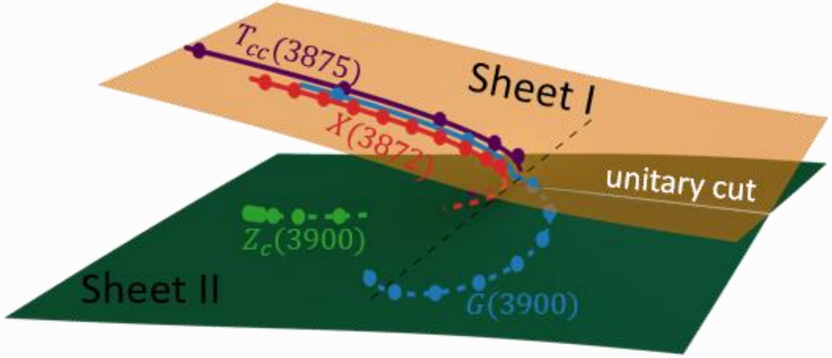
P-wave molecular resonance: $G(3900)$ & $X_1(2900)$



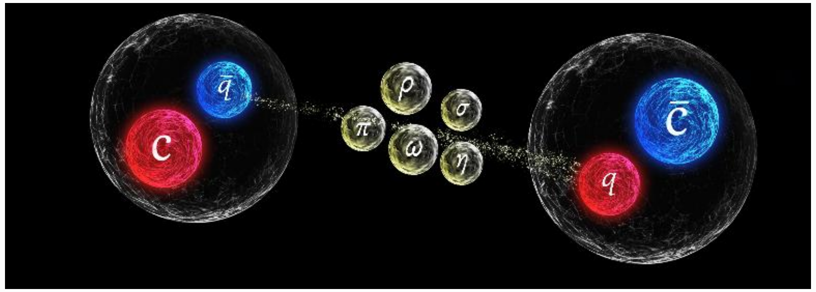
Based on
Z.-Y. Lin, J.-Z. Wang, J.-B. Cheng, L. Meng, and S.-L. Zhu, arXiv:2403.01727 (PRL, in press)
& J.-Z. Wang, Z.-Y. Lin, B. Wang, L. Meng, and S.-L. Zhu, Phys. Rev. D 110, 114003 (2024)

Speaker: Zi-Yang Lin (林子阳)

2024/12/9

[redacted]

OUTLINE

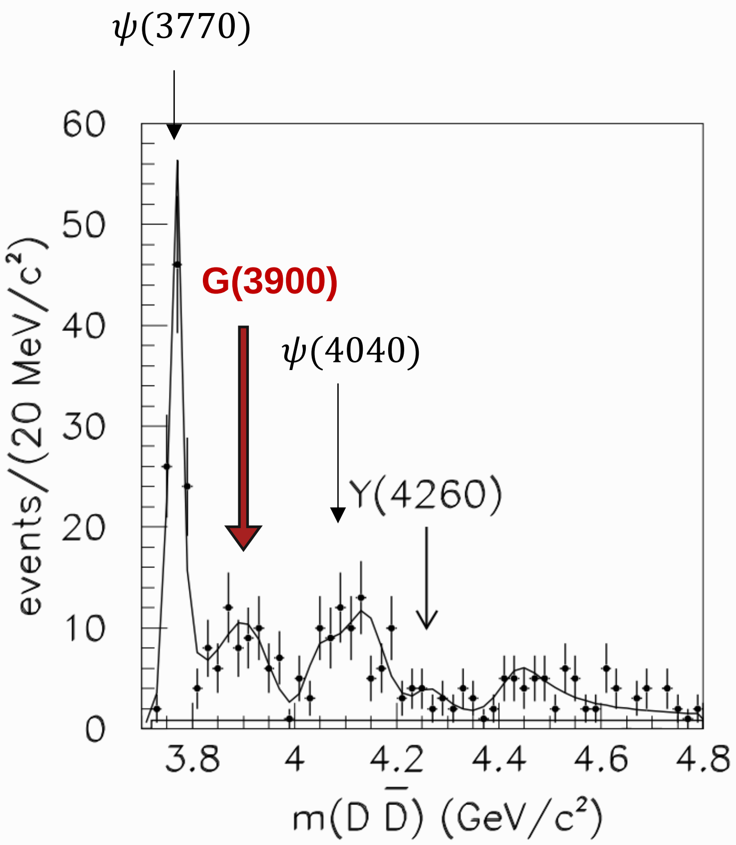


- Background ---- $G(3900)$ structure
- P-wave resonance mechanism
- Unified description of $X(3872), T_{cc}(3875)$, $Z_c(3900)$ and $G(3900)$
- $X_1(2900)$ as the P-wave $\bar{D}^*K^*$ resonance
- Predictions and discussions

Motivation--Cross Sections for $e^+e^- \rightarrow D\bar{D}$

fitted with Gaussian function

- Structure near $D\bar{D}^*$ threshold, referred to as **$G(3900)$ structure**



(BaBar Collaboration),
Phys. Rev. D 76, 111105 (2007),

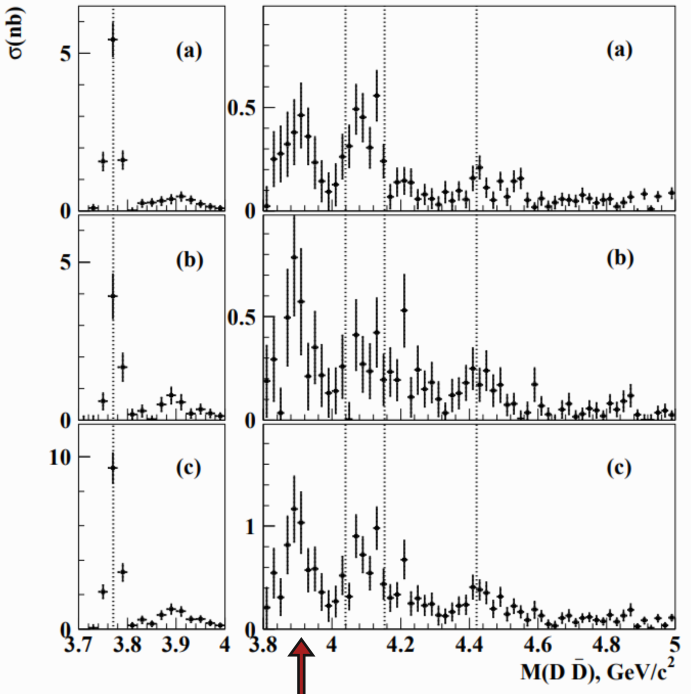


FIG. 3: The exclusive cross sections for: (a) $e^+e^- \to D^0\overline{D}^0$; (b) $e^+e^- \to D^+D^-$; (c) $e^+e^- \to D\overline{D}$. The dotted lines correspond to the $\psi(3770)$, $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$ masses [20].

(Belle Collaboration),
Phys. Rev. D 77, 011103 (2008)

Motivation--Cross Sections for $e^+e^- \to D\bar{D}$

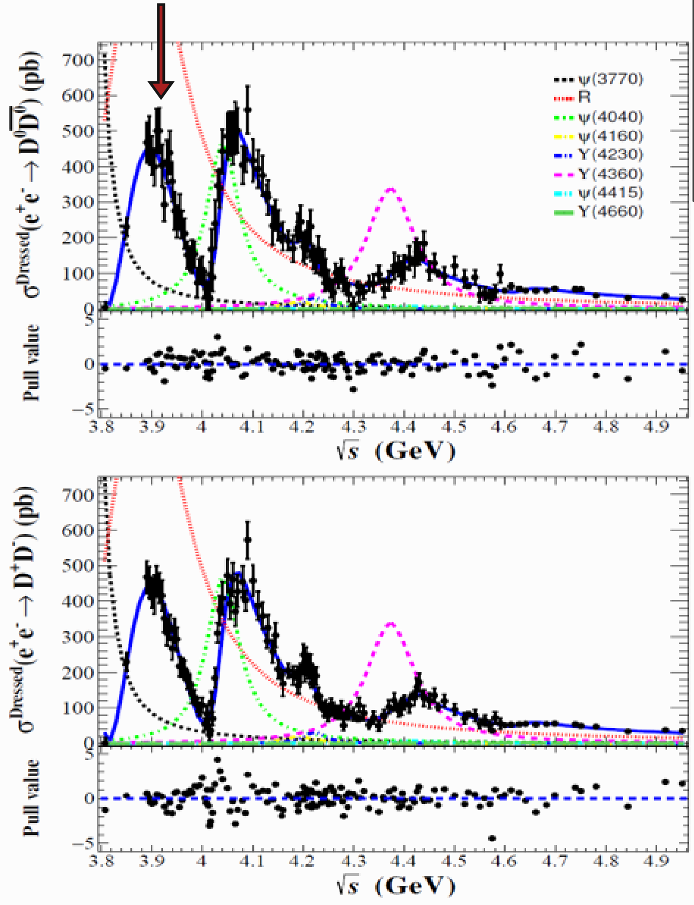
- Fit with relativistic BW

G(3900)

Mass: $3872.5 \pm 14.2 \pm 3.0$ MeV

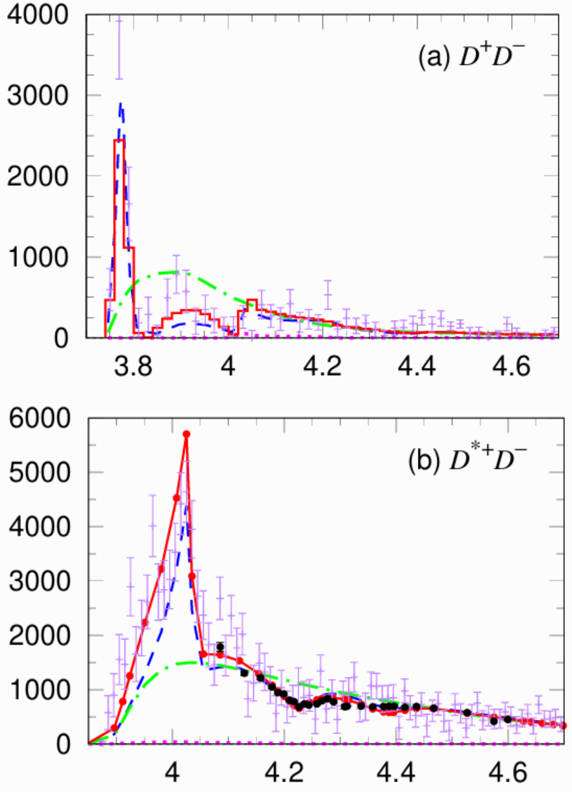
Width: $179.7 \pm 14.1 \pm 7.0$ MeV

$S(\sigma) > 20$



(BESIII Collaboration),
Phys. Rev. Lett. 133, 081901 (2024)

- Global fit, no pole for G(3900)



S.X. Nakamura, X.-H. Li, H.-P. Peng, Z.-T. Sun, and X.-R. Zhou, arxiv: 2312.17658

I Background—G(3900)/Y(3872)?

Is the $G(3900)$ structure a genuine resonance?

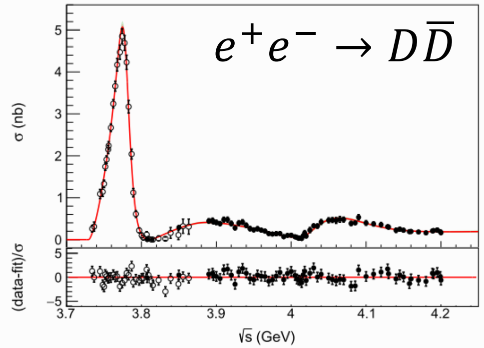
- Poltergeist?

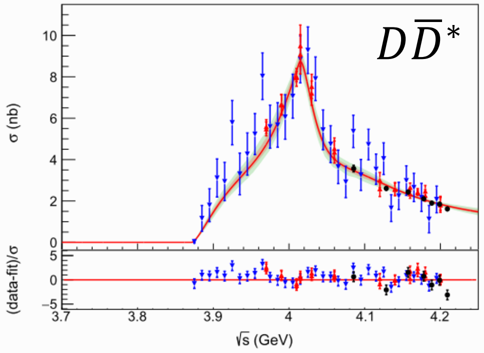
$$K_{\mu,\nu} = \sum_R \frac{g_{R:\mu} g_{R:\nu}}{m_R^2 - s} + f_{\mu,\nu}$$

$$\mathcal{M} = (1 + KC)^{-1} K$$

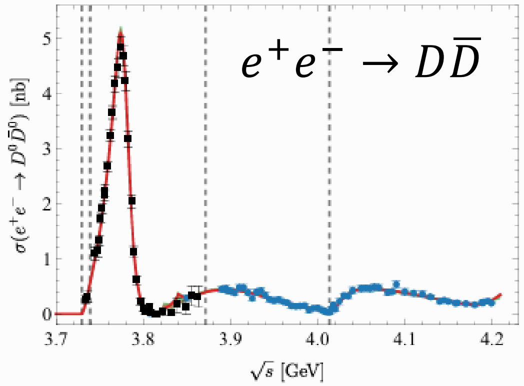
interference of $\psi(3770)$ and $\psi(4040)$ and coupled-channel effect with $D\bar{D}^*$

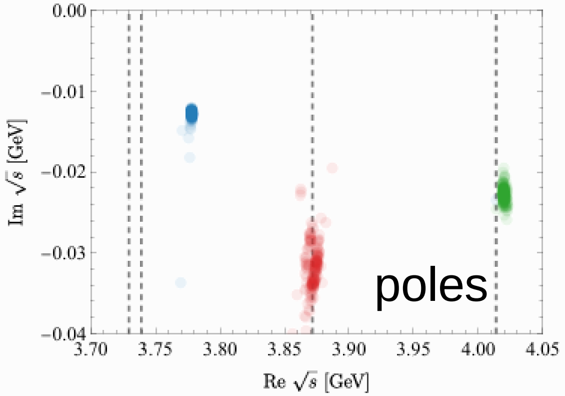
N. Hüsken et al, Phys. Rev. D 109, 114010 (2024)

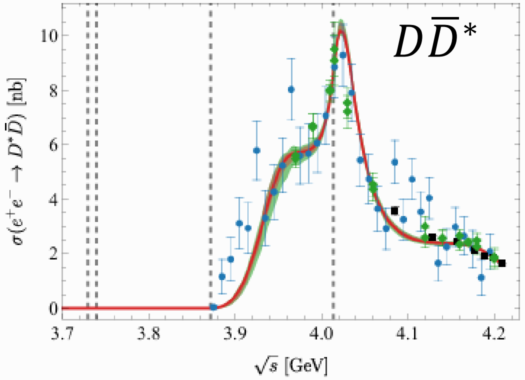




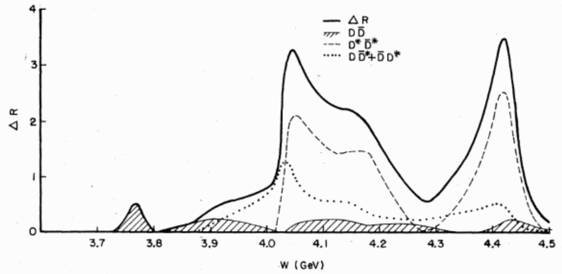
- Our *K*-matrix formalism fit with carefully done analytical continuation

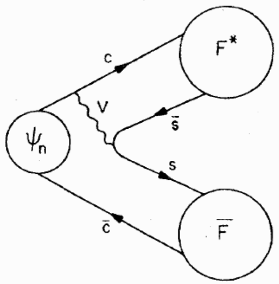






- Cornell model

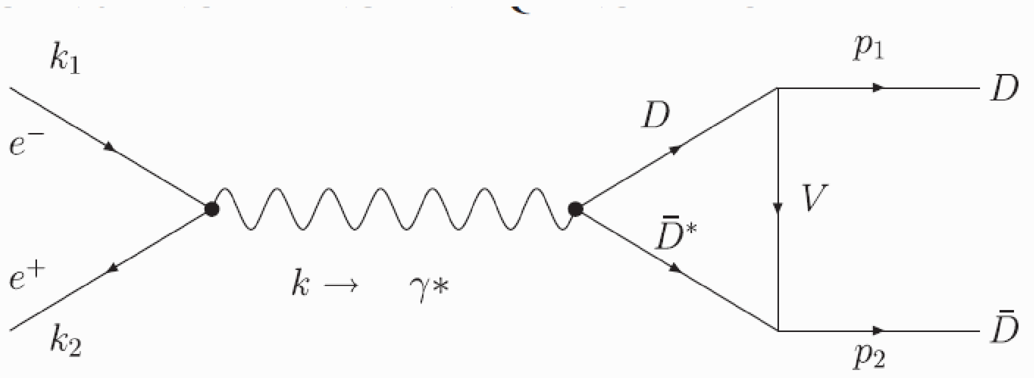




E. Eichten, K. Gottfried, T. Kinoshita, K. Lane, and T.-M. Yan, Phys. Rev. D 17, 3090 (1978)

I Background—G(3900)/Y(3872)?

- Previous investigations on $G(3900)$
- kinematic effect at the threshold



Y.J. Zhang and Q. Zhao, Phys. Rev. D 81, 034011 (2010)

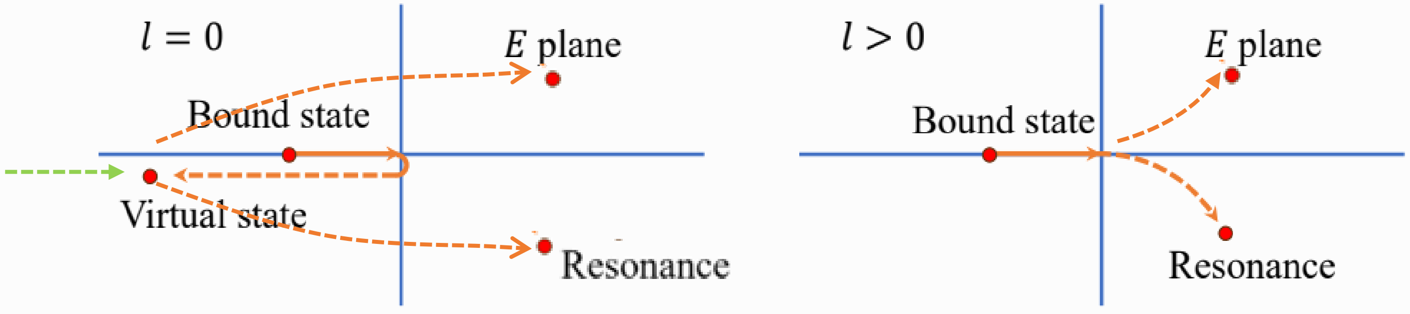
- coupled-channel effects driven by the contact interactions

Sheet	Poles (GeV)	$\|g_{D\bar{D}}\|$	$\|g_{D\bar{D}^*}\|$	$\|g_{D^*\bar{D}^*_{s=0}}\|$	$\|g_{D^*\bar{D}^*_{s=2}}\|$
II	$3.764 \pm i0.006$	13.53	9.48	5.88	16.78
III	$3.879 \pm i0.035$	4.40	10.96	7.63	18.15
IV	$4.034 \pm i0.014$	2.90	2.23	12.52	12.85

M.L. Du, U. G. Meissner and Q. Wang Phys. Rev. D 94 (2016) 9, 096006

II Formalism – P-wave resonance

⭐ P-wave resonance mechanism



(a) (b)

Jost function becomes quadratic at the lowest order

$$f_l(p) = 1 + [\alpha_l + \beta_l p^2 + O(p^4)] + i[\gamma_l p^{2l+1} + O(p^{2l+3})]$$

J. R. Taylor, Scattering Theory: The Quantum Theory of Nonrelativistic Collision

- As the attraction being weaker, p-wave bound state naturally turns resonances.
- Shape-type resonance rather than Feshbach resonance (bound states coupled with open channels)

II Formalism – OBE model

- The hadron-hadron interactions are fullfilled with pseudoscalar and vector meson exchange

$$
\begin{aligned}
\mathcal{L} = & \; g_s \mathrm{Tr}\left[\mathcal{H}\sigma\bar{\mathcal{H}}\right] + ig_a \mathrm{Tr}\left[\mathcal{H}\gamma_\mu\gamma_5\mathcal{A}^\mu\bar{\mathcal{H}}\right] \\
& + i\beta \mathrm{Tr}\left[\mathcal{H}v_\mu(\mathcal{V}^\mu - \rho^\mu)\bar{\mathcal{H}}\right] + i\lambda \mathrm{Tr}\left[\mathcal{H}\sigma_{\mu\nu}F^{\mu\nu}\bar{\mathcal{H}}\right] \\
& + g_s \mathrm{Tr}\left[\bar{\tilde{\mathcal{H}}}\sigma\tilde{\mathcal{H}}\right] + ig_a \mathrm{Tr}\left[\bar{\tilde{\mathcal{H}}}\gamma_\mu\gamma_5\mathcal{A}^\mu\tilde{\mathcal{H}}\right] \\
& - i\beta \mathrm{Tr}\left[\bar{\tilde{\mathcal{H}}}v_\mu(\mathcal{V}^\mu - \rho^\mu)\tilde{\mathcal{H}}\right] + i\lambda \mathrm{Tr}\left[\bar{\tilde{\mathcal{H}}}\sigma_{\mu\nu}F^{\mu\nu}\tilde{\mathcal{H}}\right]
\end{aligned}
$$

OBE model
$(\pi, \eta, \sigma, \rho, \omega)$

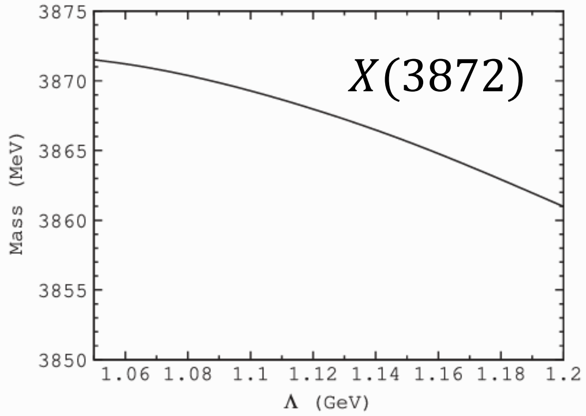
$$
\rho^\mu = \frac{ig_V}{\sqrt{2}}\begin{pmatrix} \frac{\rho_0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ \\ \rho^- & -\frac{\rho_0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \end{pmatrix}^\mu, \quad \mathbb{P} = \begin{pmatrix} \frac{\pi_0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ \\ \pi^- & -\frac{\pi_0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \end{pmatrix}.
$$

$$
\mathcal{V}^\mu = \frac{1}{2}[\xi^\dagger, \partial_\mu \xi], \quad \mathcal{A}^\mu = \frac{1}{2}\{\xi^\dagger, \partial_\mu \xi\} \qquad F^{\mu\nu} = \partial^\mu\rho^\nu - \partial^\nu\rho^\mu - [\rho^\mu, \rho^\nu]
$$

$$
\xi = \exp(i\mathbb{P}/f_\pi).
$$

succeed in decribing the $X(3872)$

N. Li and S.-L. Zhu Phys. Rev. D 86, 074022 (2012)



& predicting the T_{cc}^+

TABLE IV. The numerical results for the $D^{(*)}D^{(*)}$ system. "$* * *$" means the corresponding state does not exist due to symmetry while "$\cdots$" means there does not exist binding energy with the cutoff parameter less than 3.0 GeV. The binding energies for the states $D^{(*)}D^{(*)}[I(J^P) = 0(1^+)]$ and $D^{(*)}D^{(*)}[I(J^P) = 1(1^+)]$ are relative to the threshold of DD^* while that of the state $D^{(*)}D^{(*)}[I(J^P) = 1(0^+)]$ is relative to the DD threshold.

			$D^{(*)}D^{(*)}$							
I	J^P		OPE				OBE			
	0^+		$* * *$				$* * *$			
		Λ (GeV)	1.05	1.10	1.15	1.20	0.95	1.00	1.05	1.10
		B.E. (MeV)	1.24	4.63	11.02	20.98	0.47	5.44	18.72	42.82
		M (MeV)	3874.61	3871.22	3864.83	3854.87	3875.38	3870.41	3857.13	3833.03
		r_{rms} (fm)	3.11	1.68	1.12	0.84	4.46	1.58	0.91	0.64
0	1^+	P_1 (%)	96.39	92.71	88.22	83.34	97.97	92.94	85.64	77.88
		P_2 (%)	0.73	0.72	0.57	0.42	0.58	0.55	0.32	0.15
		P_3 (%)	2.79	6.45	11.07	16.11	1.41	6.42	13.97	21.91
		P_4 (%)	0.08	0.13	0.14	0.13	0.04	0.09	0.08	0.05

N. Li, Z.-F. Sun, X. Liu, and S.-L. Zhu Phys. Rev. D 88,114008 (2013)

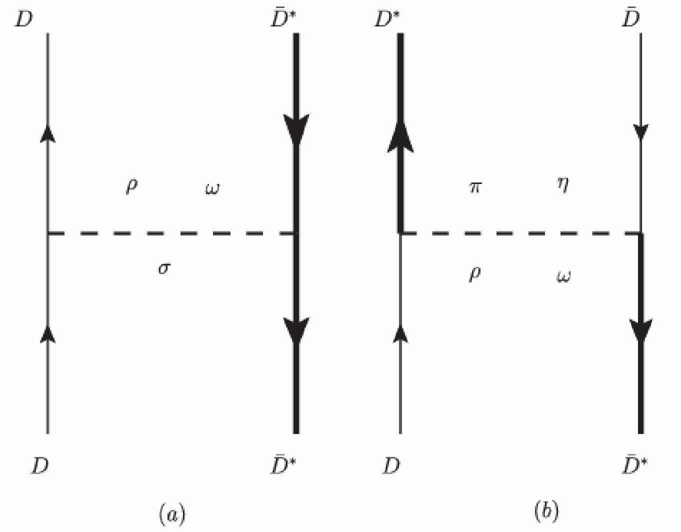
III Formalism – OBE model

- **Nonlocality** of one-pion exchange: $\vec{q} = \vec{p}_1 - \vec{p}_3$ $\quad \vec{k} = \vec{p}_1 - \vec{p}_4 = \vec{p}_1 + \vec{p}_3$

  extra minus signs in cross diagrams (u-channel) related to C-parity in odd partial waves

$$|C = \pm\rangle = \frac{1}{\sqrt{2}}(|D(\boldsymbol{p})\bar{D}^*(-\boldsymbol{p})\rangle \mp |\bar{D}(\boldsymbol{p})D^*(-\boldsymbol{p})\rangle).$$

  if mistake $\vec{k}$ to $\vec{q}$



direct diagram cross diagram

- nonlocal regulator:

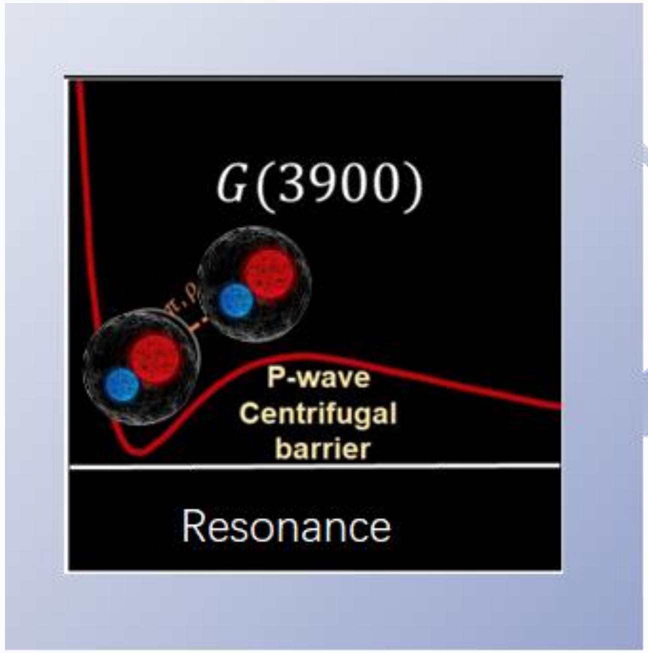
$$V(\boldsymbol{p}', \boldsymbol{p}) \to V(\boldsymbol{p}', \boldsymbol{p}) \frac{\Lambda^2}{p'^2 + \Lambda^2} \frac{\Lambda^2}{p^2 + \Lambda^2}.$$

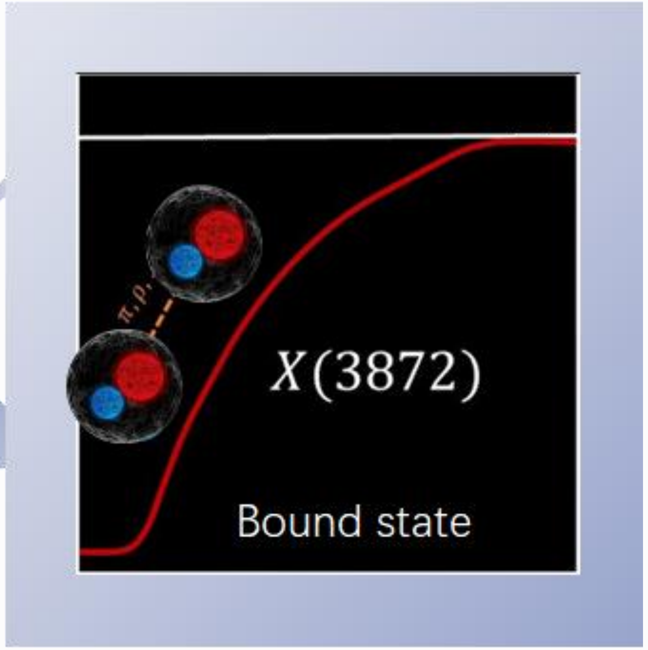
- local regulator (to test the regulator-independency)

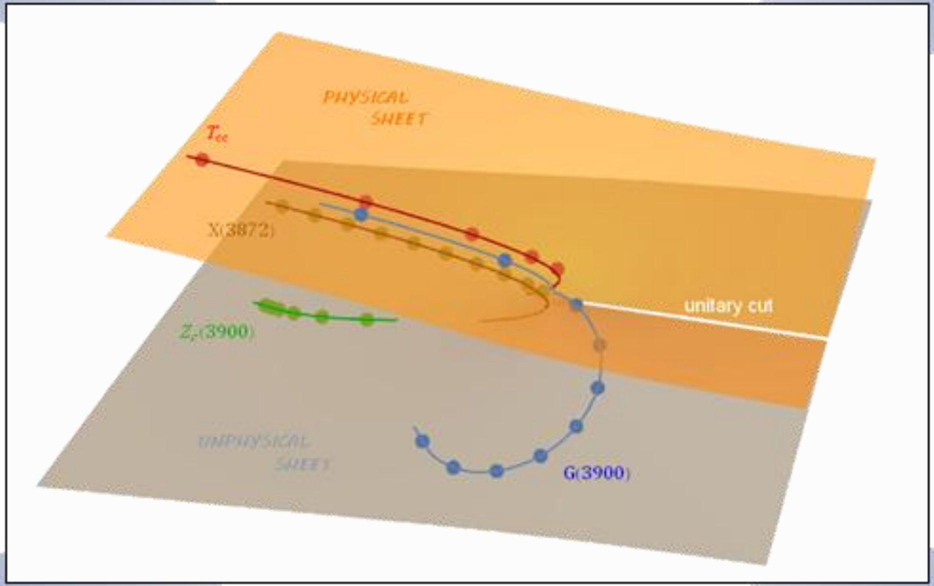
$$V^D(\boldsymbol{q}) \to V^D(\boldsymbol{q}) \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + \boldsymbol{q}^2}\right)^2, \qquad V^C(\boldsymbol{k}) \to V^C(\boldsymbol{k}) \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + \boldsymbol{k}^2}\right)^2$$

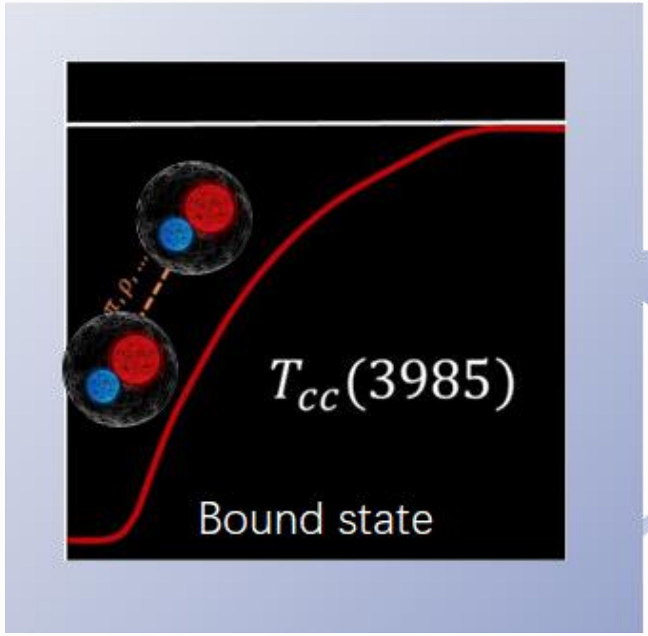
- Poles derived by complex scaling method / Lippmann-Schwinger equation

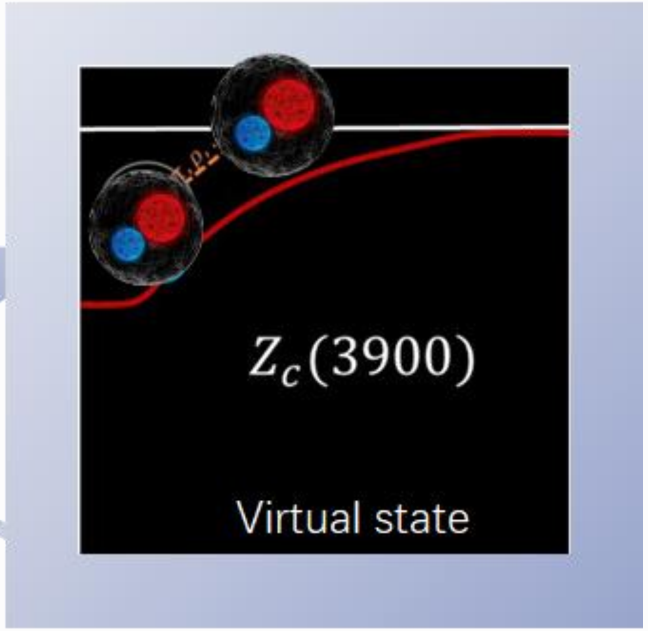
potentials in coordinate space
(local cutoff, just for display)





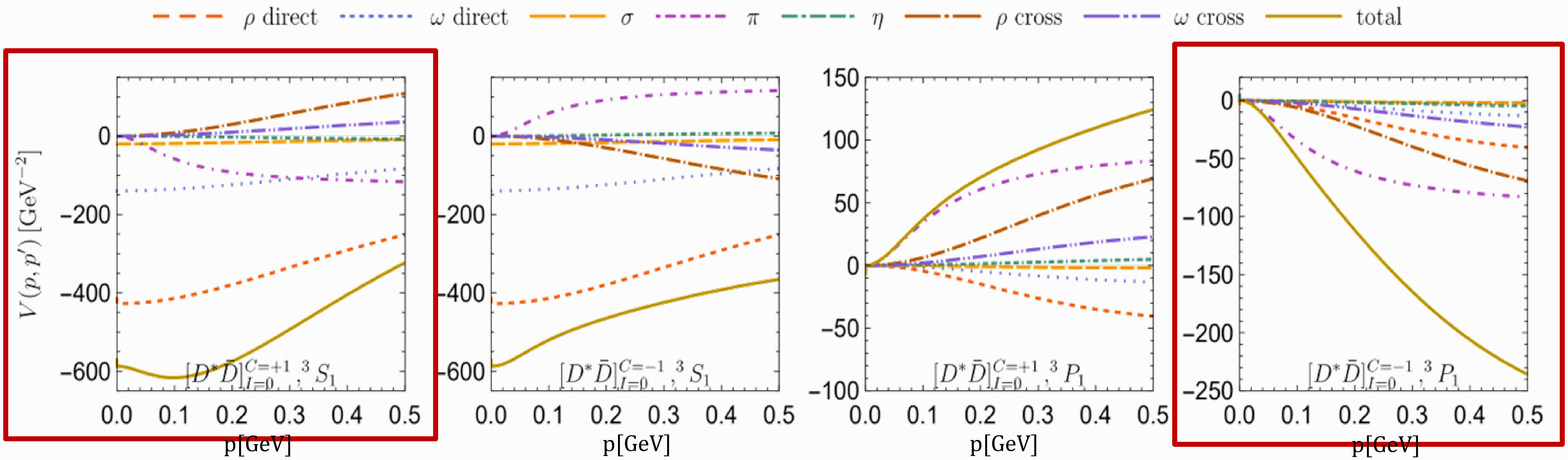






Results – Unified description of $D\bar{D}^*$ molecules

- Partial-wave potentials $V(p,p')$ for different exchanged mesons: ($p' = p$ in the figure)



$\mathbf{1^{++}}$, $\boldsymbol{X(3872)}$ | $\mathbf{1^{+-}}$ | $\mathbf{1^{-+}}$ | $\mathbf{1^{--}}$, $\boldsymbol{G(3900)}$

a little weaker than 1^{++}

S-wave

P-wave

Results – Unified description of $D\bar{D}^*/DD^*$ molecules

- Fix the cutoff to generate loosely bound $X(3872), T_{cc}^+$, we obtain

  virtual state $Z_c(3900)$

  left-hand cut problem – need special integral contour

  p-wave resonance $G(3900)$

- potentials related by G-parity rule
- the same set of parameters

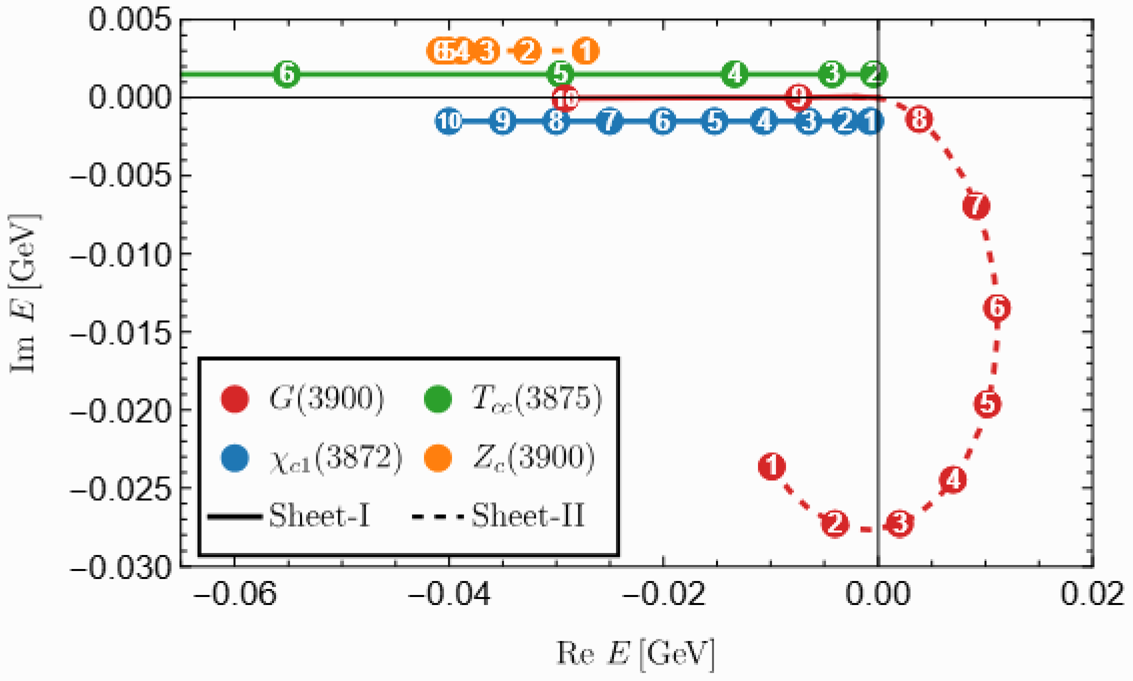
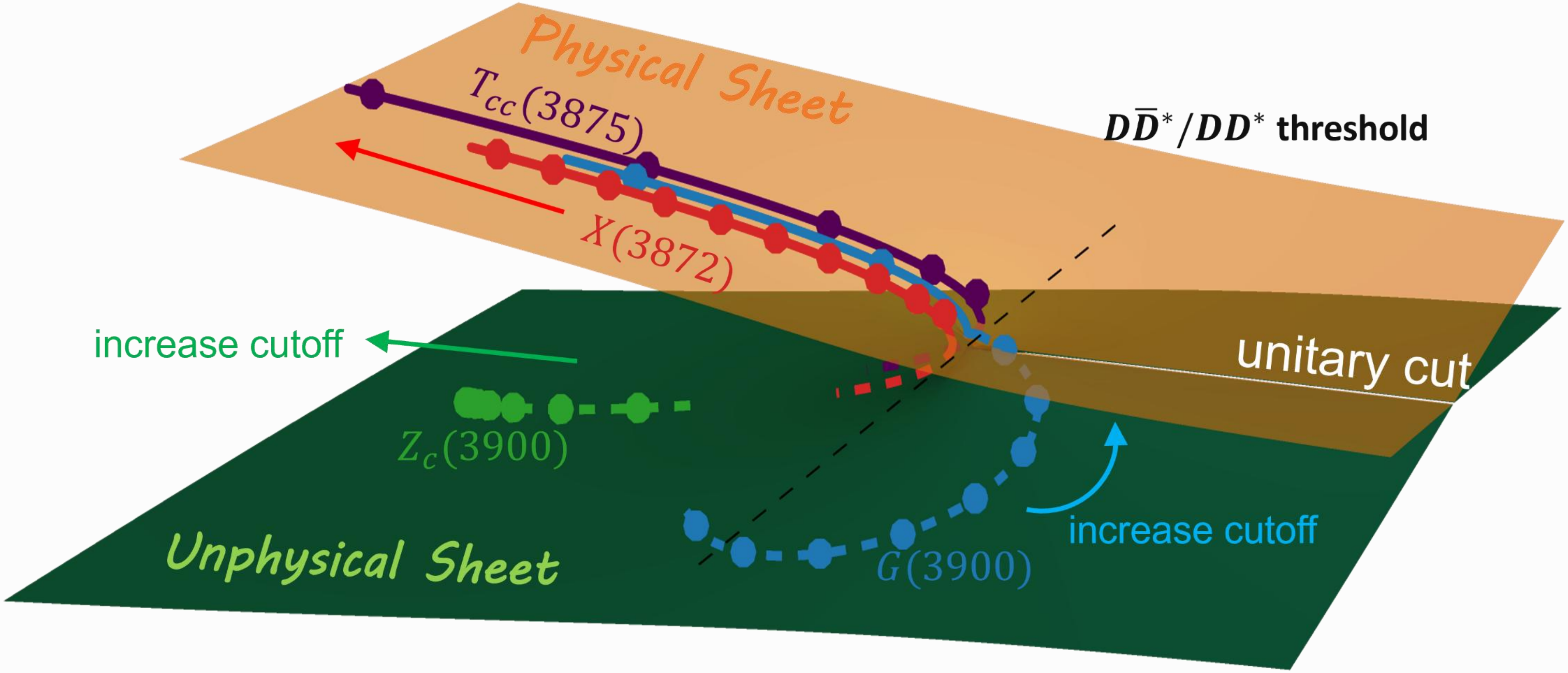


FIG. 4. The pole trajectories with the cutoff parameters correspond to $\chi_{c1}(3872)$, $T_{cc}(3875)$, $Z_c(3900)$ and the newly observed $G(3900)$ states. The circled number 1-10 represent the increasing cutoff 0.4-1.3 GeV in order. The solid (dashed) lines represent the pole trajectories in the physical (unphysical) Riemann sheets. The poles on the negative real axis are slightly shifted for transparency.

Results – Unified description of $D\bar{D}^*/DD^*$ molecules



Discussion – Robustness of our conclusion

- Higher partial waves are dominant by long-range interactions (pion exchange)
- S-wave resonance relies on a special shape of potentials, while P-wave resonances can be naturally generated thanks to the centrifugal barrier
- P-wave interactions fixed by S-wave states ------ cutoff independent
- Isospin breaking, three-body effects have little influence on $G(3900)$

III

Predictions – P-wave resonances

Nonlocal Regulator:

$$V(\mathbf{p}',\mathbf{p}) \to V(\mathbf{p}',\mathbf{p})\frac{\Lambda^2}{p'^2+\Lambda^2}\frac{\Lambda^2}{p^2+\Lambda^2}.$$

TABLE I. The poles in all channels of $D\bar{D}^*$ and DD^*, up to the orbital angular momentum $L=1$ (in unit of MeV). The B and V superscripts denote the bound state and the virtual state, respectively. Otherwise the pole refers to a resonance.

		$D\bar{D}^*, C=+$		$D\bar{D}^*, C=-$		DD^*	
		$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
$\Lambda=0.5$GeV	$1^+(^3S_1)$	-3.1^B, $\chi_{c1}(3872)$	-	-1.60^B	-34.8^V, $Z_c(3900)$	-0.41^B, $T_{cc}(3875)$	-
	$0^-(^3P_0)$	$-1.5-14.5i$	-	-	-	$-9.6-9.7i$	-
	$1^-(^3P_1)$	-	-	$-4.0-27.3i$, $G(3900)$	-	$-31.7-70.6i$	-
	$2^-(^3P_2)$	$-42.6-39.4i$	-	$-21.3-50.7i$	-	$-37.8-40.9i$	-
$\Lambda=0.6$GeV	$1^+(^3S_1)$	-6.5^B, $\chi_{c1}(3872)$	-	-5.8^B	-39.5^V, $Z_c(3900)$	-4.3^B, $T_{cc}(3875)$	-
	$0^-(^3P_0)$	$3.2-13.7i$	-	-	-	$-10.2-12.1i$	-
	$1^-(^3P_1)$	-	-	$2.0-27.3i$, $G(3900)$	-	$-33.7-84.8i$	-
	$2^-(^3P_2)$	$-44.2-49.0i$	-	$-19.3-58.8i$	-	$-37.8-49.3i$	-

P-wave resonances more likely to be observed

1^{+-} partner of $X(3872)$, bound or virtual state

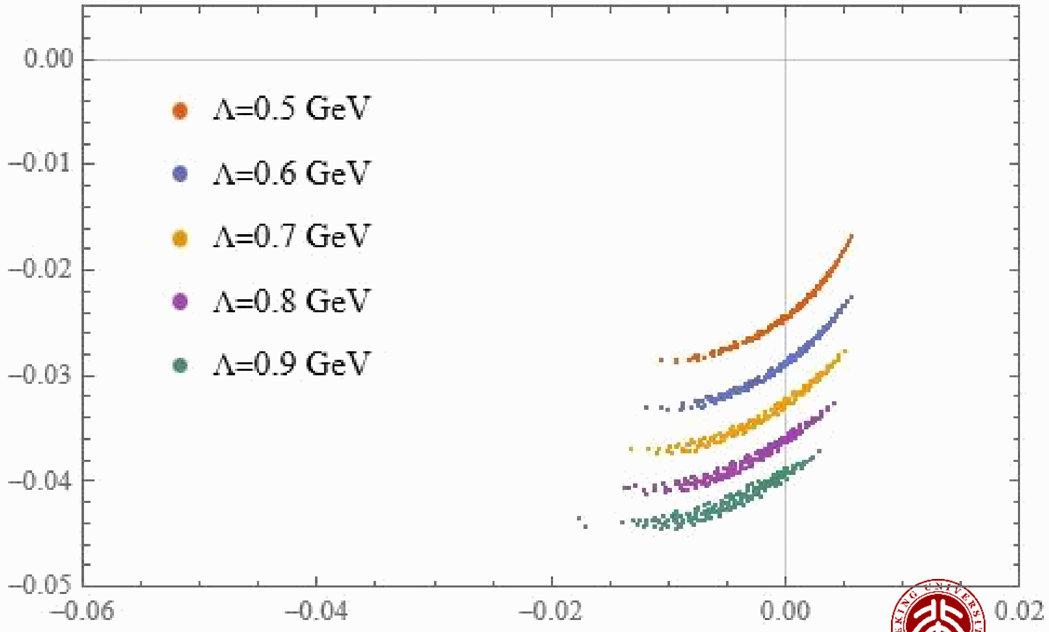
III Systematic uncertainty – coupling constants

- Randomly adjust the coupling constants in OBE to reproduce a shallow bound T_{cc} and $X(3872)$

$$\mathcal{L} = g_s \mathrm{Tr}\left[\mathcal{H}\sigma\bar{\mathcal{H}}\right] + i g_a \mathrm{Tr}\left[\mathcal{H}\gamma_\mu\gamma_5\mathcal{A}^\mu\bar{\mathcal{H}}\right] + i\beta \mathrm{Tr}\left[\mathcal{H}v_\mu(\mathcal{V}^\mu - \rho^\mu)\bar{\mathcal{H}}\right] + i\lambda \mathrm{Tr}\left[\mathcal{H}\sigma_{\mu\nu}F^{\mu\nu}\bar{\mathcal{H}}\right]$$

- Z_c as a virtual state ranged from -35 to -15 MeV (which mainly the σ coupling g_s)
- pion coupling g_a fixed by $D^* \to D\pi$
- then we obtain a distribution of $G(3900)$

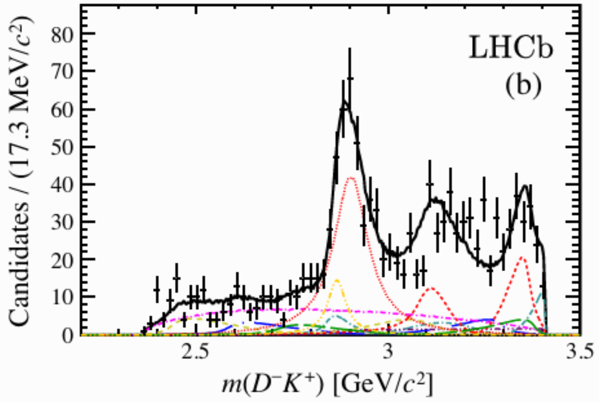
robust near-threshold resonance



IV Extension – $X_1(2900)$ as the P-wave $\bar{D}^*K^*$ resonance

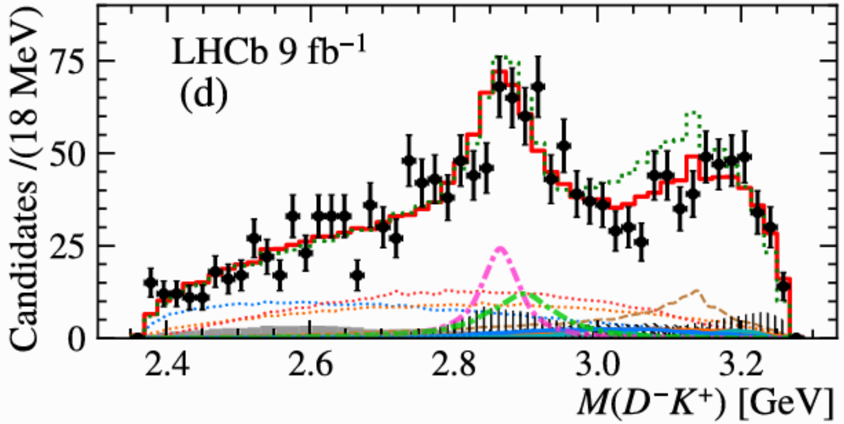
- Question: does this P-wave mechanism exist in other systems?
  Is such a wide resonance below threshold important?

- $X_0(2900)$ & $X_1(2900)$ seem a pair of S-wave and P-wave molecules like $X(3872)$ & $G(3900)$



	$T^*_{\bar{c}\bar{s}0}(2900)^0$	$T^*_{\bar{c}\bar{s}1}(2900)^0$
m_0	2866 ± 7	2904 ± 5
Γ_0	57 ± 13	110 ± 12

(LHCb Collaboration), Phys. Rev. D 102, 112003 (2020)



	$T^*_{\bar{c}\bar{s}0}(2900)^0$	$T^*_{\bar{c}\bar{s}1}(2900)^0$
m_0	2887 ± 10	2914 ± 19
Γ_0	92 ± 23	128 ± 32

(LHCb Collaboration), Phys. Rev. Lett. 133, 131902 (2024)

IV Extension – $X_1(2900)$ as the P-wave $\bar{D}^*K^*$ resonance

- Interactions with $K^*(K)$ similar to charmed mesons
- g' (π coupling) determined by $K^* \to K\pi$
- β' (ρ/ω coupling) from local hidden gauge
- unknown parameter λ' varies

$$\mathcal{L}_{K^{(*)}K^{(*)}P} = +\frac{2g'}{f_\pi}(K_b K^{*\dagger}_{a\lambda} + K^*_{b\lambda}K^\dagger_a)\partial^\lambda P_{ba} + i\frac{2g'}{f_\pi} \times v'^\alpha \varepsilon_{\alpha\mu\nu\lambda} K^{*\mu}_b K^{*\lambda\dagger}_a \partial^\nu P_{ba},$$

$$\mathcal{L}_{K^{(*)}K^{(*)}V} = +\sqrt{2}\beta' g'_V K_b K^\dagger_a v' \cdot V_{ba} - 2\sqrt{2}\lambda' g'_V \times \epsilon_{\lambda\mu\alpha\beta} v'^\lambda (K_b K^{*\mu\dagger}_a + K^{*\mu}_b K^\dagger_a)(\partial^\alpha V^\beta_{ba}) - \sqrt{2}\beta' g'_V K^*_b \cdot K^{*\dagger}_a v' \cdot V_{ba} - i2\sqrt{2}\lambda' g'_V K^{*\mu}_b K^{*\nu\dagger}_a (\partial_\mu V_\nu - \partial_\nu V_\mu)_{ba},$$

- More coupled channels due to the total spin

0^+: ${}^1S_0$ $\quad\quad$ 1^-: ${}^1P_1, {}^3P_1, {}^5P_1$

IV Results – $X_1(2900)$ as the P-wave $\bar{D}^*K^*$ resonance

- $X_0(2900), X_1(2900)$ and an additional 1^- state

$$T_{cs0}(2900) \sim {}^1S_0$$
$$T_{cs1}(2900) \sim {}^1P_1 + {}^5P_1$$
$$T'_{cs1}(2900) \sim {}^3P_1$$

- Dependency on the unknown parameter λ'

States	$\lambda' = 0.56$	$\lambda' = 0.28$	$\lambda' = 0.84$
$T_{cs0+}(2900)$	2.859^*	2.878^*	2.833^*
	$2.857 - 0.012i$	$2.876 - 0.016i$	$2.832 - 0.008i$
$T_{cs1-}(2900)$	$2.834 - 0.037i^*$	$2.835 - 0.052i^*$	$2.840 - 0.028i^*$
	$2.828 - 0.054i$	$2.827 - 0.069i$	$2.834 - 0.045i$
$T'_{cs1-}(2900)$	$2.868 - 0.028i^*$	$2.869 - 0.033i^*$	$2.869 - 0.024i^*$
	$2.861 - 0.049i$	$2.862 - 0.054i$	$2.862 - 0.045i$

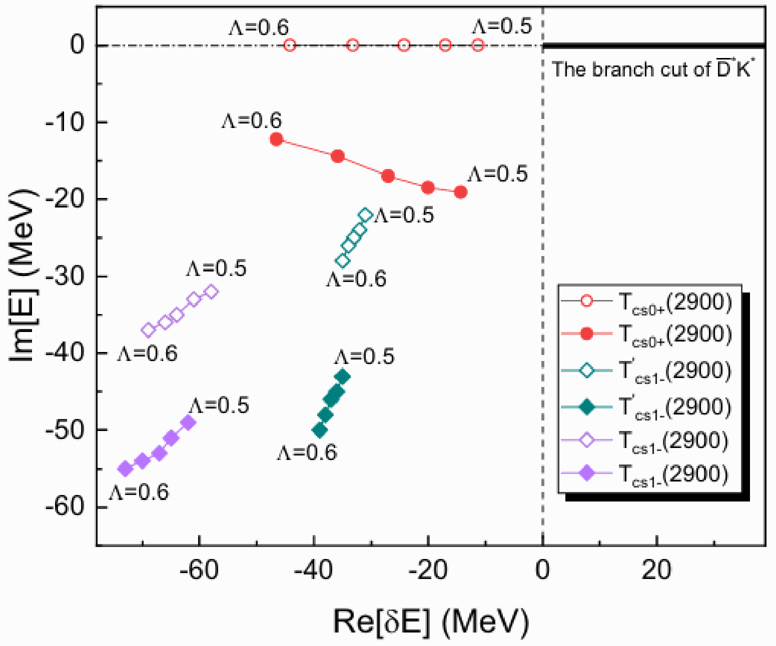


FIG. 2. The pole trajectories of S-wave $T_{cs0+}(2900)$, P-wave $T_{cs1-}(2900)$ and $T'_{cs1-}(2900)$ with the varying cutoff parameter Λ from 0.5 to 0.6 GeV. Here, the circle and diamond points correspond to the poles in the physical and unphysical Riemann sheets, respectively. The hollow and solid points represent the results without and with the width effect of the K^* meson, respectively.

V Summary and perspective

- Model-based calculation: unified description of $X(3872), T_{cc}, Z_c(3900)$ and $G(3900)$, with additional states predicted

- Robustness from the pion exchange and P-wave mechanism

- The energy of P-wave molecule is not necessarily much higher than its S-wave partner

- More possible p-wave molecular states like $X_1(2900)$

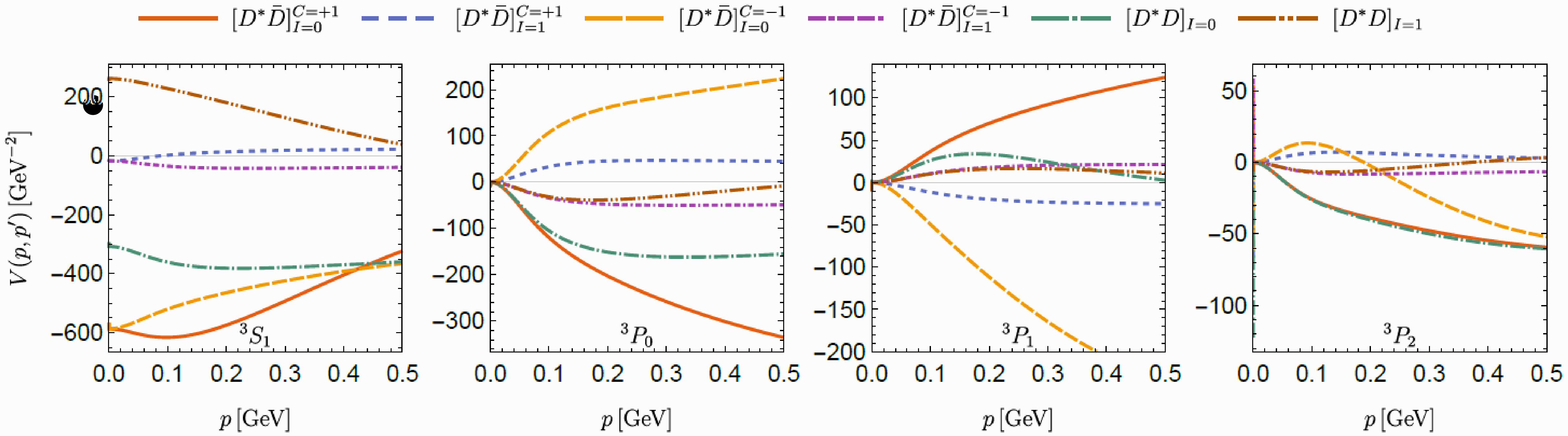
- Open question: what is the role of wide and near/below-threshold resonance?
  For example, even in nucleon-nucleon system?

Thank you for your attention!

Backup – Unified description of $D\bar{D}^*/DD^*$ potentials

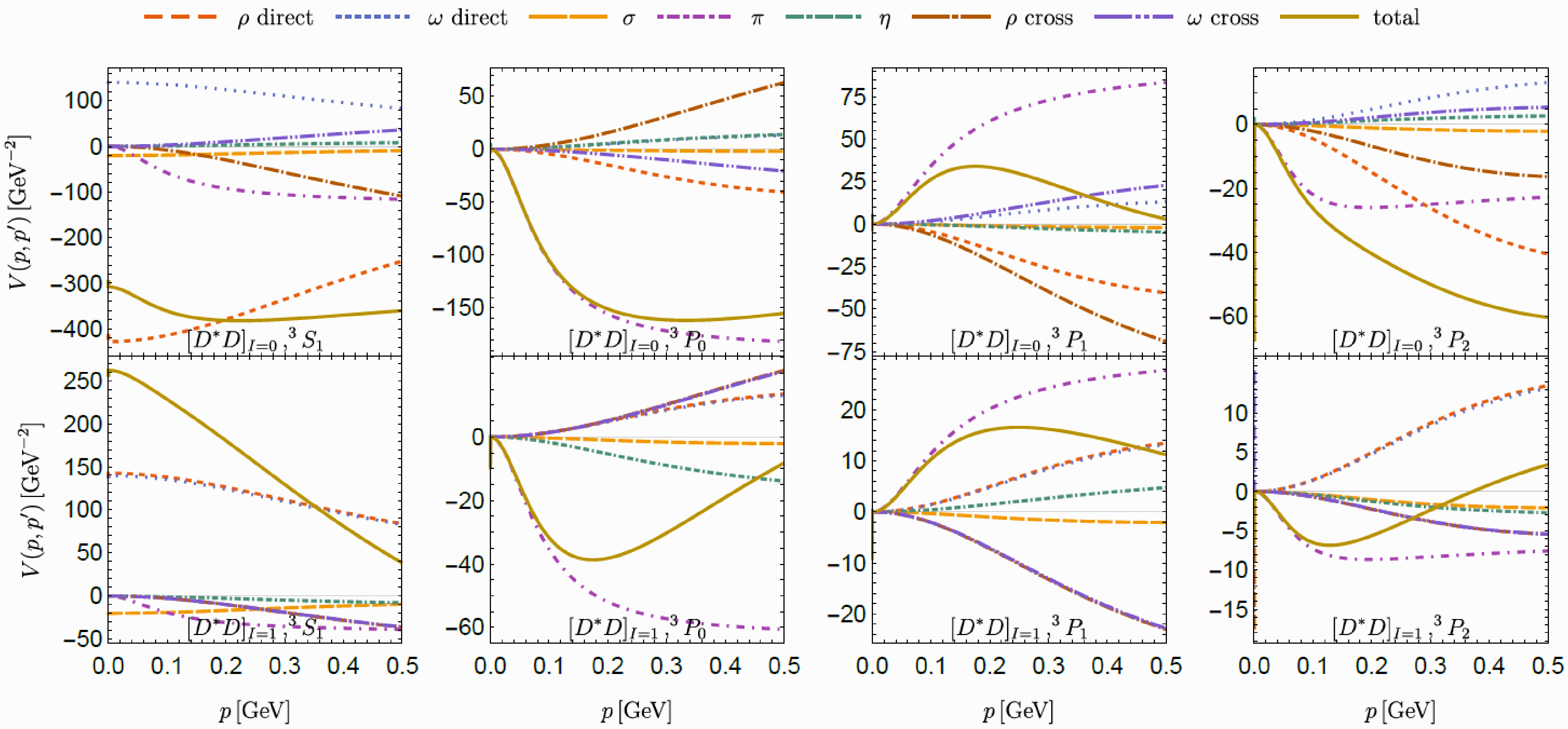
- Total potentials in different channels:

weak $I = 1$ interactions, due to the cancel between ρ and ω exchange



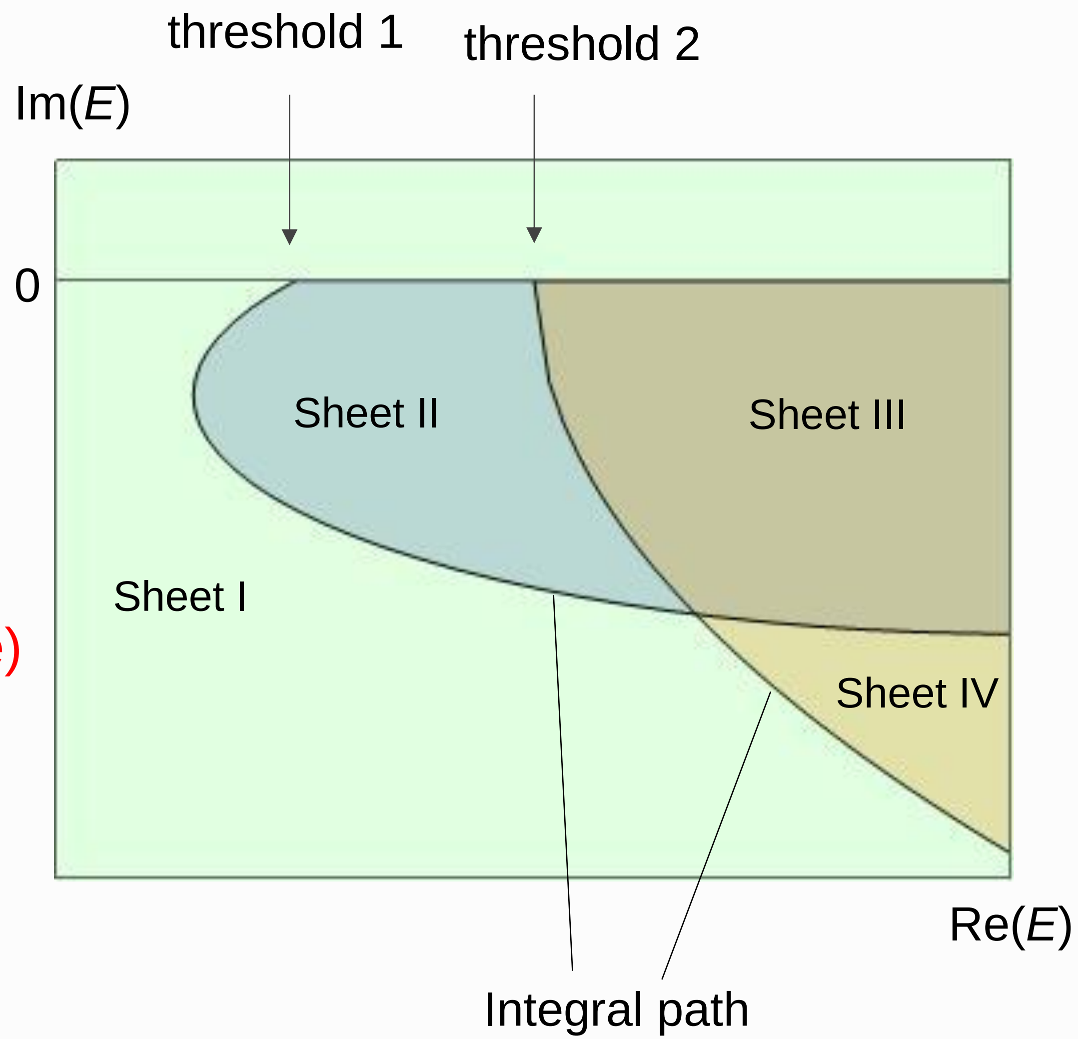
- Strong attraction: $1^+(DD^*)$、1^{++}、$1^{+-}(D\bar{D}^*)$
- Weak attraction: $0^-(DD^*)$、0^{-+}、$1^{--}(D\bar{D}^*)$ weaker: $2^-(DD^*)$、2^{--}、$2^{-+}(D\bar{D}^*)$

Backup – Unified description of $D\bar{D}^*/DD^*$ potentials



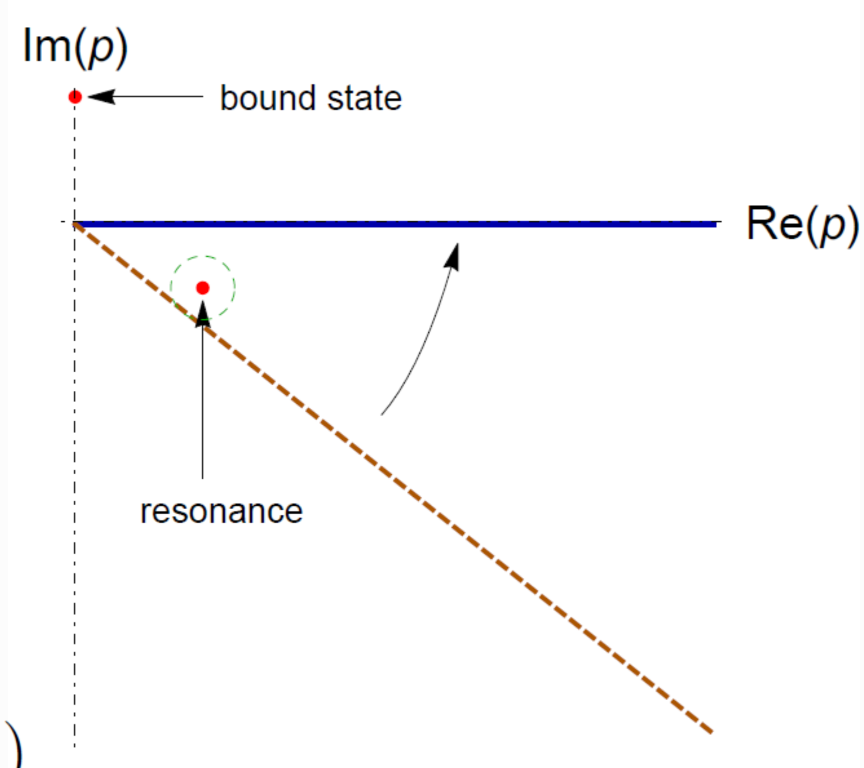
Backup —— Schrödinger equation on the 2nd Riemann sheet

- 2-channel example
- $k_1 = \sqrt{2\mu_1(E - E_{th1})} \quad k_2 = \sqrt{2\mu_2(E - E_{th2})}$
- Sheet I: $\mathrm{Im}(k_1) > 0 \quad \mathrm{Im}(k_2) > 0$
  bound state
- Sheet II: $\mathrm{Im}(k_1) < 0 \quad \mathrm{Im}(k_2) < 0$
  quasibound state (Feshbach-type resonance)
- Sheet III: $\mathrm{Im}(k_1) < 0 \quad \mathrm{Im}(k_2) < 0$
  resonance
- Sheet IV: $\mathrm{Im}(k_1) < 0 \quad \mathrm{Im}(k_2) < 0$
  "threshold cusp"



Backup —— Schrödinger equation on the 2nd Riemann sheet

- 1st Riemann sheet: integrate along the real axis
- 2nd Riemann sheet: the residue of the pole must be include
- Or change the integral path

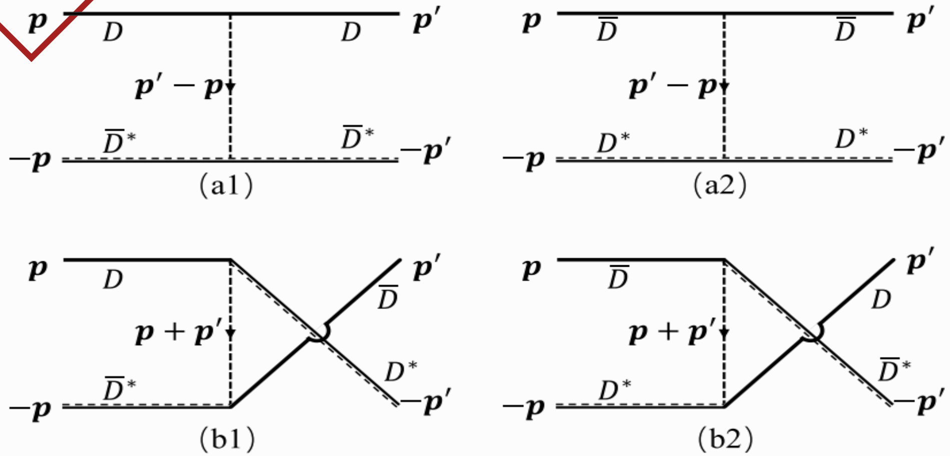


- E.g. complex scaling method:

$$
\begin{aligned}
E\tilde{\phi}_l(p) &= \frac{p^2 e^{-2i\theta}}{2m}\tilde{\phi}_l(p) \\
&+ \int \frac{p'^2 e^{-3i\theta} dp'}{(2\pi)^3} V_{l,l'}(pe^{-i\theta}, p'e^{-i\theta})\tilde{\phi}_{l'}(p')
\end{aligned}
$$

- Avoid the branch cut in the potential

Backup – OBE potential and partial-wave expansion



$$V_\sigma^D(\boldsymbol{p}', \boldsymbol{p}) = -\frac{g_s^2}{\boldsymbol{q}^2 + m_\sigma^2},$$

$$V_\pi^C(\boldsymbol{p}', \boldsymbol{p}) = -\frac{g^2}{2 f_\pi^2} \frac{(\boldsymbol{\epsilon} \cdot \boldsymbol{k})(\boldsymbol{\epsilon}' \cdot \boldsymbol{k})}{\boldsymbol{k}^2 - k_0^2 + m_\pi^2} \tau \cdot \tau,$$

$$V_\eta^C(\boldsymbol{p}', \boldsymbol{p}) = -\frac{g^2}{6 f_\pi^2} \frac{(\boldsymbol{\epsilon} \cdot \boldsymbol{k})(\boldsymbol{\epsilon}' \cdot \boldsymbol{k})}{\boldsymbol{k}^2 - k_0^2 + m_\eta^2} \mathbb{1} \cdot \mathbb{1},$$

$$V_{\rho/\omega}^D(\boldsymbol{p}', \boldsymbol{p}) = \frac{\frac{1}{4}\beta^2 g_V^2 (\boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}')}{\boldsymbol{q}^2 + m_{\rho/\omega}^2} \times \begin{cases} \tau \cdot \tau, & \text{for } \rho, \\ \mathbb{1} \cdot \mathbb{1}, & \text{for } \omega, \end{cases}$$

$$V_{\rho/\omega}^C(\boldsymbol{p}', \boldsymbol{p}) = \frac{\lambda^2 g_V^2}{\boldsymbol{k}^2 - k_0^2 + m_{\rho/\omega}^2} \{(\boldsymbol{k} \cdot \boldsymbol{\epsilon})(\boldsymbol{k} \cdot \boldsymbol{\epsilon}') - \boldsymbol{k}^2 (\boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}')\} \times \begin{cases} \tau \cdot \tau, & \text{for } \rho, \\ \mathbb{1} \cdot \mathbb{1}, & \text{for } \omega, \end{cases}$$

Partial wave decomposition of $(\boldsymbol{\epsilon} \cdot \boldsymbol{k})(\boldsymbol{\epsilon}' \cdot \boldsymbol{k}) D(p', p, z)$

$$V_S^{J=0} = \frac{2\pi}{3} \int_{-1}^{1} D(p', p, z)(p^2 + p'^2 + 2pp'z) dz,$$

$$V_P^{J=0} = 2\pi \int_{-1}^{1} D(p', p, z)\{(p^2 + p'^2)z + pp'(1 + z^2)\} dz,$$

$$V_P^{J=1} = 2\pi \int_{-1}^{1} D(p', p, z) \frac{1}{2}(z^2 - 1) pp' dz,$$

$$V_P^{J=2} = \frac{2\pi}{5} \int_{-1}^{1} D(p', p, z)\{2(p^2 + p'^2)z + \frac{1}{2} pp'(1 + 7z^2)\} dz. \qquad \text{(B3)}$$

Backup – Cutoff-independency

Local Regulator:

$$V^D(\boldsymbol{q}) \to V^D(\boldsymbol{q}) \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + \boldsymbol{q}^2}\right)^2,$$

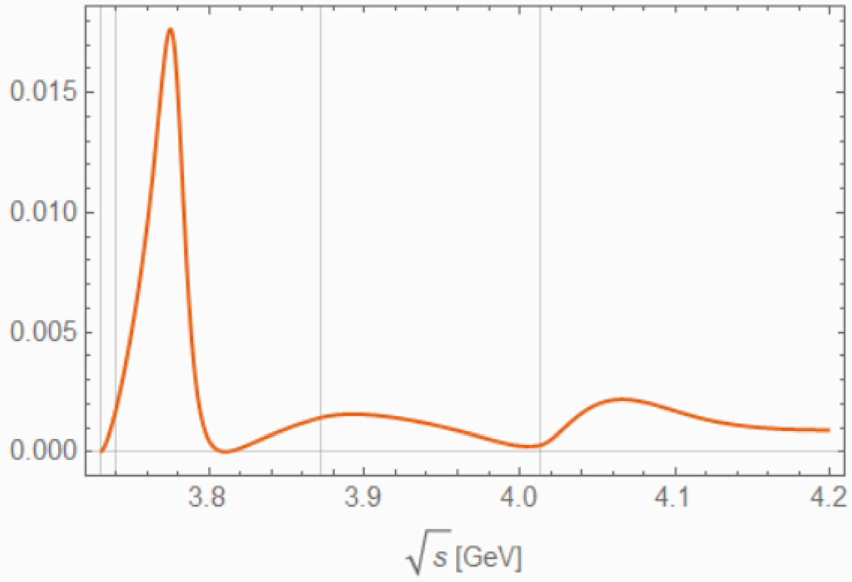
$$V^C(\boldsymbol{k}) \to V^C(\boldsymbol{k}) \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + \boldsymbol{k}^2}\right)^2$$

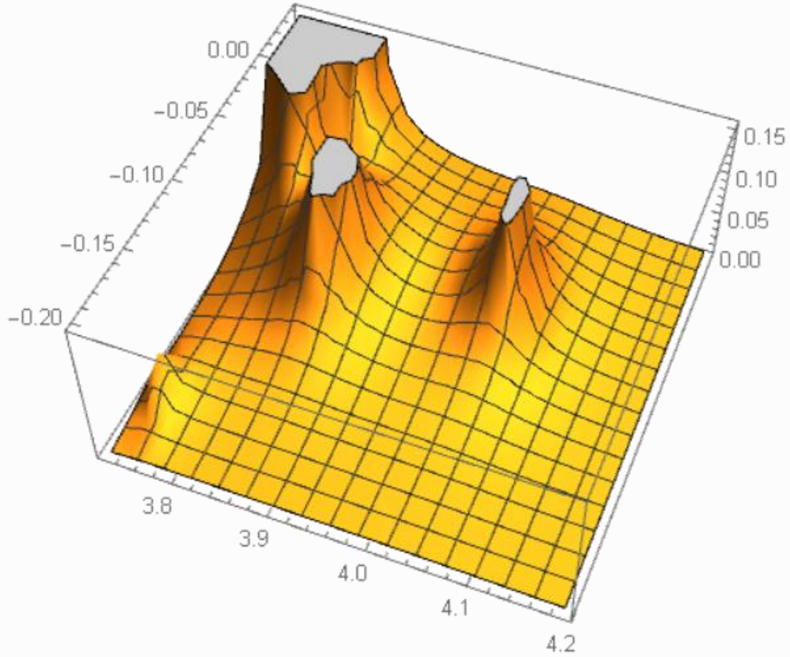
TABLE II. The poles in all channels of $D\bar{D}^*$ and DD^*, up to the orbital angular momentum $L = 1$ with the regularization in Eq. (C2). The B and V superscripts denote the bound state and the virtual state, respectively. Otherwise the pole refers to a resonance.

		$D\bar{D}^*, C=+$		$D\bar{D}^*, C=-$		DD^*	
		$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
$\Lambda = 1.25$ GeV	$1^+(^3S_1)$	-0.40^B, $\chi_{c1}(3872)$	-	-25.0^V	-39.6^V, $Z_c(3900)$	-0.79^B, $T_{cc}(3875)$	-
	$0^-(^3P_0)$	$3.3 - 17.2i$	-	-	-	$-11.2 - 16.7i$	-
	$1^-(^3P_1)$	-	-	$4.4 - 39.9i$, $Y(3872)$	-	$-96.6 - 87.3i$	-
	$2^-(^3P_2)$	$-71.2 - 63.5i$	-	$-31.0 - 96.5i$	-	$-61.3 - 53.6i$	-
$\Lambda = 1.35$ GeV	$1^+(^3S_1)$	-2.8^B, $\chi_{c1}(3872)$	-	-2.2^V	-38.5^V, $Z_c(3900)$	-8.8^B, $T_{cc}(3875)$	-
	$0^-(^3P_0)$	$6.6 - 11.6i$	-	-	-	$-10.2 - 18.0i$	-
	$1^-(^3P_1)$	-	-	$10.2 - 33.7i$, $Y(3872)$	-	$-92.9 - 97.7i$	-
	$2^-(^3P_2)$	$-68.0 - 75.4i$	-	$-23.3 - 97.2i$	-	$-58.4 - 59.6i$	-

Backup – G(3900)/Y(3872)

- pole search for the K matrix method
- 1st sheet for $D^*\bar{D}^*$, 2nd sheet for $D\bar{D}^*$ and $D\bar{D}$



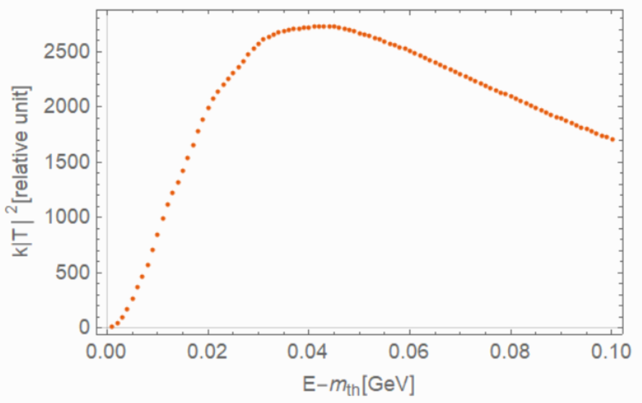


G(3900) related pole:

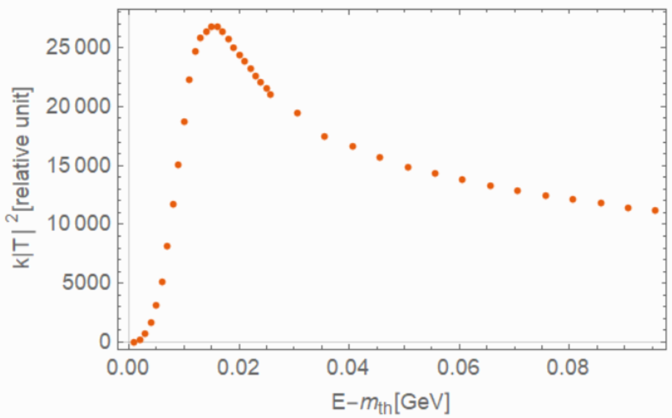
$3864 - 65i$ MeV

Backup – Line shape/residue

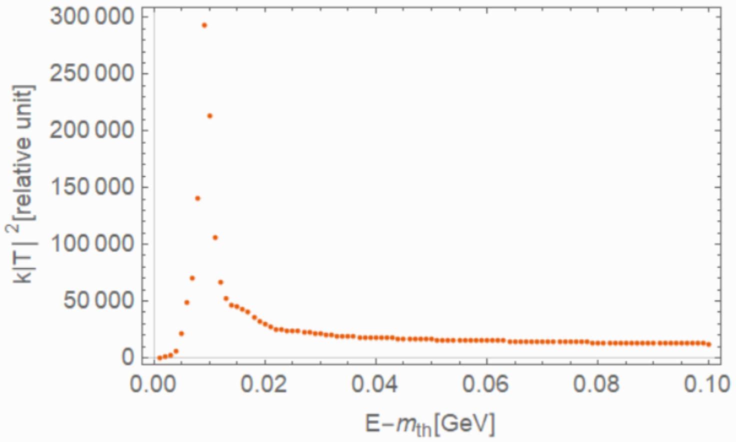
- Line shape of $G(3872)$ in p-wave $D\bar{D}^* \to D\bar{D}^*$



$\Lambda = 0.5$ GeV, $E = -4.0 - 27.3i$ MeV



$\Lambda = 1.0$ GeV, $E = 9.1 - 6.9i$ MeV



$\Lambda = 1.1$ GeV, $E = 3.8 - 1.4i$ MeV

- Residues of S-matrix

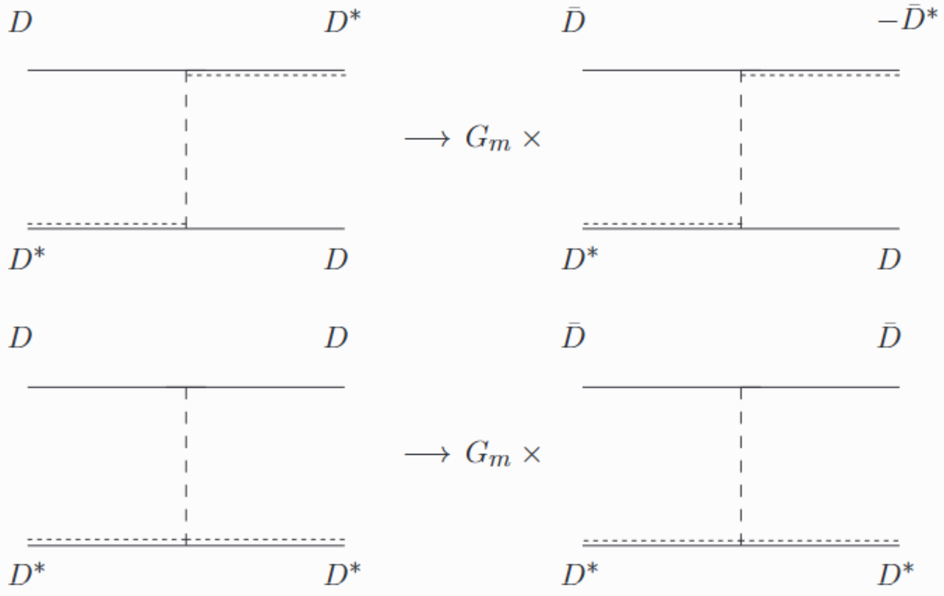
TABLE IV. Add residue values |Res{Pole}| of $I = 0$ DD^* cases.

		$\Lambda = 0.5$ GeV	$\Lambda = 0.6$ GeV
Nonlocal	$1^+(^3S_1)$	-0.48^B (Res:2.26), T_{cc}	-4.3^B (Res:27.1), T_{cc}
	$0^-(^3P_0)$	$-9.8 - 9.7i$ (Res:20.1)	$-10.1 - 12.2i$ (Res:24.6)
	$1^-(^3P_1)$	$-31.6 - 70.7i$ (Res:129.7)	$-33.7 - 84.8i$ (Res:189.1)
	$2^-(^3P_2)$	$-37.7 - 40.9i$ (Res:20.2)	$-37.8 - 49.2i$ (Res:26.7)

Backup – G-parity rule

$$D = (-D^+, D^0) \xrightarrow{G} \bar{D} = (\bar{D}^0, D^-) \xrightarrow{G} -D,$$
$$D^* = (-D^{*+}, D^{*0}) \xrightarrow{G} -\bar{D}^* = -(\bar{D}^{*0}, D^{*-}) \xrightarrow{G} -D^*$$

$$|D\bar{D}^*/\bar{D}D^*, G = \pm\rangle = \frac{1}{\sqrt{2}}(|D\bar{D}^*\rangle \pm |\bar{D}D^*\rangle)$$



$$V^C_{\bar{D}D^* \to D\bar{D}^*} = (-G_m)V^C_{DD^* \to DD^*}$$
$$V^D_{\bar{D}D^* \to \bar{D}D^*} = V^D_{DD^* \to DD^*}.$$

$$V_{D\bar{D}^*/\bar{D}D^*, G_{MM}} = G_m V^D_{DD^* \to DD^*} - G_m G_{MM} V^C_{DD^* \to DD^*}$$

Backup —— three-body effect in OPE

- Different choice of q_0 in the denominator

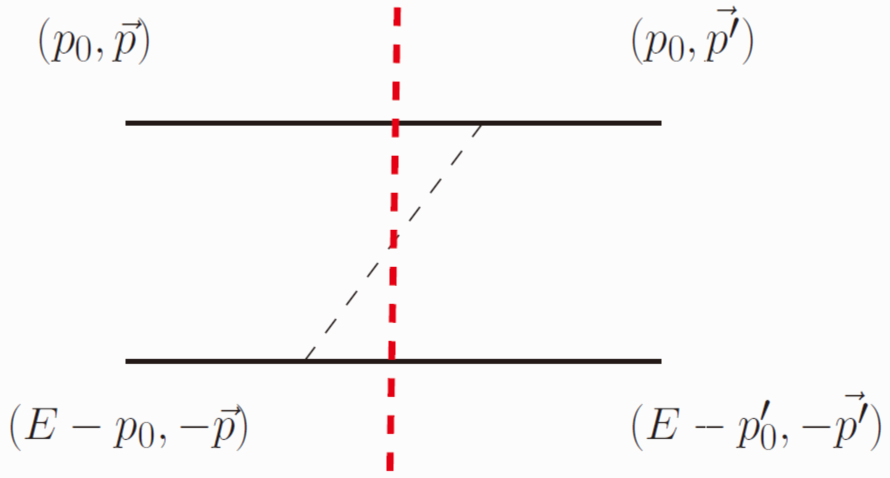
$$\frac{i}{q_0^2 - \boldsymbol{q}^2 - m_\pi^2}$$

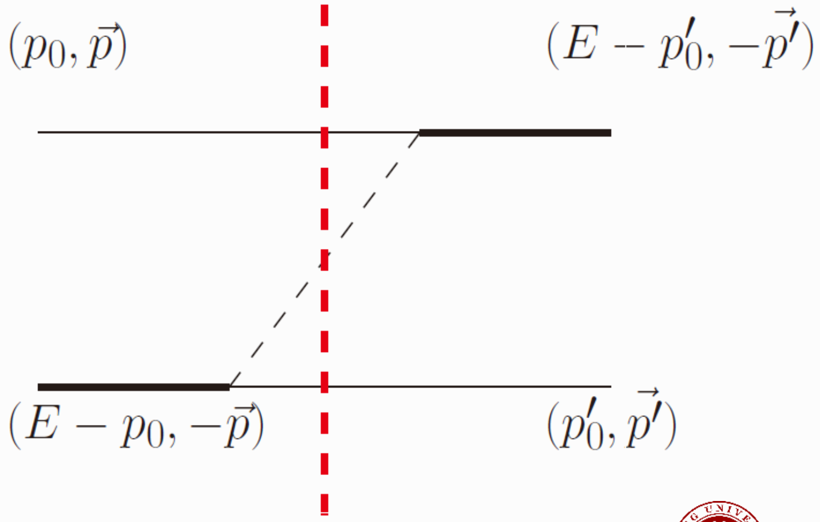
- Direct diagram (P_c)

$$q_0 = p_0 - p'_0$$

- Cross diagram (T_{cc}^+)

$$q_0 = E - p_0 - p'_0$$





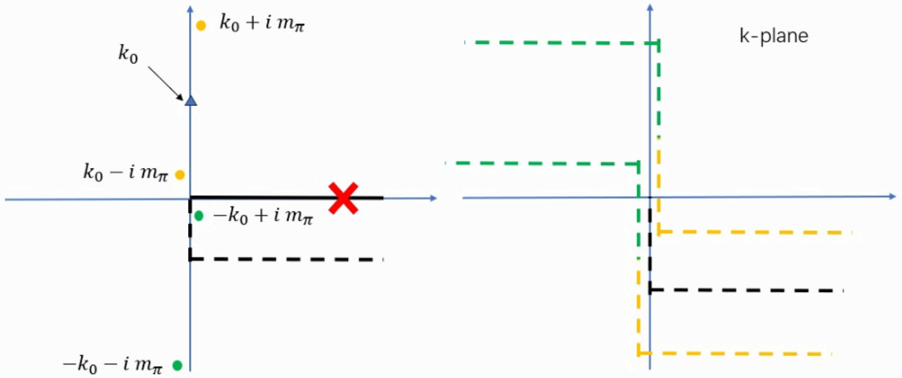
Backup —— left-hand cut & virtual state

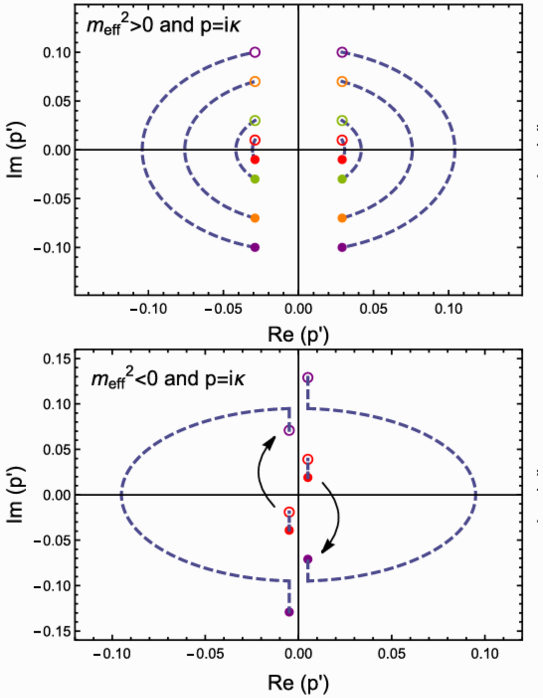
- Partial-wave potential

$$V_{OPE}(p',p) = 2\pi \left(2 + \frac{(-m_\pi^2 + i\epsilon)\ln\frac{(p+p')^2+m_\pi^2-i\epsilon}{(p-p')^2+m_\pi^2-i\epsilon}}{2pp'} \right)$$

- Branch point at $p = p' \pm i\, m_\pi$

- Special contour





- CSM result