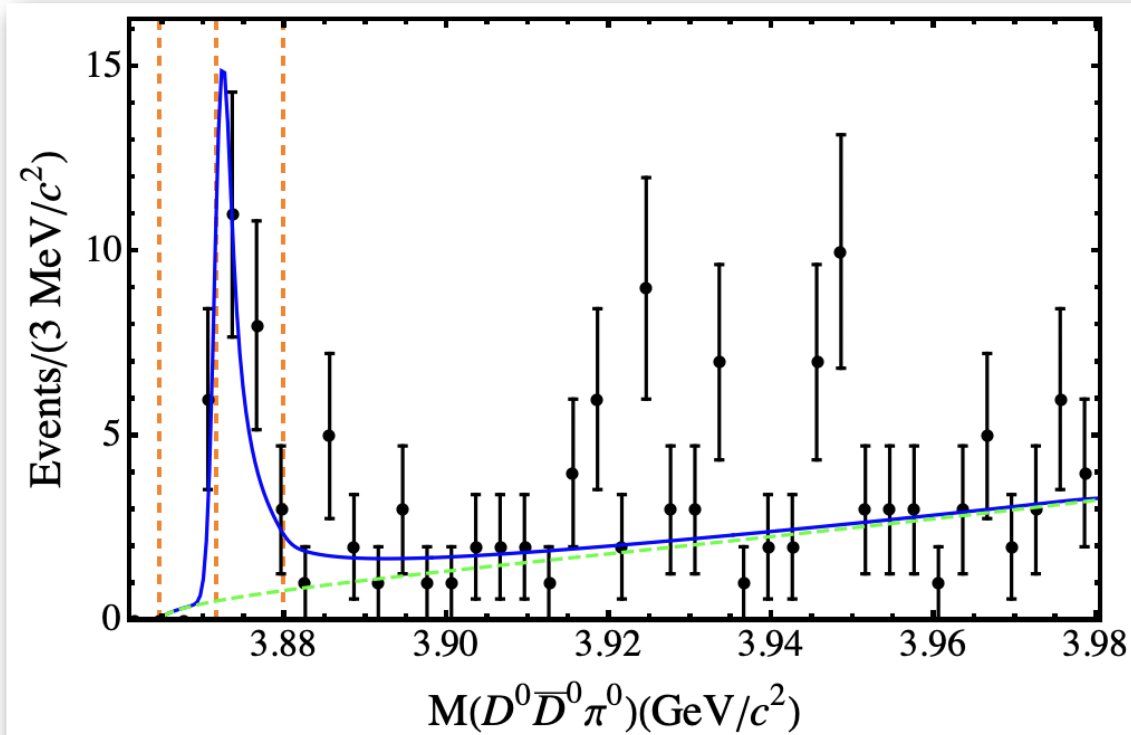
We obtain the distribution of the  $\pi\pi$  **without any fitting** except a normalization factor !



# Preliminary results of $X(3872)$ decay

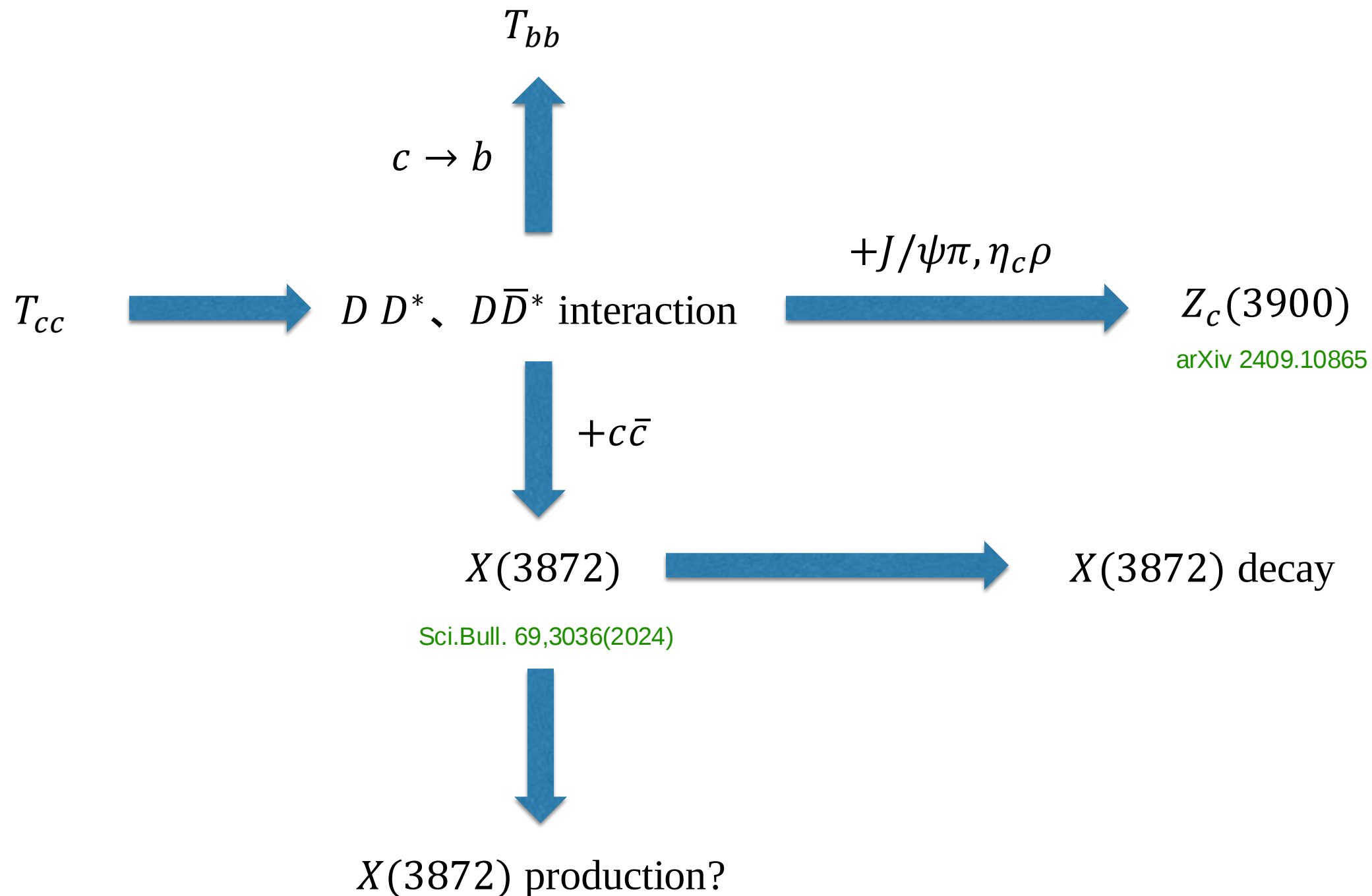


Without any fitting except a normalization factor!



BESIII, Phys. Rev. Lett. 124, 242001 (2020)

PRD110, 074007(2024)



- What important role the  $c\bar{c}$  core can play in the production of the  $X(3872)$ ?

The probability of the  $c\bar{c}$  component in the  $X(3872)$  can be obtained from fitting its production:

$$d\sigma(pp \rightarrow X(J/\psi\pi^+\pi^-)) = d\sigma(pp \rightarrow \chi'_{c1}) \cdot k, \quad k = Z_{c\bar{c}} \cdot Br_0$$

$$\longrightarrow Z_{c\bar{c}} = (28-44)\%$$

C. Meng, H. Han, K.T. Chao, Phys. Rev. D 96, 074014

Thank you !

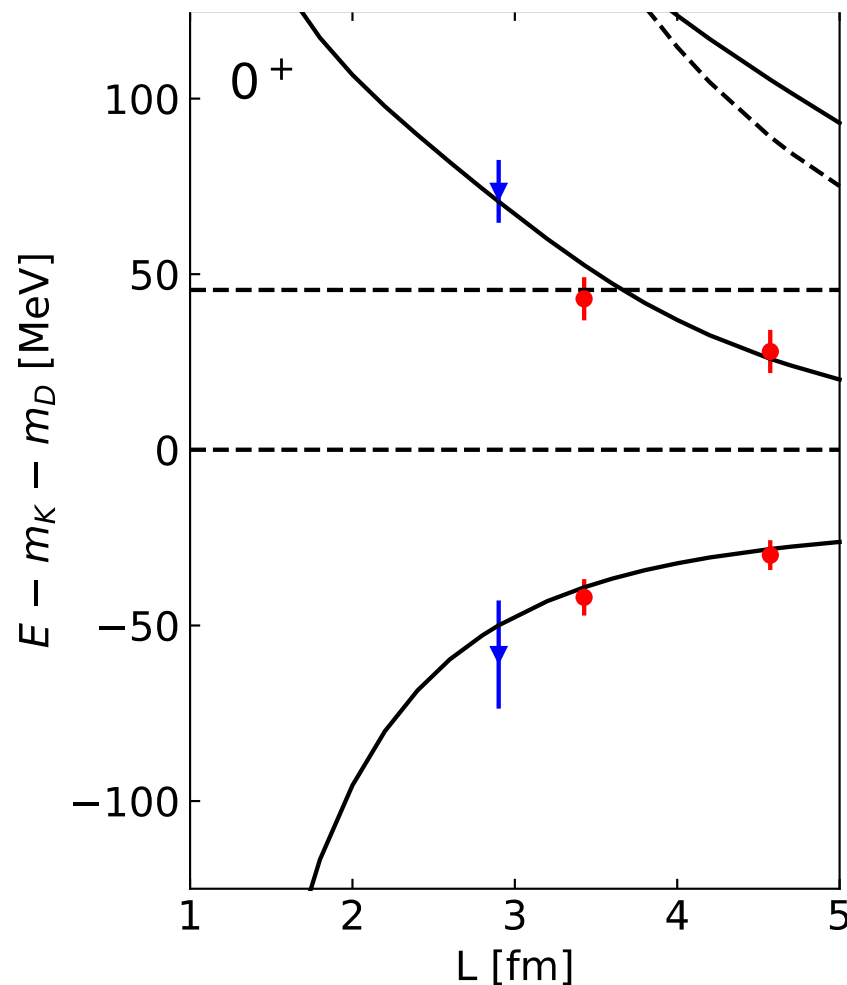
- With fixed  $\Lambda = 1.0$  GeV,  $\chi^2/\text{dof} = 0.95$

$$g_c = 4.2_{-3.1}^{+2.2}, \Lambda' = 0.323_{-0.031}^{+0.033} \text{ GeV}$$

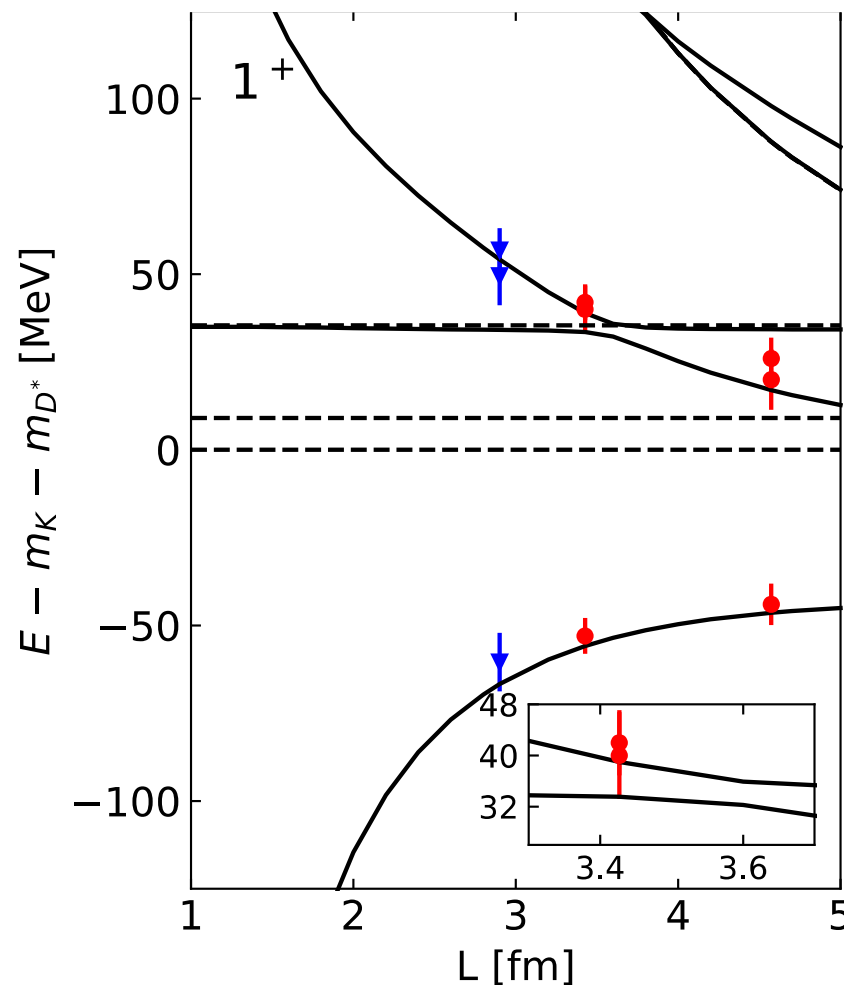
$$\gamma = 10.3_{-1.0}^{+1.1}$$

Lattice data from: C. B. Lang et al., [Phys. Rev. D 90, 034510 \(2014\)](#);  
G. S. Bali et al., [Phys. Rev. D 96, 074501 \(2017\)](#)

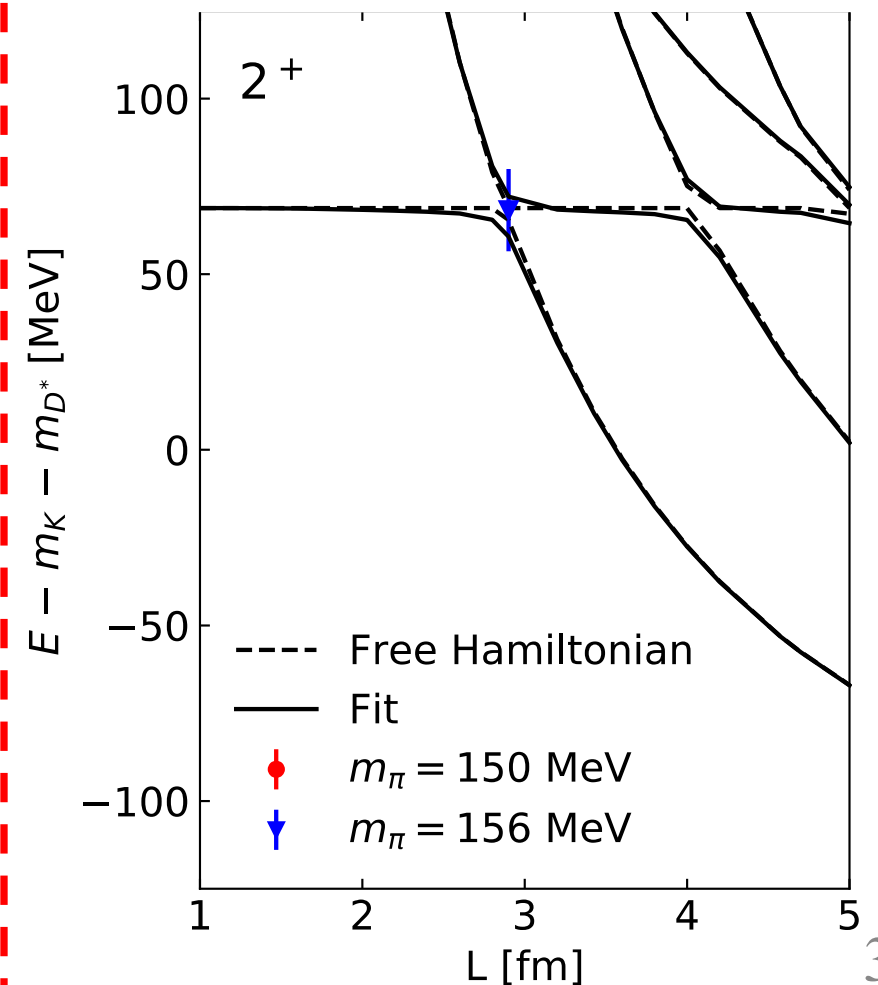
Fit



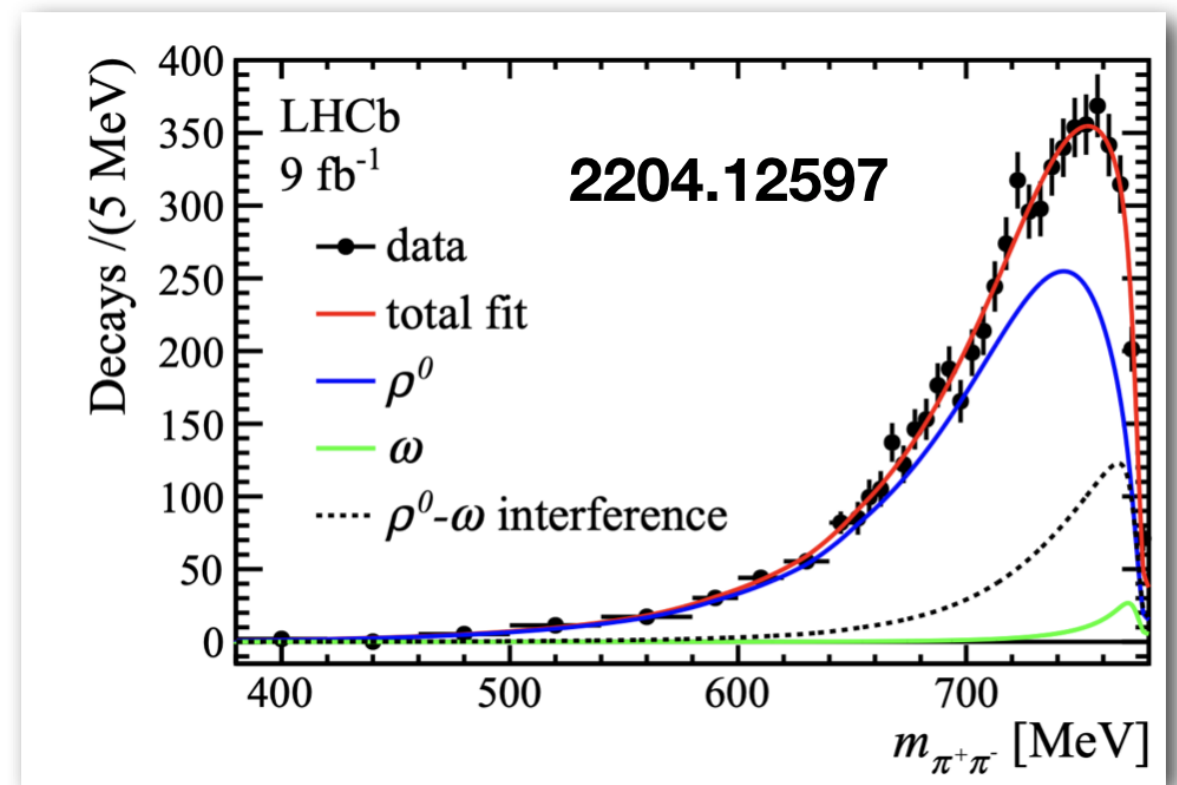
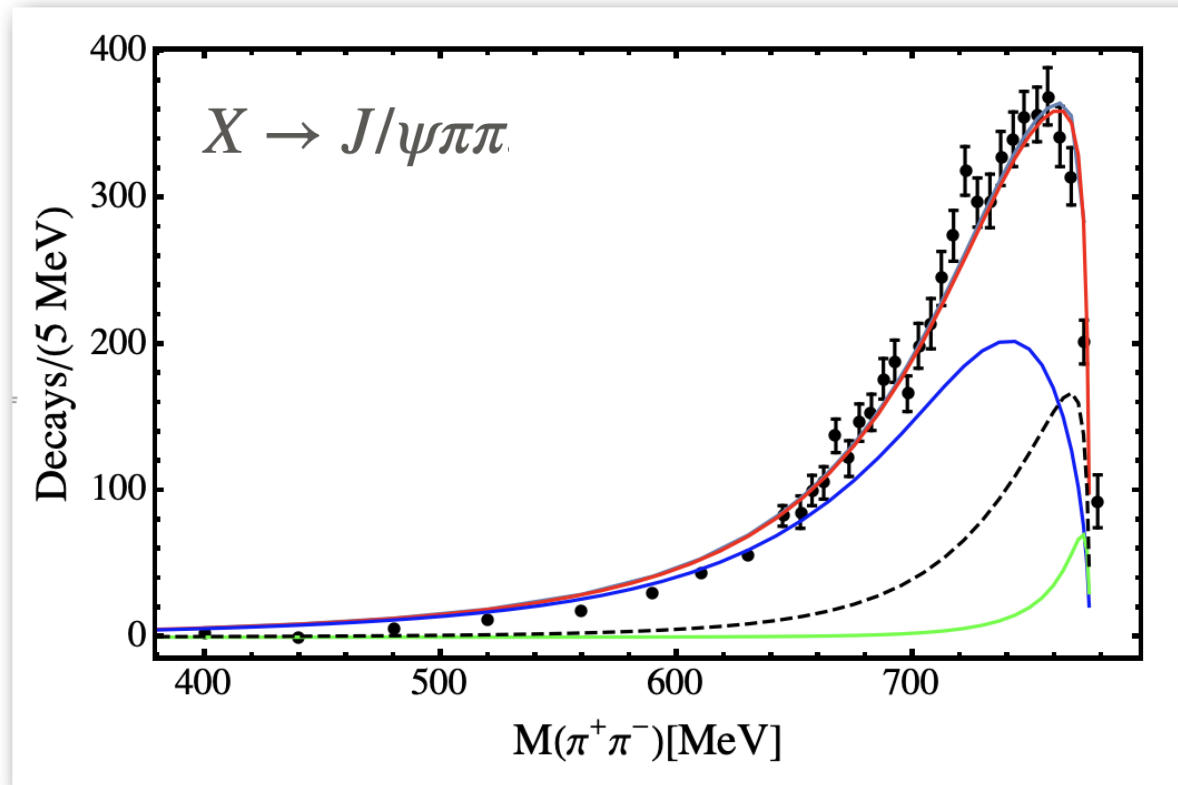
$E - m_K - m_{D^*}$  [MeV]



$E - m_K - m_{D^*}$  [MeV]



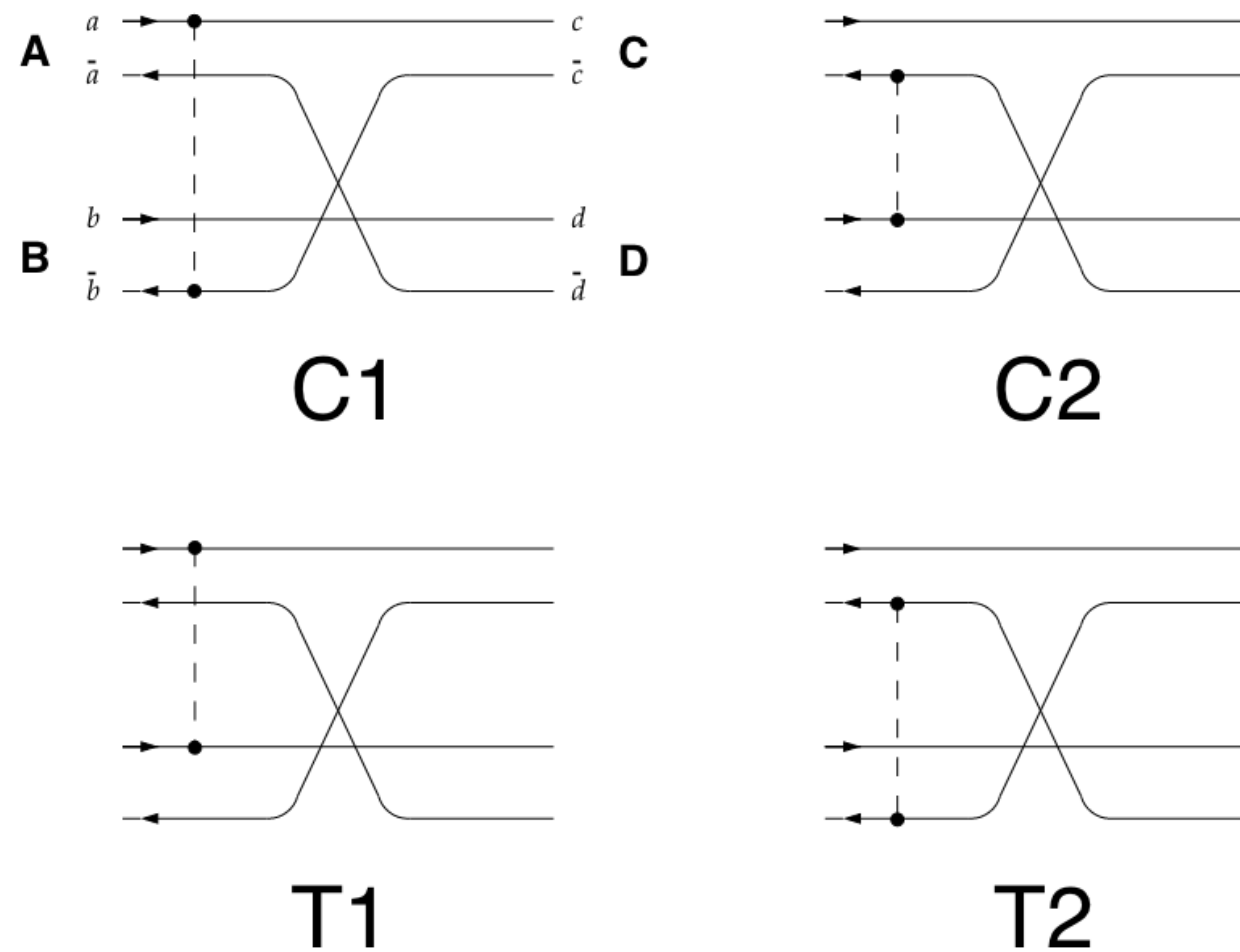
Postpredict



$$R_X \sim 0.21$$

$$A_{\omega \rightarrow 2\pi} = \varepsilon G_\rho A_{\rho \rightarrow 2\pi}$$

$$\varepsilon \approx \sqrt{m_\omega m_\rho \Gamma_\rho \Gamma_{\omega \rightarrow 2\pi}} \approx 3.4 \times 10^{-3} \text{ GeV}^2$$



C.-Y. Wong, E.S. Swanson, Ted Barnes, PhysRevC.66.029901;  
 Z.-Y. Zhou, M.-T. Yu, Z.-G. Xiao, PhysRevD.100.094025;  
 G.-J. Wang, X.-H. Liu et al, Eur.Phys.J. C79 (2019) no.7, 567