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**Chiral study of the axion and
light-flavor meson interactions**



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Introduction

Axion: an elegant solution to strong CP

[Peccei, Quinn, PRL '77]
[Weinberg, PRL '78] [Wilzeck, PRL '78]

$$\mathcal{L}_{\text{QCD}}^{\text{axion}} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \dots$$

Spontaneous breaking of $U(1)_{\text{PQ}}$ symmetry & anomalous term: axion (pseudo-NGB)

- f_a : the axion decay constant. Invisible axion: $f_a \gg v_{\text{EW}}$. Axion interactions with SM particles are accompanied with the factors of $(1/f_a)^n$.
- Stringent constraints for $m_{a,0} = 0$ (QCD axion) $m_a^2 = m_{a,0}^2 + \frac{F_\pi^2 m_\pi^2}{4f_a^2}$

[Di Luzio, et al., Phy.Rep'20] [Sikivie, RMP'21] [Irastorza, Redondo, PPNP'18]

Cosmology, Astronomy, Colliders, Quantum precision measurements, Cavity Haloscope,

- It is possible to introduce other model-dependent axion interactions, such as axion-quark, axion-photon and axion-lepton terms.

We will focus on the QCD-like axion: $m_{a,0} (\neq 0) \ll f_a$ with model-independent $aG\tilde{G}$ interaction, i.e., the **MODEL INDEPENDENT QCD axion** interactions.

- Axion-hadron interactions are relevant at low energies.

Axion chiral perturbation theory ($A\chi$ PT)

$$\mathcal{L}_{\text{QCD}}^{\text{axion}} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \boxed{\frac{a}{f_a} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}}$$

Two ways to proceed:

(1) Remove the $aG\tilde{G}$ term via the quark axial transformation

$$\begin{array}{l} \text{Tr}(Q_a) = 1 \\ \begin{array}{l} \curvearrowright \\ \curvearrowleft \end{array} \end{array} \quad q \rightarrow e^{i\frac{a}{2f_a}\gamma_5} Q_a q$$

$$-\frac{a\alpha_s}{8\pi f_a} G\tilde{G} - \frac{\partial_\mu a}{2f_a} \bar{q}\gamma^\mu \gamma_5 Q_a q \quad M_q \rightarrow M_q(a) = e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a}$$

Mapping to χ PT

$$\mathcal{L}_2 = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger + \chi_a U^\dagger + U \chi_a^\dagger \rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\text{LO}}$$

$$\chi_a = 2B_0 e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \quad J_A^\mu|_{\text{LO}} = -i\frac{F^2}{2} \langle Q_a (\partial^\mu U U^\dagger + U^\dagger \partial^\mu U) \rangle$$

- $Q_a = M_q^{-1}/\text{Tr}(M_q^{-1})$ is usually taken to eliminate the LO mass mixing between axion and pion [Georgi,Kaplan,Randall, PLB'86], though any other hermitian Q_a should lead to the same physical quantities.
- $J_A^\mu \partial_\mu a$ will cause the kinematical mixing. Should be consistently included. [Bauer, et al., PRL'21]

(2) Explicitly keep the $aG\tilde{G}$ term and match it to χ PT

Reminiscent:

QCD $U(1)_A$ anomaly that is caused by topological charge density $\omega(x) = \alpha_s G_{\mu\nu} \tilde{G}^{\mu\nu} / (8\pi)$ is responsible for the massive singlet η_0 .

Axion could be similarly included as the η_0 via the $U(3)$ χ PT:

$$\mathcal{L}^{\text{LO}} = \frac{F^2}{4} \langle u_\mu u^\mu \rangle + \frac{F^2}{4} \langle \chi_+ \rangle + \frac{F^2}{12} M_0^2 X^2$$

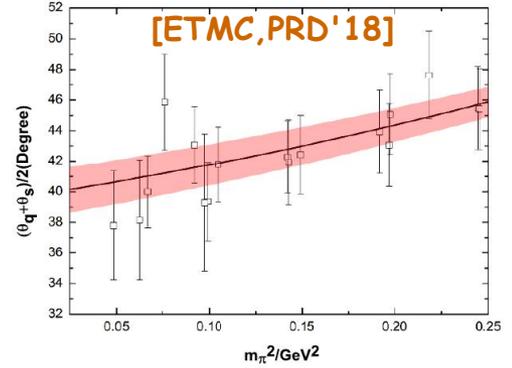
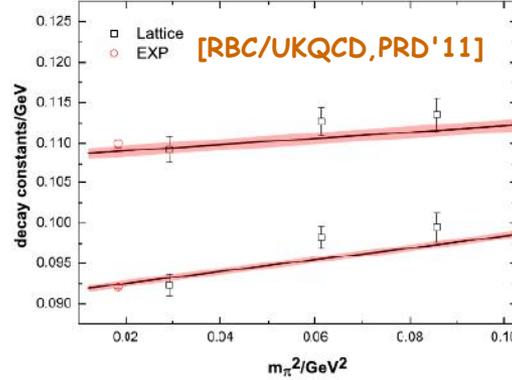
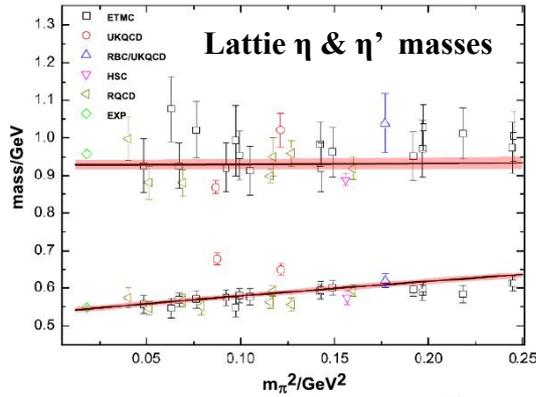
$$U = u^2 = e^{i\frac{\sqrt{2}\Phi}{F}}, \quad \chi = 2B(s + ip), \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u,$$

$$u_\mu = iu^\dagger D_\mu U u^\dagger, \quad D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$$

$$X = \log(\det U) + i\frac{a}{f_a} \quad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

- Q_a is not needed in $U(3)$ χ PT.
- $M_0^2 = 6\tau/F^2$, with τ the topological susceptibility. Note that $M_0^2 \sim \mathcal{O}(1/N_c)$.
- δ expansion scheme: $\delta \sim \mathcal{O}(p^2) \sim \mathcal{O}(m_q) \sim \mathcal{O}(1/N_c)$.
- Axion interactions enter via the axion-meson mixing terms.

Parameters	NLO Fit
F (MeV)	$91.05^{+0.42}_{-0.44}$
$10^3 \times L_5$	$1.68^{+0.05}_{-0.06}$
$10^3 \times L_8$	$0.88^{+0.04}_{-0.04}$
Λ_1	$-0.17^{+0.05}_{-0.05}$
Λ_2	$0.06^{+0.08}_{-0.09}$
$\chi^2/(d.o.f.)$	219.9/(111-5)



$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = M^{\text{LO+NLO}} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix} \quad M^{\text{LO+NLO}} = \begin{pmatrix} 1 + (0.015 \pm 0.001) & 0.017 + (-0.010 \pm 0.001) & 0.009 + (-0.007 \pm 0.001) & \frac{12.1 + (0.48 \pm 0.08)}{f_a} \\ -0.019 + (0.007 \pm 0.001) & 0.94 + (0.21 \pm 0.01) & 0.33 + (-0.22 \pm 0.03) & \frac{34.3 + (0.9 \pm 0.2)}{f_a} \\ -0.003 + (-0.003 \pm 0.000) & -0.33 + (-0.18 \pm 0.03) & 0.94 + (0.13 \pm 0.02) & \frac{25.9 + (-0.5 \pm 0.1)}{f_a} \\ \frac{-12.1 + (-0.20 \pm 0.03)}{f_a} & \frac{-23.8 + (1.6^{+0.8}_{-0.8})}{f_a} & \frac{-35.7 + (-5.7^{+1.6}_{-1.7})}{f_a} & 1 + \frac{27.6 \pm 1.0}{f_a^2} \end{pmatrix}$$

Two-photon couplings

$$\mathcal{L}_{\text{WZW}}^{\text{LO}} = -\frac{3\sqrt{2}e^2}{8\pi^2 F} \varepsilon_{\mu\nu\rho\sigma} \partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2 \Phi \rangle \quad \mathcal{L}_{\text{WZW}}^{\text{NLO}} = it_1 \varepsilon_{\mu\nu\rho\sigma} \langle f_+^{\mu\nu} f_+^{\rho\sigma} \chi_- \rangle + k_3 \varepsilon_{\mu\nu\rho\sigma} \langle f_+^{\mu\nu} f_+^{\rho\sigma} \rangle X$$

Note: one needs the π - η - η' - a mixing as input to calculate $g_{a\gamma\gamma}$.

$$F_{\pi^0\gamma\gamma}^{\text{Exp}} = 0.274 \pm 0.002 \text{ GeV}^{-1},$$

$$F_{\eta\gamma\gamma}^{\text{Exp}} = 0.274 \pm 0.006 \text{ GeV}^{-1},$$

$$F_{\eta'\gamma\gamma}^{\text{Exp}} = 0.344 \pm 0.008 \text{ GeV}^{-1}.$$

$$t_1 = -(4.4 \pm 2.3) \times 10^{-4} \text{ GeV}^{-2},$$

$$k_3 = (1.25 \pm 0.23) \times 10^{-4}$$

$$g_{a\gamma\gamma} = 4\pi\alpha_{em} F_{a\gamma\gamma} = -\frac{\alpha_{em}}{2\pi f_a} (1.63 \pm 0.01)$$

which can be compared to: 1.92 ± 0.04 [Grilli de Cortona, et al., JHEP'16] and 2.05 ± 0.03 [Lu, et al., JHEP'20]

- IB corrections could cause visible effects (working in progress).

Cosmology constraints on axion thermalization rate

Axion thermal production in the early Universe : Extra radiation (ΔN_{eff})

Extra effective number of relativistic d.o.f :

$$\Delta N_{\text{eff}} \simeq \frac{4}{7} \left(\frac{43}{4g_{\star s}(T_D)} \right)^{\frac{4}{3}}$$

$g_{\star s}(T)$: effective number of entropy d.o.f at temperature T

T_D : axion decoupling temperature from the thermal medium

➤ CMB constraint (Planck'18) [Aghanim et al., 2020] : $\Delta N_{\text{eff}} \leq 0.28$

➤ T_D : Instantaneous decoupling approximation

$$\Gamma_a(T_D) = H(T_D)$$

Axion thermalization rate

Hubble expansion parameter

$$\Gamma_a(T) = \frac{1}{n_a^{\text{eq}}} \int d\tilde{\Gamma} \sum |\mathcal{M}_{a\text{-SM}}|^2 n_B(E_1)n_B(E_2) [1 + n_B(E_3)][1 + n_B(E_4)]$$

$$H(T) = T^2 \sqrt{4\pi^3 g_*(T)/45} / m_{\text{Pl}}$$

Axion-SM particle scattering amplitudes

Key thermal channels of axion-SM scatterings at different temperatures

☞ $T_D \gtrsim 1 \text{ GeV}$: $ag \leftrightarrow gg$.

[Masso et al., 2002, Graf and Steffen, 2011]

☞ $T_D \lesssim 1 \text{ GeV}$: Hadrons need to be included.

☞ $T_D \lesssim 200 \text{ MeV}$: $a\pi \leftrightarrow \pi\pi$.

[Chang and Choi, 1993, Hannestad et al., 2005,
Giusarma et al., 2014, D'Eramo et al., 2022]

☐ **Reliable $a\pi$ interaction is crucial to determine Γ_a for $T_D < T_c \approx 155 \text{ MeV}$**

➤ For a long time, only the LO $a\pi \leftrightarrow \pi\pi$ amplitude is employed to calculate Γ_a , e.g.,

[Chang, Choi, PLB'93] [Hannestad, et al., JCAP'05] [Hannestad, et al., JCAP'05] [D'Eramo, et al., PRL'22]

➤ Recent NLO calculation of Γ_a shows that the reliable chiral expansion is only valid for $T_\chi \leq 70 \text{ MeV}$ for perturbative amplitudes at NLO [Di Luzio, et al., PRL'21]

➤ Chiral unitarization approach is adopted to extend the applicable ranges of energy and temperature. [Di Luzio, et al., PRD'23]

➤ However, to our knowledge, all the previous works have ignored the thermal corrections to the $a\pi \leftrightarrow \pi\pi$ amplitudes. We give the first estimation of such effects on the determination of axion parameters. [Wang, ZHG, Zhou, PRD'24]

$$\Gamma_a(T) = \frac{1}{n_a^{\text{eq}}} \int d\tilde{\Gamma} \sum_{\text{channels}} |\mathcal{M}_{a\pi,\pi\pi}|^2 n_B(E_1) n_B(E_2) [1 + n_B(E_3)][1 + n_B(E_4)]$$

Calculation of thermal $a\pi \leftrightarrow \pi\pi$ amplitudes at one-loop level

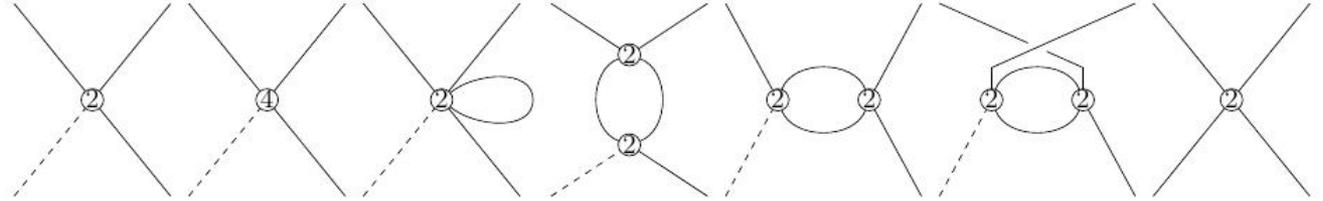
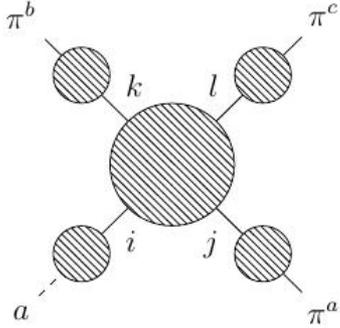
- Finite-temperature effects are included by imaginary time formalism (ITF), where [Kapusta and Gale, 2011, Bellac, 2011, Laine and Vuorinen, 2016]

$$p^0 \rightarrow i\omega_n, \quad \text{with } \omega_n = 2\pi nT, n \in \mathbb{Z},$$

$$-i \int \frac{d^d q}{(2\pi)^d} \rightarrow -i \int_{\beta} \frac{d^d q}{(2\pi)^d} \equiv T \sum_n \int \frac{d^{d-1} q}{(2\pi)^{d-1}}.$$

- Compute the thermal Green functions in ITF

$$G_{a\pi^a; \pi^b \pi^c}^T(p_1, p_2; p_3, p_4) = \sum_{i,j,k,l} G_{ai}(p_1^2) G_{\pi^a j}(p_2^2) G_{k\pi^b}(p_3^2) G_{l\pi^c}(p_4^2) A_{ij;kl}(p_1, p_2; p_3, p_4).$$



Feynman diagrams for amputated functions up to NLO.

- The effective Lagrangian at $\mathcal{O}(p^4)$

$$\mathcal{L}_4 \supset \frac{l_3 + l_4}{16} \langle \chi_a U^\dagger + U \chi_a^\dagger \rangle \langle \chi_a U^\dagger + U \chi_a^\dagger \rangle + \frac{l_4}{8} \langle \partial_\mu U \partial^\mu U^\dagger \rangle \langle \chi_a U^\dagger + U \chi_a^\dagger \rangle$$

$$- \frac{l_7}{16} \langle \chi_a U^\dagger - U \chi_a^\dagger \rangle \langle \chi_a U^\dagger - U \chi_a^\dagger \rangle + \frac{h_1 - h_3 - l_4}{16} \left[\left(\langle \chi_a U^\dagger + U \chi_a^\dagger \rangle \right)^2 \right.$$

$$\left. + \left(\langle \chi_a U^\dagger - U \chi_a^\dagger \rangle \right)^2 - 2 \langle \chi_a U^\dagger \chi_a U^\dagger + U \chi_a^\dagger U \chi_a^\dagger \rangle \right] + \frac{\partial_\mu a}{2f_a} J_A^\mu |_{\text{NLO}},$$

$$J_A^\mu |_{\text{NLO}} \supset -i l_1 \langle Q_a \{ \partial^\mu U, U^\dagger \} \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle$$

$$- i \frac{l_2}{2} \langle Q_a \{ \partial_\nu U, U^\dagger \} \rangle \langle \partial^\mu U \partial^\nu U^\dagger + \partial^\nu U \partial^\mu U^\dagger \rangle$$

$$- i \frac{l_4}{4} \langle Q_a \{ \partial^\mu U, U^\dagger \} \rangle \langle \chi_a U^\dagger + U \chi_a^\dagger \rangle.$$

Unitarization of the partial-wave $a\pi \leftrightarrow \pi\pi$ amplitude

Inverse amplitude method (IAM)

$$\mathcal{M}_{a\pi;IJ}^{\text{IAM}} = \frac{\left(\mathcal{M}_{a\pi;IJ}^{(2)}\right)^2}{\mathcal{M}_{a\pi;IJ}^{(2)} - \mathcal{M}_{a\pi;IJ}^{(4)}}$$

$$\mathcal{M}_{a\pi;IJ}(E_{cm}) = \frac{1}{2} \int_{-1}^{+1} d\cos\theta \mathcal{M}_{a\pi;I}(E_{cm}, \cos\theta) P_J(\cos\theta)$$

$$\text{Im}\mathcal{M}_{a\pi;IJ}(E_{cm}) \stackrel{s}{=} \frac{1}{2} \rho_{\pi\pi}^T(E_{cm}) \mathcal{M}_{\pi\pi;\pi\pi}^{IJ*} \mathcal{M}_{a\pi;IJ}, \quad (E_{cm} > 2m_\pi)$$

$$\rho_{\pi\pi}^T(E_{cm}) = \frac{\sigma_\pi(E_{cm}^2)}{16\pi} \left[1 + 2n_B\left(\frac{E_{cm}}{2}\right) \right], \quad \sigma_\pi(s) = \sqrt{1 - \frac{4m_\pi^2}{s}}, \quad n_B(E) = \frac{1}{e^{E/T} - 1}$$

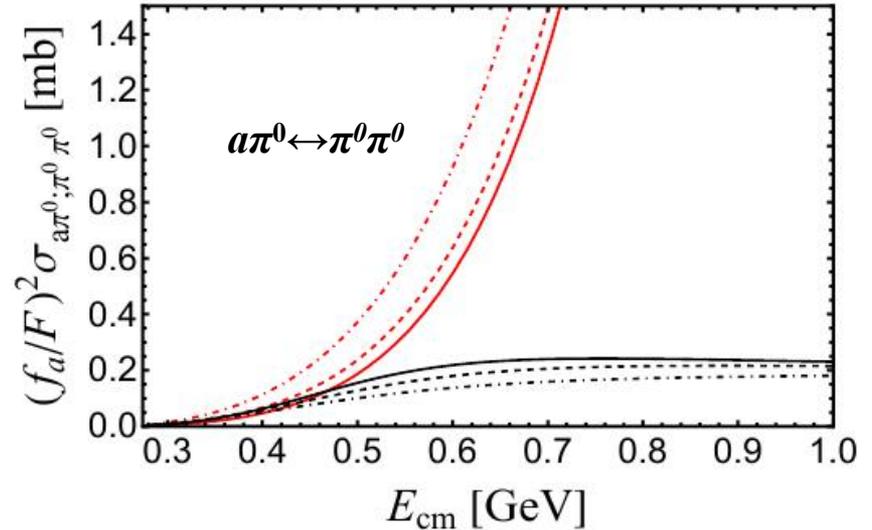
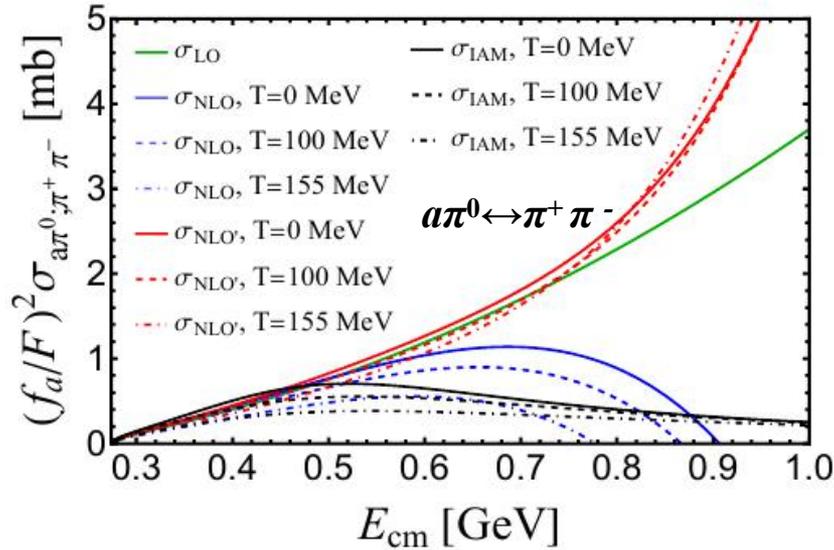
- Resonances poles on the second Riemann sheet

	$f_0(500)/\sigma$		$\rho(770)$	
	$M_\sigma \pm i\frac{\Gamma_\sigma}{2}$	$ f_a g_{\sigma a\pi} $	$M_\rho \pm i\frac{\Gamma_\rho}{2}$	$ f_a g_{\rho a\pi} $
$T = 0 \text{ MeV}$	$422 \pm i240 \text{ MeV}$	0.032 GeV^2	$739 \pm i72 \text{ MeV}$	0.035 GeV^2
$T = 100 \text{ MeV}^*$	$368 \pm i310 \text{ MeV}$	0.037 GeV^2	$744 \pm i77 \text{ MeV}$	0.036 GeV^2

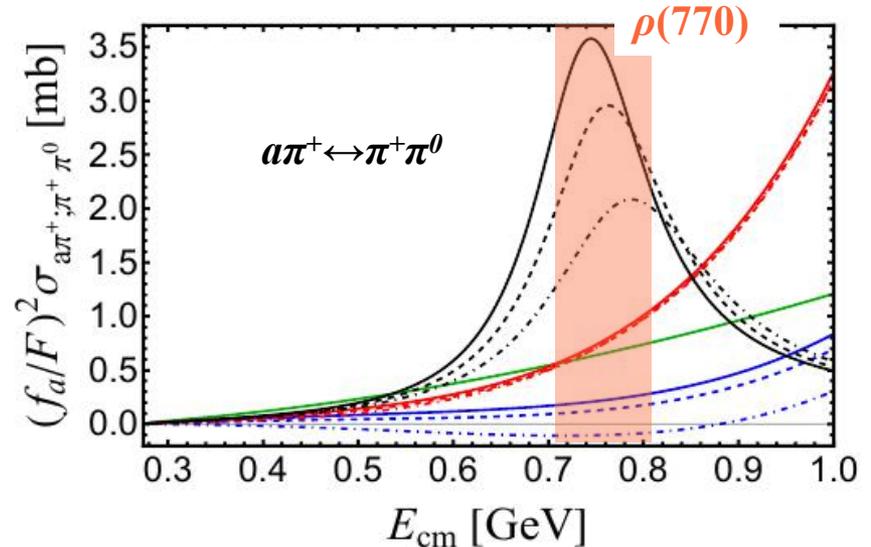
*Only include s -channel unitary thermal correction.

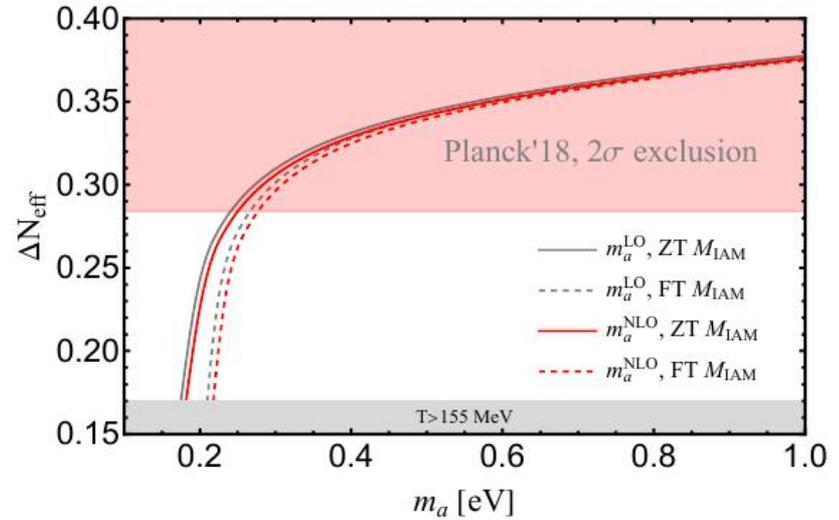
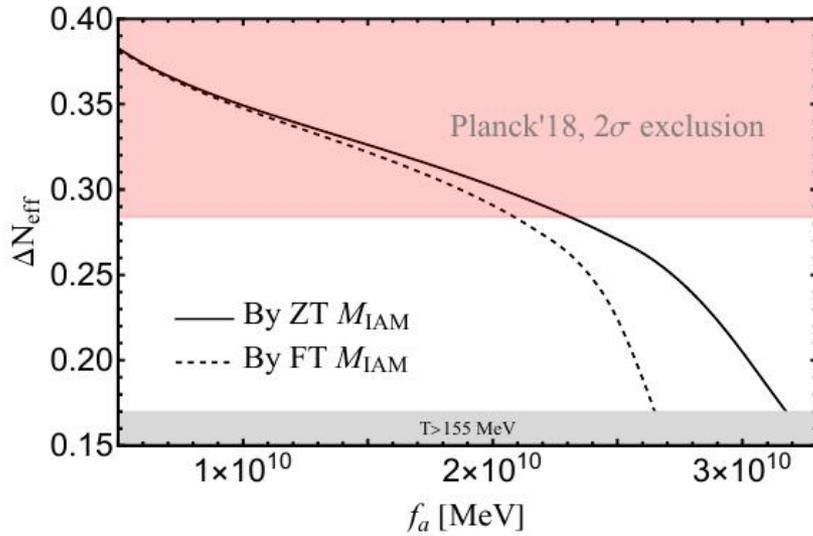
$a\pi \leftrightarrow \pi\pi$ cross-sections at finite temperatures

$$|\mathcal{M}^{(2)} + \mathcal{M}^{(4)}|^2 = \left(\mathcal{M}^{(2)}\right)^2 + 2\mathcal{M}^{(2)}\text{Re}\left(\mathcal{M}^{(4)}\right) + |\mathcal{M}^{(4)}|^2.$$



- The chiral perturbative amplitudes up to NLO begin to be unreliable around $E \simeq 500$ MeV.





□ The constraints **10% corrections are observed**

	lower limit of f_a	upper limit of m_a by m_a^{LO}	upper limit of m_a by m_a^{NLO}
ZT	2.3×10^7 GeV	0.24 eV	0.25 eV
FT	2.1×10^7 GeV	0.27 eV	0.28 eV

□ The QCD axion mass up to LO & NLO

$$m_a^2|_{\text{LO}} = \gamma_{ud} m_\pi^2 \frac{F^2}{f_a^2}, \quad \text{where } \gamma_{ud} = \frac{m_u m_d}{(m_u + m_d)^2},$$

$$m_a^2|_{\text{NLO}} = \gamma_{ud} m_\pi^2 \frac{F^2}{f_a^2} \left\{ 1 - 2 \frac{m_\pi^2}{(4\pi F)^2} \log \frac{m_\pi^2}{\mu^2} + 2 \left[h_1^r(\mu^2) - h_3 \right] \frac{m_\pi^2}{F^2} - 8l_7 \gamma_{ud} \frac{m_\pi^2}{F^2} \right\}$$

Axion production from $\eta \rightarrow \pi\pi a$ decay in SU(3) χ PT

Why focus on axion in η decay:

- ✓ Valuable channel to search axion @colliders: many available experiments with large data samples of η/η' [BESIII, STCF, JLab, REDTOP,]
- ✓ $\eta \rightarrow \pi\pi\pi$ (IB suppressed), $\eta \rightarrow \pi\pi a$ (no IB suppression)
- ✓ $\eta \rightarrow \pi\pi a$: theoretically easier to handle than $\eta' \rightarrow \pi\pi a$ (next step)

Previous works:

- ❖ Most of them rely on leading-order χ PT
- ❖ Possible issue: bulk contributions @LO χ PT are constant terms, and potential large corrections from higher orders may result.
- ❖ Hadron resonance effects may lead to enhancements.

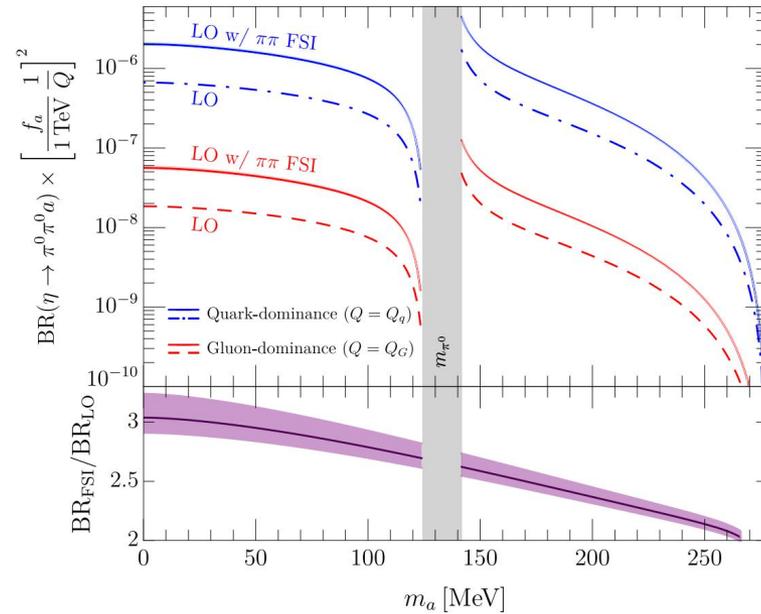
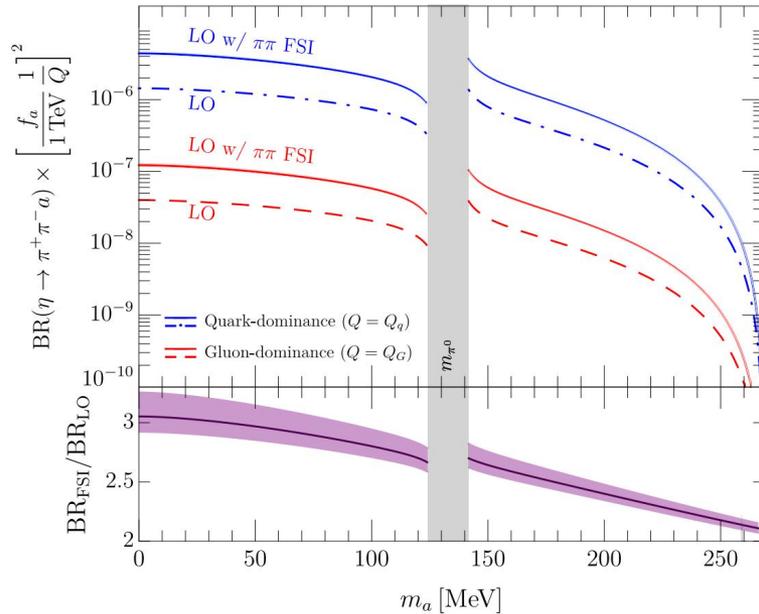
Advances in our work :

- Study of renormalization of $\eta \rightarrow \pi\pi a$ @1-loop level in SU(3) χ PT
- To implement unitarization to the $\eta \rightarrow \pi\pi a$ χ PT amplitude
- Uncertainty analyses in the phenomenological discussions

$$M_0(s) = P(s)\Omega_0^0(s)$$

$\eta \rightarrow \pi\pi a$ LO amplitude

Omnes function: $\pi\pi$ FSI



Our improvements:

- NLO perturbative decay amplitude include s - and $t(u)$ -channel interactions perturbatively.
- The unitarized decay amplitude will be constructed to account for the s -channel $\pi\pi$ final state interaction (FSI) effect that respect the chiral symmetry.
- Dalitz plots will be explored to decode the dynamics in $\eta \rightarrow \pi\pi a$.

LO χ PT Lagrangian

$$\mathcal{L}_2 = \frac{F^2}{4} \langle \partial_\mu U \partial^\mu U^\dagger + \chi_a U^\dagger + U \chi_a^\dagger \rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu \Big|_{\text{LO}} + \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2$$

$$\chi_a = 2B_0 M(a) \quad M(a) \equiv \exp\left(-i \frac{a}{2f_a} Q_a\right) M \exp\left(-i \frac{a}{2f_a} Q_a\right) \quad J_A^\mu \Big|_{\text{LO}} = -i \frac{F^2}{2} \langle Q_a \{ \partial^\mu U, U^\dagger \} \rangle$$

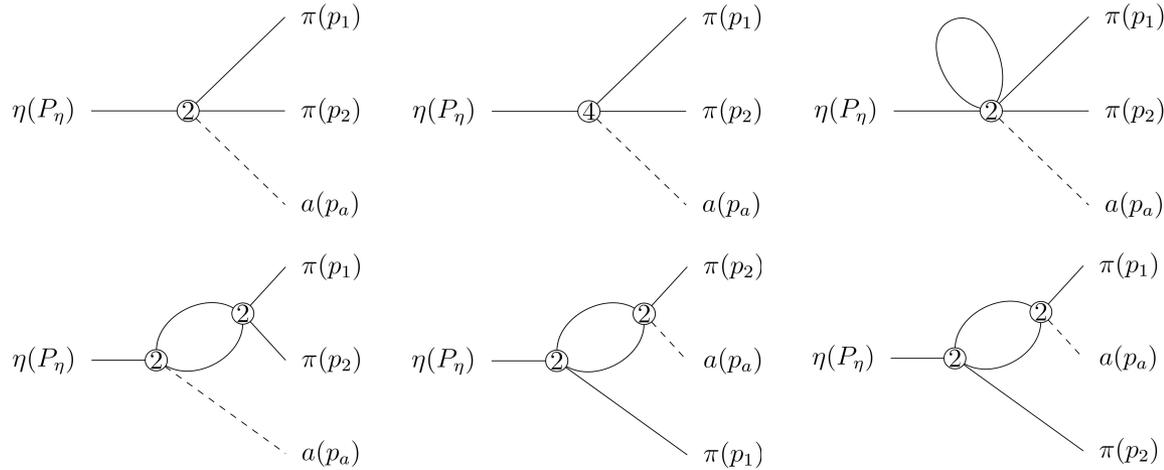
Note: we consider the octet part (\bar{Q}_a) of Q_a in SU(3) χ PT

NLO χ PT Lagrangian

$$\mathcal{L}_4 = L_1 \langle \partial_\mu U \partial^\mu U^\dagger \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \dots + \frac{\partial_\mu a}{2f_a} J_A^\mu \Big|_{\text{NLO}},$$

$$J_A^\mu \Big|_{\text{NLO}} = -4iL_1 \langle \bar{Q}_a \{ U^\dagger, \partial^\mu U \} \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \dots$$

Feynman diagrams up to NLO



Parameters

Masses and F_π [MeV]				LECs $L_i^r(\mu)$ at $\mu = 770$ MeV (in unit of 10^{-3})							
m_π	m_K	m_η	F_π	L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
137	496	548	92.1	1.0(1)	1.6(2)	-3.8(3)	0.0(3)	1.2(1)	0.0(4)	-0.3(2)	0.5(2)

[J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. 64, 149 (2014)]

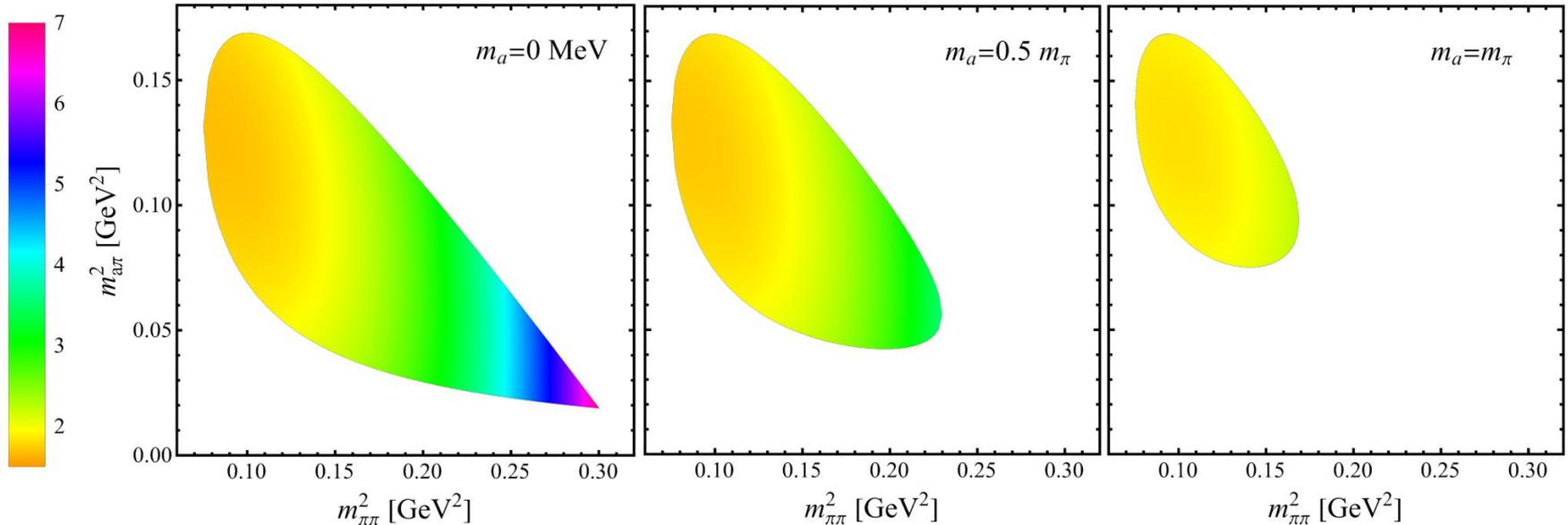
✓ Renormalization condition is verified to be consistent with conventional ChPT.

Observations:

- Strong isospin breaking effects enter the $\eta \rightarrow \pi\pi a$ amplitudes at the order of $(m_u - m_d)^2$
- In the isospin limit ($m_u = m_d$), the amplitudes with $\pi^+\pi^-$ and $\pi^0\pi^0$ in $\eta \rightarrow \pi\pi a$ processes are identical.

● Dalitz plots

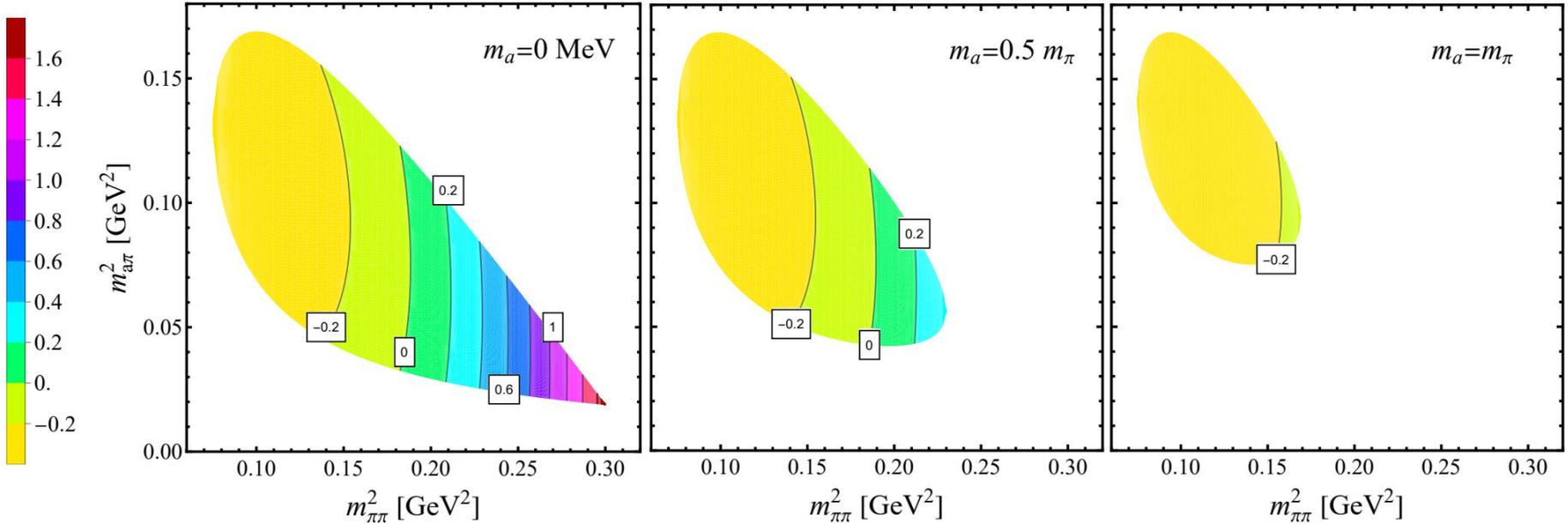
$$10^6 f_a^2 \frac{d^2\Gamma_{\eta \rightarrow \pi^+\pi^- a}}{ds dt} = \frac{10^6 f_a^2}{32(2\pi)^3 m_\eta^3} \left(|\mathcal{M}_{\eta;\pi\pi a}^{(2)}|^2 + 2\mathcal{M}_{\eta;\pi\pi a}^{(2)} \text{Re}(\mathcal{M}_{\eta;\pi\pi a}^{(4)}) + |\mathcal{M}_{\eta;\pi\pi a}^{(4)}|^2 \right) \quad \text{in unit of GeV}^{-1}$$



[Wang,ZHG,Lu,Zhou, 2403.16064]

● **Dalitz plots to show the NLO/LO convergence**

$$\left(2\mathcal{M}_{\eta;\pi\pi a}^{(2)} \text{Re} \left(\mathcal{M}_{\eta;\pi\pi a}^{(4)} \right) + \left| \mathcal{M}_{\eta;\pi\pi a}^{(4)} \right|^2 \right) / \left| \mathcal{M}_{\eta;\pi\pi a}^{(2)} \right|^2$$



Important lessons:

- **Non-perturbative effect in the $\pi\pi$ subsystem can be important.**
- **Perturbative treatment of the $a\pi$ subsystem is justified.**

- **Unitarization of the partial-wave $\eta \rightarrow \pi\pi a$ amplitude**

$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni}}(s) = \frac{\mathcal{M}_{\eta;\pi\pi a}^{00,\text{L}}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)},$$

$$G_{\pi\pi}(s) = -\frac{1}{(4\pi)^2} \left(\log \frac{m_\pi^2}{\mu^2} - \sigma_\pi(s) \log \frac{\sigma_\pi(s) - 1}{\sigma_\pi(s) + 1} - 1 \right),$$

$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{L}}(s) = \mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s) + \mathcal{M}_{\eta;\pi\pi a}^{00,(4)}(s) - G_{\pi\pi}(s) \mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s) T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s).$$

The unitarized amplitude satisfies the relation

$$\text{Im} \mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni}}(s) = \rho_{\pi\pi}(s) \mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni}}(s) \left(T_{\pi\pi \rightarrow \pi\pi}^{00,\text{Uni}}(s) \right)^*, \quad (2m_\pi < \sqrt{s} < 2m_K)$$

with the unitarized PW $\pi\pi$ amplitude $T_{\pi\pi \rightarrow \pi\pi}^{00,\text{Uni}}(s) = \frac{T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)}$

- **Unitarized PW amplitude based on LO $\eta \rightarrow \pi\pi a$ amplitude**

$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni-LO}}(s) = \frac{\mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)}.$$

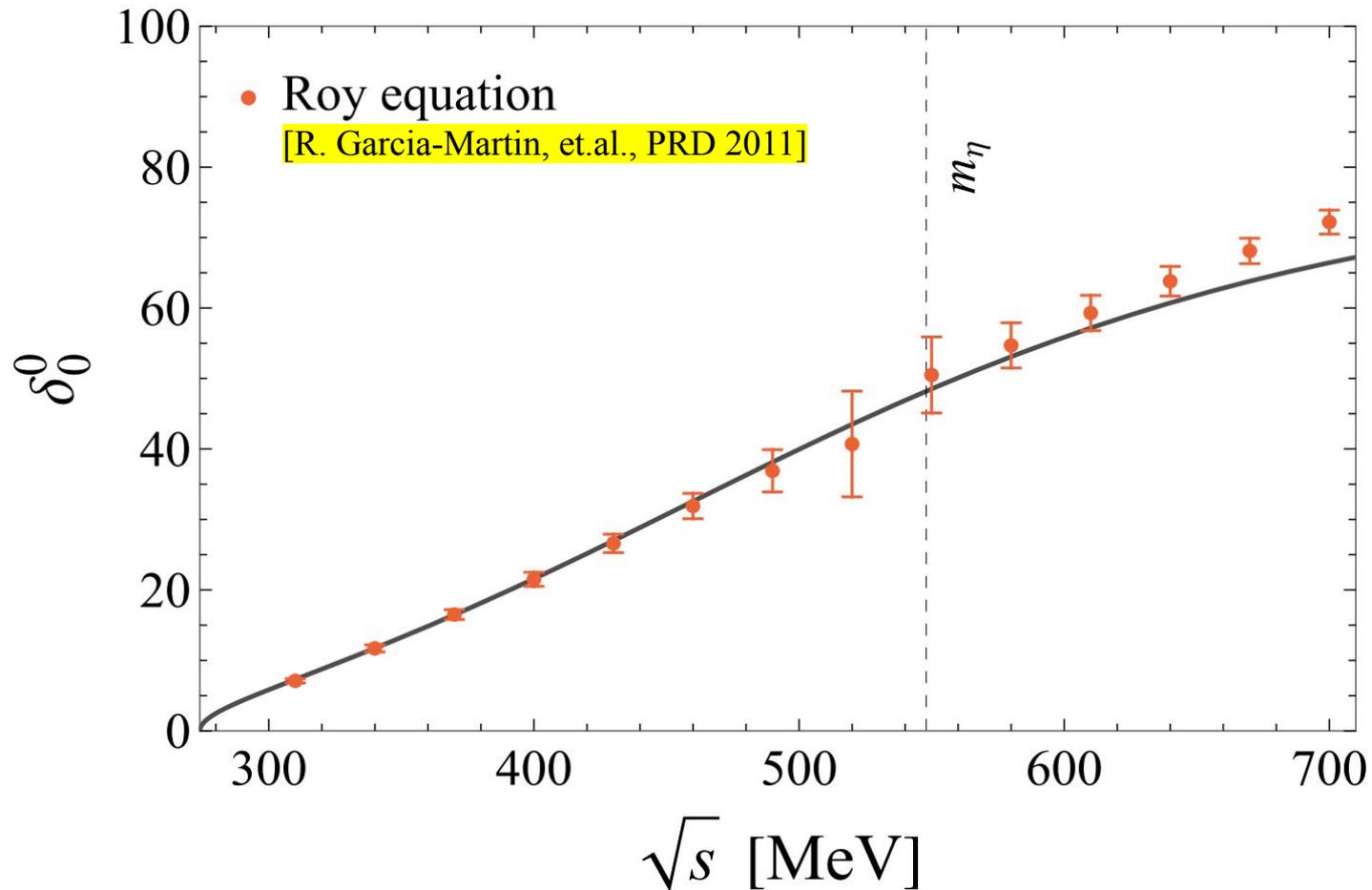
Resemble the method:

**Alves and Sergi,
arXiv:2402.02993 [hep-ph].**

$$M_0(s) = P(s)\Omega_0^0(s)$$

Phase shifts from the unitarized PW $\pi\pi$ amplitude

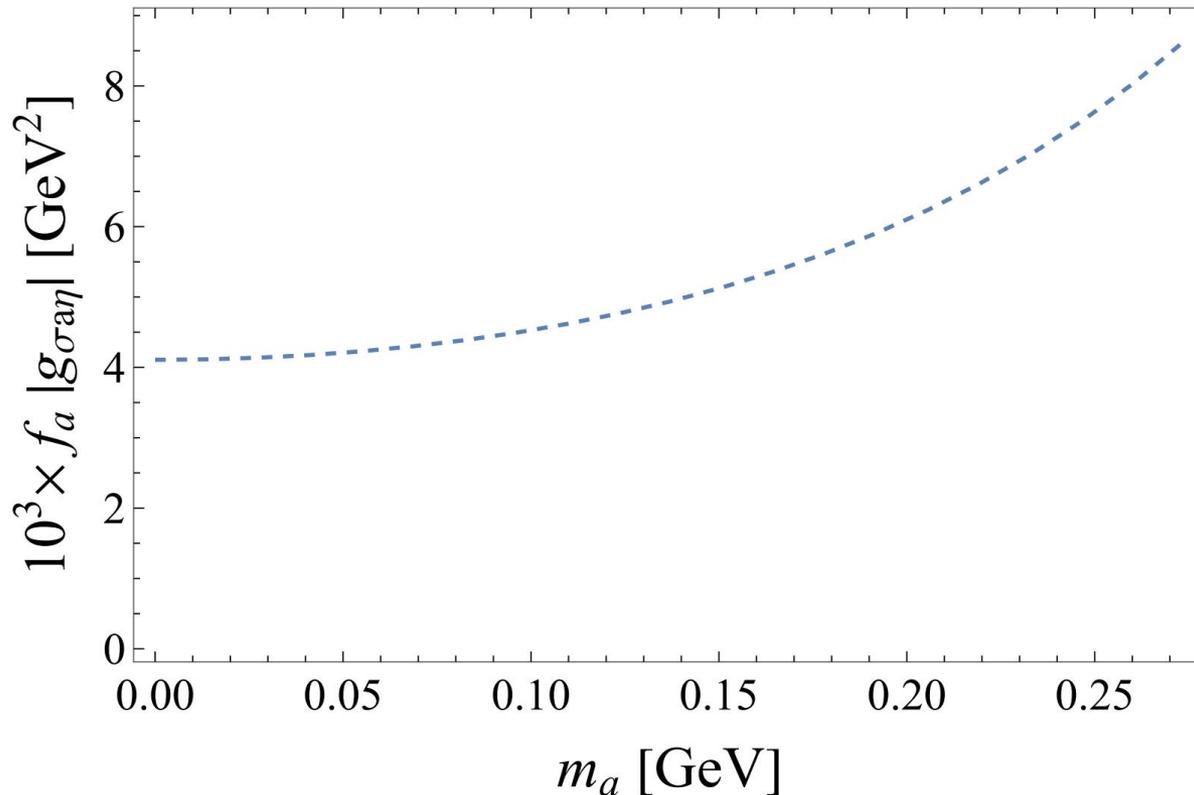
$$T_{\pi\pi\rightarrow\pi\pi}^{00,\text{Uni}}(s) = \frac{T_{\pi\pi\rightarrow\pi\pi}^{00,(2)}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi\rightarrow\pi\pi}^{00,(2)}(s)}$$



- Pole position of $f_0(500)/\sigma$ on the second Riemann sheet $\sqrt{s_\sigma} = 457 \pm i251$ MeV

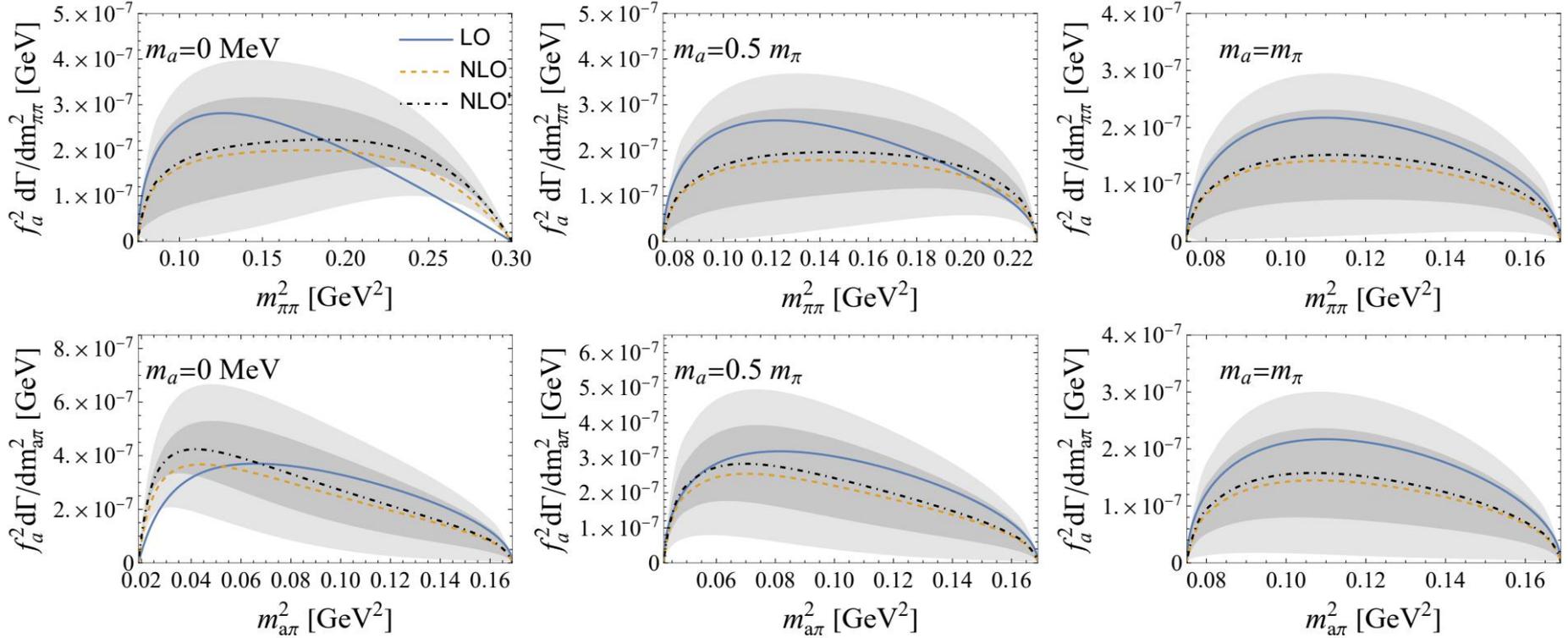
- σ - $a\eta$ coupling varying with m_a

$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni,II}}(s) \Big|_{s \rightarrow s_\sigma} \sim -\frac{g_{\sigma\pi\pi}g_{\sigma a\eta}}{s - s_\sigma}$$



Predictions of the $\pi\pi$ and $a\pi$ invariant-mass distributions at different axion masses

$$|\mathcal{M}|^2 = \left(\mathcal{M}^{(2)}\right)^2 + 2\mathcal{M}^{(2)}\text{Re}\left(\mathcal{M}^{(4)}\right) + |\mathcal{M}^{(4)}|^2$$



Uncertainty bands:

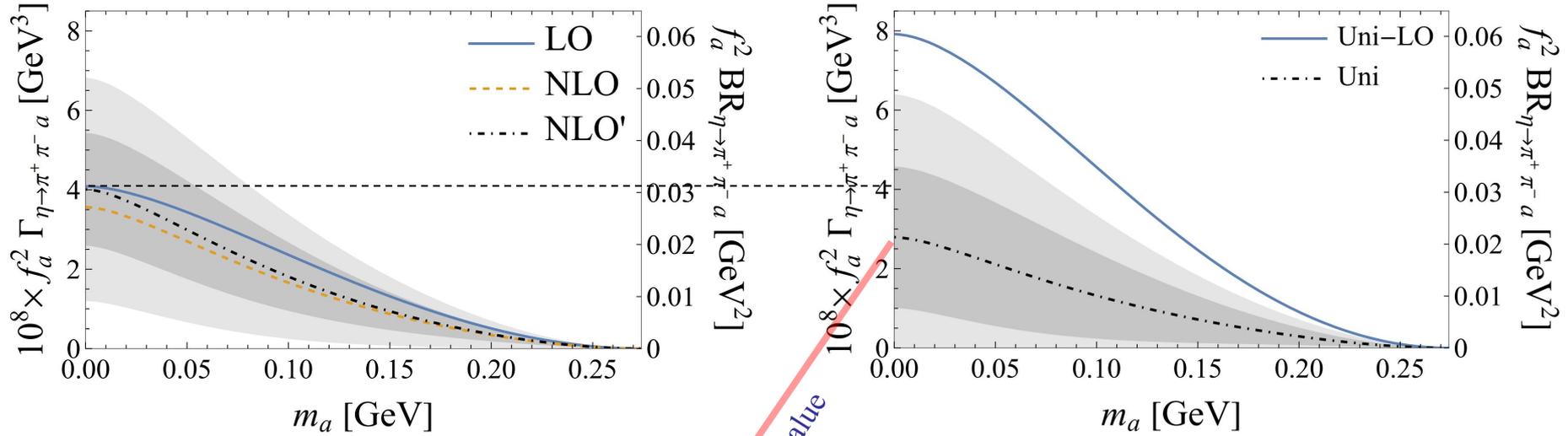
➤ **Lighter regions:**

	L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
	1.0(1)	1.6(2)	-3.8(3)	0.0(3)	1.2(1)	0.0(4)	-0.3(2)	0.5(2)

➤ **Darker regions:** freeze the $1/N_c$ suppressed ones (L_4, L_6, L_7)

Predictions of the $\eta \rightarrow \pi\pi a$ branching ratios by varying m_a

[Wang,ZHG,Lu,Zhou, 2403.16064]



$$\text{BR}_{\eta \rightarrow \pi^+ \pi^- a} \Big|_{m_a \rightarrow 0} = 2.1 \times 10^{-2} \left(\frac{\text{GeV}^2}{f_a^2} \right)$$

Possible detection channels: $a \rightarrow \gamma\gamma$, $a \rightarrow e^+e^-$, $a \rightarrow \mu^+\mu^-$

Summary

- Chiral perturbation theory provides a systematical and useful framework to study the axion-meson interactions .
- π - η - η' - a mixing and $g_{a\gamma\gamma}$ are predicted in U(3) $\Lambda\chi$ PT by taking the various hadronic and lattice inputs.
- Thermal $a\pi \leftrightarrow \pi\pi$ amplitudes in SU(2) $\Lambda\chi$ PT are worked out. Thermal corrections to amplitudes cause around 10% shift of the axion parameters.
- Axion production from the $\eta \rightarrow \pi\pi a$ decay is calculated. Large uncertainties from higher-order LECs are found.

谢谢！