A new method to access heavy meson light-cone distribution amplitudes from first-principle

徐吉

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强子物理在线论坛

In collaboration with:

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Outline

- **Introduction to B-meson LCDA**
  1. Importance of B-meson LCDA
  2. Definition and properties of B-meson LCDA
- **Introduction to LaMET**
  1. Introduction of Larger Momentum Effective Theory (LaMET)
  2. Recent progress in the frame of LaMET
- **Accessing B-meson LCDA**
  1. Our previous attempts
  2. Two-step factorization to access LCDA
- **Proposals for future research**

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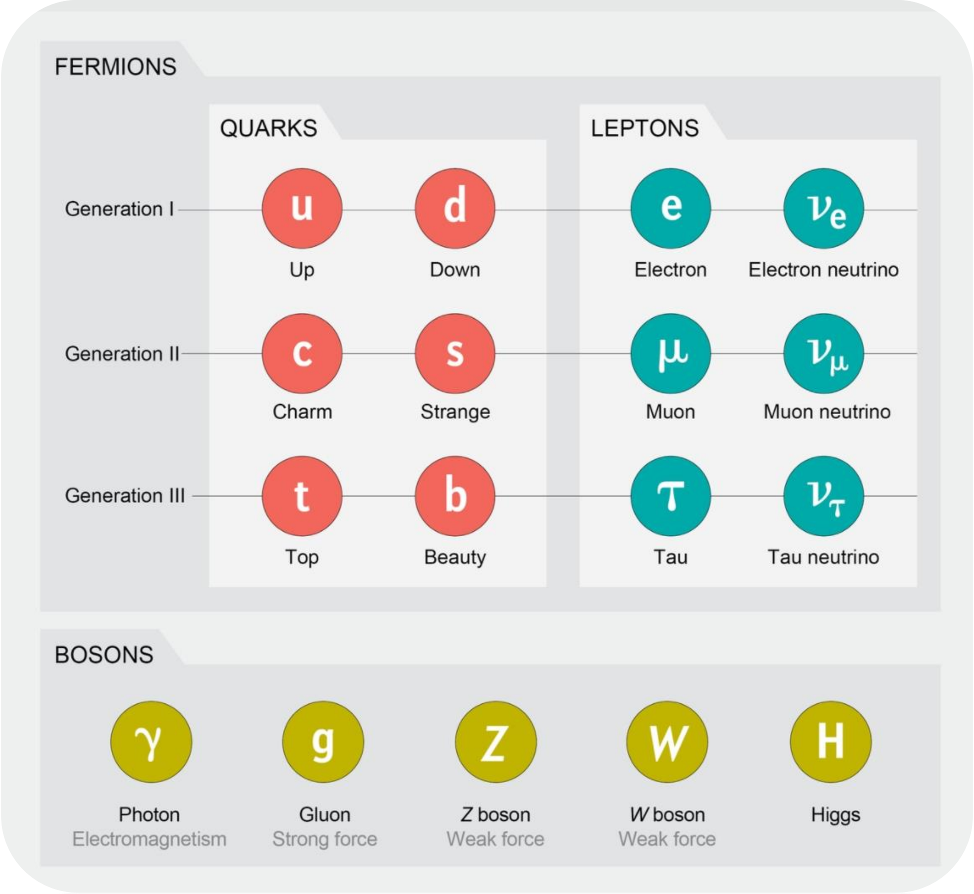
Why B-meson LCDA important?

➢ The study of flavor physics plays a very important role in the development of particle physics.



https://www.scientificamerican.com/article/beautiful-physics-the-search-for-new-particles-at-lhcb/

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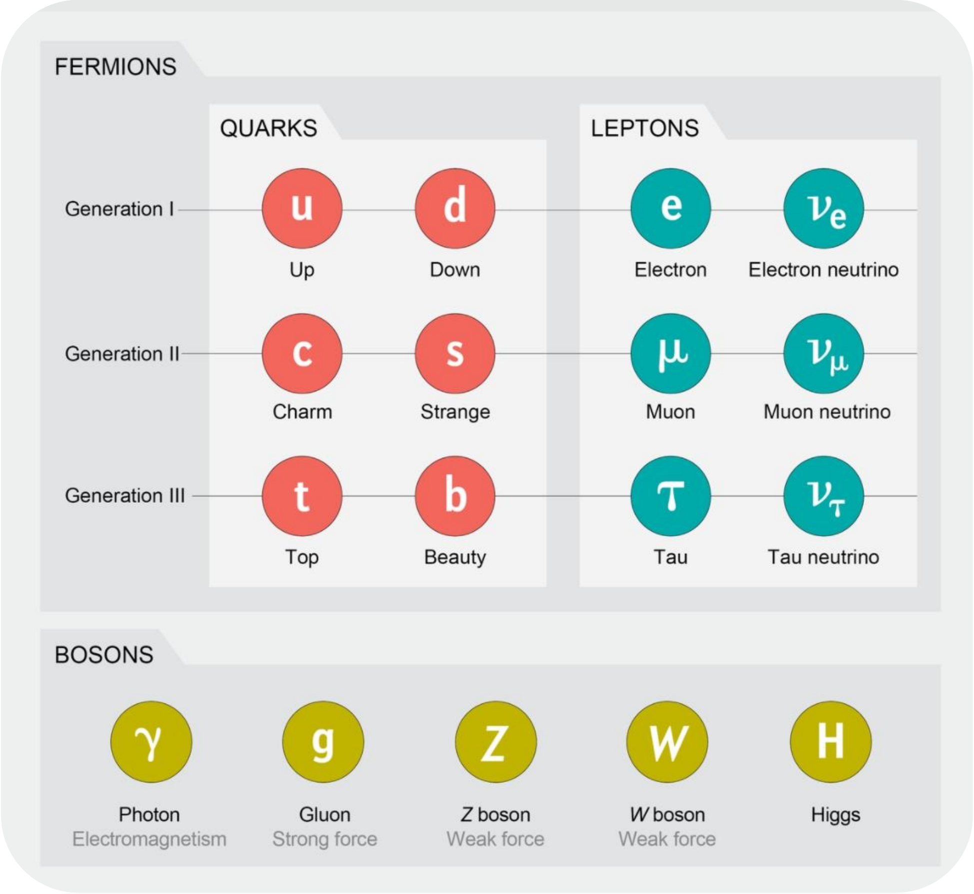

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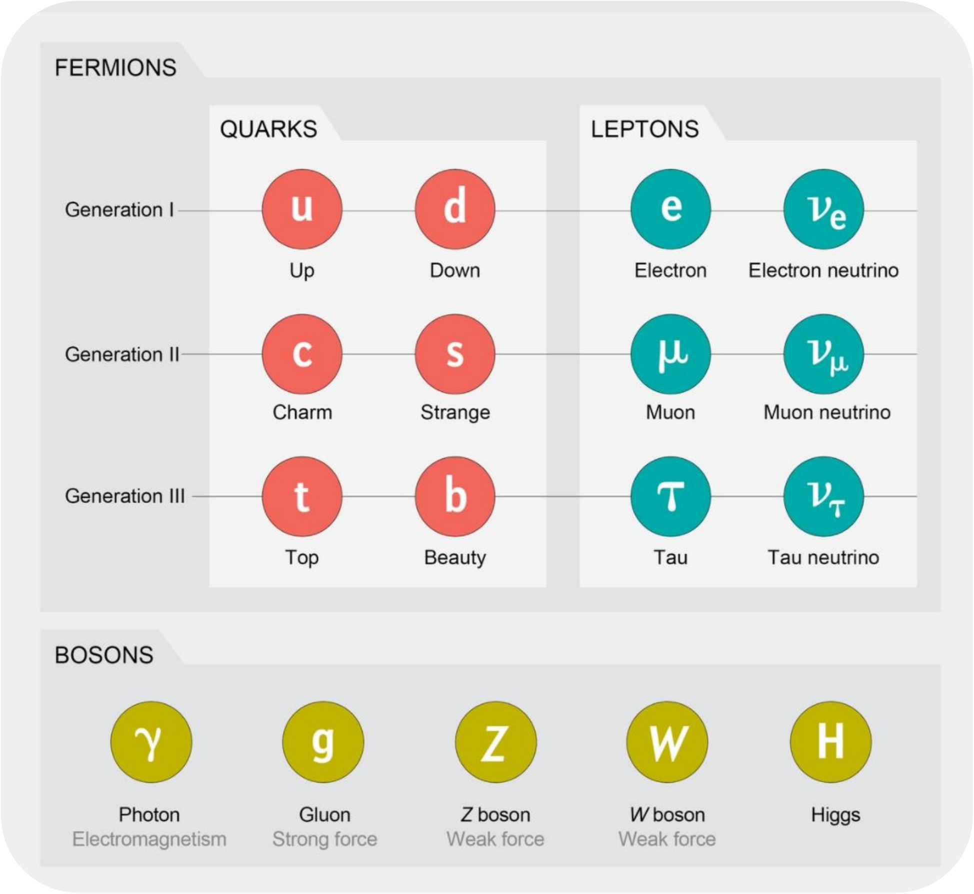

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- Heavy Flavor Physics: $b,\ c,\ \tau.$

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- Understanding the strong interaction dynamics of heavy quark decays.

  - ✓ $B \to \pi\,\pi$ Phys. Rev. Lett. 83, 1914 (1999) 1423 citations in INSPIRE
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"The hadronic matrix elements that enter B meson decays into two light mesons can be computed from first principles … and expressed in terms of form factors and meson light-cone distribution amplitudes."

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- Accurate measurement of standard model parameters: V_{ub} and V_{cb}

  - ✓ $B \to \pi\,\ell\,\nu$ Phys. Lett. B 633, 61 (2006) 215 citations in INSPIRE
  - ✓ $B \to D\,\ell\,\nu$ Phys. Rev. D 92, 054510 (2015) 387 citations in INSPIRE

  "The uncertainty in their prediction is dominated by the uncertainty in the light-cone distribution amplitudes (LCDAs) of the B- and π-mesons."

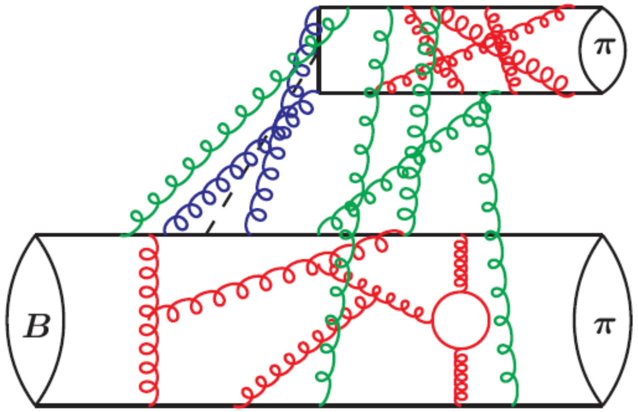
- Understanding the strong interaction dynamics of heavy quark decays.

$$\langle \pi(p')\pi(q)|Q_i|\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\varphi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

$B \to \pi$ form factor

Hard kernel

B-meson LCDA



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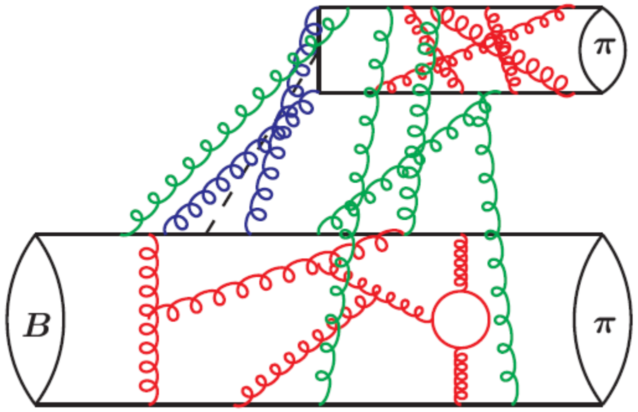
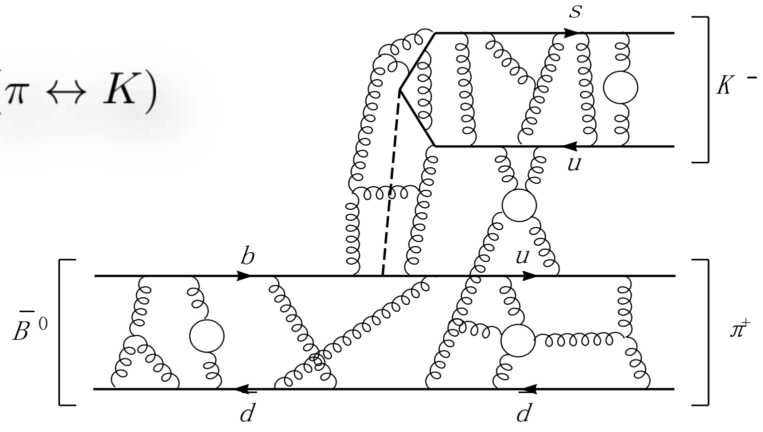


$$\mathcal{A}(\bar{B}^0 \to \pi^+ K^-) = \frac{G_F}{\sqrt{2}}\sum_{ij} V_{\rm CKM}\left(C_i^{\rm SM} + C_i^{\rm NP}\right)\Big[F_j^{B\to\pi}(m_K^2)\int_0^1 du\, T_{ij}^{\rm I}(u)\phi_K(u) + (\pi \leftrightarrow K) + \int_0^1 d\xi\, du\, dv\, T_i^{\rm II}(\xi,u,v)\varphi_B(\xi)\phi_\pi(v)\phi_K(u)\Big]$$

Hard kernel

B-meson LCDA



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- The light-ray HQET matrix element [Grozin, Neubert, 1997; Beneke, Feldmann, 2000]

$$\langle 0\,|\bar{q}_\beta(z)[z,0]h_{v\alpha}(0)|\,\bar{B}(v)\rangle = -\frac{i\tilde{f}_B m_B}{4}\left[\frac{1+\not{v}}{2}\left\{2\tilde{\varphi}_B^+(t,\mu) + \frac{\tilde{\varphi}_B^-(t,\mu)-\tilde{\varphi}_B^+(t,\mu)}{t}\not{z}\right\}\gamma_5\right]_{\alpha\beta} .$$

Leading twist Sub-leading twist

We assume that $z^2 = 0$, define $t = v.z$ and the path-ordered exponential

$$[z,0] = \mathrm{P}\exp\left(ig_s\int_{z_2}^{z_1} dz^\mu A_\mu(z)\right) .$$

The prefactor is chosen in such a way that for $z = 0$ one obtains

$$\langle 0|\bar{q}_\beta\,[\gamma^\mu\gamma_5]_{\beta\alpha}\,b_\alpha|\bar{B}(v)\rangle = if_B m_B v^\mu .$$

If $\tilde{\phi}_B^+(t=0) = \tilde{\phi}_B^-(t=0)$=1.

- The exponential models of B-meson LCDA in momentum space

$$\varphi_B^+(\omega) = \frac{\omega}{\omega_0^2} e^{-\frac{\omega}{\omega_0}} \,, \qquad \varphi_B^-(\omega) = \frac{1}{\omega_0} e^{-\frac{\omega}{\omega_0}} \,.$$

[Phys. Rev. D 55 (1997) 272-290] [Phys. Rev. D 69, 034014(2004)]

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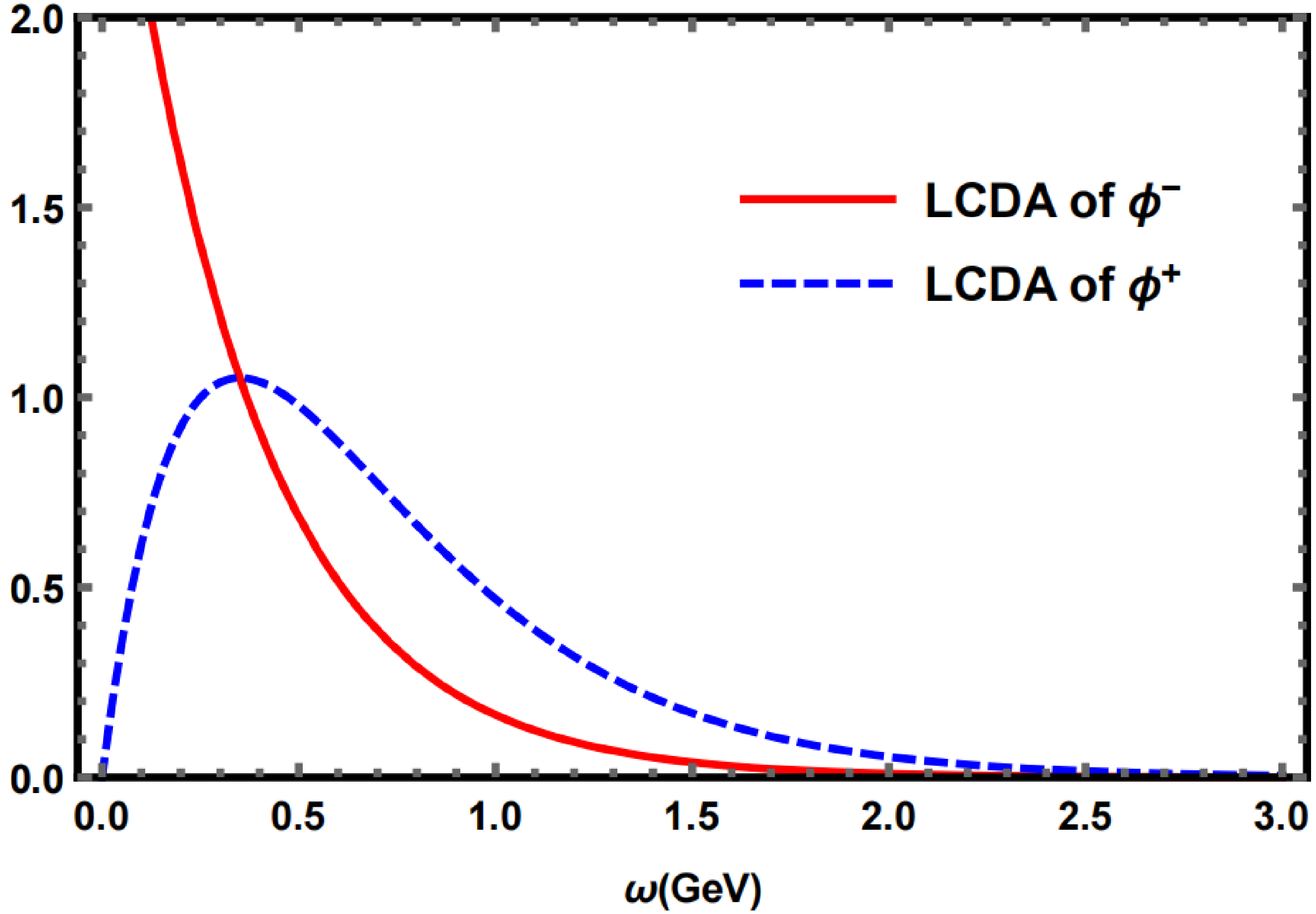

https://forsal.pl/biznes/energetyka/artykuly/8390201,maly-atom-duze-korzysci.html

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- Progress is mainly concentrated on the perturbative aspect.

- Evolution equations of $\tilde{\phi}_+^B$ and $\tilde{\phi}_-^B$ *[Lange, Neubert, 2003; Bell, Feldmann, 2008]*
- Solution of evolution equations. *[Bell, Feldmann, Yu-Ming Wang and Yip, 2013; Braun, Manashov, 2014]*
- RG equations of $\phi_B^+(\omega, \mu)$ at two-loops. *[Braun, Ji, Manashov, 2019; Liu, Neubert, 2020]*
- NNLO QCD correction to relevant hadronic B-meson decays. *[Bell, Beneke, Huber, Xin-qiang Li, 2020]*
- Non-leptonic two-body decays of B-meson. *[Ya-Dong Yang, Xin-qiang Li, 2005]*

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- The LCDAs encode the information about the probabilities of finding the light quark carrying certain momentum inside heavy meson.
- Progress is mainly concentrated on the perturbative aspect.
- The studies on the shape of B-meson LCDAs are quite model dependent.
- Lattice QCD provides a systematic ab initio calculations of the non-perturbative strong interactions.

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Difficulties

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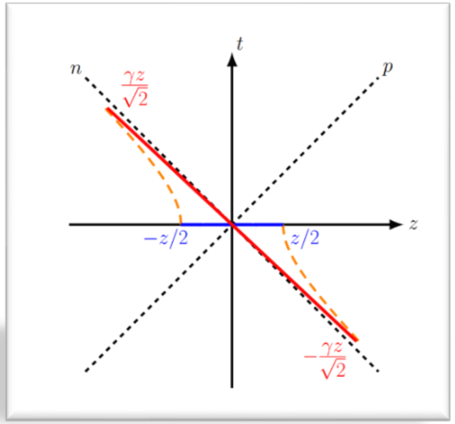

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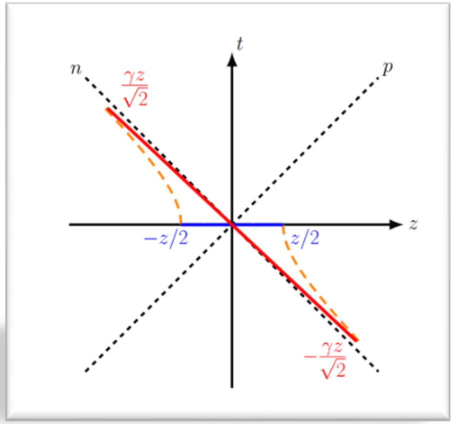

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- Non-negative moments $\int dk\, k^n \varphi_B^+(k)$ for $n = 0, 1, 2 \ldots$ are not related to matrix elements of local operators and in fact do not exist. [Phys.Rev.D 69, 034014 (2004)]

$$[\bar{q}(tn)\not{n}[tn,0]\Gamma h_v(0)]_R \neq \sum_{p=0}^{\infty} \frac{t^p}{p!} \left[\bar{q}(0)(\overleftarrow{D} \cdot n)^p h_v(0)\right]_R .$$

Cannot get φ_B^+ by their moments

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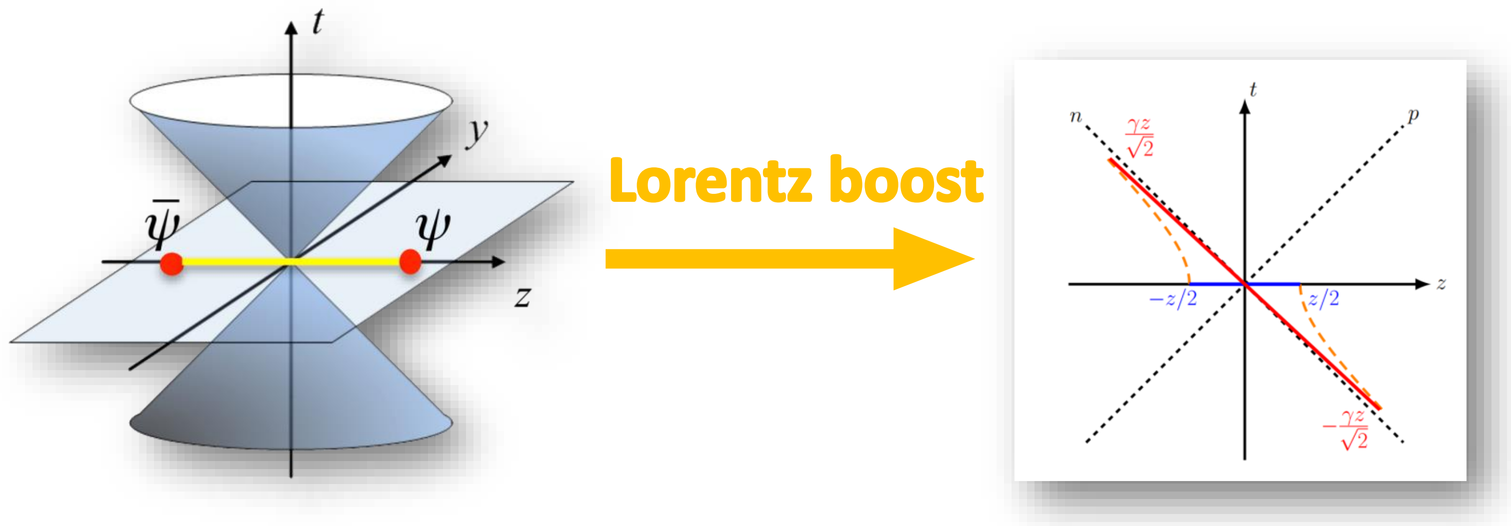
Large Momentum Effective Theory

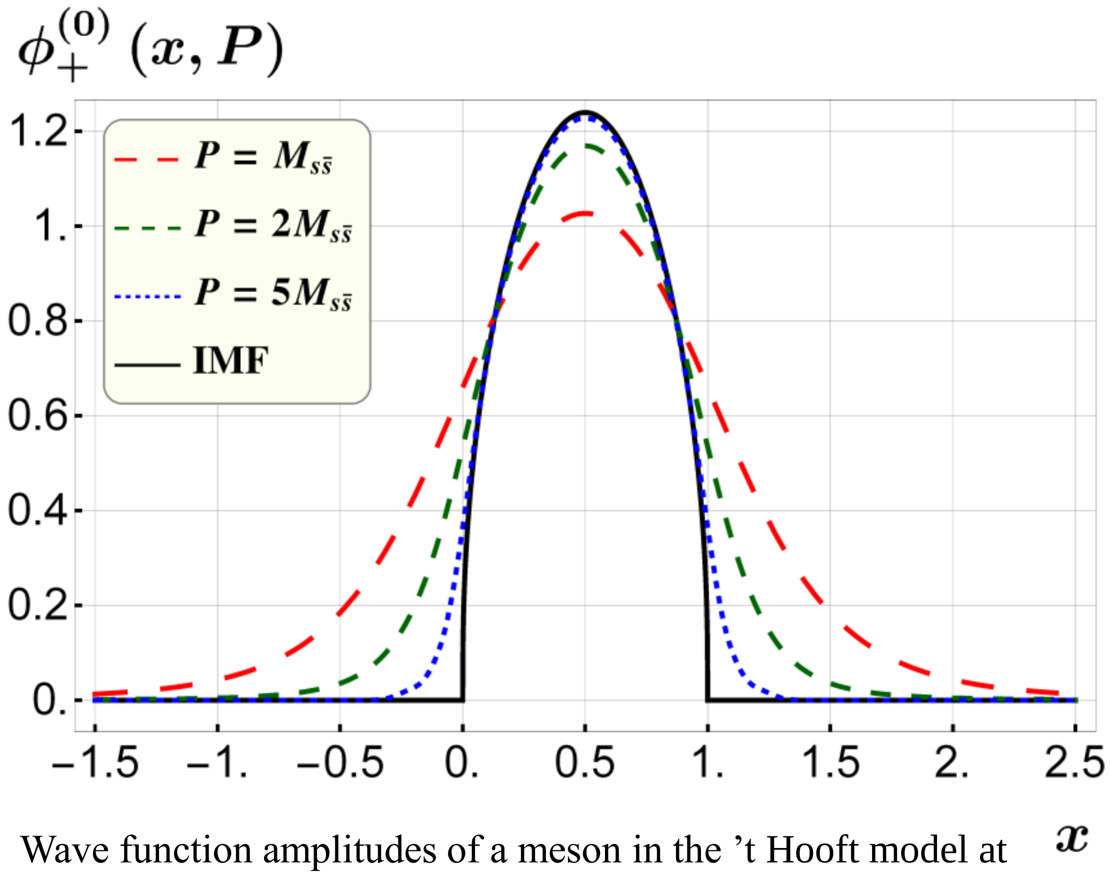
X. Ji, Phys. Rev. Lett. 110 (2013)
X. Ji, Sci.China Phys.Mech.Astron. 57 (2014)



- LaMET: light-cone can be accessed by simulating correlation functions with a large but finite P_z.
- With this approach, parton physics can be extracted from the physical properties of the proton at a moderately-large momentum.

Thus, the theory has been named as **large-momentum effective theory (LaMET)**

- It is generally expected that the large momentum limit of the proton state exists and is smooth, and some small parameters such as Λ/P^z control the limiting process. This is true in certain simple QFT models such as 't Hooft model. [Ji, Liu, Liu, Zhang, Zhao, 2021]



Wave function amplitudes of a meson in the 't Hooft model at different external momenta. [Jia, Liang, Li, Xiong, 2017]

- **However, in QFTs, UV divergences bring in complications.**
  - ✓ The physically relevant one is clearly $\Lambda_{UV} \gg P \to \infty$, as discussed in the previous subsection.
  - ✓ Historically, It was found that taking $P \to \infty$ by ignoring the UV divergences considerably simplifies the perturbation theory rules.

- In asymptotically free theories such as QCD, differences (or discontinuities) in taking the limits of $P \gg \Lambda_{UV}$ and $\Lambda_{UV} \gg P \to \infty$ are perturbatively calculable, as only the high-momentum modes matter. The differences are called **matching coefficients.**

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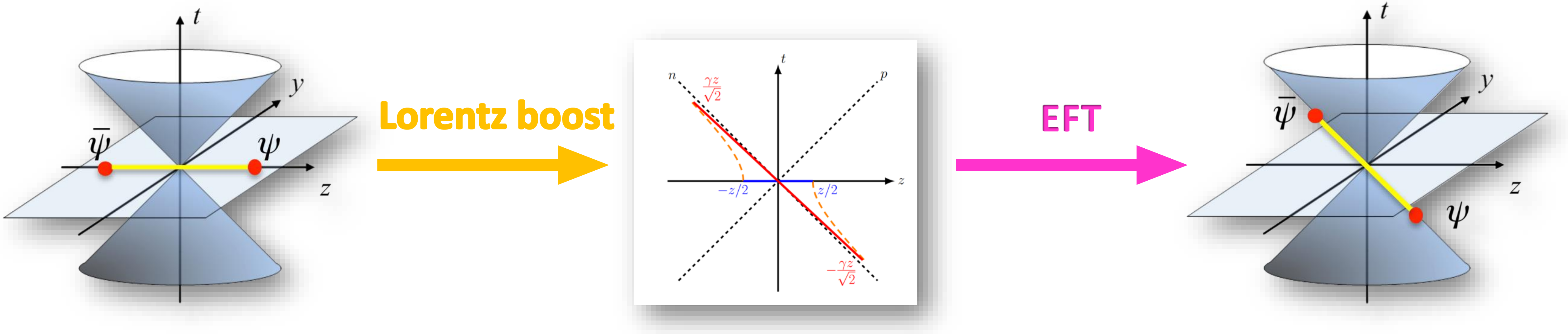

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Lattice calculation

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Lattice calculation $\xrightarrow[\text{LQCD}]{}$ **Quasi-quantity**

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- **Step2：** Extracting quasi-quantities.

- **Step3：** Extracting the light-cone physics from the lattice ME (Matching).

Lattice calculation —LQCD→ **Quasi-quantity** —Matching→ **LC-quantity**

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LaMET

X. Ji, Phys. Rev. Lett. 110 (2013) *660 citations in INSPIRE*

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Theoretical research

Quark PDF:

- ✓ X. Xiong, X. Ji, J. H. Zhang and Y. Zhao, Phys. Rev. D 90, 014051 (2014);
- ✓ L. B. Chen, W. Wang and R. Zhu, Phys. Rev. Lett. 126, 072002 (2021);

Gluon PDF:

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- ✓ J. H. Zhang et al. [LP3 Collaboration], Nucl. Phys. B 939, 429 (2019);

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Related theories

Pseudodistribution:

- ✓ A. V. Radyushkin, Phys. Rev. D 96, 034025 (2017);
- ✓ K. Orginos, A. V. Radyushkin, J. Karpie and S. Zafeiropoulos, Phys. Rev. D 96, 094503 (2017);
- ✓ S. Zhao and A. V. Radyushkin, Phys. Rev. D 103, 054022 (2021);
- ✓ I. Balitsky, W. Morris, A. V. Radyushkin, JHEP 02, 193 (2022);

Lattice cross sections:

- ✓ Y. Q. Ma and J. W. Qiu, Phys. Rev. Lett. 120, 022003 (2018);
- ✓ R. S. Sufian, J. Karpie, C. Egerer, K. Orginos and J. W. Qiu, Phys. Rev. D 99, 074507 (2019);
- ✓ Z. Y. Li, Y. Q. Ma and J. W. Qiu, Phys. Rev. Lett. 126, 072001 (2021);
- ✓ J. Bringewatt, N. Sato, W. Melnitchouk, J. W. Qiu and F. Steffens et al., Phys. Rev. D 103, 016003 (2021);

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- Our previous attempts

  [Phys.Rev.D 102, 011502 (2020)]

Quasi-DA in HQET

LCDA in HQET

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  [Phys.Rev.D 102, 011502 (2020)]

Quasi-DA in HQET

LCDA in HQET

$$\tilde{\varphi}_B^+(\xi,\mu) = \int_0^\infty d\omega\, H(\xi,\omega,n_z\cdot v,\mu)\,\varphi_B^+(\omega,\mu) + O\left(\frac{\Lambda_{\rm QCD}}{n_z\cdot v\,\xi}\right)$$

$$i\tilde{f}_B(\mu)\, m_B\, \tilde{\varphi}_B^+(\xi,\mu) = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi}\, e^{i\,n_z\cdot v\,\xi\,\tau} \langle 0|\, (\bar{q}_s\, W_c)\,(\tau\, n_z)\; \not{n}_z\, \gamma_5\; (W_c^\dagger\, h_v)\,(0)|\bar{B}(v)\rangle$$

- **Can we utilize the heavy meson quasi-DA in HQET to obtain LCDA in HQET ?**

- Our previous attempts

  [Phys.Rev.D 102, 011502 (2020)]

  [Phys.Rev.D 106, 114019 (2022)]

  [Phys.Rev.D 106, L011503 (2022)]

  [Phys.Rev.D 109, 034001 (2024)]

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Quasi-DA in HQET

LCDA in HQET

- We currently lack configurations under the HQET Lagrangian.
- Difficult to realize the boosted HQET field on lattice QCD.

Outline

- **Introduction to B-meson LCDA**
  1. Importance of B-meson LCDA
  2. Definition and properties of B-meson LCDA
- **Introduction to LaMET**
  1. Introduction of Larger Momentum Effective Theory (LaMET)
  2. Recent progress in the frame of LaMET
- **Accessing B-meson LCDA**
  1. Our previous attempts
  2. Two-step factorization to access LCDA
- **Proposals for future research**

➢ Two-step factorization to access heavy meson LCDA

Quasi-DA in QCD

LCDA in HQET

LCDA in QCD

- From quasi-DA in QCD to LCDA in QCD:

[Phys.Rev.D 97, 114026 (2018)]
[Phys.Rev.D 99, 094036 (2019)]
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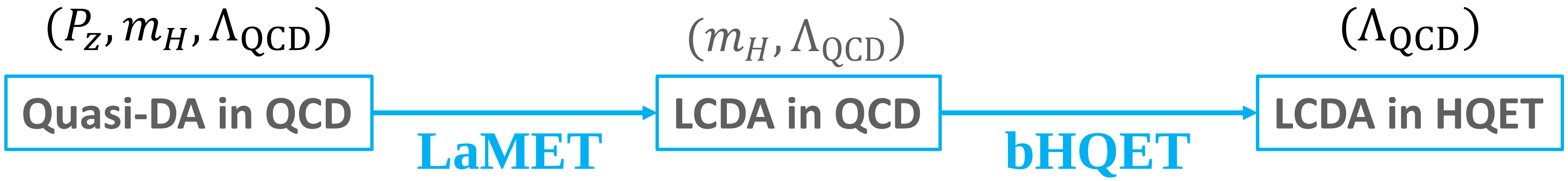
➢ Two-step factorization to access heavy meson LCDA

➢ A multi-scale process:

- LaMET requires: $\Lambda_{\rm QCD}, m_H \ll P_z$ and integrate out P_z;
- bHQET requires: $\Lambda_{\rm QCD} \ll m_H$ and finally integrate out m_H;

$\Rightarrow$ Hierarchy $\Lambda_{\rm QCD} \ll m_H \ll P_z$.


$(P_z, m_H, \Lambda_{\text{QCD}})$
Quasi-DA in QCD
LaMET
$(m_H, \Lambda_{\text{QCD}})$
LCDA in QCD
bHQET
(Λ_{QCD})
LCDA in HQET

$(P_z, m_H, \Lambda_{\rm QCD})$ → **Quasi-DA in QCD** —**LaMET**→ $(m_H, \Lambda_{\rm QCD})$ **LCDA in QCD** —**bHQET**→ $(\Lambda_{\rm QCD})$ **LCDA in HQET**

$$\tilde{\phi}(x, P^z; m_H) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu; m_H) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right),$$

[Phys.Rev.D 99, 094036 (2019)]

[arXiv:2403.17492]

$(P_z, m_H, \Lambda_{\text{QCD}})$ **Quasi-DA in QCD** → **LaMET** → $(m_H, \Lambda_{\text{QCD}})$ **LCDA in QCD** → **bHQET** → (Λ_{QCD}) **LCDA in HQET**

$$\tilde{\phi}(x, P^z; m_H) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu; m_H) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{(xP^z, \bar{x}P^z)^2}\right),$$

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$$\phi(y, \mu; m_H) = \frac{\tilde{f}_H}{f_H} J_{\text{peak}} m_H \varphi^+(\omega, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_H}\right),$$

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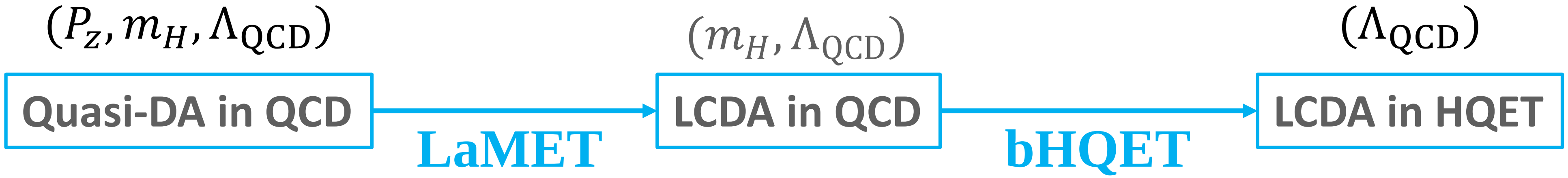
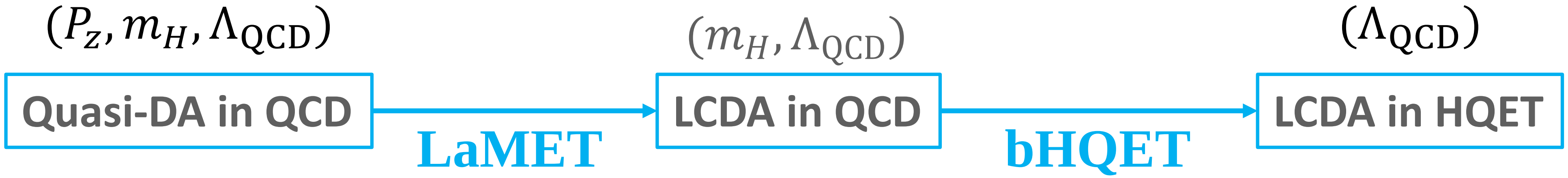


⇒ **Hierarchy $\boldsymbol{\Lambda_{QCD} \ll m_H \ll P_z}$:** A big challenge for lattice simulation.



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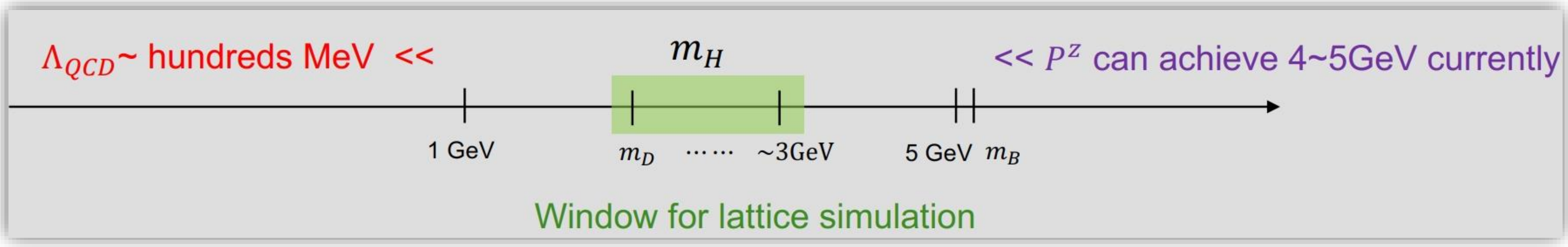

- Under current conditions, the heavy meson can be a D, but cannot be a B meson.

➢ A fine CLQCD ensemble for the lattice QCD verification

- ✓ H48P32: $n_s^3 \times n_t = 48^3 \times 144$, $a = 0.05187$fm.
- ✓ Coulomb gauge fixed grid source with grid $= 1 \times 1 \times n_s$; 549 configurations × 8 measurements;
- ✓ $m_\pi \simeq 317$ MeV, $m_{\eta_s} \simeq 700$ MeV;
- ✓ Determine the charm quark mass by tuning $m_{J/\psi}$ to its physical value, then $m_D \simeq 1.90$ GeV;
- ✓ Boost momenta $P^z = \{2.99,\ 3.49,\ 3.98\}$ GeV, spatial separation $z = 0\sim12a$.

➢ **The quasi-DA defined as**

$$\tilde{\phi}(x, P^z; m_H) = \int \frac{dz}{2\pi} e^{-ixP^z z} \frac{\tilde{M}^0(z, P^z; \gamma^z\gamma_5, m_H)}{\tilde{M}^0(z, 0; \gamma^t\gamma_5, m_H)}$$

$$\tilde{M}^0(z, P^z; \Gamma, m_H) = \frac{\langle 0 | \bar{q}(z)\Gamma W_c(z,0) Q(0) | H(P^z)\rangle}{\langle 0 | \bar{q}(0)\Gamma Q(0) | H(P^z)\rangle}$$

$\tilde{M}^0(z, 0; \gamma^t\gamma^5, m_H)$ in denominator is used to renormalize the bare matrix element $\tilde{M}^0(z, P^z; \gamma^z\gamma^5, m_H)$.

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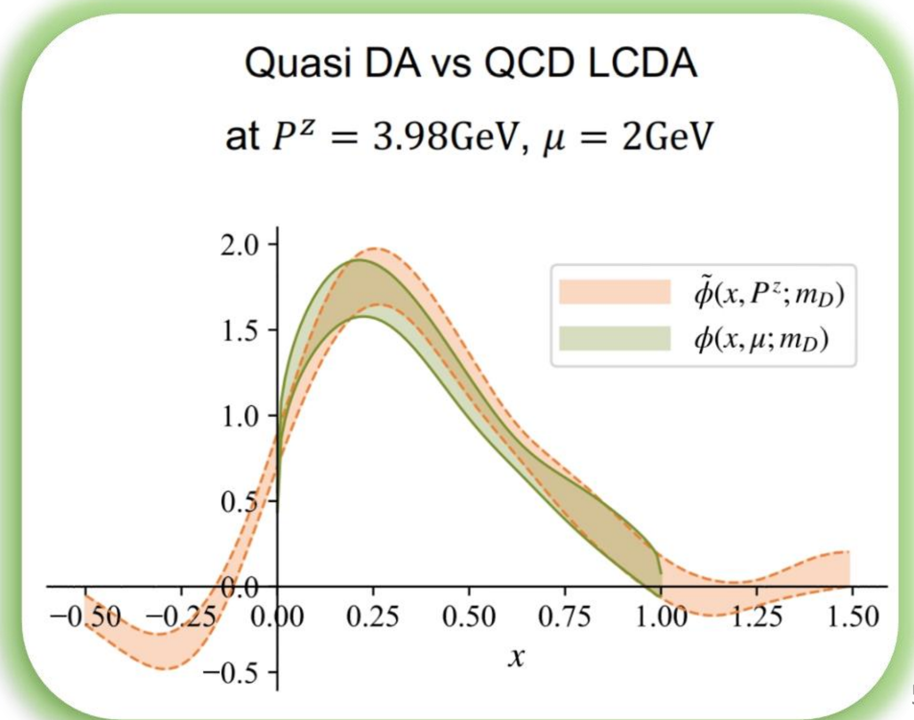
- Previously mentioned matching formula I

$$\tilde{\phi}(x, P^z; m_H) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu; m_H) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

STEP MATCHING I



- **The LCDA in QCD defined as**

$$\phi(y,\mu;m_H) = \frac{1}{if_H}\int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}\,e^{iyP_H\tau n_+}\times\langle 0\,|\bar{q}(\tau n_+)\not{n}_+\gamma_5 W_c(\tau n_+,0)Q(0)|\,H(P_H)\rangle$$

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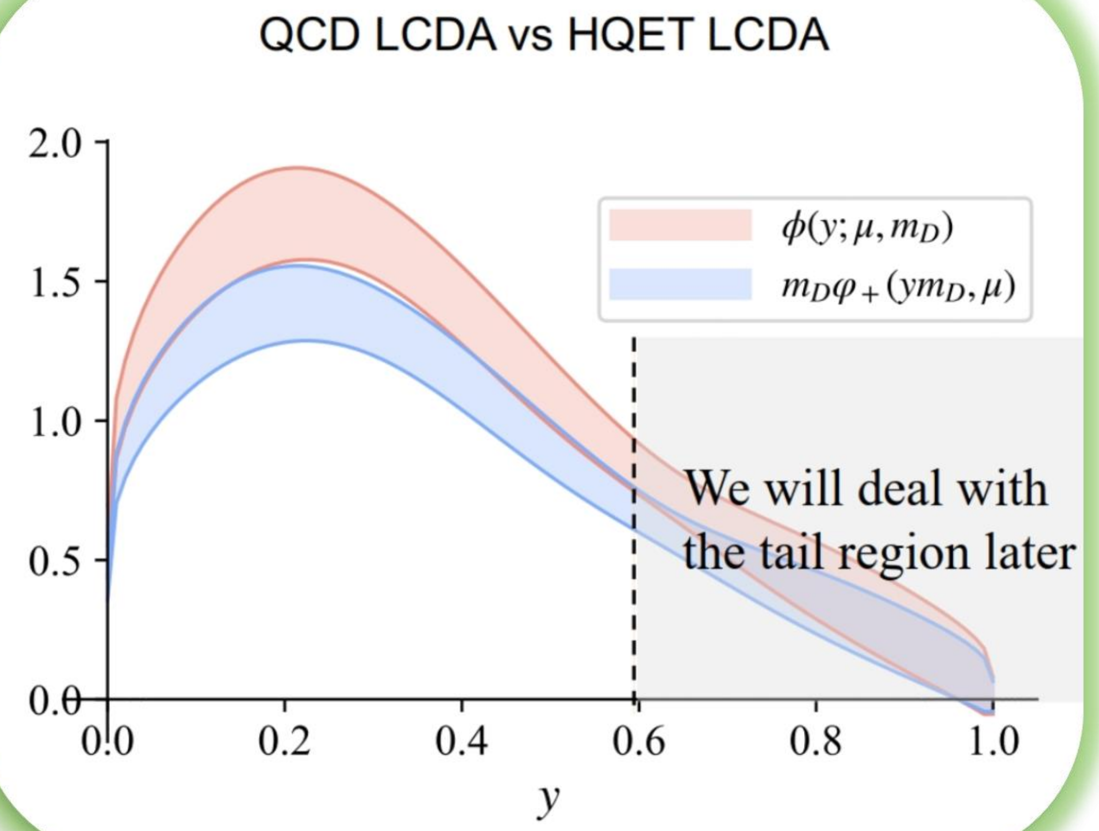
- Previously mentioned matching formula II

$$\phi(y,\mu;m_H) = \frac{\tilde{f}_H}{f_H} J_{\rm peak} m_H \varphi^+(\omega,\mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_H}\right)$$

STEP MATCHING II



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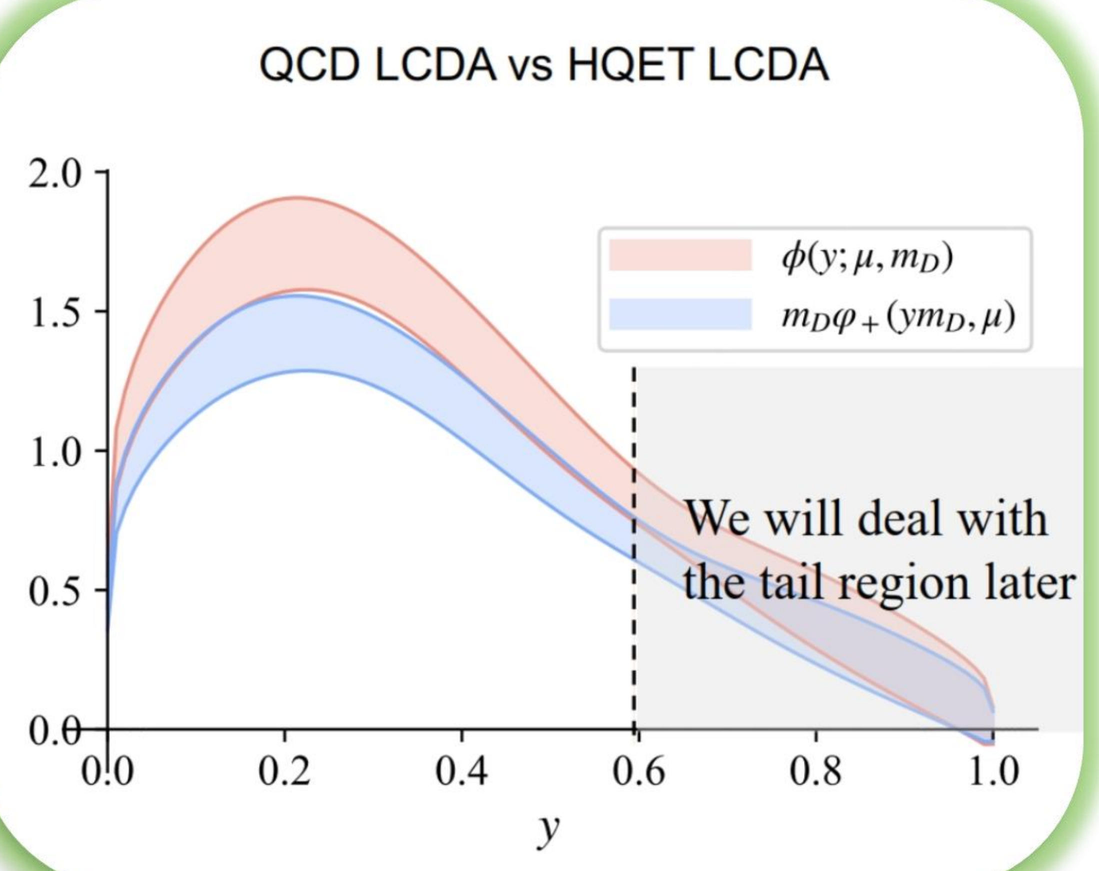


STEP MATCHING II

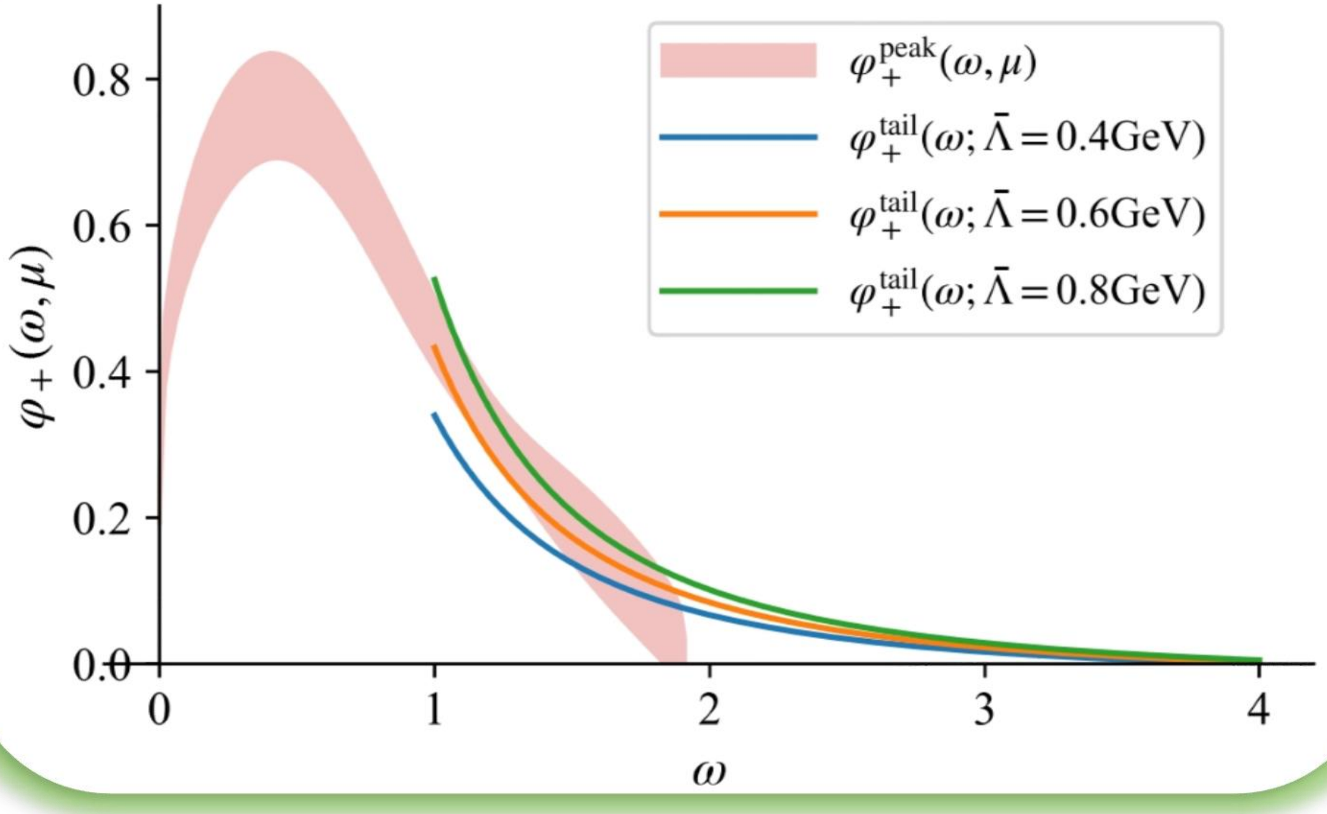
- **Peak region: y $\simeq \Lambda_{QCD}/m_H$**
- **Tail region: y $\simeq$ 1**

➢ The tail region of HQET LCDA is perturbatively calculable [Phys·Rev·D 72, 094082 (2005)]

$$\varphi^{+}_{\text{tail}}(\omega,\mu) = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right],$$

where $\bar{\Lambda} \equiv m_H - m_H^{\text{pole}}$ denotes the effective light quark mass. In the figure below, we show this result with $\bar{\Lambda} = \{0.4,\ 0.6,\ 0.8\}$ GeV.

The final result of HQET LCDA will merge the peak (from LQCD) and tail region (from 1-loop calculation).



➢ Compare with several phenomenological models

[Nucl.Phys.B 898, 563-604 (2015)]; [JHEP 07, 154 (2018)]; [JHEP 05, 024 (2022)].

$$\varphi_{\mathrm{I}}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}\,,$$

$$\varphi_{\mathrm{II}}^{+}(\omega,\mu_0)=\frac{4}{\pi\omega_0}\frac{k}{k^2+1}\left[\frac{1}{k^2+1}-\frac{2\left(\sigma_B^{(1)}-1\right)}{\pi^2}\ln k\right]\,,$$

$$\varphi_{\mathrm{III}}^{+}(\omega,\mu_0)=\frac{2\omega^2}{\omega_0\omega_1^2}e^{-(\omega/\omega_1)^2}\,,$$

$$\varphi_{\mathrm{IV}}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0\omega_2}\frac{\omega_2-\omega}{\sqrt{\omega\left(2\omega_2-\omega\right)}}\theta\left(\omega_2-\omega\right)\,,$$

$$\varphi_{\mathrm{V}}^{+}(\omega,\mu_0)=\frac{\Gamma(\beta)}{\Gamma(\alpha)}\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}U(\beta-\alpha,3-\alpha,\omega/\omega_0)\,,$$

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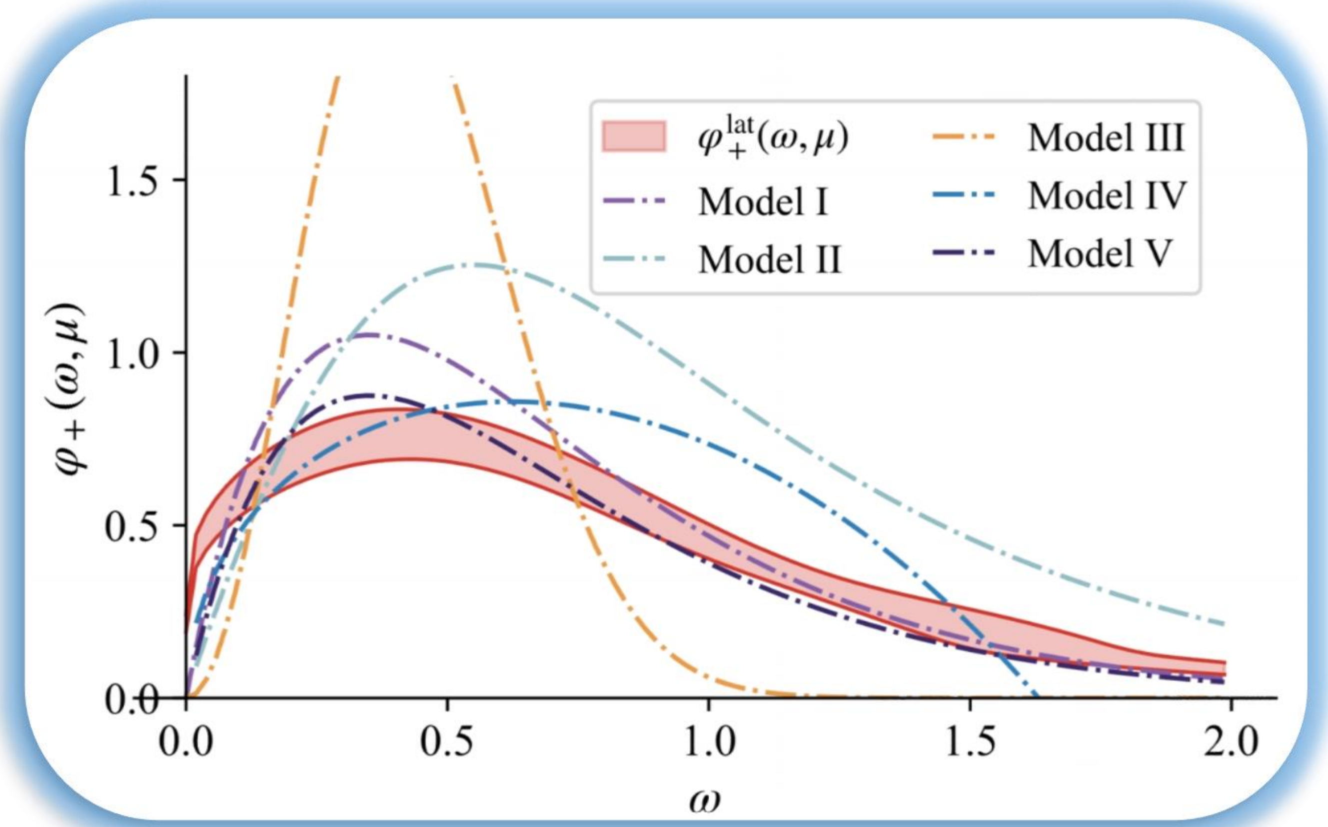

The final result of HQET LCDA

First inverse moment (IM)

$$\lambda_B^{-1}(\mu) \equiv \int_{-\infty}^{\infty} d\omega \frac{\phi_B^+(\omega,\mu)}{\omega}.$$

Models	I	II	III	IV	V
Parameters	$\omega_0 = 0.433(23)\mathrm{GeV}$	$\omega_0 = 0.682(45)\mathrm{GeV}$	——	$\omega_0 = 0.427(21)\mathrm{GeV}$	$\omega_0 = 0.449(42)\mathrm{GeV}$
		$\sigma_B^{(1)} = 2.78(48)$			
fit range	$\omega \in [0.2, 1.4]\mathrm{GeV}$	$\omega \in [0.2, 1.4]\mathrm{GeV}$		$\omega \in [0.4, 0.8]\mathrm{GeV}$	$\omega \in [0.2, 1.4]\mathrm{GeV}$
χ^2/d.o.f	1.4	1.2		2.1	1.0

- ✓ The first inverse moment is a crucial quantity in light-cone sum-rule studies and QCD factorization theorems in heavy flavor physics.
- ✓ The current numerical results are unable to accomplish the integration over full-ω range.
- ✓ We determine the IM by fitting the parameterization forms of different model.

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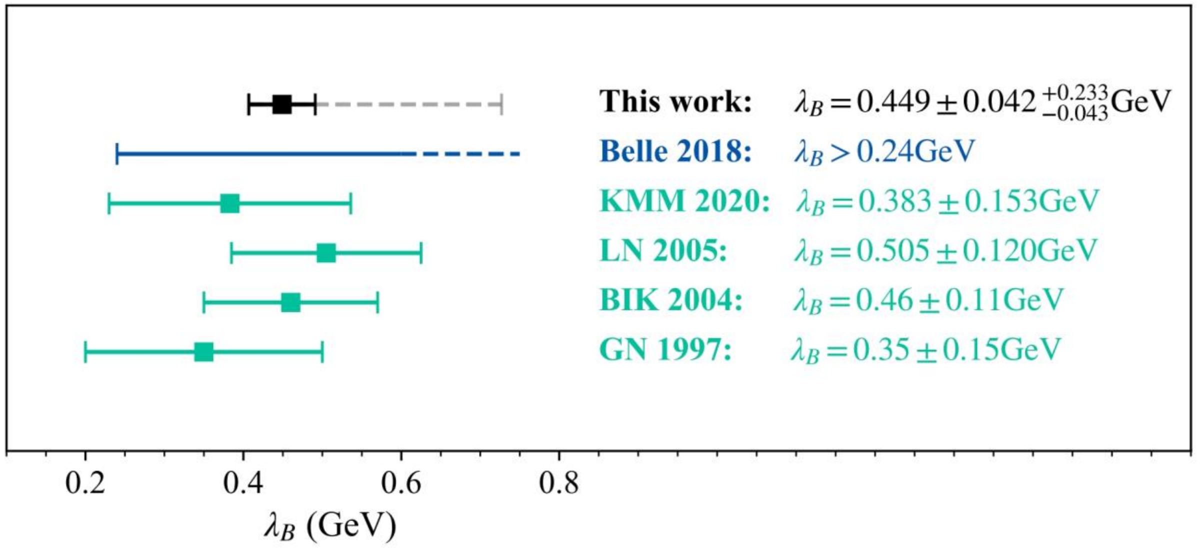

[arXiv: 2403.17492]
[Phys.Rev.D 98, 112016 (2018)]
[JHEP 10, 043 (2020)]
[Phys.Rev.D 72, 094082 (2005)]
[Phys.Rev.D 69, 034014 (2004)]
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Two-loop corrections to matching coefficient

“Two-loop evolution equation for the B-meson distribution amplitude” *[Braun, Ji, Manashov, 2019]*

- **Status for quark PDF**

L.-B. Chen, W.Wang and R.Zhu, Phys.Rev.Lett. 126 (2021)

Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, Phys.Rev.Lett. 126 (2021)

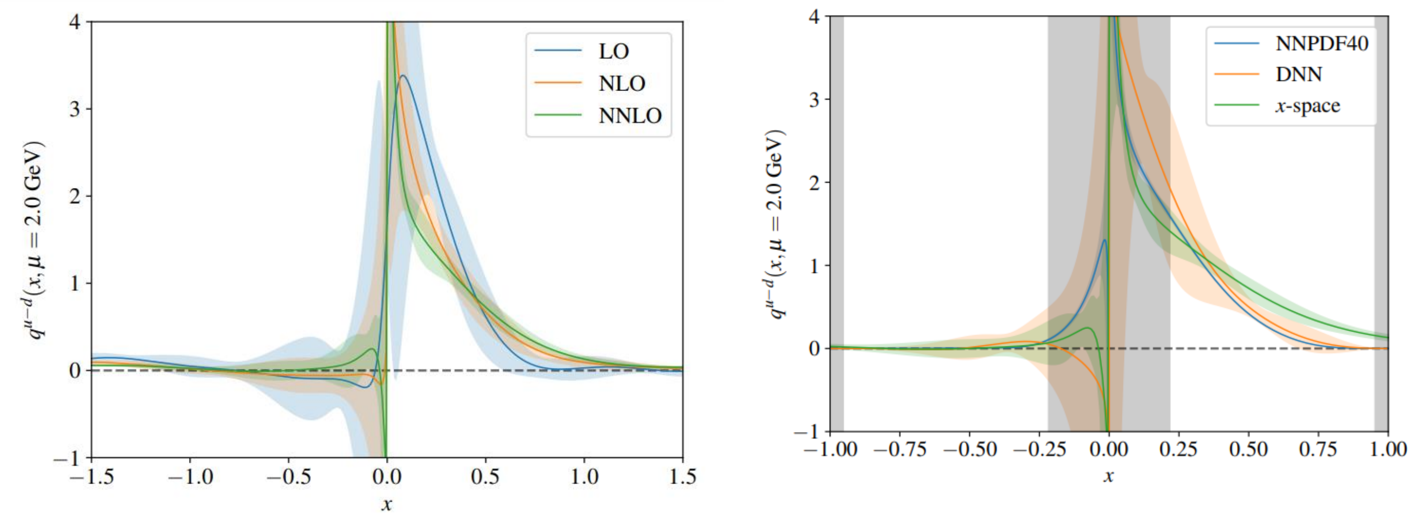

FIG. 15: Dependence of the x-space matched isovector-quark PDF on the perturbative order used in the matching kernel. The results shown use the largest value of momentum computed in this work (i.e. $P_z = 1.53$ GeV).

FIG. 17: Comparison of the x-dependence of the isovector-quark PDF from the global analysis of NNPDF4.0, the DNN, and x-space matching. The gray bands correspond to the regions of x where we do not rigorously trust the results of LaMET.

Phys. Rev. D 107, 074509 (2023)

Power corrections

➢ quark-PDF

$$\tilde{q}(y, P^z, \mu) = \int_{-1}^{1} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) q(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(yP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{[(1-y)P^z]^2}\right),$$

➢ Light-meson

$$\tilde{\mathcal{F}}(\Gamma, x, P^z, \tilde{\mu}) = \int_0^1 dy \tilde{C}_\Gamma\left(x, y, \frac{\tilde{\mu}}{\mu}, \frac{P^z}{\mu}\right) \mathcal{F}(\bar{\Gamma}, y, \mu) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(P^z)^2}\right),$$

➢ B-meson

$$\varphi_B^+(\xi, \mu) = \int_0^\infty d\omega\, H(\xi, \omega, n_z \cdot v, \mu) \phi_B^+(\omega, \mu) + O\left(\frac{\Lambda_{\rm QCD}}{n_z \cdot v \xi}\right).$$

One-loop:

Phys.Rev.D 98, 056004 (2018)

Two-loop:

Phys.Rev.Lett. 126 (2021)

However, no power correction considered

Summary

- We propose a practical method to calculate the heavy meson LCDA in HQET.
- We use a CLQCD ensemble to verify the feasibility of the method.
- The first set of preliminary results has been showcased.

Please stay tuned for our first result for LCDA!

全国第二十届重味物理和CP破坏研讨会，复旦大学，上海

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Thanks!