

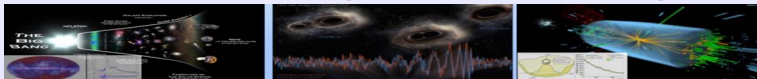
# Angular correlation and deformed Hellings-Downs curve by spin-2 ultralight dark matter

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Chinese Academy of Sciences

*Based on: PRD('24) by R.G. Cai(ITP), J.R. Zhang(HIAS) & Y.L. Zhang  
Phys.Rev.D 106, 066006, by S. Sun(BIT), X.Y. Yang(KIAS) & Y.L. Zhang*

2024, Jul 23@ZhangJiajie [email: zhangyunlong@nao.cas.cn]



# Motivation: new physics in ultra-low energy

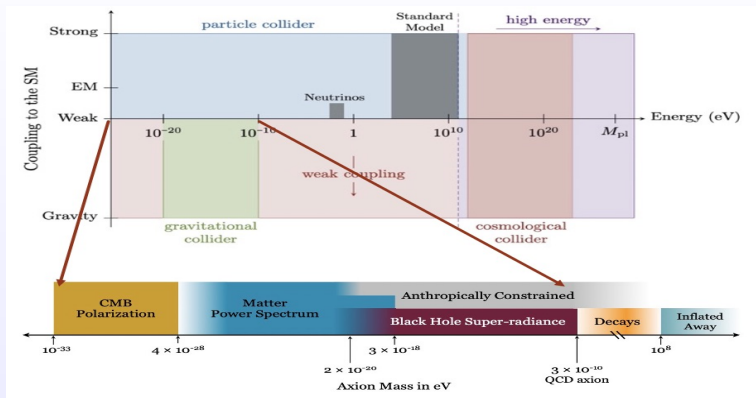
GW & ULDM

Ultralight DM

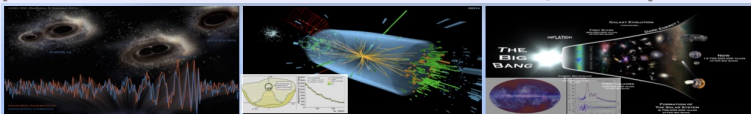
Axion-like DM  
Resonance

GW and DM

spin-0 DM  
spin-2 DM



[cf. Baumann-Chia-Porto-Stout, Gravitational Collider Physics, 2019]



# Spectrum of gravitational wave and axion mass

GW & ULDM

Ultralight DM

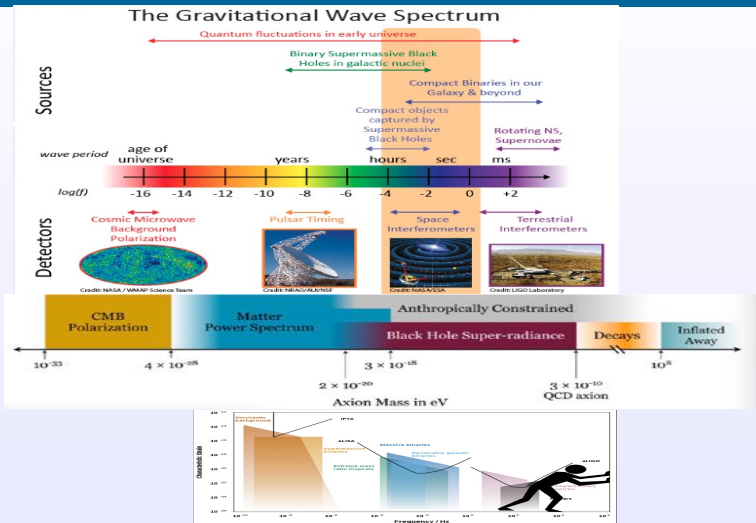
Axion-like DM

Resonance

GW and DM

spin-0 DM

spin-2 DM



[cf. LISA/Ultra-High-Frequency Gravitational Waves Initiative]

# Axion annihilation and Stochastic GWs

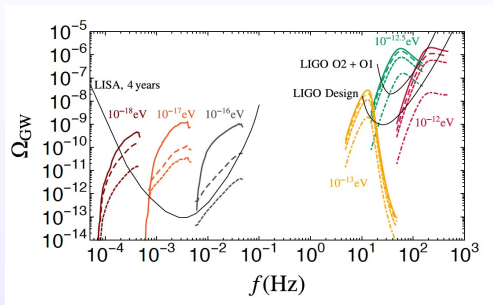
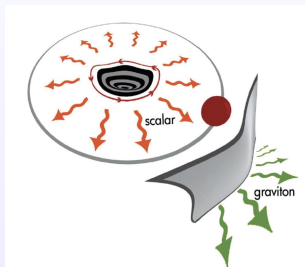
GW & ULDM

Ultralight DM

Axion-like DM  
Resonance

GW and DM

spin-0 DM  
spin-2 DM



- Axion annihilation  $\vartheta + \vartheta \rightarrow hh$ , Strain  $h \sim 10^{-21} - 10^{-32}$ .
- Stochastic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

# Energy level transition and Monochromatic GW

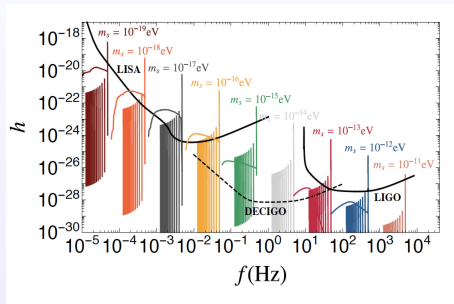
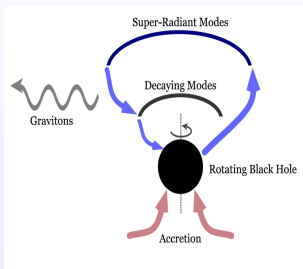
GW & ULDM

Ultralight DM

Axion-like DM  
Resonance

GW and DM

spin-0 DM  
spin-2 DM



- Energy transition  $\vartheta^+ \rightarrow \vartheta^- + h$ , Strain  $h \sim 10^{-19} - 10^{-27}$
- Monochromatic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

# Branch Ratio of EM & GW ( $\vartheta \rightarrow \gamma\gamma$ & $\vartheta \rightarrow hh$ )

GW & ULDM

Ultralight DM

Axion-like DM

Resonance

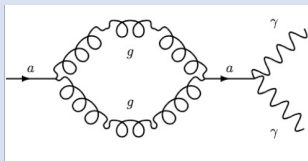
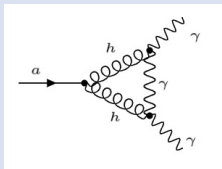
GW and DM

spin-0 DM

spin-2 DM

- The triangle Feynman diagram: where the axion-photon coupling is generated from Chern-Simon gravity coupling.

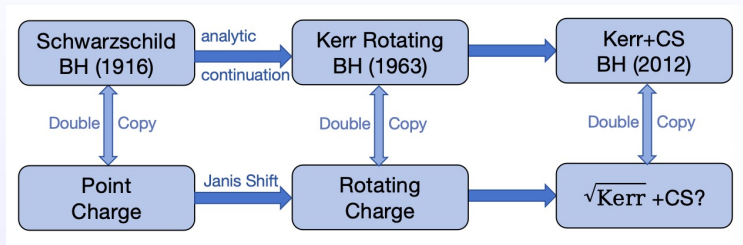
$$\mathcal{L}_{\vartheta F \tilde{F}} = -\frac{\alpha_\gamma}{4} \vartheta F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad \mathcal{L}_{\vartheta R \tilde{R}} = \frac{\alpha_g}{4} \vartheta R_{\alpha\gamma\delta}^{\beta} \tilde{R}^{\alpha\gamma\delta}_{\beta}.$$



- The triangle diagram is divergent as  $\alpha_\gamma \sim \alpha_g (\Lambda_{CS}/M_{pl})^4$ , where  $\Lambda_{CS}$  is the cut-off for Chern-Simons theory.
- Two powers of  $M_{pl}$  from  $h_{\mu\nu} T^{\mu\nu}$  coupling.

[cf. S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps, PRD'21]

# Double Copy Relation



- Weyl double copy: Gravity theory = (Gauge theory)<sup>2</sup>
- Correspondence: Gravity solutions  $\leftrightarrow$  Gauge solutions

[cf. Y. R. Liu, J. R. Zhang, Y. L. Zhang, "Slowly rotating charges from Weyl double copy for Kerr black hole with Chern-Simons correction," CTP(2024)]

# Joint analysis & Branch ratio ( $\vartheta \rightarrow \gamma\gamma$ & $\vartheta \rightarrow hh$ )

GW & ULDM

Ultralight DM

Axion-like DM  
Resonance

GW and DM

spin-0 DM  
spin-2 DM

## The interaction

- $S_{total} = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa_4} R - \frac{1}{4} F^2 + \mathcal{L}_\vartheta + \mathcal{L}_{\vartheta F\tilde{F}} + \mathcal{L}_{\vartheta R\tilde{R}} \right)$
- EM field  $\mathcal{L}_\vartheta = -\frac{1}{2}(\partial\vartheta)^2 - \frac{1}{2}m_\vartheta^2\vartheta^2$ .
- $\mathcal{L}_{\vartheta F\tilde{F}} = -\frac{\alpha_\gamma}{4}\vartheta F_{\mu\nu}\tilde{F}^{\mu\nu}$ ,  $\mathcal{L}_{\vartheta R\tilde{R}} = \frac{\alpha_g}{4}\vartheta R^\beta_{\alpha\gamma\delta}\tilde{R}^{\alpha\gamma\delta}_\beta$ .

## Equation of motion

- $\square h_{ij} = \kappa_4 \alpha_g \tilde{\epsilon}^{pk}_{(i} [\dot{\vartheta}(\partial_p \square h_{j)k}) - \ddot{\vartheta}(\partial_p \partial_t h_{j)k})] - 2\kappa_4 (T_{ij}^{(\gamma)} + T_{ij}^{(\vartheta)})$
- axion like field  $(\square - m_\vartheta^2)\vartheta = \frac{\alpha_\gamma}{4} F\tilde{F} - \frac{\alpha_g}{4} R\tilde{R}$
- Polarization  $\nabla_\mu F^{\mu\nu} = -\alpha_\gamma \partial_\mu \vartheta \tilde{F}^{\mu\nu}$

# Fast GW burst from axion decay $\vartheta \rightarrow hh$

GW & ULDM

Ultralight DM

Axion-like DM

Resonance

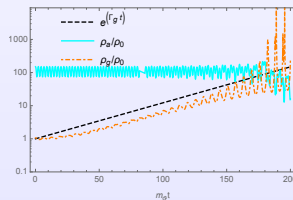
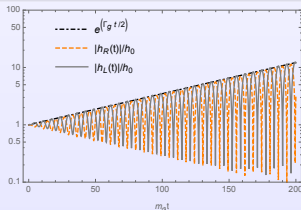
GW and DM

spin-0 DM

spin-2 DM

## EOM of GW( $I = L, R$ )

- Coupling term  $\mathcal{L}_{\vartheta R\tilde{R}} = \frac{\alpha_g}{4} \vartheta R_{\alpha\gamma\delta}^{\beta} \tilde{R}_{\beta}^{\alpha\gamma\delta}$ ,
- $[\ddot{h}_I(t) + k^2 h_I(t)] [1 - \varepsilon_I \kappa_4 \alpha_g k \dot{\vartheta}(t)] = \varepsilon_I \kappa_4 \alpha_g k \ddot{\vartheta}(t) \dot{h}_I(t)$ .
- Factor  $e^{\Gamma_g t_g}$ ,  $\frac{\Gamma_g}{m_{\vartheta}} \sim \left( \frac{\kappa_4 \alpha_g}{1\text{eV}^{-3}} \right) \left( \frac{m_{\vartheta}}{10^{-9}\text{eV}} \right)^2 \left( \frac{\vartheta_0}{10^9\text{GeV}} \right)$ .

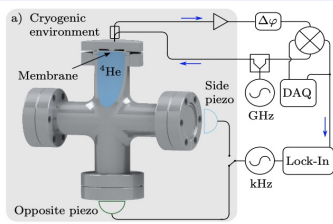
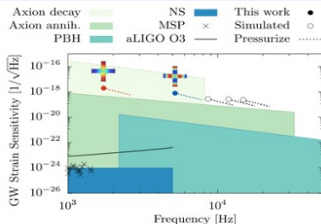


[cf. S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps, PRD'21]

# GW burst & high frequency detection

## Branch Ratio and GWs

- $\frac{\text{Br}(\vartheta \rightarrow gg)}{\text{Br}(\vartheta \rightarrow \gamma\gamma)} \simeq \frac{\alpha_g^2}{\alpha_\gamma^2} \simeq \left(\frac{M_{pl}}{\Lambda_{cs}}\right)^8$ , (Power of FRB  $P_{(\gamma)} \sim 10^{42} \text{ergs/s}$ ).
- High frequency  $h_{(g)} \sim 10^{-26} \left(\frac{1\text{GHz}}{\nu}\right) \left(\frac{P_{(g)}}{P_{(\gamma)}}\right)^{1/2} \left(\frac{1\text{kpc}}{L}\right)$
- Low freq.  $h_{(g)} \sim 10^{-21} \left(\frac{10^{-2}\text{Hz}}{\nu}\right)^{1/2} \left(\frac{M_{BH}}{10^7 M_\odot}\right)^{1/2} \left(\frac{1\text{kpc}}{L}\right)$



cf. PRD'21, S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps.  
PRD'21, V. Vadakkumbatt et al, Prototype superfluid gravitational wave detector.

L.H. Du's group, Evidence for chiral graviton modes in fractional quantum Hall liquids

# Ultralight dark matter and gravitational wave

GW & ULDM

Ultralight DM

Axion-like DM  
Resonance

GW and DM

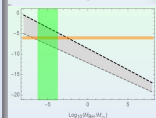
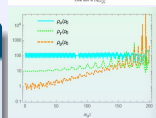
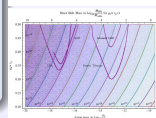
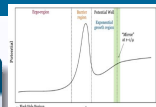
spin-0 DM  
spin-2 DM

## GW & EM signals from axion like DM

- Axion annihilation  $\vartheta + \vartheta \rightarrow h$  (Stochastic GW)  
Energy transition  $\vartheta^+ \rightarrow \vartheta^- + h$  (Monochromatic)
- Superradiance  $\alpha \equiv \frac{R_{BH}}{\lambda_{\vartheta}} \simeq \left( \frac{M_{BH}}{M_{\odot}} \right) \left( \frac{m_{\vartheta}}{10^{-10} \text{eV}} \right)$
- Fast Radio Burst from Axion  $\sim \vartheta F \tilde{F}$  ( $\vartheta \rightarrow \gamma\gamma$ )
- GW burst from Axion  $\sim \vartheta R \tilde{R}$  ( $\vartheta \rightarrow hh$ )

## GW detection and Ultra-light DM

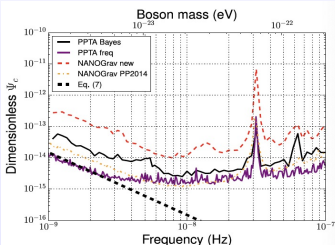
- Tabletop exp: QCD axion & GW burst ( $\sim$  GHz)
- LISA & LVK: BH Superradiance ( $\sim$  mHz - kHz)
- PTA & SKA : Ultra-light DM ( $\sim$  nHz)
- LSS & CMB : DE & Modified gravity ( $\sim$  nnHz)



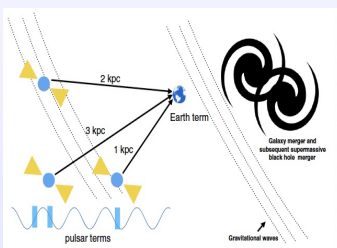
# Timing residuals & Ultralight dark matter

## PTA & Ultra light dark matter

- Fuzzy like DM  $f_c = \frac{m}{\pi} \simeq 4.8 \text{ nHz} \left( \frac{m}{10^{-23} \text{ eV}} \right)$
- Coherence length:  $\lambda_{\text{dB}} = \frac{2\pi\hbar}{mv} \simeq 4 \text{ kpc} \left( \frac{10^{-23} \text{ eV}}{m} \right) \left( \frac{10^{-3}}{v} \right)$
- Pulsar timing residuals:  $R_c(f) = \frac{1}{\sqrt{3}} \frac{h_c(f)}{2\pi f} \left( \frac{f_s}{f} \right)^{1/2}$



[cf. X. Xue, X. J. Zhu et al. 2018] & [cf. Burke-Spolaor, et al. 2019]



# The timing residuals of nHz gravitational waves

GW & ULDM

Ultralight DM

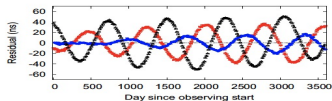
Axion-like DM

Resonance

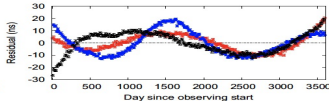
GW and DM

spin-0 DM

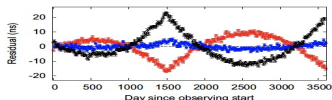
spin-2 DM



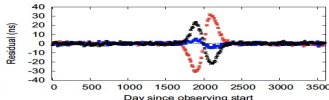
(a) Continuous wave



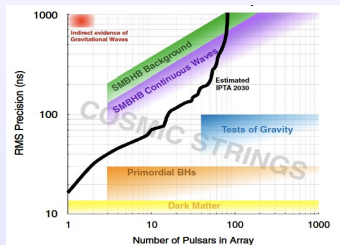
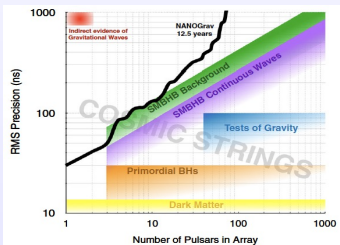
(b) Background



(c) Burst with Memory



(d) Burst



[cf. Burke-Spolaor, *et al.*, "The astrophysics of nanohertz gravitational waves"]

# Oscillation of fuzzy dark matter

GW & ULDM

Ultralight DM

Axion-like DM  
Resonance

GW and DM

spin-0 DM  
spin-2 DM

## DM oscillation induced time residual

- Metric:  $ds^2 = -(1 + 2\Phi) dt^2 + [(1 - 2\Psi) \delta_{ij} + h_{ij}] dx^i dx^j$ .
- e.g. the scalar field  $\phi(x, t) = \phi(x) \cos[mt + \theta_0(x)]$ ,
- Oscillating potential  $\Psi \simeq \bar{\Psi}(x) + \Psi_\phi \cos[2(mt + \theta_0(x))]$
- Doppler effect:  $z_\phi(t) \equiv \frac{\omega_0 - \omega_\phi(t)}{\omega_0} \simeq \Psi(x_\phi, t_\phi) - \Psi(x_0, t_0)$ .
- Timing residual in the pulse  $R_\phi(t) = \int_0^t z_\phi(t') dt'$
- **Strain**  $h_\phi = 2\sqrt{3} \Psi_\phi = \frac{\sqrt{3}}{4M_{pl}^2} \frac{\rho_\phi}{m^2} \simeq 5.2 \times 10^{-17} \alpha_0 \left(\frac{f_{yr}}{f}\right)^2$ ,
- **GW Timing residual**  $R_c(f) \equiv \sqrt{\frac{S_c(f)}{T_s}} = \frac{1}{\sqrt{3}} \frac{h_c(f)}{2\pi f} \left(\frac{f_s}{f}\right)^{1/2}$

[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014) ]

# Pulsar timing constraints on ULDM

GW & ULDM

Ultralight DM

Axion-like DM  
Resonance

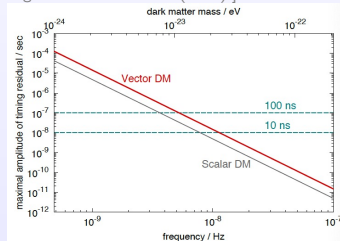
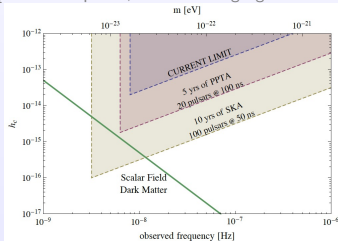
GW and DM

spin-0 DM  
spin-2 DM

## DM oscillation induced time residual

- Spin-0: massive scalar field  $\mathcal{L}_{(0)} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$
- Spin-1: massive vector field  $\mathcal{L}_{(1)} = -\frac{1}{4}F^2 - \frac{1}{4}m^2A^2$

[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]



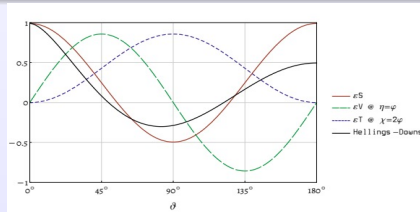
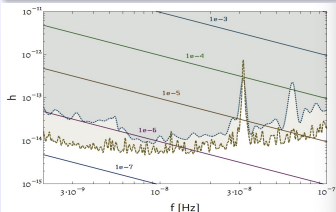
[cf. Nomura-Itoy-Soda, "Pulsar timing residual induced by ultralight vector DM" PRD(2020)]

# Pulsar timing constraints on spin-2 ULDM

## spin-2 ultralight fields

- Spin-2: massive tensor field(Fierz-Pauli): Bi-metric gravity,  

$$\mathcal{L}_{(2)} = \frac{1}{2} M_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} M_{\rho\sigma} - \frac{1}{4} m^2 (M_{\mu\nu} M^{\mu\nu} - M^2)$$
- The oscillating solution  $M_{ij} = \mathcal{M} \cos[mt + \theta_2(x)] \varepsilon_{ij}$
- Effective metric perturbations:  $\tilde{g}_{ij} = \delta_{ij} + \frac{\alpha_2}{M_{pl}} M_{ij}$
- The redshift  $z(t) = \frac{\omega(t) - \omega_0}{\omega_0} = \frac{\alpha_2}{2M_{pl}} \int dt \omega_0 \partial_t M_{ij} n^i n^j$



[cf. Armaleo-Nacir-Urbanb, "Pulsar timing array constraints on spin-2 ULDM" JCAP(2020)  
 Gromov-Son, "Bimetric Theory of Fractional Quantum Hall States" PRX(2017)]

# Gravitational effects & coupling effects

GW & ULDM

Ultralight DM

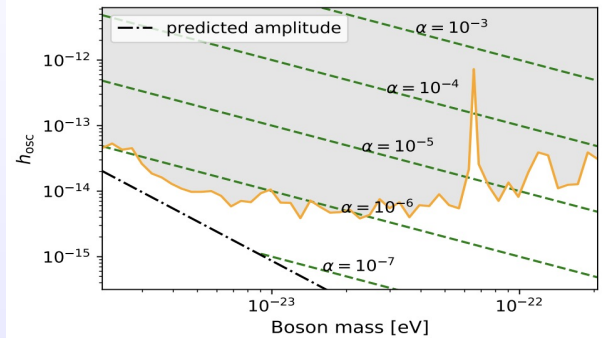
Axion-like DM  
Resonance

GW and DM

spin-0 DM  
spin-2 DM

$$S = \frac{M_{\text{Pl}}^2}{1 + \alpha^2} \int d^4x \left[ \sqrt{|g|} R(g) + \alpha^2 \sqrt{|f|} R(f) - 2 \frac{\alpha^2 M_{\text{Pl}}^2}{1 + \alpha^2} \sqrt{|g|} V(g, f; \beta_n) \right] + \int d^4x \sqrt{|g|} \mathcal{L}_m(g, \Psi)$$

$$S^{(2)} = \int d^4x \sqrt{|\bar{g}|} \left[ \mathcal{L}_{\text{GR}}^{(2)}(\mathcal{G}) + \mathcal{L}_{\text{FP}}^{(2)}(M) - \frac{1}{M_{\text{Pl}}} (\mathcal{G}_{\mu\nu} - \alpha M_{\mu\nu}) T^{\mu\nu}(\Psi) \right],$$



[cf. Y. M. Wu, Z. C. Chen, Q. G. Huang, JCAP **09**, 021 (2023)]

"Pulsar timing residual induced by ultralight tensor dark matter," ]

# Angular correlation & Hellings-Downs curves

GW & ULDM

Ultralight DM

Axion-like DM  
Resonance

GW and DM

spin-0 DM  
spin-2 DM

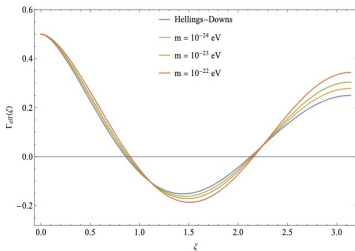


FIG. 2: Effective cross-correlation curves with  $\alpha = 10^{-6}$  and mass ranging from  $10^{-24}$  to  $10^{-22}$ . It can be seen that in this range, the deformation of spin-2 dark matter on the Hellings-Downs curve is relatively small.

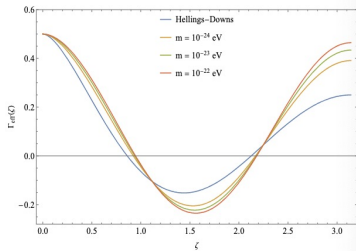


FIG. 3: Effective cross-correlation curves with  $\alpha = 10^{-5.5}$ . The deformation is very strong in this range, suggesting that if the coupling constant  $\alpha$  is above this magnitude, existing ultralight mass spin-2 dark matter would have considerable effects on the deformation of the Hellings-Downs curve at corresponding frequency.

$$S = \frac{M_{\text{Pl}}^2}{1 + \alpha^2} \int d^4x \left[ \sqrt{|g|} R(g) + \alpha^2 \sqrt{|f|} R(f) - 2 \frac{\alpha^2 M_{\text{Pl}}^2}{1 + \alpha^2} \sqrt{|g|} V(g, f; \beta_n) \right] + \int d^4x \sqrt{|g|} \mathcal{L}_m(g, \Psi)$$

$$S^{(2)} = \int d^4x \sqrt{|\bar{g}|} \left[ \mathcal{L}_{\text{GR}}^{(2)}(\mathcal{G}) + \mathcal{L}_{\text{FP}}^{(2)}(M) + \frac{1}{M_{\text{Pl}}} (\mathcal{G}_{\mu\nu} - \alpha M_{\mu\nu}) T^{\mu\nu}(\Psi) \right],$$

[cf. R.G.Cai, J.R.Zhang, Y.L. Zhang, arXiv: 2402.03984,

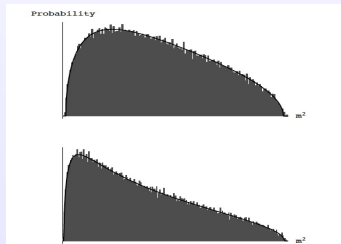
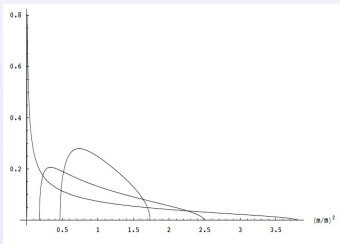
Angular correlation and deformed Hellings-Downs curve by spin-2 ultralight dark matter]

# Multi fields: Marcenko-Pastur distribution

## Mass spectrum and ultralight fields

- Marcenko-Pastur:  $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$ ,
- Energy density:  $\rho_\phi \equiv \int dm \tilde{\rho}(m) = \int dm \frac{1}{2} m^2 \tilde{\phi}(m)^2 P(m)$ .
- Convenient choice:  $\tilde{\rho}(m) \simeq \rho_\phi P(m)$ ,  $\int dm P(m) = 1$ .

[cf. Marcenko-Pastur, "Distributions of Eigenvalues for Some Sets of Random Matrices," (1967) ]



[cf. Easthera-McAllister, "Random Matrices and the Spectrum of N-flation" JCAP(2006)

Cai-Hu-Piao, "Entropy Perturbations in N-flation" PRD(2009) ]

# Phenomenological fitting results

GW & ULDM

Ultralight DM

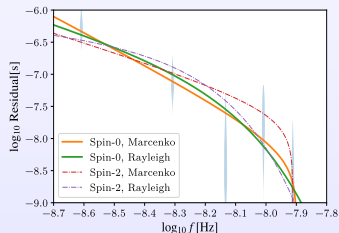
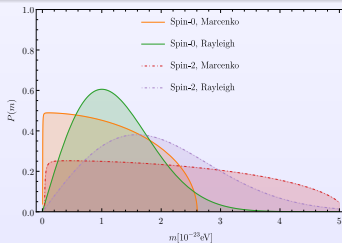
Axion-like DM  
Resonance

GW and DM

spin-0 DM  
spin-2 DM

## Mass spectrum and ultralight fields

- Marcenko-Pastur:  $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$ ,
- Rayleigh distribution:  $P_\sigma(m) = \frac{m}{\sigma^2} e^{-\frac{m^2}{2\sigma^2}}$ .



[Sun-Yang-Zhang, PRD(2022) "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter" ]

# Corner Figures of Bayesian Fitting

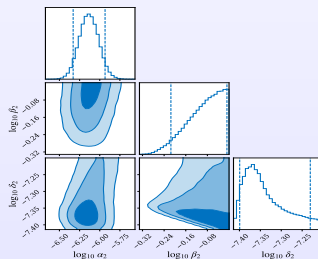
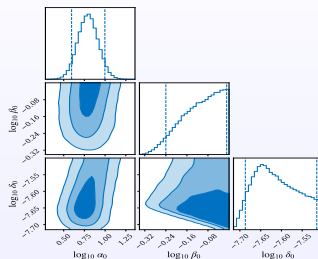
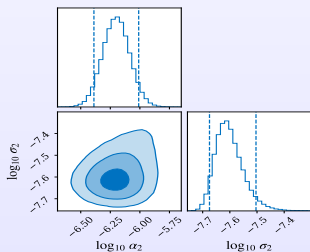
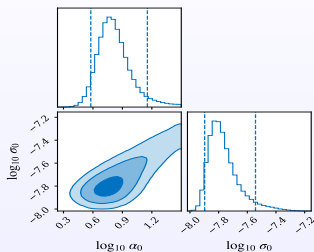
GW & ULDM

Ultralight DM

Axion-like DM  
Resonance

GW and DM

spin-0 DM  
spin-2 DM



# Phenomenological fitting results

## The effective strain

- $h_c^\phi(f) = \frac{\alpha_0}{M_{pl}^2} \frac{\sqrt{3}\rho_{DM}}{4\pi f} P(\pi f)$
- $h_c^M(f) = \frac{\alpha_2}{M_{pl}} \frac{mMP(m)}{\sqrt{5}} = \frac{\alpha_2}{M_{pl}} \frac{2\sqrt{\rho_M}}{\sqrt{5}} P(2\pi f).$

|          | Parameters                     | spin-0                             | spin-1                              | spin-2                             |
|----------|--------------------------------|------------------------------------|-------------------------------------|------------------------------------|
| Marcenko | $\alpha_i$                     | $5.9^{+1.9}_{-1.3}$                | $\sim 3\alpha_0$                    | $7.6^{+2.2}_{-1.7} \times 10^{-7}$ |
|          | $m_-^i/(10^{-23}\text{eV})$    | $2.9^{+3.6}_{-0.3} \times 10^{-3}$ | $\sim \delta_0(1 - \sqrt{\beta_0})$ | $6.3^{+6.0}_{-1.7} \times 10^{-3}$ |
|          | $m_+^i/(10^{-23}\text{eV})$    | $2.61^{+0.21}_{-0.01}$             | $\sim \delta_0(1 + \sqrt{\beta_0})$ | $5.08^{+0.02}_{-0.01}$             |
| Rayleigh | $\alpha_i$                     | $5.6^{+3.8}_{-1.0}$                | $\sim 3\alpha_0$                    | $6.1^{+2.1}_{-1.3} \times 10^{-7}$ |
|          | $\sigma_i/(10^{-23}\text{eV})$ | $1.0^{+0.4}_{-0.1}$                | $\sim \sigma_0$                     | $1.6^{+0.3}_{-0.1}$                |

[Sun-Yang-Zhang, PRD(2022), "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter" ]

# Summary and Outlook

GW & ULDM

Ultralight DM

Axion-like DM  
Resonance

GW and DM

spin-0 DM  
spin-2 DM

## ULDM with spin 0,1,2 & GW effects

- Spin-2 ultralight dark matter & Bi-metric gravity
- **Wideband mass spectrum** extension of ULDM
- Search/constraint for ULDM with GW detection
- Angular correlation & **Deformed Hellings-Downs**

## GW detection & Ultra-light DM with multi band

- Tabletop exp: GW burst & axion ( $\sim$  kHz - GHz)
- LVK & LISA: BH Superradiance ( $\sim$  mHz - kHz)
- PTA & SKA : Fuzzy dark matter ( $\sim$  nHz)
- LSS & CMB : DE & Modified gravity ( $\sim$  nHz)

