East Asian Workshop on Exotic Hadrons 2024

Study of 1^{-+} light, Charmonium-like and Fully-charm Tetraquark Spectroscopy

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Outline

① Introduction

② Construction of tetraquark wave functions

③ Theoretical model establishment

④ Mass spectrum

⑤ Summary

- 3 states observed in experiments with the exotic quantum number $I^G J^{PC} = 1^- 1^{-+}$,

① $\pi(1400)$ (PLB 205 (1988) 397)

② $\pi(1600)$ (PRL 81 (1998) 5760-5763),

③ $\pi(2015)$ (PLB 595 (2004) 109-117)

- candidates of hybrid mesons (*Rept.Prog.Phys.* 86 (2023) 2, 026201)

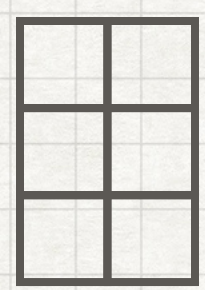
- $\eta_1(1855)$ observed by BESIII with the exotic quantum number $I^G J^{PC} = 0^+ 1^{-+}$ in $\eta\eta'$ decay process (PRL129 (19) (2022) 192002, PRD 106 (7) (2022) 072012)
- candidate of hybrid mesons (*Rept.Prog.Phys.* 86 (2023) 2, 026201)

- Tetraquark are states of two quarks and two antiquarks. (eg. $qc\bar{q}\bar{c}$, $qq\bar{q}\bar{q}$, $cc\bar{c}\bar{c}$)

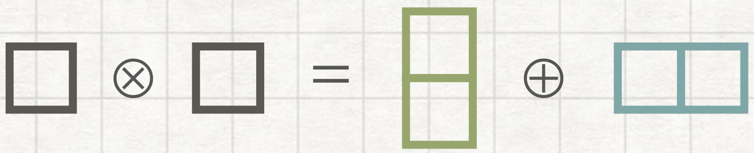
- The construction of tetraquark wave function is guided by:
- The tetraquark wave function should be a color singlet. (for $qc\bar{q}\bar{c}$, $qq\bar{q}\bar{q}$, $cc\bar{c}\bar{c}$)
- The tetraquark wave function should be antisymmetric under any permutation between identical quarks. (for $qq\bar{q}\bar{q}$, $cc\bar{c}\bar{c}$)

- It demands that the color part of tetraquark wave function must be [222] singlet.

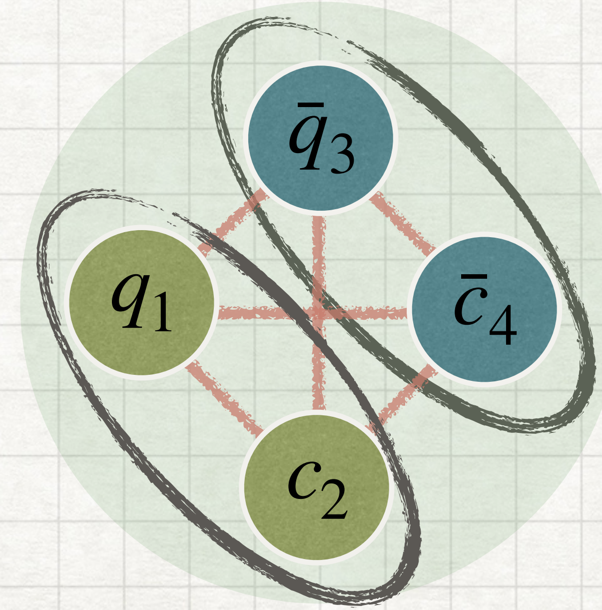
$$\psi^{c}_{[222]} =$$



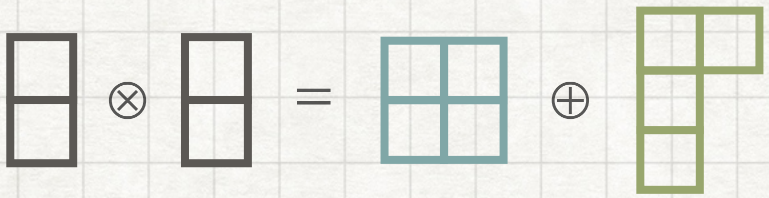
- The color part for two quarks in tetraquark states is



$$3 \otimes 3 = \bar{3} \oplus 6 \quad (1)$$

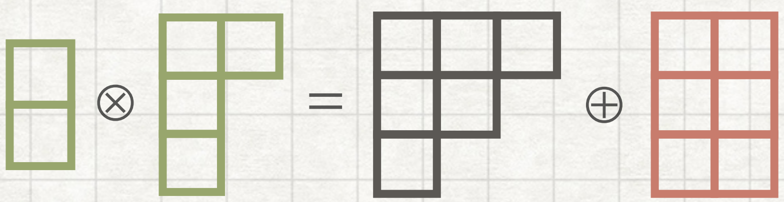


- The color part for two anti-quarks in tetraquark states is

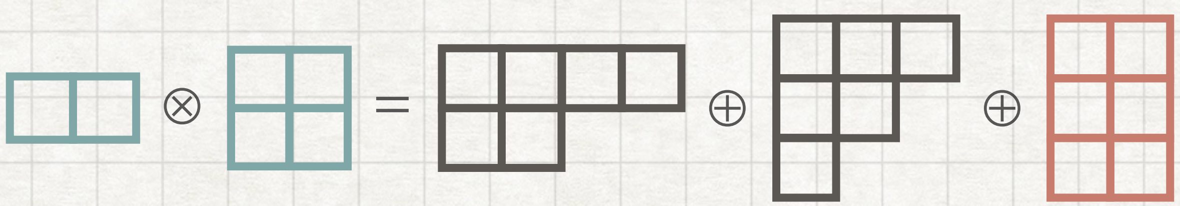


$$3 \otimes 3 = \bar{6} \oplus 3 \quad (2)$$

- The color wave function should be a color singlet.



$$\bar{3} \otimes 3 = 8 \oplus 1 \quad (3)$$



$$6 \otimes \bar{6} = 27 \oplus 8 \oplus 1 \quad (4)$$

Color wave function

antitriplet-triplet state: $$\psi^{c}_{\bar{3}-3} = \frac{1}{3}[\frac{1}{2}(rg - gr)(\bar{r}\bar{g} - \bar{g}\bar{r}) + \frac{1}{2}(br - rb)(\bar{b}\bar{r} - \bar{r}\bar{b}) + \frac{1}{2}(gb - bg)(\bar{g}\bar{b} - \bar{b}\bar{g})] \quad (5)$$

sextet-antisextet state: $$\psi^{c}_{6-\bar{6}} = \frac{1}{\sqrt{6}}[rr\bar{r}\bar{r} + gg\bar{g}\bar{g} + bb\bar{b}\bar{b} + \frac{1}{2}(rg + gr)(\bar{r}\bar{g} + \bar{g}\bar{r}) + \frac{1}{2}(rb + br)(\bar{r}\bar{b} + \bar{b}\bar{r}) + \frac{1}{2}(gb + bg)(\bar{g}\bar{b} + \bar{b}\bar{g})] \quad (6)$$

- The total wave function for qq or cc can be represented as

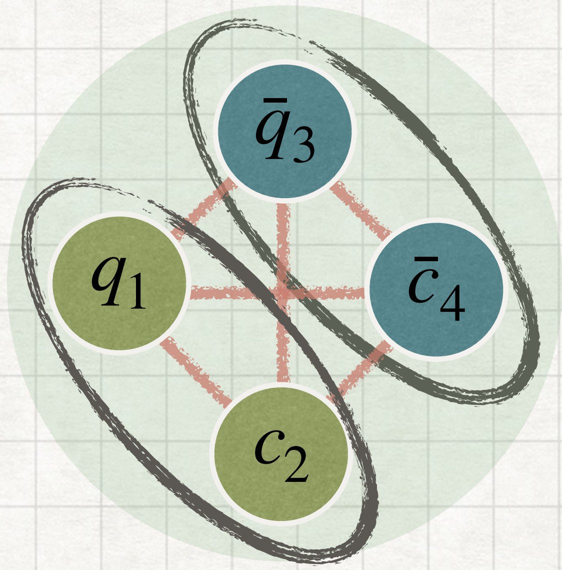
$$\psi_{total} = \psi_{spatial}\psi_{spin}\psi_{flavor}\psi_{color} \tag{7}$$

- The total wave function should be antisymmetric for qq or cc cluster.
- Listed in tables below are all the possible configurations of spatial-spin-flavor part of the qq and cc cluster.

$\psi^c_{[2]}(qq) = \Box\Box \;\Rightarrow$	$\psi^c_{[2]}\psi^{osf}_{[11]}$	$\psi^c_{[2]}\psi^o_{[2]}\psi^{sf}_{[11]}$	$\psi^c_{[2]}\psi^o_{[2]}\psi^s_{[11]}\psi^f_{[2]}$ $\psi^c_{[2]}\psi^o_{[2]}\psi^s_{[2]}\psi^f_{[11]}$
$\psi^c_{[11]}(qq) = \begin{array}{c}\Box\\\Box\end{array} \;\Rightarrow$	$\psi^c_{[11]}\psi^{osf}_{[2]}$	$\psi^c_{[11]}\psi^o_{[2]}\psi^{sf}_{[2]}$	$\psi^c_{[11]}\psi^o_{[2]}\psi^s_{[11]}\psi^f_{[11]}$ $\psi^c_{[11]}\psi^o_{[2]}\psi^s_{[2]}\psi^f_{[2]}$
$\psi^c_{[2]}(cc) = \Box\Box \;\Rightarrow$	$\psi^c_{[2]}\psi^{osf}_{[11]}$	$\psi^c_{[2]}\psi^o_{[2]}\psi^{sf}_{[11]}$	$\psi^c_{[2]}\psi^o_{[2]}\psi^s_{[11]}\psi^f_{[2]}$
$\psi^c_{[11]}(cc) = \begin{array}{c}\Box\\\Box\end{array} \;\Rightarrow$	$\psi^c_{[11]}\psi^{osf}_{[2]}$	$\psi^c_{[11]}\psi^o_{[2]}\psi^{sf}_{[2]}$	$\psi^c_{[11]}\psi^o_{[2]}\psi^s_{[2]}\psi^f_{[2]}$

(8)

Spin wave function



- For $qc\bar{q}\bar{c}$ and $qq\bar{q}\bar{q}$, the possible spin combinations are:

$$\left[\psi^{q_1q_2}_{[s=1]} \otimes \psi^{\bar{q}_3\bar{q}_4}_{[s=1]}\right]_{S=0,1,2}, \psi^{q_1q_2}_{[s=1]} \otimes \psi^{\bar{q}_3\bar{q}_4}_{[s=0]}, \text{ and } \psi^{q_1q_2}_{[s=0]} \otimes \psi^{\bar{q}_3\bar{q}_4}_{[s=0]} \tag{9}$$

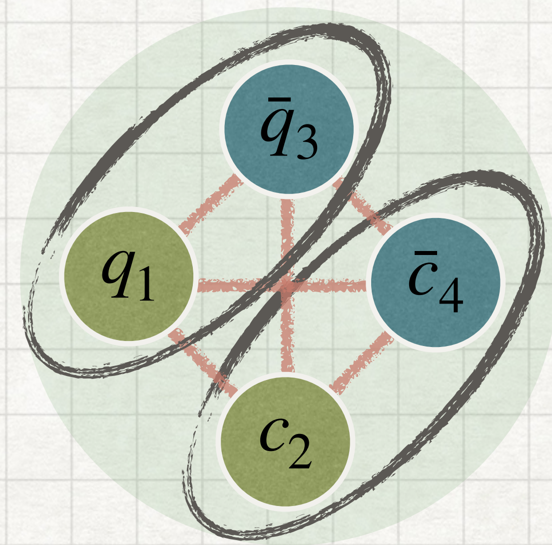
- For $cc\bar{c}\bar{c}$, the possible spin combination of color $(6 \otimes \bar{6})_c$ configuration is

$$\psi^{cc}_{[s=0]} \otimes \psi^{\bar{c}\bar{c}}_{[s=0]} \tag{10}$$

- For $cc\bar{c}\bar{c}$, the possible spin combinations of color $(\bar{3} \otimes 3)_c$ configuration is:

$$\left[\psi^{cc}_{[s=1]} \otimes \psi^{\bar{c}\bar{c}}_{[s=1]}\right]_{S=0,1,2} \tag{11}$$

Jacobi coordinate for tetraquark (PRC 75, 045206)



$$x_1 = \frac{1}{\sqrt{2}}(r_1 - r_3) \quad x_2 = \frac{1}{\sqrt{2}}(r_2 - r_4)$$

$$x_3 = \frac{m_1 r_1 + m_3 r_3}{m_1 + m_3} - \frac{m_2 r_2 + m_4 r_4}{m_2 + m_4} \tag{12}$$

Reduced mass:

$$m_{x_1} = \frac{2 m_1 m_3}{m_1 + m_3} \quad m_{x_2} = \frac{2 m_2 m_4}{m_2 + m_4} \quad m_{x_3} = \frac{(m_1 + m_3)(m_2 + m_4)}{m_1 + m_2 + m_3 + m_4} \tag{13}$$

for $qc\bar{q}\bar{c}$: $\quad m_1 = m_3 = m_u \quad m_2 = m_4 = m_c$

$$x_1 = \frac{1}{\sqrt{2}}(r_1 - r_3) \quad x_2 = \frac{1}{\sqrt{2}}(r_2 - r_4) \quad x_3 = \frac{1}{2}(r_1 + r_3 - r_2 - r_4) \tag{14}$$

$$m_{x_1} = m_u \quad m_{x_2} = m_c \quad m_{x_3} = \frac{2 m_u m_c}{m_u + m_c} \tag{15}$$

[7]E. Santopinto and G. Galatà. Phys. Rev. C75, 045206(2007).

Spatial wave function

- We construct the complete bases by using the harmonic oscillator wave function.

- The total spatial wave function of tetraquark, coupling among the x_1, x_2 and x_3 harmonic oscillator wave functions, may take the general form,

$$\psi_{NL} = \sum_{\{n_i,l_i\}} A(n_1, n_2, n_3, l_1, l_2, l_3) \times \psi_{n_1 l_1}(x_1) \otimes \psi_{n_2 l_2}(x_2) \otimes \psi_{n_3 l_3}(x_3) \tag{16}$$

- For example, the complete bases of the P-wave tetraquark are listed as follows:

ψ_{11}	$\Psi_{01}(x_1)\Psi_{00}(x_2)\Psi_{00}(x_3)$	$\Psi_{00}(x_1)\Psi_{01}(x_2)\Psi_{00}(x_3)$	$\Psi_{00}(x_1)\Psi_{00}(x_2)\Psi_{01}(x_3)$
ψ_{31}	$\Psi_{11}(x_1)\Psi_{00}(x_2)\Psi_{00}(x_3)$	$\Psi_{10}(x_1)\Psi_{01}(x_2)\Psi_{00}(x_3)$	$\Psi_{10}(x_1)\Psi_{00}(x_2)\Psi_{01}(x_3)$
	$\Psi_{01}(x_1)\Psi_{10}(x_2)\Psi_{00}(x_3)$	$\Psi_{00}(x_1)\Psi_{11}(x_2)\Psi_{00}(x_3)$	$\Psi_{00}(x_1)\Psi_{10}(x_2)\Psi_{01}(x_3)$
	$\Psi_{01}(x_1)\Psi_{00}(x_2)\Psi_{10}(x_3)$	$\Psi_{00}(x_1)\Psi_{01}(x_2)\Psi_{10}(x_3)$	$\Psi_{00}(x_1)\Psi_{00}(x_2)\Psi_{11}(x_3)$

The non-relativistic Hamiltonian we study multiquark system reads:

$$H = \sum_{k=1}^{N}\left(\frac{1}{2}M_k^{ave} + \frac{p_k^2}{2m_k}\right) + \sum_{i<j}^{N}\left(-\frac{3}{16}\lambda_i^c \cdot \lambda_j^c\right)\left(V(r_{ij}) + H_{SS} + H_{LS}\right) \quad (17)$$

Here M_k^{ave} denotes the spin-averaged mass as $\frac{1}{4}M_{PS} + \frac{3}{4}M_V$, m_k are the constituent quark masses, and λ_i^c is the color operator in SU(3)

The Cornell potential and the spin dependent interaction:

$$V(r_{ij}) = A_{ij}r_{ij} - \frac{B_{ij}}{r_{ij}} \quad (18)$$

where A_{ij} is string tension coefficient, and B_{ij} is Coulomb coefficient.

$$H_{SS} = \frac{1}{6m_im_j}\Delta V_V(r)\vec{\sigma}_i \cdot \vec{\sigma}_j = \frac{2B_{ij}e^{-\sigma_{ij}{}^2 r_{ij}^2}\sigma_{ij}{}^3}{3m_im_j\sqrt{\pi}}\vec{\sigma}_i \cdot \vec{\sigma}_j \quad (19)$$

where m_i and m_j are the constituent quark masses of ith and 4th quark. $\vec{\sigma}_i$ is the spin operator in SU(2), and σ_{ij} is hyperfine interaction coefficient.

$$H_{LS} = \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_i m_j}\right)\frac{-B_{ij}\sigma_{ij}}{2\sqrt{\pi}}\frac{e^{-\sigma_{ij}{}^2 r_{ij}^2}}{r_{ij}^2}\overrightarrow{L}_{ij}\cdot(\vec{s}_i + \vec{s}_j) + \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_i m_j}\right)\frac{-B_{ij}}{4}\frac{Erf[\sigma_{ij} r_{ij}]}{r_{ij}^3}\overrightarrow{L}_{ij}\cdot(\vec{s}_i + \vec{s}_j)$$
$$+\frac{-A_{ij}}{2}\left(\frac{1}{r_{ij}}\frac{\overrightarrow{L}_{ij}\cdot\vec{s}_i}{m_i^2} + \frac{1}{r_{ij}}\frac{\overrightarrow{L}_{ij}\cdot\vec{s}_j}{m_j^2}\right) \quad (20)$$

where the relative orbital operator $\overrightarrow{\boldsymbol{L}}_{ij}$ is defined as $\overrightarrow{\boldsymbol{L}}_{ij} = \vec{\boldsymbol{r}}_{ij} \times \overrightarrow{\boldsymbol{p}}_{ij} = \vec{\boldsymbol{r}}_{ij} \times \frac{m_i \overrightarrow{\boldsymbol{p}}_i - m_j \overrightarrow{\boldsymbol{p}}_j}{m_i + m_j}$. $\vec{\boldsymbol{s}}_i$ is the spin operator.

- 3 coefficients are proposed to be mass-dependent:

$$A_{ij} = a + b m_{ij} \qquad B_{ij} = B_0\sqrt{\frac{1}{m_{ij}}} \qquad \sigma_{ij} = \sigma_0 m_{ij}, \quad (21)$$

where m_{ij} is the reduced mass of ith and jth quarks, $m_{ij} = 2\frac{m_i m_j}{m_i + m_j}$. a, b, B_0, and σ_0 are constants.

- 3 coupling constants and 4 quark masses are imported (PRD 103, 116027), and σ_0 is fixed:

$$a = 67413(\mathrm{MeV}^2) \qquad b = 35(\mathrm{MeV}) \qquad B_0 = 31.7(\mathrm{MeV}^{1/2}) \qquad \sigma_0 = 0.85$$
$$m_u = m_d = 420\mathrm{MeV} \qquad m_s = 550\mathrm{MeV} \qquad m_c = 1270\mathrm{MeV} \qquad m_b = 4180\mathrm{MeV} \quad (22)$$

$$H_{LS} = \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_i m_j}\right) \frac{-B_{ij}\sigma_{ij}}{2\sqrt{\pi}} \frac{e^{-\sigma_{ij}{}^2 r_{ij}^2}}{r_{ij}^2} \overrightarrow{L}_{ij} \cdot (\vec{s}_i + \vec{s}_j) + \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_i m_j}\right) \frac{-B_{ij}}{4} \frac{Erf[\sigma_{ij} r_{ij}]}{r_{ij}^3} \overrightarrow{L}_{ij} \cdot (\vec{s}_i + \vec{s}_j)$$

$$+ \frac{-A_{ij}}{2} \left(\frac{1}{r_{ij}} \frac{\overrightarrow{L}_{ij} \cdot \vec{s}_i}{m_i^2} + \frac{1}{r_{ij}} \frac{\overrightarrow{L}_{ij} \cdot \vec{s}_j}{m_j^2}\right) \quad (20)$$

where the relative orbital operator $\overrightarrow{L}_{ij}$ is defined as $\overrightarrow{L}_{ij} = \vec{r}_{ij} \times \overrightarrow{p}_{ij} = \vec{r}_{ij} \times \frac{m_i \overrightarrow{p}_i - m_j \overrightarrow{p}_j}{m_i + m_j}$. $\vec{s}_i$ is the spin operator.

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Meson states applied to fit the model parameters.

S-wave (MeV)	0^{-+}				1^{--}					
	n=0 (1S)		n=1 (2S)		n=0 (1S)		n=1 (2S)		n=2 (3S)	
	Ours	PDG	Ours	PDG	Ours	PDG	Ours	PDG	Ours	PDG
$b\bar{b}$	9383	9399	10019	9999	9468	9460	10055	10023	–	–
$c\bar{c}$	2927	2984	3601	3638	3097	3097	3658	3686	4030	4040
$B_s(s\bar{b})$	5349	5367	–	–	5422	5415	–	–	–	–
$B(u\bar{b})$	5276	5279	–	–	5341	5325	–	–	–	–
$D_s(c\bar{s})$	1953	1968	–	–	2125	2112	2707	2708	–	–
$D(c\bar{u})$	1870	1865	2578	2549	2030	2007	2626	2627	–	–
$s\bar{s}$	–	–	–	–	1034	1020	1632	1680	–	–
$q\bar{q}$	–	–	–	–	782	770	1401	1450	–	–

P-Wave (MeV)	1^{+-}		0^{++}		1^{++}		2^{++}	
	n=0 (1P)		n=0 (1P)		n=0 (1P)		n=0 (1P)	
	Ours	PDG	Ours	PDG	Ours	PDG	Ours	PDG
$c\bar{c}$	3518	3525	3446	3415	3493	3510	3556	3556
$D(c\bar{u})$	2465	2420	2404	2343	2451	2412	2517	2460
$q\bar{q}$	1198	h1(1170)/b1(1235)	1137	1200-1500	1200	a1(1260)/f1(1285)	1294	f2(1270)/a2(1320)

Tetraquark Parity (PRC 75, 045206)

- The total angular momentum $|L-S| \leq J \leq |L+S|$
- The parity for tetraquark $P=(-1)^{L}$
- The C-parity for tetraquark $C=(-1)^{L+S}$
- L is the total orbital angular momentum S is the total spin angular momentum

	J	L	S	P	C	J^{PC}
S-wave	0	0	0	+	+	0^{++}
	1	0	1	+	−	1^{+-}
	2	0	2	+	+	2^{++}
P-wave	1	1	0	−	−	1^{--}
	0	1	1	−	+	0^{-+}
	1	1	1	−	+	1^{-+}
	2	1	1	+	−	2^{+-}
	1	1	2	−	−	1^{--}
	2	1	2	−	−	2^{--}
	3	1	2	−	−	3^{--}

- PRD 103, 116027
- PRD 105, 036001

1^{-+} Light Tetraquark States

- $|L-S| \leq J \leq |L+S|,\ P=(-1)^L,\ C=(-1)^{L+S}$ (PRC 75, 045206)

J	L	S	P	C	J^{PC}
1	1	1	–	+	1^{-+}

1^{-+}	E1	E2	E3
$\vert\psi^c_{6\otimes\bar{6}}\psi^{S=1}_{(1\otimes1)}\rangle$	**1976**	**2029**	**2187**
$\vert\psi^c_{6\otimes\bar{6}}\psi^{S=1}_{(1\otimes0)}\rangle$	**2059**	**2113**	**2257**
$\vert\psi^c_{\bar{3}\otimes3}\psi^{S=1}_{(1\otimes0)}\rangle$	**2122**	**2187**	**2224**
$\vert\psi^c_{\bar{3}\otimes3}\psi^{S=1}_{(1\otimes1)}\rangle$	**2159**	**2219**	**2221**

1^{-+} Charmoniumlike Tetraquark States

- Eigenstates for S=1 are linear combinations of $\psi^{c}_{\bar{3}\otimes 3}\psi^{S=1}_{(1\otimes 0)}$, $\psi^{c}_{\bar{3}\otimes 3}\psi^{S=1}_{(1\otimes 1)}$, $\psi^{c}_{6\otimes \bar{6}}\psi^{S=1}_{(1\otimes 0)}$, and $\psi^{c}_{6\otimes \bar{6}}\psi^{S=1}_{(1\otimes 1)}$.
- Mixed state $|\psi^{c}_{\bar{3}\otimes 3}\psi^{S=1}_{(1\otimes 0)}, \psi^{c}_{\bar{3}\otimes 3}\psi^{S=1}_{(1\otimes 1)}, \psi^{c}_{6\otimes \bar{6}}\psi^{S=1}_{(1\otimes 0)}, \psi^{c}_{6\otimes \bar{6}}\psi^{S=1}_{(1\otimes 1)}\rangle$

1^{-+}	E1	E2	E3	E4	E5	E6
Mass	4301	4329	4366	4387	4391	4432

1^{-+} Fully-Charm Tetraquark states

1^{-+}	E1	E2	E3
$\lvert\psi^{c}_{\bar{3}\otimes 3}\psi^{S=1}_{(1\otimes 1)}\rangle$	6762	6814	6828

- The lowest 1^{-+} light tetraquark state is around 2 GeV, which is significantly far from the observed $\pi_1(1400)$, $\pi_1(1600)$ and $\eta_1(1855)$ state.

- The observed state $\pi_1(2015)$ is situated within the lowest 1^{-+} light tetraquark range. More experimental data and theoretical studies are essential for making unambiguous assignments.

- The lowest 1^{-+} charmoniumlike tetraquark state is around 4.3 GeV, might be observed in $\pi\chi_{c1}$, $\eta\chi_{c1}$, and γh_c decay processes.

- The lowest 1^{-+} fully-charm tetraquark state is around 6.8 GeV, might be observed in $J/\psi h_c$ decay process.

Thank you.

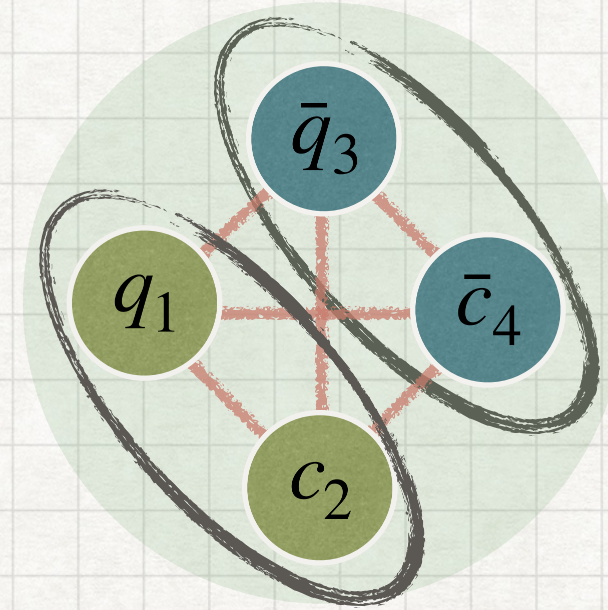
- For the charmoniumlike tetraquark, the hyperfine interaction H_{hyp} in Hamiltonian may mix up different color-spin configurations due to the cross terms,

$$H_{SS} = \frac{2B_{ij}e^{-\sigma_{ij}{}^2 r_{ij}^2}\sigma_{ij}{}^3}{3m_i m_j\sqrt{\pi}}\vec{\sigma}_i \cdot \vec{\sigma}_j$$

- Eigenstates for S=1 are linear combinations of $\psi^c_{\bar{3}\otimes3}\psi^{S=1}_{(1\otimes0)}$, $\psi^c_{\bar{3}\otimes3}\psi^{S=1}_{(1\otimes1)}$, $\psi^c_{6\otimes\bar{6}}\psi^{S=1}_{(1\otimes0)}$, and $\psi^c_{6\otimes\bar{6}}\psi^{S=1}_{(1\otimes1)}$.

$\vec{\lambda}_i\vec{\lambda}_j\vec{\sigma}_i\vec{\sigma}_j$	$\lvert\psi^c_{\bar{3}\otimes3}\psi^{S=1}_{(1\otimes0)}\rangle$	$\lvert\psi^c_{\bar{3}\otimes3}\psi^{S=1}_{(1\otimes1)}\rangle$	$\lvert\psi^c_{6\otimes\bar{6}}\psi^{S=1}_{(1\otimes0)}\rangle$	$\lvert\psi^c_{6\otimes\bar{6}}\psi^{S=1}_{(1\otimes1)}\rangle$
$\lvert\psi^c_{\bar{3}\otimes3}\psi^{S=1}_{(1\otimes0)}\rangle$	$\left(-\frac{8}{3},0,0,0,0,8\right)$	$\left(0,-\frac{4\sqrt{2}}{3},-\frac{4\sqrt{2}}{3},\frac{4\sqrt{2}}{3},\frac{4\sqrt{2}}{3},0\right)$	$(0,0,0,0,0,0)$	$(0,-4,4,-4,4,0)$
$\lvert\psi^c_{\bar{3}\otimes3}\psi^{S=1}_{(1\otimes1)}\rangle$	$\left(0,-\frac{4\sqrt{2}}{3},-\frac{4\sqrt{2}}{3},\frac{4\sqrt{2}}{3},\frac{4\sqrt{2}}{3},0\right)$	$\left(-\frac{8}{3},\frac{4}{3},\frac{4}{3},\frac{4}{3},\frac{4}{3},-\frac{8}{3}\right)$	$(0,-4,4,-4,4,0)$	$(0,2\sqrt{2},-2\sqrt{2},-2\sqrt{2},2\sqrt{2},0)$
$\lvert\psi^c_{6\otimes\bar{6}}\psi^{S=1}_{(1\otimes0)}\rangle$	$(0,0,0,0,0,0)$	$(0,-4,4,-4,4,0)$	$\left(-\frac{4}{3},0,0,0,0,-4\right)$	$\left(0,-\frac{10\sqrt{2}}{3},-\frac{10\sqrt{2}}{3},\frac{10\sqrt{2}}{3},\frac{10\sqrt{2}}{3},0\right)$
$\lvert\psi^c_{6\otimes\bar{6}}\psi^{S=1}_{(1\otimes1)}\rangle$	$(0,-4,4,-4,4,0)$	$(0,2\sqrt{2},-2\sqrt{2},-2\sqrt{2},2\sqrt{2},0)$	$\left(0,-\frac{10\sqrt{2}}{3},-\frac{10\sqrt{2}}{3},\frac{10\sqrt{2}}{3},\frac{10\sqrt{2}}{3},0\right)$	$\left(\frac{4}{3},\frac{10}{3},\frac{10}{3},\frac{10}{3},\frac{10}{3},\frac{4}{3}\right)$

Jacobi coordinate for tetraquark (PRC 75, 045206)



$$x_1 = \frac{1}{\sqrt{2}}(r_1 - r_2) \quad x_2 = \frac{1}{\sqrt{2}}(r_3 - r_4)$$

$$x_3 = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} - \frac{m_3 r_3 + m_4 r_4}{m_3 + m_4} \tag{12}$$

Reduced mass:

$$m_{x_1} = \frac{2m_1 m_2}{m_1 + m_2} \quad m_{x_2} = \frac{2m_3 m_4}{m_3 + m_4} \quad m_{x_3} = \frac{(m_1 + m_2)(m_3 + m_4)}{m_1 + m_2 + m_3 + m_4} \tag{13}$$

for $qc\bar{q}\bar{c}$: $m_1 = m_3 = m_u \quad m_2 = m_4 = m_c$

$$x_1 = \frac{1}{\sqrt{2}}(r_1 - r_3) \quad x_2 = \frac{1}{\sqrt{2}}(r_2 - r_4) \quad x_3 = \frac{m_u r_1 + m_c r_2 - m_u r_3 - m_c r_4}{m_c + m_u} \tag{14}$$

$$m_{x_1} = \frac{2m_u m_c}{m_u + m_c} \quad m_{x_2} = \frac{2m_u m_c}{m_u + m_c} \quad m_{x_3} = \frac{(m_u + m_c)^2}{2m_u + 2m_c} \tag{15}$$

[7]E. Santopinto and G. Galatà. Phys. Rev. C75, 045206(2007).