Glueballs in QCD sum rules

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Based on: PRD103 (2021) L091503;PRD104(2021) 094050;RPP86(2023) 026201

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East Asian Workshop on Exotic Hadrons 2024

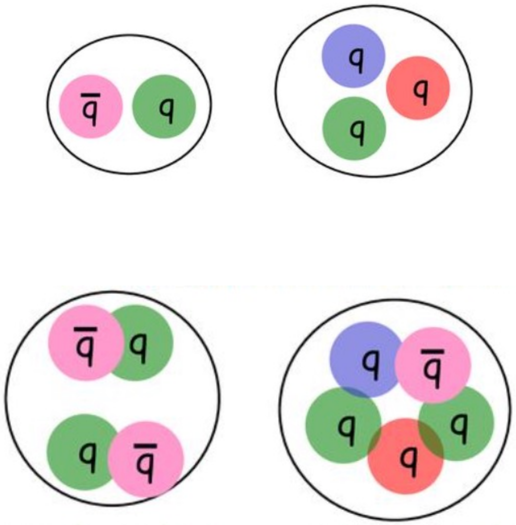
2024.12.8-12 SEU • Nanjing

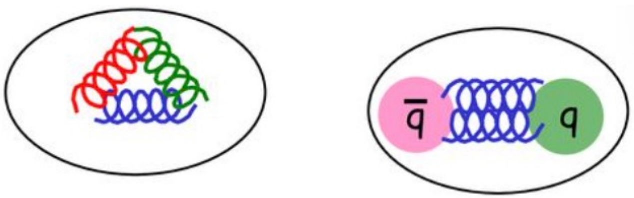
Outline

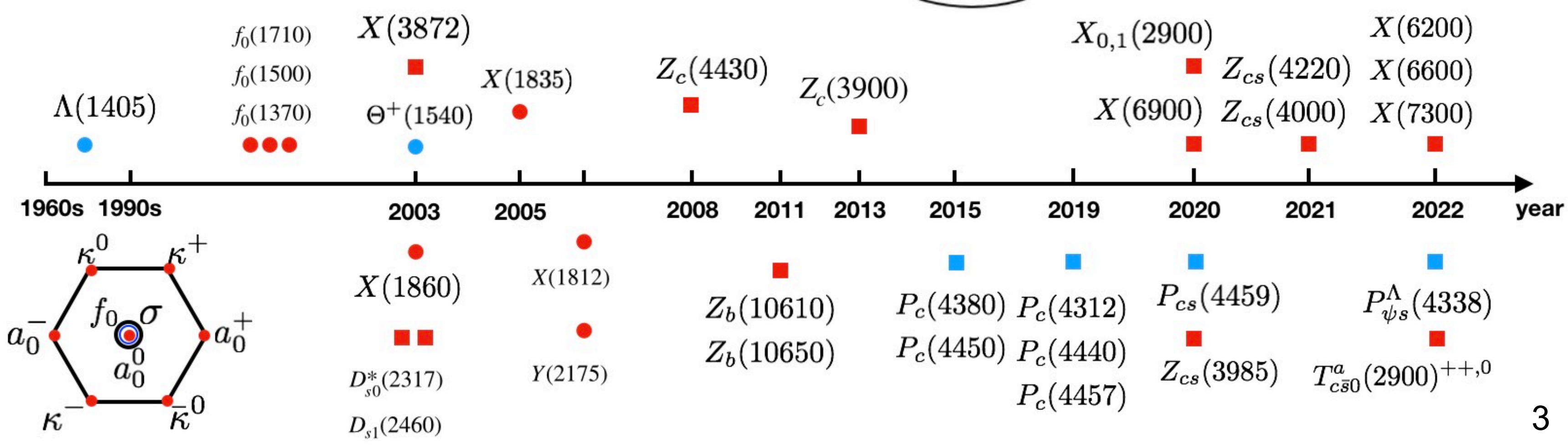
- Research progresses on glueballs
- Two-gluon glueballs
- Three-gluon glueballs
- Summary

Quark model and Exotic hadrons

- **Quark Model:** $q\bar{q}$ mesons and qqq baryons
- **Exotic Hadrons:** hadrons beyond QM, such as multiquarks, hybrids, glueballs...
- **Hybrids and Glueballs:** very distinctive predictions of QCD!

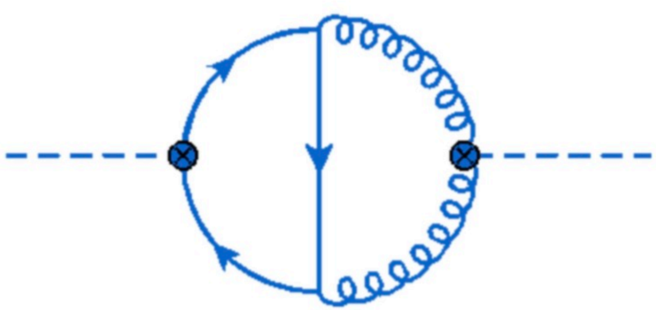


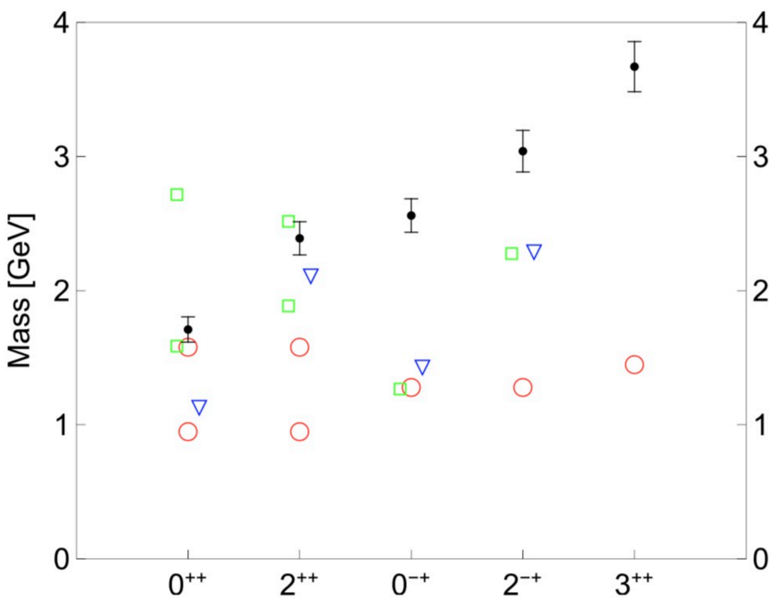




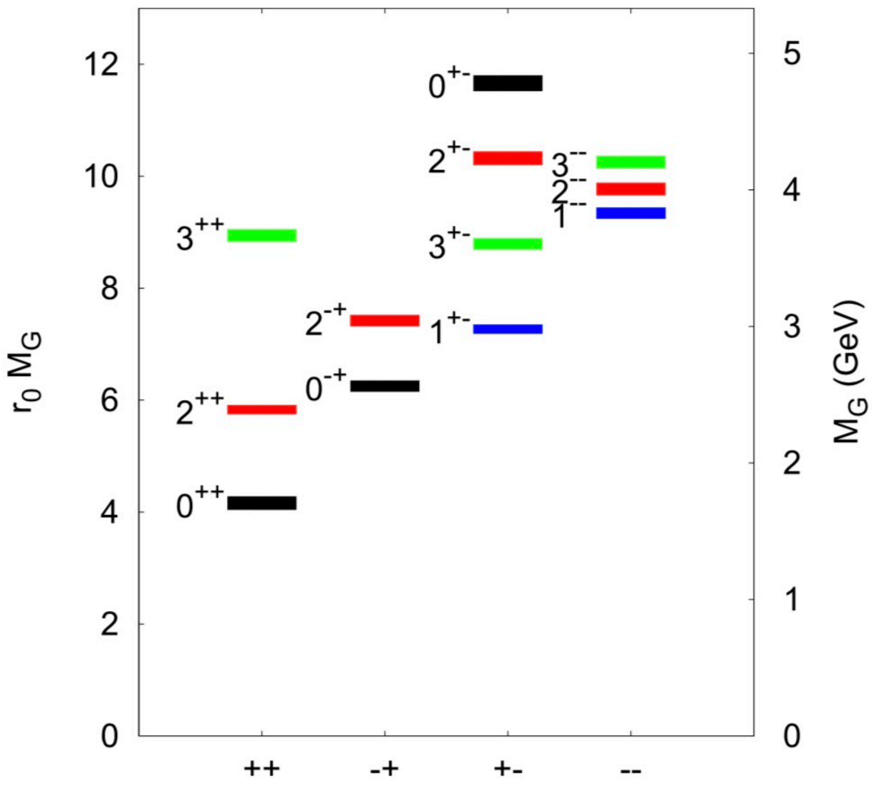
Glueballs: colorless bound states of gluons as gluons have a self-coupling

- Mixing with normal $q\bar{q}$ states, it is hard to isolate the pure glueballs experimentally.
- No universal definition of constituent gluon: massless or massive?
- Plenty of theoretical studies in the past half century based on various methods.





Mass spectrum of glueballs in LQCD



Lightest glueballs:

$J^{PC}=0^{++}$: $1710 \pm 50 \pm 80$ MeV

$J^{PC}=2^{++}$: $2390 \pm 30 \pm 120$ MeV

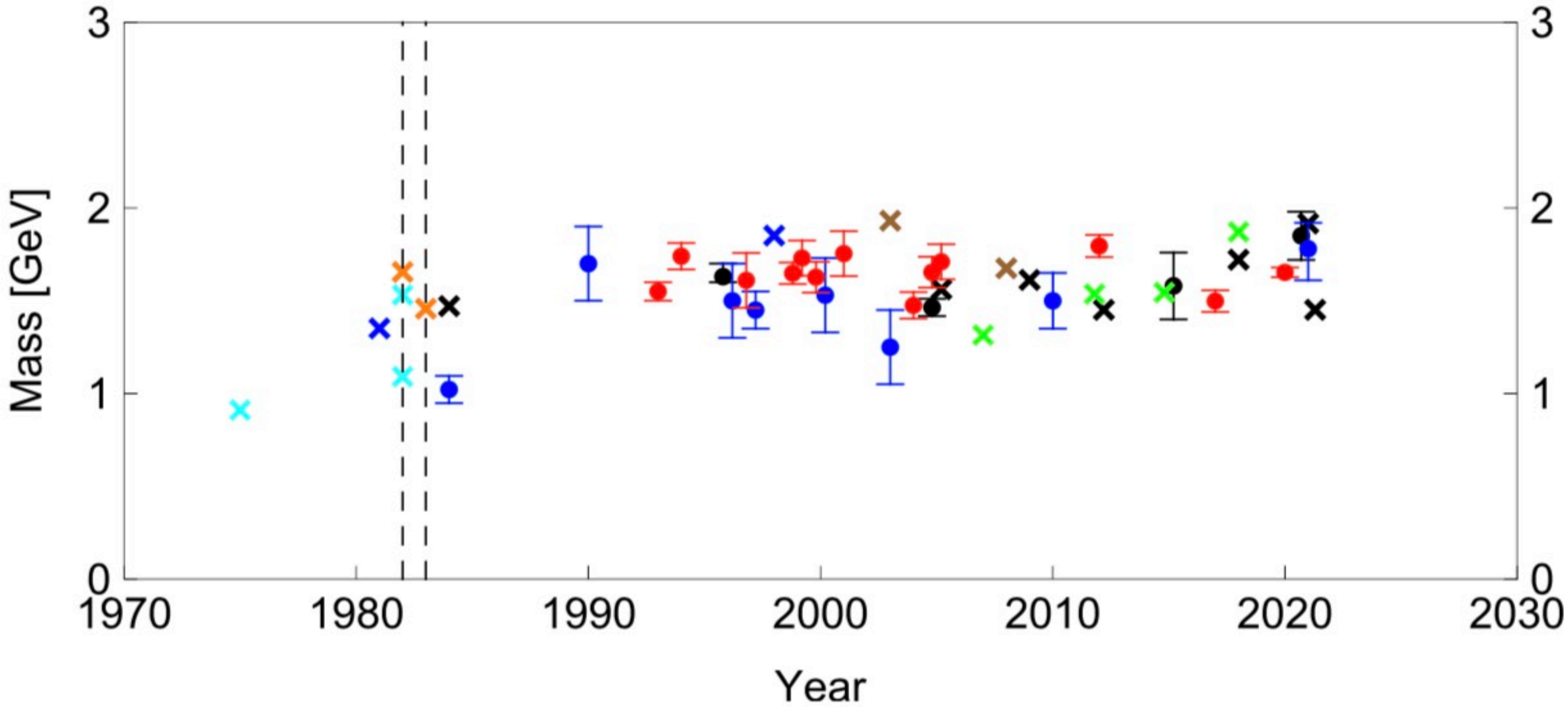
$J^{PC}=0^{-+}$: $2560 \pm 35 \pm 120$ MeV

There are some candidates in these channels!

Y. Chen et. al., PRD73(2006)014516

Scalar glueball:

H.X.Chen et. al., RPP86(2023) 026201



Average mass predictions:

$$M_{|gg;0^{++}\rangle} \approx 1650 \text{ MeV}$$

Candidates: scalar light mesons

$M_{|gg;0^{++}\rangle} \approx 1650$ MeV

Name	$f_0(500)$	$f_0(1370)$	$f_0(1710)$	$f_0(2020)$	$f_0(2200)$
M (MeV)	410 ± 20	1370 ± 40	1700 ± 18	1925 ± 25	2200 ± 25
	$400 \to 550$	$1200 \to 1500$	1704 ± 12	1992 ± 16	2187 ± 14
Γ (MeV)	480 ± 30	390 ± 40	255 ± 25	320 ± 35	150 ± 30
	$400 \to 700$	$100 \to 500$	123 ± 18	442 ± 60	~ 200

Name	$f_0(980)$	$f_0(1500)$	$f_0(1770)$	$f_0(2100)$	$f_0(2330)$
M (MeV)	1014 ± 8	1483 ± 15	1765 ± 15	2075 ± 20	2340 ± 20
	990 ± 20	1506 ± 6		2086^{+20}_{-24}	~ 2330
Γ (MeV)	71 ± 10	116 ± 12	180 ± 20	260 ± 25	165 ± 25
	$10 \to 100$	112 ± 9		284^{+60}_{-32}	250 ± 20

Production rates in the gluon-rich J/ψ radiative decay processes:

$$\mathcal{B}(J/\psi \to \gamma f_0(1500)) \sim 0.29 \times 10^{-3},$$
$$\mathcal{B}(J/\psi \to \gamma f_0(1710)) \sim 2.2 \times 10^{-3},$$

BESIII:*Natl.Sci.Rev.* 8 (2021) nwab198

$$\mathcal{B}(J/\psi \to \gamma |gg;0^{++}\rangle) = (3.8 \pm 0.9) \times 10^{-3}.$$

PRL110(2010)021601

Mixing scheme:

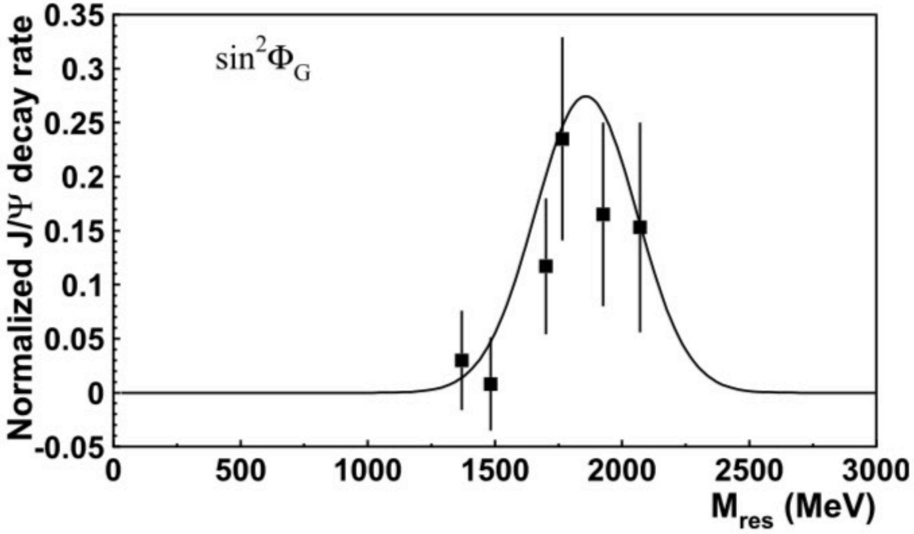
PLB826(2022)36906

$$\mathcal{M} = \begin{pmatrix} M_{gg} & f & \sqrt{2}f \\ f & M_{s\bar{s}} & 0 \\ \sqrt{2}f & 0 & M_{n\bar{n}} \end{pmatrix}$$

$$H' = (|n\bar{n}\rangle \cos\varphi' - |s\bar{s}\rangle \sin\varphi') \cos\phi^H + |gg\rangle \sin\phi^H,$$
$$L' = (|n\bar{n}\rangle \sin\varphi' + |s\bar{s}\rangle \cos\varphi') \cos\phi^L + |gg\rangle \sin\phi^L,$$

$$|n\bar{n}\rangle = \frac{|u\bar{u} + d\bar{d}\rangle}{\sqrt{2}}, \qquad f = \langle s\bar{s}|V|gg\rangle = \langle n\bar{n}|V|gg\rangle/\sqrt{2}$$

$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(1770)$	$f_0(2020)$	$f_0(2100)$
$(5 \pm 4)\%$	$< 5\%$	$(12 \pm 6)\%$	$(25 \pm 10)\%$	$(16 \pm 9)\%$	$(17 \pm 8)\%$.



Fractional glueball contents:

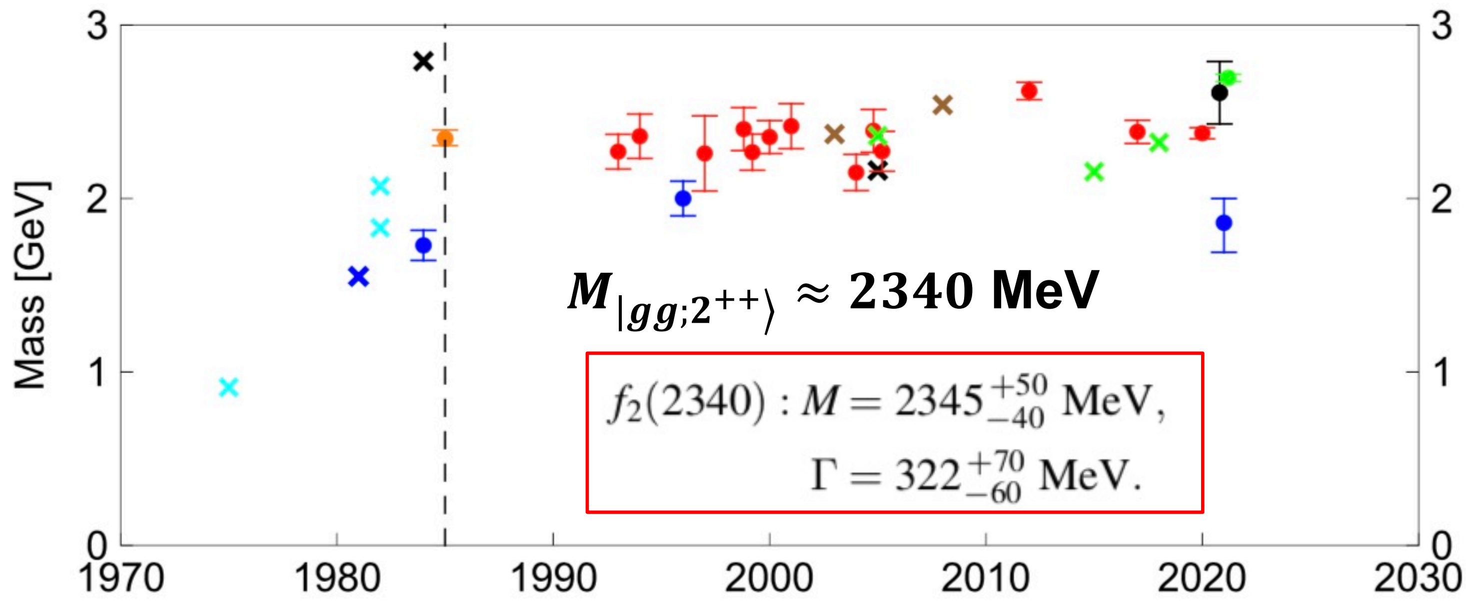
$(\mathbf{78 \pm 18})\%$

$M \approx$ **1865 MeV**

$\Gamma \approx$ **370 MeV**

Tensor glueball:

H.X.Chen et. al., RPP86(2023) 026201



$M_{|gg;2^{++}\rangle} \approx$ **2340 MeV**

$$f_2(2340): M = 2345^{+50}_{-40}\ \text{MeV},$$
$$\Gamma = 322^{+70}_{-60}\ \text{MeV}.$$

$$\mathcal{B}(J/\psi \to \gamma f_2(2340) \to \gamma K\bar{K}) = (5.54^{+0.34}_{-0.40}{}^{+3.82}_{-1.49}) \times 10^{-5},$$
$$\mathcal{B}(J/\psi \to \gamma f_2(2340) \to \gamma\eta\eta) = (5.60^{+0.62}_{-0.65}{}^{+2.37}_{-2.07}) \times 10^{-5},$$
$$\mathcal{B}(J/\psi \to \gamma f_2(2340) \to \gamma\phi\phi) = (1.91 \pm 0.14^{+0.72}_{-0.73}) \times 10^{-5},$$

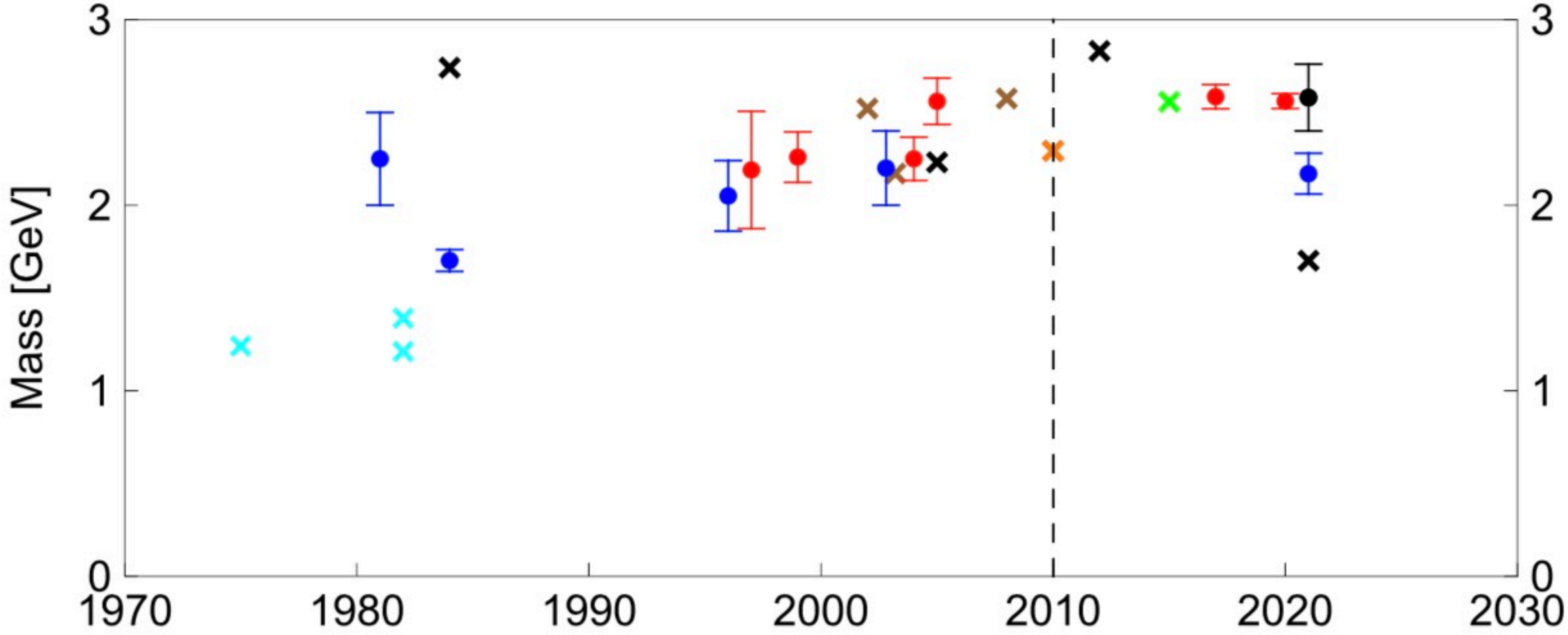
BESIII's measurements

$$\mathcal{B}(J/\psi \to \gamma|gg;2^{++}\rangle) = (1.1 \pm 0.2 \pm 0.1) \times 10^{-2}$$

PRL113(2013)091601

Pseudoscalar glueball:

RPP86(2023) 026201

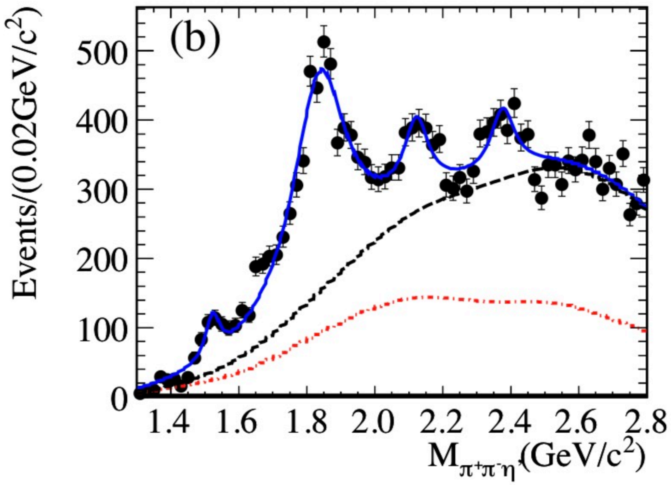


Average mass predictions:

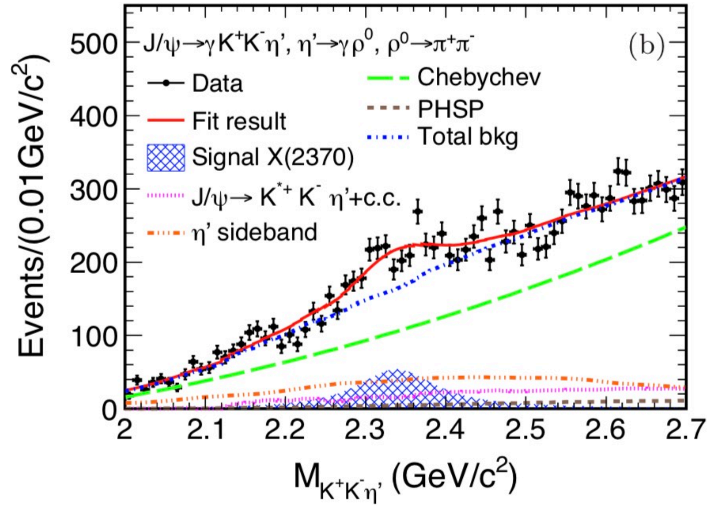
$$M_{|gg;0^{-+}\rangle} \approx \mathbf{2360\ MeV}$$

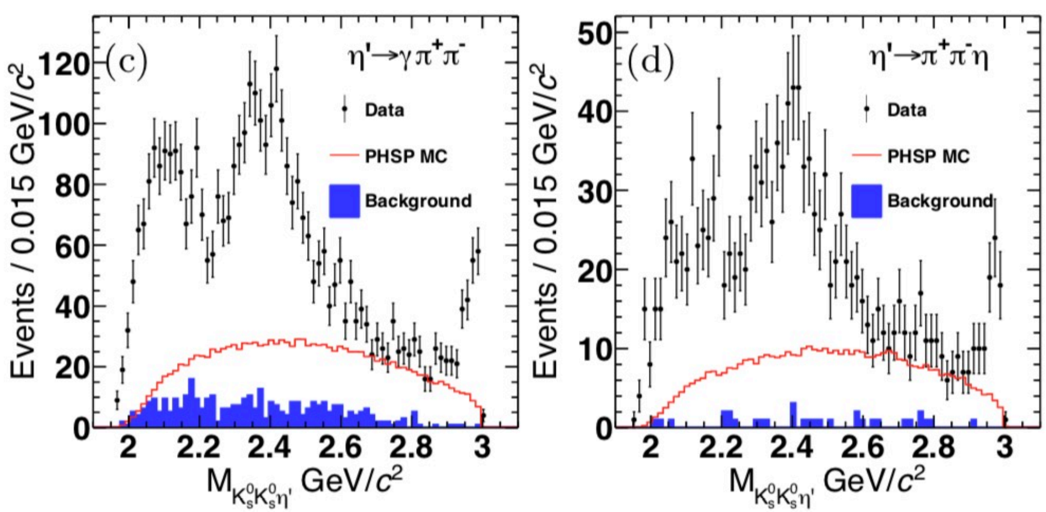
Pseudoscalar candidate X(2370): BESIII

PRL106(2011)072002: $J/\psi \rightarrow \gamma\pi\pi\eta'$



EPJC80(2020)746: $J/\psi \rightarrow \gamma KK\eta'$





PRL132(2024)181901:

$J/\psi \rightarrow \gamma K_S K_S \eta'$

$M = 2395 \pm 11^{+26}_{-94}\ MeV$

$\Gamma = 188^{+18+124}_{-17-33}\ MeV$

$J^{PC} = 0^{-+}$

Production rates in the radiative J/ψ decays:

$$\mathcal{B}(J/\psi \to \gamma X(2370) \to \gamma K^+ K^- \eta')$$
$$= (1.79 \pm 0.23 \pm 0.65) \times 10^{-5},$$
$$\mathcal{B}(J/\psi \to \gamma X(2370) \to \gamma K_S^0 K_S^0 \eta')$$
$$= (1.18 \pm 0.32 \pm 0.39) \times 10^{-5}.$$

EPJC80(2020)746;
PRD93(2016)112011

$$\mathcal{B}(J/\psi \to \gamma X(2500) \to \gamma \phi\phi) = (1.7 \pm 0.2^{+0.2}_{-0.8}) \times 10^{-5}.$$

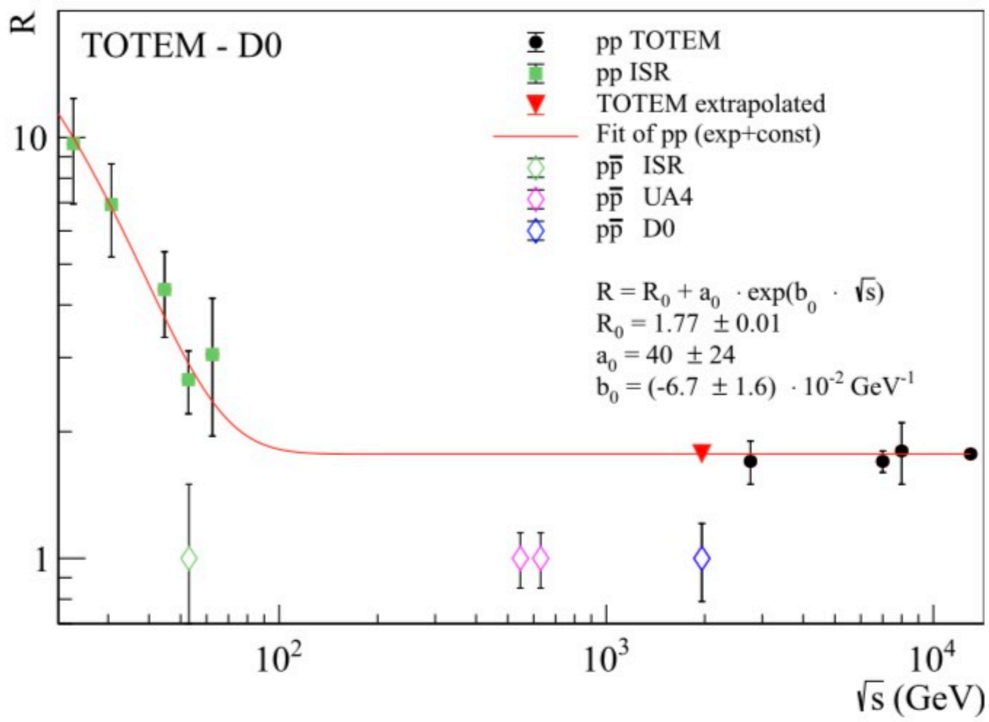
Close to the LQCD prediction of the pseudoscalar glueball

$$\mathcal{B}(J/\psi \to \gamma |gg; 0^{-+}\rangle) = (2.31 \pm 0.80) \times 10^{-4}.$$

PRD9100(2019)054511

Evidence for a t-channel exchanged odderon

PRL127(2021)062003



The discrepancy suggests the existence of a t-channel exchanged odderon with C=-. **If exist, it should be a three-gluon glueball!**

QCD sum rule calculations of glueballs

The relativistic two-gluon and three-gluon operators:

C=+

$$J_0 = g_s^2 G_a^{\mu\nu} G^a_{\mu\nu},$$

$$\tilde{J}_0 = g_s^2 G_a^{\mu\nu} \tilde{G}^a_{\mu\nu},$$

$$J_1^{\alpha\beta} = g_s^2 G_a^{\alpha\mu} \tilde{G}_\mu^{a,\beta} - \{\alpha \leftrightarrow \beta\},$$

$$J_2^{\alpha_1\alpha_2,\beta_1\beta_2} = \mathcal{S}[g_s^2 G_a^{\alpha_1\beta_1} G^{a,\alpha_2\beta_2}],$$

$$\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2} = \mathcal{S}[g_s^2 G_a^{\alpha_1\beta_1} \tilde{G}^{a,\alpha_2\beta_2}].$$

$$\eta_0 = f^{abc} g_s^3 G_a^{\mu\nu} G_{b,\nu\rho} G^\rho_{c,\mu},$$

$$\tilde{\eta}_0 = f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} \tilde{G}_{b,\nu\rho} \tilde{G}^\rho_{c,\mu},$$

$$\eta_1^{\alpha\beta} = f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} G_{b,\mu\nu} \tilde{G}_c^{\alpha\beta},$$

$$\tilde{\eta}_1^{\alpha\beta} = f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} G_{b,\mu\nu} G_c^{\alpha\beta},$$

0

$$\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2} = f^{abc} \mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\mu} G^{\beta_2}_{c,\mu} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

$$\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2} = f^{abc} \mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\mu} \tilde{G}^{\beta_2}_{c,\mu} - \{\alpha_2 \leftrightarrow \beta_2\}].$$

All spin-1 two- and three-gluon operators with C=+ vanish!

PRD103 (2021) L091503;PRD104(2021) 094050

Three-gluon operators:

C=-

$$\xi_1^{\alpha\beta} = d^{abc} g_s^3 G_a^{\mu\nu} G_{b,\mu\nu} G_c^{\alpha\beta},$$

$$\tilde{\xi}_1^{\alpha\beta} = d^{abc} g_s^3 G_a^{\mu\nu} G_{b,\mu\nu} \tilde{G}_c^{\alpha\beta},$$

$$\xi_2^{\alpha_1\alpha_2,\beta_1\beta_2} = d^{abc} \mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} G_b^{\alpha_2\mu} \tilde{G}_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

$$\tilde{\xi}_2^{\alpha_1\alpha_2,\beta_1\beta_2} = d^{abc} \mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\mu} G_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

$$\xi_3^{\cdots} = d^{abc} \mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\beta_2} G_c^{\alpha_3\beta_3}],$$

$$\tilde{\xi}_3^{\cdots} = d^{abc} \mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\beta_2} \tilde{G}_c^{\alpha_3\beta_3}],$$

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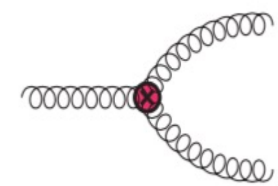
Glueball sum rules:

Two-point correlation function:

$$\Pi^{\alpha\beta,\alpha'\beta'}(q^2) \equiv i \int d^4x e^{iqx} \langle 0|\mathbf{T}[J_1^{\alpha\beta}(x) J_1^{\alpha'\beta'\dagger}(0)]|0\rangle$$
$$= (g^{\alpha\alpha'} g^{\beta\beta'} - g^{\alpha\beta'} g^{\beta\alpha'})\Pi(q^2),$$

Gluon field strength tensor

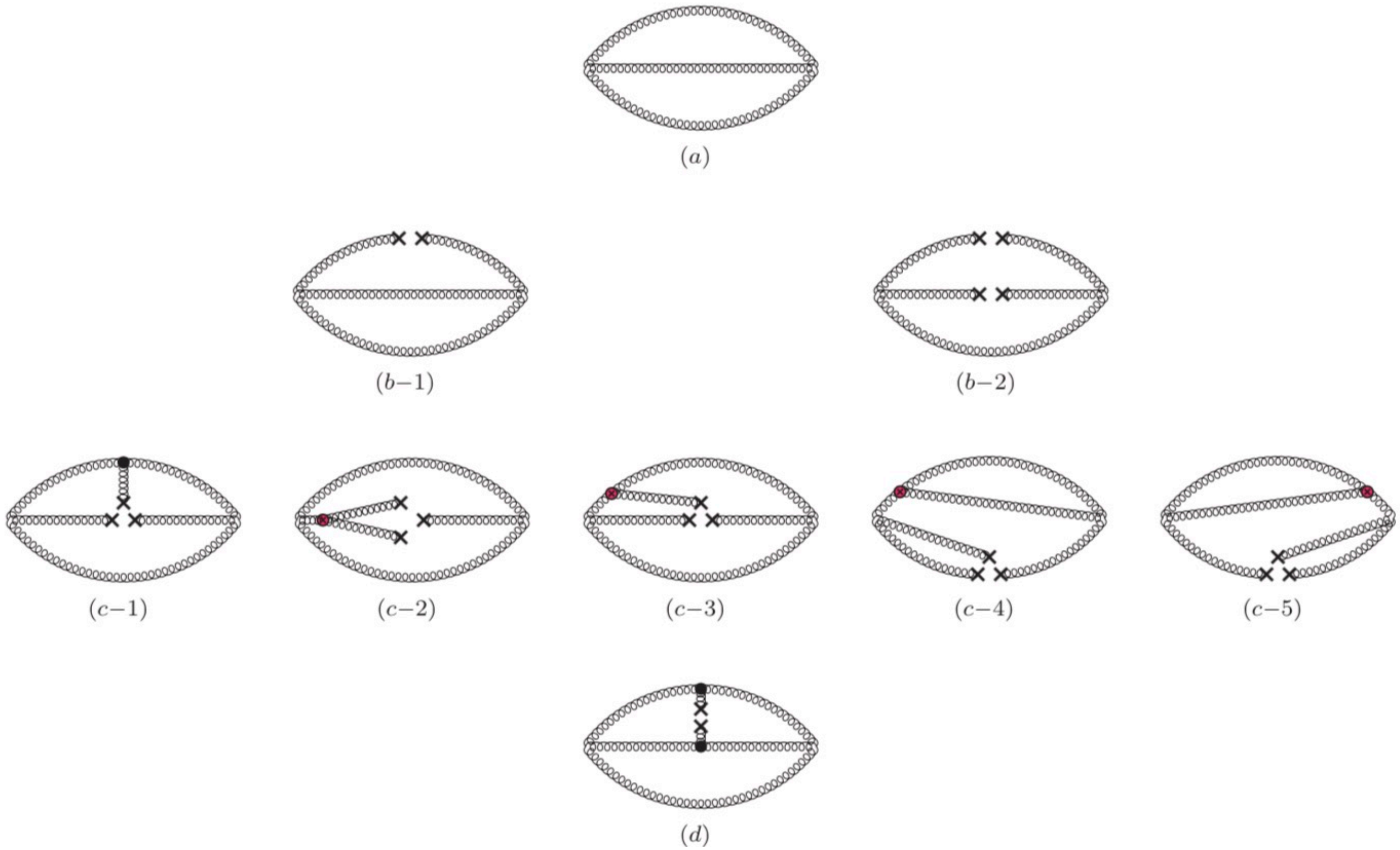
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A_{b,\mu} A_{c,\nu},$$



Full gluon field propagator in the fixed point gauge

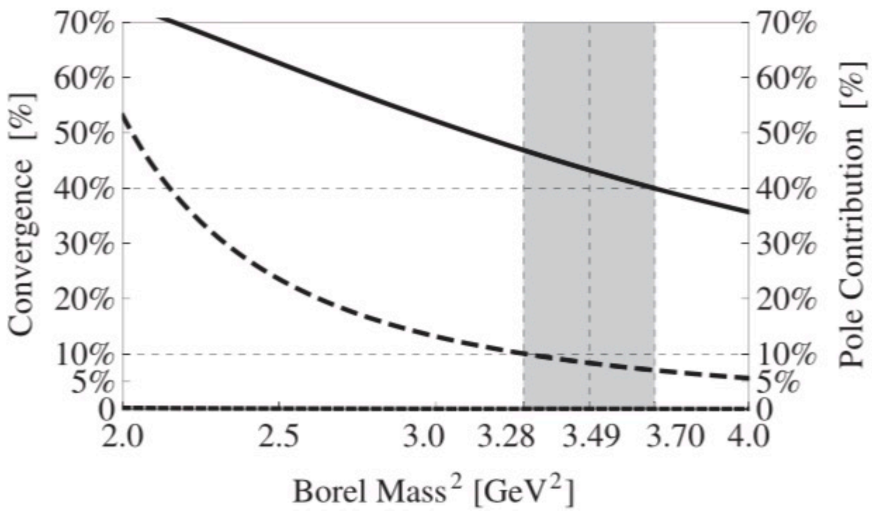
$$\langle 0|\mathbf{T}[A^a_\mu(x) A^b_\nu(y)]|0\rangle = \frac{\delta^{ab} g_{\mu\nu}}{4\pi^2 (x-y)^2}$$
$$+ \frac{g_s \ln(-(x-y)^2)}{8\pi^2} f^{abc} G_{c,\mu\nu}$$
$$- \frac{g_s g_{\mu\nu} x^\alpha y^\beta}{8\pi^2 (x-y)^2} f^{abc} G_{c,\alpha\beta}(0).$$

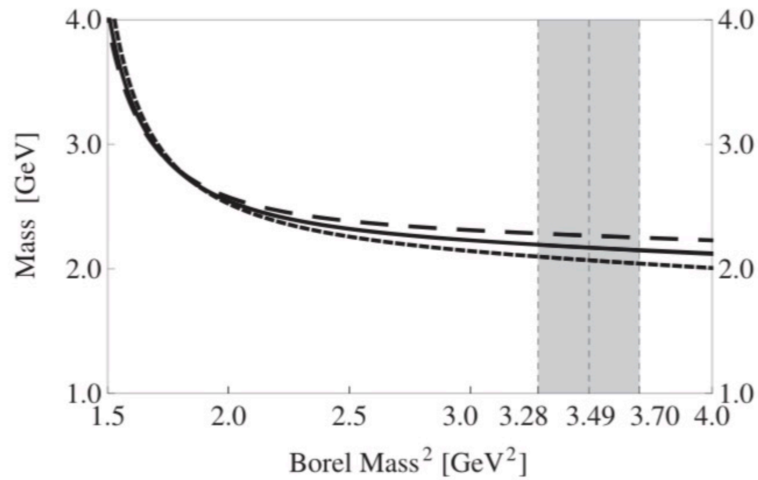
OPEs are calculated up to d=8 condensates



Pseudoscalar glueball

PRD104(2021) 094050





Mass prediction:

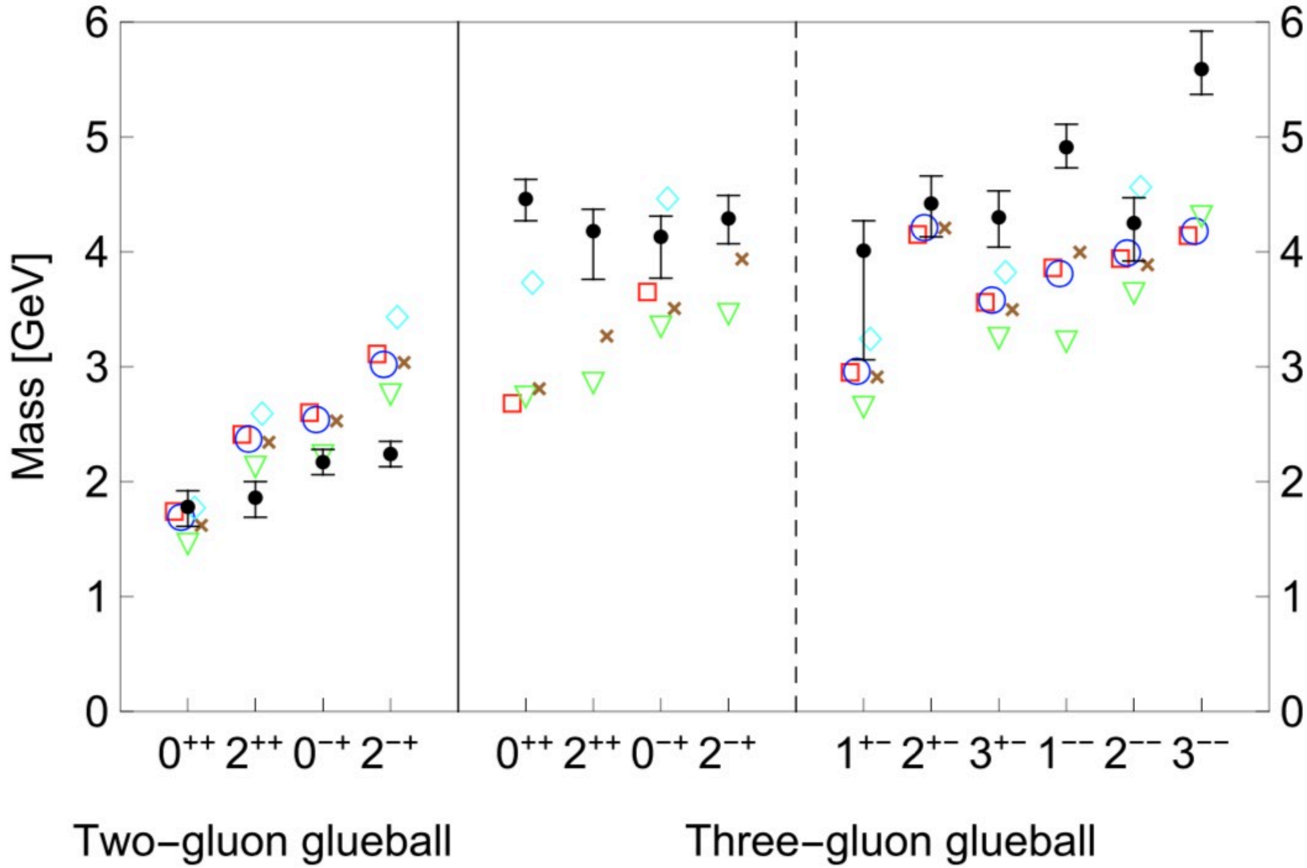
$$M_{|GG;0^{-+}\rangle} = 2.17 \pm 0.11 \text{ GeV}.$$

Glueball mass spectrum

Glueball	Current	$s_0^{\min}$ [GeV^2]	Working Regions s_0 [GeV^2]	Working Regions M_B^2 [GeV^2]	Pole [%]	Mass [GeV]
$\lvert GG;0^{++}\rangle$	J_0	7.8	9.0 ± 1.0	3.70–4.19	40–48	$1.78^{+0.14}_{-0.17}$
$\lvert GG;2^{++}\rangle$	$J_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	8.5	10.0 ± 1.0	3.99–4.60	40–50	$1.86^{+0.14}_{-0.17}$
$\lvert GG;0^{-+}\rangle$	$\tilde{J}_0$	8.2	9.0 ± 1.0	3.28–3.70	40–47	$2.17^{+0.11}_{-0.11}$
$\lvert GG;2^{-+}\rangle$	$\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	8.1	10.0 ± 1.0	3.27–4.20	40–55	$2.24^{+0.11}_{-0.11}$
$\lvert GGG;0^{++}\rangle$	η_0	31.6	33.0 ± 3.0	7.25–7.61	40–44	$4.46^{+0.17}_{-0.19}$
$\lvert GGG;2^{++}\rangle$	$\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	16.0	35.0 ± 3.0	4.77–9.04	40–90	$4.18^{+0.19}_{-0.42}$
$\lvert GGG;0^{-+}\rangle$	$\tilde{\eta}_0$	17.0	33.0 ± 3.0	4.48–8.13	40–88	$4.13^{+0.18}_{-0.36}$
$\lvert GGG;2^{-+}\rangle$	$\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	33.1	35.0 ± 3.0	8.10–8.53	40–44	$4.29^{+0.20}_{-0.22}$
$\lvert GGG;1^{+-}\rangle$	$\xi_1^{\alpha\beta}$	9.0	34.0 ± 4.0	3.16–9.09	40–99	$4.01^{+0.26}_{-0.95}$
$\lvert GGG;2^{+-}\rangle$	$\xi_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	32.7	35.0 ± 4.0	7.53–8.09	40–46	$4.42^{+0.24}_{-0.29}$
$\lvert GGG;3^{+-}\rangle$	$\xi_3^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}$	30.2	33.0 ± 4.0	7.69–8.40	40–47	$4.30^{+0.23}_{-0.26}$
$\lvert GGG;1^{--}\rangle$	$\tilde{\xi}_1^{\alpha\beta}$	31.2	34.0 ± 4.0	5.81–6.77	40–51	$4.91^{+0.20}_{-0.18}$
$\lvert GGG;2^{--}\rangle$	$\tilde{\xi}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	19.7	36.0 ± 4.0	5.80–9.47	40–81	$4.25^{+0.22}_{-0.33}$
$\lvert GGG;3^{--}\rangle$	$\tilde{\xi}_3^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}$	35.8	38.0 ± 4.0	6.15–7.22	40–49	$5.59^{+0.33}_{-0.22}$

PRD103 (2021) L091503;
PRD104(2021) 094050

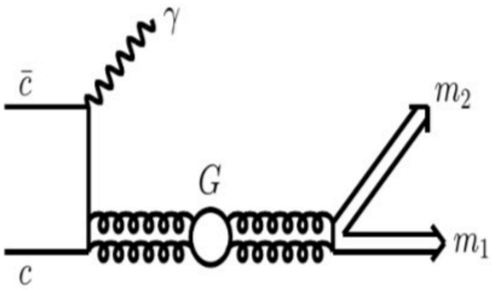
Comparing to LQCD's results



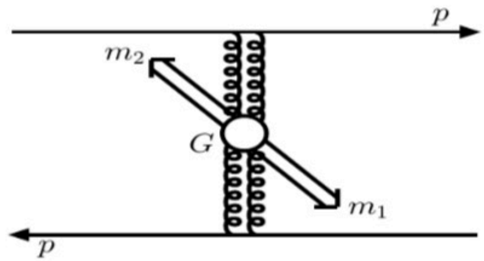
RPP86(2023) 026201

Productions and decays

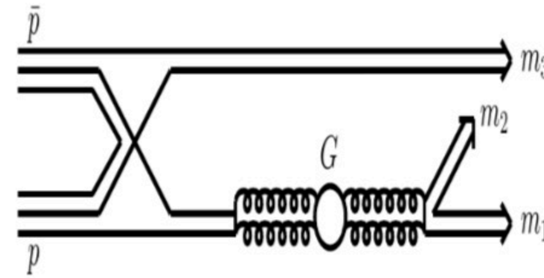
Phys. Rept., 454:1–202, 2007



J/ψ radiative decay

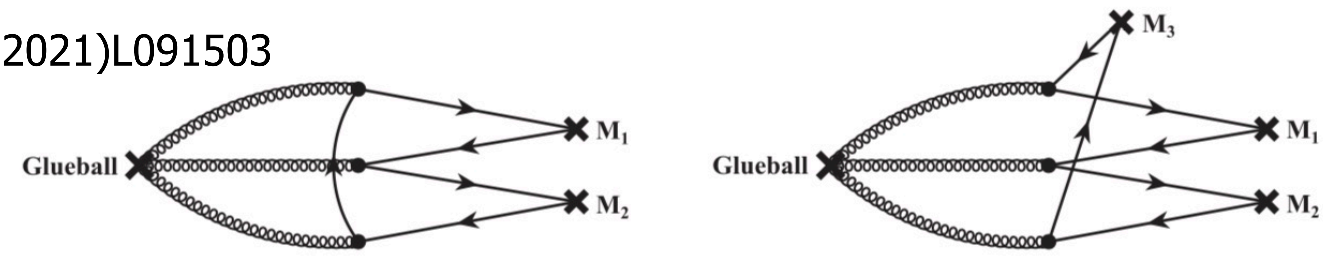


double pomeron exchange



$p\bar{p}$ annhilation

PRD103(2021)L091503



(a) (b)

$0^- \to$	PPP, VVP, VVV	(S-wave)	&	VP, VV	(P-wave),
$0^+ \to$	VPP, VVP, VVV	(P-wave)	&	PP, VV	(S-wave),
$1^- \to$	VPP, VVP, VVV	(S-wave)	&	PP, VP, VV	(P-wave),
$1^+ \to$	PPP, VPP, VVP, VVV	(P-wave)	&	VP, VV	(S-wave),
$2^- \to$	VVP, VVV	(S-wave)	&	VP, VV	(P-wave),
$2^+ \to$	VPP, VVP, VVV	(P-wave)	&	VV	(S-wave),
$3^- \to$	VVV	(S-wave)	&	VV	(P-wave),
$3^+ \to$	VVP, VVV	(P-wave)	&	VP, VV	(D-wave).

Summary

- The existence of glueball is one of the most distinctive prediction of QCD, and essential to the confirmation of the theory!
- **We systematically calculated the mass spectra of the two-gluon and three-gluon glueballs.**
- **The ground state of spin-1 glueballs with C=+ do not exist in the relativistic framework.**
- Some gluon-rich processes are expected to detect glueballs.

Thank you