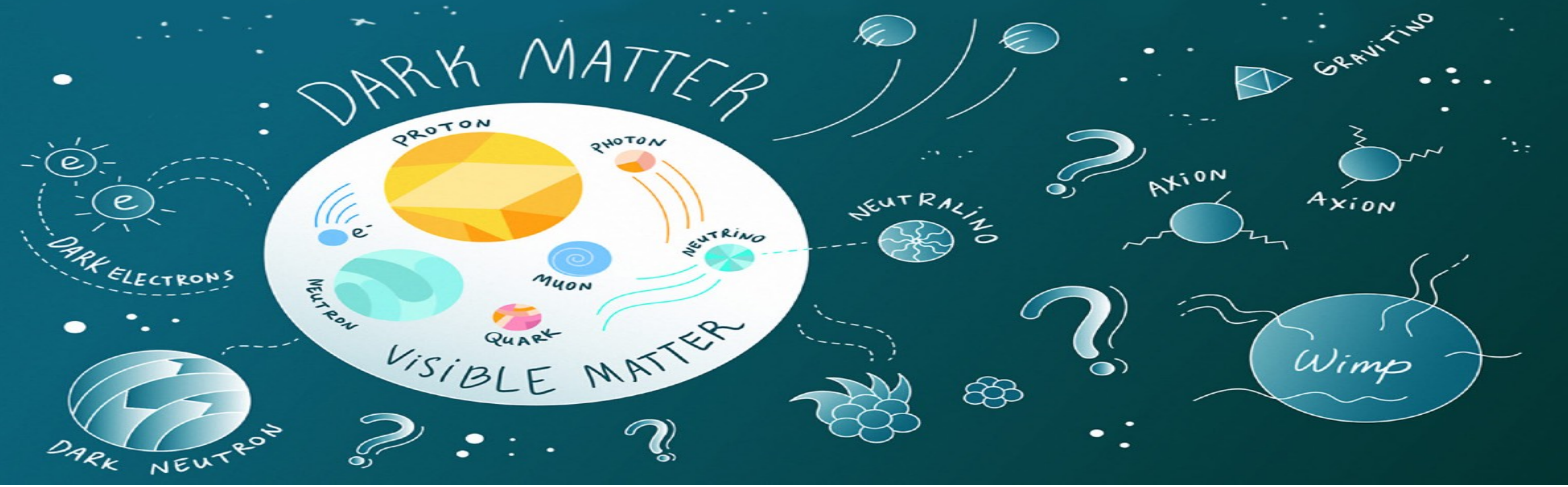


Axion, muon g-2 and New Physics

Xiao-Ping Wang（王小平）

July, 22@ Axion 2024

Base On: Phys.Rev.D 107 (2023) 9, 095016[arXiv:2210.09335]
arXiv:2405.02084

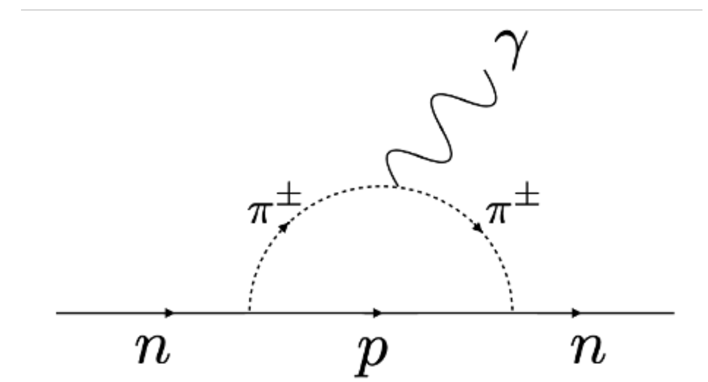
Collaborated with: Rundong Fang, Ji-Heng Guo, Jia Liu, Xiao-lin Ma and Lian-tao Wang

SM and New Physics

Strong CP problem

- Low-energy QCD done predict

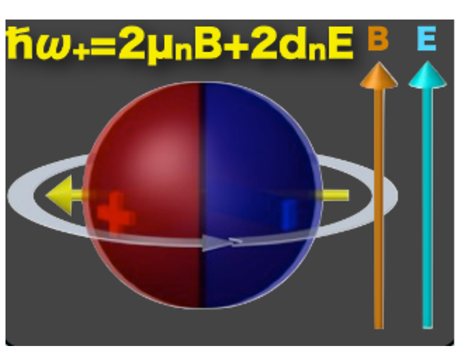
$$\mathscr{L} \supset \frac{\bar{\theta} g_s^2}{32\pi^2} G\tilde{G}$$



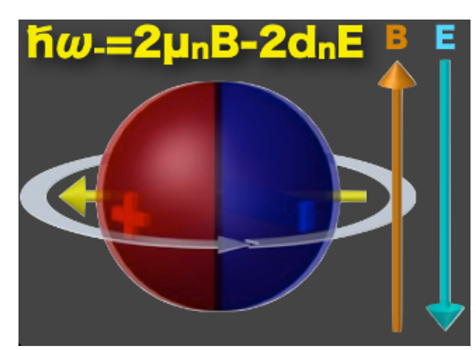
- Based on above QCD, we can build a theory for meson, then induce the neutron eDM at nucleon level

$$d_n = \frac{e\bar{\theta} g_A c_+ \mu}{8\pi^2 f_\pi^2} \log \frac{\Lambda^2}{m_\pi^2} \sim 3 \times 10^{-16} \bar{\theta} \text{ e cm}$$

- Pression measurement



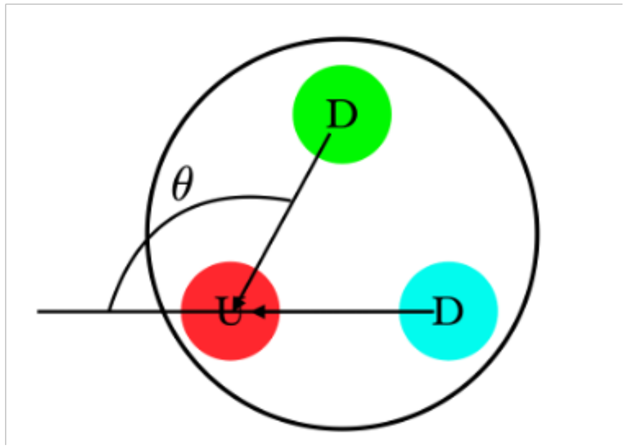
$$d_n = \frac{\hbar \Delta\omega}{4E} \leq 10^{-26} \text{ e cm}$$



Solution to Strong CP problem

- The neutron eDM in classical formula

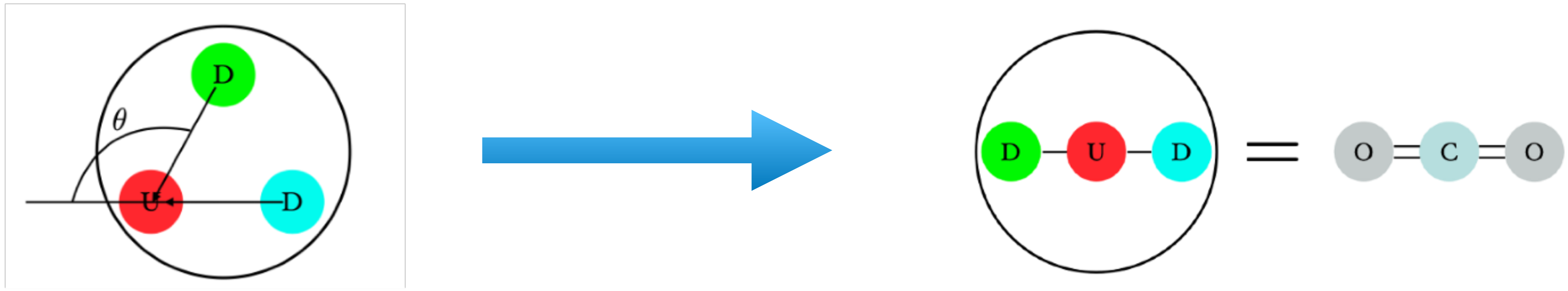
$$\vec{d} = \sum q\vec{r}$$



- Use the neutron has a size $r_n \sim 1/m_\pi$

$$|d_n| \approx 10^{-13}\sqrt{1-\cos\theta}e \text{ cm}$$

- Comparing to the experiment results, we need $\cos\theta = 1$

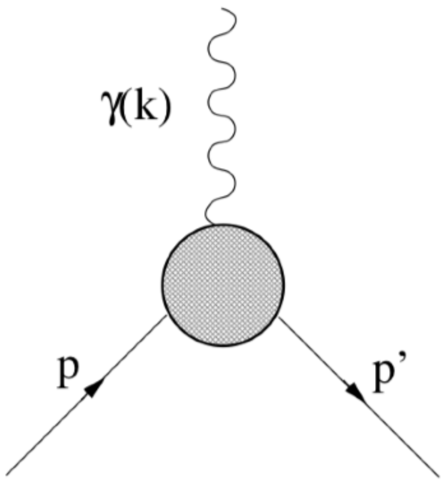


- The EFT consists of a single new particle, the axion (a), and a single new coupling (f_a)

$$\mathscr{L} \supset \left(\frac{a}{f_a} + \bar{\theta}\right)\frac{1}{32\pi^2}G\tilde{G} \quad \longrightarrow \quad d_n \propto \frac{a}{f_a} + \bar{\theta} = 0$$

Anomalous Magnetic Moment

- In Quantum Field Theory (with C, P invariance)



$$= (-ie)\bar{u}(p')\left[\gamma^\mu F_1(k^2) + \frac{i\sigma^{\mu\nu}k_\nu}{2m} F_2(k^2)\right] u(p)$$

$$F_1(0) = 1 \qquad F_2(0) = a$$

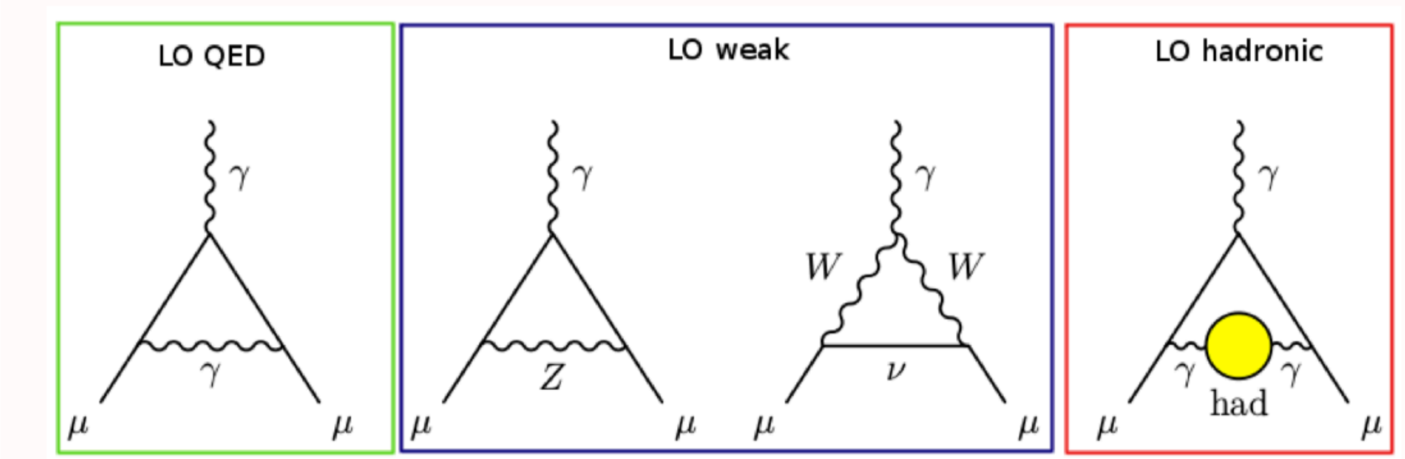
- The anomaly is defined through the quantity $a_\ell = (g_\ell - 2)/2$. Total anomaly can be written as:

$$a_\ell = \underbrace{a_\ell^{QED} + a_\ell^{hadronic} + a_\ell^{weak}}_{\text{Standard Model}} + \underbrace{a_\ell^{BSM}}_{\text{New Physics}}$$

Anomalous Magnetic Moment

- **Standard Model (SM)**

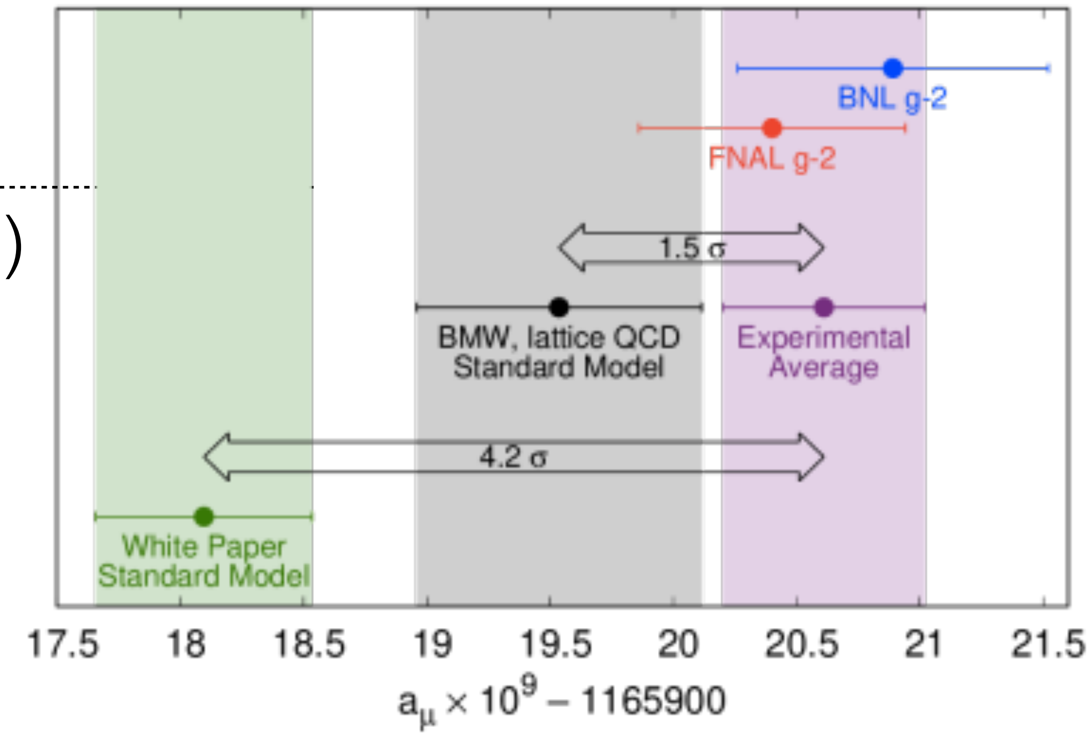
$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{had}}$$



$\times 10^{-10}$

QED Contribution	11 658 471.895 (0.015)
EW Contribution	15.4 (0.1)
Hadronic Contribution	
LO hadronic	694.9(4.3)
NLO hadronic	-9.8(0.1)
Light-by-light	10.5(2.6)
Theory Total	11659182.3(4.9)

- **Anomalous Muon g-2**



Axion Like Paticle

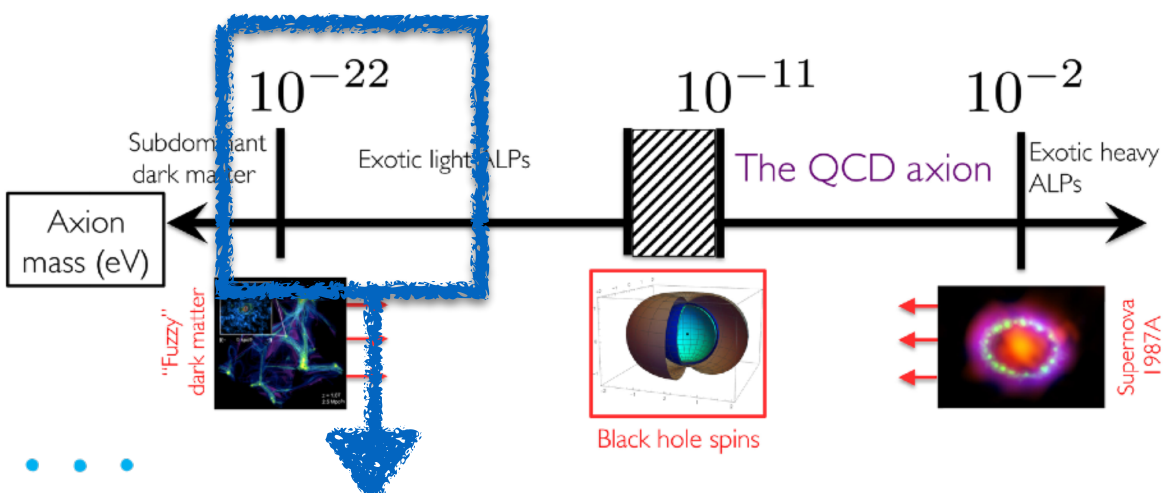
- The ALP Lagrangian

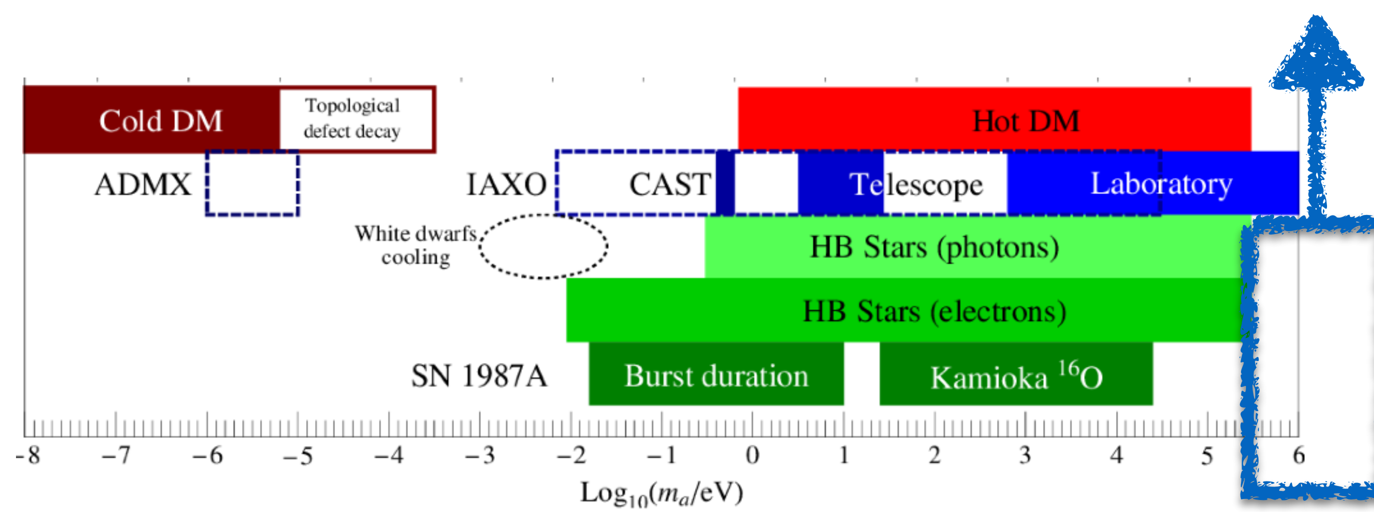
$$\mathscr{L}_a^{\text{int}} \supset \frac{\alpha}{8\pi}\frac{C_{a\gamma}}{f_a} aF\tilde{F} + C_{af}\frac{\partial_\mu a}{2f_a}\bar{f}\gamma^\mu\gamma_5 f + \frac{C_{a\pi}}{f_a f_\pi}\partial_\mu a[\partial\pi\pi\pi]^\mu - \frac{i}{2}\frac{C_{an\gamma}}{m_n}\frac{a}{f_a}\bar{n}\sigma_{\mu\nu}\gamma_5 n F^{\mu\nu}$$

Coupling to photon	Coupling to fermions	Coupling to mesons	Coupling to nucleon EDM

- The ALP Mass

via colliders



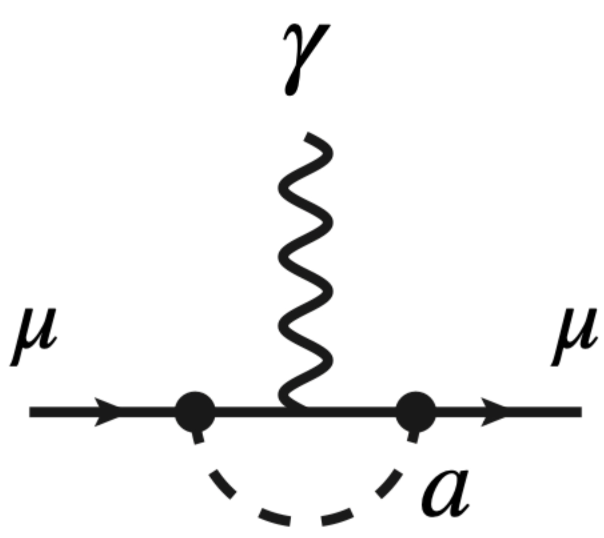


via neutrino experiments

Heavy ALP for Muon g-2

The (pseudo)scalar Yukawa coupling to lepton

$$\mathscr{L}_{\rm yuk} = \phi\bar{\ell}\left(g_R + i g_I \gamma_5\right)\ell$$



The 1-loop contribution to g-2

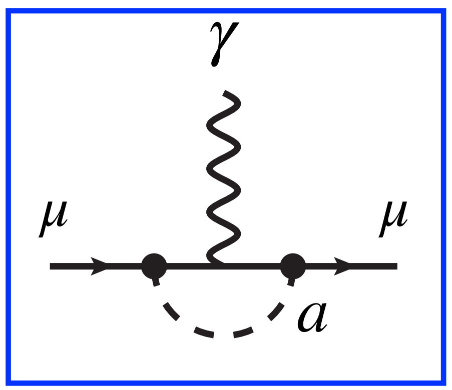
$$\Delta a_\ell = \frac{1}{8\pi^2}\int_0^1 dx \frac{(1-x)^2\left((1+x)g_R^2 - (1-x)g_I^2\right)}{(1-x)^2 + x\left(m_\phi/m_\ell\right)^2}$$

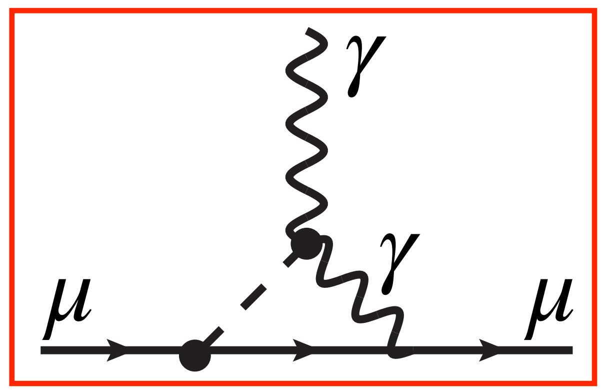
- For scalar, $\Delta a_\ell > 0$
- For (psudo)scalar, $\Delta a_\ell < 0$

The pseudo-scalar solution

Further requirement for pseudo-scalar

$$\mathscr{L} = i y_{a\psi} a \bar{\psi} \gamma_5 \psi + \frac{1}{4} g_{a\gamma\gamma} \tilde{F} F$$



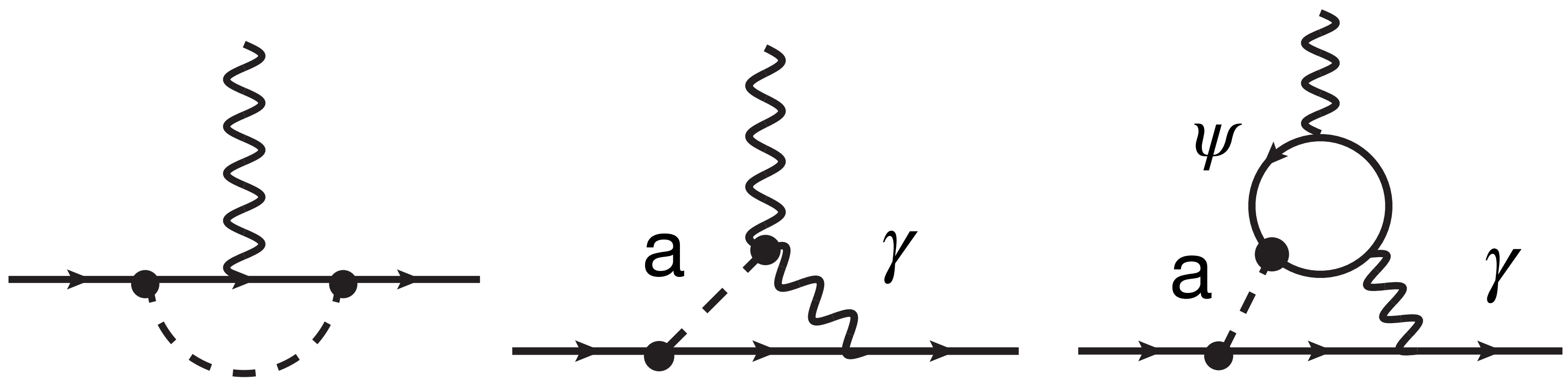


- Assumes $g_{a\gamma\gamma}$ remains essentially constant throughout the integration over virtual photon-loop momentum
- $g_{a\gamma\gamma}$ and $y_{a\ell}$ can adjust its sign to give positive result

Complete calculation for ALP

The axion-like particle Lagrangian

$$\mathscr{L}_{\text{eff}}^{D\le 5} = \sum_f \frac{C_{ff}}{2}\frac{\partial^\mu a}{f_a}\bar{f}\gamma_\mu\gamma_5 f + \frac{\alpha C_{\gamma\gamma}}{4\pi}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\alpha C_{\gamma Z}}{2\pi s_w c_w}\frac{a}{f_a}F_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{\alpha C_{ZZ}}{4\pi s_w^2 c_w^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{Z}^{\mu\nu}$$
$$+ \frac{\alpha C_{WW}}{\pi s_w^2}\frac{a}{f_a}\epsilon_{\mu\nu\rho\sigma}\partial^\mu W_+^\nu \partial^\rho W_-^\sigma + \ldots$$



$$\Delta a_\mu^{(1)} \propto -\frac{c_{\mu\mu}^2}{16\pi^2} \qquad \Delta a_\mu^{(2)} \propto -\frac{c_{\mu\mu}c_{\gamma\gamma}\alpha}{16\pi^3} \qquad \Delta a_\mu^{(3)} \propto -\frac{c_{\mu\mu}c_{ii}\alpha}{16\pi^3}$$

- Different sign for $c_{\mu\mu}$ and $c_{\gamma\gamma}$ is needed
- The 3rd diagram subtlety:

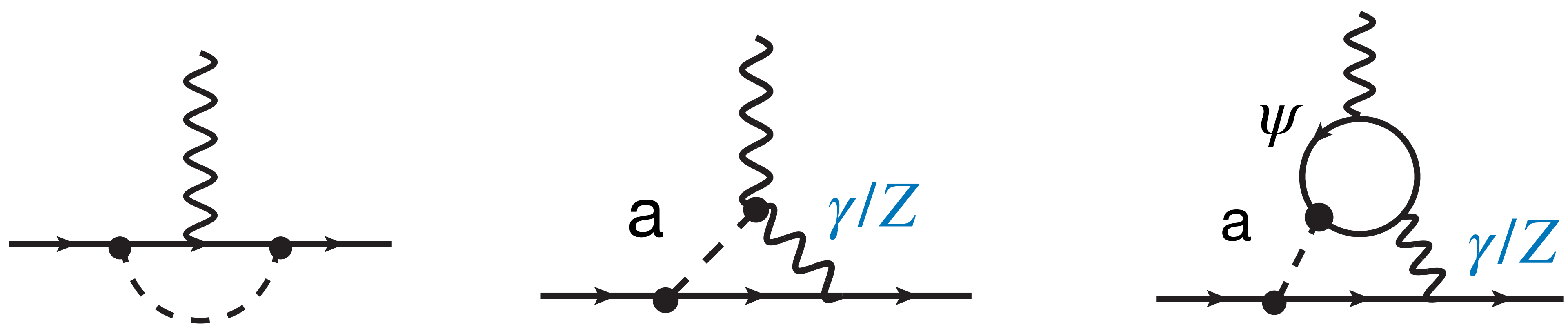
Complete calculation for ALP

The axion-like particle Lagrangian

$$\mathscr{L}_{\text{eff}}^{\text{D}\le 5} = \sum_f \frac{C_{ff}}{2}\frac{\partial^\mu a}{f_a}\bar{f}\gamma_\mu\gamma_5 f + \frac{\alpha C_{\gamma\gamma}}{4\pi}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\alpha C_{\gamma Z}}{2\pi s_w c_w}\frac{a}{f_a}F_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{\alpha C_{ZZ}}{4\pi s_w^2 c_w^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{Z}^{\mu\nu}$$

$$+\frac{\alpha C_{WW}}{\pi s_w^2}\frac{a}{f_a}\epsilon_{\mu\nu\rho\sigma}\partial^\mu W_+^\nu \partial^\rho W_-^\sigma + \dots$$

$$C_{\gamma\gamma} = C_{WW} + C_{BB} \qquad C_{\gamma Z} = c_w^2 C_{WW} - s_w^2 C_{BB} \qquad C_{ZZ} = c_w^4 C_{WW} + s_w^4 C_{BB}$$

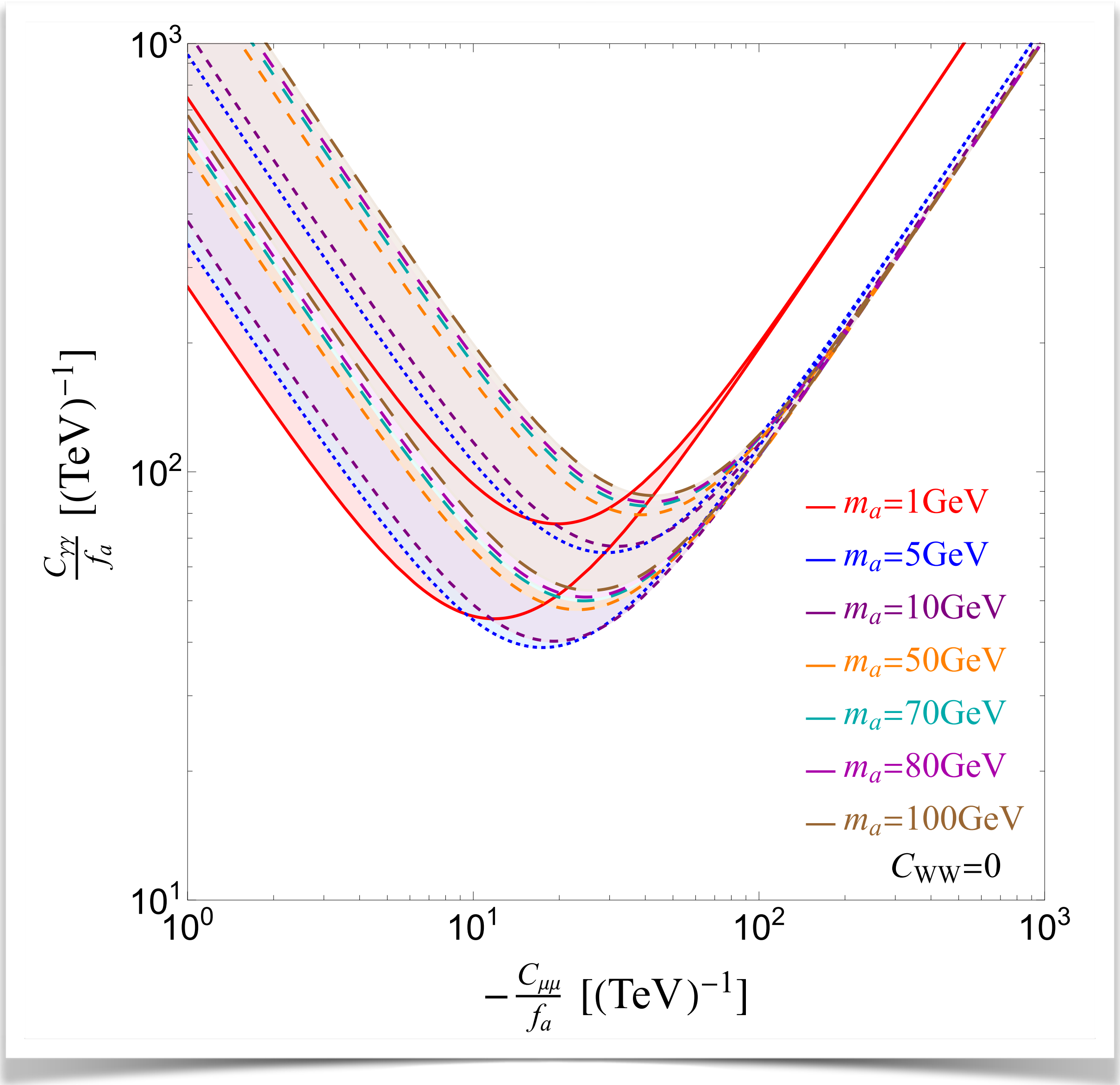


$$\Delta a_{2Z} = \frac{\alpha c_{\gamma z} c_{\mu\mu} m_\mu^2 \left(4 s_w^2 - 1\right)}{32\pi^3 c_w^2 s_w^2 f_a^2} \cdot h_Z(x, y, \mu)$$

$$m_\psi \to \infty,\ \Delta a_3 \to 0$$

$$\Delta a_{3Z} = -\frac{\alpha c_{\mu\mu}^2 m_\mu^2 \left(4 s_w^2 - 1\right)^2}{128\pi^3 c_w^2 s_w^2 f_a^2}\left[H_Z(\textbf{two loop}) + h_Z(\textbf{counter term})\right]$$

Muon g-2 solution $C_{ww} = 0$



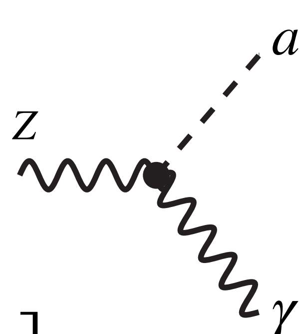
- In g-2 solution region, mostly decay to $a \to \mu^+\mu^-$
- The inclusion of Z diagram makes some difference for large m_a
- Exotic Z decay should happen

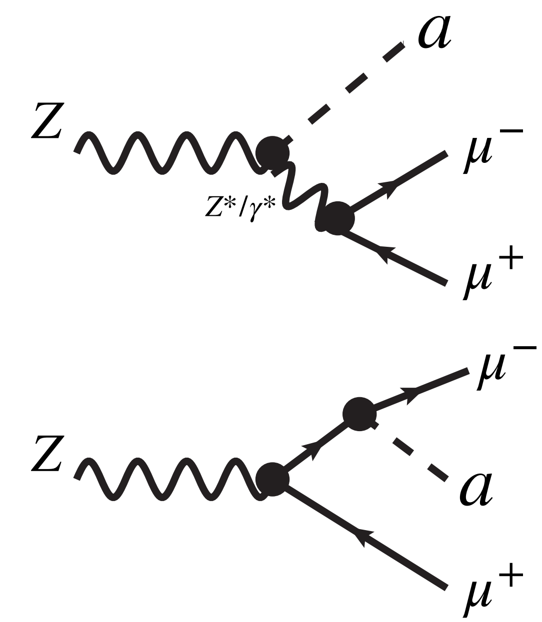
Z decay $C_{ww} = 0$

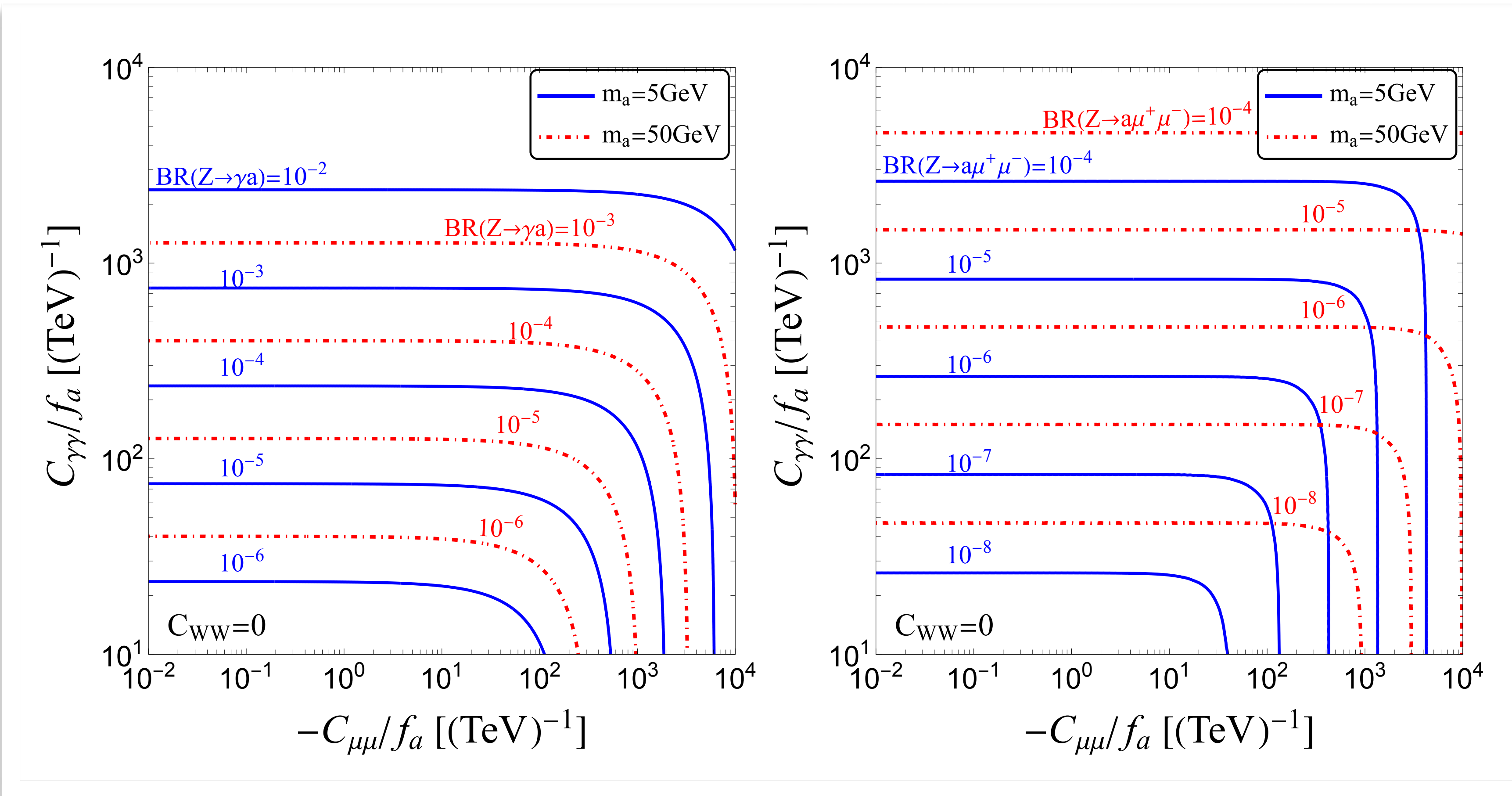
ALP production from Z decay

$$\Gamma(Z \to \gamma a) = \frac{\alpha^2(m_Z)\, m_Z^3}{96\pi^3 s_w^2 c_w^2 f_a^2} \left| C_{\gamma Z}^{\text{eff}} \right|^2 \left(1 - \frac{m_a^2}{m_Z^2}\right)^3$$

$$C_{\gamma Z}^{\text{eff}} = C_{\gamma Z} + C_{\mu\mu}\left(\frac{1}{4} - s_w^2\right)\left[1 + \frac{m_\mu^2}{m_a^2 - m_Z^2}\cdot\left(\mathscr{F}\left(\frac{m_a^2}{m_\mu^2}\right) - \mathscr{F}\left(\frac{m_Z^2}{m_\mu^2}\right)\right)\right]$$







ALP decay $C_{ww} = 0$

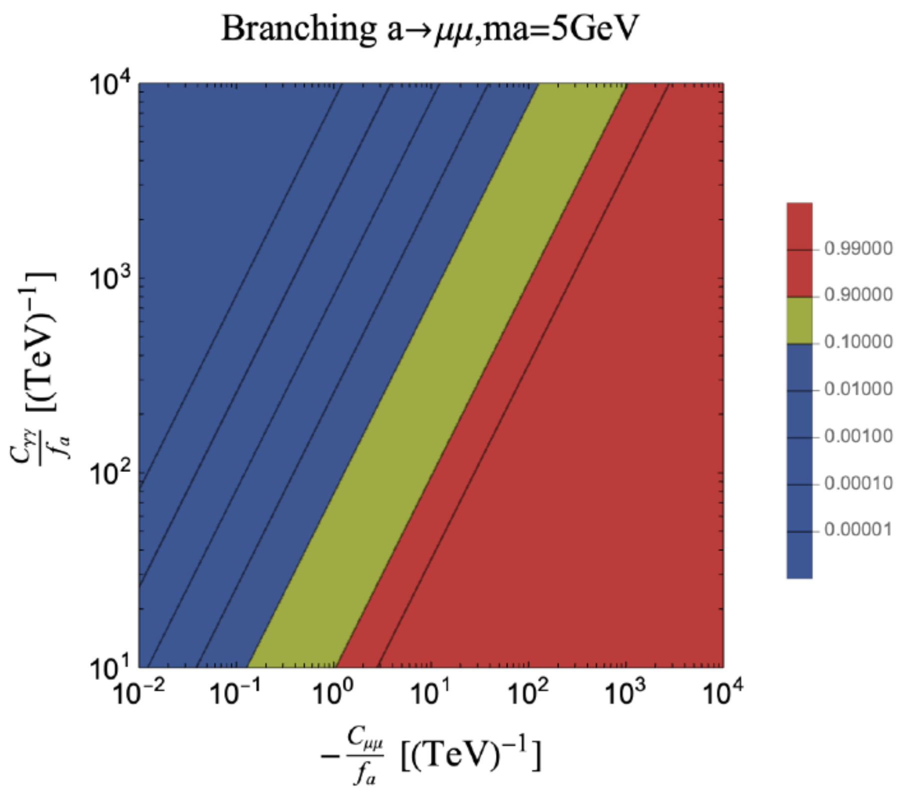
- ALP decay

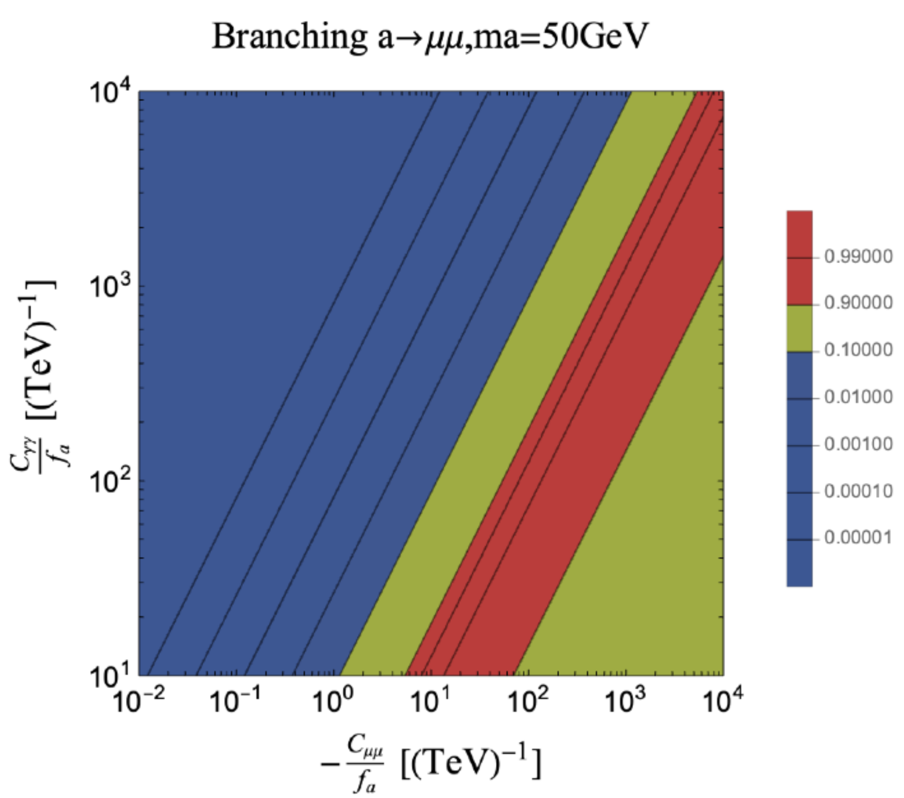
$$\Gamma\left(a \rightarrow \mu^+\mu^-\right) = \frac{m_a m_\mu^2}{8\pi f_a^2}\left|C_{\mu\mu}^{\text{eff}}\right|^2 \sqrt{1 - \frac{4m_\mu^2}{m_a^2}}$$

$$\Gamma(a \rightarrow \gamma\gamma) = \frac{\alpha^2 m_a^3}{64\pi^3 f_a^2}\left|C_{\gamma\gamma}^{\text{eff}}\right|^2$$

$$C_{\gamma\gamma}^{\text{eff}} = C_{\gamma\gamma} + C_{\mu\mu}\left[1 + \frac{m_\mu^2}{m_a^2}\cdot\mathscr{F}\left(\frac{m_a^2}{m_\mu^2}\right)\right] + \mathcal{O}(\alpha)$$

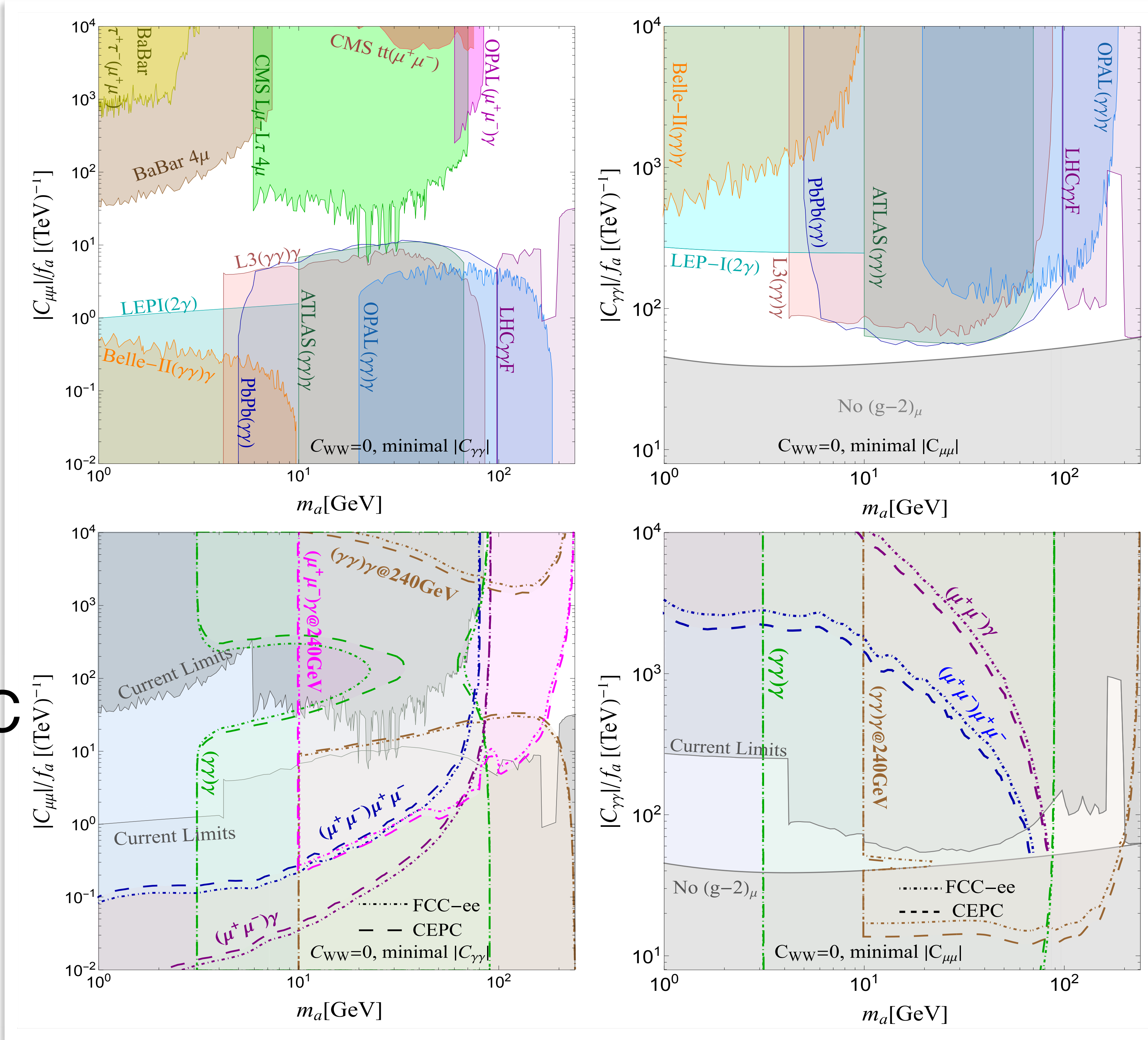
$$C_{\mu\mu}^{\text{eff}} = C_{\mu\mu} + \mathcal{O}\left(\alpha^2\right)$$



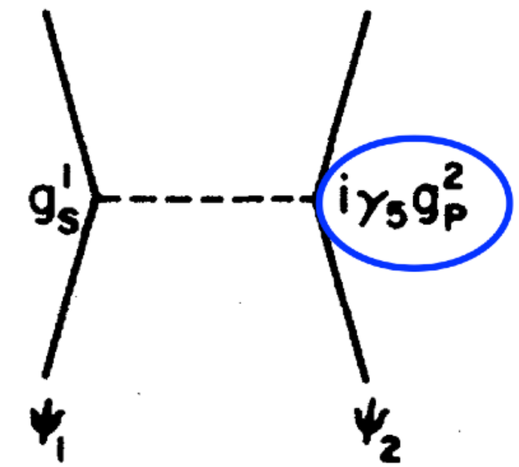


Searching at colliders $C_{ww} = 0$

- Current
- Current+ CEPC



Ultra-light ALP for Muon g-2



- The Lagrangian for ALP is

$$\mathscr{L}_{\text{int}} = g_a \partial_\alpha \phi \bar{\psi} \gamma^\alpha \gamma^5 \psi \Rightarrow \begin{aligned} \mathscr{H}_{\text{LMSF}} &= -g_a \partial_\alpha \phi \bar{\psi} \gamma^\alpha \gamma^5 \psi \\ H_{\text{LMSF}} &= -\frac{g_a \partial_\alpha \phi}{(2\pi)^3} \int \frac{d^3 p}{2E_\mu} \sum_{s,s'} a_p^{s\dagger} \bar{u}^s(p) \gamma^\alpha \gamma^5 a_p^{s'} u^{s'}(p) \end{aligned}$$

- In the rotated muon rest frame

$$H_{\text{LMSF}} = -g_a \vec{\nabla} \phi \cdot \hat{\vec{\sigma}}$$

$$\frac{d\vec{S}}{dt} = \vec{\omega} \times \vec{S} \Rightarrow \Delta\left(\frac{dS_i}{dt}\right) = \epsilon^{jki} \delta\omega_j S_k \quad (1)$$

- Based on the Heisenberg equations, the spin operators evolve

$$\frac{d\hat{S}_i}{dt} = i\left[H, \hat{S}_i\right] \Rightarrow \Delta\left(\frac{dS_i}{dt}\right) = \left\langle \mu \left| -g^a \epsilon^{jki} \partial_j a \hat{\sigma}_k \right| \mu \right\rangle = -2 g_a \epsilon^{jki} \partial_j \phi S_k \quad (2)$$

- Compare (1) and (2) $\quad \delta\omega_j = -2 g_a \partial_j \phi$

Ultra-light ALP for Muon g-2

- In the LMSF, the interaction between ϕ and nucleons is

$$\mathscr{L} = g_s \phi \bar{N} N \quad \Rightarrow \quad \delta\omega_{\mathrm{RMRF}} = -2 g_a \frac{d\phi}{dr} = \frac{g_s g_a N_E}{2\pi r_E^2}$$

- Boosting back to the lab frame

$$\delta\omega = \frac{g_s g_a N_E}{2\pi \gamma r_E^2}$$

- Relating the frequency change to the muon g-2 results,

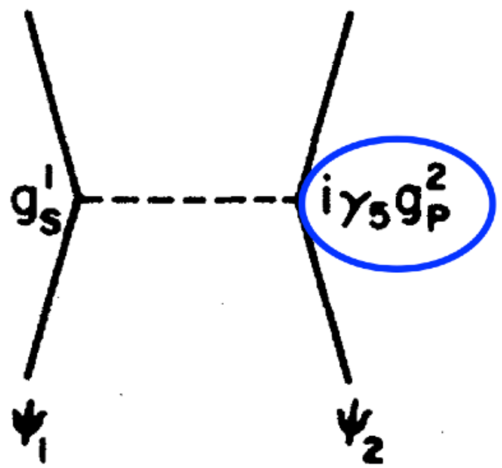
$$\frac{\delta\omega}{\omega} \approx \frac{\Delta a_\mu}{a_\mu}$$

- $\Delta a_\mu = (249 \pm 48) \times 10^{-11}$ indicate

$$g_a g_s \in \left[4.6 \times 10^{-29}, 1.7 \times 10^{-28}\right] \mathrm{GeV}^{-1}$$

Phys.Rev.Lett. 130 (2023) 18

Spin dependent Scalar Force



→ CP violating coupling

$$\mathscr{L} = \partial_\mu \phi \left(\bar{\nu}_L^i \boldsymbol{\kappa}_\nu^{ij} \nu_L^j + \bar{e}_L^i \boldsymbol{\kappa}_L^{ij} e_L^j + \bar{e}_R^i \kappa_R^{ij} e_R^j \right) + g_s \phi \bar{N} N$$

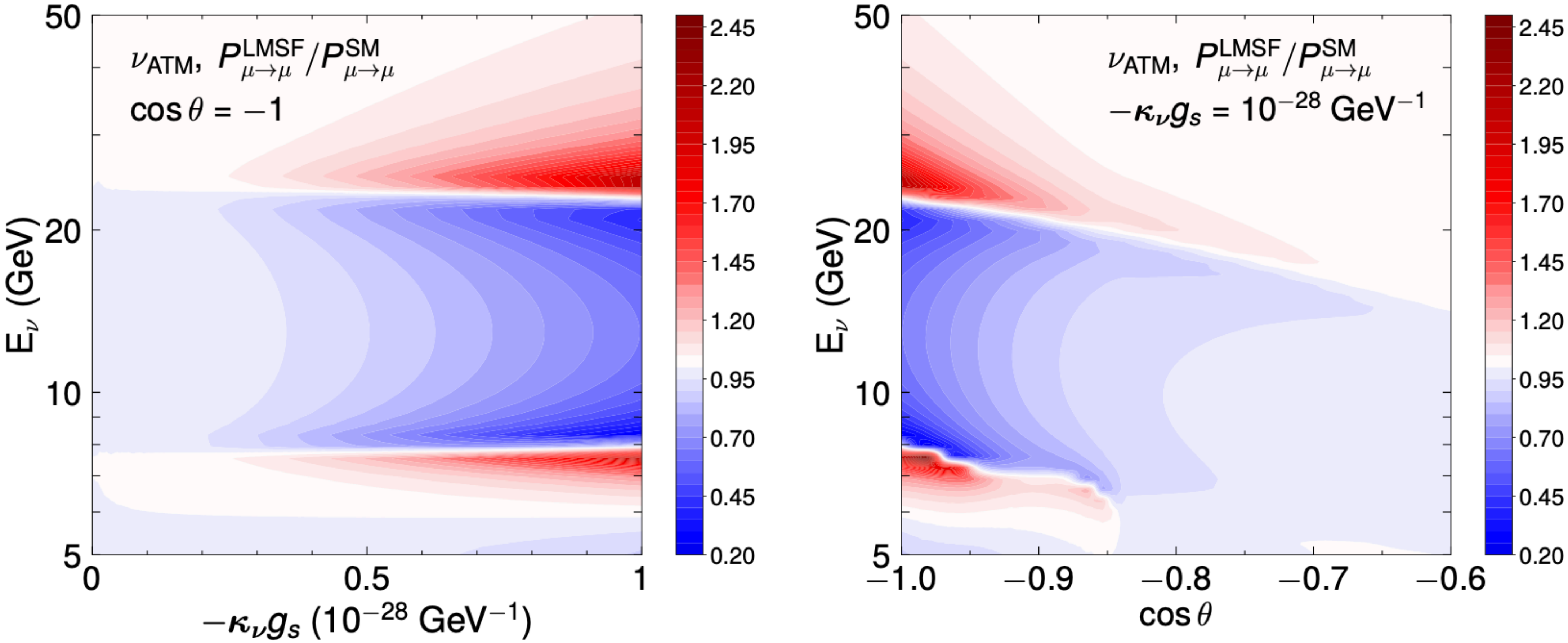
- This leads to a gradient interaction between neutrinos and matter at leading order

$$\Delta H = \boldsymbol{\kappa}_\nu \nabla \phi \cdot \overrightarrow{p} / | \overrightarrow{p} |$$

- The potential for the matter source (Sun/ Earth)

$$\phi(r) = \frac{g_s}{r} \int_0^r n_N(\ell) \ell^2 d\ell$$

The oscillation probability

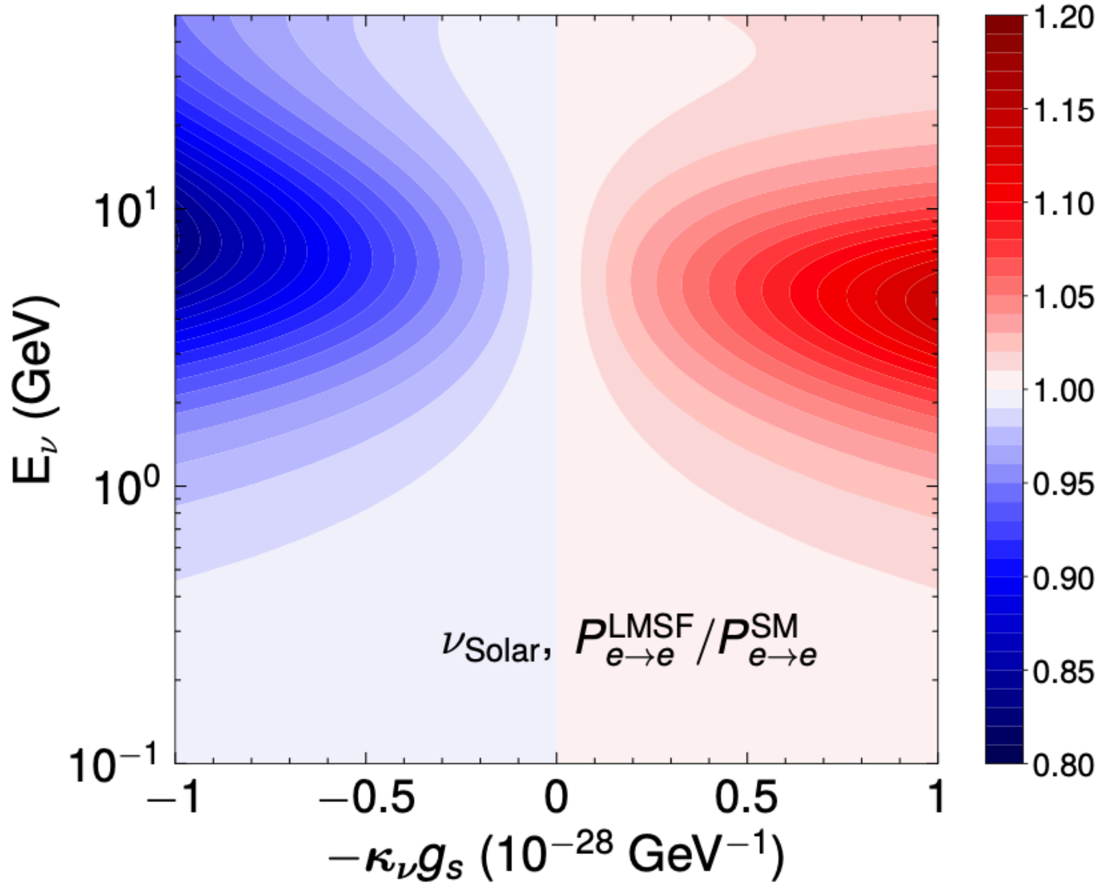


- χ^2 method for the atmospheric neutrino data

$$\chi^2(N,O) = 2\sum_{ij\alpha}\left(N_{ij\alpha} - O_{ij\alpha} + O_{ij\alpha}\ln\frac{O_{ij\alpha}}{N_{ij\alpha}}\right)$$

i and j refer to the bin indices for neutrino energy Eν and incoming angle cosθ

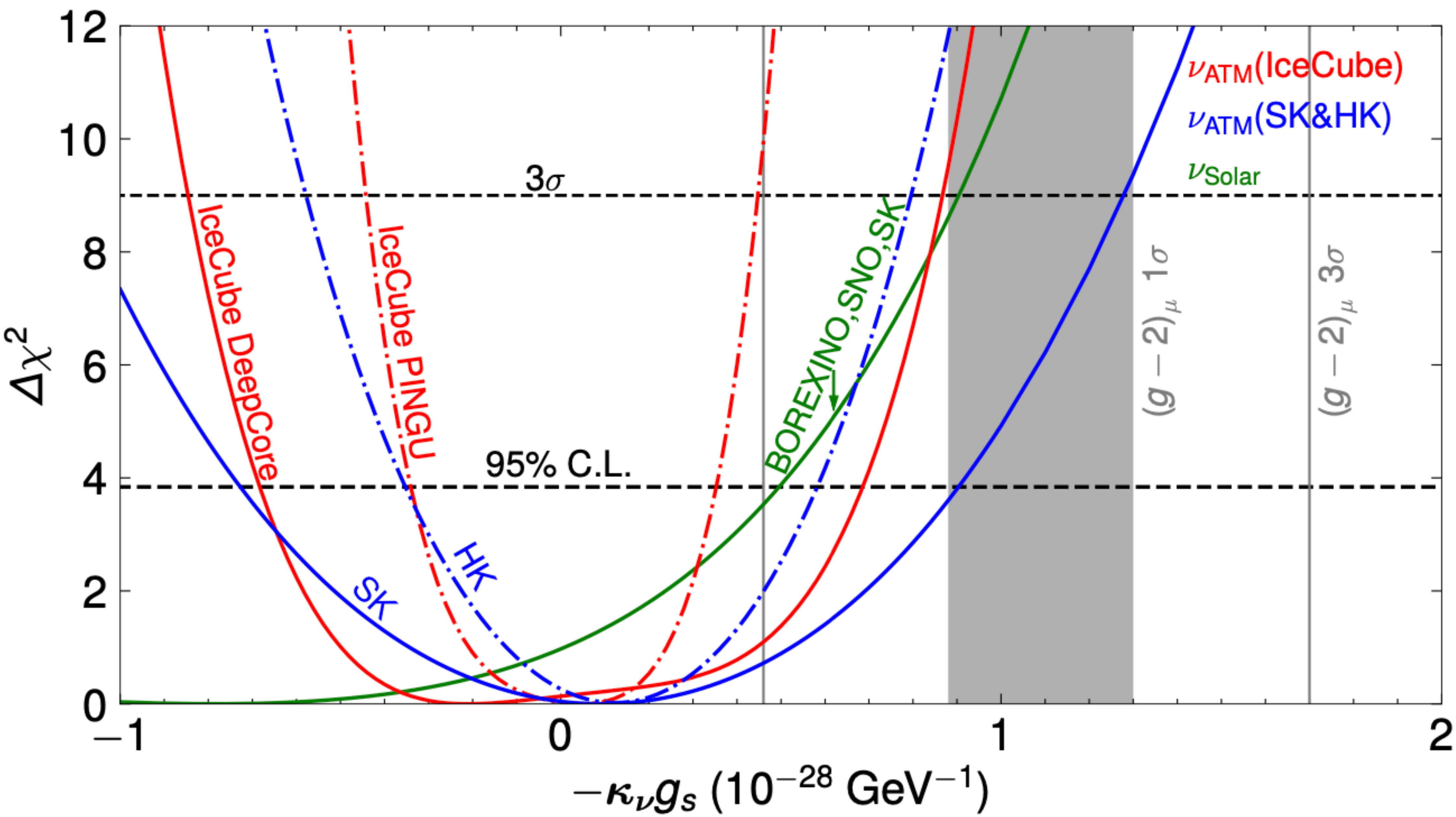
The oscillation probability



- χ^2 method for the solar neutrino data

$$\chi^2 = \sum_i \frac{\left(P_{ee}\left(E_i\right) - P_{ee}^{\rm obs}\left(E_i\right)\right)^2}{\left(\delta P_{ee}^{\rm obs}\left(E_i\right)\right)^2}$$

Final Results for the long-range force



Summary

- ALP have potential to solve strong CP problem
- Muon g-2 can be solved by ALP
- Heavy ALP can be searched via colliders
- Neutrino oscillation experiments can be used to detect long-range muon spin force which can explain muon g-2.

Thank you!