Exotic hadrons with heavy quarks Part 3: Applications

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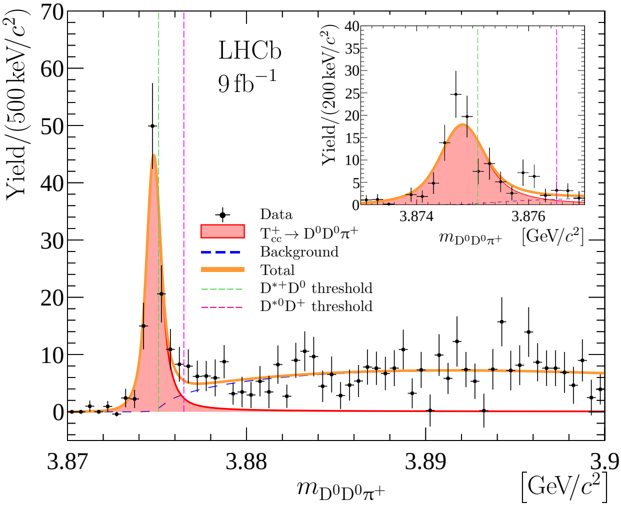
Double-charm state T_{cc}^{+}

$$I = 0 \quad J^P = 1^+$$

Minimal quark content: $cc\bar{u}\bar{d}$

$$T_{cc}^{+} \to D^0 D^0 \pi^+$$

T_{cc}^+ @ LHCb (Nature Phys. 18 (2022) 7, 751)



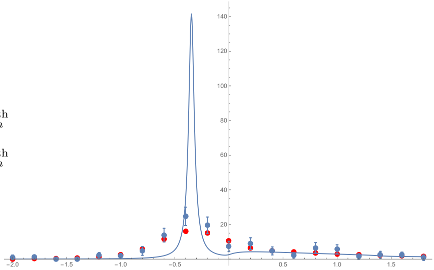
$$\delta m_{\mathrm{BW}} = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV} \quad \Gamma_{\mathrm{BW}} = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}$$

Simple Flatté fit $(\chi^2/N_{\rm dof} \approx 1)$

$$\mathcal{A} = \frac{\sqrt{\mathcal{N}}}{E - E_f + \frac{i}{2}\left[g(\tilde{k}_1 + \tilde{k}_2) + \Gamma_0\right]}$$

$$\tilde{k}_n = \begin{cases} \sqrt{\mu_n\left(\sqrt{(E-E_n^{\rm th})^2 + \frac{1}{4}\Gamma_{D^*}^2} + (E-E_n^{\rm th})\right)}, & E > E_n^{\rm th} \\ -i\sqrt{\mu_n\left(\sqrt{(E-E_n^{\rm th})^2 + \frac{1}{4}\Gamma_{D^*}^2} - (E-E_n^{\rm th})\right)}, & E < E_n^{\rm th} \end{cases}$$

$\Gamma_0^{\rm fit} = 0 \implies$ No compact component



Pole position:

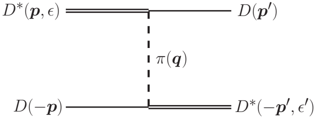
$$E_{\rm pole} = (-347 - i31)\ \text{keV}$$

In neglect of D^* width:

$$X_1 = \frac{\sqrt{E_B + \Delta}}{\sqrt{E_B} + \sqrt{E_B + \Delta}} \qquad X_2 = \frac{\sqrt{E_B}}{\sqrt{E_B} + \sqrt{E_B + \Delta}}$$

For $E_B = 347$ keV and $\Delta = 1.41$ MeV: $X_1 = 0.7$ $X_2 = 0.3$

Pion exchange in $I = 0$ DD^* system



$$V_\pi(\boldsymbol{p}, \boldsymbol{p}') = \left(\frac{g_c}{2f_\pi}\right)^2 \langle \boldsymbol{\tau} \cdot \boldsymbol{\tau} \rangle \frac{(\boldsymbol{\epsilon} \cdot \boldsymbol{q})(\boldsymbol{q} \cdot \boldsymbol{\epsilon}'^*)}{u - m_\pi^2}$$

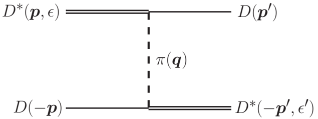
$$\underset{\substack{I\,=\,0\\ \text{central}\\ \cancel{\text{recoil}}}}{\Longrightarrow} \quad \left(\frac{g_c}{2f_\pi}\right)^2 \left(-1 + \overbrace{\frac{\mu_\pi^2}{\boldsymbol{q}^2 + \underbrace{[m_\pi^2 - (m_{D^*} - m_D)^2]}_{\text{Effective mass } \mu_\pi^2}}}^{\text{Long-range OPE}}\right)$$

- Short-range OPE absorbed by (re-fitted) contact interaction
- Perturbative long-range OPE as per

$$\alpha_\pi^{\text{eff}} = \frac{g_c^2 |\mu_\pi^2|}{f_\pi^2} \ll 1$$

(XEFT: Voloshin'2004,Fleming et al.'2007,...)

Pion exchange in $I = 0$ DD^* system



$$V_\pi(\boldsymbol{p}, \boldsymbol{p}') = \left(\frac{g_c}{2f_\pi}\right)^2 \langle \boldsymbol{\tau}\cdot\boldsymbol{\tau}\rangle \frac{(\boldsymbol{\epsilon}\cdot\boldsymbol{q})(\boldsymbol{q}\cdot\boldsymbol{\epsilon}'^*)}{u - m_\pi^2}$$

$$\Longrightarrow \left(\frac{g_c}{2f}\right)^2 \left(-1 + \overbrace{\frac{\mu_\pi^2}{\cdots}}^{\text{Long-range OPE}}\right)$$

Is pion exchange essential in T_{cc}^{+}?

- Short-range OPE absorbed by (re-fitted) contact interaction
- Perturbative long-range OPE as per

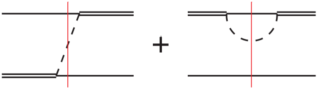
$$\alpha_\pi^{\text{eff}} = \frac{g_c^2|\mu_\pi^2|}{f_\pi^2} \ll 1$$

(XEFT: Voloshin'2004,Fleming et al.'2007,...)

Comment on pion exchange in T_{cc}^+

- Physical T_{cc}^+ ($m_\pi < m_{D^*} - m_D \Longrightarrow \mu_\pi^2 < 0$ & $|\mu_\pi| \ll m_\pi$):

$\Longrightarrow$ 3-body unitarity:



$\Longrightarrow$ T_{cc}^+ spin partner at D^*D^* threshold

$$\alpha_\pi^{D\text{-wave}} \simeq g_c^2 q_{\rm typ}^2/f_\pi^2 \simeq g_c^2 m_D(m_{D^*} - m_D)/f_\pi^2 > 1$$

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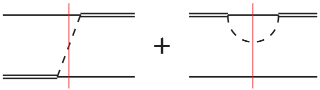

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- Lattice T_{cc}^+ ($m_\pi^{\text{lat}} > m_{D^*}^{\text{lat}} - m_D^{\text{lat}} \Longrightarrow (\mu_\pi^{\text{lat}})^2 > 0$ & $\mu_\pi^{\text{lat}} > m_\pi^{\text{ph}}$):

$\Longrightarrow$ $\alpha_\pi = g_c^2 \mu_\pi^2 / f_\pi^2 \sim 1$

$\Longrightarrow$ Left-hand cut in partial-wave amplitudes

$$\int d\Omega_{\hat{\boldsymbol{k}}\hat{\boldsymbol{k}}'} V_\pi(\boldsymbol{k} - \boldsymbol{k}') \sim \log \frac{\mu_\pi^2 + (k+k')^2}{\mu_\pi^2 + (k-k')^2} \underset{k'=k=p}{\Longrightarrow} \log\left(1 + \frac{4p^2}{\mu_\pi^2}\right)$$

EFT approach to T_{cc}^+

$$\left.\begin{array}{l} \gamma_B = \sqrt{m_D E_B} \simeq 25\ \text{MeV} \\ p_{\text{data}}^{\text{max}} = \sqrt{m_D\, \Delta E_{\text{data}}} \simeq 100\ \text{MeV} \\ p_{\text{coupl.ch.}} = \sqrt{m_D(m_{D^*} - m_D)} \simeq 500\ \text{MeV} \end{array}\right\} \Longrightarrow$$

$\Lambda = 500$ MeV
Potential at LO
OPE included
No couple channels

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- Lippmann-Schwinger equation for scattering amplitude (1 free parameter)

$$T(M,p,p') = V(M,p,p') - \int \frac{d^3q}{(2\pi)^3} V(M,p,q)G(M,q)T(M,q,p')$$

$$V(M,p,p') = v_0 + V_{\text{OPE}}$$

- Production amplitude (1 additional free parameter: P = point-like source)

$$U(M,p) = P - \int \frac{d^3q}{(2\pi)^3} T(M,p,q)G(M,q)P$$

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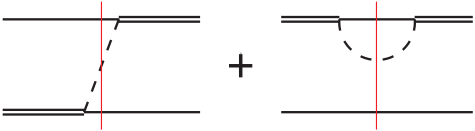
- Lippmann-S … parameter)

$T(M,$ … $T(M,q,p')$

3-body effects:



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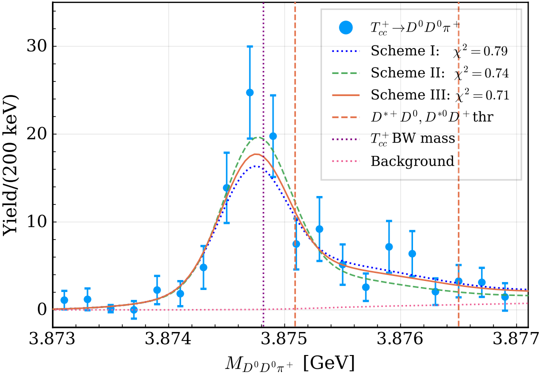
Fitting schemes, results, and conclusions

	Γ_{D^*}=const, $\cancel{\text{OPE}}$	$\Gamma_{D^*}(p,M)$, $\cancel{\text{OPE}}$	$\Gamma_{D^*}(p,M)$, OPE
$\chi^2/\text{d.o.f.}$	0.79	0.74	0.71
v_0 [GeV^{-2}]	-23.34 ± 0.08	$-22.88^{+0.08}_{-0.06}$	$-5.04^{+0.10}_{-0.08}$
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$



- (Quasi)bound state just below $D^{*+}D^0$ threshold
- Compositeness: 70% & 30%

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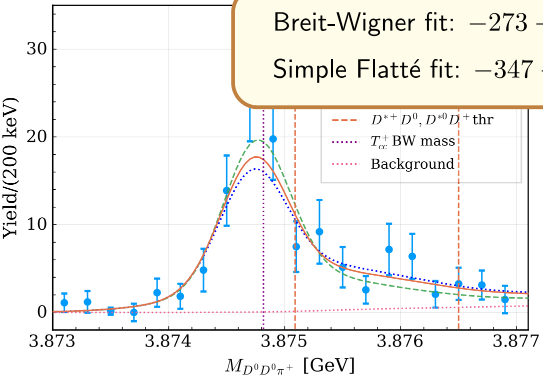
Breit-Wigner fit: $-273 - i410$ keV

Simple Flatté fit: $-347 - i31$ keV



- (Quasi)bound state just below $D^{*+}D^0$ threshold
- Compositeness: 70% & 30%

Spin partner T_{cc}^{*+}

HQSS: $V^{I=0}(D^*D^* \to D^*D^*, 1^+) = V^{I=0}(D^*D \to D^*D, 1^+) = v_0$

T_{cc}^+ at D^*D threshold hints existence of T_{cc}^{*+} at D^*D^* threshold

Scheme I: $\delta_{cc}^{*+} = -1.4$ MeV

Scheme II: $\delta_{cc}^{*+} = -1.1$ MeV

Scheme III: $\delta_{cc}^{*+} = -0.5$ MeV

where $\delta_{cc}^{*+} = m_{T_{cc}^{*+}} - m_c^* - m_0^*$

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where $\delta_{cc}^{*+} = m_{T_{cc}^{*+}} - m_c^* - m_0^*$

Disclaimer:

- Coupled-channel effects D^*D-D^*D^* neglected
- Multi-body effects & OPE included not selfconsistently

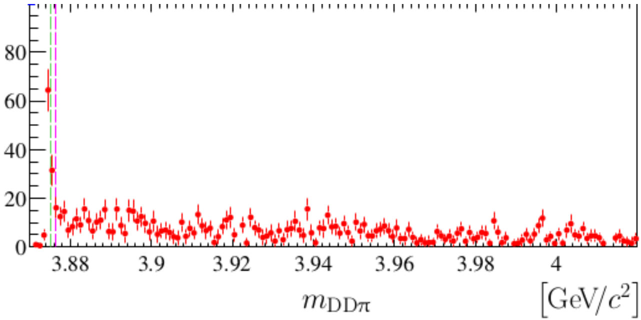
Conclusion: T_{cc}^{*+} is likely to exist but no reliable prediction is possible yet

Spin partner T_{cc}^{*+}

HQSS: $V^{I=0}(D^*D^* \to D^*D^*, 1^+) = V^{I=0}(D^*D \to D^*D, 1^+) = v_0$

T_{cc}^{+} at

LHCb Collab., Nature Communications, **13**, 3351 (2022)



Disclai

- C
- M

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T_{cc}^{+} in lattice QCD

- "Signature of a Doubly Charm Tetraquark Pole in DD^* Scattering on Lattice,"
  M. Padmanath and S. Prelovsek,
  Phys. Rev. Lett. **129**, 032002 (2022)
  S. Collins, A. Nefediev, M. Padmanath and S. Prelovsek
  arXiv:2402.14715 [hep-lat], Phys. Rev. D, in press

  $m_\pi = 280$ MeV   5 points in m_c

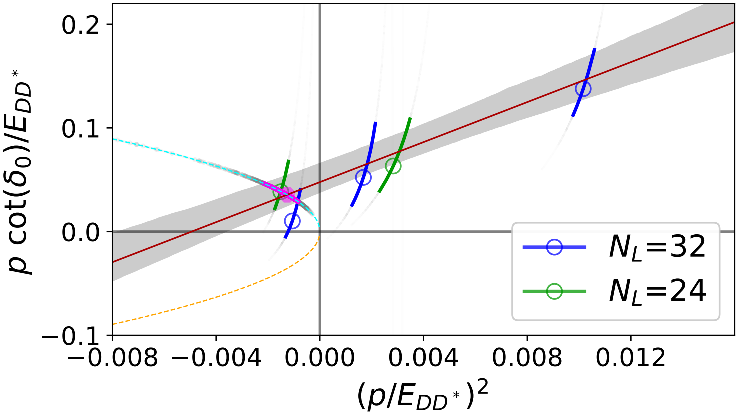
- "$T_{cc}^{+}(3875)$ relevant DD^* scattering from $N_f = 2$ lattice QCD,"
  S. Chen, C. Shi, Y. Chen, M. Gong, Z. Liu, W. Sun and R. Zhang,
  Phys. Lett. B **833**, 137391 (2022)

  $m_\pi = 348$ MeV

- "Doubly Charmed Tetraquark T_{cc}^{+} from Lattice QCD near Physical Point,"
  Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda and J. Meng,
  Phys. Rev. Lett. **131**, 161901 (2023)

  $m_\pi = 146$ MeV   HALQCD technique

ERE analysis of lattice data for T_{cc}^+



(Padmanath & Prelovsek'2022)

$$-\frac{2\pi}{\mu}T^{-1}(E) = p\cot\delta - ip = \frac{1}{a_0} + \frac{1}{2}r_0p^2 - ip$$

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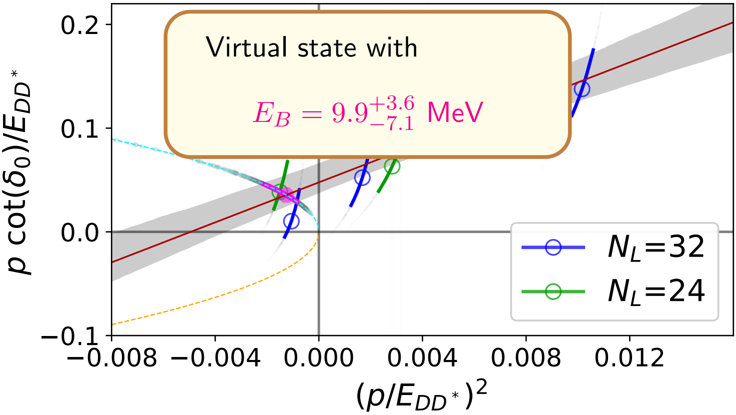

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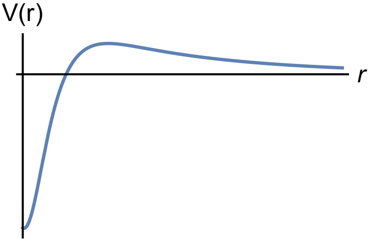
EFT analysis of lattice data for T_{cc}^+

Lippmann–Schwinger equation

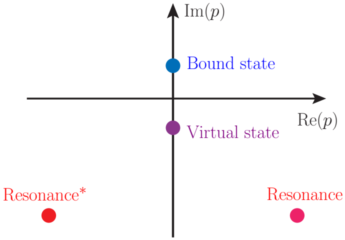
$$T(\boldsymbol{p},\boldsymbol{p}';E) = V(\boldsymbol{p},\boldsymbol{p}') - \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p},\boldsymbol{k}) G(\boldsymbol{k};E) T(\boldsymbol{k},\boldsymbol{p}';E)$$

$$V(\boldsymbol{p},\boldsymbol{p}') = \underbrace{\left[2c_0 + 2c_2(p^2+p'^2)\right]}_{\text{Contact interactions}} + \underbrace{V_\pi^S(p,p')}_{S\text{-wave OPE}}$$

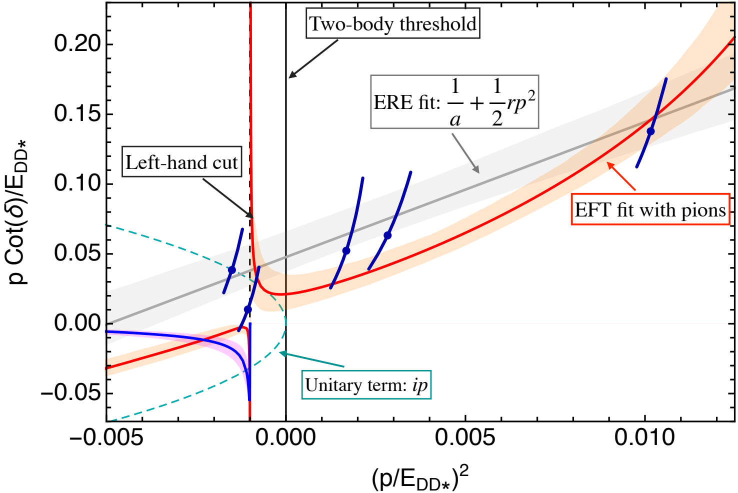
Sketch of full potential



Types of supported poles



EFT analysis of lattice data for T_{cc}^+



Lattice data: Padmanath & Prelovsek, Phys.Rev.Lett. 129 (2022), 032002
Theoretical curve: Du et al., Phys.Rev.Lett. 131 (2023), 131903

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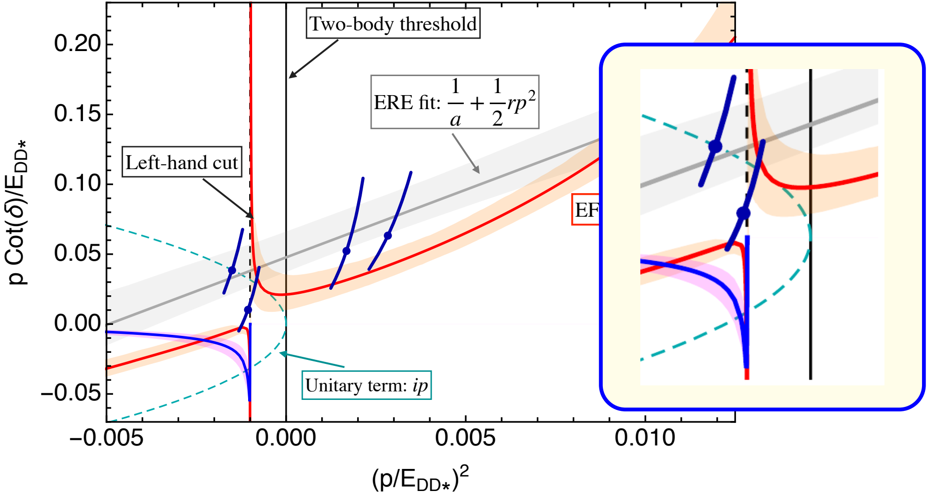

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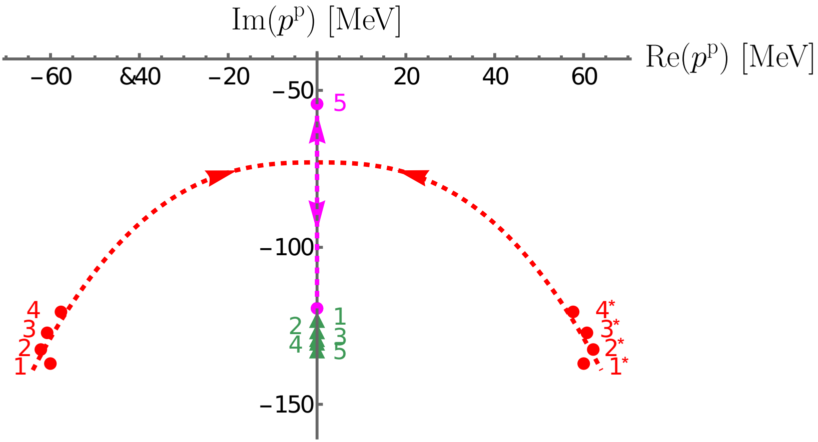
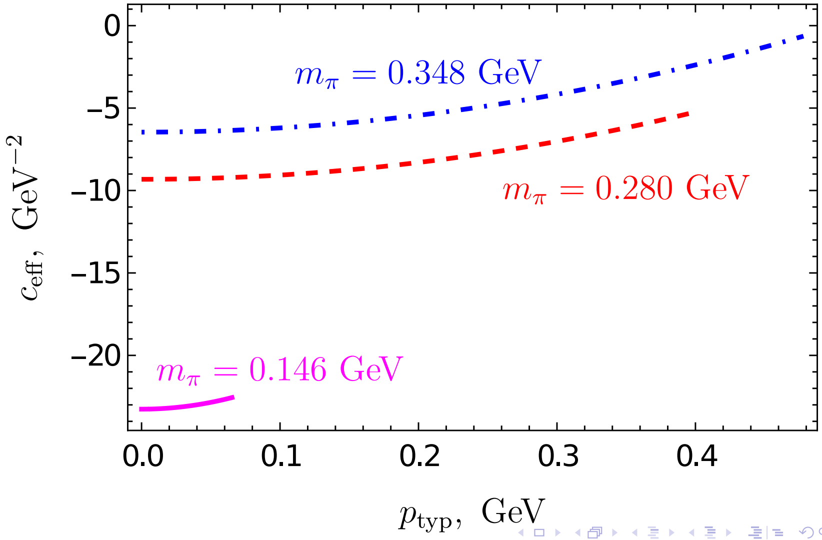
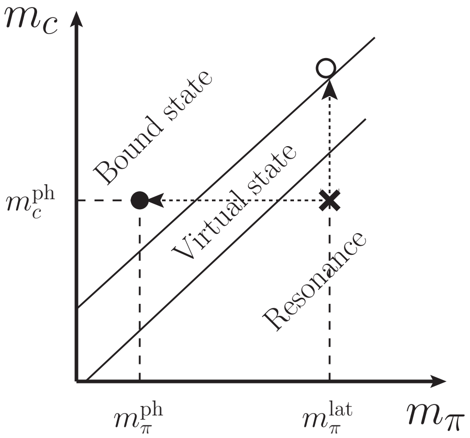
Lattice T_{cc}^{+} pole dependence on m_c



Comment on lattice T_{cc}^{+} pole dependence on m_π



T_{cc}^{+} pole motion across (m_c, m_π) plane



- Filled circle — physical T_{cc}^{+}
- Cross — starting lattice point
- Open circle — lattice T_{cc}^{+} as shallow bound state

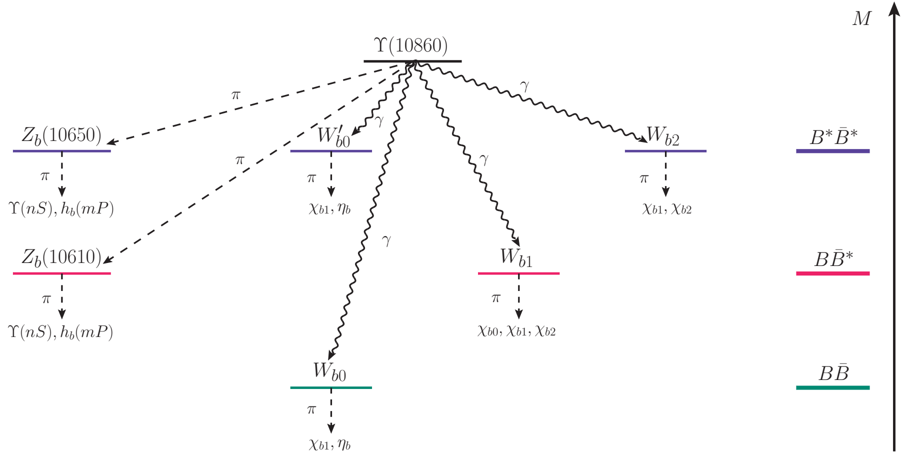
Twins $Z_b(10610)$ & $Z_b(10650)$

$$I = 1 \quad J^{PC} = 1^{+-}$$

Minimal quark content: $\bar{b}b\bar{q}q$

$$\Upsilon(10860) \to \pi Z_b^{(\prime)} \to \pi\left[B\bar{B}^{(*)}\right]$$
$$\Upsilon(10860) \to \pi Z_b^{(\prime)} \to \pi\left[\pi h_b(1,2P)\right]$$
$$\Upsilon(10860) \to \pi Z_b^{(\prime)} \to \pi\left[\pi\Upsilon(1,2,3S)\right]$$

Z_b's ($J^{PC}=1^{+-}$) and W_{bJ}'s ($J^{PC}=J^{++}$) in decays of $\Upsilon(10860)$



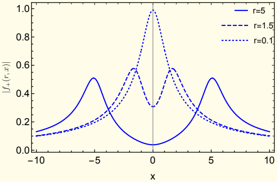
$$Z_b(10610) \quad\sim\quad B\bar{B}^* \sim 0^-_{\bar{q}b}\otimes 1^-_{\bar{b}q} \quad\sim\quad 1^-_{\bar{b}b}\otimes 0^-_{\bar{q}q} + 0^-_{\bar{b}b}\otimes 1^-_{\bar{q}q}$$

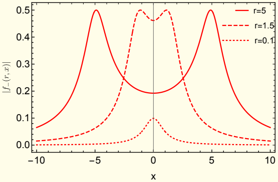
$$Z'_b(10650) \quad\sim\quad B^*\bar{B}^* \sim 1^-_{\bar{q}b}\otimes 1^-_{\bar{b}q} \quad\sim\quad 1^-_{\bar{b}b}\otimes 0^-_{\bar{q}q} - 0^-_{\bar{b}b}\otimes 1^-_{\bar{q}q}$$

(Bondar et al'2011,Voloshin'2011,...)

Z_b's ($J^{PC}=1^{+-}$) and W_{bJ}'s ($J^{PC}=J^{++}$) in decays of $\Upsilon(10860)$

M





$\Longrightarrow$ Constructive interference between Z_b & Z_b' in $\pi\pi\Upsilon$ channels

$\Longrightarrow$ Destructive interference between Z_b & Z_b' in $\pi\pi h_b$ channels

$\Longrightarrow$ Relevant (HQSS breaking!) parameter $r=(m_{z'}-m_z)/\Gamma_z$ ($r_{\rm phys}\approx 3$)

$\Longrightarrow$ $\text{Br}(\pi\pi h_b)[r_{\rm phys}]/\text{Br}(\pi\pi\Upsilon)[r_{\rm phys}]\sim 1$

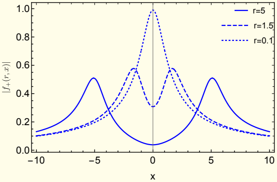
$$Z_b(10610) \sim B\bar{B}^* \sim 0^-_{\bar{q}b}\otimes 1^-_{\bar{b}q} \sim 1^-_{\bar{b}b}\otimes 0^-_{\bar{q}q}+0^-_{\bar{b}b}\otimes 1^-_{\bar{q}q}$$

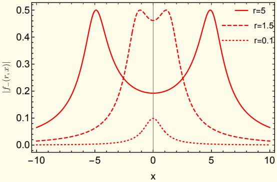
$$Z_b'(10650) \sim B^*\bar{B}^* \sim 1^-_{\bar{q}b}\otimes 1^-_{\bar{b}q} \sim 1^-_{\bar{b}b}\otimes 0^-_{\bar{q}q}-0^-_{\bar{b}b}\otimes 1^-_{\bar{q}q}$$

(Bondar et al'2011,Voloshin'2011,...)

Z_b's ($J^{PC} = 1^{+-}$) and W_{bJ}'s ($J^{PC} = J^{++}$) in decays of $\Upsilon(10860)$

M





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$$p_{\text{coupl.ch.}} = \sqrt{m_B(m_{B^*} - m_B)} \approx 500 \text{ MeV} \Longrightarrow \begin{cases} \Lambda \simeq 1 \text{ GeV} \\ \text{Potential at NLO} \\ \text{OPE included } (D \text{ waves!}) \end{cases}$$

Z_b's in EFT approach

$B^{(*)}\bar{B}^*$ potential:

$$V = V_{\rm CT}(\text{to order } O(p^0))$$

Coupled channels:

$$1^{+-} : B\bar{B}^*(^3S_1, -), B^*\bar{B}^*(^3S_1)$$
$$0^{++} : B\bar{B}(^1S_0), B^*\bar{B}^*(^1S_0)$$
$$1^{++} : B\bar{B}^*(^3S_1, +)$$
$$2^{++} : B^*\bar{B}^*(^5S_2)$$

Z_b's in EFT approach

$B^{(*)}\bar{B}^{*}$ potential:

$$V = V_{\rm CT}(\text{to order } O(p^2)) + V_\pi$$

Coupled channels:

$$\begin{aligned}
1^{+-} &: B\bar{B}^*({}^3S_1,-), B^*\bar{B}^*({}^3S_1), B\bar{B}^*({}^3D_1,-), B^*\bar{B}^*({}^3D_1)\\
0^{++} &: B\bar{B}({}^1S_0), B^*\bar{B}^*({}^1S_0), B^*\bar{B}^*({}^5D_0)\\
1^{++} &: B\bar{B}^*({}^3S_1,+), B\bar{B}^*({}^3D_1,+), B^*\bar{B}^*({}^5D_1)\\
2^{++} &: B^*\bar{B}^*({}^5S_2), B\bar{B}({}^1D_2), B\bar{B}^*({}^3D_2),\\
&\quad B^*\bar{B}^*({}^1D_2), B^*\bar{B}^*({}^5D_2), \cancel{B^*\bar{B}^*({}^5G_2)}
\end{aligned}$$

Lippmann-Schwinger equation $(\alpha,\beta,\gamma = (B\bar{B}^*, B^*\bar{B}^*)\otimes(L=0, L=2))$:

$$T_{\alpha\beta}(M,\boldsymbol{p},\boldsymbol{p}') = V^{\rm eff}_{\alpha\beta}(\boldsymbol{p},\boldsymbol{p}') - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V^{\rm eff}_{\alpha\gamma}(\boldsymbol{p},\boldsymbol{q}) G_\gamma(M,\boldsymbol{q}) T_{\gamma\beta}(M,\boldsymbol{q},\boldsymbol{p}')$$

$B^{(*)}\bar{B}^{*}$ pot

Coupled ch

1 $^{3}D_{1})$

0

1

2

$_{2})$

Lippmann-S $, L=2))$:

$T_{\alpha\beta}(M,\boldsymbol{p},$ $_{\beta}(M,\boldsymbol{q},\boldsymbol{p}')$

Free parameters:

- Contact potentials (4)
- Couplings to hidden-bottom channels (5)
- Overall normalisations (7)
- π-π interaction (6)

Total: 22

$B^{(*)}\bar{B}^*$ pot

Coupled ch

1

0

1

2

$^3D_1)$

$2)$

Lippmann-S

$, L = 2))$:

$T_{\alpha\beta}(M, \boldsymbol{p},$ … $_\beta(M, \boldsymbol{q}, \boldsymbol{p}')$

Free parameters:

- Contact potentials (4)
- Couplings to hidden-bottom channels (5)
- Overall normalisations (7)
- π-π interaction (6)

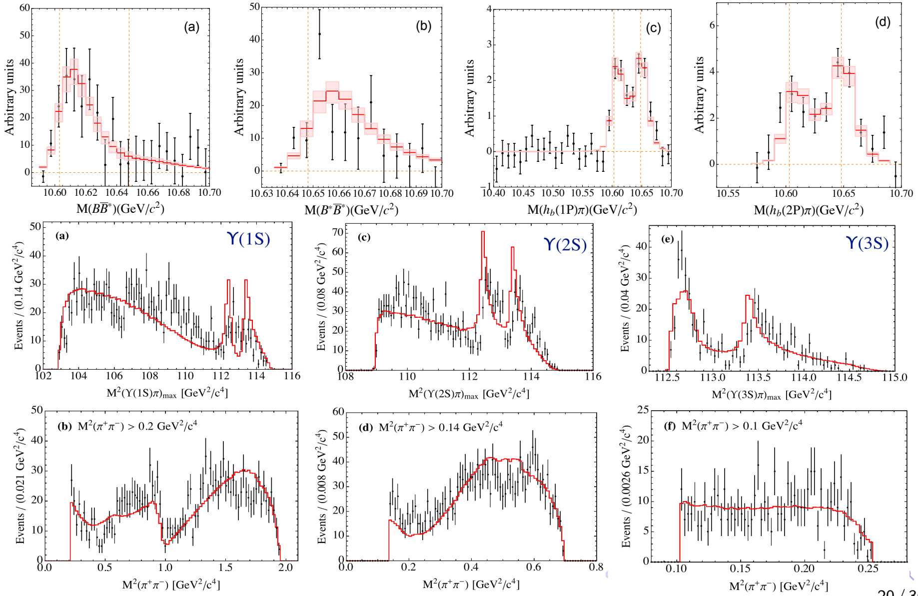
Total: 22

Naive sum of BW's:

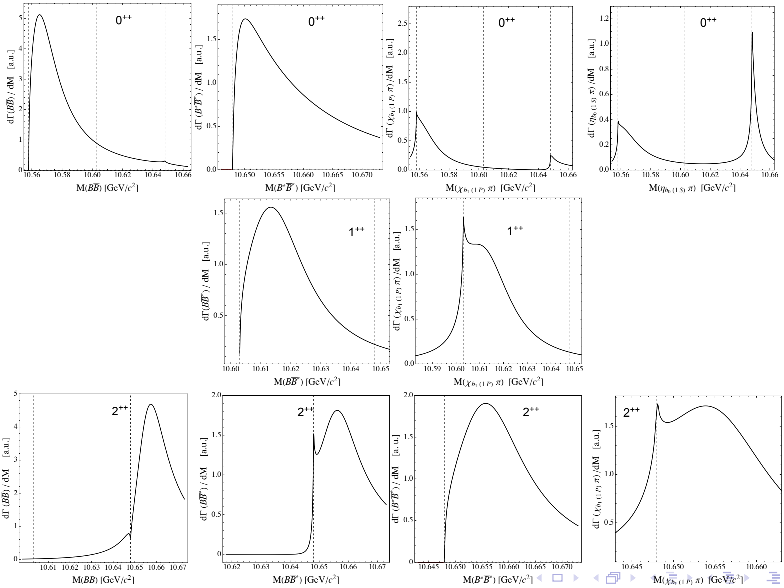
- Masses $(2 \times 7 = 14)$
- Widths $(2 \times 7 = 14)$
- Relative phases (7)
- Overall normalisations (7)
- π-π interaction (?)

Total: > 42

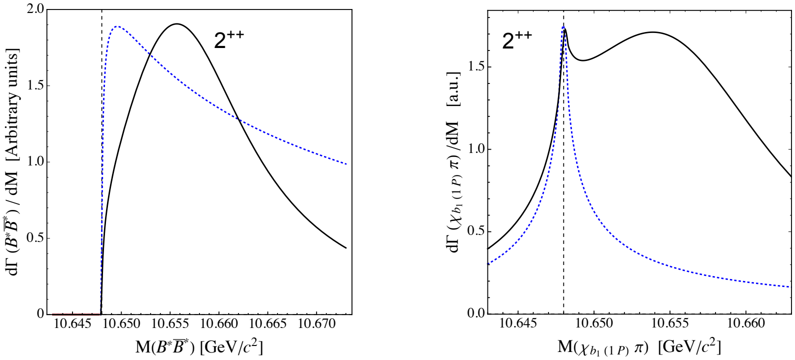
Fitted line shapes for Z_b's



Predicted line shapes for W_{bJ}'s



Role of pions



- Blue dashed line — pionless theory
- Black solid line — full theory with pions

Strange $Z_{cs}(3982)$

$$J^{PC} = 1^{+-}$$

Minimal quark content: $c\bar{c}s\bar{q}$

$$e^+e^- \to K^+\left[D_s^- D^{*0} + D_s^{*-} D^0\right]$$

Expectations

- Twin bottomonium-like Z_b states $(I = 1,\ J^{PC} = 1^{+-})$ as $B^{(*)}\bar{B}^*$ molecules

$$
\begin{aligned}
Z_b(10610) &\sim B\bar{B}^* \sim 0^-_{\bar{q}b} \otimes 1^-_{\bar{b}q} \sim 1^-_{\bar{b}b} \otimes 0^-_{\bar{q}q} + 0^-_{\bar{b}b} \otimes 1^-_{\bar{q}q} \\
Z'_b(10650) &\sim B^*\bar{B}^* \sim 1^-_{\bar{q}b} \otimes 1^-_{\bar{b}q} \sim 1^-_{\bar{b}b} \otimes 0^-_{\bar{q}q} - 0^-_{\bar{b}b} \otimes 1^-_{\bar{q}q}
\end{aligned}
$$

- Similar pattern in the spectrum of charmonium

$$
\begin{aligned}
Z_c(3900) &\sim D\bar{D}^* \\
Z'_c(4020) &\sim D^*\bar{D}^*
\end{aligned}
$$

- Flavour $SU(3)$ for light quarks

$\Longrightarrow$ Accurate for couplings & potentials

$\Longrightarrow$ Explicit breaking via $m_s \gg m_{u,d}$

$\Longrightarrow$ Simple relation between potentials in $I = 1/2$ and $I = 1$ channels

Expectations

- Twin bottomonium-like Z_b states $(I = 1,\ J^{PC} = 1^{+-})$ as $B^{(*)}\bar{B}^*$ molecules

$$Z_b(10610) \quad \sim \quad B\bar{B}^* \sim 0^-_{\bar{q}b} \otimes 1^-_{\bar{b}q} \sim 1^-_{\bar{b}b} \otimes 0^-_{\bar{q}q} + 0^-_{\bar{b}b} \otimes 1^-_{\bar{q}q}$$
$$Z_b'(10650) \quad \sim \quad B^*\bar{B}^* \sim 1^-_{\bar{q}b} \otimes 1^-_{\bar{b}q} \sim 1^-_{\bar{b}b} \otimes 0^-_{\bar{q}q} - 0^-_{\bar{b}b} \otimes 1^-_{\bar{q}q}$$

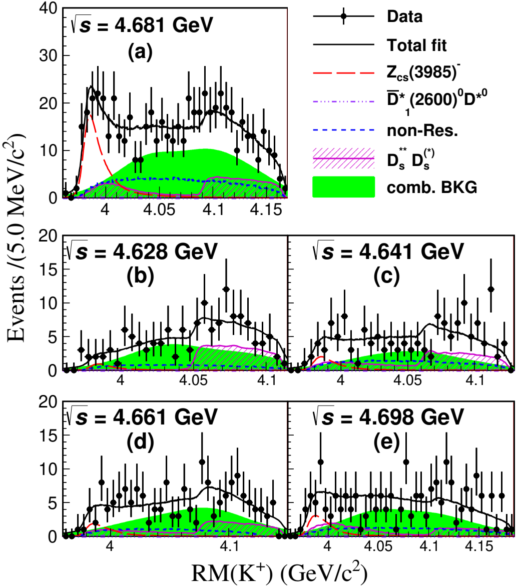
- Similar pattern in the spectrum of charmonium

$$Z_c(3900) \quad \sim \quad D\bar{D}^*$$
$$Z_c'(4020) \quad \sim \quad D^*\bar{D}^*$$

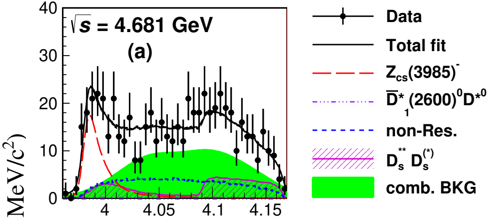
- Flavour $SU(3)$ for light quarks
  - $\Longrightarrow$ Accurate for couplings & potentials
  - $\Longrightarrow$ Explicit breaking via $m_s \gg m_{u,d}$
  - $\Longrightarrow$ Simple relation between potentials in $I = 1/2$ and $I = 1$ channels

Expect: Z_{bs} ($\sqrt{s} \gtrsim 11.2$ GeV) and Z_{cs} ($\sqrt{s} \gtrsim 4.5$ GeV) molecular states exist

Z_{cs} @ BES III (Phys.Rev.Lett. 126 (2021) 10, 102001)



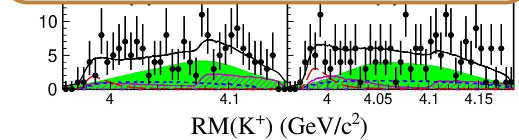
Z_{cs} @ BES III (Phys.Rev.Lett. 126 (2021) 10, 102001)



Discovery of $Z_{cs}(3982)$

$$M = 3982.5^{+1.8}_{-2.6} \pm 2.1 \text{ MeV}$$

$$\Gamma = 12.8^{+5.3}_{-4.4} \pm 3.0 \text{ MeV}$$



Theoretical framework

- Effective Field Theory (EFT) approach $\Longrightarrow$ LO short-range potential
- Heavy Quark Spin Symmetry (HQSS) $\Longrightarrow$ Multiplets of particles
- Flavour $SU(3)$ $\Longrightarrow$ symmetric potential + explicit breaking via masses
- Number-of-events distribution

$$\frac{dN}{dm_{23}} = \frac{d\sigma}{dm_{23}}\bar{\epsilon}\,\mathcal{L}_{\rm int}\,f_{\rm corr}$$

$\bar{\epsilon}$ – efficiency, $\mathcal{L}_{\rm int}$ – integrated luminosity, $f_{\rm corr}$ – radiative & vacuum polarisation correction

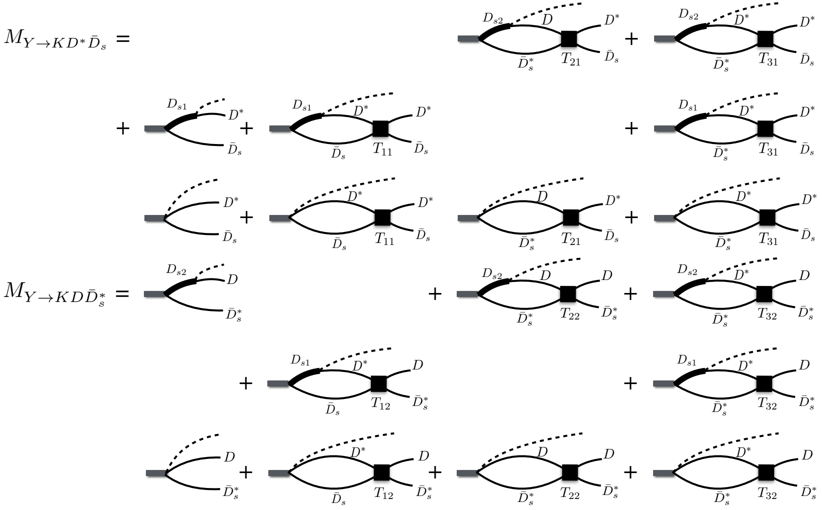
- Maximum likelihood fit

$$-2\log\mathcal{L} = 2\sum_i\left(\mu_i - n_i + n_i\log\frac{n_i}{\mu_i}\right)$$

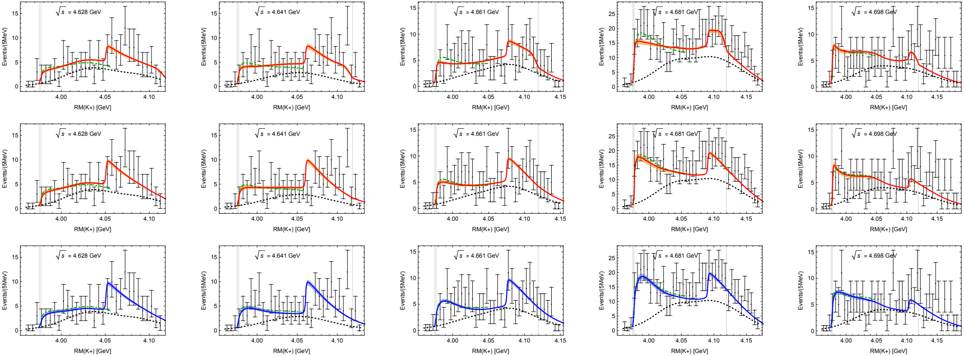
n_i – number of events, μ_i – theoretical signal function

- Combined fit of 5 distributions with 5 fitting parameters

Amplitudes


$M_{Y \to K D^* \bar{D}_s} =$
$M_{Y \to K D \bar{D}_s^*} =$
D_{s2}
D_{s1}
D
D^*
$\bar{D}_s$
$\bar{D}_s^*$
T_{11}
T_{12}
T_{21}
T_{22}
T_{31}
T_{32}

Fit results and different scenarios

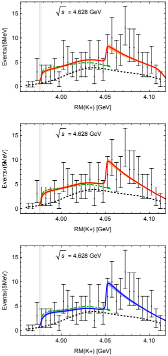


Scenario 1

$$\begin{tikzcd} Z_c(3900) \arrow[r, leftrightarrow, "\mathrm{HQSS}"] \arrow[d, leftrightarrow, "\mathrm{SU(3)}"'] & Z_c(4020) \arrow[d, leftrightarrow, "\mathrm{SU(3)}"] \\ Z_{cs}(3982) \arrow[r, leftrightarrow, "\mathrm{HQSS}"] & Z'_{cs} \end{tikzcd}$$

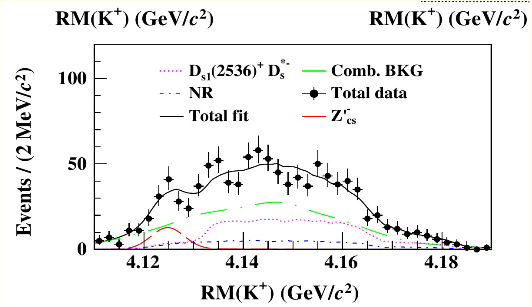
Scenario 2

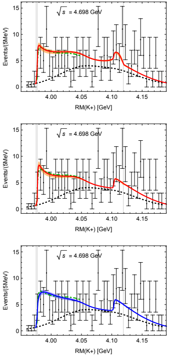
$$\begin{tikzcd} Z_c(3900) \arrow[r, leftrightarrow, "\cancel{\mathrm{HQSS}}"] \arrow[d, leftrightarrow, "\mathrm{SU(3)}"'] & Z_c(4020) \\ Z_{cs}(3982) & \text{no } Z'_{cs} \end{tikzcd}$$



BES III: Chin. Phys. C **47**, 033001 (2023)

$e^+e^- \to K^+ D_s^{*-} D^{*0} + c.c.$





$$\begin{tikzcd}
Z_c(3900) \arrow[r, leftrightarrow, "\mathrm{HQSS}"] \arrow[d, leftrightarrow, "\mathrm{SU(3)}"'] & Z_c(4020) \arrow[d, leftrightarrow, "\mathrm{SU(3)}"] \\
Z_{cs}(3982) \arrow[r, leftrightarrow, "\mathrm{HQSS}"] & Z'_{cs}
\end{tikzcd}$$

$$\begin{tikzcd}
Z_c(3900) \arrow[r, leftrightarrow, "\cancel{\mathrm{HQSS}}"] \arrow[d, leftrightarrow, "\mathrm{SU(3)}"'] & Z_c(4020) \\
Z_{cs}(3982) & \text{no } Z'_{cs}
\end{tikzcd}$$

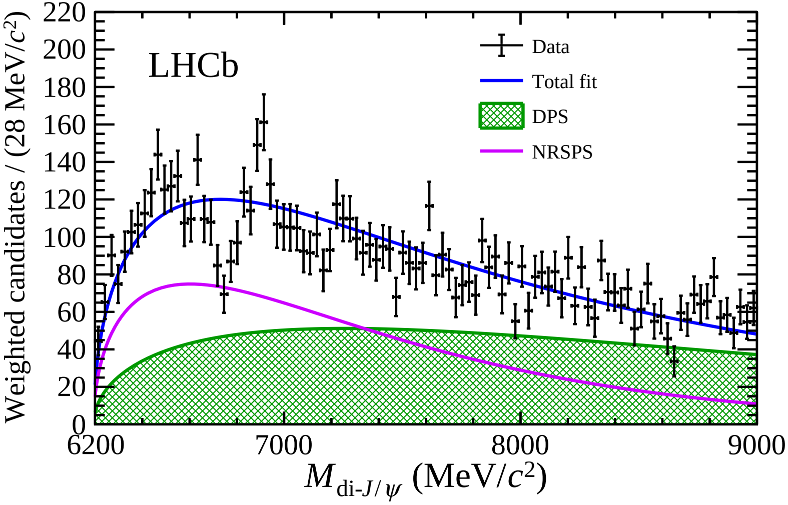
Double-J/ψ spectrum

$X(6200)$ vs $X(6900)$

$$I = 0 \quad J^{PC} = 0^{++}/2^{++}$$

Minimal quark content: $\bar{c}\bar{c}cc$

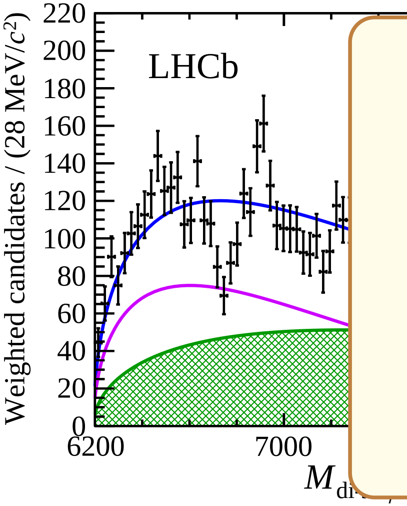
LHCb: nonresonant production



NRSPS=NonResonant Single Parton Scattering

DPS=Double Parton Scattering

LHCb: conclusions from analysis



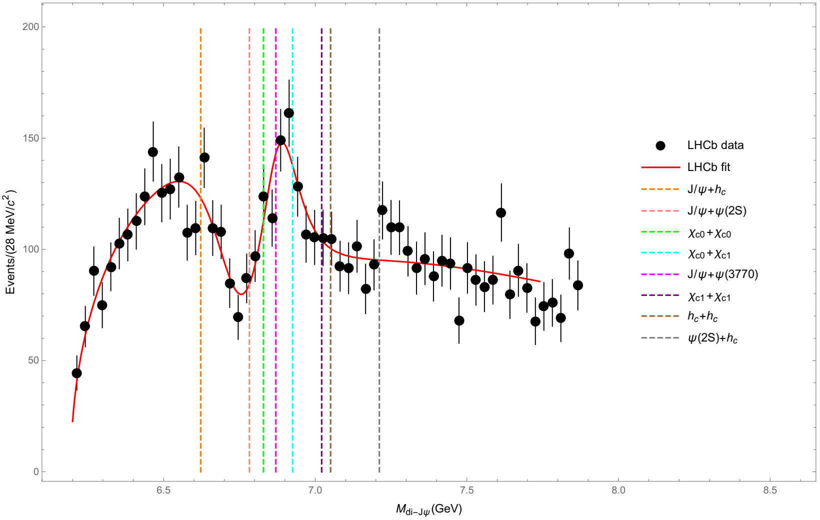
Thus LHCb reports:

- A narrow resonance-like structure at 6.9 GeV
- A broad structure just above double-J/ψ threshold
- 5σ deviation from nonresonant double-J/ψ production

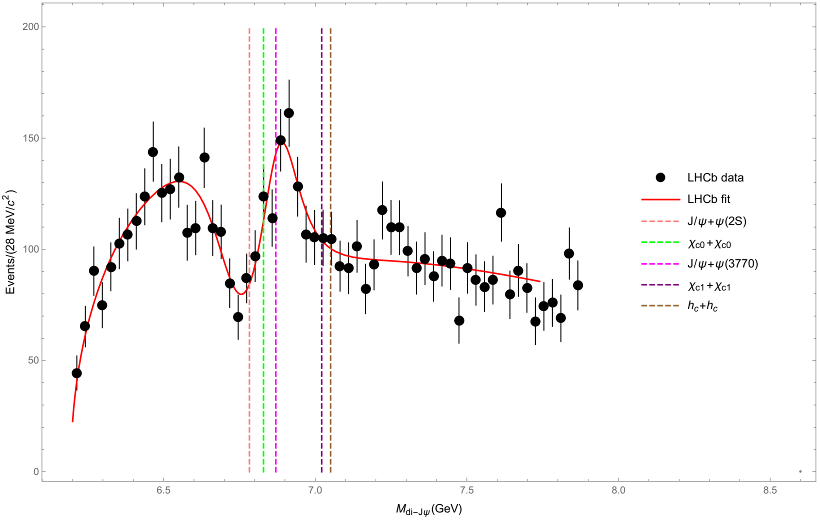
Choosing relevant channels

- Restrict ourselves to thresholds in the range 6.2-7.2 GeV
- Consider only S-wave channels
- Compatible with light exchanges
  - $J/\psi J/\psi \;\Leftrightarrow\; \chi_{cJ}\chi_{cJ}\;(J=0,1)$
    Lowest exchange particle (ω) is (relatively) heavy $\Longrightarrow$ suppression
  - $J/\psi J/\psi \;\Leftrightarrow\; \psi(2S)J/\psi, \psi(3770)J/\psi, \ldots$
    Mediated by soft gluons (two pions) $\Longrightarrow$ no suppression
- Retain only HQSS-allowed channels
  - $J/\psi J/\psi \;\Leftrightarrow\; h_c h_c$
    Heavy quark spin flip needed $\Longrightarrow$ suppressed by $\Lambda_{\rm QCD}/m_c \ll 1$ (HQSS)
  - $J/\psi J/\psi \;\Leftrightarrow\; \psi(2S)J/\psi, \psi(3770)J/\psi$
    No c-quark spin flip needed $\Longrightarrow$ HQSS-allowed

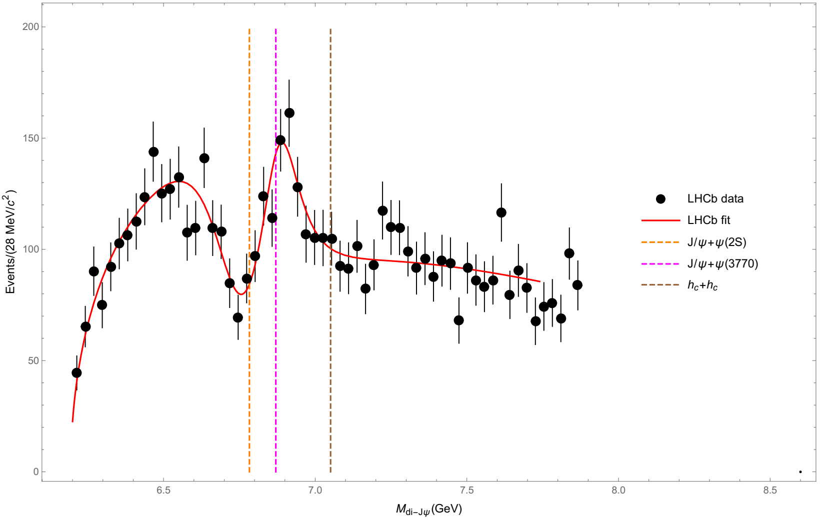
All channels



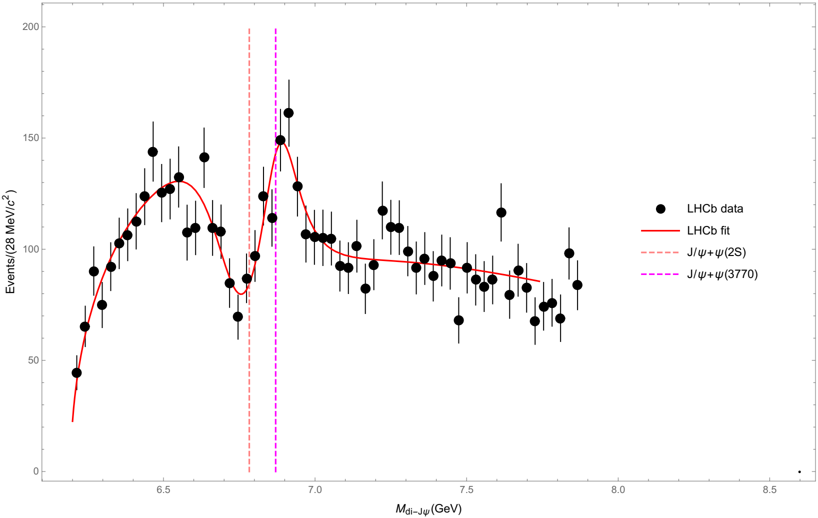
Only S-wave channels (no $J/\psi h_c$, $\psi(2S)h_c$, $\chi_{c0}\chi_{c1}$)



No heavy exchanges (no $\chi_{c0}\chi_{c0}$, $\chi_{c1}\chi_{c1}$)



Only HQSS-allowed channels (no $h_c h_c$)



The models

Two-channel model (7 parameters)
$J/\psi J/\psi$ & $\psi(2S)J/\psi$

$$V_{2\mathrm{ch}}(E) = \begin{pmatrix} a_1 + b_1 k_1^2 & c \\ c & a_2 + b_2 k_2^2 \end{pmatrix}$$

Three-channel model (8 parameters)
$J/\psi J/\psi,\ \psi(2S)J/\psi$ & $\psi(3770)J/\psi$

$$V_{3\mathrm{ch}}(E) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

Lippmann-Schwinger equation

$$T(E) = V(E) \cdot [1 - G(E)V(E)]^{-1}$$

Production amplitude in $J/\psi J/\psi$ channel (channel 1):

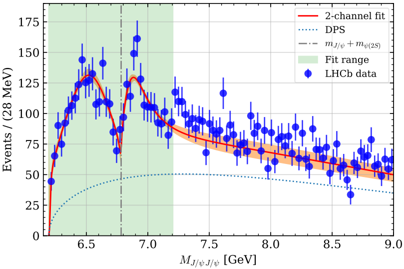
$$\mathcal{M}_1 = \alpha e^{-\beta E^2}\left[b + G_1(E)T_{11}(E) + G_2(E)T_{21}(E) + r_3 G_3(E)T_{31}(E)\right]$$

Slope β fixed to double-parton scattering (DPS): $\beta = 0.0123$ GeV^{-2}

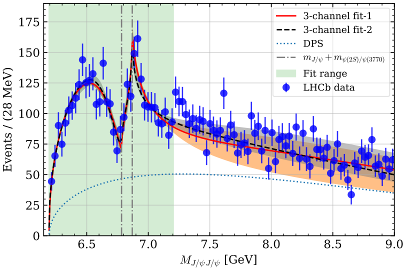
$$r_3 = \begin{cases} 0 & \text{2ch model} \\ 1 & \text{3ch model} \end{cases}$$

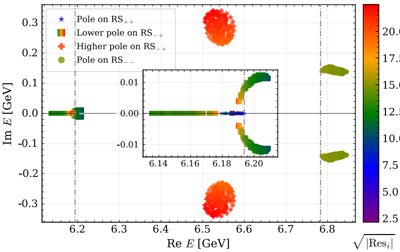
Fits & poles

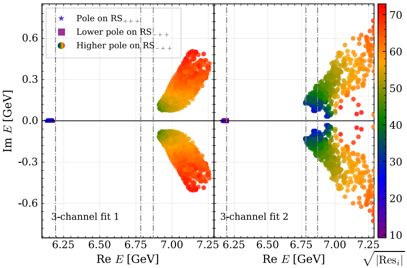
$J/\psi J/\psi$ & $J/\psi\psi(2S)$



$J/\psi J/\psi$, $J/\psi\psi(2S)$ & $J/\psi\psi(3770)$







$X(6200)$ vs $X(6900)$

- Poles above the double-J/ψ threshold ($X(6900)$) are badly determined
- Pole near the double-J/ψ threshold ($X(6200)$) is robust

$$E_0^{\rm 2ch} = 6203^{+\ 6}_{-27} - i\,12^{+\ 1}_{-12}\ ({\rm RS}_{-+})\ \text{or}\ [6179, 6194]\ ({\rm RS}_{++})$$
$$E_0^{\rm 3ch}[\text{Fit 1}] = 6163^{+18}_{-32}\ ({\rm RS}_{+++})$$
$$E_0^{\rm 3ch}[\text{Fit 2}] = 6189^{+\ 5}_{-10}\ ({\rm RS}_{-++})\ \text{or}\ [6159, 6194]\ ({\rm RS}_{+++})$$

- Compositeness of $X(6200)$ is large $\Longrightarrow$ hint for a molecule

$$T(k) \approx -8\pi\sqrt{s}\left[\frac{1}{a_0} + \frac{1}{2}r_0 k^2 - ik\right]^{-1}$$

$$\bar{X}_A = (1 + 2|r_0/a_0|)^{-1/2} \sim 1$$

Matuschek.et al. Eur.Phys.J. **A57** (2021) 101

Conclusions

- Collider experiments at energies above open-flavour thresholds started new era in hadronic physics $\Longrightarrow$ Many surprises from experiment
- Many prominent candidates for hadronic molecules
- Lattice QCD $\Longrightarrow$ alternative source of information on exotic hadrons
- Well established theoretical tools $\Longrightarrow$ reliable conclusions/predictions
- Upgraded/future experimental facilities $\Longrightarrow$ Quo vadis?
- Deuteron(90), X(20), Z(10), pentaquarks(9), di-J/ψ(3), T_{cc}^{+}(2),...

  to be continued

Backup

Pole positions & Branching fractions

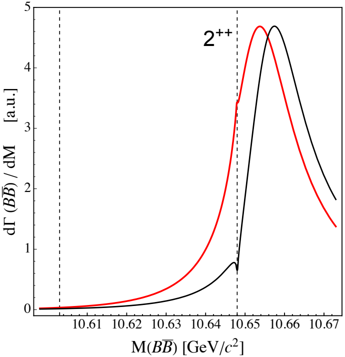
Extracted pole positions for Z_b's and W_{bJ}'s:

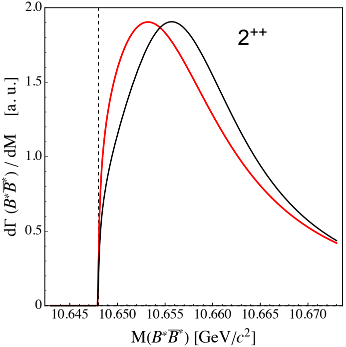
J^{PC}	State	Threshold	E_B w.r.t. threshold, [MeV]
1^{+-}	Z_b	$B\bar{B}^*$	$(-2.3 \pm 0.5) - i(1.1 \pm 0.1)$
1^{+-}	Z_b'	$B^*\bar{B}^*$	$(1.8 \pm 2.0) - i(13.6 \pm 3.1)$
0^{++}	W_{b0}	$B\bar{B}$	$(2.3 \pm 4.2) - i(16.0 \pm 2.6)$
0^{++}	W_{b0}'	$B^*\bar{B}^*$	$(-1.3 \pm 0.4) - i(1.7 \pm 0.5)$
1^{++}	W_{b1}	$B\bar{B}^*$	$(10.2 \pm 2.5) - i(15.3 \pm 3.2)$
2^{++}	W_{b2}	$B^*\bar{B}^*$	$(7.4 \pm 2.8) - i(9.9 \pm 2.2)$

Predicted partial branching fractions for W_{bJ}'s:

J^{PC}	$B\bar{B}$	$B\bar{B}^*$	$B^*\bar{B}^*$	$\chi_{b0}(1P)\pi$	$\chi_{b0}(2P)\pi$	$\chi_{b1}(1P)\pi$	$\chi_{b1}(2P)\pi$	$\chi_{b2}(1P)\pi$	$\chi_{b2}(2P)\pi$	$\eta_{b0}(1S)\pi$	$\eta_{b0}(2S)\pi$
0^{++}	0.73	—	0.14	—	—	0.05	0.06	—	—	0.002	0.01
1^{++}	—	0.76	—	0.03	0.06	0.02	0.04	0.04	0.05	—	—
2^{++}	0.06	0.07	0.54	—	—	0.03	0.06	0.09	0.16	—	—

Theoretical uncertainty estimate



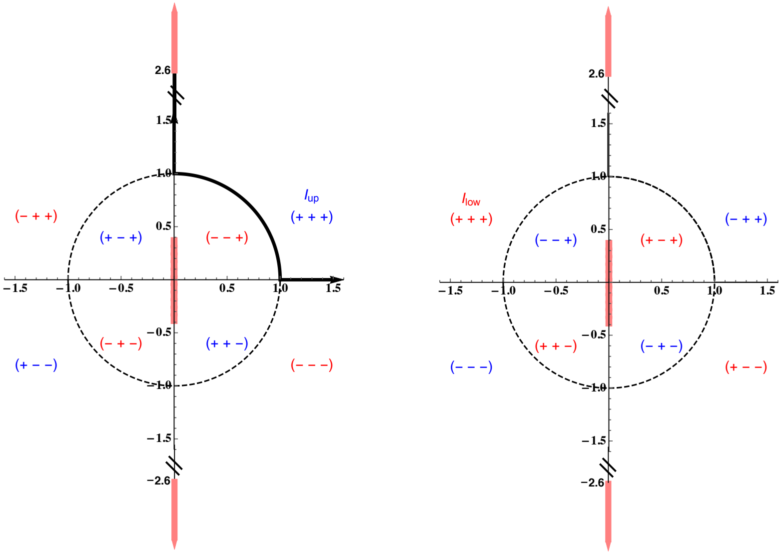


Red curve: complete LO

Black curve: (almost) complete NLO

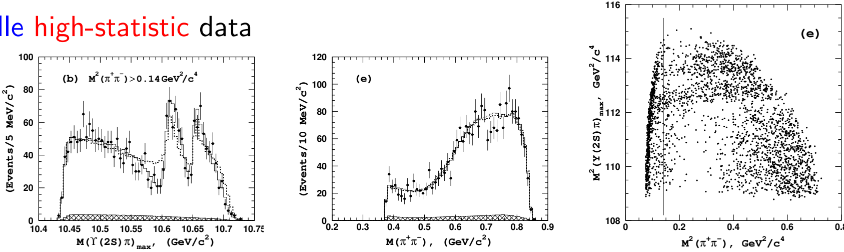
$$X^{(\nu)}(Q) = \sum_{n=0}^{\nu} \alpha_n \left(\frac{p_{\text{typ}}}{\Lambda}\right)^n \quad \underset{\text{NLO vs LO}}{\Longrightarrow} \quad \delta E \simeq E_{\text{typ}} \frac{p_{\text{typ}}}{\Lambda} \simeq 15 \frac{500}{1000} \simeq 7.5 \text{ MeV}$$

Complex ω-plane

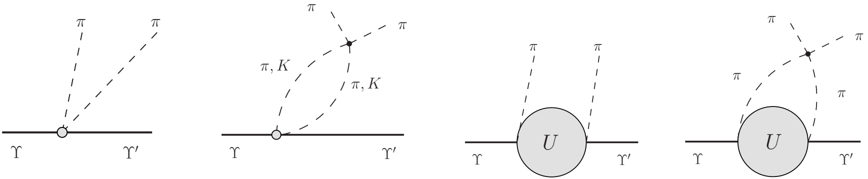


$\Upsilon(10860) \to \pi^+\pi^-\Upsilon(nS)$ $(n = 1, 2, 3)$ **decays**

Belle high-statistic data



Pions (kaons) FSI is included via dispersive technique

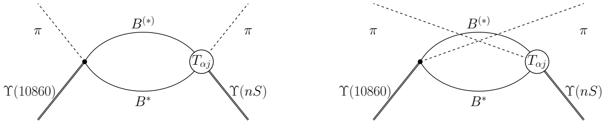


with all Im parts under theoretical control (real fitting parameters)

$\Upsilon(10860) \to \pi^+\pi^-\Upsilon(nS)$ $(n = 1, 2, 3)$ decays

- No-FSI amplitude: left-hand cuts only, U — the full coupled-channel amplitude

$$M_{\text{no-FSI}}(t,u) = U(t) + U(u) = M_0^L + M_{\text{higher}} = \frac{1}{2}\int_{-1}^{1} dz\, M_{\text{no-FSI}}(t,u) + M_{\text{higher}}$$



- Amplitude with FSI: right-hand cut included, M_0^R restored dispersively from M_0^L

$$M(s,t,u) = M_{\text{no-FSI}}(t,u) + \frac{\Omega_0(s)}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\Omega_0^{-1}(s')T(s')\sigma(s')M_0^L(s')}{s'-s-i\epsilon}$$

$$\Omega_0(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{T^*(s')\sigma(s)\Omega_0(s')}{s'-s-i\epsilon} \qquad T(s) = \begin{pmatrix} T_{\pi\pi\to\pi\pi} & T_{\pi\pi\to K\bar{K}} \\ T_{K\bar{K}\to\pi\pi} & T_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$

$\Upsilon(10860) \to \pi^+\pi^-\Upsilon(nS)$ $(n = 1, 2, 3)$ decays

- Diminish dependence on the large-s tail of Omnés

  $\Longrightarrow$ Two subtractions in dispersive integral

  $\Longrightarrow$ Second order polynomial $c_1 + c_2 s$ added to amplitude

- $\mathrm{Im}M_0^L$ under control (including anomalous pieces)

  $\Longrightarrow$ Real subtraction constants c_1 and c_2 [as opposed to Molnar et al'2019]

- Low-energy $\pi\Upsilon$ scattering is described by chiral Lagrangian

  $\Longrightarrow$ Matching c_1 and c_2 to chiral expansion

$\Upsilon(10860) \to \pi^+\pi^-\Upsilon(nS)$ $(n = 1, 2, 3)$ **decays**

- Diminish dependence on the large-s tail of Omnés

  $\Longrightarrow$ Two subtractions in dispersive integral

  $\Longrightarrow$ Second order polynomial $c_1 + c_2 s$ added to amplitude

- ImM_0^L under control (including anomalous pieces)

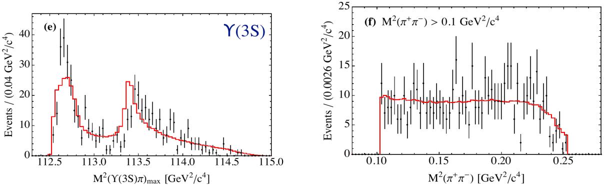
  $\Longrightarrow$ Real subtraction constants c_1 and c_2 [as opposed to Molnar et al'2019]

- Low-energy $\pi\Upsilon$ scattering is described by chiral Lagrangian

  $\Longrightarrow$ Matching c_1 and c_2 to chiral expansion

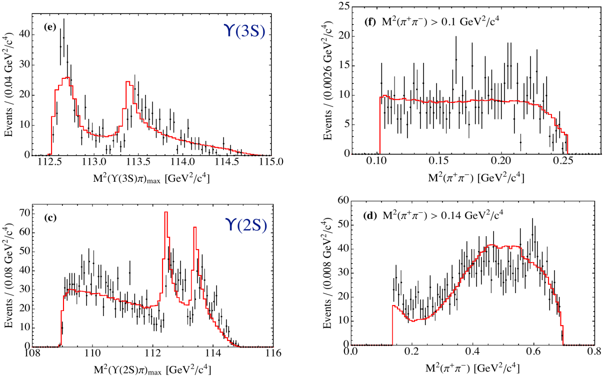
3 real fitting parameters: c_1, c_2, $\mathcal{N}$

Fit to data on $\Upsilon(10860) \to \pi^+\pi^-\Upsilon(nS)$ ($n = 1, 2, 3$)



- Data dominated by Z_b's
- No structures in $M_{\pi\pi}$

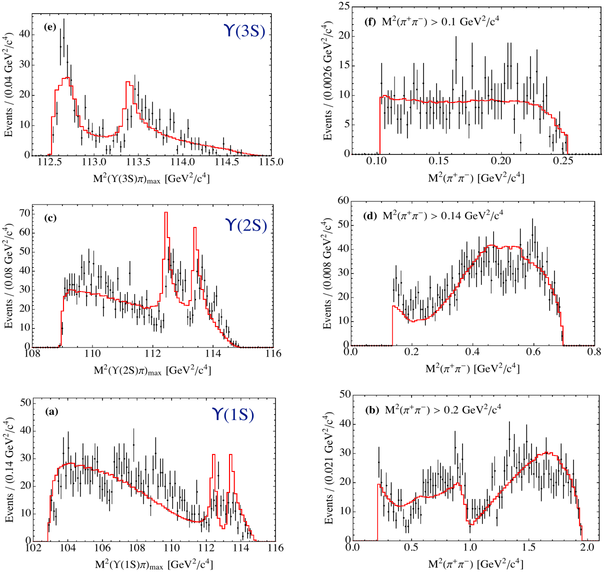
Fit to data on $\Upsilon(10860) \to \pi^+\pi^-\Upsilon(nS)$ ($n = 1, 2, 3$)



- Data dominated by Z_b's
- No structures in $M_{\pi\pi}$

- Additional structures in both $M^2(\pi\Upsilon)$ and $M^2(\pi\pi)$
- $\pi\pi$ FSI captures gross features

Fit to data on $\Upsilon(10860) \to \pi^+\pi^-\Upsilon(nS)$ ($n = 1, 2, 3$)



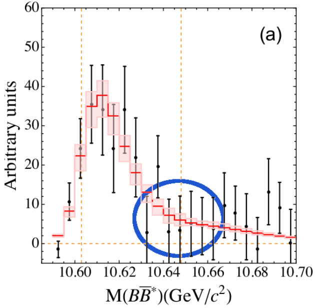
- Data dominated by Z_b's
- No structures in $M_{\pi\pi}$

- Additional structures in both $M^2(\pi\Upsilon)$ and $M^2(\pi\pi)$
- $\pi\pi$ FSI captures gross features

- Left shoulder in $M^2(\pi\Upsilon)$
- Highly nontrivial $M^2(\pi\pi)$
- $\pi\pi$ & $K\bar{K}$ FSI important

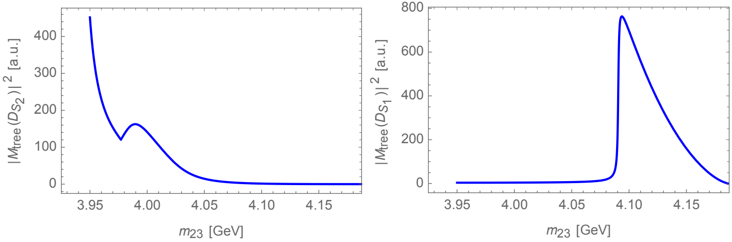
Conclusions from Z_b's analysis

- EFT approach provides good simultaneous description of all data
- Parameters are extracted directly from data
- Data are compatible with HQSS
- Parameter-free predictions for spin partners are made
- Effect from (long range) pion exchange is visible
- Puzzle of $B\bar{B}^*$-$B^*\bar{B}^*$ transitions:
  - Enhanced by pions
  - Not supported by data (surprise!)
  - EFT is flexible to adapt

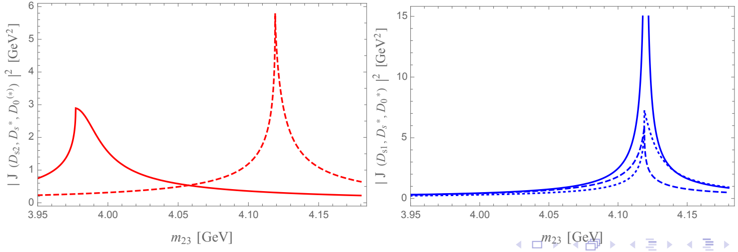


Relevance of various contributions

Tree-level diagrams with D_{sJ} $(J = 1, 2)$ exchanges



Triangle diagrams



Results

Fit	$\mathcal{C}_d$, fm^2	$\mathcal{C}_f$, fm^2	$g_{D_{s1}}/g_{D_{s2}}$	$g/g_{D_{s2}}$	$\mathcal{N}$, $10^{-2}\,\frac{\text{pb}}{\text{GeV}}$	$-2\log\mathcal{L}$
fit 1	-0.51 ± 0.02	0.18 ± 0.02	0.26 ± 0.02	-2.5 ± 0.3	0.46 ± 0.05	138
fit 1′	-0.24 ± 0.05	-0.1 ± 0.05	0.37 ± 0.03	-2.8 ± 0.6	0.35 ± 0.04	144
fit 2	0.50	-1.04 ± 0.01	-0.44 ± 0.03	-6.5 ± 2.5	0.28 ± 0.03	146

- Scenario 1

$J^{P(C)}$	State	Threshold, MeV		RS	Poles fit 1	RS	Poles fit 1′
1^+	$Z_{cs}(3982)$	$\bar{D}_sD^*/\bar{D}_s^*D$	$3975.2/3977.0$	$(+++)$	3942 ± 11	$(--+)$	3937^{+5}_{-29}
1^+	$Z_{cs}(3982)$	$\bar{D}_sD^*/\bar{D}_s^*D$	$3975.2/3977.0$	$(--+)$	3971 ± 2	$(--+)$	3972 ± 2
1^+	Z'_{cs}	$\bar{D}_s^*D^*$	4119.1	$(--+)$	$4115\pm2-(10\pm2)i$	$(++-)$	$4087^{+10}_{-10}+0^{+45}_{-0}i$
1^{+-}	$Z_c(3900)$	$(D\bar{D}^*,-)$	3871.7	$(++)$	3841 ± 11	$(-+)$	3832^{+25}_{-36}
1^{+-}	$Z_c(4020)$	$\bar{D}^*D^*$	4013.7	$(-+)$	$4009\pm18-(9\pm2)i$	$(+-)$	$3975^{+15}_{-10}+0^{+43}_{-0}i$

- Scenario 2

$J^{P(C)}$	State	Threshold, MeV		RS	Poles fit 2
1^+	$Z_{cs}(3982)$	$\bar{D}_sD^*/\bar{D}_s^*D$	$3975.2/3977.0$	$(+++)$	3954 ± 2
1^+	$Z_{cs}(3982)$	$\bar{D}_sD^*/\bar{D}_s^*D$	$3975.2/3977.0$	$(--+)$	$3959\pm7-(47\pm16)i$
1^+	Z'_{cs}	$\bar{D}_s^*D^*$	4119.1		No state/not spin partner
1^{+-}	$Z_c(3900)$	$(D\bar{D}^*,-)$	3871.7	$(-+)$	$3864\pm7-(58\pm13)i$
1^{+-}	$Z_c(4020)$	$\bar{D}^*D^*$	4013.7		Not spin partner

Values of the parameters found in the fits

Parameters of the two-channel model ($[\bar{a}_i]$=GeV^{-2}, $[\bar{b}_j]$=GeV^{-4}, $[\bar{c}]$=GeV^{-2})

$\bar{a}_1$	$\bar{a}_2$	$\bar{c}$	$\bar{b}_1$	$\bar{b}_2$	α	b
$0.2^{+0.6}_{-0.5}$	-4.2 ± 0.7	$2.94^{+0.36}_{-0.29}$	$-1.8^{+0.4}_{-0.5}$	-7.1 ± 0.4	70^{+8}_{-7}	3.3 ± 0.4

Parameters of the three-channel model ($[\bar{a}_{ij}]$=GeV^{-2})

$\bar{a}_{11}$	$\bar{a}_{12}$	$\bar{a}_{13}$	$\bar{a}_{22}$	$\bar{a}_{23}$	$\bar{a}_{33}$	α	b
$6.0^{+2.2}_{-1.6}$	$10.3^{+3.4}_{-2.8}$	$-0.2^{+1.9}_{-1.3}$	13^{+5}_{-4}	$-2.6^{+2.4}_{-1.3}$	$-2.3^{+1.5}_{-1.1}$	250^{+70}_{-60}	$-0.12^{+0.21}_{-0.22}$
$7.8^{+3.4}_{-2.0}$	16 ± 4	$0.9^{+2.3}_{-2.5}$	$26^{+12}_{-\ 6}$	-3^{+4}_{-5}	$-2.5^{+2.1}_{-1.0}$	144^{+67}_{-27}	$-0.7^{+0.5}_{-0.4}$

Each parameter with bar needs to be multiplied by $\prod_{i=1}^{4} \sqrt{2m_i}$, where m_i's are the involved charmonium masses

$$m_{J/\psi} = 3.0969 \text{ GeV} \quad m_{\psi(2S)} = 3.6861 \text{ GeV}$$

Compositeness of $X(6200)$

$$T(k) = -8\pi\sqrt{s}\left[\frac{1}{a_0} + \frac{1}{2}r_0 k^2 - i\,k + \mathcal{O}(k^4)\right]^{-1}$$

$$\bar{X}_A = (1 + 2|r_0/a_0|)^{-1/2}$$

	2-ch. fit	3-ch. fit 1	3-ch. fit 2
$a_0(\mathrm{fm})$	≤ -0.49 or ≥ 0.48	$-0.61^{+0.29}_{-0.32}$	≤ -0.60 or ≥ 0.99
$r_0(\mathrm{fm})$	$-2.18^{+0.66}_{-0.81}$	$-0.06^{+0.03}_{-0.04}$	$-0.09^{+0.08}_{-0.05}$
$\bar{X}_A$	$0.39^{+0.58}_{-0.12}$	$0.91^{+0.04}_{-0.07}$	$0.95^{+0.04}_{-0.06}$