Global coupled-channel analysis of $e^+e^- \to c\bar{c}$ data

arXiv: 2312.17658v3 (v3 to be posted soon)

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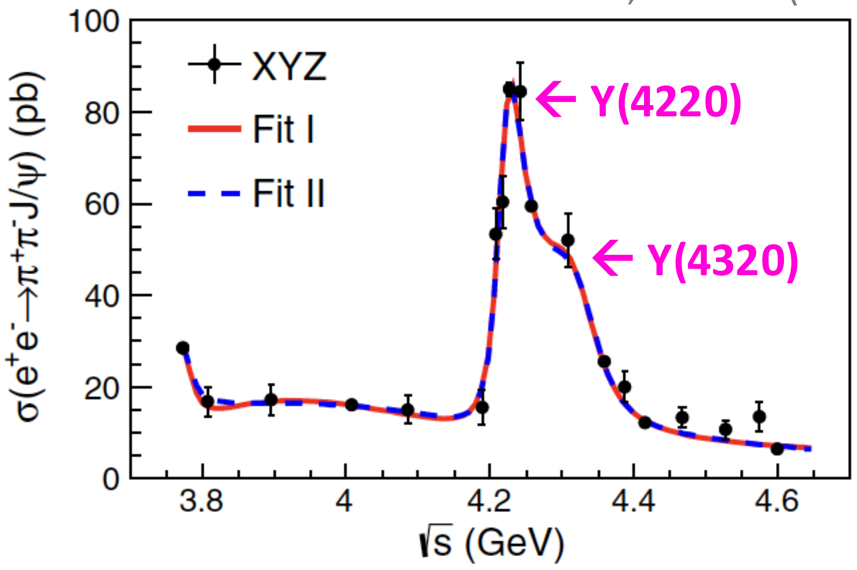
Introduction

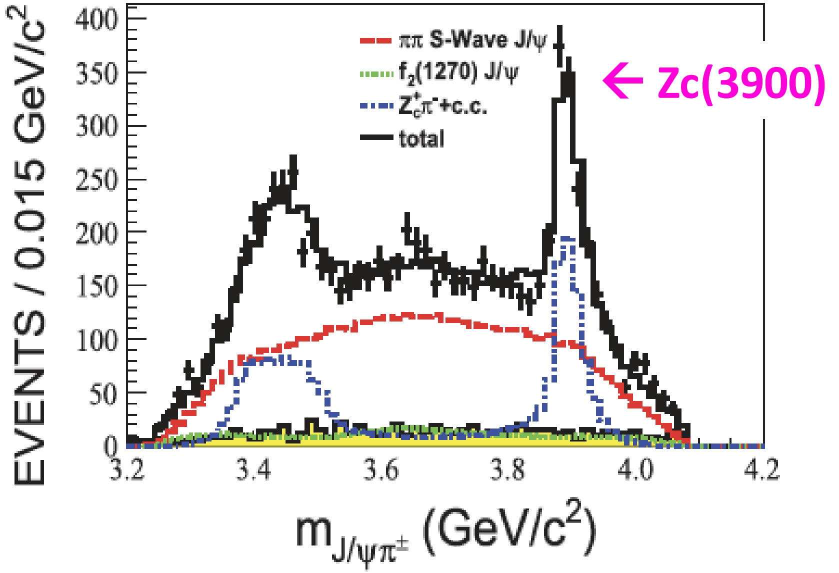
BESIII data for XYZ physics (only a few from many)

$e^+e^- \rightarrow J/\psi\, \pi^+\pi^-$ →

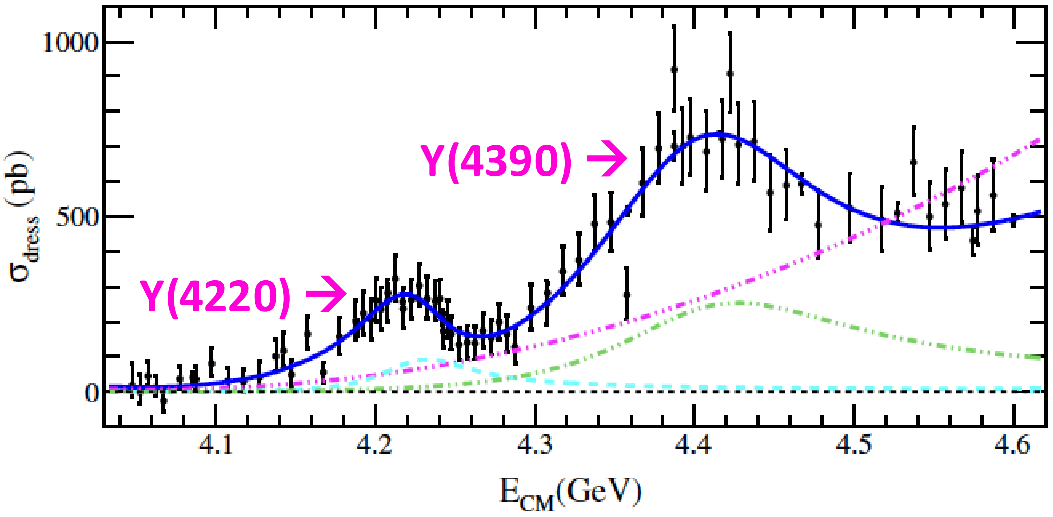
PRL 118, 092001 (2017)



PRL 119, 072001 (2017)

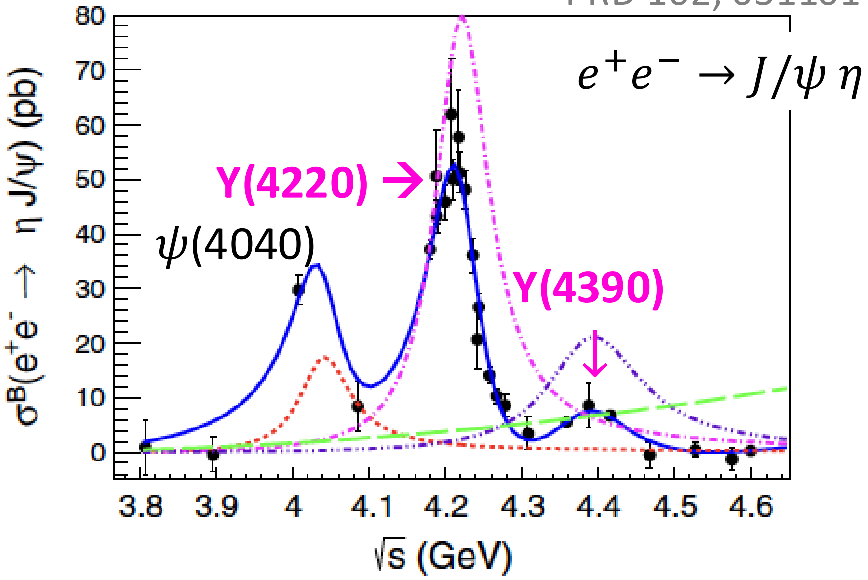


$e^+e^- \rightarrow \pi^+ D^0 D^{*-}$ PRL 122, 102002 (2019)



PRD 102, 031101 (2020)

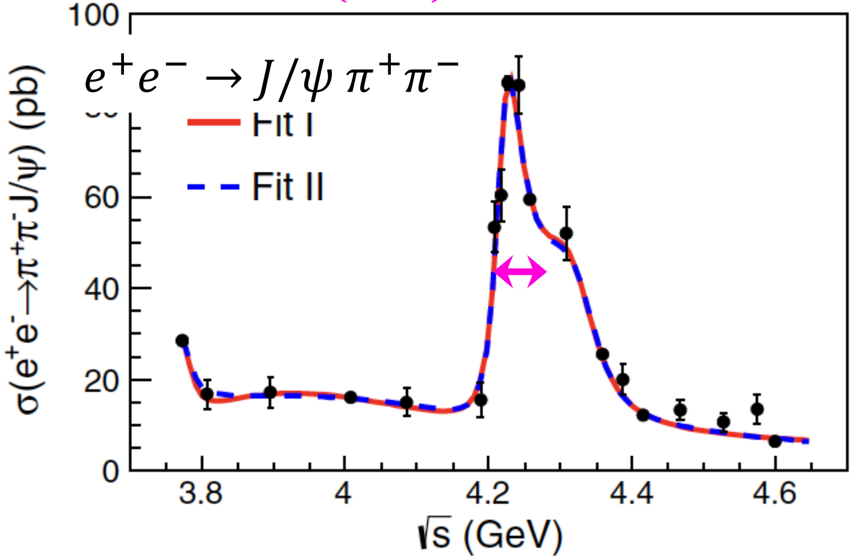
$e^+e^- \rightarrow J/\psi\, \eta$



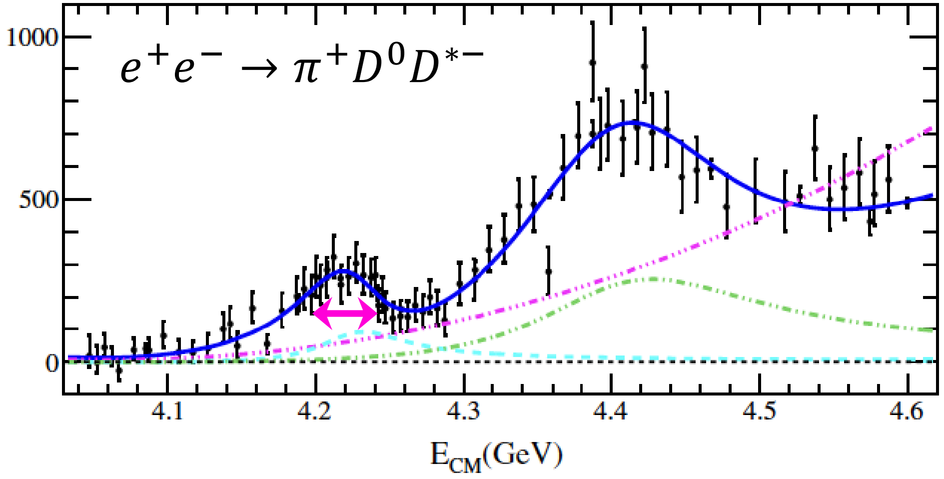
Outstanding question in XYZ physics : Y width problem

Why Y states seem to have different widths for different final states ?

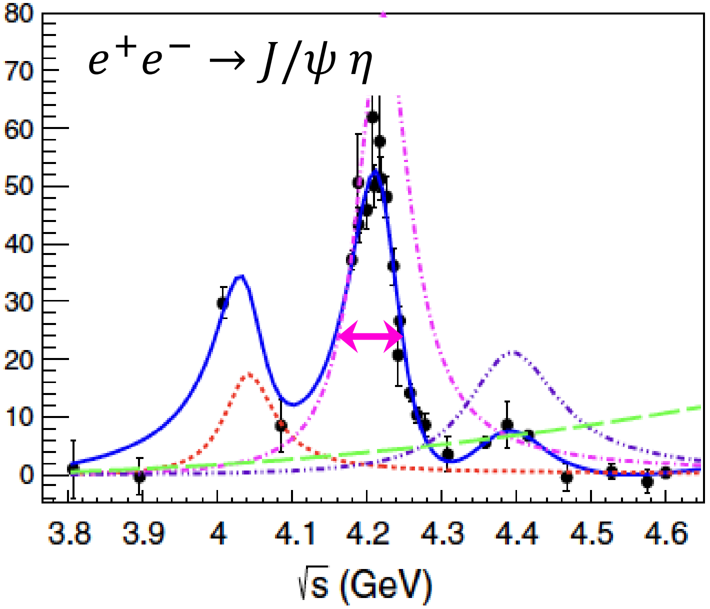
$\Gamma_{Y(4220)} \sim 44\pm4$ MeV



$\Gamma_{Y(4220)} \sim 77\pm7$ MeV



$\Gamma_{Y(4220)} \sim 82\pm6$ MeV



How to find solution to Y width problem ?

- Analyze different final states with different models (usual experimental analysis; single-channel analysis)

  → no simple relation between resonance parameters from different models

  → Y width problem created

  Y-width problem is artifact of single-channel analysis

How to find solution to Y width problem ?

😃 Analyze different final states simultaneously with a unified and (semi-)unitary model

(global coupled-channel analysis)

* how various charmonia interfere to create different lineshapes in different final states
* kinematical effects (threshold opening, triangle singularity) change lineshapes in some processes

→ Solution of the Y width problem

At the same time, global analysis determines:

(i) vector charmonium pole structure (pole locations)

(ii) couplings of the poles with decay channels (residues)

Now is the time to conduct global analysis of $e^+e^- \to c\bar{c}$ data, and determine vector charmonium poles and residues

BESIII accumulated high-quality data for various $e^+e^- \to c\bar{c}$ processes over wide energy region covering *Y*

$e^+e^- \to D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi\, \eta^{(\prime)}, \chi_{c0}\omega, \Lambda_c\bar{\Lambda}_c$ (two-body final states)

$e^+e^- \to \pi D^{(*)}\bar{D}^{(*)},\ J/\psi\pi\pi,\ \psi'\pi\pi, h_c\pi\pi, J/\psi K\bar{K}$ (three-body final states)

$e^+e^- \to \eta_c\rho\pi\ (\rho \to \pi\pi)$ (four-body final states)

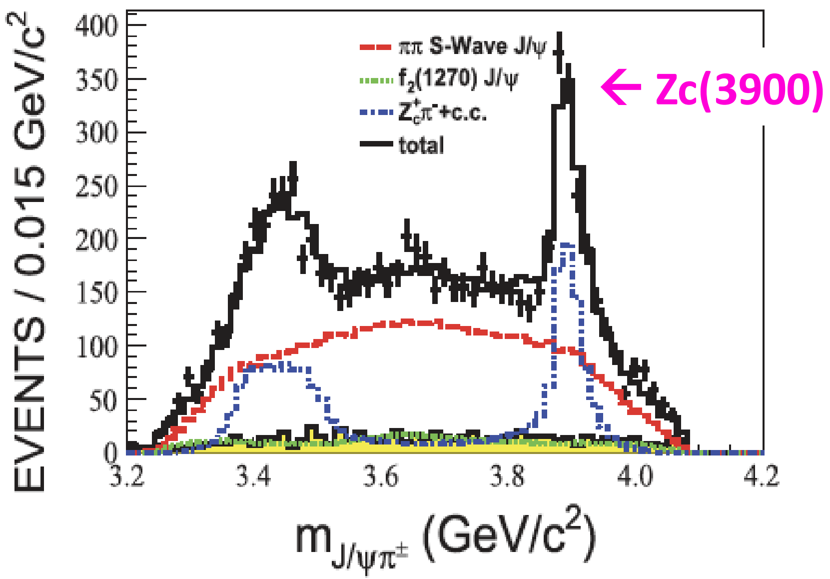
The global analysis is important not only for *Y* but also for well-established ψ(4040), ψ(4160), ψ(4415) because:

- Their properties were previously determined by simple Breit-Wigner fit to inclusive (e^+e^- → hadrons) *R* values
- Analyzing precise exclusive data → More detailed and precise information

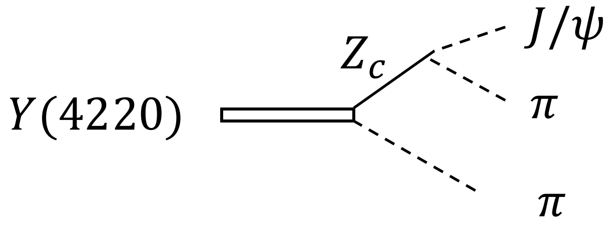
Understanding Y inevitably involves understanding Zc

Zc(3900), Zc(4020) : outstanding exotic candidates including $c\bar{c}u\bar{d}$

$e^+e^- \to J/\psi\,\pi^+\pi^-$ at Y(4220) region →



Zc appears as:



→ Y and Zc properties should be highly correlated

Global $e^+e^- \to c\bar{c}$ analysis consider Zc signals → address Y and Zc properties simultaneously

This work

- Global analysis of BESIII and Belle data in $3.75 \leq \sqrt{s} \leq 4.7$ GeV with a unified coupled-channel model

  $e^+e^- \to D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi\,\eta^{(\prime)}, \chi_{c0}\omega\,, \Lambda_c\bar{\Lambda}_c$ (10 two-body final states)

  $e^+e^- \to \pi D^{(*)}\bar{D}^{(*)},\ J/\psi\pi\pi,\ \psi'\pi\pi, h_c\pi\pi, J/\psi K\bar{K}$ (7 three-body final states)

  $e^+e^- \to \eta_c\rho\pi\ (\rho \to \pi\pi)$ (1 four-body final states)

- Approximate three-body unitarity

- Fit both total cross sections and invariant mass distributions

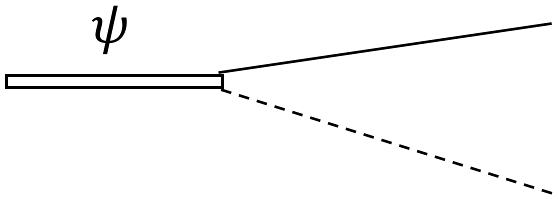
- Extract vector charmonium (ψ, Y) and Zc poles (mass, width)

Near-future work → Extraction of residues (branching fractions) and solution of Y width problem

MODEL

Coupled-channels

(quasi) two-body channels included; $J^{PC} = 1^{--}$



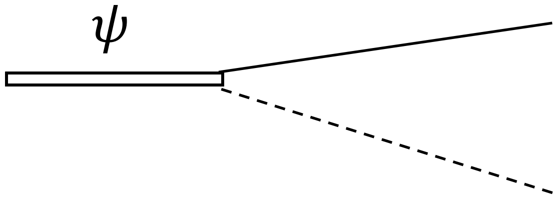
$D_1(2420)\bar{D}^{(*)}$, $D_1(2430)\bar{D}^{(*)}$, , $D_2^*(2460)\bar{D}^{(*)}$, $D^{(*)}\bar{D}^{(*)}$, $D_{s1}(2536)\bar{D}_s$

$\omega\chi_{c0}$

$D_1(2420), D_1(2430)$, $D_2^*(2460), D^*, D_{s1}(2536), \omega$ → Breit-Wigner (BW) propagators; mass and width from PDG

BW partially violate three-body unitarity in our three-body calculation

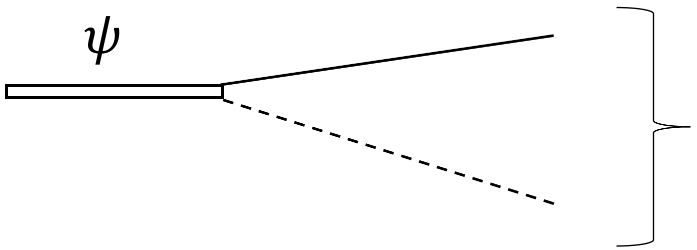
Otherwise the model is manifestly three-body unitary



$D_s^{(*)}\bar{D}_s^{(*)}$, $J/\psi\eta$, $J/\psi\eta'$, $\Lambda_c\bar{\Lambda}_c$ treated as stable particles

Coupled-channels

(quasi) two-body channels included; $J^{PC} = 1^{--}$

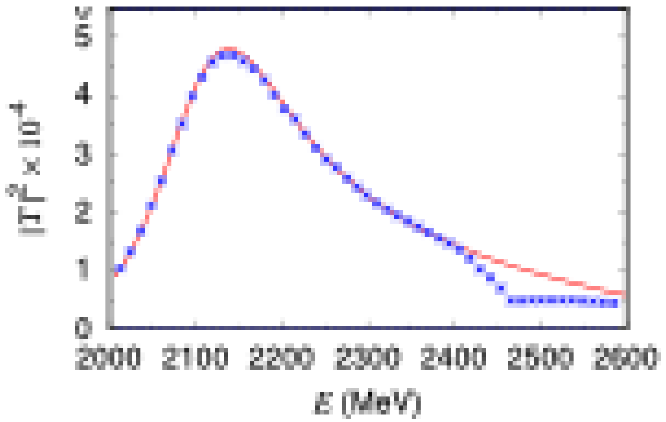


$D_0^*(2300)\bar{D}^*$, $f_0 J/\psi$, $f_2 J/\psi$, $f_0\psi'$, $f_0 h_c$, $Z_c\pi$, $Z_{cs}\bar{K}$

$D_0^*(2300), f_0, f_2, Z_c, Z_{cs}$ as (virtual) poles in two-body scattering amplitudes

$D\pi$ s-wave amplitude fitting LQCD-based amplitude

Albaladejo et al. PLB 767 (2017)

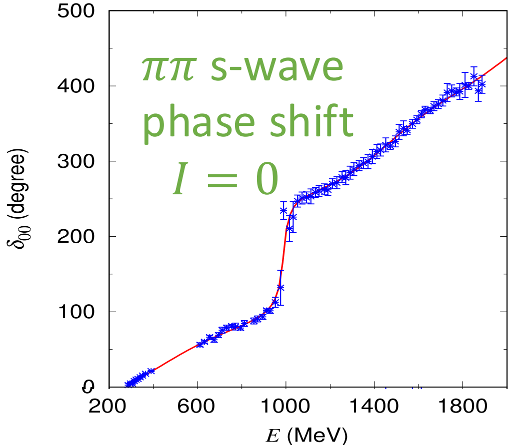


D_0^* pole :

$2104 - i\,100$ MeV (ours)

$2105^{+6}_{-8} - i\,102^{+10}_{-12}$ MeV (Albaladejo et al.)

$\pi\pi$ s[d]-wave amplitude fitting empirical amplitude



$f_0(500), f_0(980),$

$f_0(1370), f_2(1270)$ poles

→ consistent with PDG

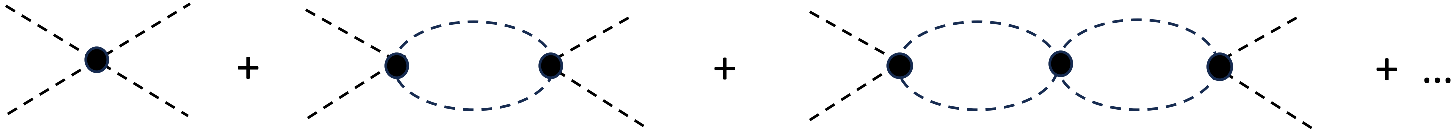
$Z_{c(s)}$ amplitude

Z_c : $J^{PC} = 1^{+-}$ $D^*\bar{D} - D^*\bar{D}^* - J/\psi\pi - \psi'\pi - h_c\pi - \eta_c\rho$ couple—channel scattering amplitude

Z_{cs} : $J^{PC} = 1^{+-}$ $D_s^*\bar{D} - D_s\bar{D}^* - J/\psi K$

driven by contact interactions; s-wave interactions except $h_c\pi$ p-wave interaction

$Z_{c(s)}$ amplitude



intermediate loops include

all possible coupled-channels

$v_{[D^*\bar{D}],[D^*\bar{D}]} = v_{D^*\bar{D}^*,D^*\bar{D}^*} = v_{[D_s^*\bar{D}],[D_s^*\bar{D}]}$ (HQSS, SU(3))

$v_{[D^*\bar{D}],J/\psi\pi} = v_{[D_s^*\bar{D}],J/\psi K}$ (SU(3))

$C = -1$ basis $[D^*\bar{D}] = \frac{1}{\sqrt{2}}(D^*\bar{D} - D\bar{D}^*)$

$[D_s^*\bar{D}] = \frac{1}{\sqrt{2}}(D_s^*\bar{D} - D_s\bar{D}^*)$

SU(3)

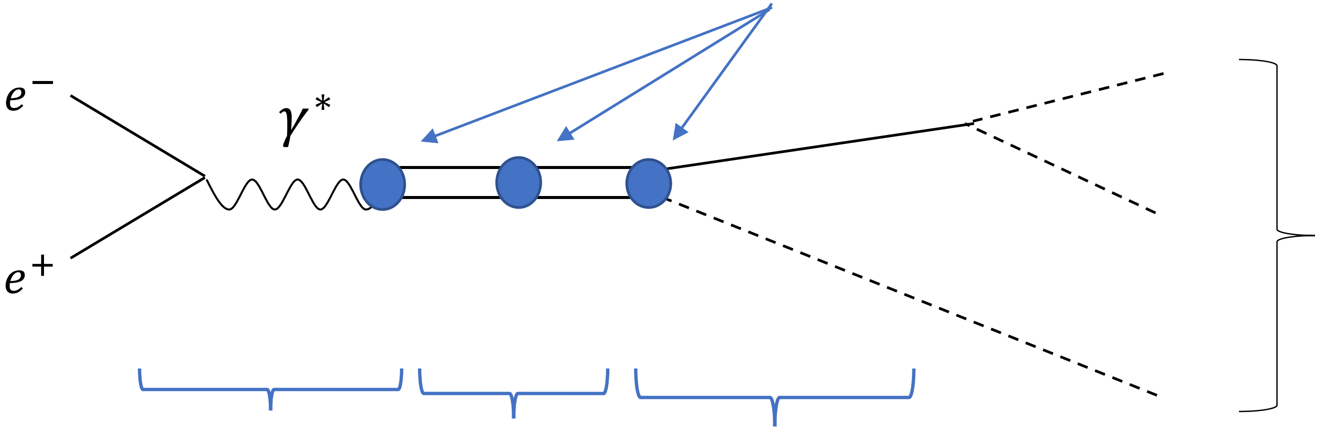
no coupling between hidden-charm channels (e.g. $v_{J/\psi\pi,J/\psi\pi} = v_{J/\psi\pi,\psi'\pi} = 0$)

Nonzero couplings are determined by the global fit → poles may be generated if needed by data

Full amplitude for $e^+e^- \to$ three-body final states

Dressed vertices (propagator) : bare vertices (propagator) dressed by hadron scattering

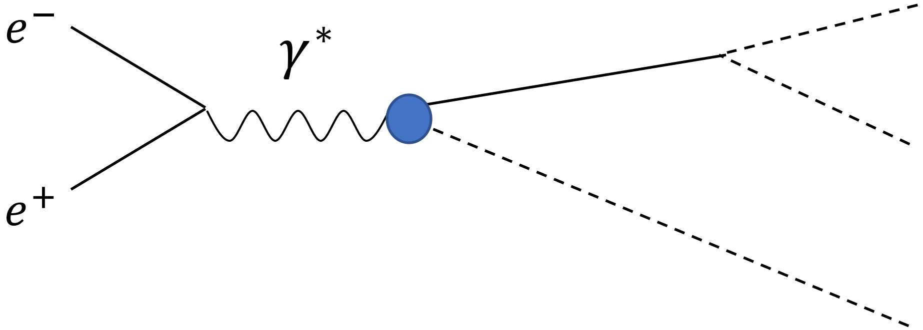
Unitarity requirement



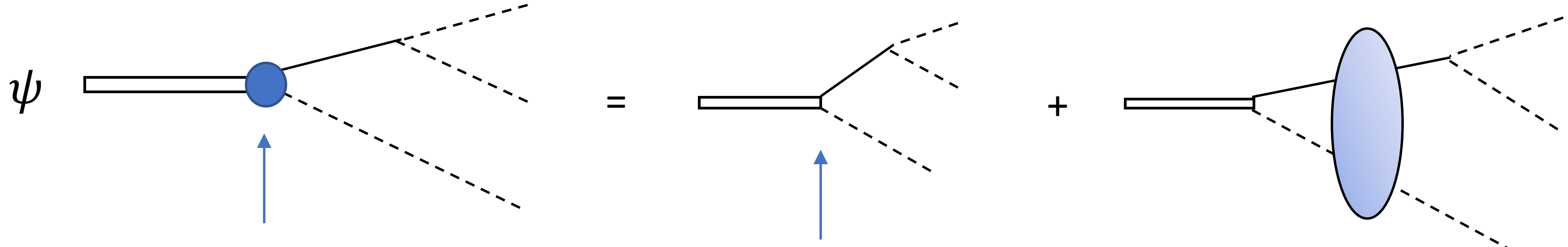
$\pi D^{(*)}\bar{D}^{(*)},\ J/\psi\pi\pi,\ \psi'\pi\pi, h_c\pi\pi, \eta_c\rho\pi, J/\psi K\bar{K}$

ψ production, propagation, decay

Non-resonant mechanisms are also included



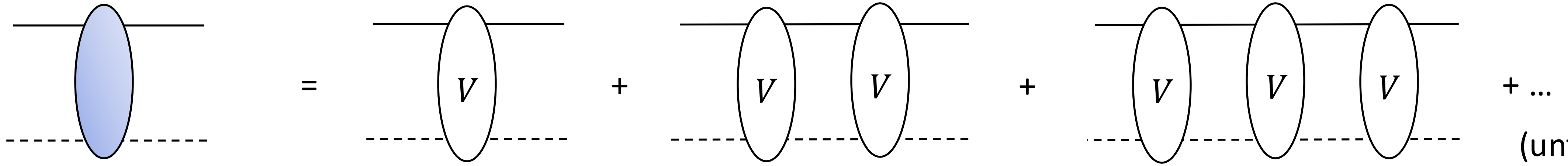
Three-body decays of ψ



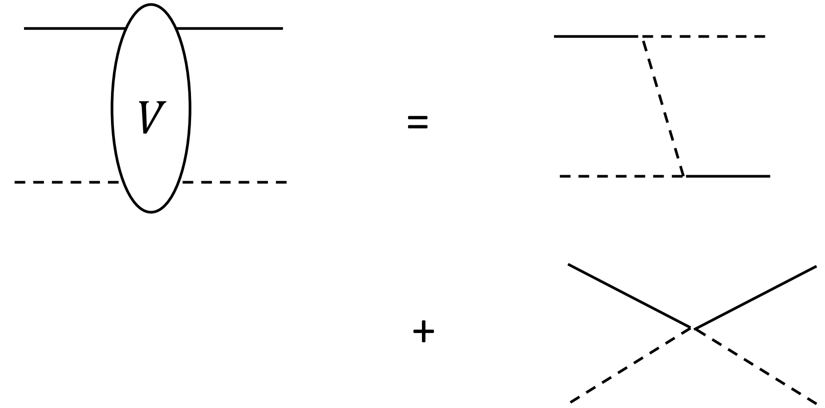
dressed decay vertex

bare vertex

rescattering terms (final-state interactions)



(until infinite loops)

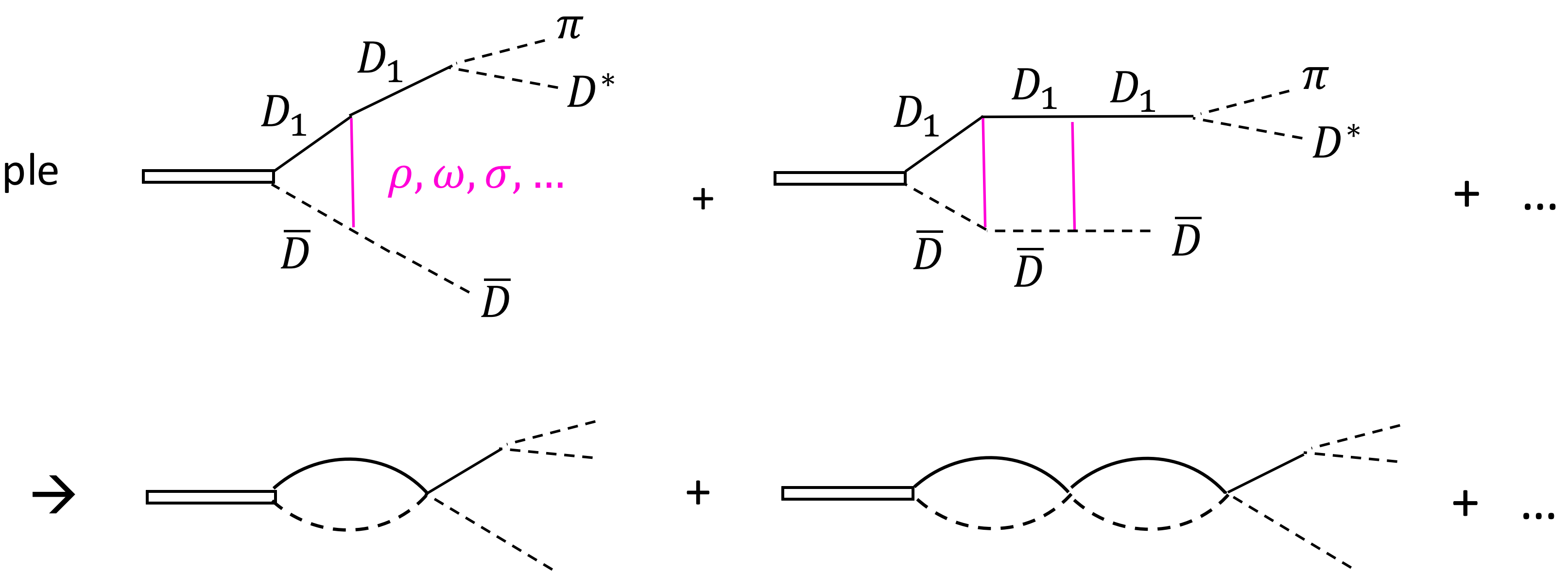


(on-shell) particle-exchange mechanisms → three-body unitarity

Short-range mechanisms among open-charm channels

Short-range mechanisms among open-charm channels

example



(Our model)

Contact interactions among $D_1\bar{D}^{(*)}, D_2^*\bar{D}^*, D^{(*)}\bar{D}^{(*)}, D_{s1}\bar{D}_s, D_s^{(*)}\bar{D}_s^{(*)}, \Lambda_c\bar{\Lambda}_c$ channels

→ fitted to data (advantage of separable interactions)

High-precision BESIII data require these contributions (threshold cusps)

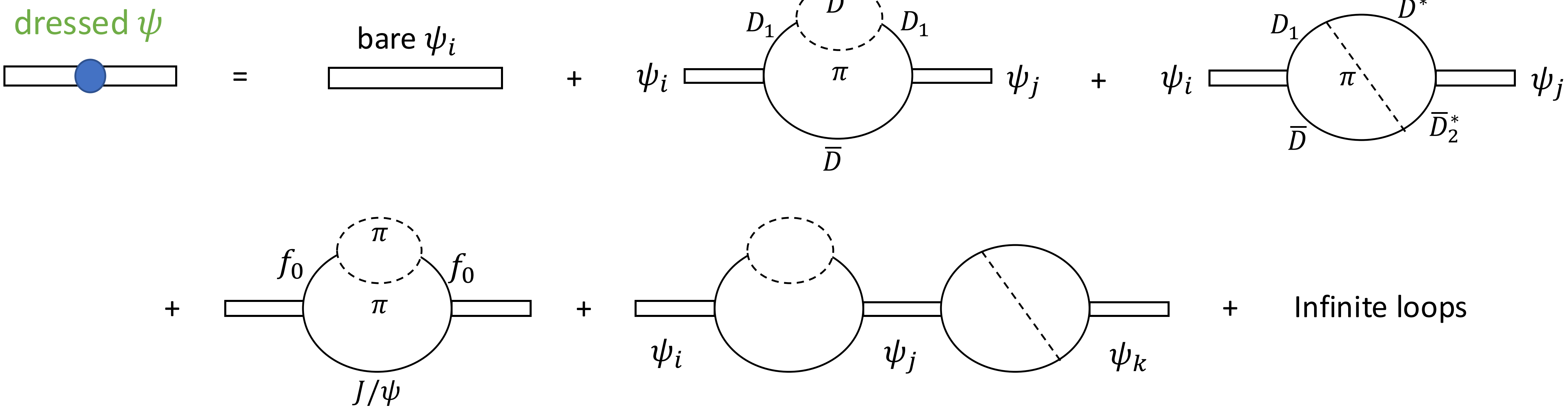
We can examine Y(4220) as $D_1\bar{D}$ molecule and Y(4360) as $D_1\bar{D}^*$ molecule from global analysis

→ To be done

ψ propagator

(we do not use BW)

($D^*\pi$-loop is replaced by D_1 BW)



Charmonium poles are formed by non-perturbative couplings between bare ψ and $D_1\bar{D}$, $f_0\,J/\psi$, ...

(= poles of dressed ψ propagator)

Unitary coupled-channel model : resonance pole (mass, width) and decay dynamics are explicitly related.

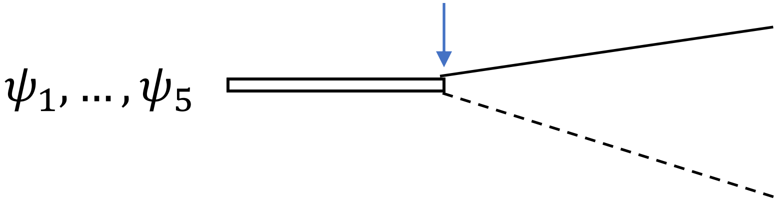
(unitarity requirement)

Breit-Wigner model : decay dynamics are simulated by BW mass and width parameters

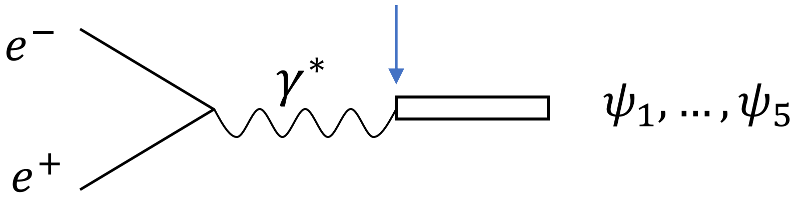
Fitting parameters in global analysis

* bare ψ masses (5 bare states)

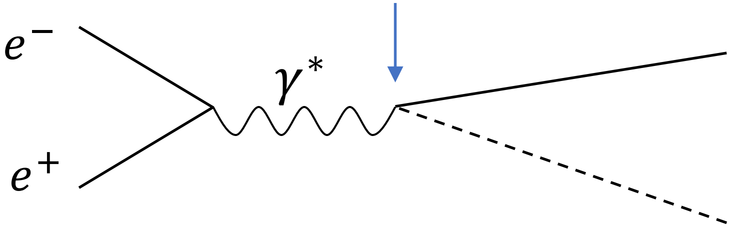
* bare ψ coupling constants (real)



* bare photon-ψ coupling constants (real)



* non-resonant photon coupling constants (real)



* $\psi(4660)$, $\psi(4710)$ Breit-Wigner mass, width, vertices

* coupling constants in Z_c amplitude :

  $v_{D^*\bar{D},D^*\bar{D}}$, $v_{D^*\bar{D},J/\psi\pi}$, $v_{D^*\bar{D},\psi'\pi}$ etc.

* Contact-interaction strengths among open-charm channels

* Cutoffs in non-resonant vertices for

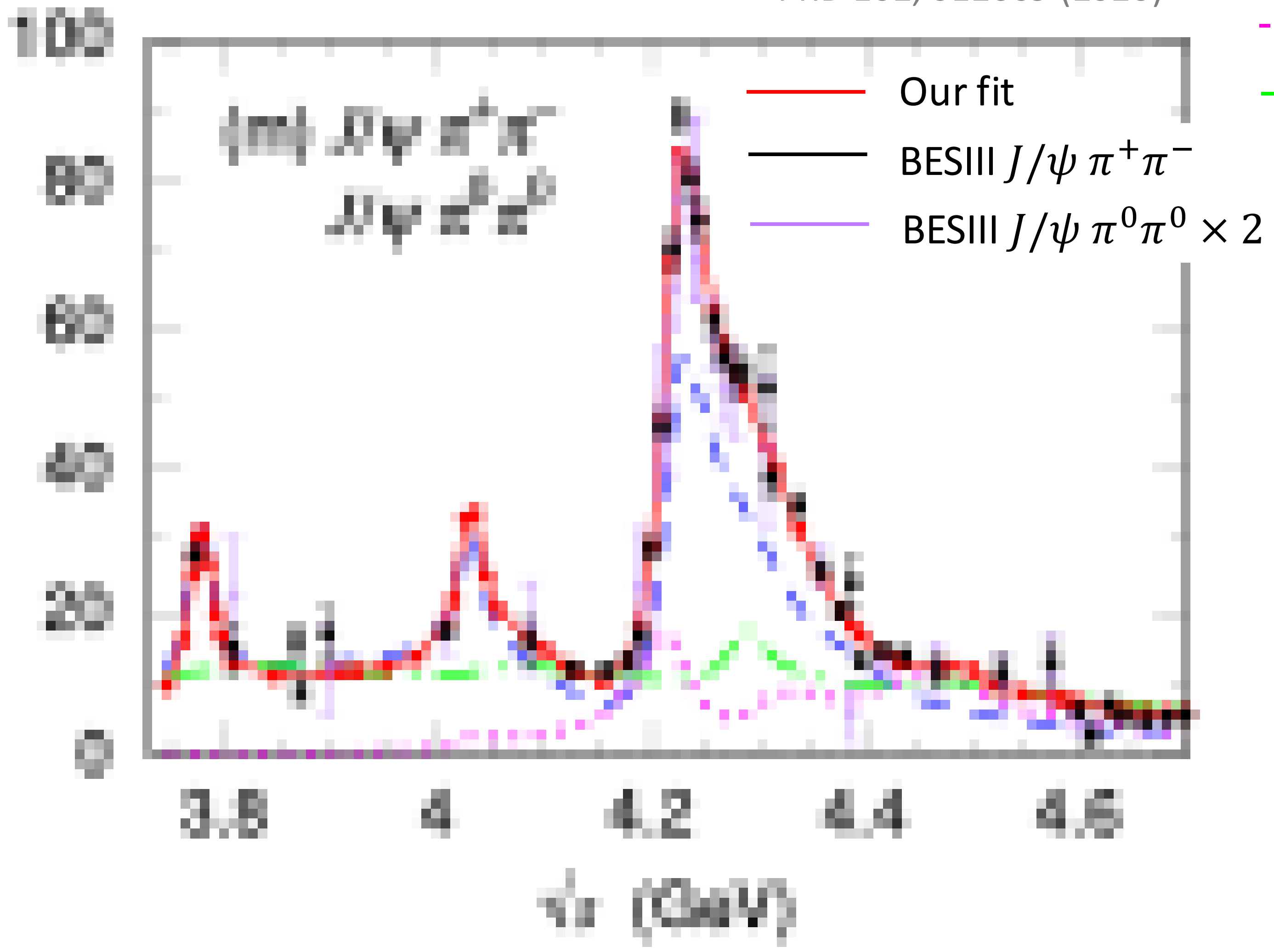
  $\gamma^* \to D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, \Lambda_c\bar{\Lambda}_c$

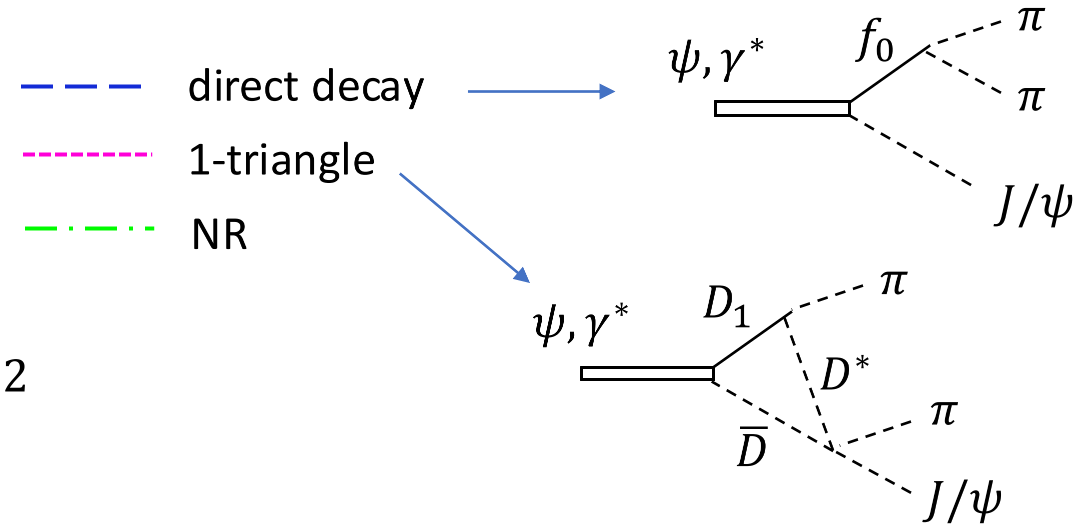
In total, 205 fitting parameters

Selected fit results

$e^+e^- \to J/\psi\,\pi^+\pi^-, J/\psi\,\pi^0\pi^0$

Data: BESIII,
PRD 106, 072001 (2022)
PRD 102, 012009 (2020)





- Overall good agreement with data

  our model is isospin symmetric

  $\sigma(J/\psi\pi^+\pi^-) = 2 \times \sigma(J/\psi\pi^0\pi^0))$

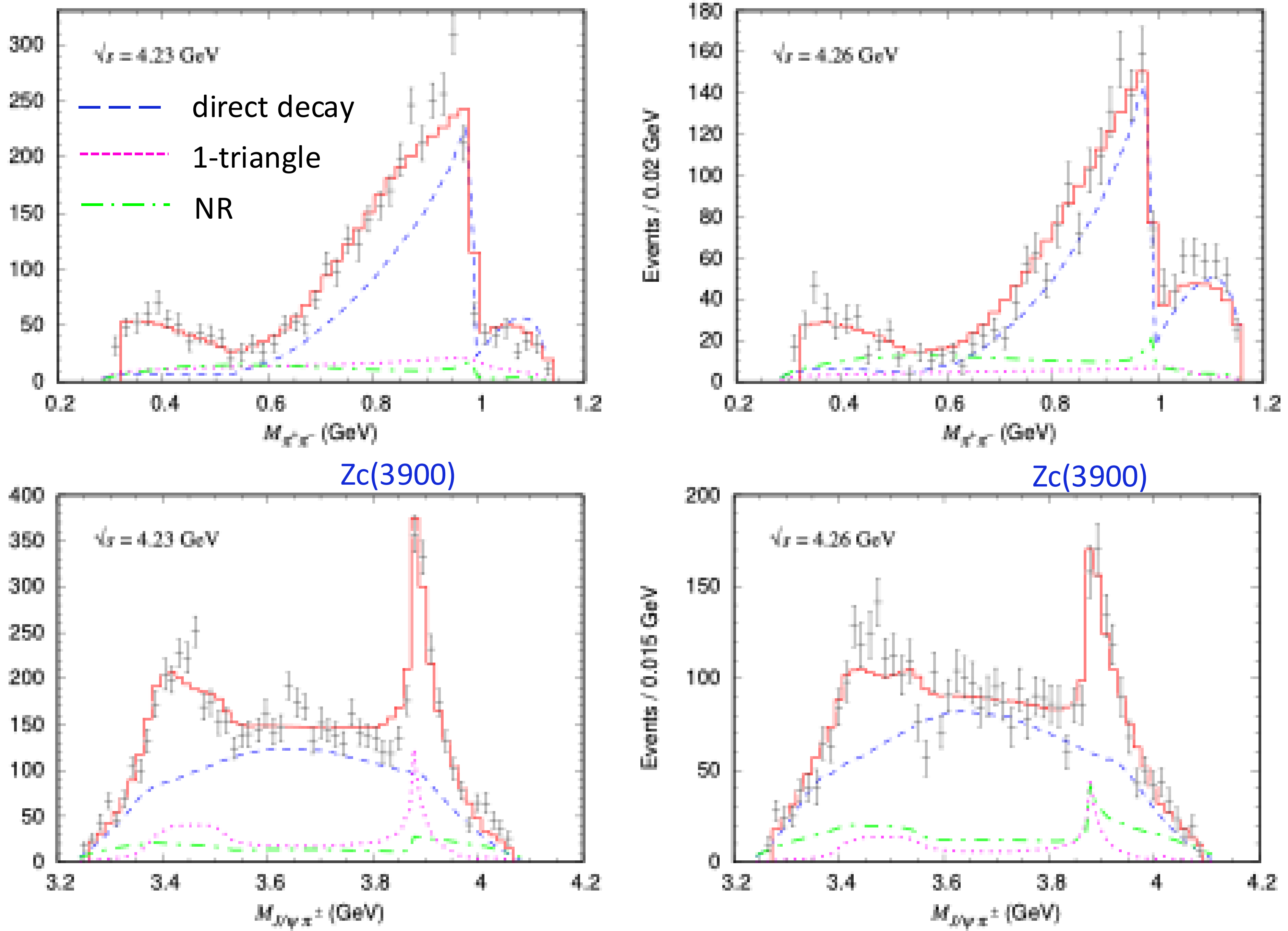
- Triangle singularity effect is seen

  in NR contribution at $\sqrt{s} \sim 4.28$ GeV

- Peaking structure at $\sqrt{s} \sim 4$ GeV is from $\psi(4040)$; consequence of the combined fit

$e^+e^- \to J/\psi\, \pi^+\pi^-$

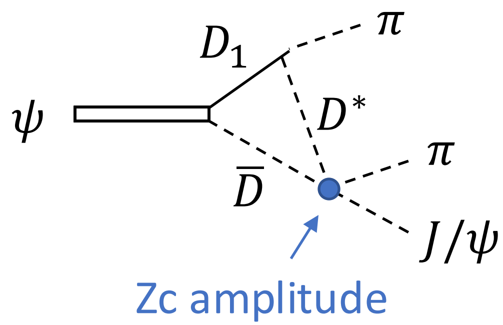
Data: BESIII, PRL 119, 072001 (2017)



Fit to invariant masses

Zc(3900) peaks are well fitted

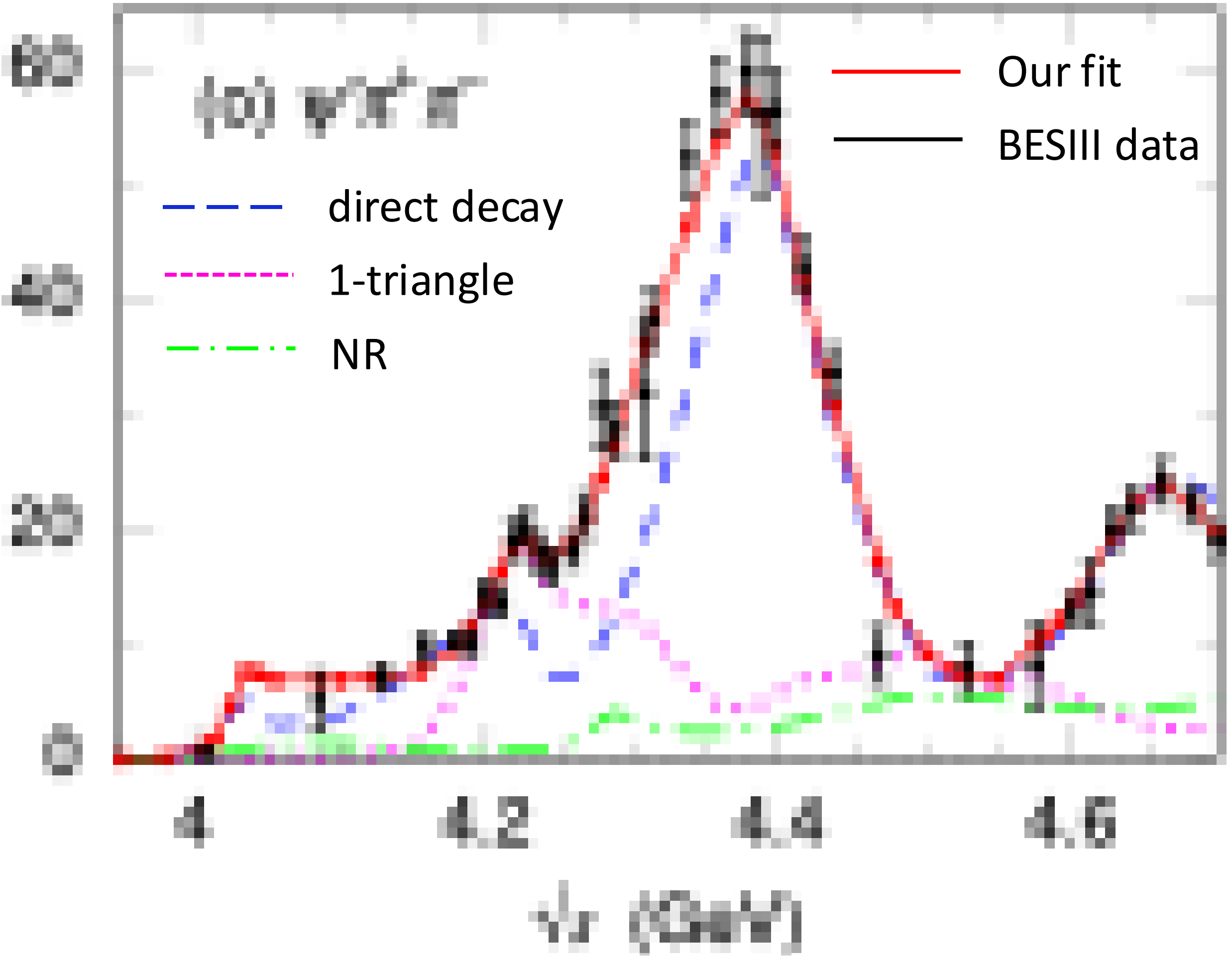
1-loop causes $D^*\bar{D}$ thres. cusp enhanced by a possible pole (a bit off TS condition)



We will examine Zc(3900) pole

$e^+e^- \to \psi' \pi^+\pi^-$

Data: BESIII, PRD 104, 052012 (2021)



- Overall good fit
- Enhancement at ~ 4.03 GeV is from $\psi(4040)$

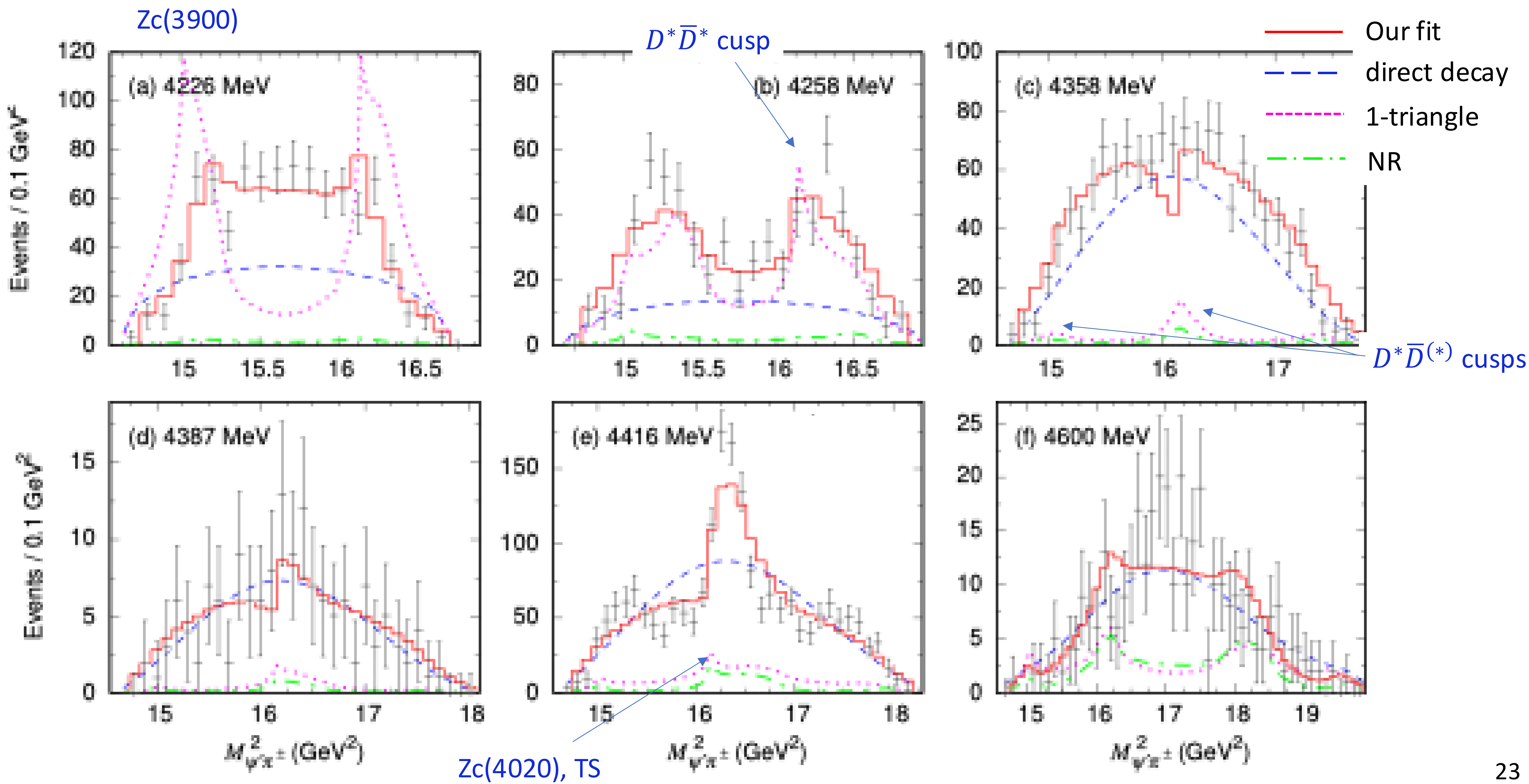
  ← consequence of coupled-channel fit
- 1-triangle contribution is large at $\psi(4220)$ peak
- TS effect seen at ~ 4.28 GeV → $D_1(2420)\bar{D}$ threshold

  in NR contribution

$e^+e^- \to \psi' \pi^+\pi^-$

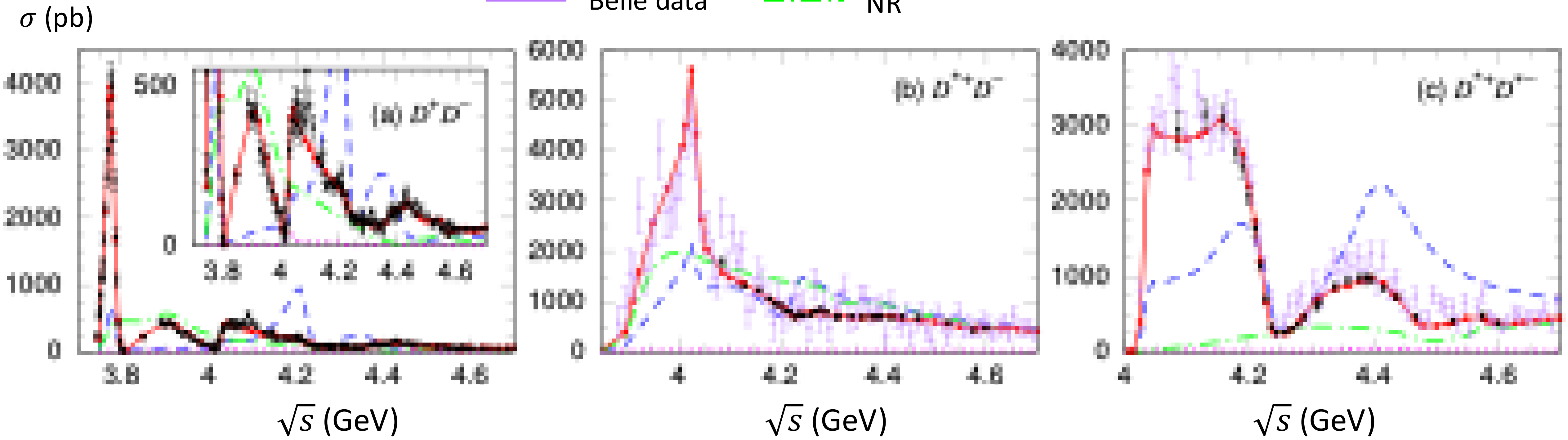
Fit to $\psi'\pi$ invariant mass distributions; Zc, cusp, TS effects

Data: BESIII, PRD 96, 032004 (2017)

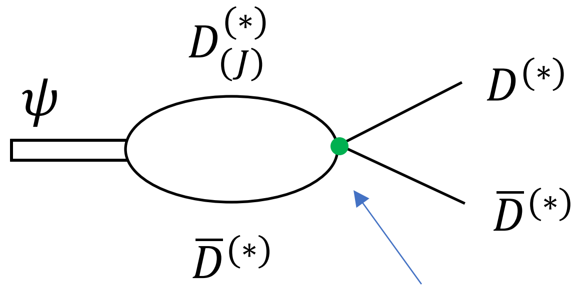


$e^+e^- \to D^{(*)}\bar{D}^{(*)}$

Data: BESIII, PRL 133, 081901 (2024)
JHEP 05 (2022) 155.
Belle, PRD 97, 012002 (2018)

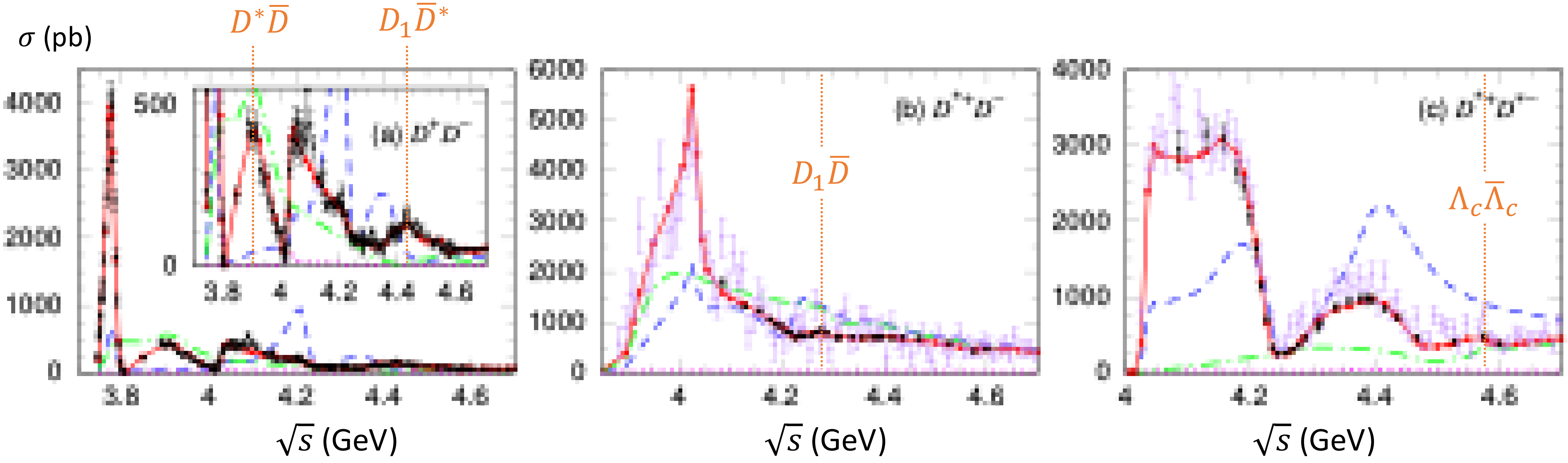


- *Precise* BESIII data are well fitted
- Contact interactions among open-charm channels important (difference between blue and red curves above)
- 1-triangle (particle exchange) is small

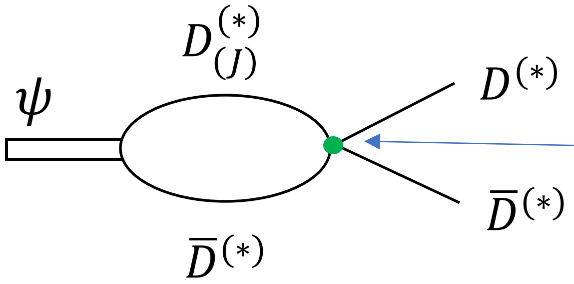


Open-charm rescattering by short-range interactions

$e^+e^- \to D^{(*)}\bar{D}^{(*)}$



Fitting precise data, we need threshold cusps from



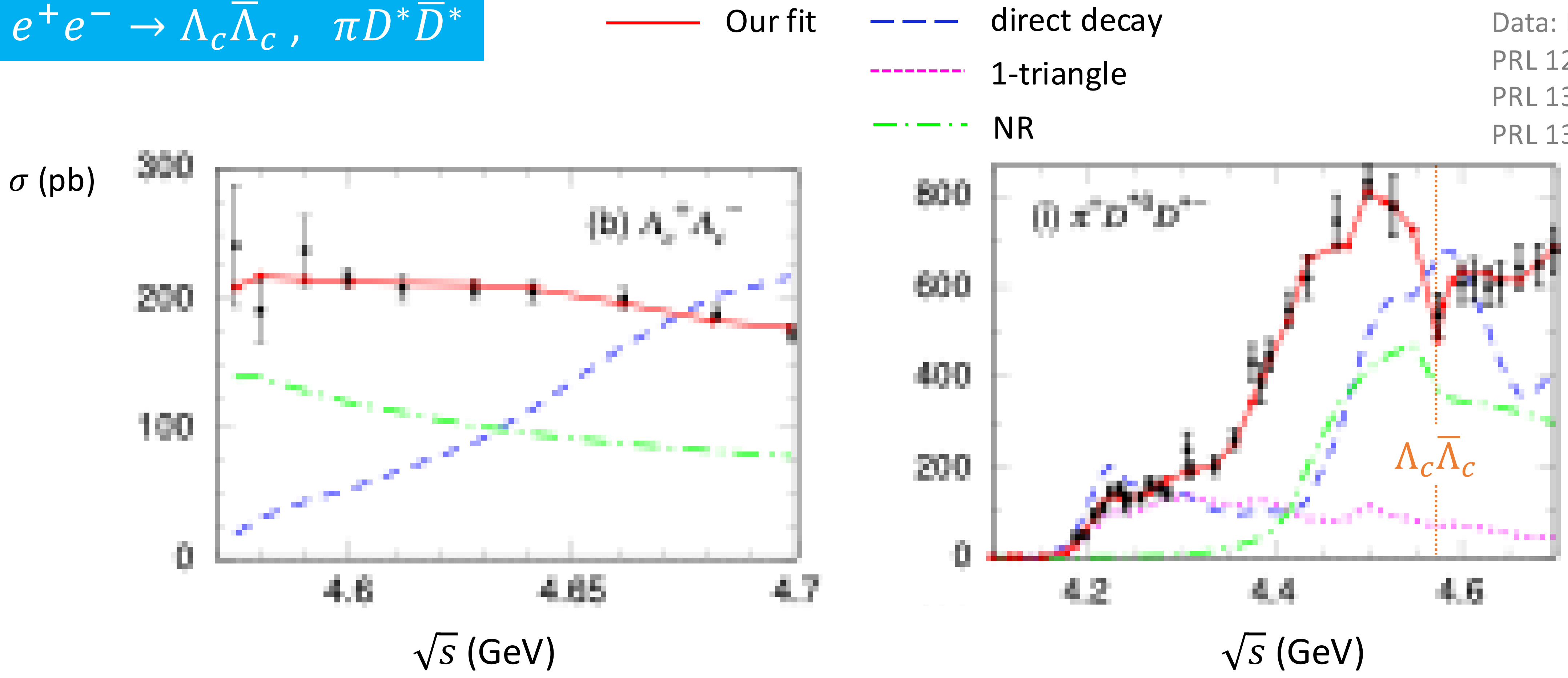
Open-charm rescattering by short-range interactions

Fitting cusps → good constraints on interactions among open-charm channels

→ good constraints on existence of $D^{(*)}_{(J)}\bar{D}^{(*)}$ molecules

$e^+e^- \to \Lambda_c\bar{\Lambda}_c\,,\ \ \pi D^*\bar{D}^*$

Data: BESIII,
PRL 120, 132001 (2018)
PRL 131, 191901 (2023)
PRL 130, 121901 (2023)



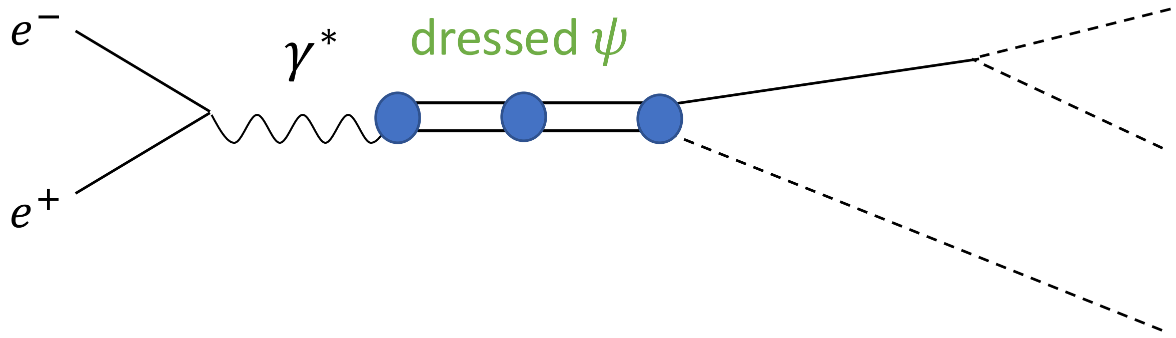
- Non-zero $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ cross section at threshold ← Sommerfeld factor
- $\Lambda_c\bar{\Lambda}_c$ threshold enhancement ← attractive $\Lambda_c\bar{\Lambda}_c$ interaction (pole near threshold is likely)
- $\Lambda_c\bar{\Lambda}_c$ threshold cusp is important to fit $e^+e^- \to \pi D^*\bar{D}^*$ data at $\sqrt{s} \sim 4.57$ GeV

Poles and resonance properties

ψ poles from their dressed propagator

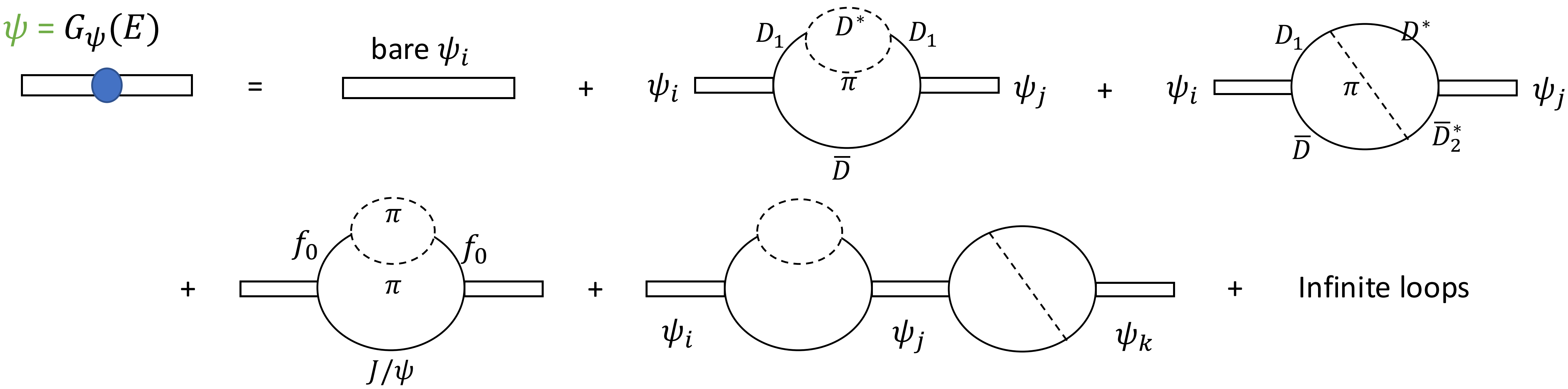
(we are not using BW)

Full amplitude



+ non-resonant

dressed ψ = $G_\psi(E)$



Search complex energy E_ψ where $G_\psi(E_\psi) = \infty$ (E_ψ: pole energy, pole position) by analytical continuation of $G_\psi(E)$

Resonance parameters (final check to be done)

$M = \mathrm{Re}[E_\psi]$
$\Gamma = -2 \times \mathrm{Im}[E_\psi]$

This work		PDG [4] + $Y(4320)$ [16]		
M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	
3780.0 ± 0.5	30.3 ± 1.7	3778.1 ± 0.7	27.5 ± 0.9	$\psi(3770)$
4029.2 ± 0.3	27.9 ± 0.7	4039 ± 1	80 ± 10	$\psi(4040)$
4187.8 ± 1.7	127.4 ± 3.5	4191 ± 5	70 ± 10	$\psi(4160)$
4228.3 ± 0.7	44.1 ± 1.7	4222.5 ± 2.4	48 ± 8	$\psi(4230)$
4305.7 ± 1.6	128.7 ± 3.5	4298 ± 12	127 ± 17	$Y(4320)$ ← Not in PDG
4353.8 ± 5.6	123.4 ± 4.5	4374 ± 7	118 ± 12	$\psi(4360)$
4388.1 ± 2.1	107.5 ± 4.8	4421 ± 4	62 ± 20	$\psi(4415)$
4655.4 ± 2.5	135.0 ± 6.8	4630 ± 6	72^{+14}_{-12}	$\psi(4660)$ ← BW fit

7 poles from 5 bare states; Number of poles form our analysis is consistent with PDG + Y(4320)

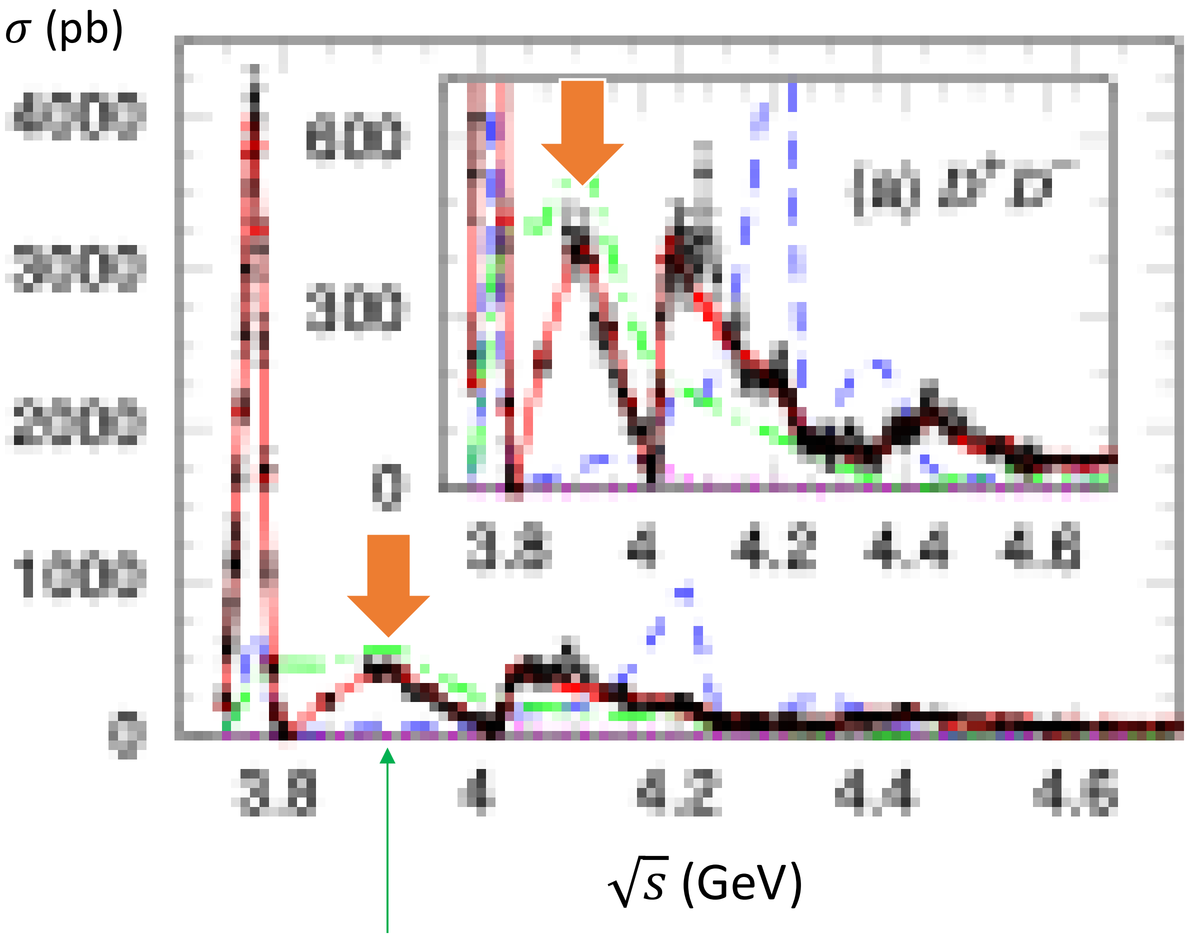
Noticeable differences from PDG

Mass : $\psi(4415)$

Width: $\psi(4040), \psi(4160), \psi(4415), \psi(4660)$

No Y-width puzzle by construction: same Y-widths for all final states

G(3900)



resonance-like peak at 3.9 GeV called G(3900)

G(3900) state claimed in BESIII analysis of $e^+e^- \to D\bar{D}$

BESIII, PRL 133, 081901 (2024)

G(3900) state predicted by meson-exchange model

Z.-Y. Li et al. 2403.01727; PRL

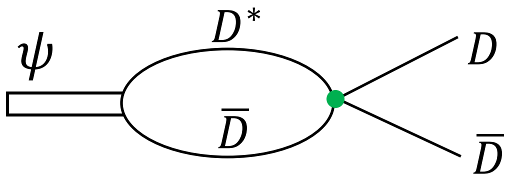
No G(3900) pole by K-matrix analysis of $e^+e^- \to D^{(*)}\bar{D}^{(*)}$

N. Husken et al., PRD 109, 11401 (2024)

No G(3900) pole from our analysis

G(3900) peak in our model

Interference between ψ(3770), ψ(4040) and non-resonant amplitudes + $D^*\bar{D}$ threshold cusp



Is Y(4220) $D_1\bar{D}$ molecule ?

Is Y(4360) $D_1\bar{D}^*$ molecule ?

→ To be examined soon

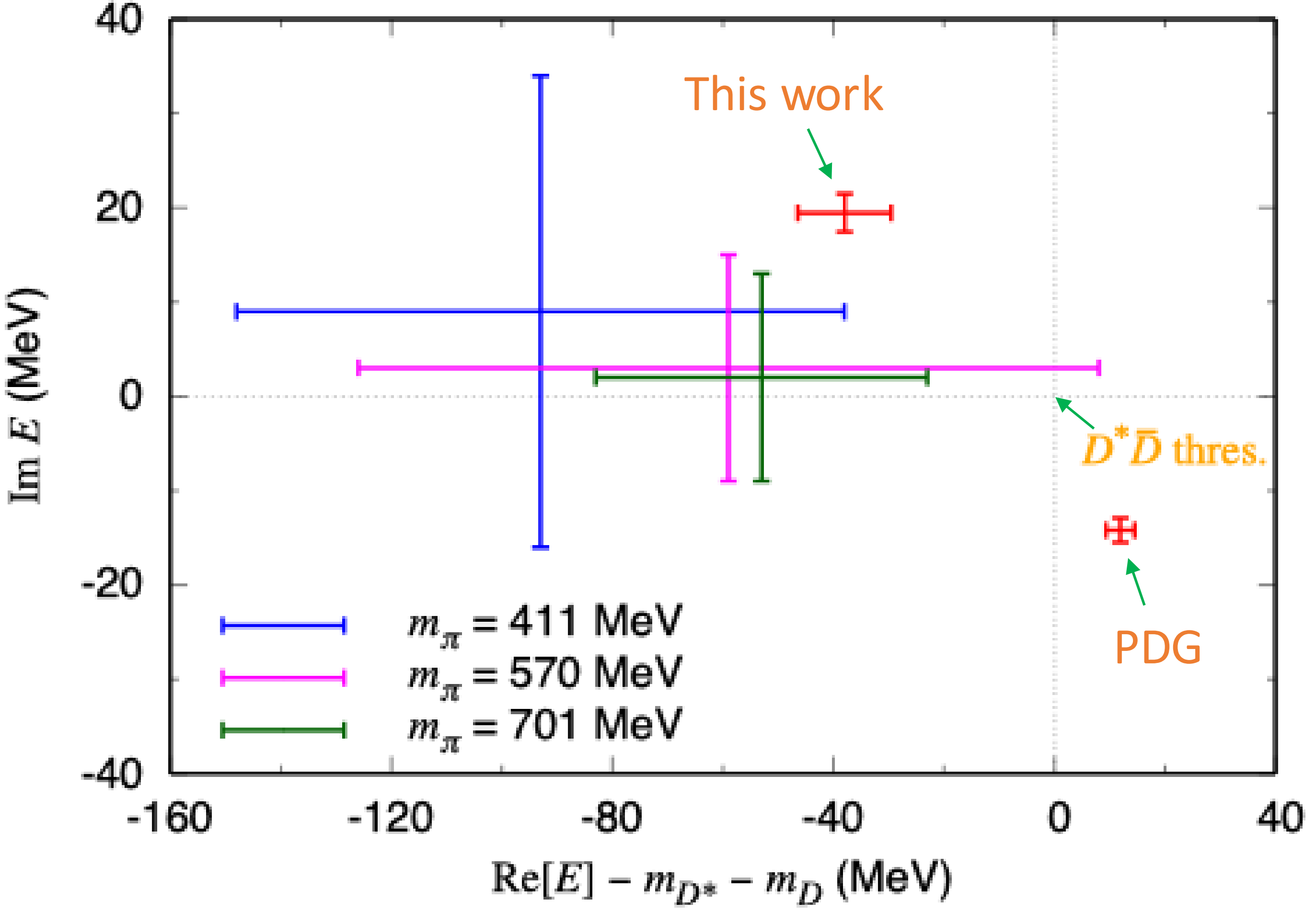
Zc poles

from $J^{PC}=1^{+-}$ $D^*\bar{D}-D^*\bar{D}^*-J/\psi\pi-\psi'\pi-h_c\pi-\eta_c\rho$ couple—channel amplitude

This work	PDG [4]		
E_{pole} (MeV)	M (MeV)	Γ (MeV)	
$(3837.7\pm8.4)+(19.4\pm2.0)i$	3887.1 ± 2.6	28.4 ± 2.6	$Z_c(3900)$
$(3989.9\pm5.9)+(26.1\pm5.1)i$	4024.1 ± 1.9	13 ± 5	$Z_c(4020)$

Zc(3900) pole: comparison with LQCD result

$D^*\overline{D}$ unphysical sheet (complex E-plane)



LQCD ($m_\pi = 411$ MeV)
HAL QCD, J. Phys. G 45, 024002 (2018)

$m_{D^*} + m_D - (93 \pm 55 \pm 21) + (9 \pm 25 \pm 7)i$ MeV

$S(\{-k_i^*\}) = S^*(\{k_i\})$ applied; PRD 105, 014034 (2022)

This work

$m_{D^*} + m_D - (38 \pm 7) + (19 \pm 1)i$ MeV

PDG

$m_{D^*} + m_D + (11.9 \pm 2.6) - (14.2 \pm 1.3)i$ MeV

LQCD and this work are fairly consistent (virtual poles)

Summary and perspective

Summary

- Conducted global coupled-channel analysis of most of available $e^+e^- \to c\bar{c}$ data in $\sqrt{s} = 3.75 - 4.7$ GeV

  Global coupled-channel analysis is common for N*. The $e^+e^- \to c\bar{c}$ analysis now gets closer to the standard !
- Reasonable fits are obtained overall
- Vector charmonium and Zc poles extracted

  -- 7 poles from 5 bare states; # of poles consistent with PDG + Y(4320); no G(3900) pole

  -- Zc poles are virtual poles at ~ 40 MeV below $D^*\bar{D}^{(*)}$ thresholds, consistent with LQCD results

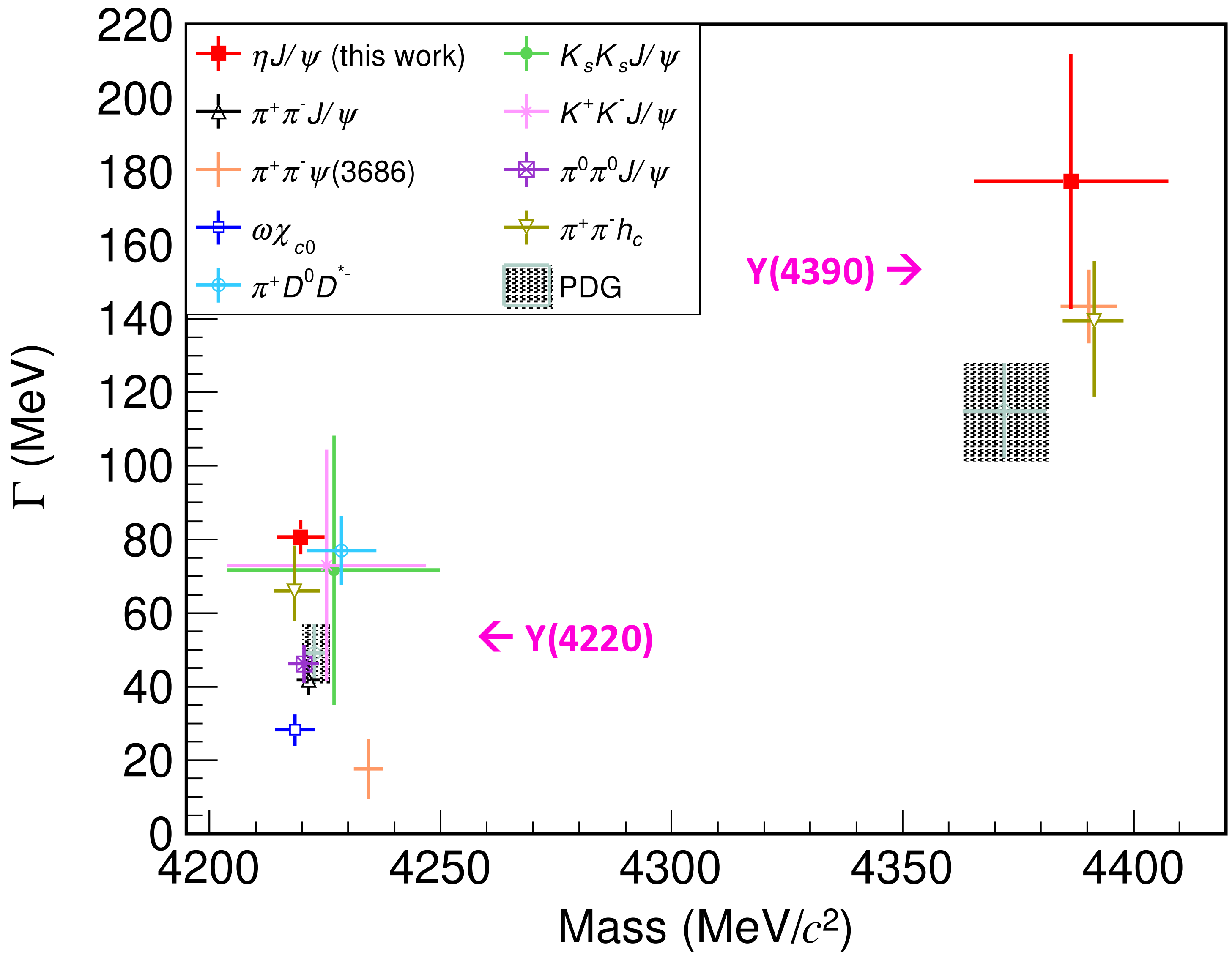
Future

- Pole residues will be extracted → address Y width problem, structure of exotic candidates *Y*
- Fit efficiency-corrected, background-free Dalitz plots (not 1D fit) to fully consider experimental constraints on charmonium and Zc properties
- Include $e^+e^- \to K\bar{D}_s^{(*)}D^{(*)}$ cross sections when available → include higher charmonium states

  → address Zcs(3985) from global analysis

Backup

Outstanding question in XYZ physics : Y width problem



BESIII, arXiv:2310.03361

Previous coupled-channel analyses for Y-width puzzle

Three-body model

* M. Cleven, Q. Wang, F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Zhao, PRD 90, 074039 (2014)

Analysis of $e^+e^- \to \pi D\bar{D}^*$, $J/\psi\pi\pi$, $h_c\pi\pi$ cross section and invariant mass in $4.1 \lesssim \sqrt{s} \lesssim 4.3$ GeV [Y(4230) region]

Pioneering works, but the data were very limited

* L. Detten, C. Hanhart, V. Baru, Q. Wang, D. Winney, Q.Zhao, PRD 109 , 116002 (2024)

Fitting data in Y(4230) region; more final states than the above; Y(4230) as $D_1\bar{D}$ molecule claimed

Breit-Wigner fit to cross section data

* D.-Y. Chen, X. Liu, T. Matsuki, Eur. Phys. J. C 78, 136 (2018)

Fitting $e^+e^- \to \pi D\bar{D}^*$, $J/\psi\pi\pi$, $h_c\pi\pi$ cross sections → Y(4320) and Y(4390) unnecessary

Two-body unitary model fitted to cross section data

* Z.-Y. Zhou, C.-Y. Li, Z. Xiao, arXiv:2304.07052

Fitting $e^+e^- \to D^{(*)}\bar{D}^{(*)}$, $\pi D\bar{D}$ cross sections → ψ(4160) is Y(4230)

Our analysis

- more complete dataset
- more coupled-channels
- three-body unitary

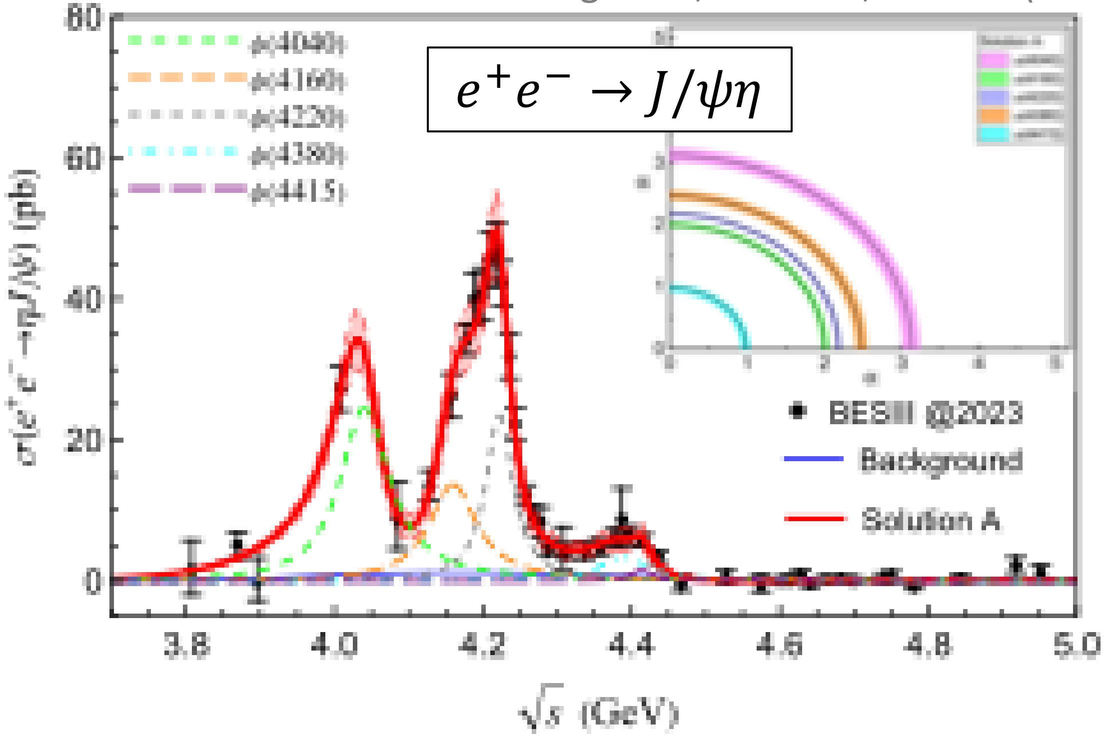
→ more reliable conclusion

How to find solution to Y width problem ?

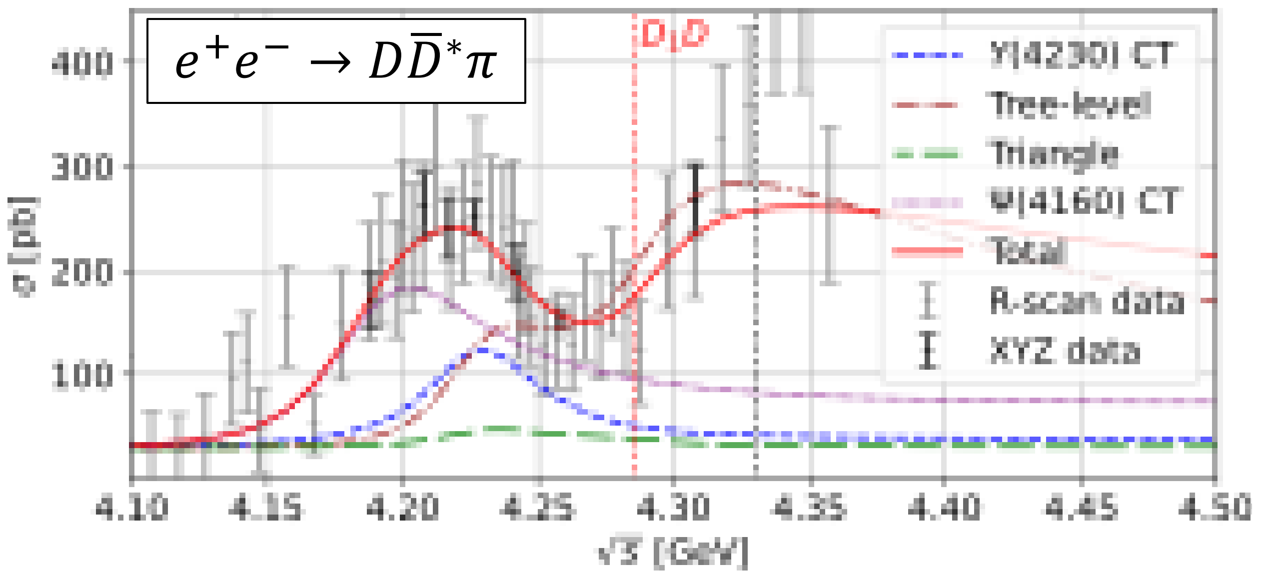
🙁 Combine a couple of charmonia to solve Y-width problem

Narrow Y(4220) from $e^+e^- \to J/\psi\pi\pi$ → narrow Y(4220) + $\psi(4160)$ → broad Y(4220) in other processes

T.-C. Peng et al., PRD 109, 094048 (2024)



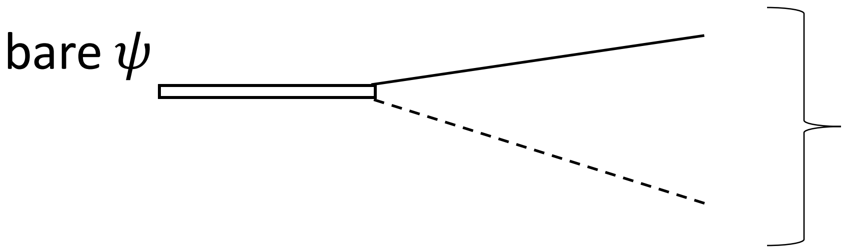
L. von Detten et al., PRD 109 , 116002 (2024)



Problem: sum of Breit-Wigner amplitudes violates unitarity; more problematic for overlapping resonances

ψ decays (bare vertices)

(quasi) two-body channels included; $J^{PC} = 1^{--}$

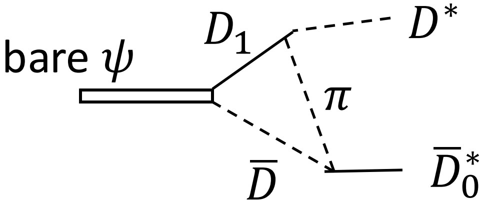


$D_0^*(2300)D^*$, $f_0 J/\psi$, $f_2 J/\psi$, $f_0 \psi'$, $f_0 h_c$, $Z_c \pi$, $Z_{cs} K$

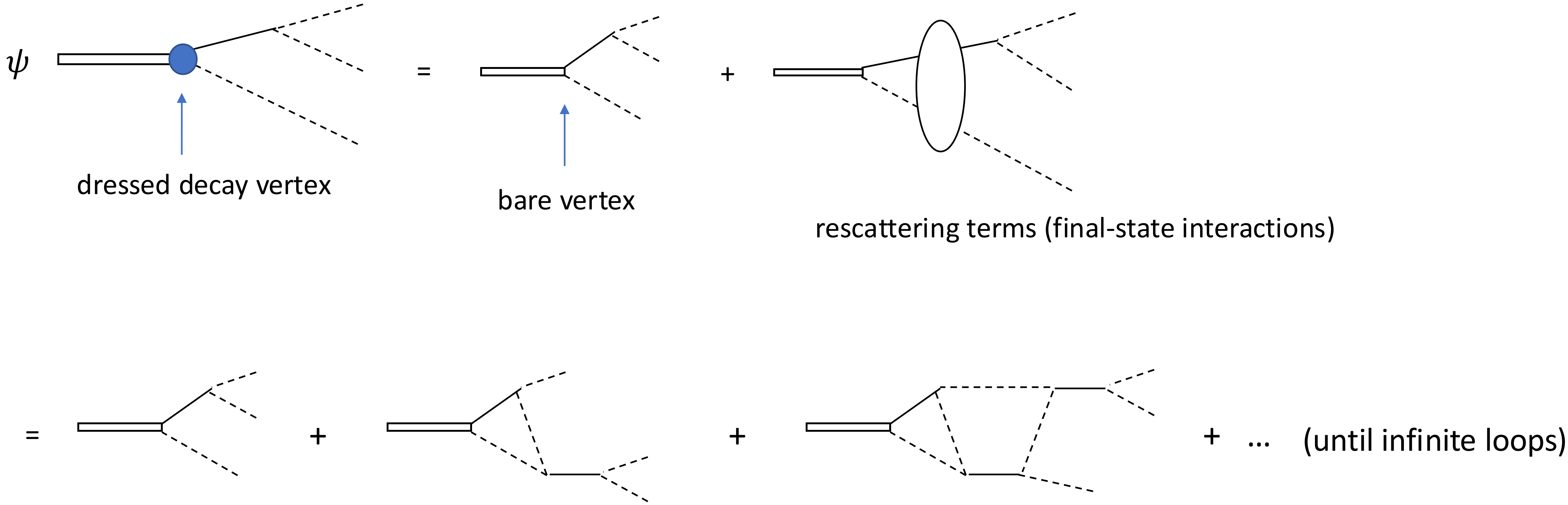
We do not include “ bare $\psi \to D_0^* \bar{D}^*,\ Z_c\pi\ ,\ Z_{cs}\bar{K}$ ”

bare ψ dominantly decays to two-body states; D_0^* and Z_c are probably not compact states

$D_0^* \bar{D}^*$ and $Z_c\pi$ channels are generated by coupled-channel effect like



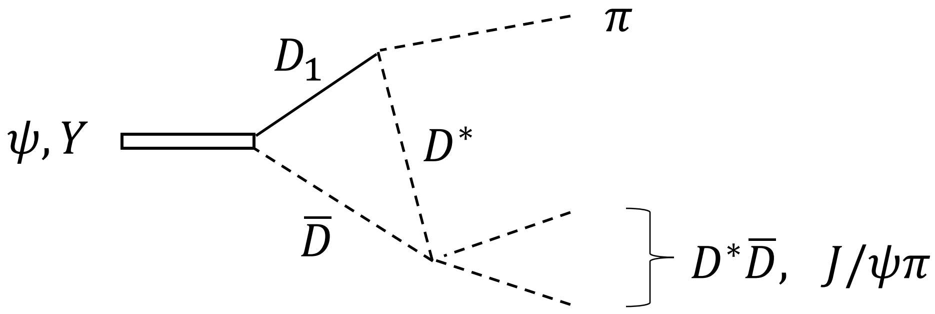
Three-body decays of ψ



Final state interactions described by solution of Faddeev equation → Coupled-channels taken into account

Rescattering mechanisms (particle exchange) required by three-body unitarity are considered

Triangle singularity (TS) from our model



Kinematical condition for TS

Energy-momentum is conserved everywhere as classical process

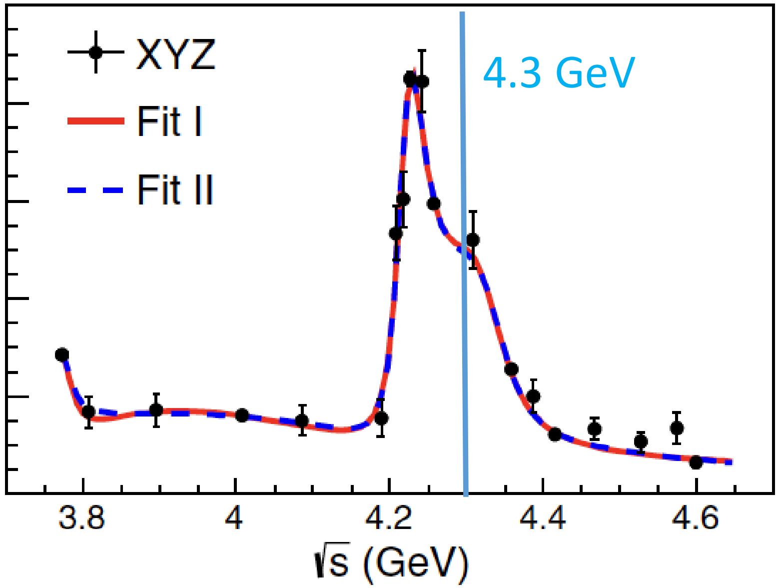
→ amplitude is significantly enhanced at

$\sqrt{s} \sim m_{D_1} + m_{\bar{D}}$ (~4.3 GeV) and $M_{D^*\bar{D}} \sim m_{D^*} + m_{\bar{D}}$ (~3.88 GeV)

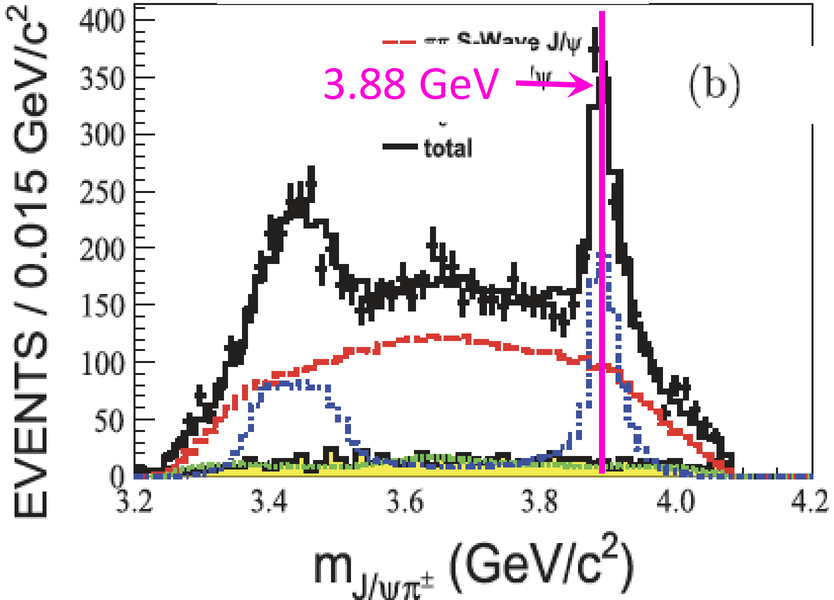
$M_{J/\psi\pi}$

Data show coincidence of Y(4320), Zc, and TS

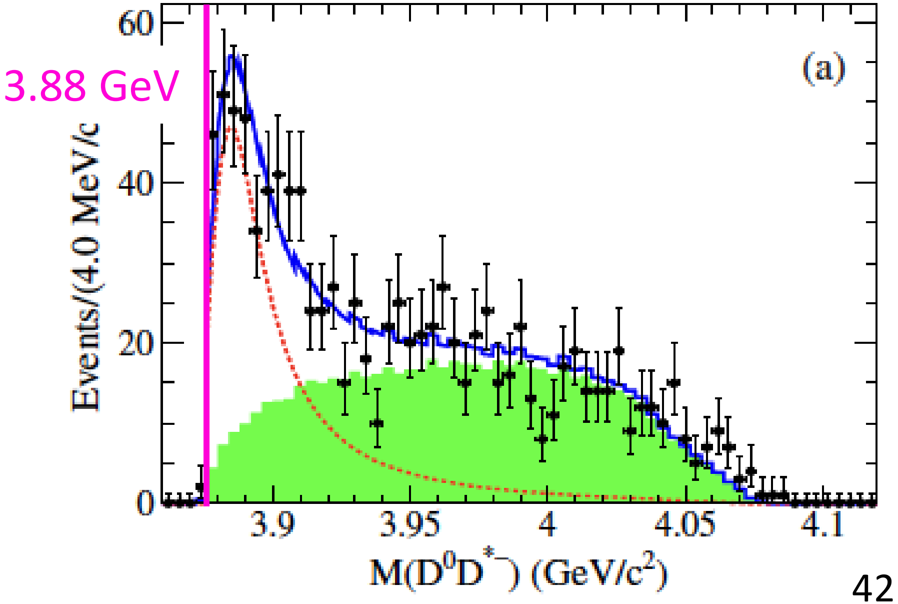
$e^+e^- \to J/\psi\,\pi^+\pi^-$



$e^+e^- \to J/\psi\,\pi^+\pi^-$

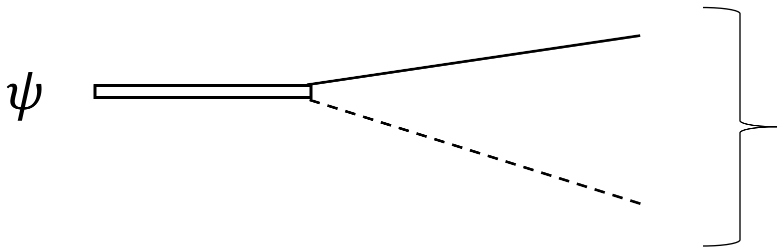


$e^+e^- \to \pi^+ D^0 D^{*-}$



Coupled-channels

(quasi) two-body channels included; $J^{PC} = 1^{--}$



$D_1(2420)\bar{D}^{(*)}$, $D_1(2430)^0\bar{D}^{(*)}$, $D_2^*(2460)\bar{D}^{(*)}$, $D^{(*)}\bar{D}^{(*)}$

$D_1(2420), D_1(2430)$, $D_2^*(2460), D^*$ → Breit-Wigner (BW) propagators; mass and width from PDG

$D_J^{(*)} \rightarrow D^{(*)}\pi$ coupling strength is determined, assuming the following decays saturate the width

$D_1(2420) \rightarrow D^*\pi$ (mainly d-wave decay); small s–wave coupling fixed by helicity angle distribution data

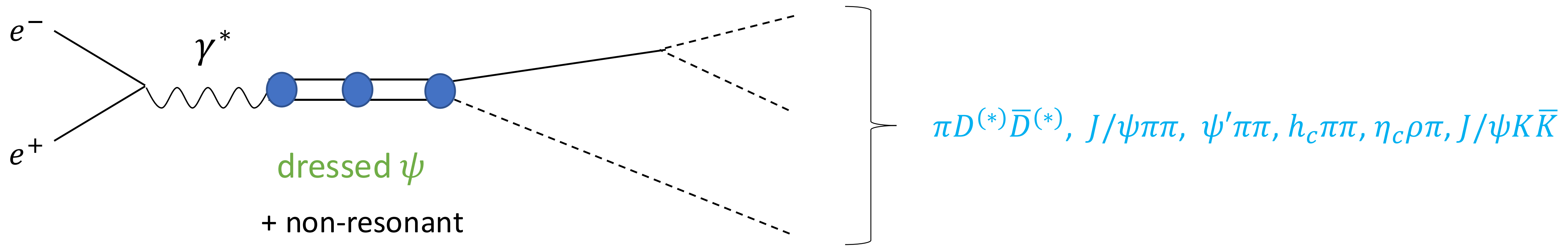
Babar, PRD 82, 111101 (2010)

$D_1(2430) \rightarrow D^*\pi$ (s−wave decay)

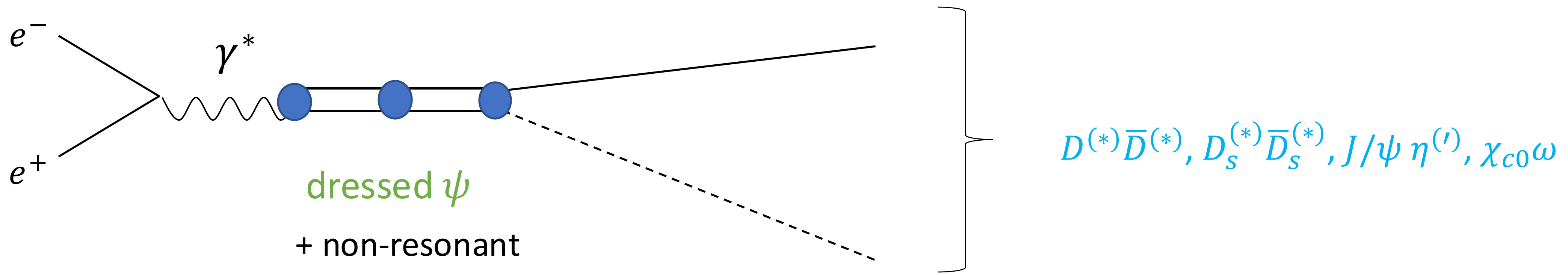
$D_2^*(2460) \rightarrow D^*\pi + D\pi;\ \Gamma(D\pi)/\Gamma(D^*\pi) \sim 1.5$

$D^{*+} \rightarrow D\pi$

Full amplitude for three-body final states $e^+e^- \to \pi D^{(*)}\bar{D}^{(*)},\ J/\psi\pi\pi,\ \psi'\pi\pi, h_c\pi\pi, \eta_c\rho\pi, J/\psi K\bar{K}$

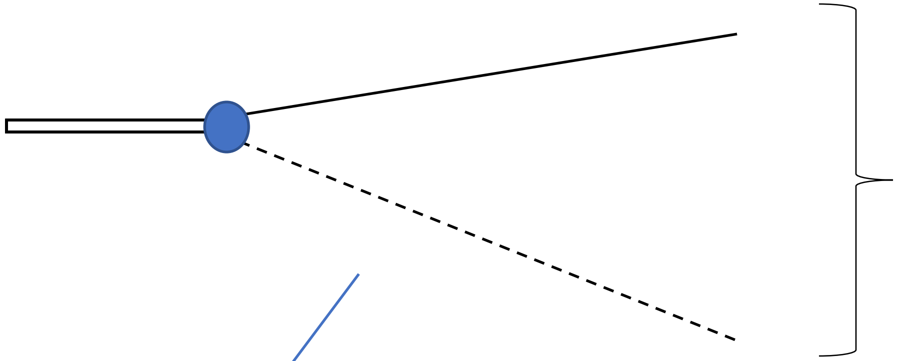


Full amplitude for two-body final states $e^+e^- \to D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi\,\eta^{(\prime)}, \chi_{c0}\omega$



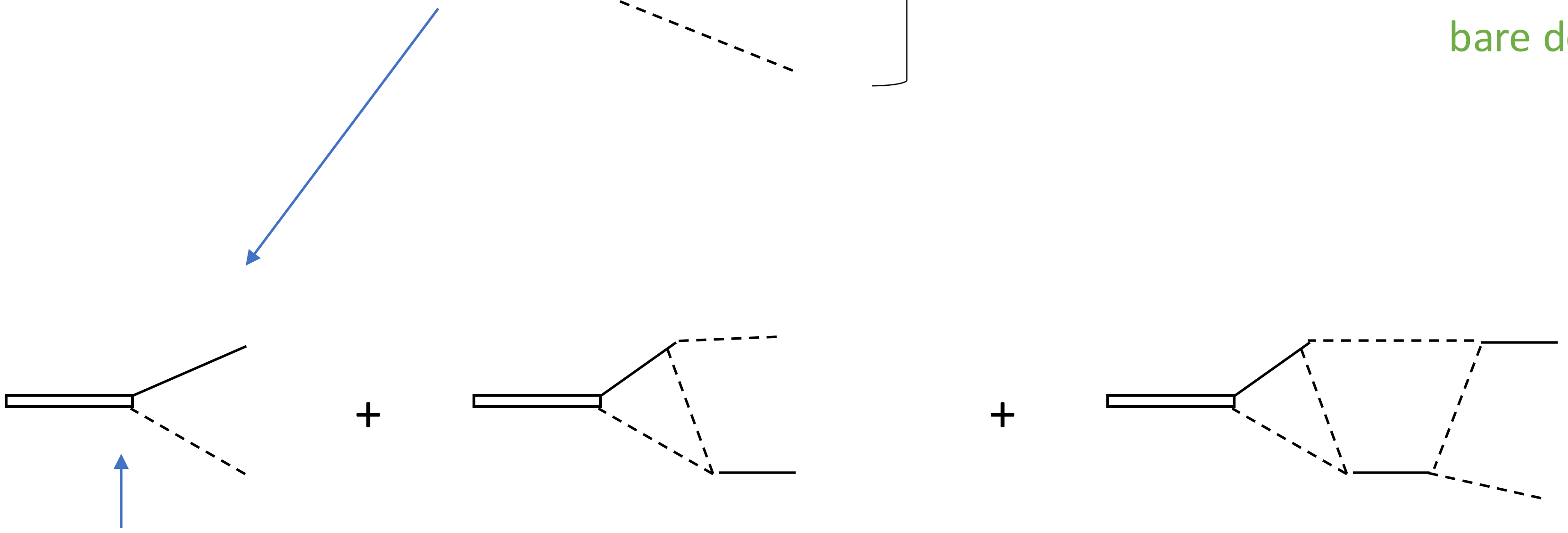
Two-body decay processes of ψ and Y

bare ψ



$D^{(*)}\overline{D}^{(*)}, D_s^{(*)}\overline{D}_s^{(*)}, J/\psi\,\eta^{(\prime)}, \chi_{c0}\omega$

bare decay mechanism only



$+ \; \dots$ (until infinite loops)

bare decay vertex

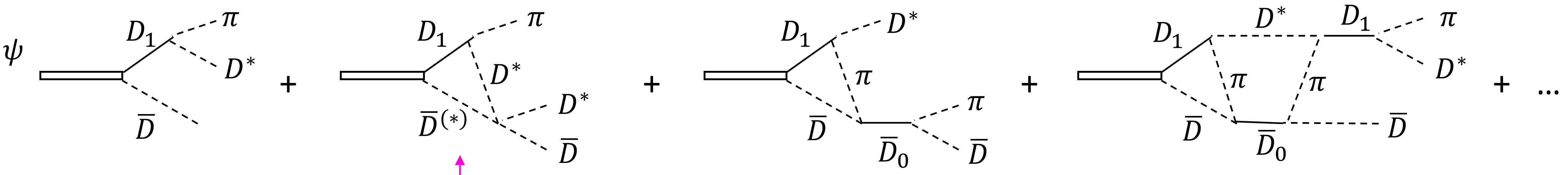
Final state interactions described by solution of Faddeev equation

Contact interactions included also

Three-body decays of ψ

(some selected diagrams)

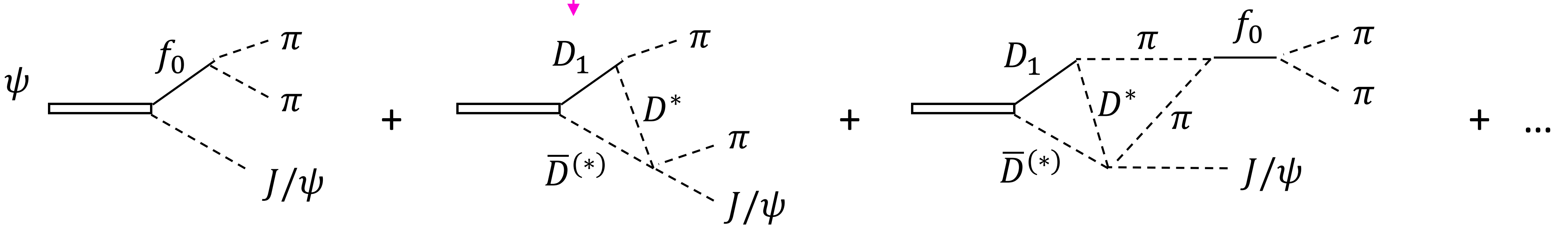
$e^+e^- \to \pi D^* \bar{D}$



(until infinite loops)

$e^+e^- \to J/\psi\pi\pi$

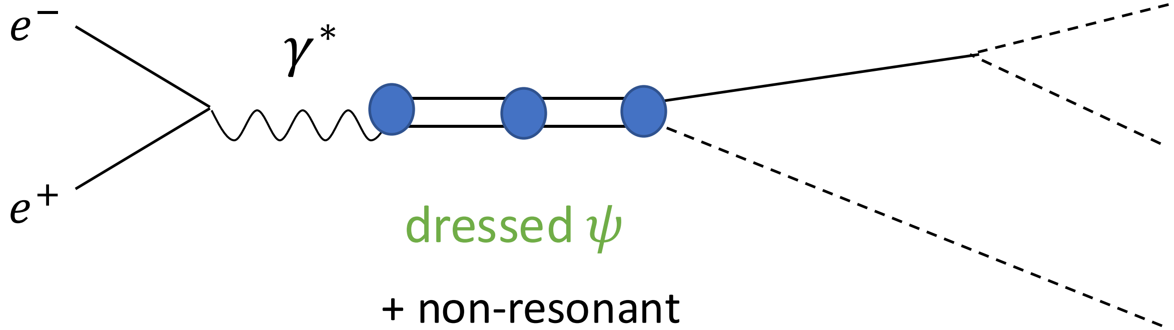
Triangle singularity loop



(until infinite loops)

Different processes share the same interactions ← unitarity requirement

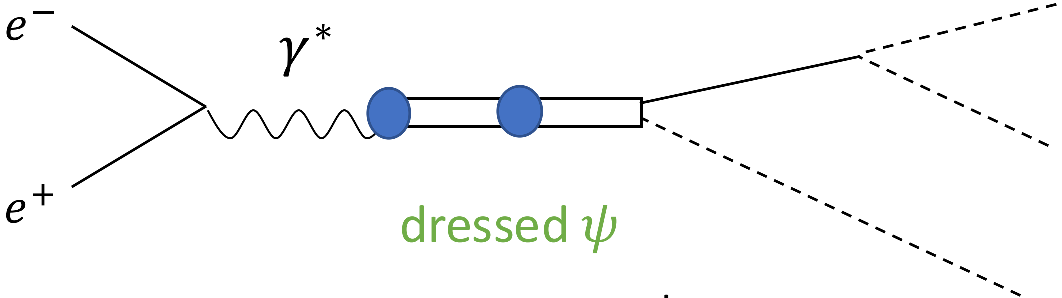
Full amplitude



dressed ψ

+ non-resonant

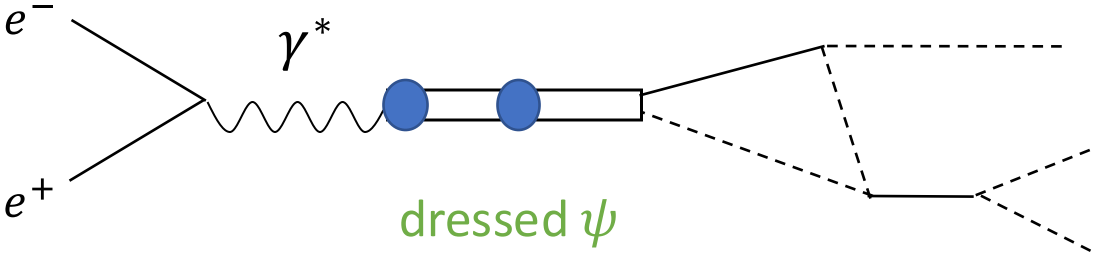
tree



dressed ψ

+ non-resonant

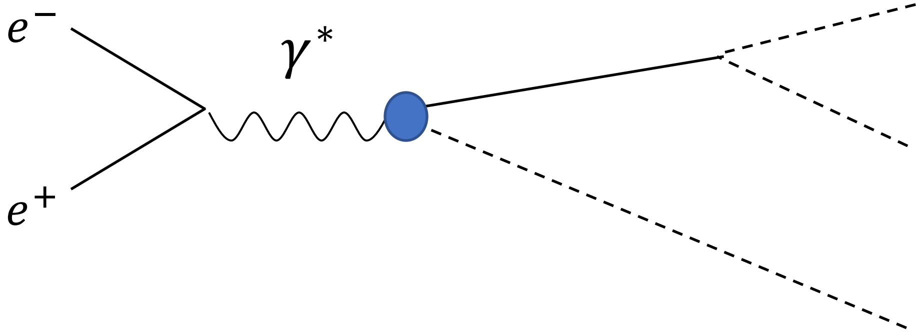
1-loop

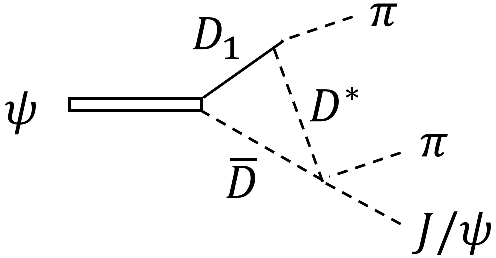


dressed ψ

+ non-resonant

NR (non-resonant)

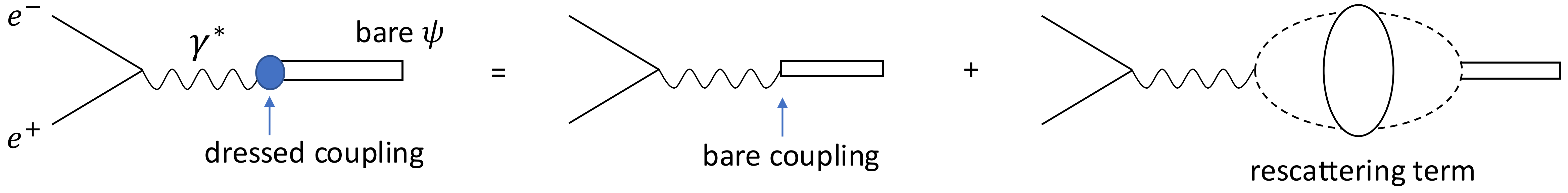




$D^*\bar{D}$ threshold cusp and/or TS occurs from 1-loop

ψ production mechanisms

$e^+e^- \to c\bar{c}$ data in $3.75 \le \sqrt{s} \le 4.7$ GeV region → Charmonium excitations are important mechanism



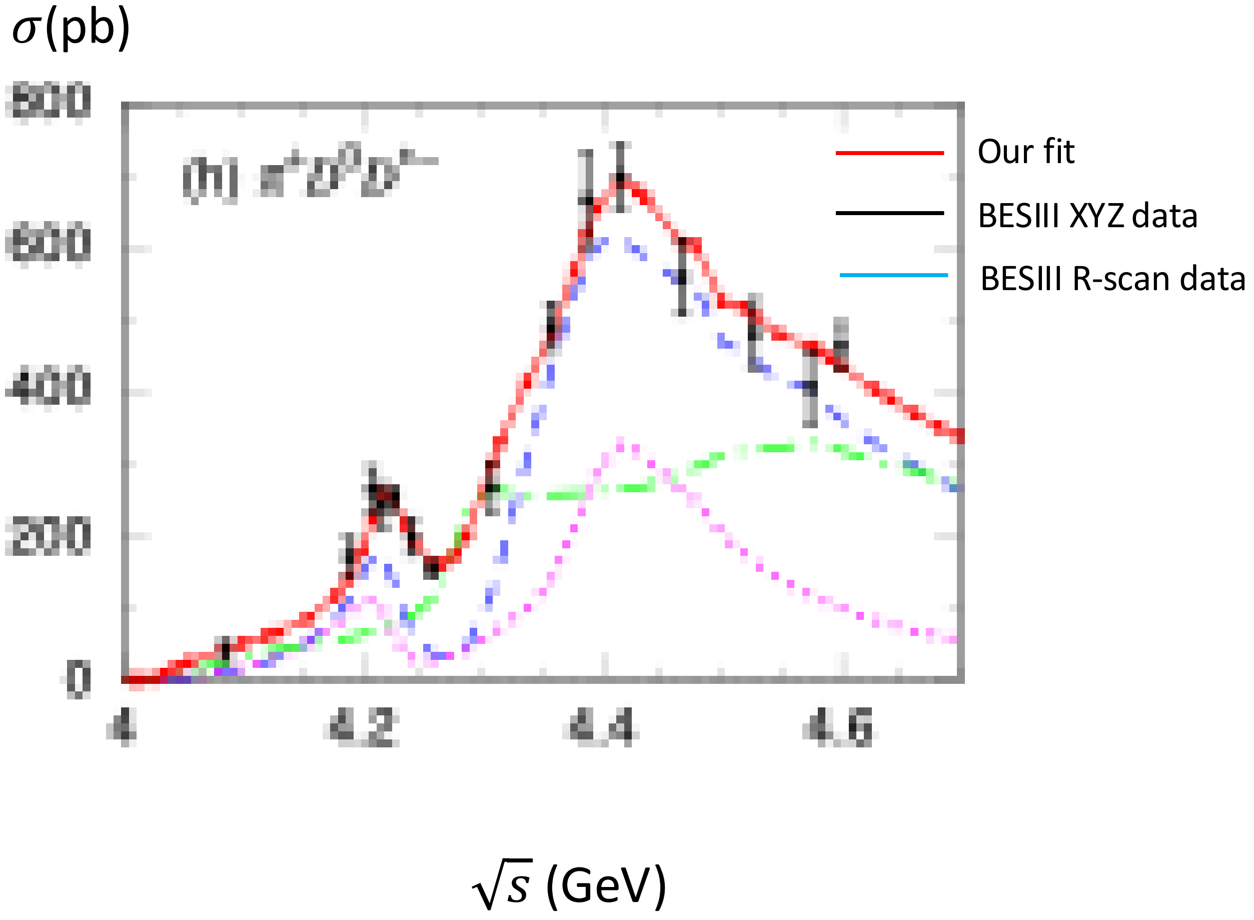
Data determine how many bare states to be included (5 bare states) and which charmonium states exist

Expected states $\psi(3770),\ \psi(4040),\ \psi(4160),\ \psi(4415),\ Y(4220),\ Y(4360)$

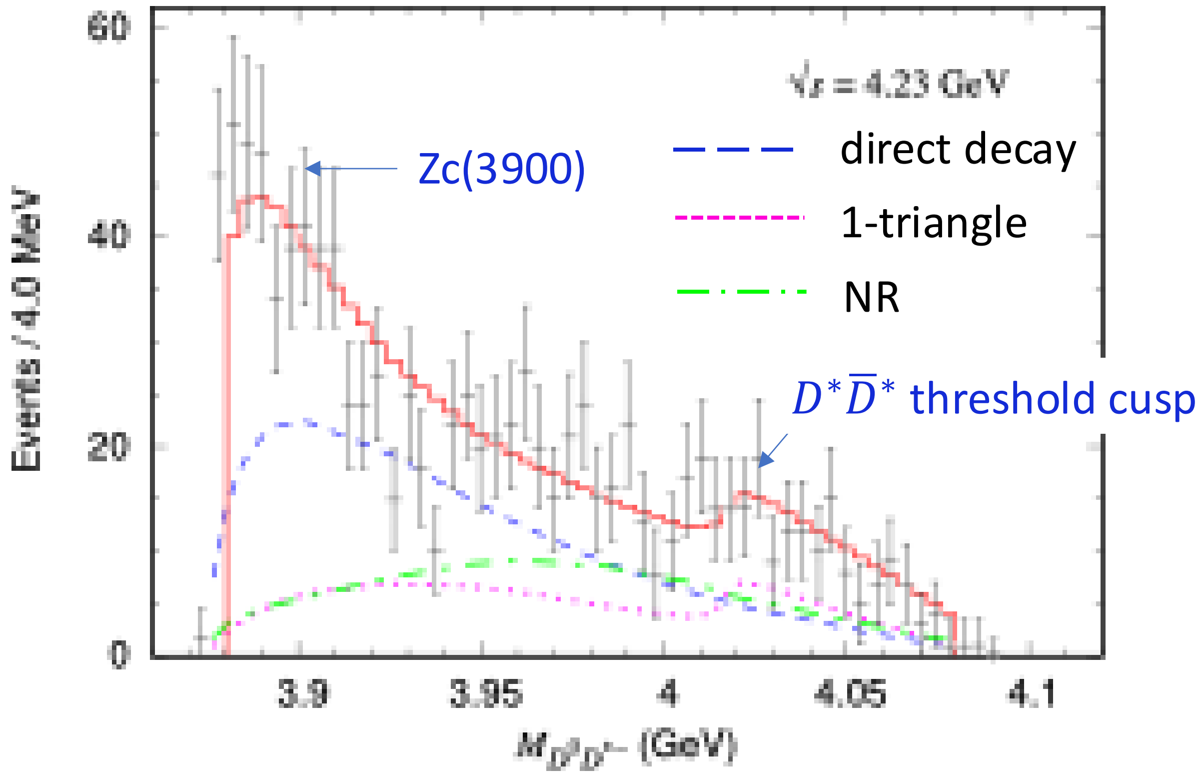
Data is not sufficient for coupled-channel analysis in $\sqrt{s} > 4.6$ GeV (three-body final states including $c\bar{c}s\bar{s}$)

$Y(4660),\ Y(4710)$ are not included in coupled-channel amplitude → included as a Breit-Wigner amplitudes

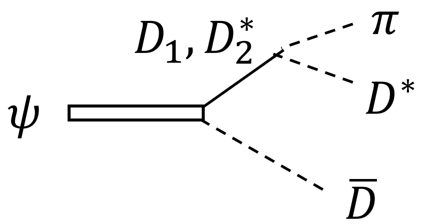
$e^+e^- \to \pi^+ D^0 D^{*-}$

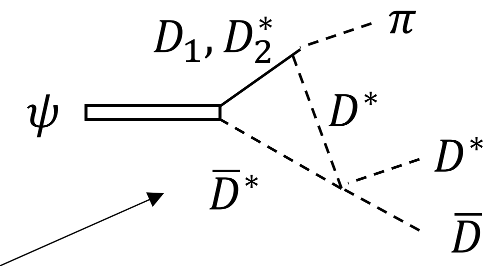


$D^0 D^{*-}$ invariant mass distributions



Tree contribution is dominant





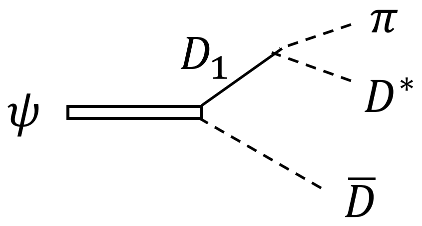
- $D^*\bar{D}^*$ threshold cusp is caused by
- $D^*\bar{D}$ threshold enhancement is mostly from tree; $\psi \to D_1 \bar{D}$

$e^+e^- \to \pi^+ D^0 D^{*-}$

Conflict with BESIII analysis result

Conclusion from BESIII PRD 92, 092006 (2015)

we conclude that the $D\bar{D}_1(2420)$ contribution to our observed Born cross section is smaller than its relative systematic uncertainty.

→ 

is very small !

Difficult to make our model consistent with this BESIII conclusion. Why ? Insufficient information !!

Hope BESIII to conduct amplitude analysis on this process, and present detailed results and/or Dalitz plots.

Without this information, $e^+e^- \to \pi^+ D^0 D^{*-}$ data cannot be well fitted, giving bad influence on the global fit overall

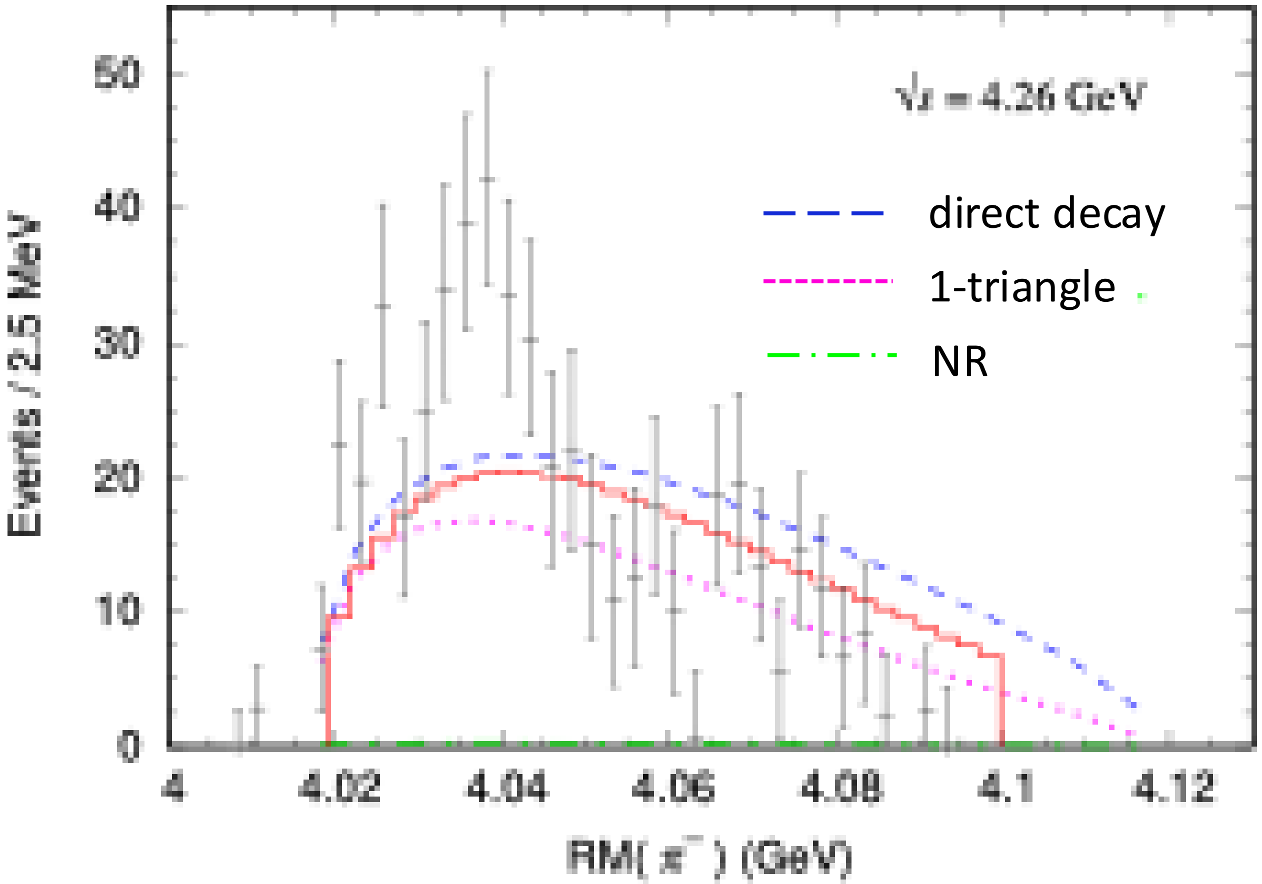
Most of previous theoretical models share the same problem

$e^+e^- \to \pi^-(D^*\bar{D}^*)^+$

$D^*\bar{D}^*$ invariant mass distributions (pion recoil mass)

Our fit

BESIII data



Fit does not seem good, however

Kinematical end of the data ~ 4.09 GeV

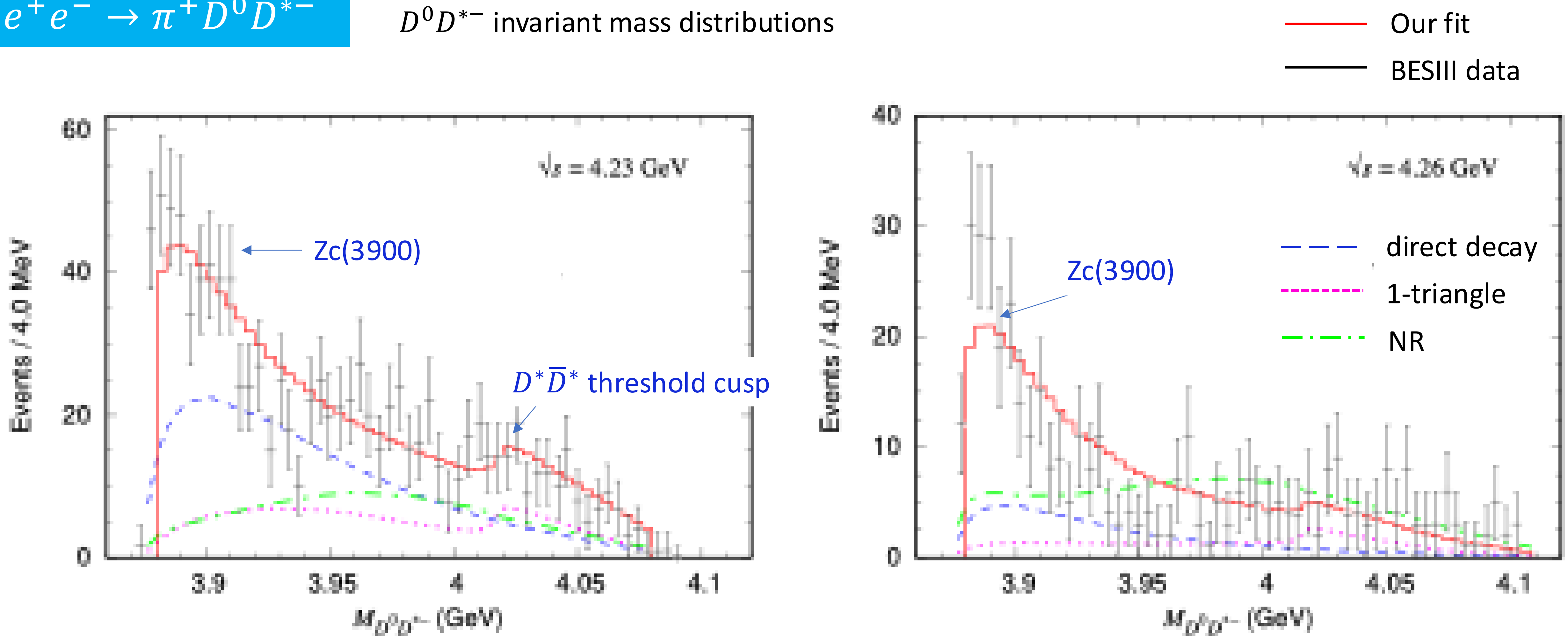
Actual kinematical end ~ 4.12 GeV

→ Efficiency correction seems significant for this lineshape data

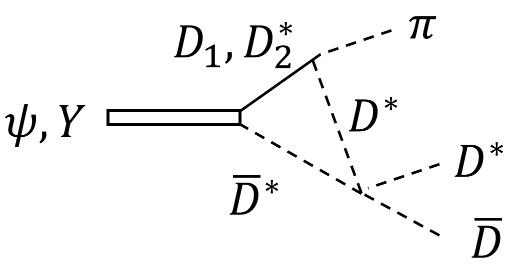
Wait for efficiency corrected data (or MC output) for future improvement of coupled-channel model

$e^+e^- \to \pi^+ D^0 D^{*-}$

$D^0 D^{*-}$ invariant mass distributions



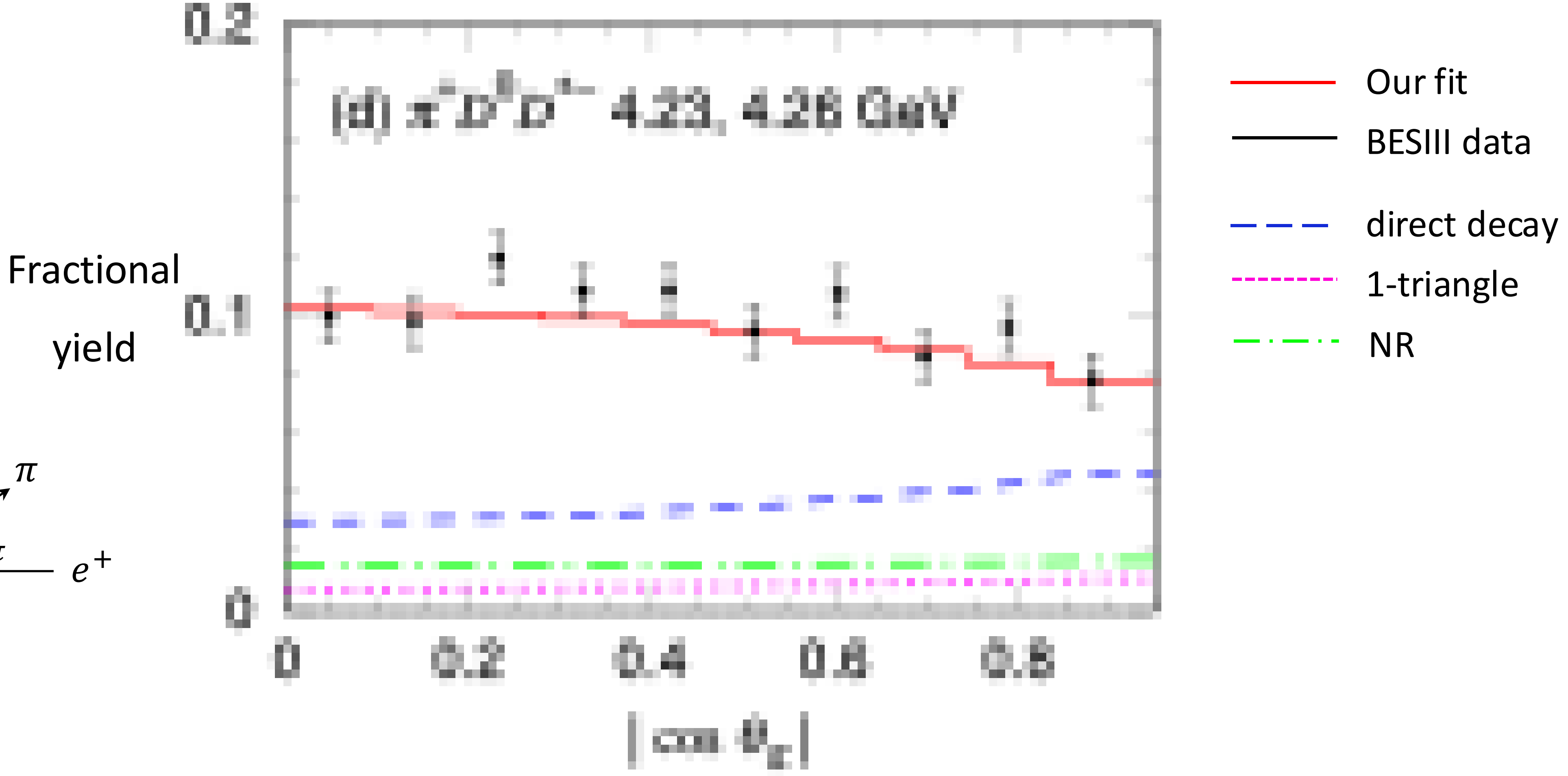
- Threshold enhancement (or Zc(3900) contribution) is fitted by the model
- $D^*\bar{D}^*$ threshold cusps are caused by

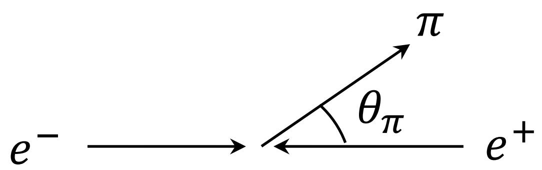


- $D^*\bar{D}$ threshold enhancement is mostly from tree; $\psi \to D_1\bar{D}$

$e^+e^- \to \pi^+ D^0 D^{*-}$

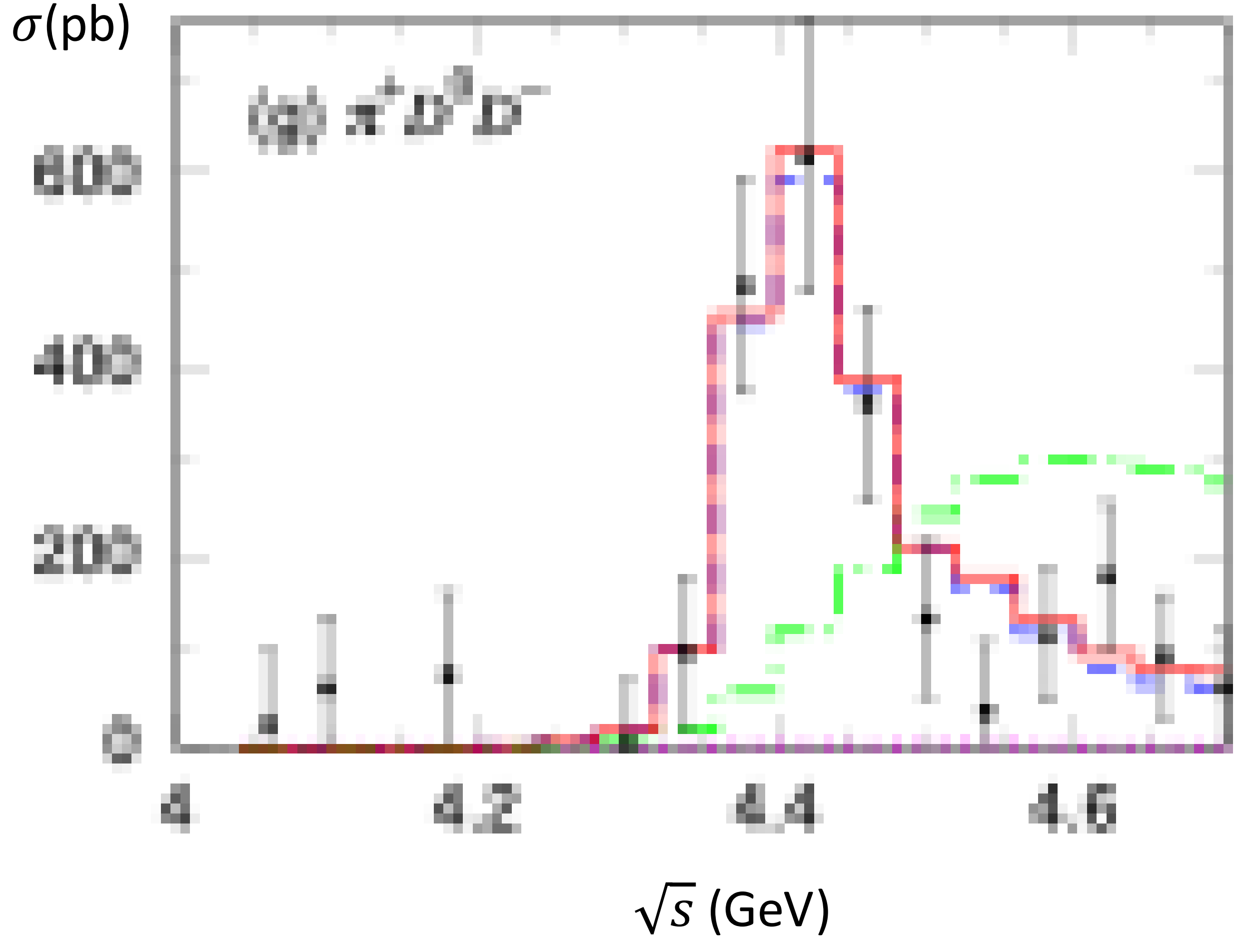
Pion angle distributions from e^+e^- beam direction in total CM frame

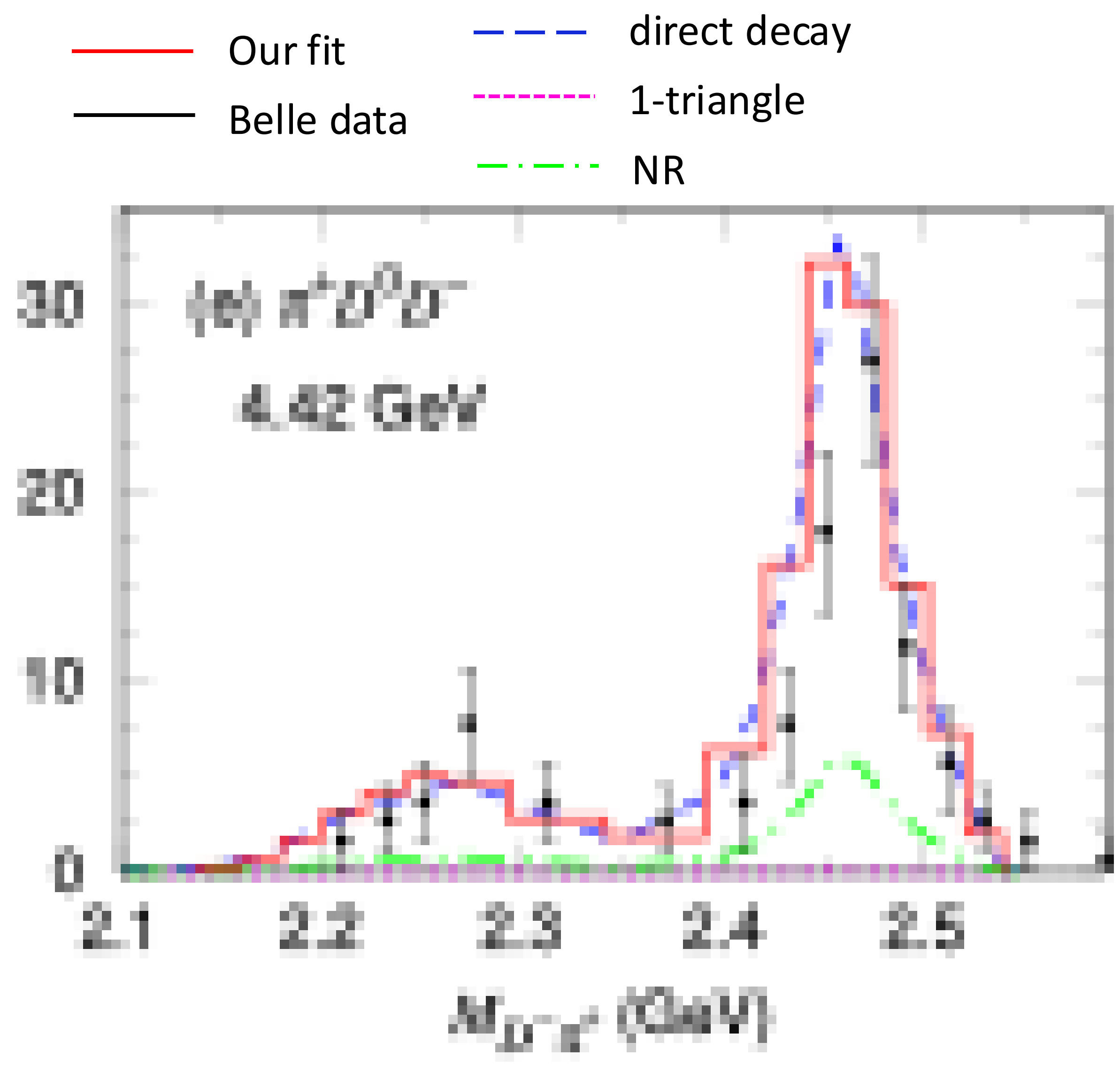




Data are average of 4.23 GeV ($N = 418$) and 4.26 GeV ($N = 239$) data

$e^+e^- \to \pi^+ D^0 D^-$



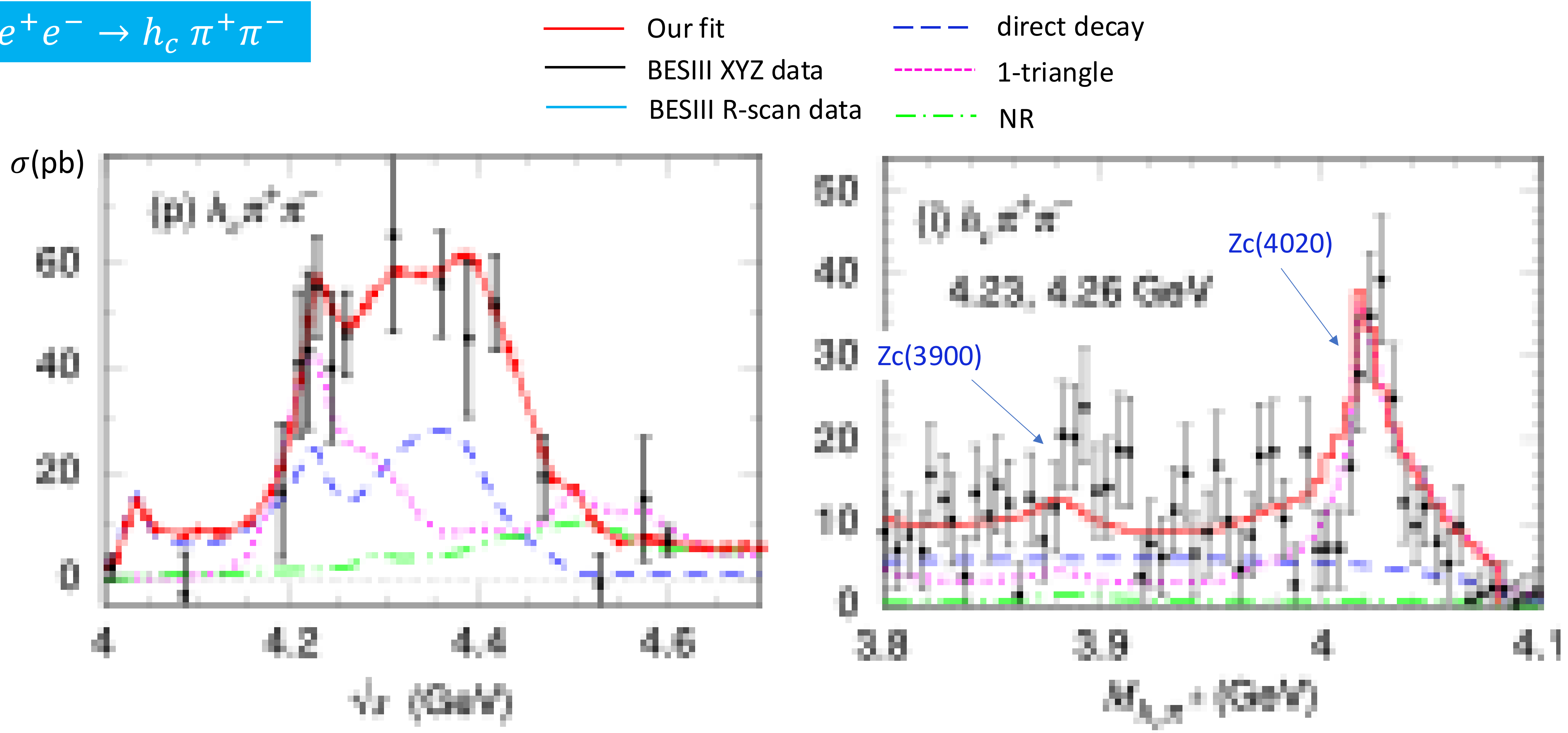


Clear $\psi(4420)$ peak is well fitted

Dominat $D_2^*(2460)$ contribution

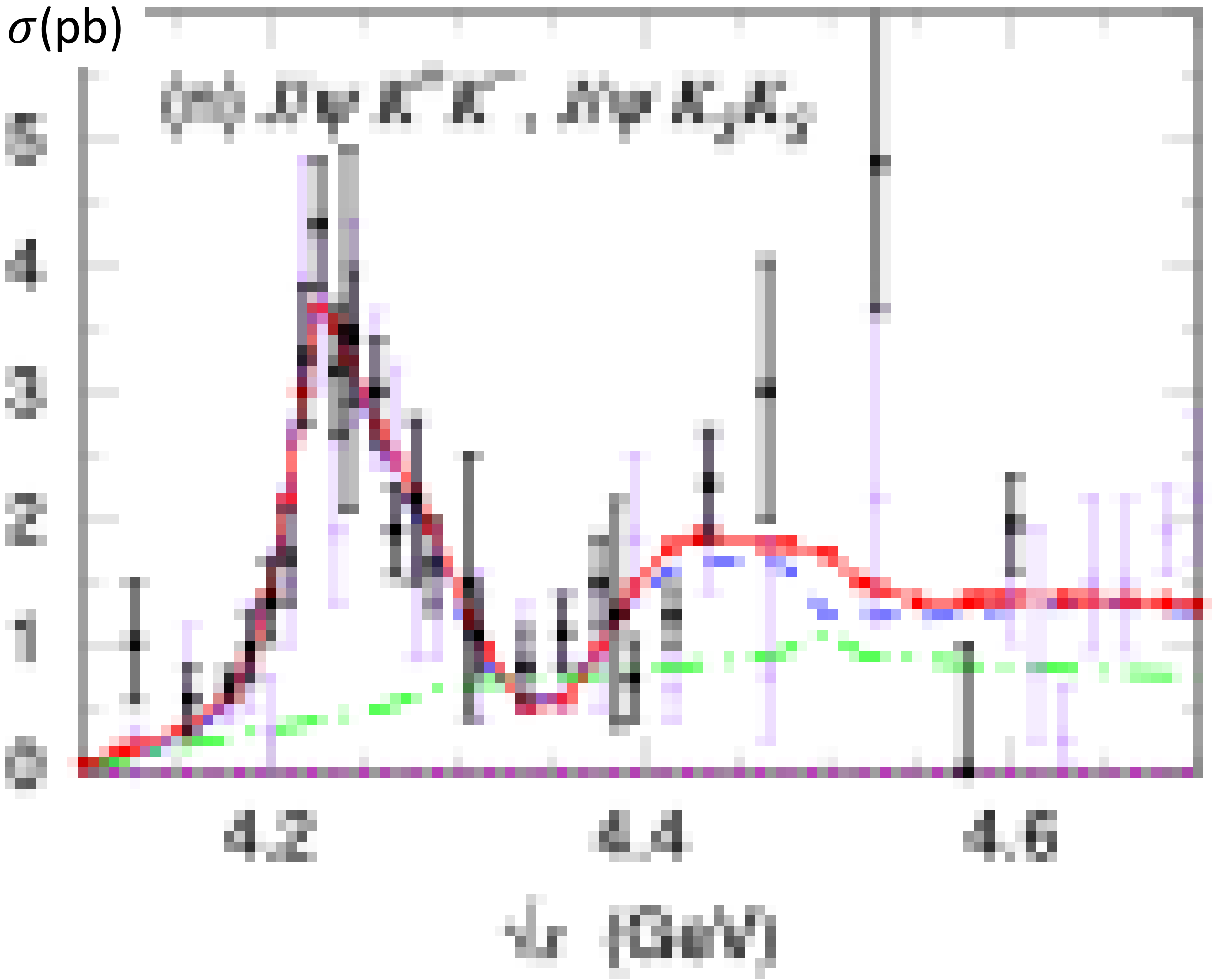
Hope to have a better quality data from BESIII ! → important for coupled-channel analysis

$e^+e^- \to h_c\,\pi^+\pi^-$



- Enhancement at ~ 4.03 GeV is from $\psi(4040)$ ← consequence of coupled-channel fit
- 1-triangle contribution causes threshold cusps, enhanced by Zc virtual poles

$e^+e^- \to J/\psi\, K^+K^-, J/\psi K_S K_S$



Our fit

BESIII $J/\psi\, K^+K^-$ data

BESIII $J/\psi K_S K_S$ data

direct decay

1-triangle

NR

- Overall good agreement with data

  (our model is isospin symmetric

  → $\sigma(J/\psi K^+K^-) = 2 \times \sigma(J/\psi K_S K_S)$)

- Model does not fit bump at ~4.5 GeV

  in $J/\psi\, K^+K^-$ data

  * $J/\psi K_S K_S$ data do not show the same bump

  * data largely fluctuate and error is large

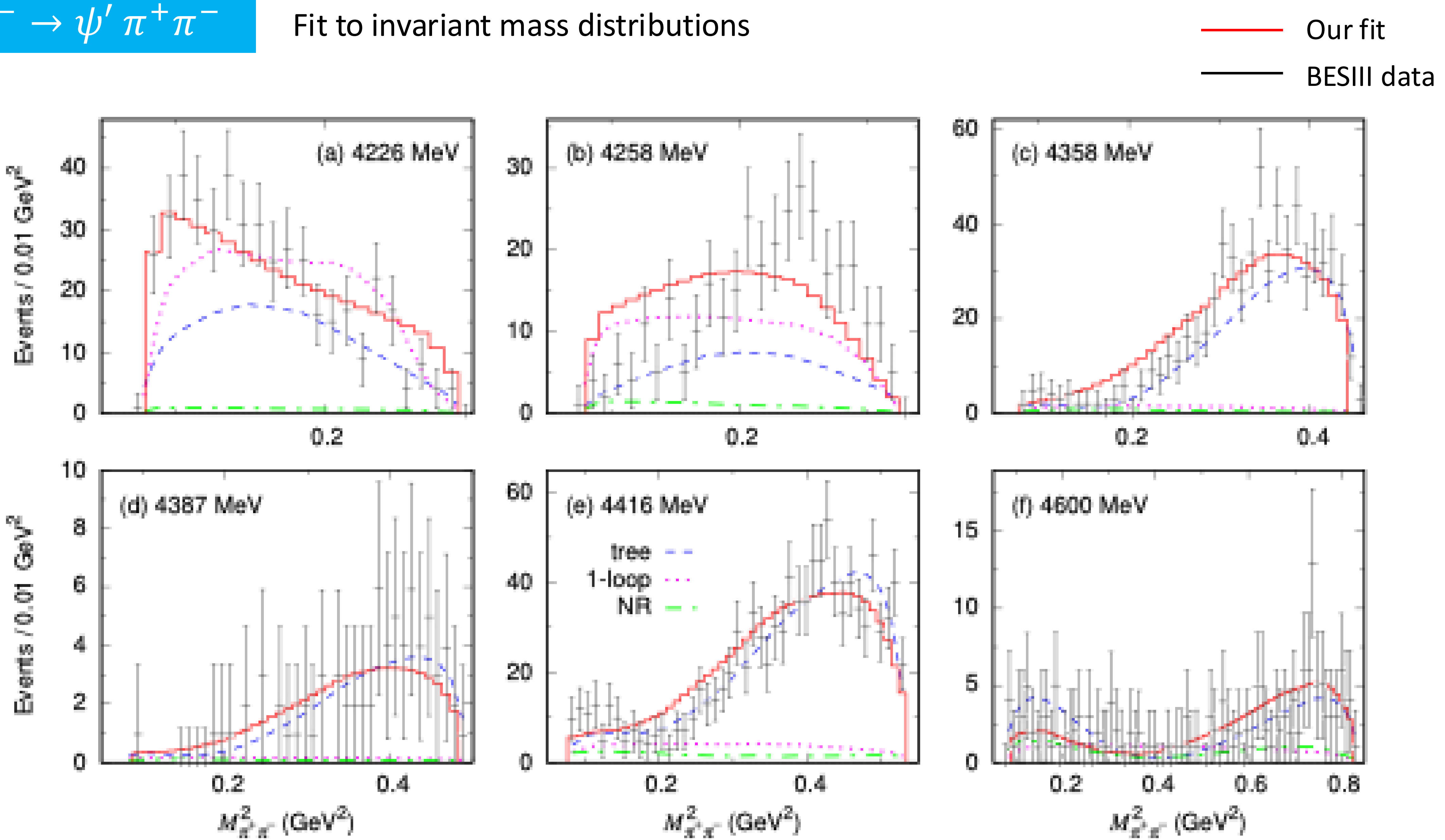
→ our model does not have Y(4500)

more precise data is important to pin-down

the existence of Y(4500)

$e^+e^- \to \psi' \, \pi^+\pi^-$

Fit to invariant mass distributions

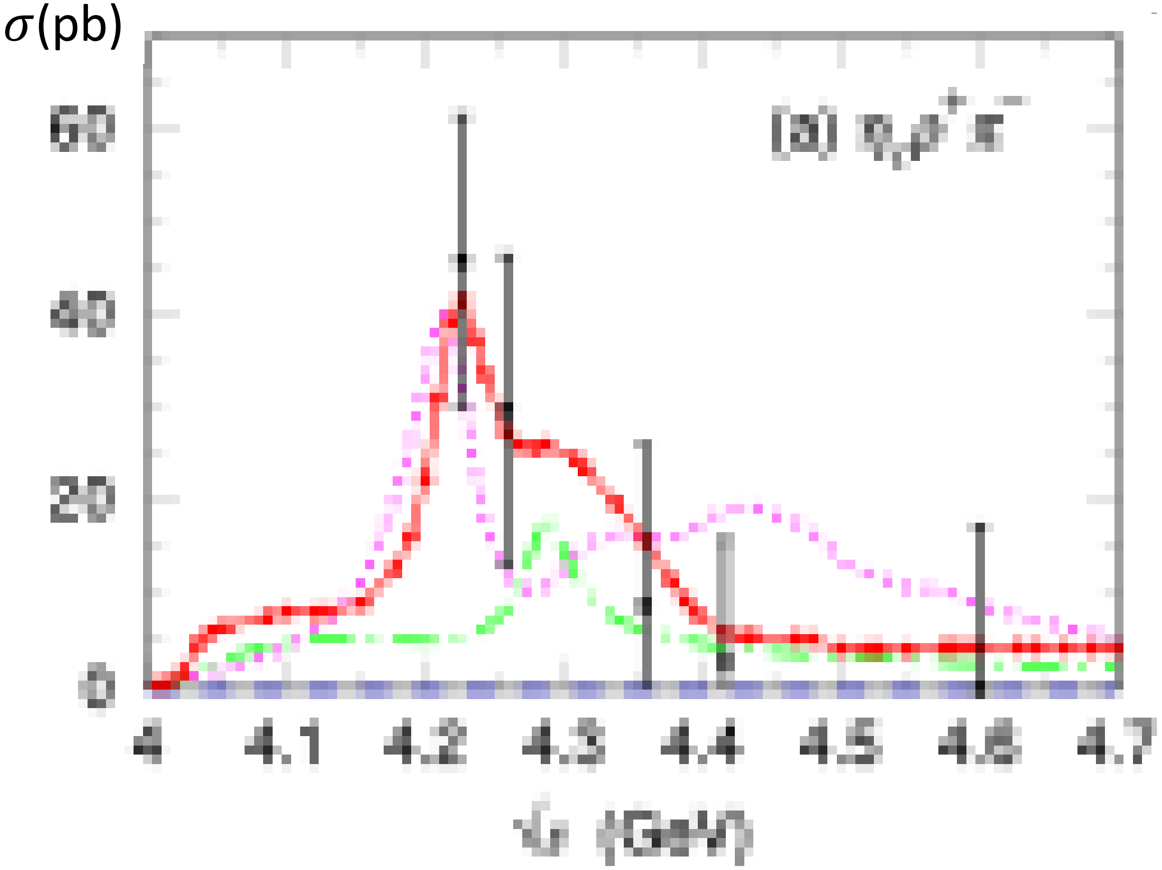


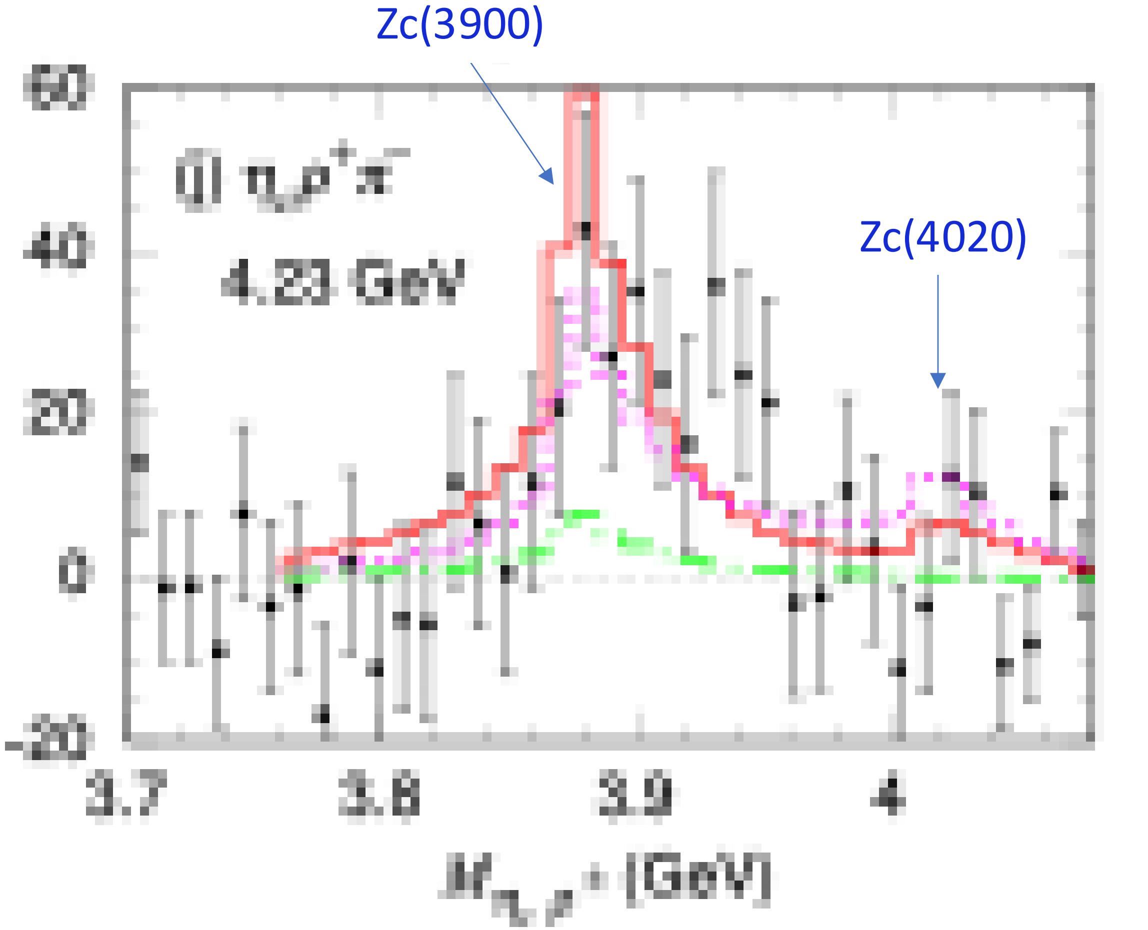
$e^+e^- \to \eta_c\, \rho^+\pi^-$

$\rho \to \pi\pi$ taken into account in calculation

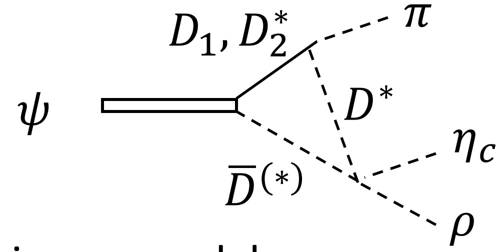
Our fit

BESIII data





Mostly from 1-triangle

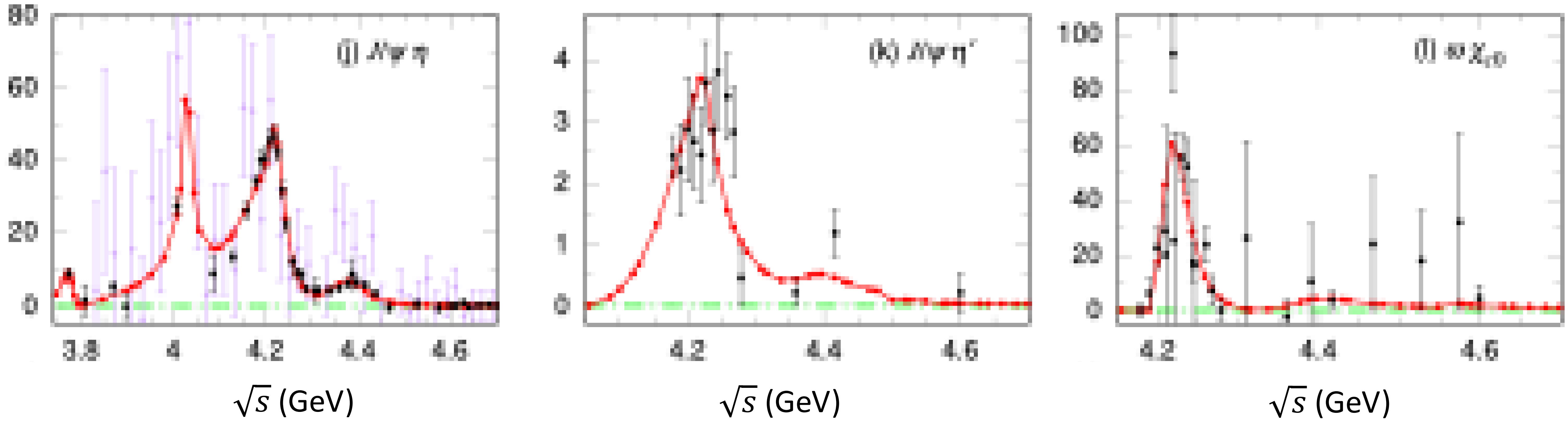


Zc(3900) peak is fitted

No direct-decay mechanism for $\eta_c\rho\pi$ in our model

$e^+e^- \to J/\psi\, \eta^{(\prime)}, \chi_{c0}\omega$

Our fit
BESIII data
Belle data

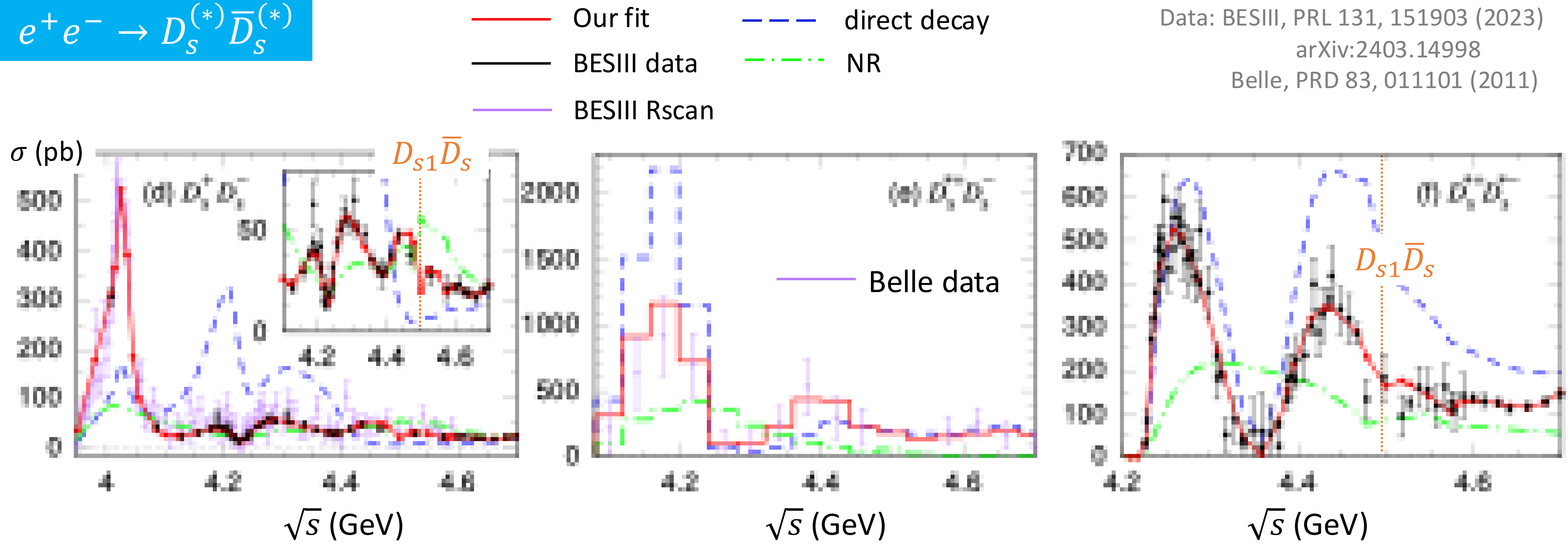


For $J/\psi\eta$, a sharp peak appears at 4.02 GeV, as a consequence of coupled-channel fit

← BESIII does not have data point, but Belle data seems to favor this result

$e^+e^- \to D_s^{(*)}\bar{D}_s^{(*)}$

Data: BESIII, PRL 131, 151903 (2023)
arXiv:2403.14998
Belle, PRD 83, 011101 (2011)

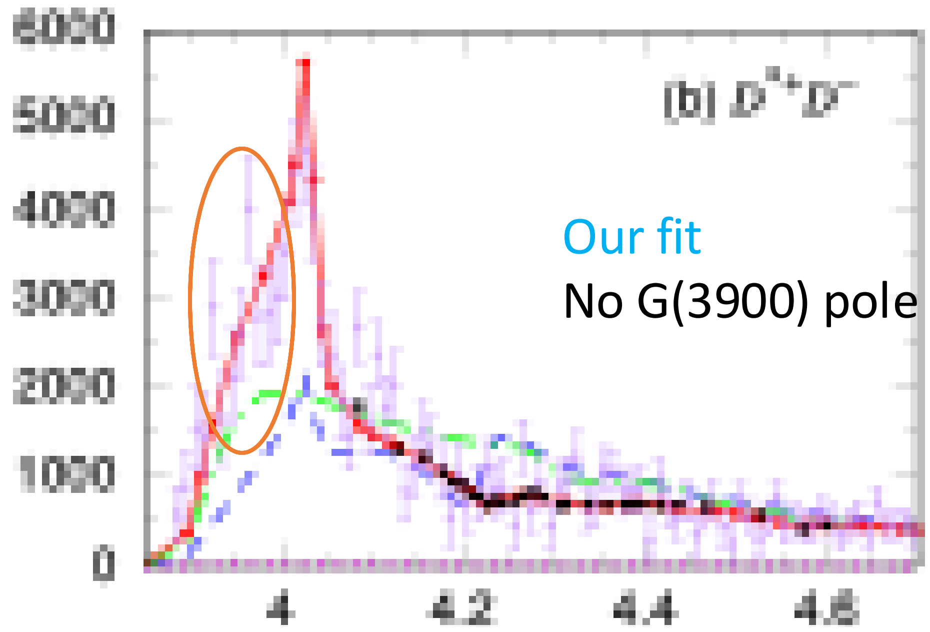


- *Precise* BESIII data are well fitted
- Contact interactions among open-charm channels important (difference between blue and red curves above)
- $D_{s1}\bar{D}_s$ threshold cusps included to fit data

How to pin-down existence of G(3900) ?

$e^+e^- \to D^*\overline{D}$ near threshold

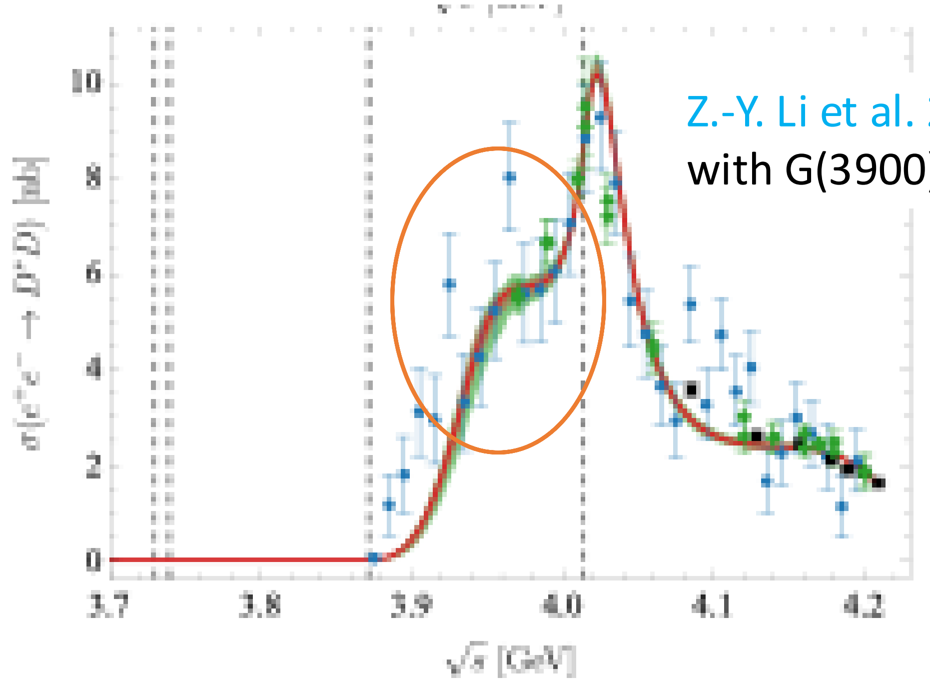
σ (pb)



Our fit

No G(3900) pole

$\sqrt{s}$ (GeV)



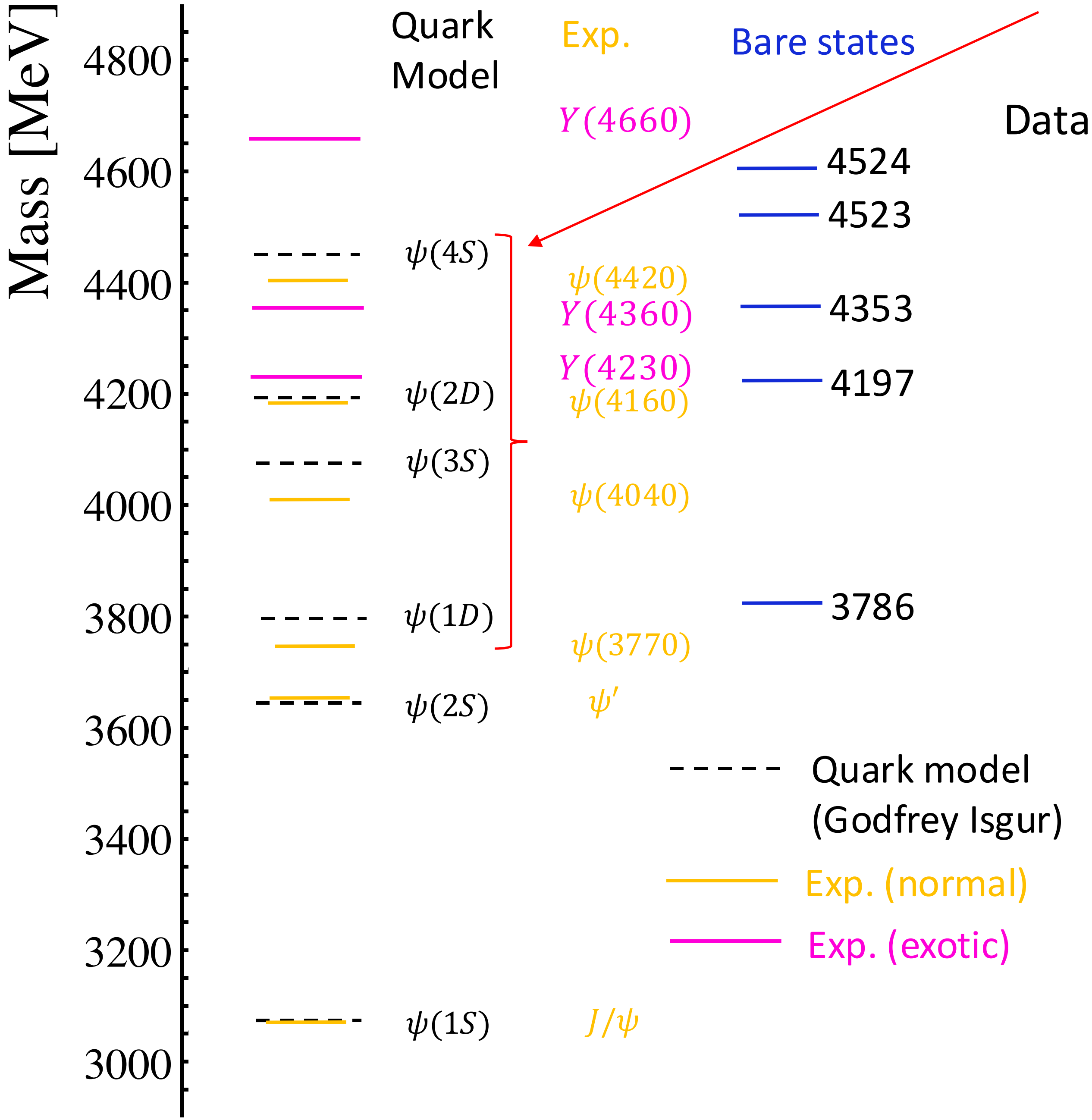
Z.-Y. Li et al. 2403.01727; PRL

with G(3900) pole

Visible difference between two fits → G(3900) effect ?

Higher precision data from BESIII could help pin-down existence of G(3900)

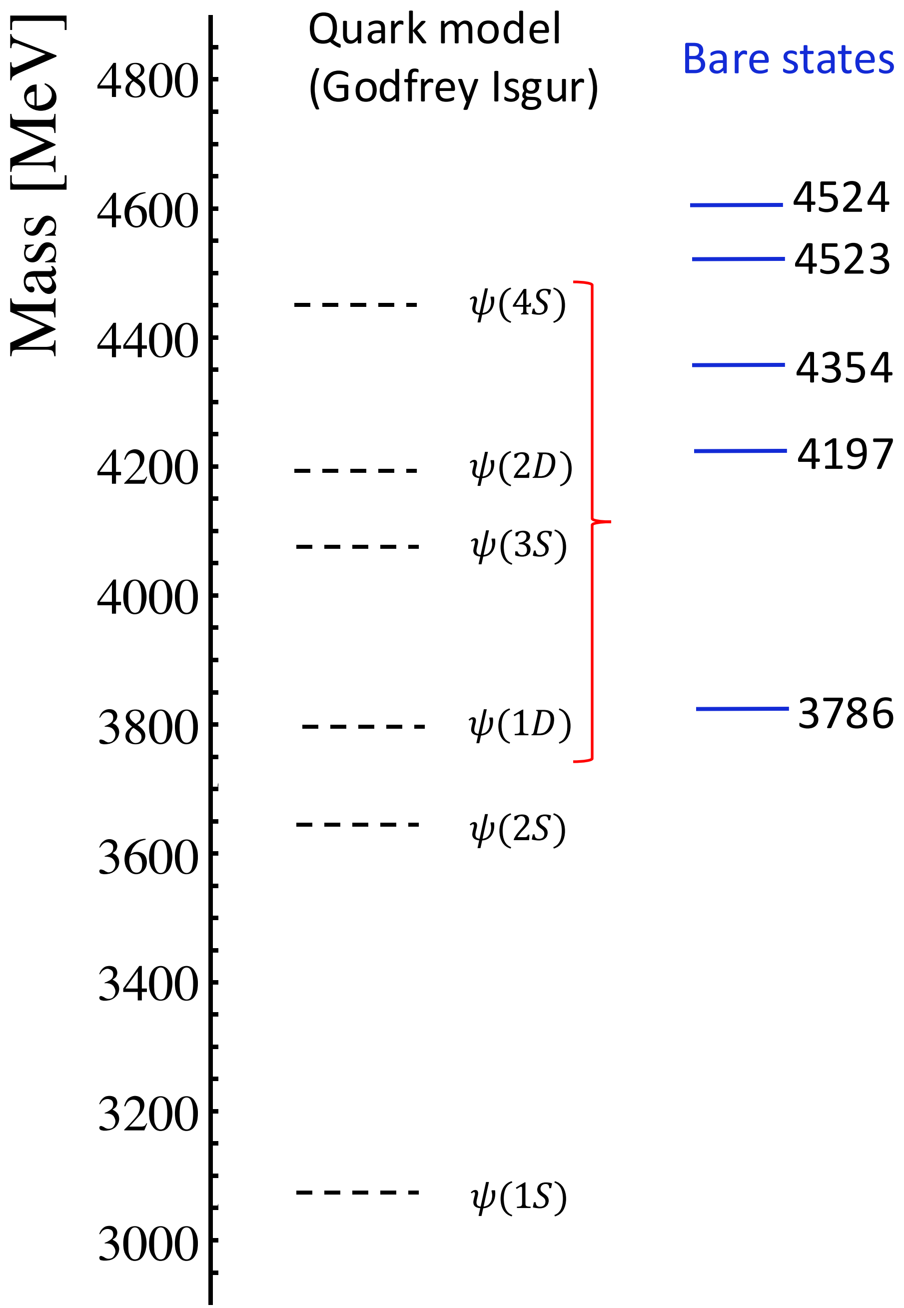
Charmonium spectrum ($J^{PC} = 1^{--}$)



Quark model predicts four states in the relevant energy region

Data require five bare states for achieving reasonable fit

Charmonium spectrum ($J^{PC} = 1^{--}$)



Quark model predicts four states in the relevant energy region

Data require five bare states for achieving reasonable fit

Conceptually, quark-model-state and our bare state is similar

→ Resonance without hadron-hadron continuum components

Very model-dependent argument/questions

One bare state is not accommodated in the quark model

→ Is it exotic bare state ?

Does it generate Y(4230) and Y(4360) after being dressed ?

Does it correspond to hybrid state predicted by LQCD ?

Liu et al., JHEP 07 (2012) 126

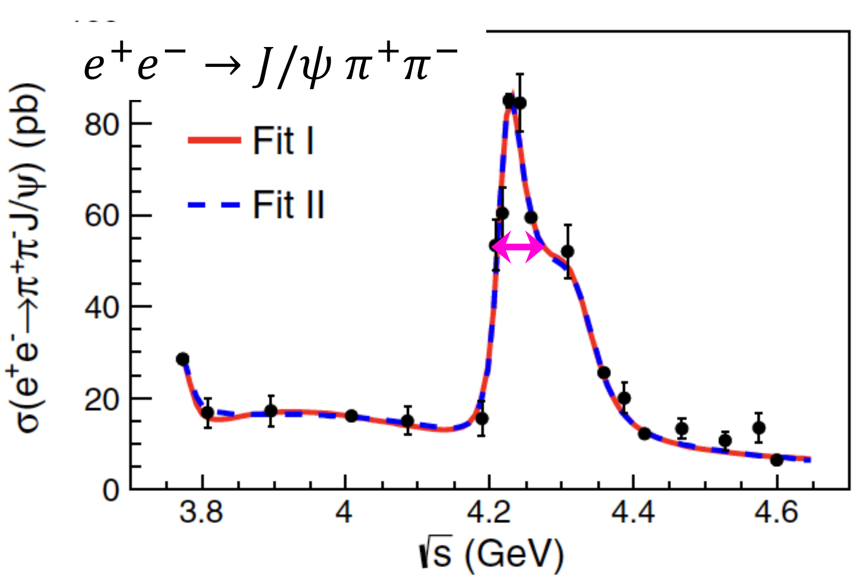
Our model alone cannot answer these interesting questions

Maybe possible by combining with structure model (quark model, etc.)

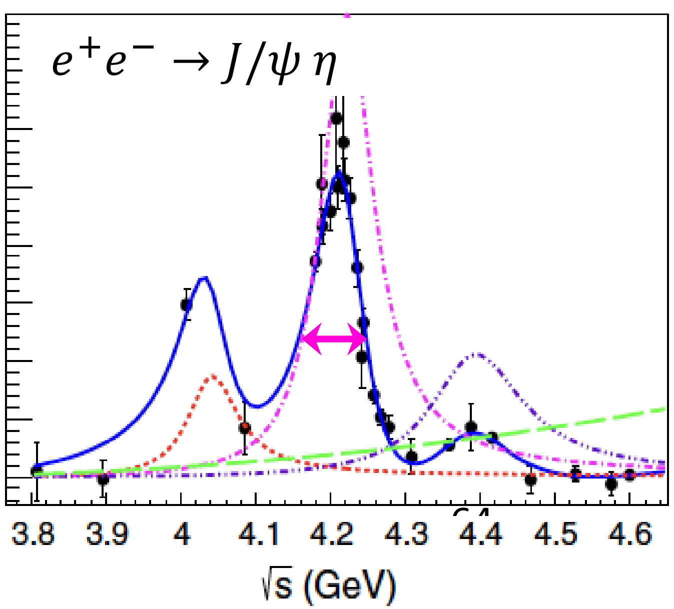
(speculation) Possible solution to Y width problem

	M (MeV)	Γ (MeV)
	3780 ± 0.5	30 ± 1.7
	4029 ± 0.3	28 ± 0.7
$\psi_{\rm wid}$	4186 ± 1.8	127 ± 2.9
$\psi_{\rm nar}$	4228 ± 0.7	44 ± 1.2
	4306 ± 2.6	129 ± 1.9
	4354 ± 3.1	123 ± 3.4
	4388 ± 1.5	107 ± 3.3

$\Gamma_{Y(4220)}$ ~ 44±4 MeV



$\Gamma_{Y(4220)}$ ~ 82±6 MeV



Two poles at M~ 4230 (4380) MeV with narrow ($\psi_{\rm nar}$) and wide ($\psi_{\rm wid}$) widths. We can explain Y widths if:

For $e^+e^- \to J/\psi\,\pi^+\pi^-$ $\quad |g_{\psi_{\rm nar}\to J/\psi\pi\pi}| \gg |g_{\psi_{\rm wid}\to J/\psi\pi\pi}|$

For $e^+e^- \to J/\psi\,\eta$ $\quad |g_{\psi_{\rm nar}\to J/\psi\eta}| \ll |g_{\psi_{\rm wid}\to J/\psi\eta}|$

$g_{\psi_{\rm nar}\to J/\psi\pi\pi}$: pole residue

Residues will be extracted in near future, and address the Y width problem

Relation between bare state and pole

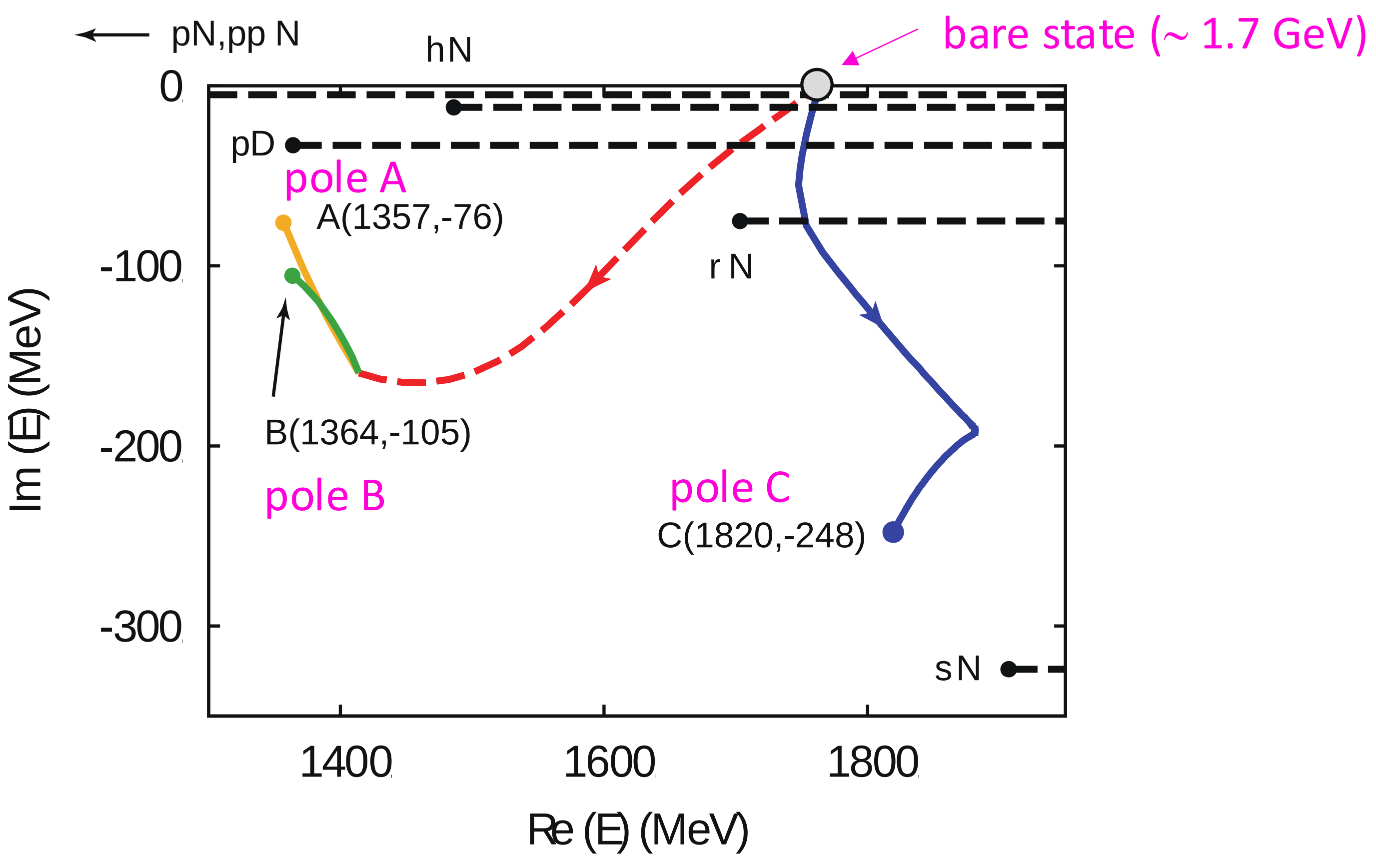
Data require five bare states

→ dressed by hadron continuum

→ seven poles

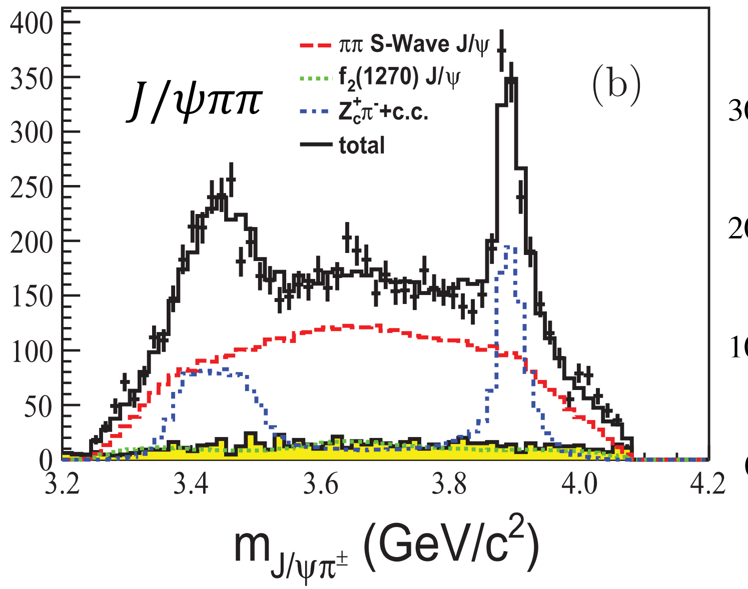
M (MeV)	Γ (MeV)
3780 ± 0.5	30 ± 1.7
4029 ± 0.3	28 ± 0.7
4188 ± 1.8	127 ± 2.9
4228 ± 0.7	44 ± 1.2
4306 ± 2.6	129 ± 1.9
4354 ± 3.1	123 ± 3.4
4388 ± 1.5	107 ± 3.3

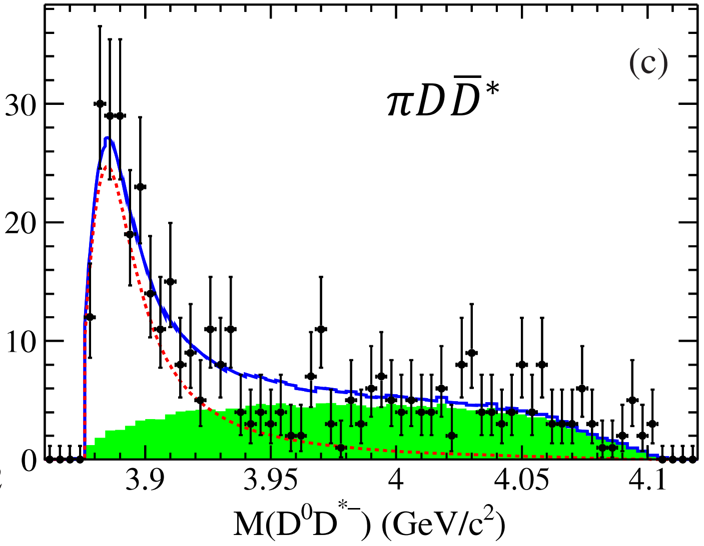
Similar finding in nucleon resonances Suzuki et al. (EBAC) PRL 104, 042302 (2010)



Future work : Which pair of poles come from the same bare state (mainly) ?

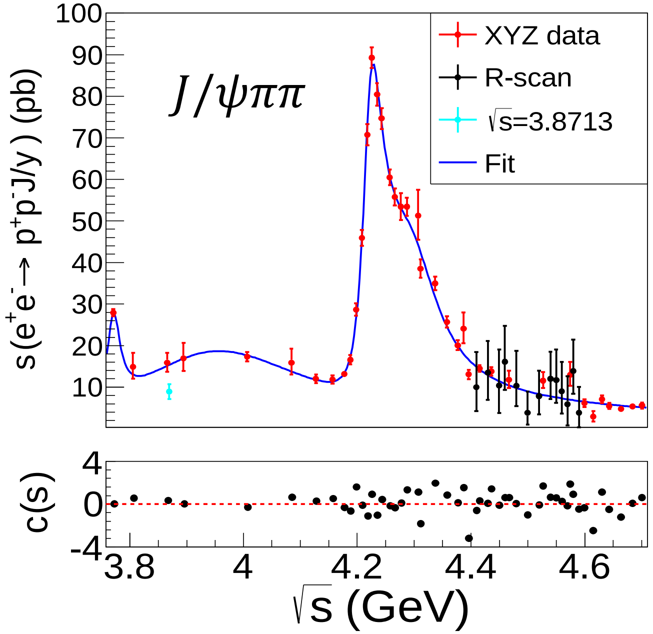
Common problem in previous theoretical analyses on Zc(3900)

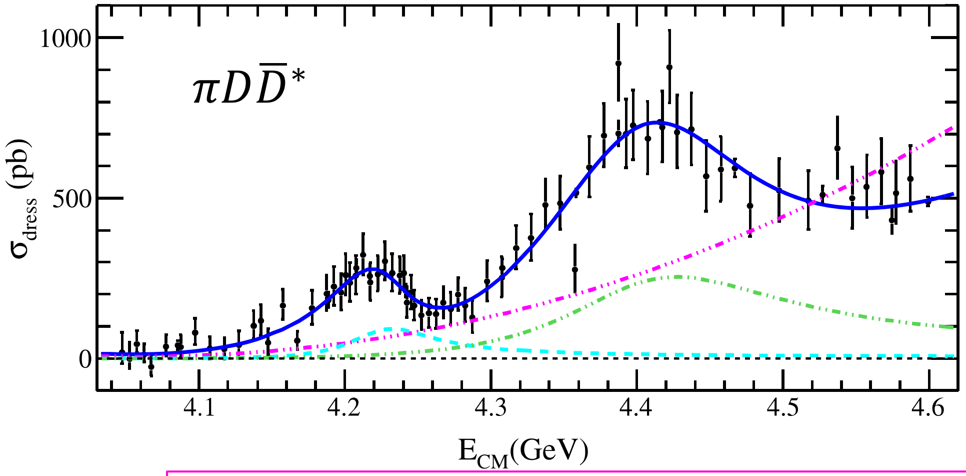




Invariant mass distributions (left, event numbers) are fitted to determine Zc(3900) pole

→ model's overall normalization is arbitrary but model has $\sigma(e^+e^- \to J/\psi\pi\pi)/\sigma(e^+e^- \to \pi D\bar{D}^*)$





In previous theoretical analyses, cross sections (left) were not considered

→ $\sigma(e^+e^- \to J/\psi\pi\pi)/\sigma(e^+e^- \to \pi D\bar{D}^*)$ from model is unchecked

Cross section data can test Zc production mechanism, Zc decay residues

$$\frac{\sigma(e^+e^- \to J/\psi\,\pi\pi)}{\sigma(e^+e^- \to \pi D\bar{D}^*)}$$ should be checked to see if models are reasonable

Our analysis cleared this problem

Present analysis result is consistent with lattice QCD

Previous LQCD analyses on $Z_c(3900)$ in:

Prelovsek et al. PLB 727, 172 (2013), PRD 91, 014504 (2015)
Chen et al. PRD 89 , 094506 (2014)
Ikeda et al. (HAL QCD) PRL 117, 242001 (2016)
Cheung et al. (Hadron spectrum Collab.) JHEP 11, 033 (2017)

LQCD conclusion : $I = 1,\ J^{PC} = 1^{+-}\ D^*\bar{D}$ s-wave interaction is very weak,

disfavoring narrow $Z_c(3900)$ pole near $D^*\bar{D}$ threshold

Most of previous determinations of Zc(3900) pole are not consistent with LQCD

Q. Can the global analysis tell Zc(3900) is resonance or virtual state ?

The presented analysis employed energy independent interactions for Zc amplitude

→ Only virtual or bound states are examined → virtual state works fine

Ongoing update

Zc amplitude with resonant Zc(3900) state is implemented in the three-body coupled-channel model

→ Its performance in the global fit will be examined

$e^+e^- \to c\bar{c}$ data and coupled-channel analyses