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Chiral EFT for nuclear forces using the Gradient Flow Method

based on work done in collaboration with Hermann Krebs, e-Print: 2311.10893; 2312.13932

- The problem
- Solution: Chiral gradient flow + Path integral approach
- Summary and outlook

European Research Council
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ERATO Exploratory Research for Advanced Technology

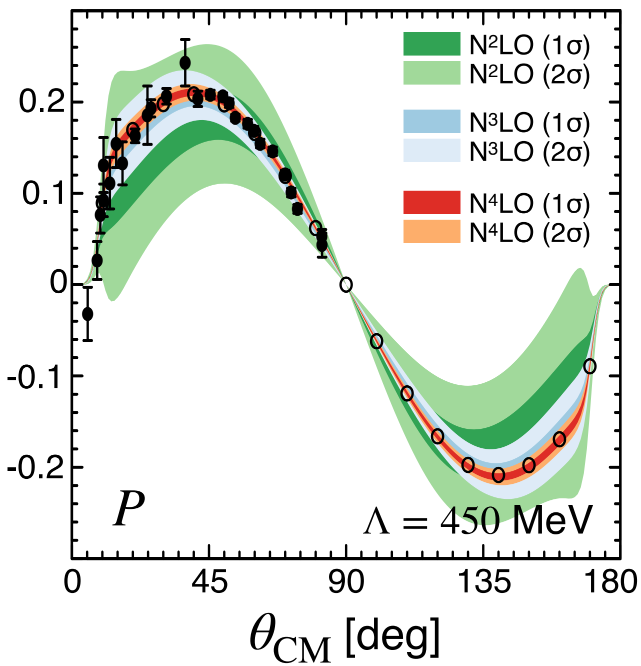
The problem

The SMS-regularized chiral NN interactions Reinert, Krebs, EE, EPJA 54 (2018) 86; PRL 126 (2021) 092501

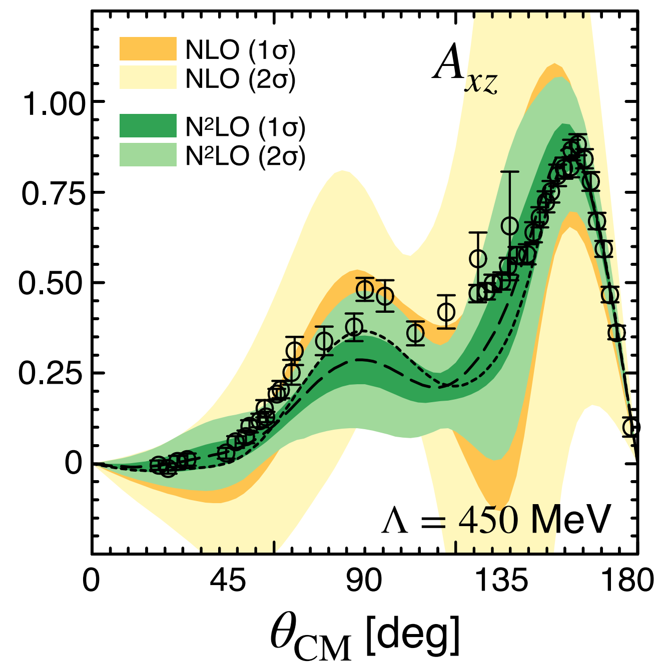
$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2} \; e^{-\frac{\vec{q}^{\,2} + M_\pi^2}{\Lambda^2}} + \text{subtraction}, \qquad V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu \frac{\rho(\mu)}{\vec{q}^{\,2} + \mu^2} \; e^{-\frac{\vec{q}^{\,2} + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

+ nonlocal (Gaussian) cutoff for contacts

2 nucleons



3 nucleons

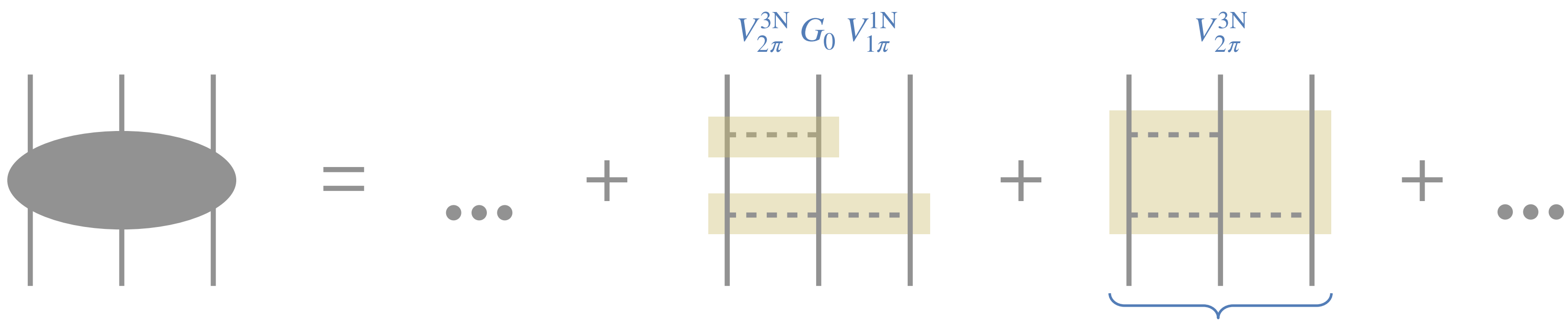


Where are calculations beyond N²LO?

The problem

Use DimReg (or equivalent) to derive 3NFs from $\mathscr{L}_{\rm eff}$, together with Cutoff Regularization in the Schrödinger equation. What about chiral symmetry?

Faddeev equation for 3N scattering:



derived using DimReg Bernard et al. ’08

$$-\Lambda \frac{g_A^4}{96\sqrt{2\pi^3}F_\pi^6}\Big[\underbrace{\tau_1\cdot\tau_3\,(\vec{q}_3\cdot\vec{\sigma}_1)}_{\text{absorbable into } c_D\text{:}} - \underbrace{\frac{4}{3}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)(\vec{q}_2\cdot\vec{\sigma}_3)}_{\text{violates chiral symmetry}}\Big]\frac{\vec{q}_3\cdot\vec{\sigma}_3}{q_3^3+M_\pi^2} + \ldots$$

$\Rightarrow$ mixing DimReg with Cutoff regularization breaks chiral symmetry EE, Krebs, Reinert ’19

$\Rightarrow$ 3NF, 4NF & MECs beyond N²LO have to be re-derived using Cutoff Reg. (2NF ok at fixed M_π)

Symmetry-preserving regularization

Hermann Krebs, EE, e-Print: 2312.13932

Want to regularize $\mathscr{L}_\pi$, $\mathscr{L}_{\pi N}$ in such a way that:

— the chiral and gauge symmetries are preserved,

— the regularized OPE potential has the SMS (Gaussian) cutoff,

— nuclear forces & currents are finite (DR still needed…) and regularized

Preliminaries

Effective Lagrangian for Goldstone bosons

Pions transform *nonlinearly* under SU(2)$_L$ × SU(2)$_R$ and via the adjoint irrep. of SU(2)$_V$ (i.e., $R = L$):

$$U(\boldsymbol{\pi}) \to R U L^\dagger, \qquad U(\boldsymbol{\pi}) = \underbrace{1 + \frac{i}{F}\boldsymbol{\tau}\cdot\boldsymbol{\pi} - \frac{\boldsymbol{\pi}^2}{2F^2} - \alpha\frac{i}{F^3}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\boldsymbol{\pi}^2 + \mathcal{O}(\boldsymbol{\pi}^4)}_{\text{most general parametrization of an SU(2) matrix in terms of } \pi\text{'s}}$$

SU(2) matrices

$$\underbrace{\mathcal{L}_\pi^{\rm E}}_{\text{in Euclidean space}} = \frac{F^2}{4}\mathrm{Tr}\Big[\overbrace{(\nabla_\mu U)^\dagger \nabla_\mu U}^{\nabla_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu} - U^\dagger \underbrace{\chi}_{\chi = 2B(s+ip)} - \chi^\dagger U\Big] = \frac{1}{2}\boldsymbol{\pi}\cdot\big(\underbrace{-\partial^2}_{\equiv \partial_\mu\partial_\mu = \partial_0^2 + \vec{\nabla}^2} + M^2\big)\boldsymbol{\pi}^2 + \ldots$$

Effective Lagrangian for pions and nucleons

Generalization to N: $u(\boldsymbol{\pi}) := \sqrt{U}, \quad u \to R u K^\dagger = K u L^\dagger \quad \text{with} \quad K(L,R,U) = \sqrt{L U^\dagger R^\dagger}\, R \sqrt{U}$

Define [CCWZ '69] $N \to KN$ and introduce $u_\mu = i u^\dagger \nabla_\mu U u^\dagger, \quad u_\mu \to K u_\mu K^\dagger, \ \ldots$

$$\Rightarrow \quad \mathcal{L}_{\pi N}^{\rm E} = N^\dagger (D_0 + g u_\mu S_\mu) N + \ldots$$

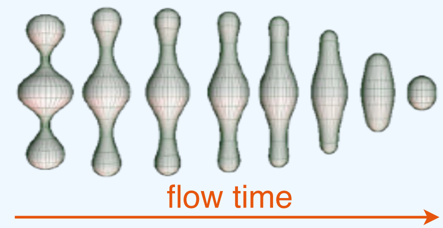
First attempt: Higher-derivative regularization of $\mathscr{L}_\pi^{\rm E}$ Slavnov '71

$$\mathcal{L}_\pi^{\rm E} \longrightarrow \mathcal{L}_{\pi,\Lambda}^{\rm E} = \frac{F^2}{4}\mathrm{Tr}\Big[\partial_\mu U^\dagger e^{-\partial^2/\Lambda^2}\partial_\mu U\Big] = \underbrace{-\frac{1}{2}\boldsymbol{\pi}\cdot\partial^2\, e^{-\partial^2/\Lambda^2}\boldsymbol{\pi}}_{\Delta_\Lambda^{\rm E} = 1/(q_0^2 + \vec{q}^{\,2})\, e^{-(q_0^2+\vec{q}^{\,2})/\Lambda^2}} + \frac{1}{2F^2}\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}\, \underbrace{e^{-\partial^2/\Lambda^2}}_{\text{de-regularization...}}\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi} + \mathcal{O}(\pi^6)$$

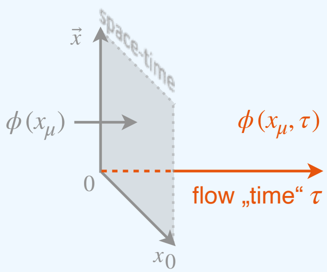
(In the absence of external sources)

Gradient flow

Gradient flows: methods for smoothing manifolds
(e.g., Ricci flow used in the proof of the Poincaré conjecture)



Gradient flow as a regulator in field theory



Flow equation: $\dfrac{\partial}{\partial \tau}\phi(x,\tau) = -\dfrac{\delta S[\phi]}{\delta \phi(x)}\bigg|_{\phi(x)\to\phi(x,\tau)}$

subject to the boundary condition $\phi(x,0) = \phi(x)$

Free scalar field:

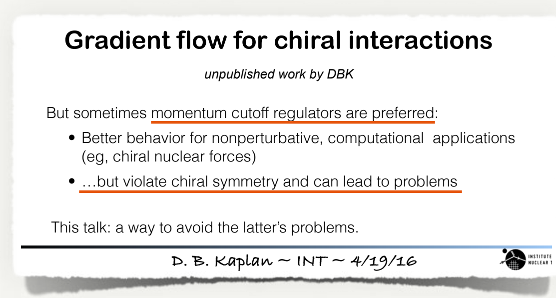
$$\left[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)\right]\phi(x,\tau) = 0 \quad \Rightarrow \quad \phi(x,\tau) = \underbrace{\int d^4y\, \overbrace{G(x-y,\tau)}^{G(x,\tau) = \frac{\theta(\tau)}{16\pi^2\tau^2} e^{-\frac{x^2+4M^2\tau^2}{4\tau}}}\phi(y)}_{\textit{heat kernel}} \quad \Rightarrow \quad \tilde{\phi}(q,\tau) = e^{-\tau(q^2+M^2)}\tilde{\phi}(q)$$

YM gradient flow Narayanan, Neuberger ’06, Lüscher, Weisz ’11: $\partial_\tau A_\mu(x,\tau) = D_\nu G_{\nu\mu}(x,\tau)$ ← extensively used in LQCD

Chiral gradient flow Krebs, EE, 2312.13932

Generalize $U(x)$ to $W(x,\tau)$: $\partial_\tau W = -i\underbrace{w}_{\sqrt{W}}\,\overbrace{\mathrm{EOM}(\tau)}^{[D_\mu, w_\mu] + \frac{i}{2}\chi_-(\tau) - \frac{i}{4}\mathrm{Tr}\,\chi_-(\tau)}\,w, \quad W(x,0) = U(x)$

We have proven $\forall\tau$: $W(x,\tau) \in \mathrm{SU}(2), \quad W(x,\tau) \;\to\; RW(x,\tau)L^\dagger$


Gradient flow for chiral interactions

unpublished work by DBK

But sometimes momentum cutoff regulators are preferred:

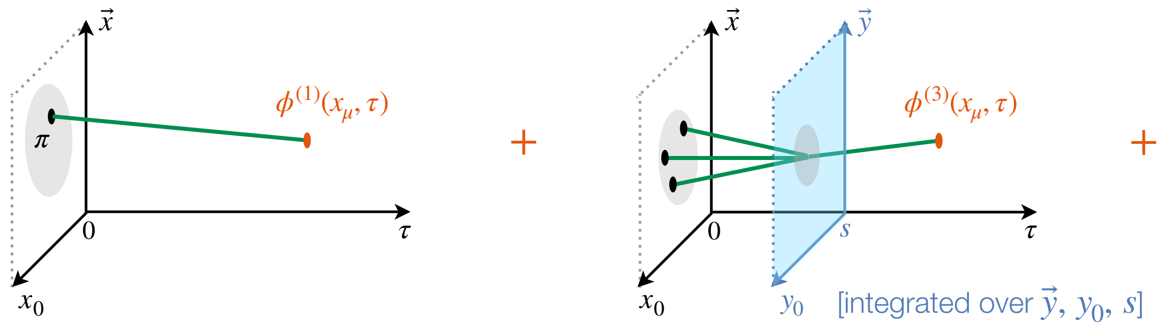
- Better behavior for nonperturbative, computational applications (eg, chiral nuclear forces)
- ...but violate chiral symmetry and can lead to problems

This talk: a way to avoid the latter's problems.

D. B. Kaplan ~ INT ~ 4/19/16

Solving the chiral gradient flow equation

$\phi(x_\mu, \tau) \;=$



$+ \;\ldots$

$$\left.\begin{aligned}\left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right]\boldsymbol{\phi}^{(1)}(x,\tau) &= 0\\ \boldsymbol{\phi}^{(1)}(x,0) &= \boldsymbol{\pi}(x)\end{aligned}\right\} \Rightarrow \quad \boldsymbol{\phi}^{(1)}(x,\tau) = \int d^4y\, \overbrace{G(x-y,\tau)}^{G(x,\tau)=\frac{\theta(\tau)}{16\pi^2\tau^2}e^{-\frac{x^2+4M^2\tau^2}{4\tau}}}\boldsymbol{\pi}(y) \quad \Rightarrow \quad \tilde{\boldsymbol{\phi}}^{(1)}(q,\tau) = e^{-\tau(q^2+M^2)}\tilde{\boldsymbol{\pi}}(q)$$

SMS regulator for $\tau = 1/(2\Lambda^2)$

$$\left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right]\phi_b^{(3)}(x,\tau) = \overbrace{(1-2\alpha)\partial_\mu\boldsymbol{\phi}^{(1)}\cdot\partial_\mu\boldsymbol{\phi}^{(1)}\phi_b^{(1)} - 4\alpha\,\partial_\mu\boldsymbol{\phi}^{(1)}\cdot\boldsymbol{\phi}^{(1)}\partial_\mu\phi_b^{(1)} + \frac{M^2}{2}(1-4\alpha)\boldsymbol{\phi}^{(1)}\cdot\boldsymbol{\phi}^{(1)}\phi_b^{(1)}}^{\equiv\, \mathrm{RHS}_b(x,\tau)}$$

$$\phi_b^{(3)}(x,0) = 0$$

$$\Rightarrow \quad \phi_b^{(3)}(x,\tau) = \int_0^\tau ds \int d^4y\, G(x-y,\tau-s)\,\mathrm{RHS}_b(y,s)$$

In momentum space, this solution takes the form:

$$\tilde{\phi}_b^{(3)}(q,\tau) = \int \prod_{i=1}^{3}\frac{d^4q_i}{(2\pi)^4}(2\pi)^4\delta^4(q-q_1-q_2-q_3)\,\underbrace{f_\Lambda(\{q_i\})}_{\frac{e^{-\tau(q^2+M^2)}-e^{-\tau\sum_{j=1}^{3}(q_j^2+M^2)}}{q_1^2+q_2^2+q_3^2-q^2+2M^2}}\left[4\alpha\, q_1\cdot q_3 - (1-2\alpha)q_1\cdot q_2 + \frac{M^2}{2}(1-4\alpha)\right]\tilde{\boldsymbol{\pi}}(q_1)\cdot\tilde{\boldsymbol{\pi}}(q_2)\,\tilde{\pi}_b(q_3)$$

Gradient flow regularization of BChPT

— The pion Lagrangian $\mathscr{L}_\pi^{\mathrm{E}}$ is left unchanged

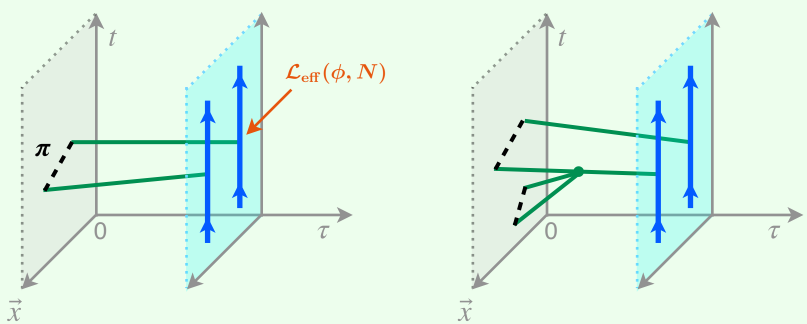
— Nucleons are ***defined*** to „live" at a fixed $\tau > 0$.

Remember, in the original formulation: $N \to KN$, where $K = \sqrt{LU^\dagger R^\dagger}R\sqrt{U}$ with $U \to RUL^\dagger$.

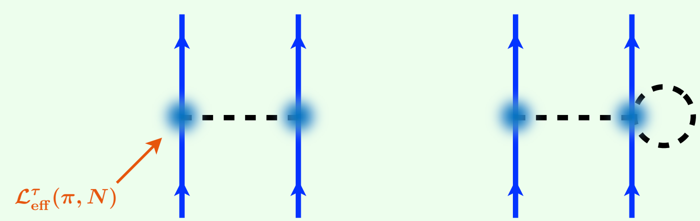
Instead, we introduce $N(\tau)$ via $N \to KN\Big|_{U\to W(\tau)}$: χ-symmetry is preserved since $W \to RWL^\dagger$.

$\Rightarrow$ BChPT using gradient flow regularization: $\mathcal{L}^{\mathrm{E}} = \mathcal{L}_\pi^{\mathrm{E}} + \mathcal{L}_{\pi N}^{\mathrm{E}}(\tau), \quad \underbrace{\mathcal{L}_{\pi N}^{\mathrm{E}}(\tau) = \mathcal{L}_{\pi N}^{\mathrm{E}}\Big|_{U\to W(\tau)}}_{\text{non-local (smeared) Lagrangian upon expressing in } \pi\text{'s}}$

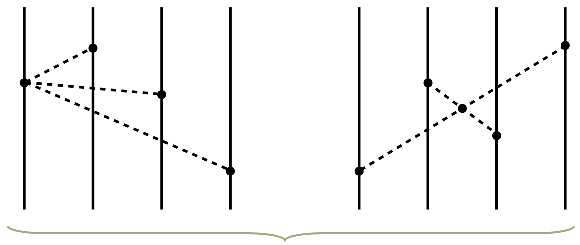
Local field theory in 5d



Smeared (non-local) theory in 4d



Chiral symmetry II: The 4N force



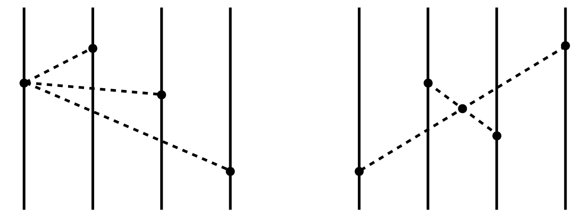
Unregularized expression for this 4NF EE, EPJA 34 (2007):

$$\hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_4 \, \vec{\sigma}_2 \cdot \vec{q}_2 \, \vec{\sigma}_3 \cdot \vec{q}_3 \, \vec{\sigma}_4 \cdot \vec{q}_4$$

$$\begin{aligned} V^{4N} &= -\frac{g^4}{64F^6} \frac{\hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]}}{(\vec{q}_2^{\,2}+M^2)(\vec{q}_3^{\,2}+M^2)(\vec{q}_4^{\,2}+M^2)} \, \vec{\sigma}_1 \cdot \vec{q}_{12} \\ &+ \frac{g^4}{128F^6} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]}}{(\vec{q}_1^{\,2}+M^2)(\vec{q}_2^{\,2}+M^2)(\vec{q}_3^{\,2}+M^2)(\vec{q}_4^{\,2}+M^2)} \left(M^2 + \vec{q}_{12}^{\,2}\right) \; + \; 23 \text{ perm.} \end{aligned}$$

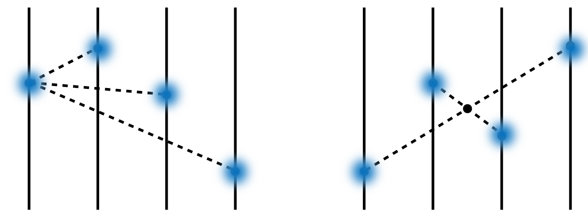
Chiral symmetry II: The 4N force

unregularized



the sum must be α-independent

regularized



the sum must be α-independent

$$
\begin{aligned}
V_\Lambda^{4N} \;=\; & \frac{g^4}{64F^6}\frac{\hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]}}{(\vec{q}_2^{\,2}+M^2)(\vec{q}_3^{\,2}+M^2)(\vec{q}_4^{\,2}+M^2)}\Big[\vec{\sigma}_1\cdot\vec{q}_1\big(2g_\Lambda-4f_\Lambda^{123}+2f_\Lambda^{134}-f_\Lambda^{234}\big)-\vec{\sigma}_1\cdot\vec{q}_2 f_\Lambda^{234} \\
& +2\vec{\sigma}_1\cdot\vec{q}_1\big(5M^2+\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+\vec{q}_3^{\,2}+\vec{q}_4^{\,2}+\vec{q}_{34}^{\,2}\big)\frac{g_\Lambda-f_\Lambda^{134}}{2M^2+\vec{q}_1^{\,2}+\vec{q}_3^{\,2}+\vec{q}_4^{\,2}-\vec{q}_2^{\,2}} \\
& -4\vec{\sigma}_1\cdot\vec{q}_1\big(3M^2+\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+\vec{q}_3^{\,2}+\vec{q}_4^{\,2}-\vec{q}_{34}^{\,2}\big)\frac{g_\Lambda-f_\Lambda^{124}}{2M^2+\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+\vec{q}_4^{\,2}-\vec{q}_3^{\,2}}\Big] \\
+\; & \frac{g^4}{128F^6}\frac{\vec{\sigma}_1\cdot\vec{q}_1\,\hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]}}{(\vec{q}_1^{\,2}+M^2)(\vec{q}_2^{\,2}+M^2)(\vec{q}_3^{\,2}+M^2)(\vec{q}_4^{\,2}+M^2)}\big(M^2+\vec{q}_{12}^{\,2}\big)\big(4f_\Lambda^{123}-3g_\Lambda\big)\;+\;23\ \text{perm.},
\end{aligned}
$$

$$f_\Lambda^{ijk}=e^{-\frac{\vec{q}_i^{\,2}+M^2}{\Lambda^2}}e^{-\frac{\vec{q}_j^{\,2}+M^2}{\Lambda^2}}e^{-\frac{\vec{q}_k^{\,2}+M^2}{\Lambda^2}}$$

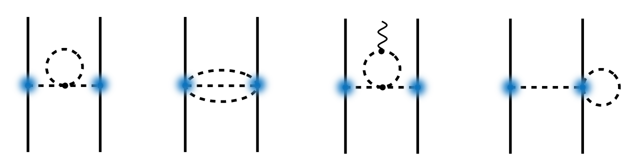
$$e^{-\frac{\vec{q}_1^{\,2}+M^2}{2\Lambda^2}}e^{-\frac{\vec{q}_2^{\,2}+M^2}{2\Lambda^2}}e^{-\frac{\vec{q}_3^{\,2}+M^2}{2\Lambda^2}}e^{-\frac{\vec{q}_4^{\,2}+M^2}{2\Lambda^2}}$$

(reduces to the unregularized result in the $\Lambda\to\infty$ limit)

— per construction, no exponentially growing factors

— nuclear forces and currents are sufficiently regularized

(some purely pionic loops are divergent and require e.g. DR)



Nuclear interactions from path integral

Hermann Krebs, EE, e-Print: 2311.10893

The considered 4NFs were calculated using Feynman diagrams. But more generally,

Potential  $\neq$ Feynman diagram 

From Lagrangian to nuclear interactions: $\mathcal{L}_{\pi N} \xrightarrow{\text{canonical quan.}} \mathcal{H}_{\pi N}$

TOPT → Weinberg, van Kolck, Pastore, Schiavilla, …

MUT → EE, Glöckle, Meißner, Krebs, Bernard, …

$$\boxed{\begin{matrix} V_{2N}, & V_{3N}, & \ldots \\ J^{\mu}_{2N}, & J^{\mu}_{3N}, & \ldots \\ A^{\mu}_{2N}, & A^{\mu}_{3N}, & \ldots \end{matrix}}$$

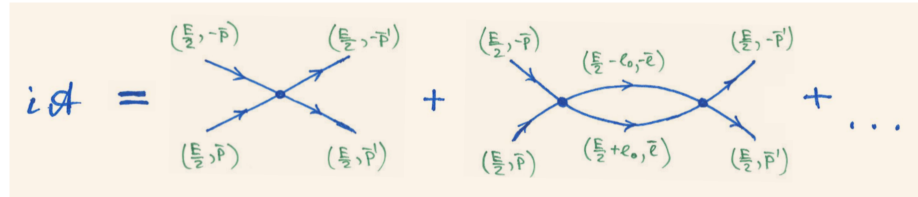
$\Rightarrow$ impractical for ***regularized*** Lagrangians that involve $e^{-\tau(-\partial_x^2+M^2)}\boldsymbol{\pi}(x)$

$\Rightarrow$ new method to derive nuclear interactions using the path integral approach

Warm-up exercise

Pion-less EFT:

$$\mathcal{L} = N^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2m_N} \right] N - \frac{C_S}{2}(N^\dagger N)^2 + \ldots$$



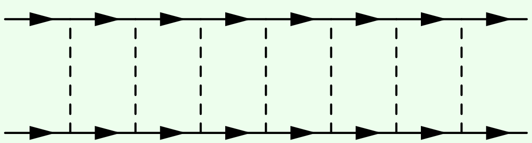
$$\Rightarrow \quad \mathcal{A}_{\text{tree}} = [C_0 + C_2(\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + \ldots]$$

Scattering amplitude to 1 loop:

$$-i\mathcal{A}_{1-\text{loop}} = \int \frac{d^4 l}{(2\pi)^4} [C_0 + C_2(\vec{p}^{\,2} + \vec{l}^{\,2}) + \ldots] \frac{1}{\left(\frac{E}{2} + l_0 - \frac{\vec{l}^{\,2}}{2m_N} + i\epsilon\right)\left(\frac{E}{2} - l_0 - \frac{\vec{l}^{\,2}}{2m_N} + i\epsilon\right)} [C_0 + \ldots]$$

$$= -i \int \frac{d^3 l}{(2\pi)^3} [C_0 + C_2(\vec{p}^{\,2} + \vec{l}^{\,2}) + \ldots] \frac{1}{E - \frac{\vec{l}^{\,2}}{m_N} + i\epsilon} [C_0 + (\vec{l}^{\,2} + \vec{p}^{\,\prime 2}) \ldots]$$

All l_0-integrals factorize $\Rightarrow$ Lippmann-Schwinger eq. $\mathcal{A} = \mathcal{V} + \mathcal{V}\, G_0\, \mathcal{A}$ with $\mathcal{V} = -\mathcal{L}_{\text{int}}$



$\longleftarrow$ But l_0-integrals do not factorize for pions due to l_0-dependence of π-propagators…

Idea: $$Z[\eta^\dagger, \eta] = A \int \mathcal{D}N^\dagger\, \mathcal{D}N\, \mathcal{D}\pi \exp\left(iS_{\text{eff}}^{\Lambda} + i \int d^4x [\eta^\dagger N + N^\dagger \eta] \right)$$

instantaneous

$$\xrightarrow{\textbf{nonlocal}\text{ redefinitions of } N,\, N^\dagger} A \int \mathcal{D}\tilde{N}^\dagger\, \mathcal{D}\tilde{N} \exp\left(i S_{\text{eff, N}}^{\Lambda} + i \int d^4x [\eta^\dagger \tilde{N} + \tilde{N}^\dagger \eta] \right)$$

Nuclear interactions from path integral

Regularized toy model: $\mathcal{L}^{\mathrm{E}}_{\pi N} = N^\dagger \left[\partial_0 - \frac{\vec{\nabla}^2}{2m} - \frac{g}{2F} \vec{\sigma} \cdot \vec{\nabla} \boldsymbol{\pi} \cdot \boldsymbol{\tau} \right] N + \frac{1}{2} \boldsymbol{\pi} \cdot \left(-\partial^2 + M^2 \right) e^{\frac{-\partial^2 + M^2}{\Lambda^2}} \boldsymbol{\pi}$

Nonlocal action S^{E}_N after integrating out pion fields (Gaussian):

$$Z[\eta^\dagger, \eta] = \int DN^\dagger DN D\boldsymbol{\pi}\, e^{-S^{\mathrm{E}}_{\pi N} + \int d^4x \left[\eta^\dagger N + N^\dagger \eta \right]} = A \int DN^\dagger DN\, e^{-S^{\mathrm{E}}_{N} + \int d^4x \left[\eta^\dagger N + N^\dagger \eta \right]}$$

where $$S^{\mathrm{E}}_N = \underbrace{N_x^\dagger \left[\partial_0 - \frac{\vec{\nabla}_x^2}{2m} \right] N_x}_{\text{integration over } d^4x \text{ not shown}} + \underbrace{\frac{g^2}{8F^2} \left[N^\dagger \vec{\sigma} \boldsymbol{\tau} N \right]_{x_1} \cdot \left[\vec{\nabla}_{x_1} \otimes \vec{\nabla}_{x_1} \overbrace{\Delta^{\mathrm{E}}_\Lambda (x_1 - x_2)}^{\text{non-static regularized pion propagator}} \right] \cdot \left[N^\dagger \vec{\sigma} \boldsymbol{\tau} N \right]_{x_2}}_{\text{integration over } d^4x_1\, d^4x_2 \text{ not shown}}$$

Rewrite the pion propagator to the static one plus rest:

$$\Delta^{\mathrm{E}}_\Lambda (x) = \int \frac{d^4 q}{(2\pi)^4} e^{iq\cdot x} \frac{e^{-\frac{q_0^2 + \vec{q}^{\,2} + M^2}{\Lambda^2}}}{q_0^2 + \vec{q}^{\,2} + M^2} = \underbrace{\Delta^{\mathrm{S}}_\Lambda (x)}_{\delta(x_0) \tilde{\Delta}^{\mathrm{S}}_\Lambda (\vec{x})} + \Delta^{\mathrm{E}}_\Lambda (x) - \Delta^{\mathrm{S}}_\Lambda (x) = \Delta^{\mathrm{S}}_\Lambda (x) + \underbrace{\partial_0^2 \Delta^{\mathrm{ES}}_\Lambda (x)}_{-\int \frac{d^4 q}{(2\pi)^4} \frac{e^{iq\cdot x}}{q_0^2} \left[\tilde{\Delta}^{\mathrm{E}}_\Lambda (q) - \tilde{\Delta}^{\mathrm{S}}_\Lambda (\vec{q}) \right]}$$

Nucleon field redefinition: $N_x = \tilde{N}_x - \frac{g^2}{8F^2} \boldsymbol{\tau} \vec{\sigma} \tilde{N}_x \cdot \underbrace{\left[\vec{\nabla}_x \otimes \vec{\nabla}_x \partial_0 \Delta^{\mathrm{ES}}_\Lambda (x - x_2) \right] \cdot \left[\tilde{N}^\dagger \vec{\sigma} \boldsymbol{\tau} \tilde{N} \right]_{x_2}}_{\text{integration over } d^4x_2 \text{ not shown}}, \quad N_x^\dagger = \ldots$

$$\Rightarrow S^{\mathrm{E}}_{\tilde{N}} = \tilde{N}_x^\dagger \left[\partial_0 - \frac{\vec{\nabla}_x^2}{2m} \right] \tilde{N}_x + \frac{g^2}{8F^2} \left[\tilde{N}^\dagger \vec{\sigma} \boldsymbol{\tau} \tilde{N} \right]_{x_1} \cdot \left[\vec{\nabla}_{x_1} \otimes \vec{\nabla}_{x_1} \Delta^{\mathrm{S}}_\Lambda (x_1 - x_2) \right] \cdot \left[\tilde{N}^\dagger \vec{\sigma} \boldsymbol{\tau} \tilde{N} \right]_{x_2} + S^{1/m}_{2\mathrm{N}} + S_{3\mathrm{N}}$$

Nuclear interactions from path integral

To summarize:

$$Z[\eta^\dagger,\eta] \;=\; \int DN^\dagger DN D\boldsymbol{\pi}\, e^{-S^{\mathrm{E}}_{\pi N}+\int d^4x\,[\eta^\dagger N+N^\dagger\eta]} \;=\; \ldots \;=\; A\int D\tilde N^\dagger D\tilde N\, e^{-S^{\mathrm{E}}_{\tilde N}+\int d^4x\,[\eta^\dagger\tilde N+\tilde N^\dagger\eta]}$$

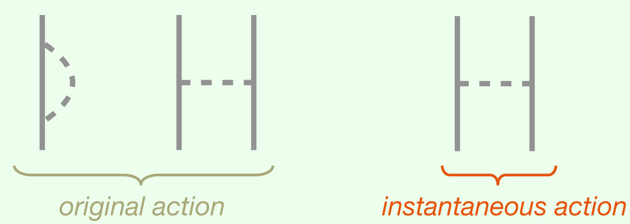
where the many-body action is now instantaneous (up to higher-order corrections):

$$S^{\mathrm{E}}_{\tilde N} \;=\; \tilde N_x^\dagger\left[\partial_0-\frac{\vec\nabla_x^{\,2}}{2m}\right]\tilde N_x \;+\; \frac{g^2}{8F^2}\left[\tilde N^\dagger\vec\sigma\boldsymbol{\tau}\tilde N\right]_{x_1}\cdot\left[\vec\nabla_{x_1}\otimes\vec\nabla_{x_1}\Delta^{\mathrm{S}}_\Lambda(x_1-x_2)\right]\cdot\left[\tilde N^\dagger\vec\sigma\boldsymbol{\tau}\tilde N\right]_{x_2} \;+\; S^{1/m}_{2\mathrm{N}} \;+\; S_{3\mathrm{N}}$$

$\Rightarrow$ read out V_{NN} directly from the action: $V^\Lambda_{2\mathrm{N}}(\vec x_{12}) = \dfrac{g^2}{4F^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,(\vec\sigma_1\cdot\vec\nabla)\,(\vec\sigma_2\cdot\vec\nabla)\,\Delta^{\mathrm{S}}_\Lambda(\vec x_{12})$

On the other hand, to order g^2:

$\Rightarrow$ something is missing…



$$\int DN^\dagger DN\, e^{(\ldots)} \;=\; \int D\tilde N^\dagger D\tilde N\;\det\left[\boldsymbol{J}_{N,N^\dagger}(\tilde N,\tilde N^\dagger)\right]e^{(\ldots)} \;=\; \int D\tilde N^\dagger D\tilde N\, e^{(\ldots)+\int d^4x\tilde N_x^\dagger\Sigma_\Lambda\tilde N_x+\ldots}$$

with $$\Sigma_\Lambda \;=\; -\frac{3g^2}{8F^2}\int\frac{d^3p}{(2\pi)^3}\,\vec p^{\,2}\,\frac{e^{-\frac{\vec p^{\,2}+M^2}{\Lambda^2}}}{\vec p^{\,2}+M^2} \;=\; -\frac{3g^2}{64\pi^{3/2}F^2}\Lambda^3 \;+\; \frac{9g^2M^2}{64\pi^{3/2}F^2}\Lambda \;-\; \underbrace{\frac{3g^2M^3}{32\pi F^2}}_{\text{the leading non-analytic contribution to } m_N} \;+\; \mathcal{O}(\Lambda^{-1})$$

Summary

New formulation of nuclear chiral EFT:

- — gradient flow regularized version of baryon ChPT
- — path integral method to reduce QFT to QM via non-local field redefinitions
  (loop contributions emerge from the functional determinant)

⇒ regularized 3N, 4N forces and currents, which are consistent with the SMS NN potentials & respect chiral and gauge symmetries Hermann Krebs, EE, in progress

Further ongoing work:

- — πN scattering inside the Mandelstam triangle using GF HBChPT (LECs)
- — NN potential and chiral extrapolations
- — Hyper-nuclear interactions

Thank you for your attention

The 11th International Workshop on Chiral Dynamics

26–30 Aug 2024
Ruhr University Bochum, Germany
Europe/Berlin timezone

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The 11th International Workshop on Chiral Dynamics (CD2024) will take place August 26-30, 2024 at the Ruhr University Bochum, Germany. This series of workshops started at MIT in 1994 and brings together theorists and experimentalists every three years to discuss the status, progress and challenges in the physics of low-energy QCD, Goldstone Boson dynamics, meson-baryon Interactions, few-body physics, lattice QCD and ChPT. Previous workshops took place in Pisa (2015), Durham, NC (2018) and Beijing (2021).

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