Searching for six quark states in lattice QCD

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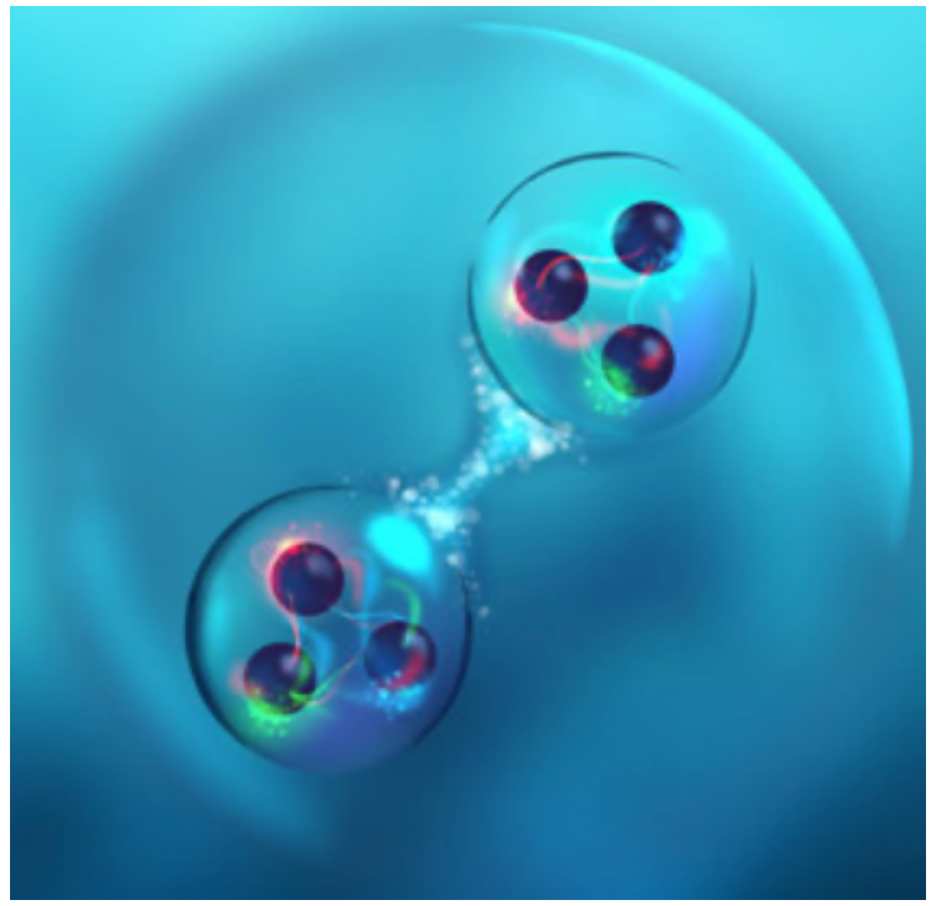
Introduction



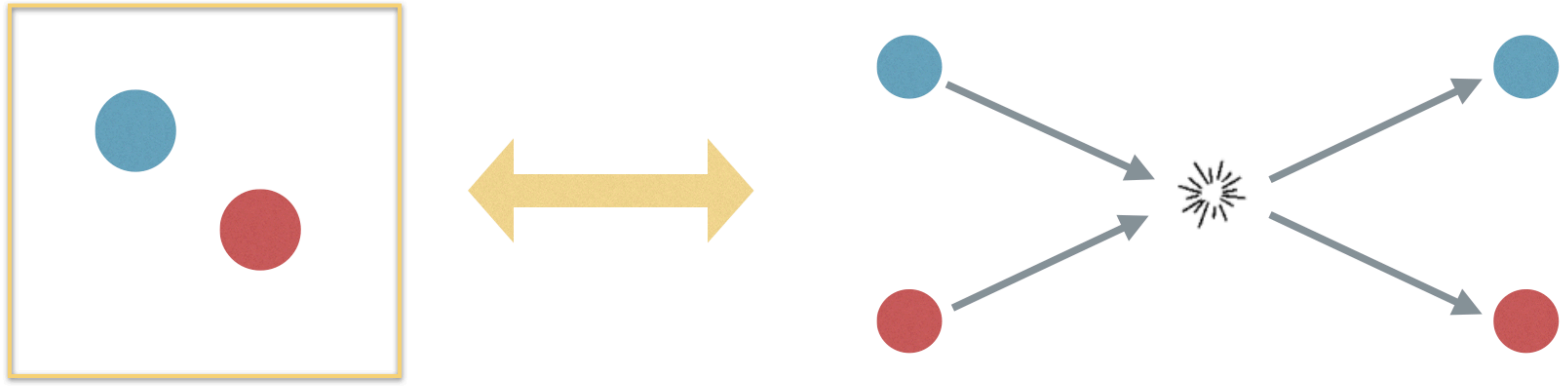
- ✦ The only known stable dibaryon — Deuteron.
  - Lattice calculation is difficult due to poor signal.
- ✦ Are there any other dibaryons ?
- ✦ H-dibaryon was predicted long time ago. (Jaffe 1977)
  - Has not been observed in experiments.
  - Lattice QCD calculations give different results.
  - How would the interaction between $\Lambda\Lambda$ depend on quark mass? — $\Lambda_c\Lambda_c$.

Outline

- ✦ Spectroscopy and scattering on lattice
- ✦ Lattice setup
- ✦ Preliminary results
  - *np* scattering
  - $\Lambda\Lambda - N\Xi$ scattering
  - $\Lambda_c\Lambda_c$ scattering

Scattering on lattice

Lüscher's finite volume method: M. Lüscher, Nucl. Phys. B354, 531(1991)



Finite volume spectrum

Infinite volume scattering parameters

Scattering on lattice

✦ General Lüscher's formula for two-body scattering:

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (1 + i\mathbf{M})] = 0$$

Diagonal matrix of phase-space factors $\rho_{ij} = \delta_{ij}\frac{2k_i}{E_{cm}}$

Infinite-volume scattering matrix

Finite volume information $M(E_{cm}, L)$

✦ Resonances/bound states are formally defined as poles in scattering amplitudes.

Scattering on lattice

Finite volume spectrum:

✦ build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \cdots\}$ with desired quantum numbers, construct the matrix of correlation function:

$$C_{ij} = < 0 | \mathcal{O}_i \mathcal{O}_j^\dagger | 0 > = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

✦ Solve the generalized eigenvalue problem(GEVP): $C_{ij} v_j^n(t) = \lambda_n(t) C_{ij}^0 v_j^n(t)$

✦ Eigenvalues: $\lambda_n(t) \sim e^{-E_n t}(1 + e^{-\Delta E\, t})$

✦ Optimal linear combinations of the operators to overlap on the n'th state:

$$\Omega_n = \sum_i v_i^n \mathcal{O}_i$$

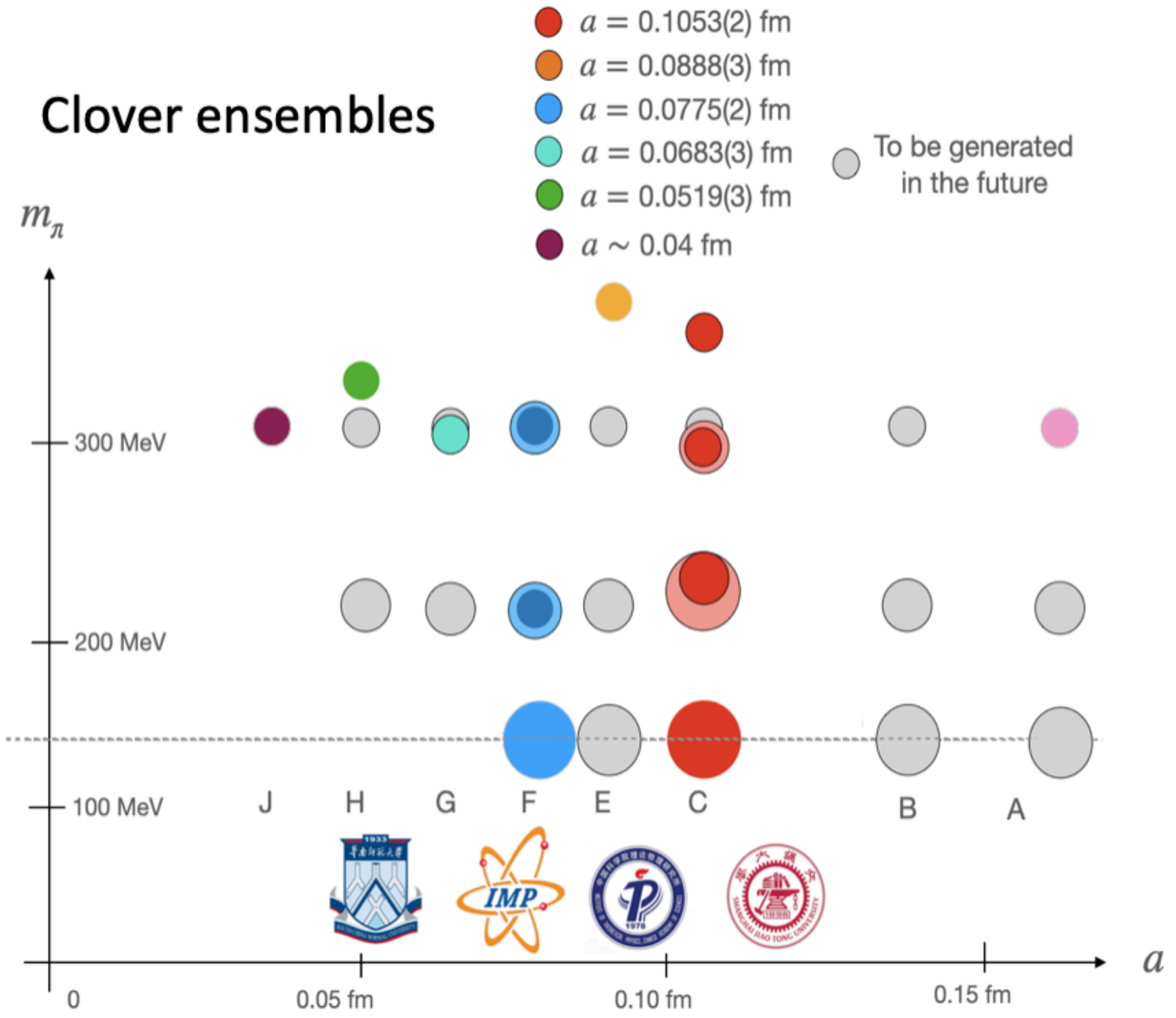
✦ Computational technique: distillation quark smearing.

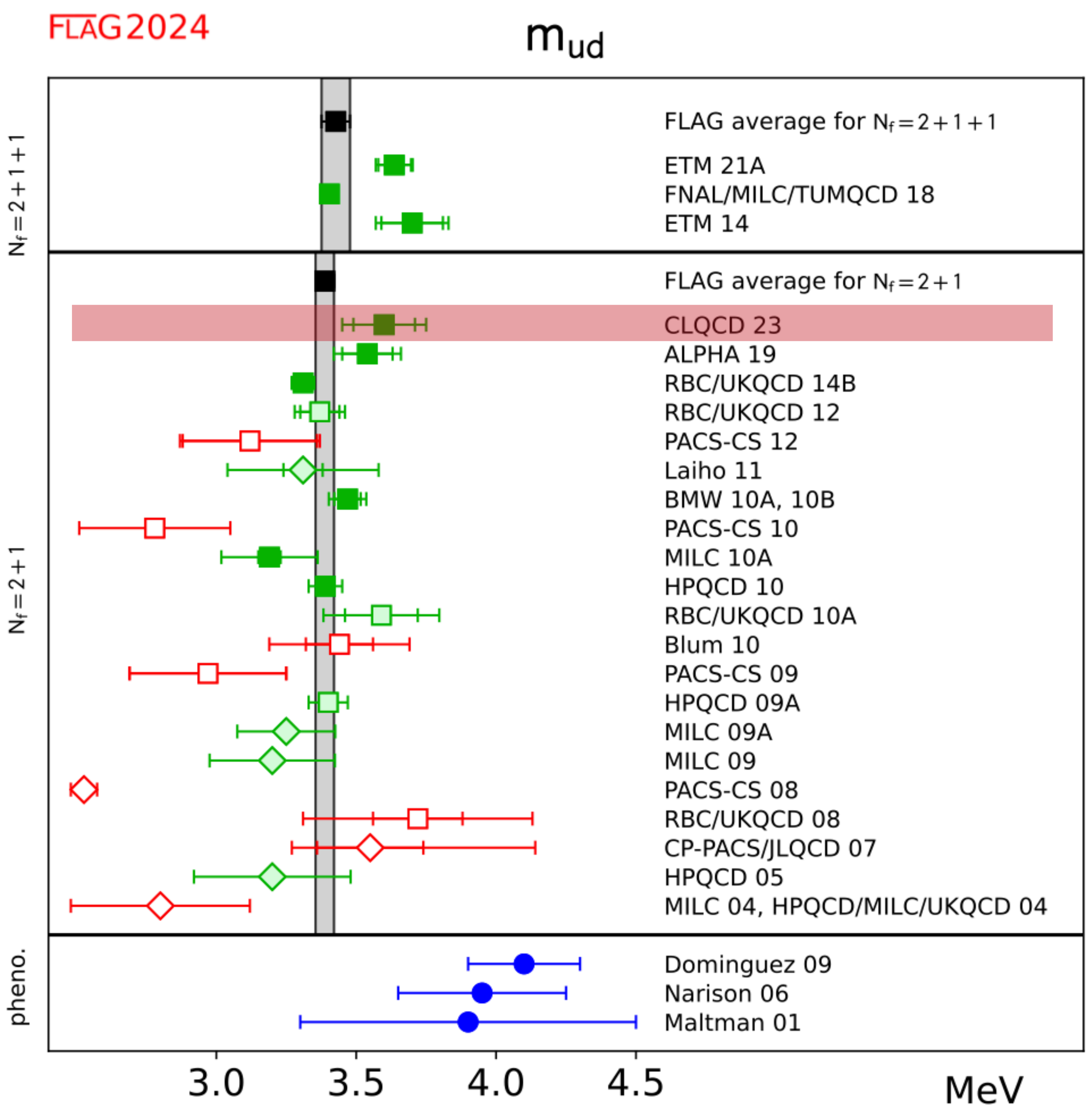
- Improve precision
- Disconneted diagrams
- Efficient for large numbers of ops

Lattice QCD configurations

- 2+1 flavor Wilson-clover configurations generated by CLQCD.

Z.-C.Hu el al., (CLQCD), Phys. Rev. D109(2024)5,054507





Deuteron: *pn* scattering

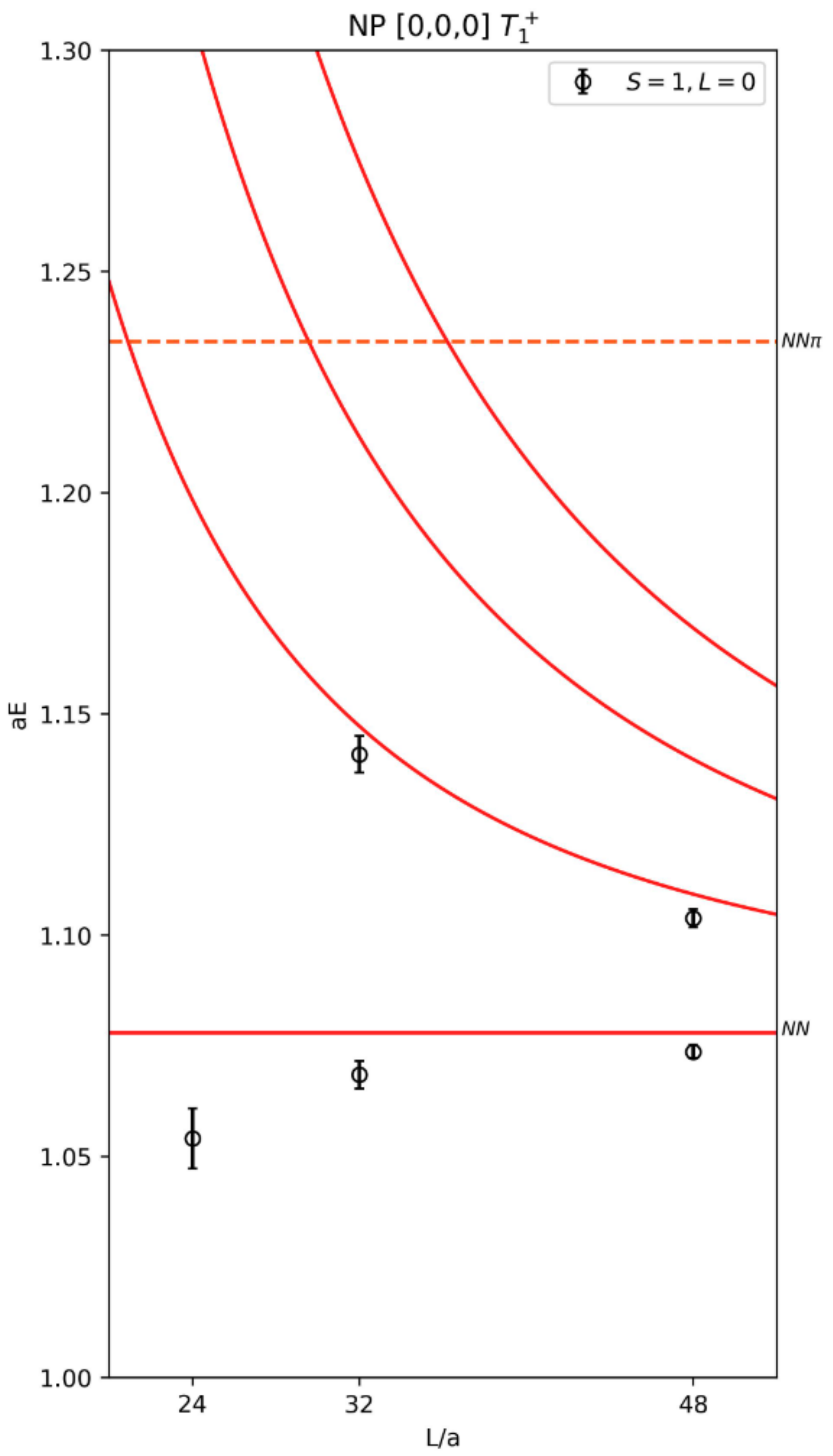
Lattice spacing	Volume($L^3 \times T$)	M_π (MeV)	$M_\pi L$	# of confs
~0.105fm	$24^3 \times 72$	290	3.7	1000
	$32^3 \times 64$	290	4.9	1000
	$48^3 \times 96$	290	7.4	280

Interpolating operators for proton and neutron:

$$p_\alpha = u_\alpha [u^T (C\gamma_5) d] \qquad n_\alpha = d_\alpha [u^T (C\gamma_5) d]$$

Deuteron operators ($I(J^P) = 0(1^+)$):

$$\mathcal{O}_D(t) = C(\alpha, \beta, \mathbf{p}_1, \mathbf{p}_2) \frac{1}{\sqrt{2}} [(p_\alpha(\mathbf{p}_1) n_\beta(\mathbf{p}_2) - n_\alpha(\mathbf{p}_1) p_\beta(\mathbf{p}_2)], \quad \mathbf{p}_1 + \mathbf{p}_2 = 0$$



The coefficients $C(\alpha, \beta, \mathbf{p}_1, \mathbf{p}_2)$ make the operators transform in the T_1^+ irrep of the cubic group and mainly overlaps with **S-wave**.

Deuteron: *pn* scattering

Scattering amplitude:

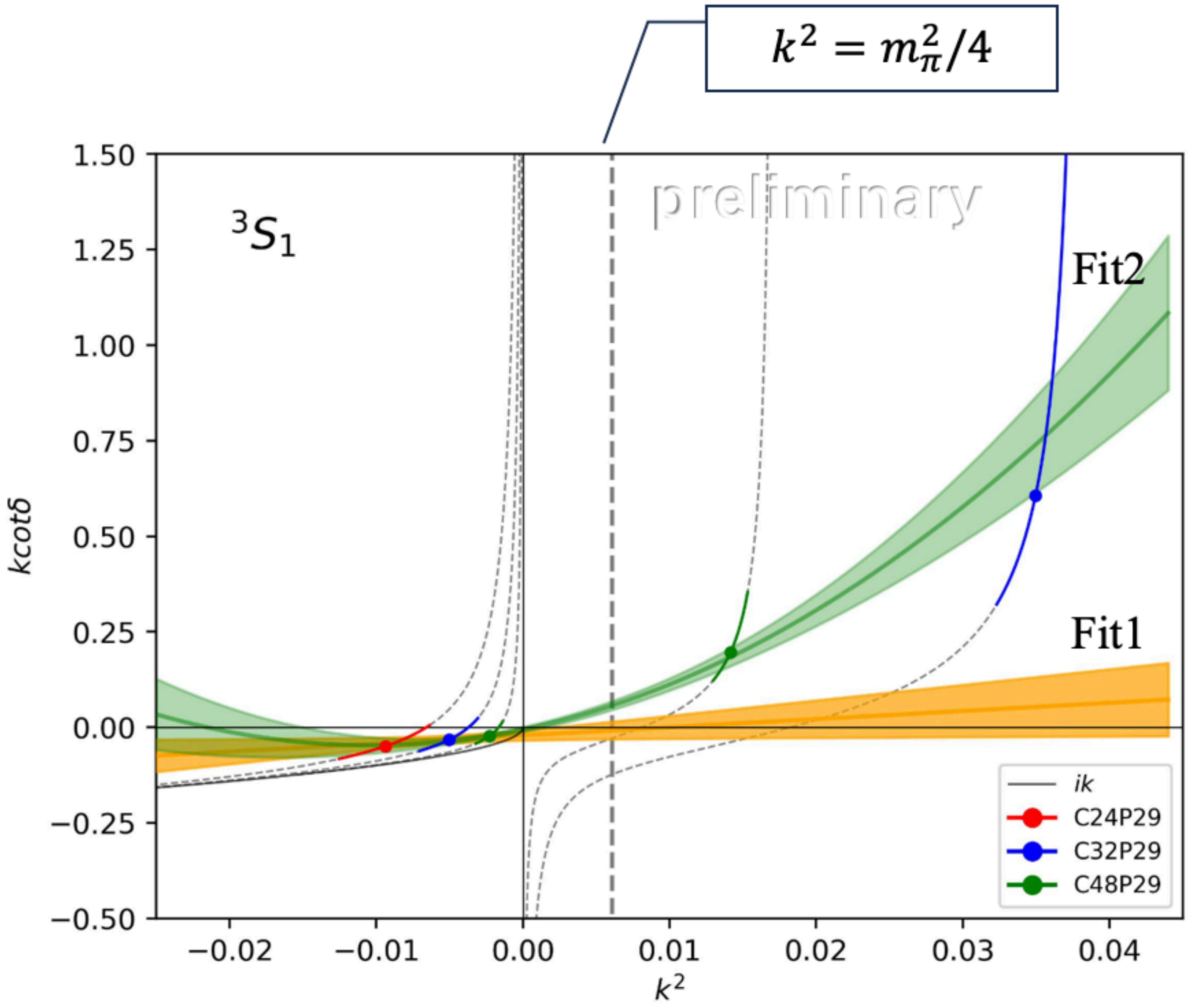
$$T \sim \frac{1}{k\cot\delta - ik}$$

Effective range expansion:

$$k\cot\delta(k) = \frac{1}{a_0} + \frac{1}{2}r_0k^2 + c^3k^4 + \cdots$$

S-wave Luscher's formula:

$$k\cot\delta(k) = \frac{2Z_{00}(1;(\frac{kL}{2\pi})^2)}{L\sqrt{\pi}}$$



	Fit 1	Fit 2
$1/a_0$ (fm^{-1})	-0.2(1)	-0.054(62)
r_0 (fm)	0.45(17)	1.67(23)
c (fm)		0.755(74)
χ^2/dof	0.70	0.59
Binding Energy (MeV)	2.4(1.9)	1.33(96)

Fit1: using three data points below the threshold, ERE up to k^2
Fit2: using five data points, ERE up to k^4

$\Lambda\Lambda$ scattering

Ensembles used in this work

Lattice spacing	Volume($L^3 \times T$)	M_π (MeV)	$M_\pi L$	# of confs
~0.105fm	$24^3 \times 72$	290	3.7	1000
	$32^3 \times 64$	290	4.9	1000
	$32^3 \times 64$	230	3.9	450
	$48^3 \times 96$	230	5.9	260
~0.077fm	$32^3 \times 96$	300	3.7	780
	$48^3 \times 96$	300	5.6	360
	$48^3 \times 96$	210	4.0	220
~0.052fm	$48^3 \times 144$	320	4.0	430

ΛΛ scattering

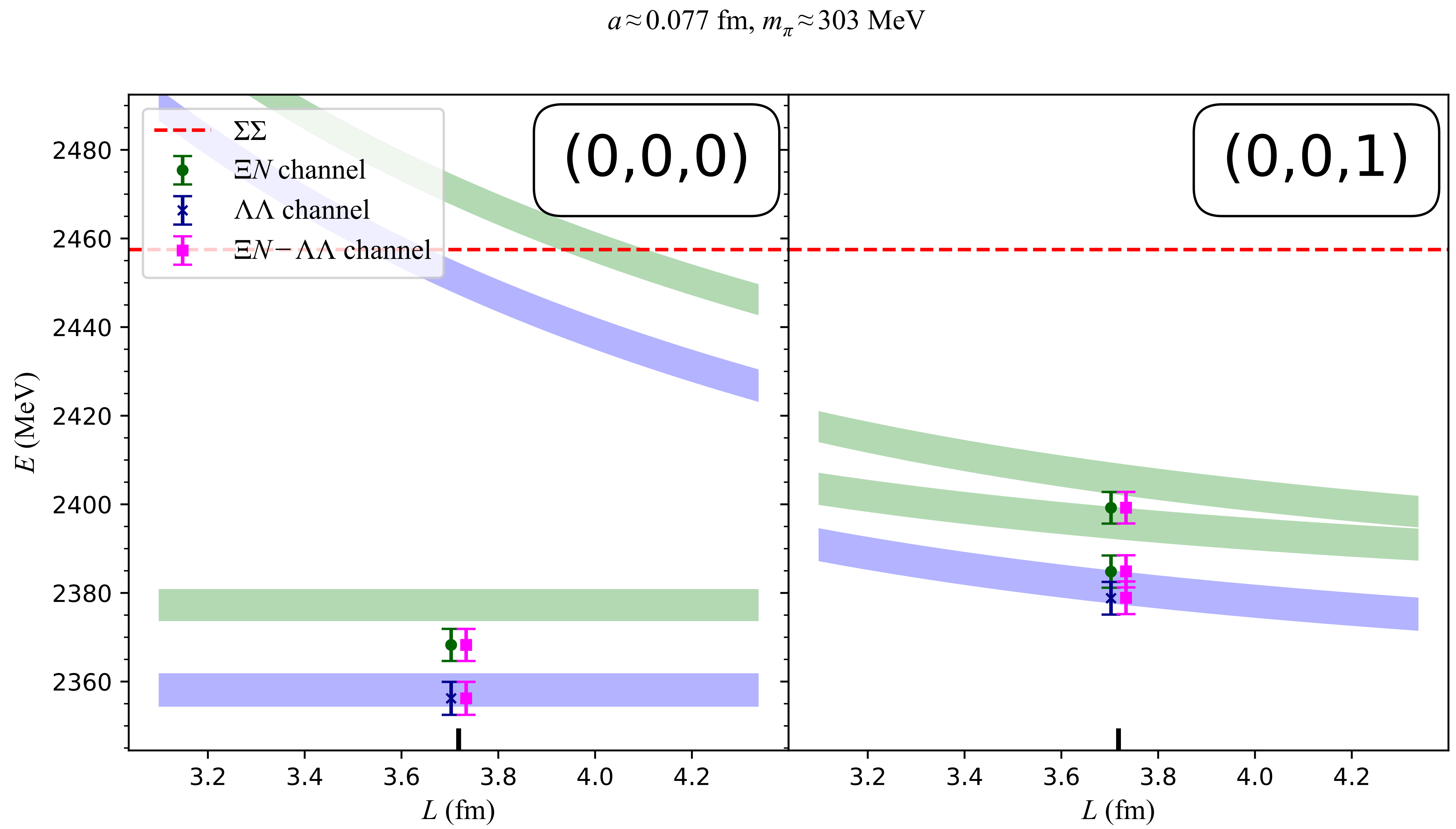
- We are intrested in the $I(J^P) = 0(0^+)$ ΛΛ scattering
- coupled channels: $\Lambda\Lambda, N\Xi, \Sigma\Sigma$
- To avoid the complexity of three coupled channels, we will keep the energy range below $\Sigma\Sigma$ threshold, and consider only $\Lambda\Lambda$ and $N\Xi$.

$$\mathcal{O}^{\mathbf{P}}(\Lambda\Lambda) = \Lambda(\mathbf{p}_1)\Lambda(\mathbf{p}_2)$$

$$\mathcal{O}^{\mathbf{P}}(N\Xi) = p(\mathbf{p}_1)\Xi^-(\mathbf{p}_2) - n(\mathbf{p}_1)\Xi^0(\mathbf{p}_2)$$

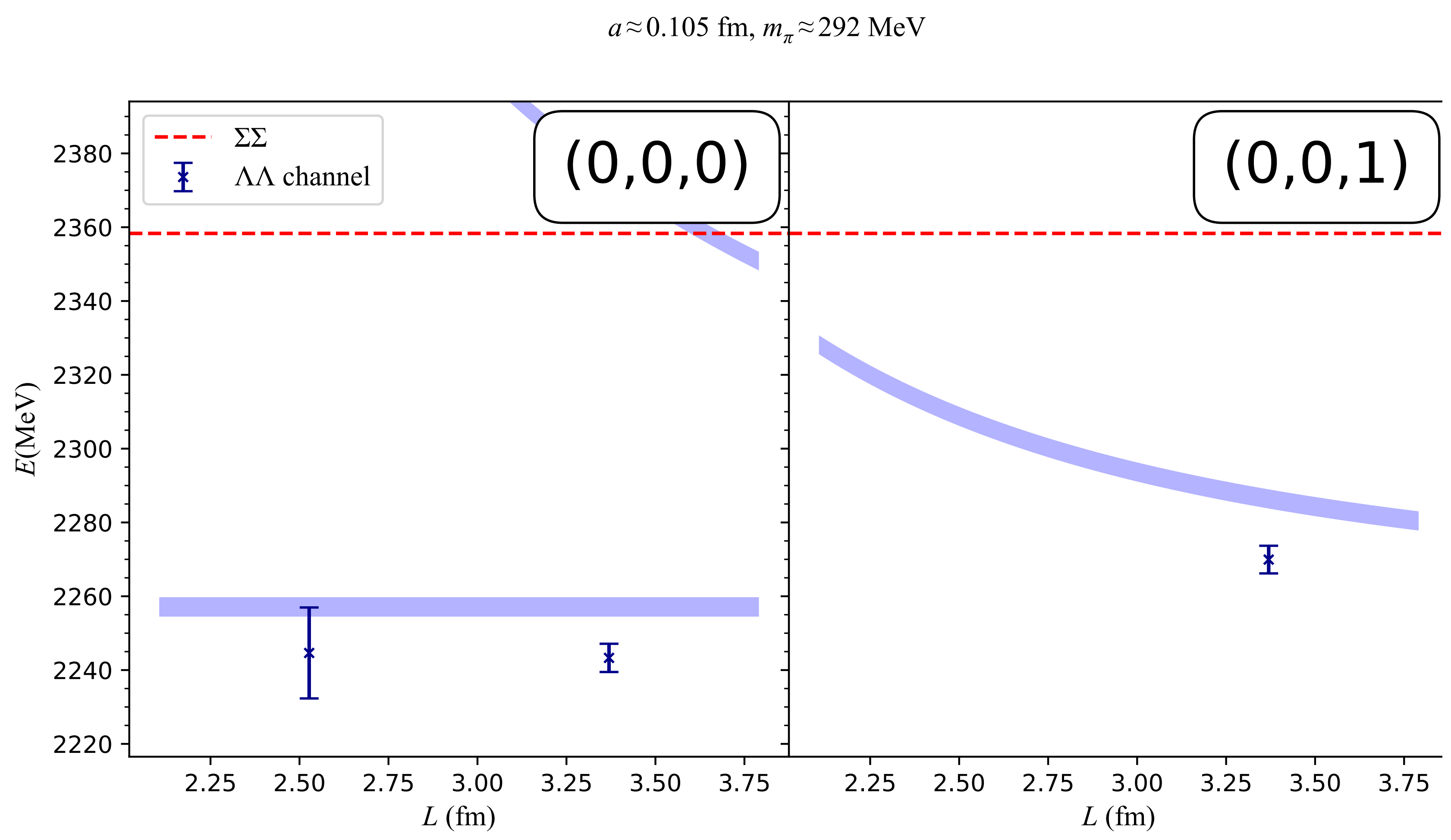
$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2, \quad \mathbf{P} = (0,0,0), (0,0,1)$$

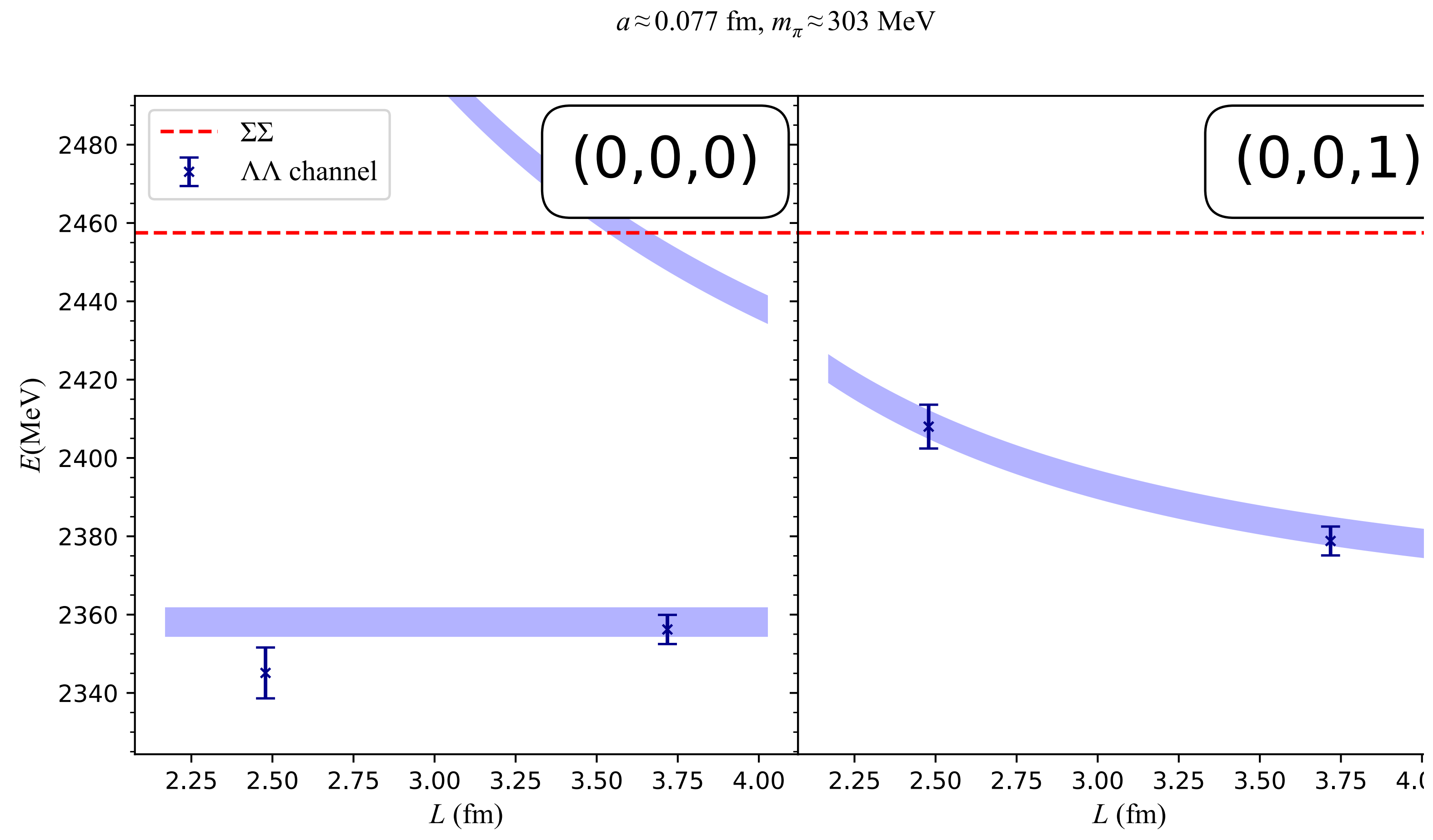
The energy levels obtained from GEVP using both $\Lambda\Lambda$ and $N\Xi$ operators are almost the same as using them separately.

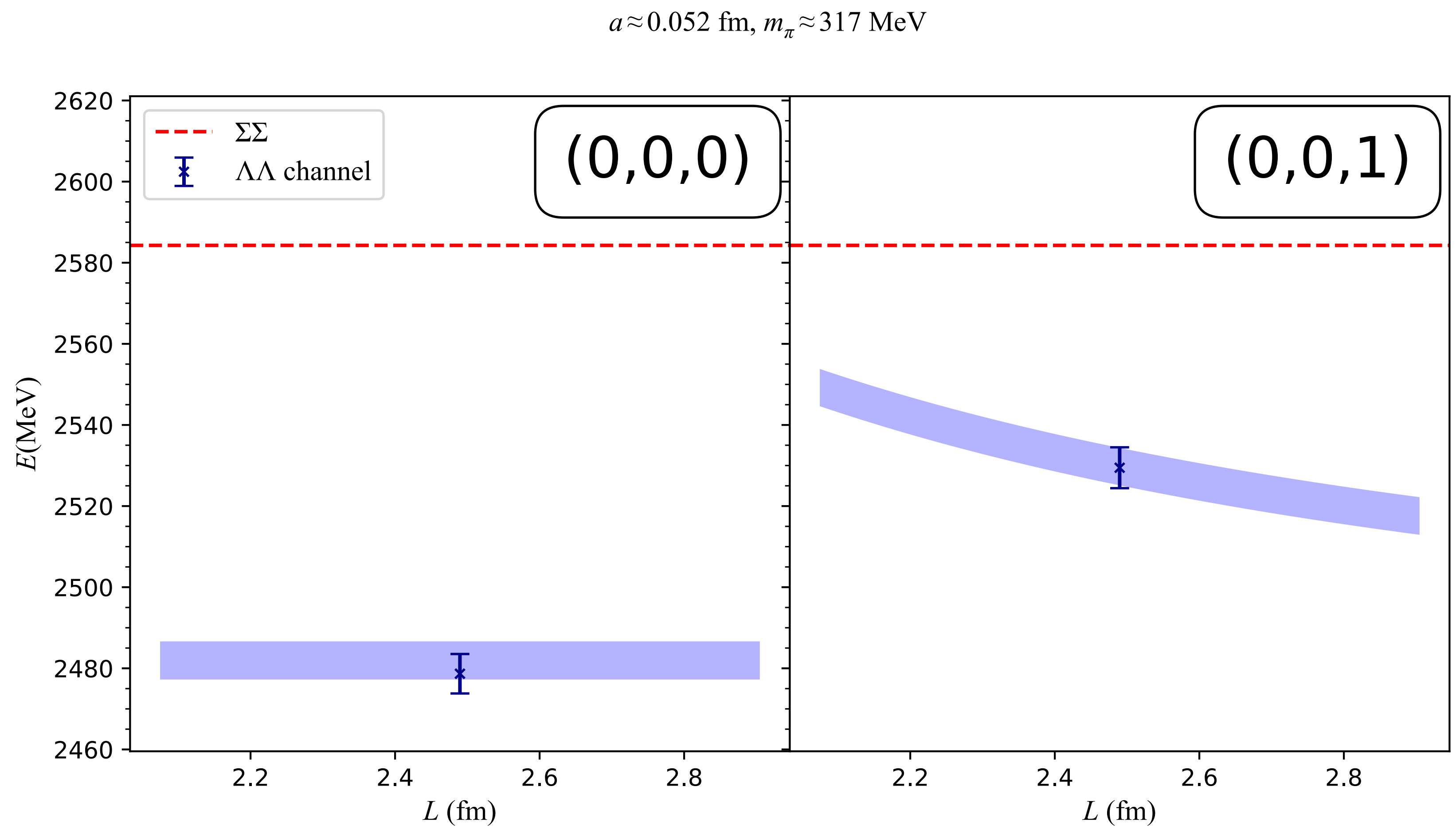


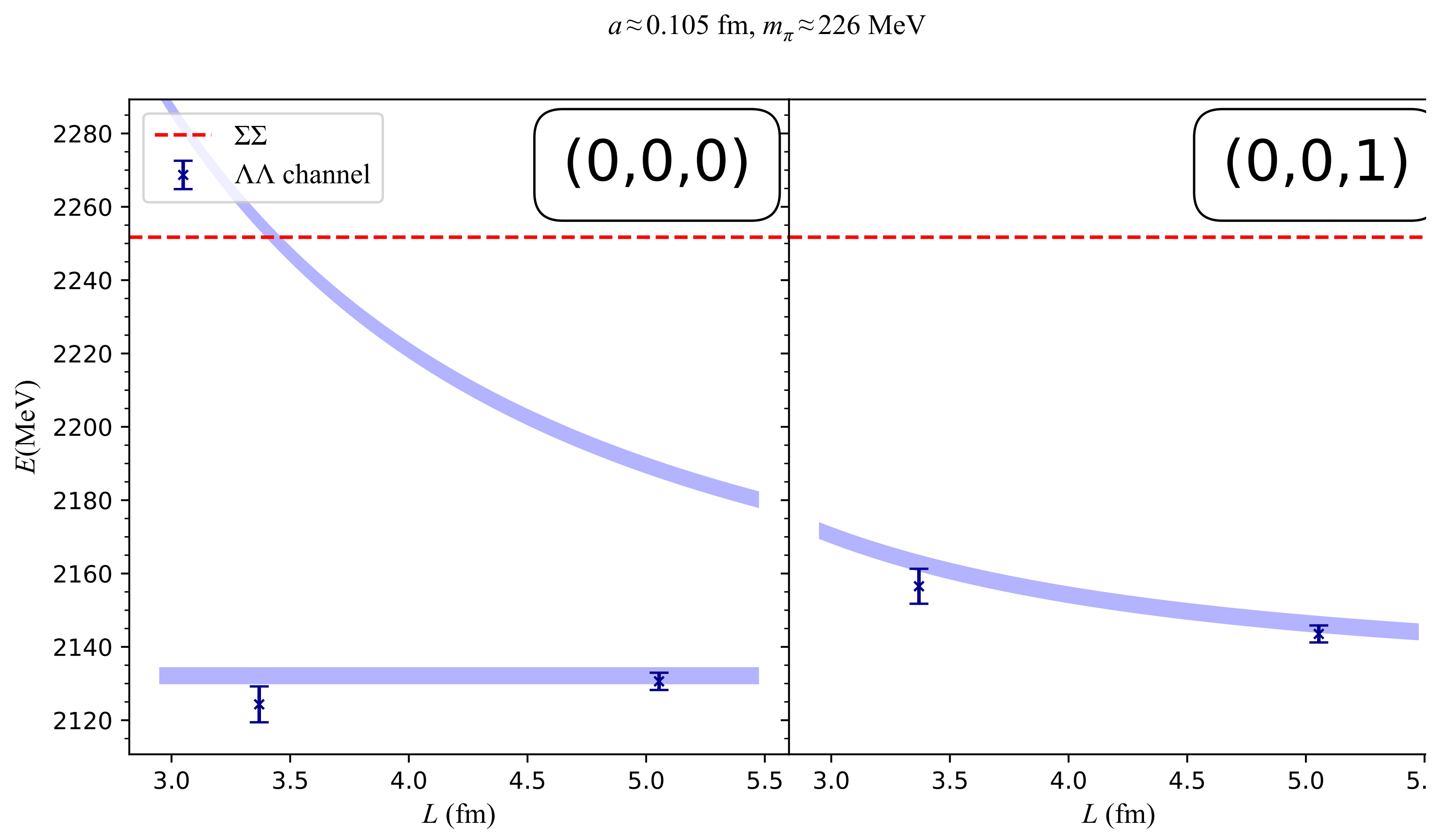
ΛΛ scattering

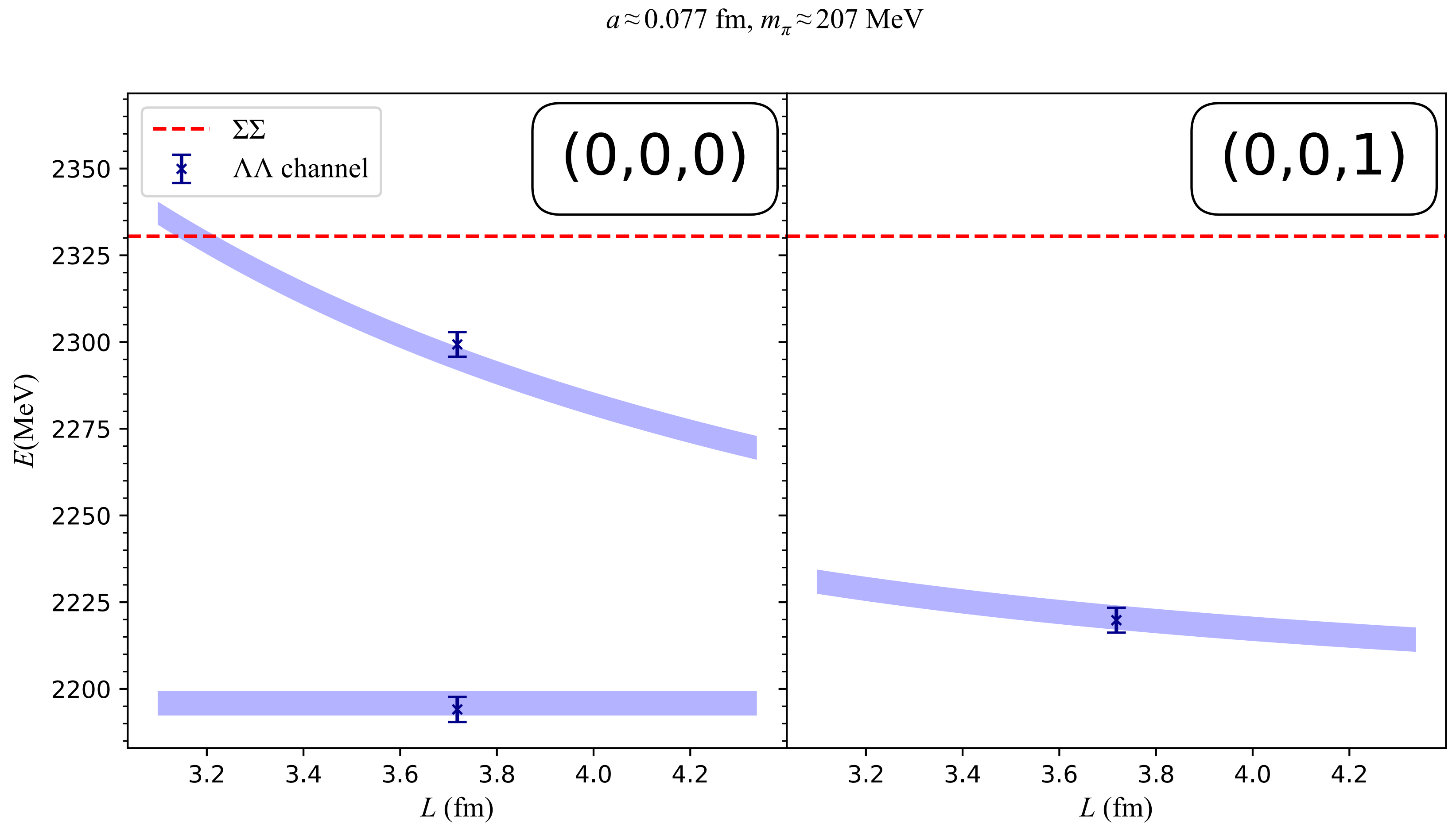
ΛΛ finite volume spectrum







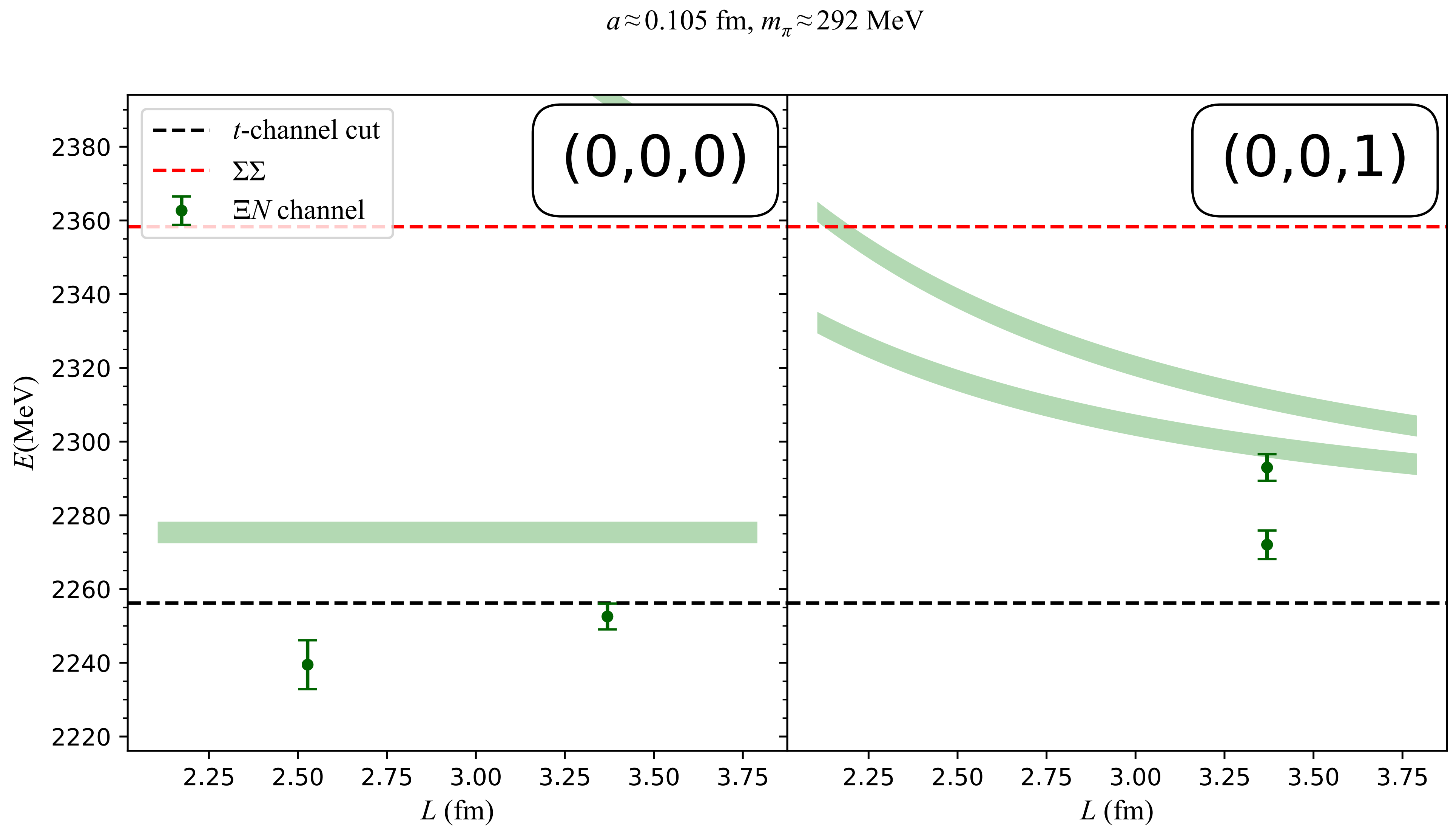


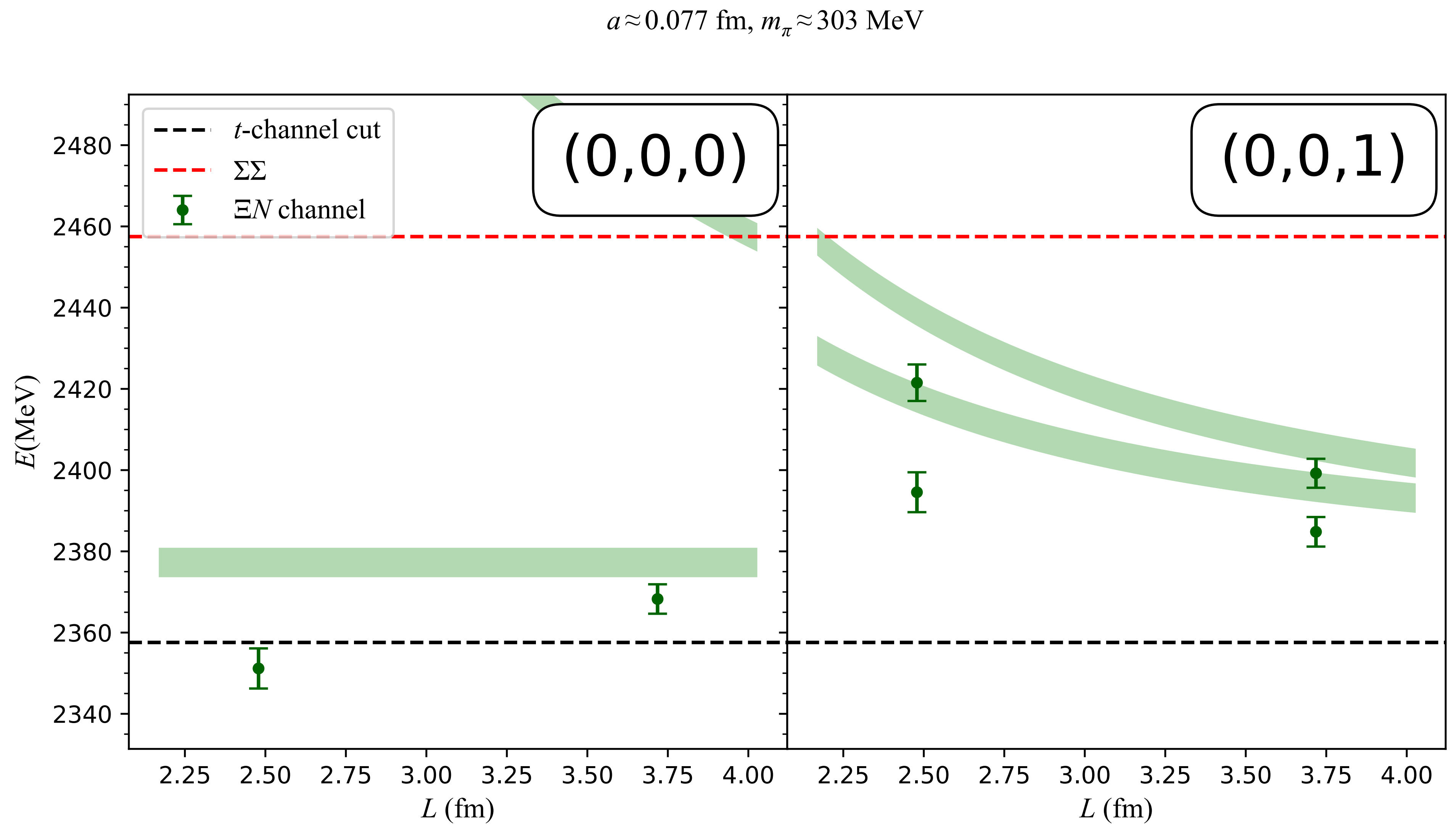


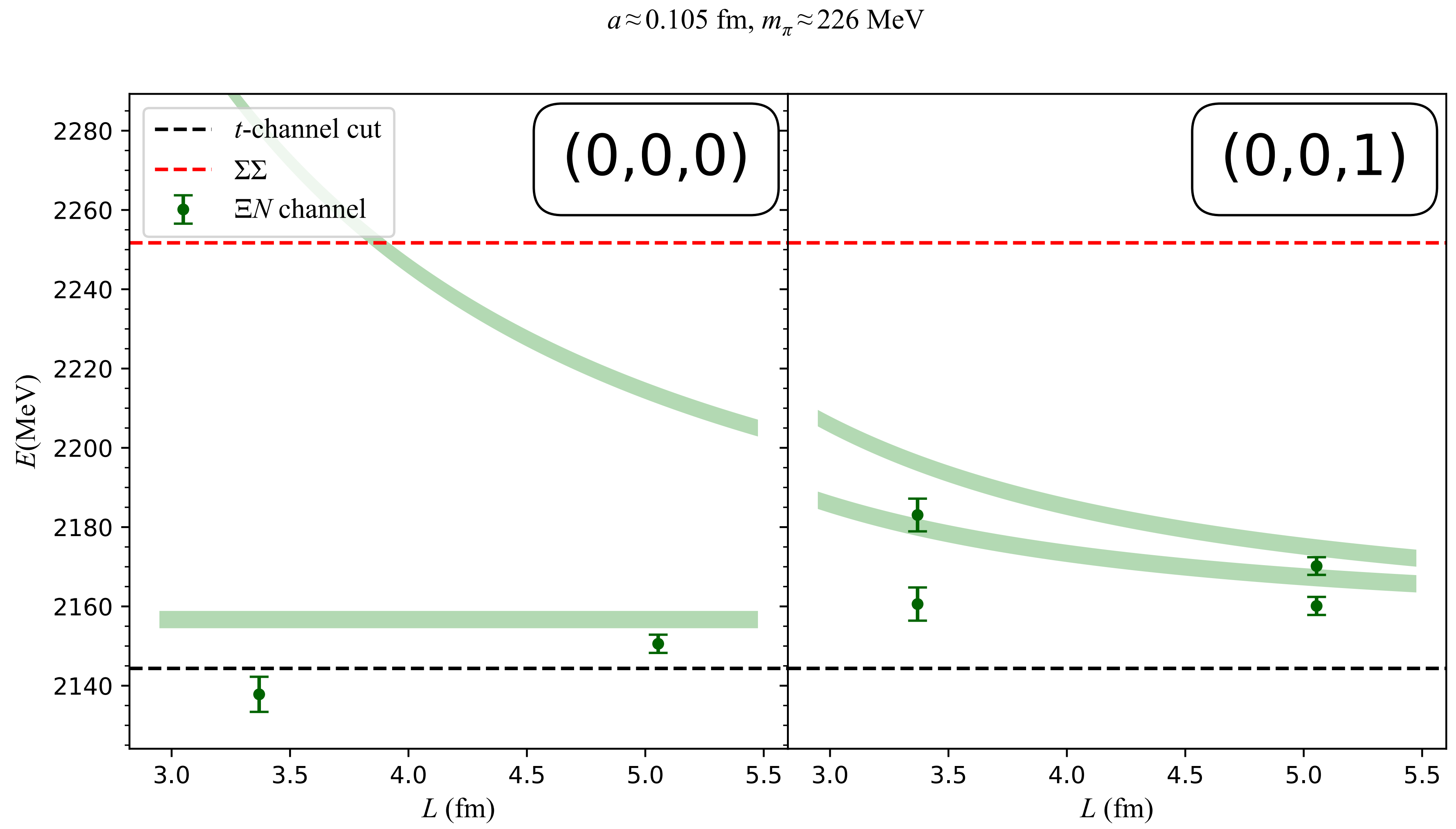
- Except for $a = 0.105$fm, $m_\pi = 292$MeV, the interacting enengies are all very close to the non-interacting energies.

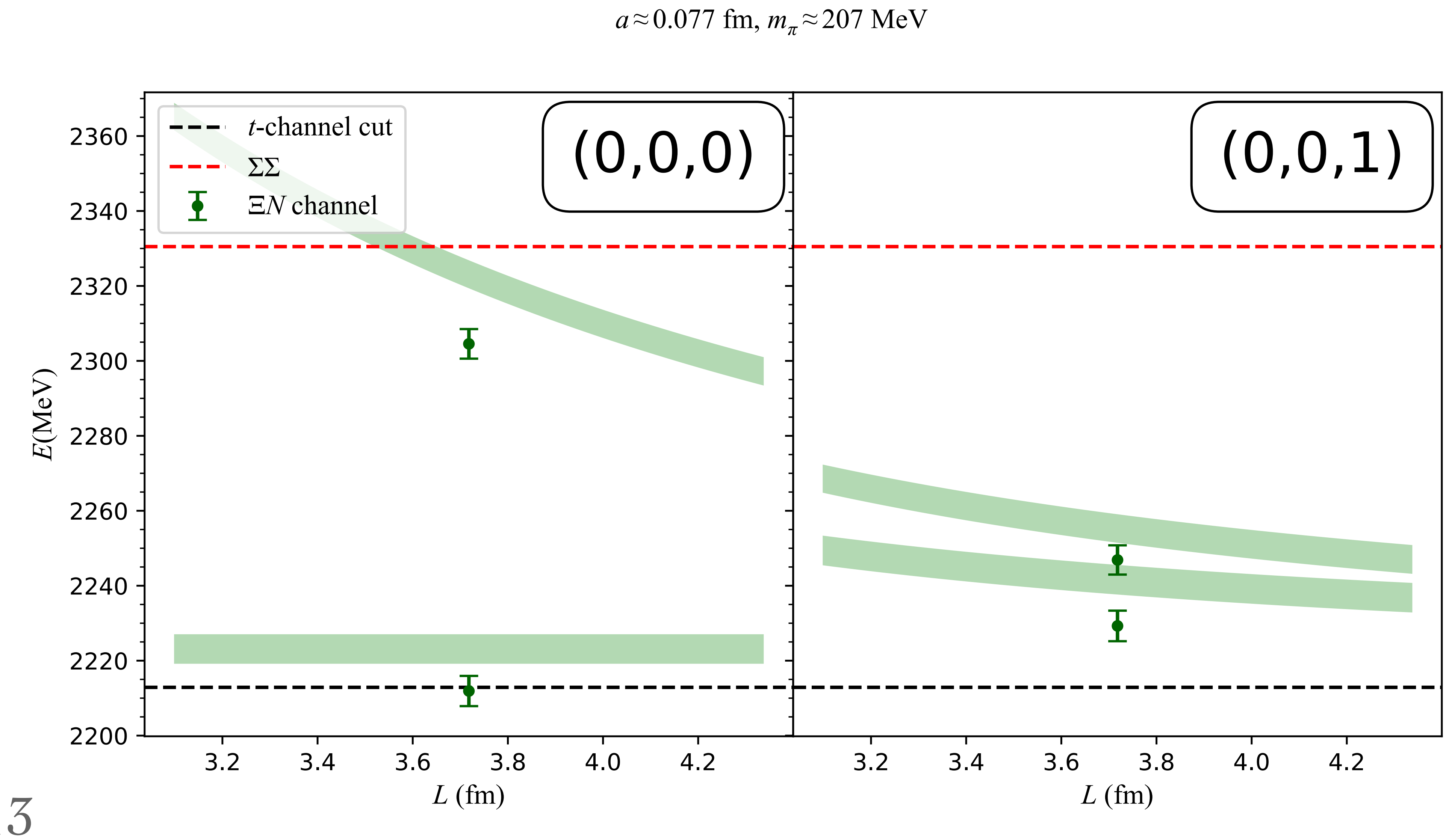
ΛΛ scattering

$N\Xi$ finite volume spectrum





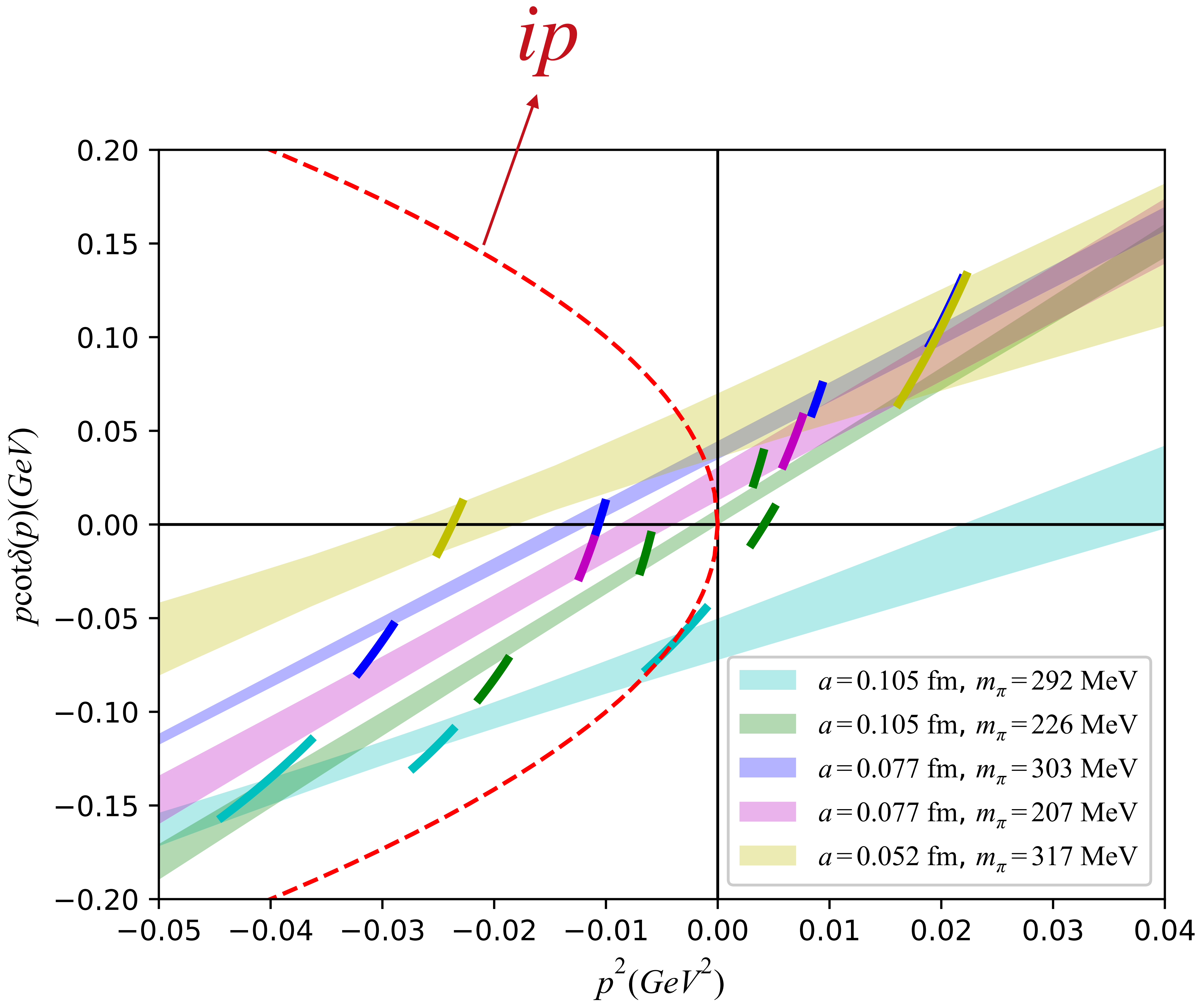




ΛΛ scattering

$N\Xi$ scattering amplitude

- The scattering amplitude is paramerized by effective range expansion: $p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$
- At $a = 0.105$ fm, $m_\pi = 292$ MeV, there is a bound state pole, when the lattice spacing becomes smaller and pion mass gets closer to the physical value, the pole becomes a virtual state pole.
- The effects of the left-hand cut has not been considered.



$\Lambda_c\Lambda_c$ scattering

Lattice spacing	Volume$(L^3 \times T)$	M_π (MeV)	$M_\pi L$	# of confs
~0.077fm	$32^3 \times 96$	300	3.7	780
	$48^3 \times 96$	300	5.6	360

- S-wave $\Lambda_c\Lambda_c$ scattering, coupled channels: $N\Xi_{cc}, \Sigma_c\Sigma_c$

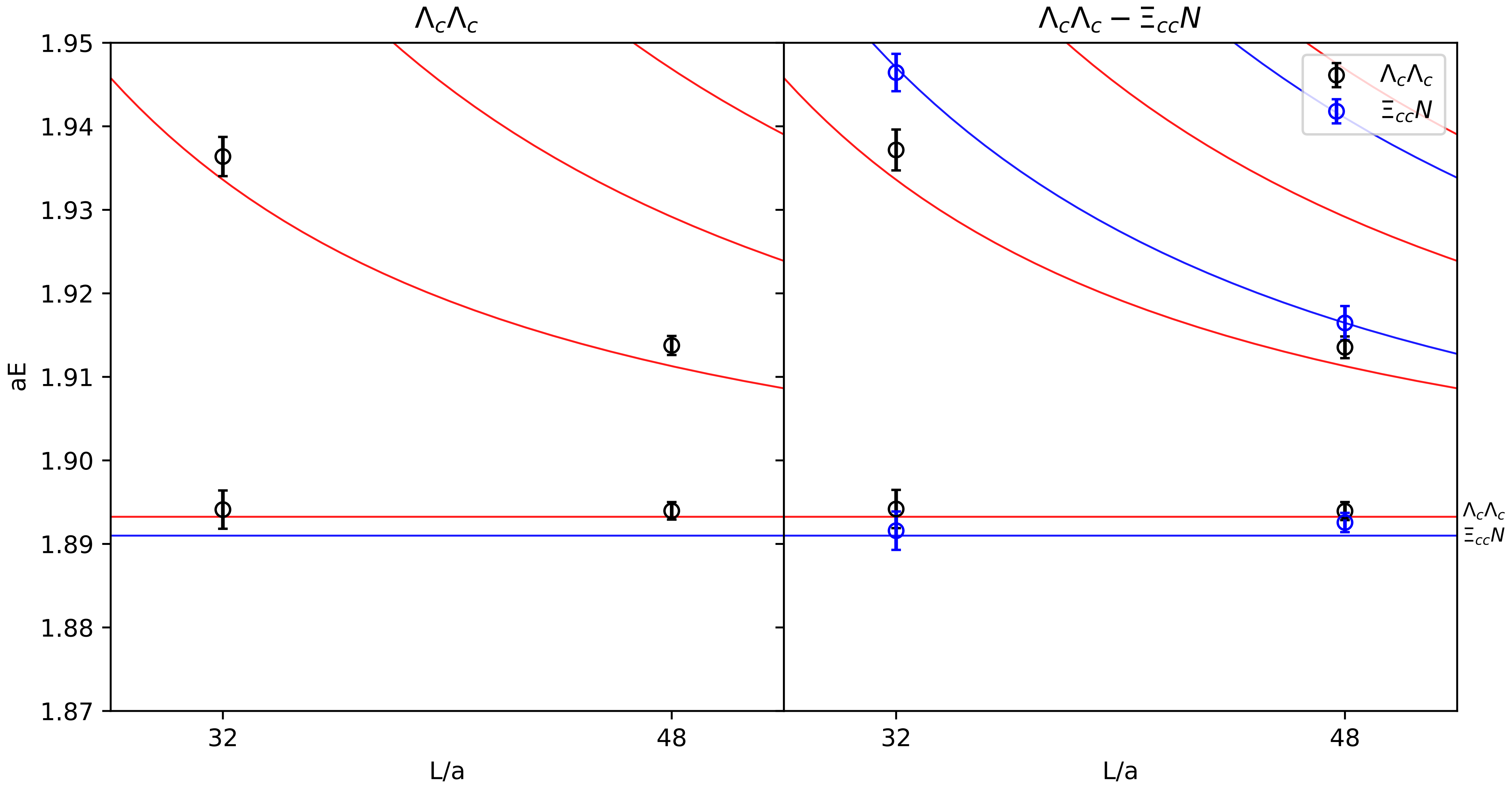
Interpolating operators$(I(J^P) = 0(0^+))$:

$$\mathcal{O}(\Lambda_c\Lambda_c) = \Lambda_c(\mathbf{p})\Lambda_c(-\mathbf{p}), \quad |\mathbf{p}|^2 = 0,1$$

$$\mathcal{O}(N\Xi_{cc}) = N(\mathbf{p})\Xi_{cc}(-\mathbf{p}), \quad |\mathbf{p}|^2 = 0,1$$

- The energy levels of $\Lambda_c\Lambda_c$ are not affected by including the $N\Xi_{cc}$ operators.

$\Lambda_c\Lambda_c - N\Xi_{cc}$ coupling



$\Lambda_c\Lambda_c$ scattering

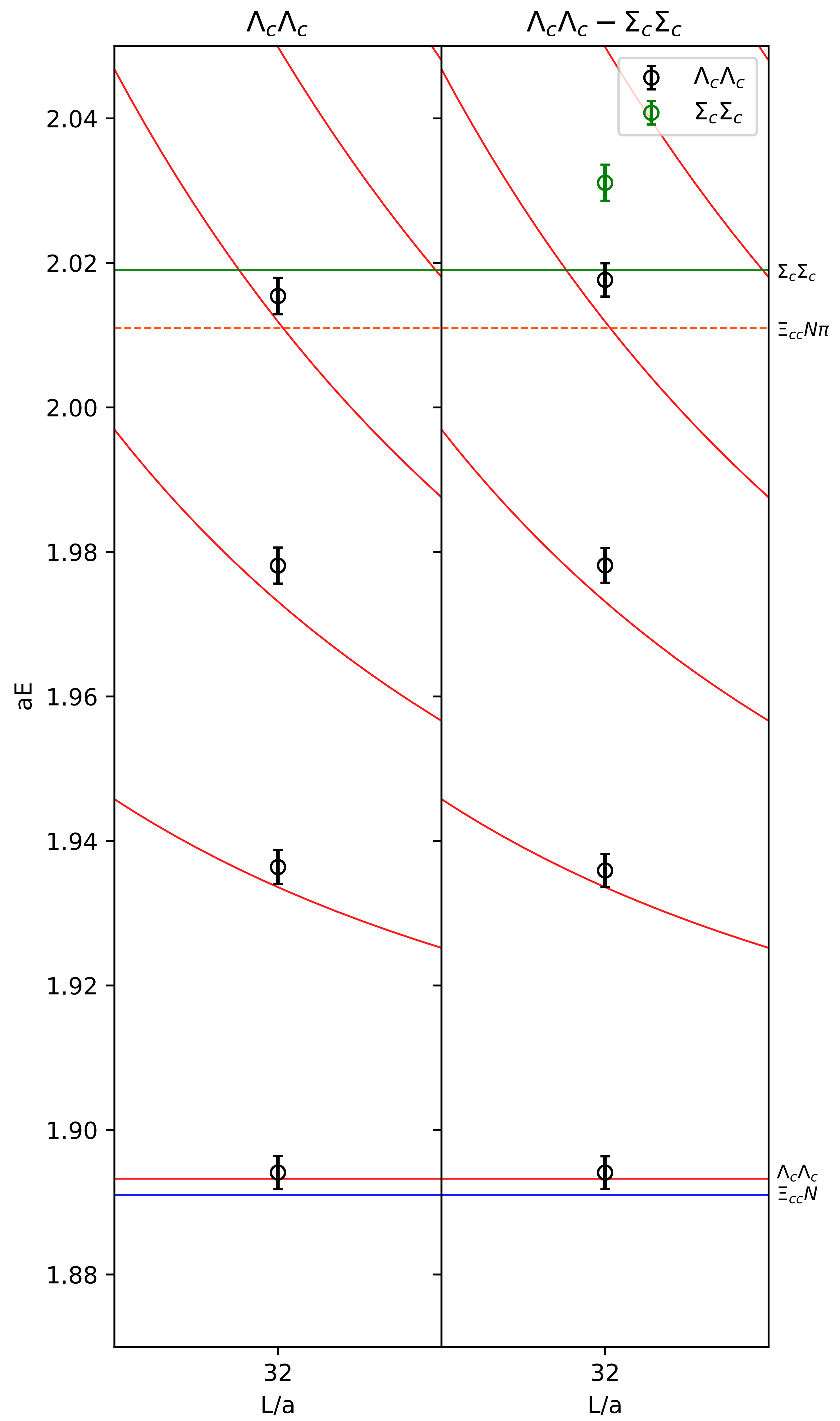
$\Lambda_c\Lambda_c - \Sigma_c\Sigma_c$ coupling

Interpolating operators
($I(J^P) = 0(0^+)$):

$$\mathcal{O}(\Lambda_c\Lambda_c) = \Lambda_c(\mathbf{p})\Lambda_c(-\mathbf{p}), \quad |\mathbf{p}|^2 = 0,1,2,3$$

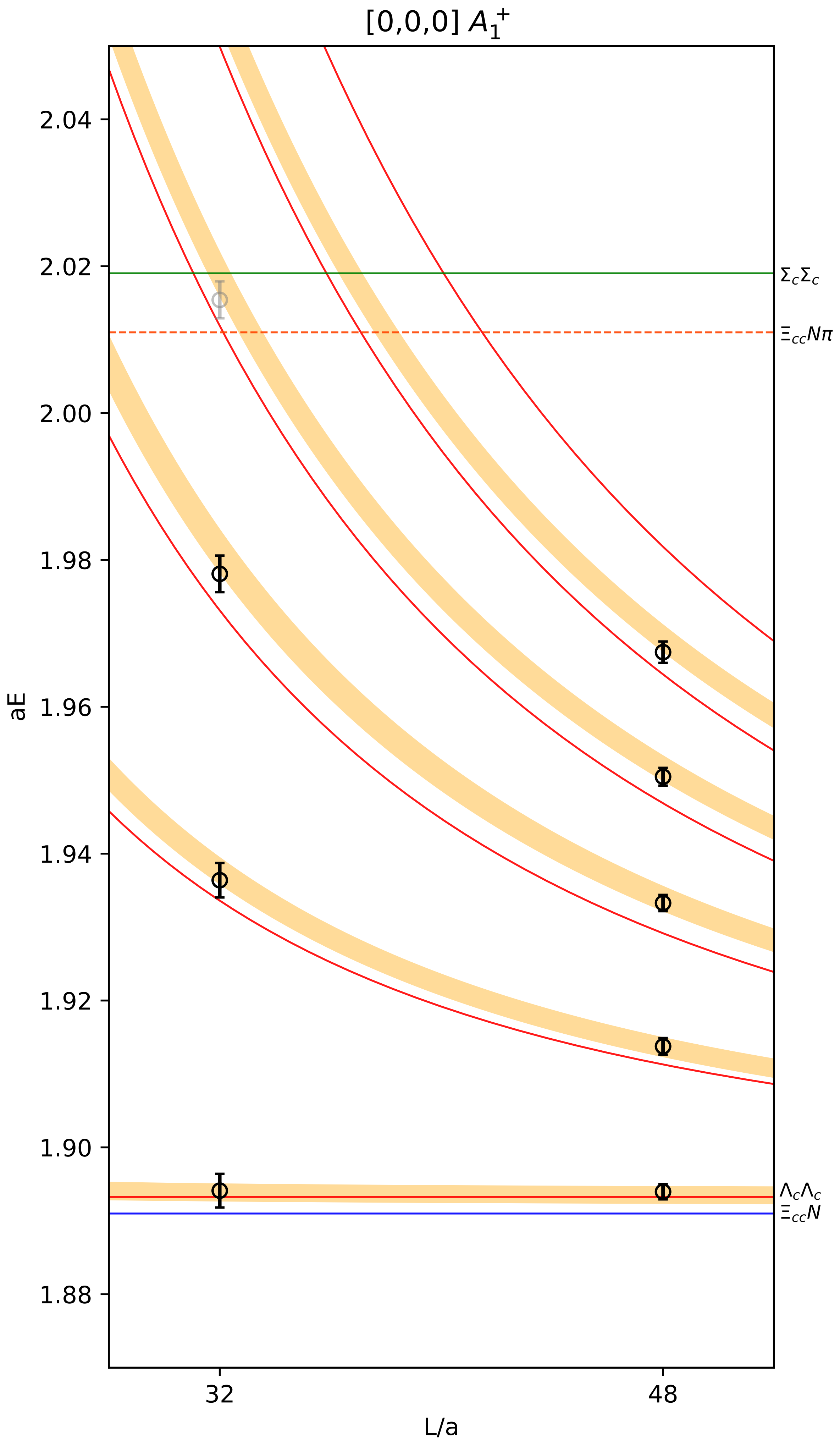
$$\mathcal{O}(\Sigma_c\Sigma_c) = \Sigma_c(\mathbf{p})\Sigma_c(-\mathbf{p}), \quad |\mathbf{p}|^2 = 0$$

- The $\Sigma_c\Sigma_c$ threshold is much higher than $\Lambda_c\Lambda_c$, the energy levels well below the $\Sigma_c\Sigma_c$ threshold is not affected by $\Sigma_c\Sigma_c$.



$\Lambda_c\Lambda_c$ scattering

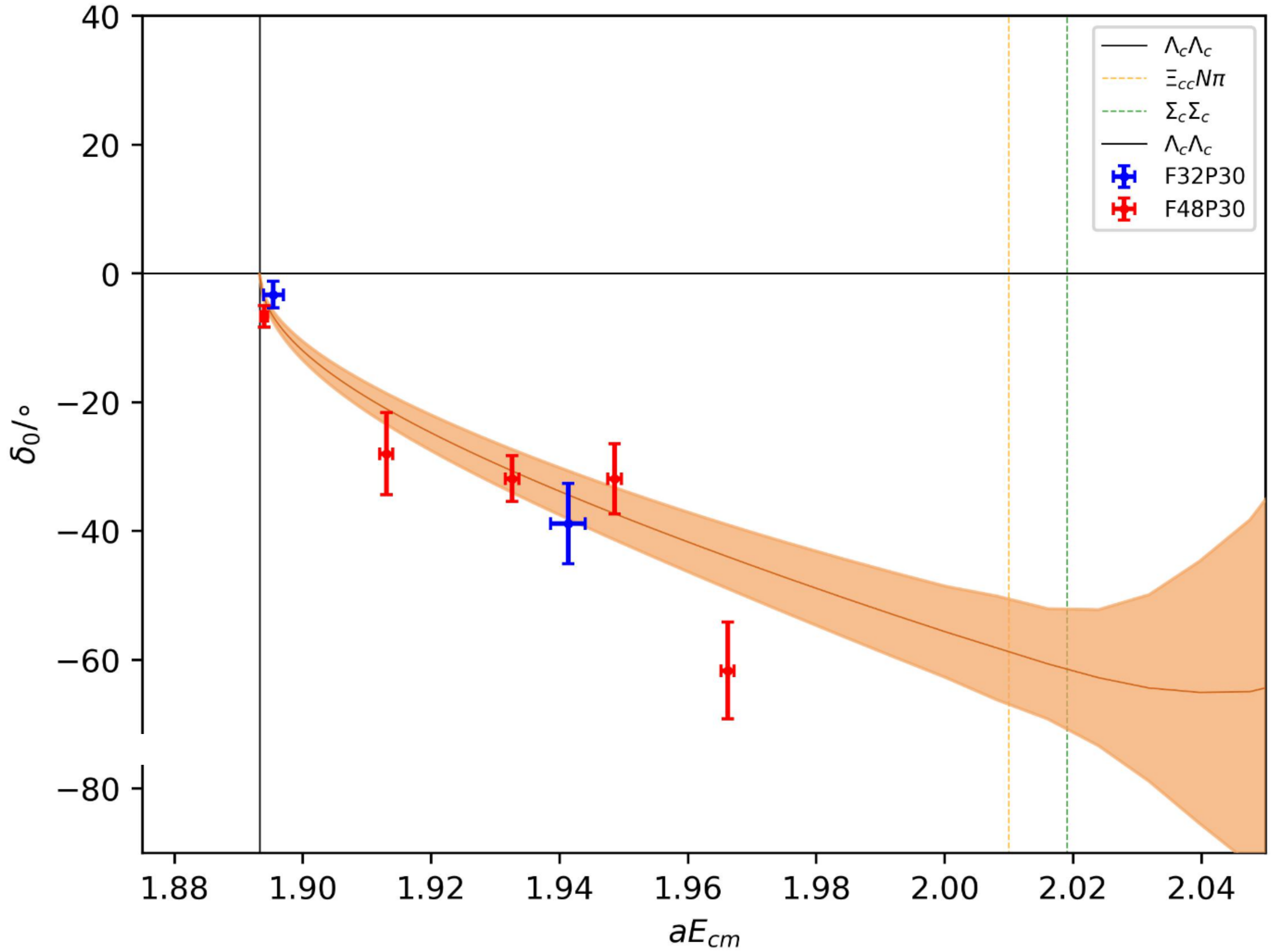
Finite-volume spectrum



effective range expansion:

$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$

Phase shift



$$a_0 = -0.225(33)\ \text{fm}$$

$$r_0 = 0.02(10)\ \text{fm}$$

Summary

- np scattering: At $m_\pi = 290$ MeV and lattice spacing $a = 0.105$fm, bound state was found in the S-wave np scattering.
- Very weak interaction in the $\Lambda\Lambda$ channel. Atractive interaction in the $N\Xi$ channel, bound/virtual state pole is found.
- Repulsive interaction in the $\Lambda_c\Lambda_c$ scattering.

- Couple channel effects need to be considered in the scattering analysis.
- The influnce of left-hand cut is ignored in the preliminaly results, need to be include in future works.

Thanks!