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Chiral dynamics of light-flavor meson and axion

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Outline:

1. **Introduction**
2. ***π-η-η'-a*** **mixing and prediction of $g_{a\gamma\gamma}$ from $g_{\pi\gamma\gamma}$ 、 $g_{\eta\gamma\gamma}$、 $g_{\eta'\gamma\gamma}$**
3. **Thermal axion-pion scattering and the implication in cosmology**
4. **Summary**

Introduction

Strong CP problem

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \sum_q \bar{q}(i\not{D} - m_q e^{i\theta_q})q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \theta\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$q \rightarrow e^{i\gamma_5\alpha}q \quad \Longrightarrow \quad \theta_q \rightarrow \theta_q + 2\alpha\,, \qquad \theta \rightarrow \theta - 2\alpha$$

implying the invariant quantity: $\bar{\theta} = \theta + \theta_q$

$$\Rightarrow \quad \mathcal{L}_{\rm QCD}^{\rm axion} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \boxed{\bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

Constraints from neutron electric dipole moment (nEDM)

$$|\bar{\theta}| \lesssim 10^{-10}$$

- **Strong CP problem: why is $\bar{\theta}$ so unnaturally tiny ?**

Axion: an elegant solution to strong CP

[Peccei,Quinn,PRL'77]

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \cdots$$

- **It is possible to introduce other model-dependent axion interactions, such as axion-quark and axion-photon.**

  We will focus on the MODEL INDEPENDENT QCD axion interactions.

- **f_a: the axion decay constant. Invisible axion: $f_a >> v_{\rm EW}$. Axion interactions with SM particles are accompanied with the factors of $(1/f_a)^n$.**

- **Stringent constraints for $m_{a,0} = 0$ (QCD axion): $m_a \sim 1/f_a$**

  QCD-like axion: $m_{a,0}\ (\neq 0) \ll f_a$ with model-independent $aG\tilde{G}$ interaction

- **Axion-hadron and axion-photon interactions are relevant at low energies.**

Axion chiral perturbation theory (AχPT)

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

Two ways to proceed:

(1) Remove the $aG\tilde{G}$ term via the quark axial transformation

$$q \to e^{i\frac{a}{2f_a}\gamma_5 Q_a} q$$

$\mathrm{Tr}(Q_a) = 1$

$$-\frac{a\alpha_s}{8\pi f_a}G\tilde{G} - \frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma_5 Q_a q$$

$$M_q \to M_q(a) = e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a}$$

Mapping to χPT

$$\mathcal{L}_2 = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi_a U^\dagger + U\chi_a^\dagger\rangle + \frac{\partial_\mu a}{2f_a}J_A^\mu|_{\rm LO}$$

$$\chi_a = 2B_0 e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \qquad J_A^\mu|_{\rm LO} = -i\frac{F^2}{2}\langle Q_a(\partial^\mu U U^\dagger + U^\dagger \partial^\mu U)\rangle$$

- **$Q_a = M_q^{-1}/\mathrm{Tr}(M_q^{-1})$ is usually taken to eliminate the LO mass mixing between axion and pion [Georgi,Kaplan,Randall, PLB'86], though any other hermitian Q_a should lead to the same physical quantities.**

- **$J_A^\mu\, \partial_\mu a$ will cause the kinematical mixing. Should be consistently included. [Bauer, et al., PRL'21]**

(2) Explicitly keep the $aG\tilde{G}$ term and match it to χPT

Reminiscent:

QCD $U(1)_A$ anomaly that is caused by topological charge density $\omega(x) = \alpha_s G_{\mu\nu}\tilde{G}^{\mu\nu}/(8\pi)$ **is responsible for the massive singlet η_0.**

Axion could be similarly included as the η_0 via the U(3) χPT:

$$\mathcal{L}^{\rm LO} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{12}M_0^2 X^2$$

$$U = u^2 = e^{i\frac{\sqrt{2}\Phi}{F}}\,, \qquad \chi = 2B(s+ip)\,, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u\,,$$

$$u_\mu = iu^\dagger D_\mu U u^\dagger\,, \quad D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$$

$$X = \log\left(\det U\right) + i\frac{a}{f_a} \qquad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

- **Q_a is not needed in U(3) χPT.**
- **$M_0^2 = 6\tau/F^2$, with τ the topological susceptibility. Note that $M_0^2 \sim O(1/N_c)$.**
- **δ expansion scheme: $\delta \sim O(p^2) \sim O(m_q) \sim O(1/N_c)$.**
- **Axion interactions enter via the axion-meson mixing terms.**

π-η-η'-a mixing in U(3) AχPT

Expansion schemes:

- **Strong isospin breaking (IB) effects: leading corrections will be kept.**
- **Effects of F/f_a : safe to keep the leading order.**
- **δ expansion: the same as the standard U(3) χPT**

Leading order results (mass mixing only)

$$\begin{pmatrix} \bar{\pi}^0 \\ \bar{\eta} \\ \bar{\eta}' \\ \bar{a} \end{pmatrix} = \begin{pmatrix} 1+O(v^2) & -v_{12} & -v_{13} & -v_{14} \\ v_{12} & 1+O(v^2) & -v_{23} & -v_{24} \\ v_{13} & v_{23} & 1+O(v^2) & -v_{34} \\ v_{14} & v_{24} & v_{34} & 1+O(v^2) \end{pmatrix} \begin{pmatrix} \pi^0 \\ \mathring{\eta} \\ \mathring{\eta}' \\ a \end{pmatrix}$$

$$\epsilon \equiv B(m_u - m_d) = m_{K^+}^2 - m_{K^0}^2 - (m_{\pi^+}^2 - m_{\pi^0}^2)$$

$$v_{12} = -\frac{\epsilon}{\sqrt{3}} \frac{c_\theta - \sqrt{2} s_\theta}{m_{\bar{\pi}}^2 - m_{\bar{\eta}}^2}\,, \quad v_{13} = -\frac{\epsilon}{\sqrt{3}} \frac{\sqrt{2} c_\theta + s_\theta}{m_{\bar{\pi}}^2 - m_{\bar{\eta}'}^2}\,, \quad v_{24} = -\frac{M_0^2 s_\theta}{\sqrt{6}(m_{a,0}^2 - m_{\bar{\eta}}^2)} \frac{F}{f_a}\,, \qquad v_{34} = \frac{M_0^2 c_\theta}{\sqrt{6}(m_{a,0}^2 - m_{\bar{\eta}'}^2)} \frac{F}{f_a}$$

$$v_{14} = \frac{M_0^2 \epsilon}{12} \frac{F}{f_a} \left\{ \frac{2\sqrt{2}}{(m_{a,0}^2 - m_{\bar{\pi}}^2)} \left[\frac{s_\theta(c_\theta - \sqrt{2} s_\theta)}{m_{\bar{\eta}}^2 - m_{\bar{\pi}}^2} - \frac{c_\theta(s_\theta + \sqrt{2} c_\theta)}{m_{\bar{\eta}'}^2 - m_{\bar{\pi}}^2} \right] + \frac{\sqrt{2} s_\theta(\sqrt{2} s_\theta - c_\theta)}{(m_{a,0}^2 - m_{\bar{\eta}}^2)(m_{\bar{\eta}}^2 - m_{\bar{\pi}}^2)} + \frac{\sqrt{2} c_\theta(\sqrt{2} c_\theta + s_\theta)}{(m_{a,0}^2 - m_{\bar{\eta}'}^2)(m_{\bar{\eta}'}^2 - m_{\bar{\pi}}^2)} \right\}$$

$$m_{\bar{a}}^2 = m_{a,0}^2 + \frac{M_0^2 F^2}{6 f_a^2} \left[1 + \frac{c_\theta^2 M_0^2}{m_{a,0}^2 - m_{\bar{\eta}'}^2} + \frac{s_\theta^2 M_0^2}{m_{a,0}^2 - m_{\bar{\eta}}^2} \right] + O(\epsilon)$$

For the QCD axion with $m_{a,0}$=0 :

$$m_{\bar{a}}^2 = \frac{M_0^2 F^2}{6 f_a^2} \left[1 - \frac{c_\theta^2 M_0^2}{m_{\bar{\eta}'}^2} - \frac{s_\theta^2 M_0^2}{m_{\bar{\eta}}^2} \right] + O(\epsilon) \quad \Longrightarrow \quad m_{\bar{a}}^2 = \frac{F^2 m_{\bar{\pi}}^2}{f_a^2} + O\left(\frac{m_{\bar{\pi}}^2}{m_{\bar{K}}^2}\right) + O\left(\frac{m_{\bar{\pi}}^2}{M_0^2}\right) + O(\epsilon)$$

[Weinberg,PRL'78]

NLO case: kinetic mixing+mass mixing terms

Relevant NLO operators to the axion-meson mixing

$$\mathcal{L}^{\rm NLO} = L_5\langle u^\mu u_\mu \chi_+\rangle + \frac{L_8}{2}\langle \chi_+\chi_+ + \chi_-\chi_-\rangle - \frac{F^2\Lambda_1}{12} D^\mu X D_\mu X - \frac{F^2\Lambda_2}{12} X\langle\chi_-\rangle$$

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = \begin{pmatrix} 1 & -y_{12} & -y_{13} & -y_{14} \\ y_{12} & 1 & -y_{23} & -y_{24} \\ y_{13} & y_{23} & 1 & -y_{34} \\ y_{14} & y_{24} & y_{34} & 1 \end{pmatrix} \times \begin{pmatrix} 1-x_{11} & -x_{12} & -x_{13} & -x_{14} \\ -x_{12} & 1-x_{22} & -x_{23} & -x_{24} \\ -x_{13} & -x_{23} & 1-x_{33} & -x_{34} \\ -x_{14} & -x_{24} & -x_{34} & 1-x_{44} \end{pmatrix} \begin{pmatrix} \overline{\pi}^0 \\ \overline{\eta} \\ \overline{\eta}' \\ \overline{a} \end{pmatrix}$$

take care of mass mixing

take care of kinetic mixing and canonicalization

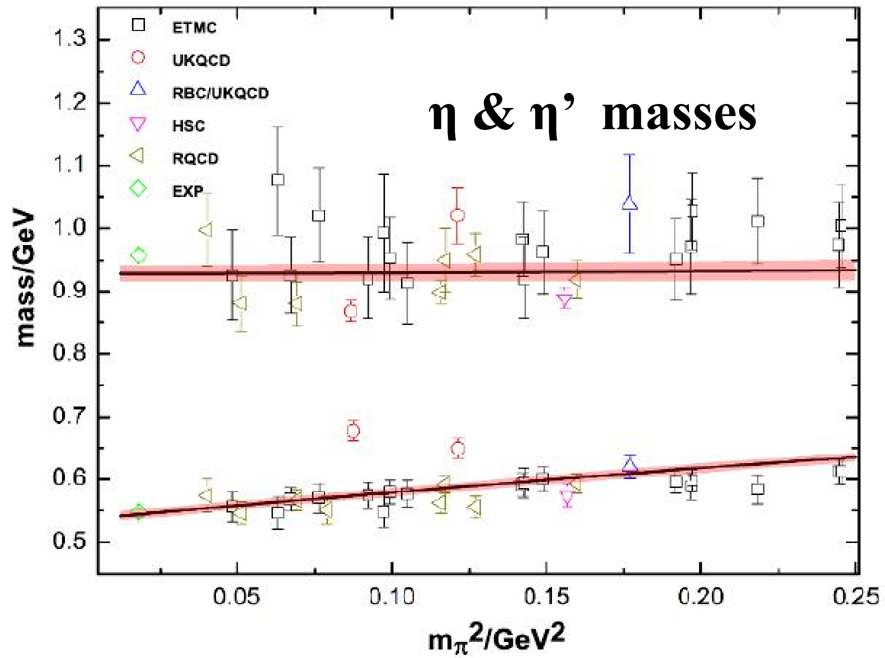
For details, see [Gao,ZHG,Oller,Zhou,JHEP'23]

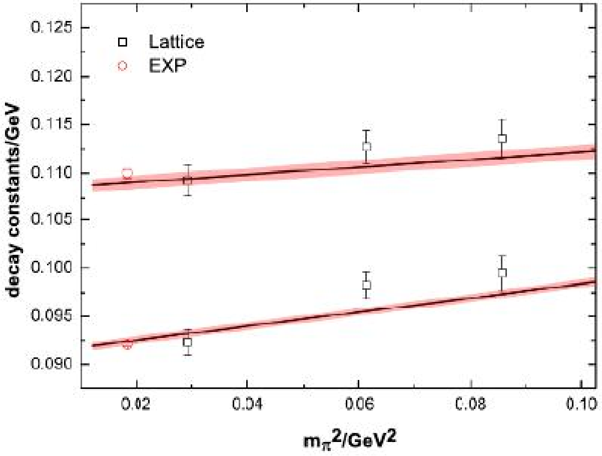
Free parameters to fix : F, L_5, L_8, Λ_1, Λ_2 , which also appear in

- **the masses of π, K, η , η'**
- **decay constants of π, K**
- **η - η' mixing related quantities**

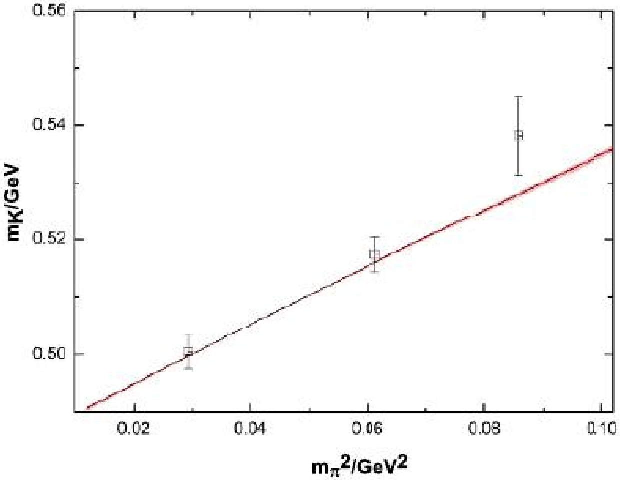
➢ Lattice simulations provide valuable data, especially m_π dependence quantities.

Parameters	NLO Fit
F(MeV)	$91.05^{+0.42}_{-0.44}$
$10^3 \times L_5$	$1.68^{+0.05}_{-0.06}$
$10^3 \times L_8$	$0.88^{+0.04}_{-0.04}$
Λ_1	$-0.17^{+0.05}_{-0.05}$
Λ_2	$0.06^{+0.08}_{-0.09}$
χ^2/(d.o.f.)	219.9/(111–5)

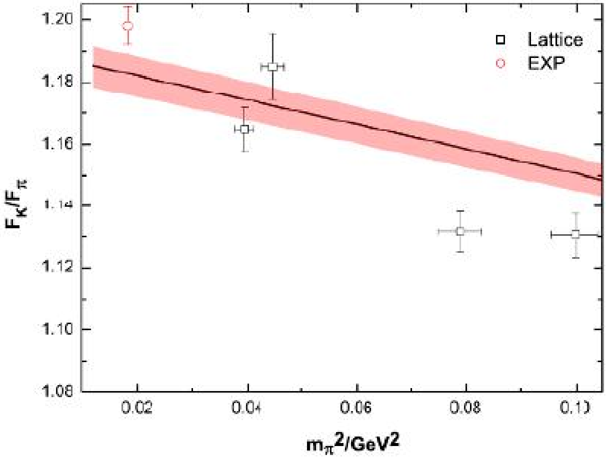




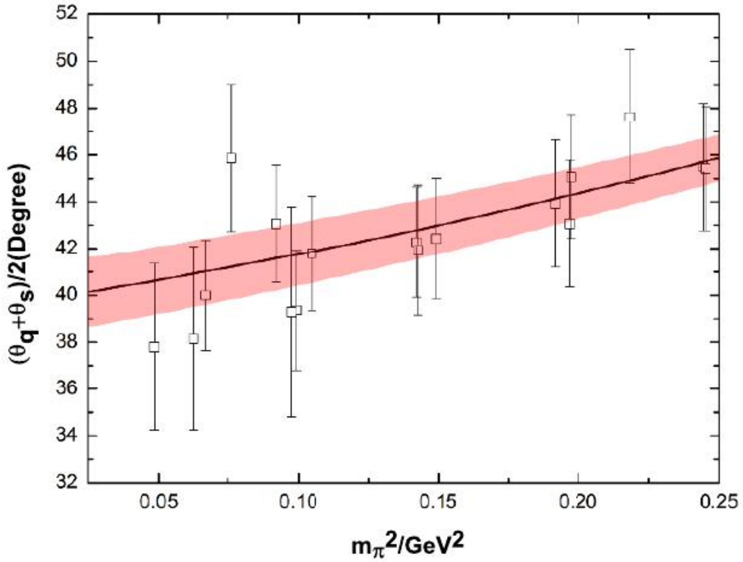
[RBC/UKQCD,PRD'11]



[RBC/UKQCD,PRD'13]



[Durr et al., PRD'10]

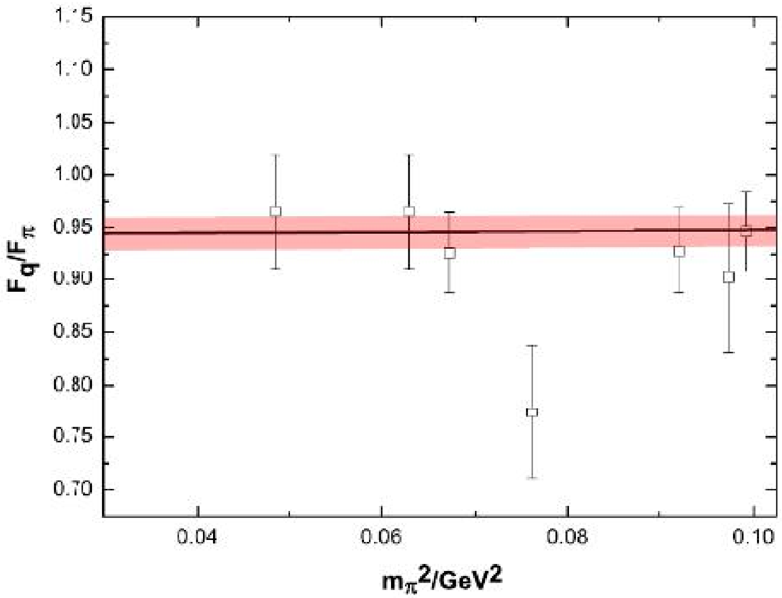


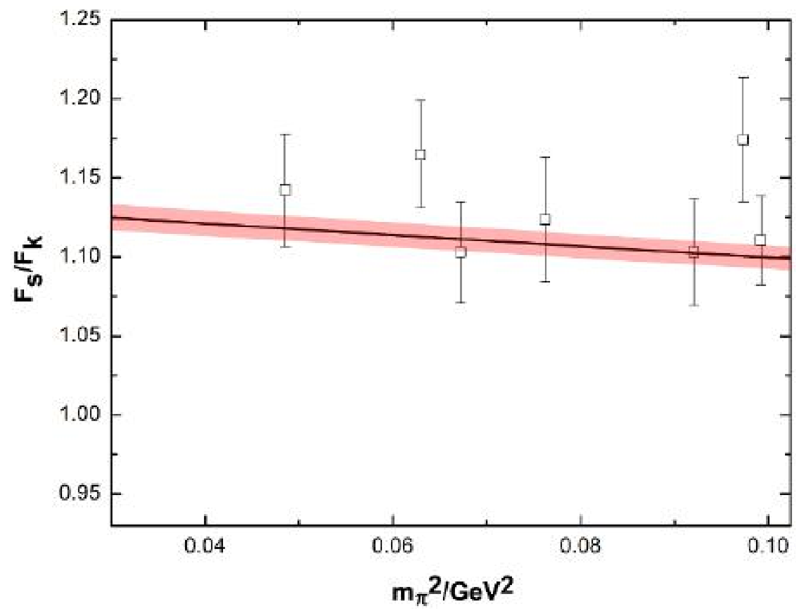
$$\begin{pmatrix} \hat\eta \\ \hat\eta' \end{pmatrix} = \frac{1}{F} \begin{pmatrix} F_8 \cos\theta_8 & -F_0 \sin\theta_0 \\ F_8 \sin\theta_8 & F_0 \cos\theta_0 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

$$\begin{pmatrix} \hat\eta \\ \hat\eta' \end{pmatrix} = \frac{1}{F} \begin{pmatrix} F_q \cos\theta_q & -F_s \sin\theta_s \\ F_q \sin\theta_q & F_s \cos\theta_s \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

$$\begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

[ETMC,PRD'18]





Predictions to the mixing pattern and axion mass

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = M^{\rm LO+NLO} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix}$$

[Gao,ZHG,Oller,Zhou,JHEP'23]

$$M^{\rm LO+NLO} = \begin{pmatrix} 1+(0.015\pm0.001) & 0.017+(-0.010\pm0.001) & 0.009+(-0.007\pm0.001) & \frac{12.1+(0.48\pm0.08)}{f_a} \\ -0.019+(0.007\pm0.001) & 0.94+(0.21\pm0.01) & 0.33+(-0.22\pm0.03) & \frac{34.3+(0.9\pm0.2)}{f_a} \\ -0.003+(-0.003\pm0.000) & -0.33+(-0.18\pm0.03) & 0.94+(0.13\pm0.02) & \frac{25.9+(-0.5\pm0.1)}{f_a} \\ \frac{-12.1+(-0.20\pm0.03)}{f_a} & \frac{-23.8+(1.6^{+0.8}_{-0.8})}{f_a} & \frac{-35.7+(-5.7^{+1.6}_{-1.7})}{f_a} & 1+\frac{27.6\pm1.0}{f_a^2} \end{pmatrix}$$

$$\begin{aligned} m_{\hat{\pi}} &= [134.90+(0.10\pm0.07)]\ \text{MeV}\,, \\ m_{\hat{K}} &= [489.2+(5.0^{+3.4}_{-3.5})]\ \text{MeV}\,, \\ m_{\hat{\eta}} &= [490.2+(60.9^{+10.2}_{-10.0})]\ \text{MeV}\,, \\ m_{\hat{\eta}'} &= [954.3+(-28.4^{+11.9}_{-12.6})]\ \text{MeV}\,, \\ m_{\hat{a}} &= [5.96+(0.12\pm0.02)]\ \mu\text{eV}\,\frac{10^{12}\ \text{GeV}}{f_a}\,, \end{aligned}$$

Two-photon couplings

$$\mathcal{L}_{\rm WZW}^{\rm LO} = -\frac{3\sqrt{2}e^2}{8\pi^2 F}\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2 \Phi\rangle$$

$$\mathcal{L}_{\rm WZW}^{\rm NLO} = i t_1 \varepsilon_{\mu\nu\rho\sigma}\langle f_+^{\mu\nu} f_+^{\rho\sigma}\chi_-\rangle + k_3 \varepsilon_{\mu\nu\rho\sigma}\langle f_+^{\mu\nu} f_+^{\rho\sigma}\rangle X$$

- **One needs the π-η-η'-*a* mixing as input.**
- **Our goal: to use the experimental inputs from $g_{\pi\gamma\gamma}$, $g_{\eta\gamma\gamma}$, $g_{\eta'\gamma\gamma}$ to predict $g_{a\gamma\gamma}$.**

LO *a*γγ coupling is purely caused by the mixing.

$$\begin{aligned} F_{a\gamma\gamma} &= \frac{(c_\theta - 2\sqrt{2}s_\theta)v_{24} + (s_\theta + 2\sqrt{2}c_\theta)v_{34}}{4\sqrt{3}\pi^2 F} \\ &+ \frac{(c_\theta - 2\sqrt{2}s_\theta)(x_{24}+y_{24}) + (s_\theta + 2\sqrt{2}c_\theta)(x_{34}+y_{34})}{4\sqrt{3}\pi^2 F} - \frac{64\sqrt{6}}{3F}k_3\left(\frac{F}{f_a} - \sqrt{6}s_\theta v_{24} + \sqrt{6}c_\theta v_{34}\right) \\ &- \frac{64}{27F}t_1\Big[-\sqrt{3}s_\theta(4\sqrt{2}v_{24}m_\pi^2 - 7v_{34}m_\pi^2 + 2\sqrt{2}v_{24}m_K^2 + 4v_{34}m_K^2) \\ &\qquad + \sqrt{3}c_\theta(4\sqrt{2}v_{34}m_\pi^2 + 7v_{24}m_\pi^2 + 2\sqrt{2}v_{34}m_K^2 - 4v_{24}m_K^2)\Big]. \end{aligned}$$

$$F_{\pi^0\gamma\gamma} = \frac{1}{4\pi^2 F} + \frac{1}{4\pi^2 F}x_{11} - \frac{64}{3F}t_1 m_\pi^2,$$

$$\begin{aligned} F_{\eta\gamma\gamma} &= \frac{c_\theta - 2\sqrt{2}s_\theta}{4\sqrt{3}\pi^2 F}(1+x_{22}) + \frac{s_\theta + 2\sqrt{2}c_\theta}{4\sqrt{3}\pi^2 F}(x_{23}-y_{23}) + \frac{64\sqrt{6}}{3F}s_\theta k_3 \\ &- \frac{64\sqrt{3}}{27F}t_1\left[c_\theta(7m_\pi^2 - 4m_K^2) - 2\sqrt{2}s_\theta(m_K^2 + 2m_\pi^2)\right], \end{aligned}$$

$$\begin{aligned} F_{\eta'\gamma\gamma} &= \frac{s_\theta + 2\sqrt{2}c_\theta}{4\sqrt{3}\pi^2 F}(1+x_{33}) + \frac{c_\theta - 2\sqrt{2}s_\theta}{4\sqrt{3}\pi^2 F}(x_{23}+y_{23}) - \frac{64\sqrt{6}}{3F}c_\theta k_3 \\ &- \frac{64\sqrt{3}}{27F}t_1\left[s_\theta(7m_\pi^2 - 4m_K^2) + 2\sqrt{2}c_\theta(m_K^2 + 2m_\pi^2)\right], \end{aligned}$$

$$F^{\mathrm{Exp}}_{\pi^0\gamma\gamma} = 0.274 \pm 0.002\,\mathrm{GeV}^{-1}\,,$$
$$F^{\mathrm{Exp}}_{\eta\gamma\gamma} = 0.274 \pm 0.006\,\mathrm{GeV}^{-1}\,,$$
$$F^{\mathrm{Exp}}_{\eta'\gamma\gamma} = 0.344 \pm 0.008\,\mathrm{GeV}^{-1}\,.$$

$$t_1 = -(4.4 \pm 2.3) \times 10^{-4}\,\mathrm{GeV}^{-2}\,,$$
$$k_3 = (1.25 \pm 0.23) \times 10^{-4}$$

This allows us to predict

$$F_{a\gamma\gamma} = -\frac{[20.1 + (0.5 \pm 0.1)] \times 10^{-3}}{f_a}$$

$$g_{a\gamma\gamma} = 4\pi\alpha_{em} F_{a\gamma\gamma} = -\frac{\alpha_{em}}{2\pi f_a}(1.63 \pm 0.01)$$

which can be compared to 1.92 ± 0.04 **and** 2.05 ± 0.03

[Grilli de Cortona, et al., JHEP'16]

[Lu, et al., JHEP'20]

- **IB corrections could cause visible effects (working in progress).**

Cosmology constraints on axion thermalization rate

Axion thermal production in the early Universe : Extra radiation (ΔN_{eff})

Extra effective number of relativisitc d.o.f :

$$\Delta N_{\rm eff} \simeq \frac{4}{7}\left(\frac{43}{4g_{\star s}(T_D)}\right)^{\frac{4}{3}}$$

$g_{\star s}(T)$: effective number of entropy d.o.f at temperature T

T_D: axion decoupling temperature from the thermal medium

- **CMB constraint (Plank'18) [Aghanim et al., 2020] : $\Delta N_{eff} \leq 0.28$**
- **T_D : Instantaneous decoupling approximation**

$$\Gamma_a(T_D) = H(T_D)$$

Axion thermalization rate

$$\Gamma_a(T) = \frac{1}{n_a^{\rm eq}}\int d\tilde{\Gamma}\sum|\mathcal{M}_{a-{\rm SM}}|^2\, n_B(E_1)n_B(E_2)[1+n_B(E_3)][1+n_B(E_4)]$$

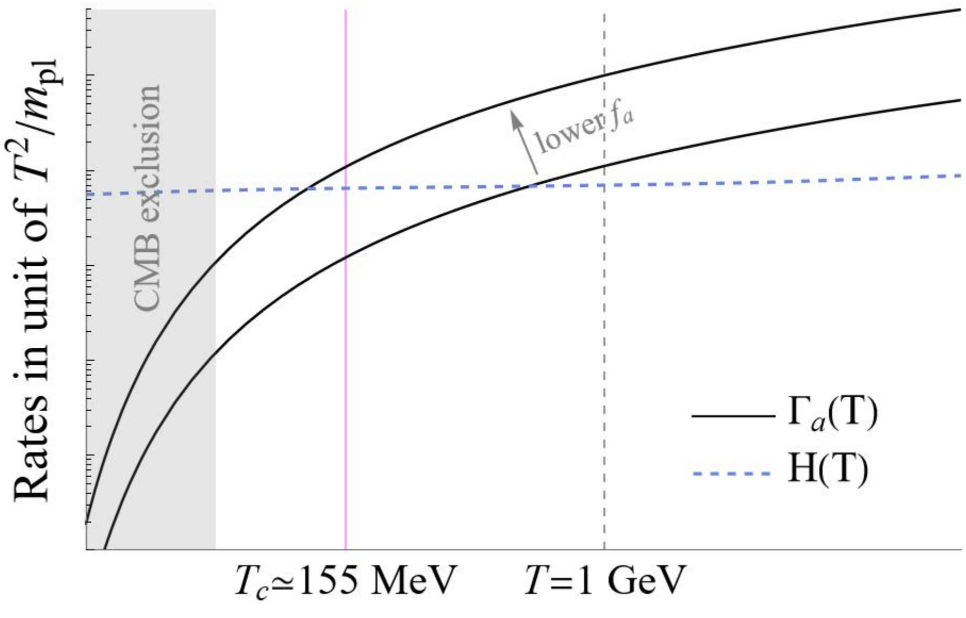
Axion-SM particle scattering amplitudes

Hubble expansion parameter

$$H(T) = T^2\sqrt{4\pi^3 g_*(T)/45}/m_{\rm Pl}$$

Key thermal channels of axion-SM scatterings at different temperatures

- ☞ $T_D \gtrsim 1$ GeV: $ag \leftrightarrow gg$.
  [Masso et al., 2002, Graf and Steffen, 2011]
- ☞ $T_D \lesssim 1$ GeV: Hadrons need to be included.
- ☞ $T_D \lesssim 200$ MeV: $a\pi \leftrightarrow \pi\pi$.
  [Chang and Choi, 1993, Hannestad et al., 2005, Giusarma et al., 2014, D'Eramo et al., 2022]



$\Delta N_{\rm eff}$ from CMB constraint (translated to the $T_{\rm D}$ limit) sets the lower bound of f_a, or equivalently upper bound of m_a in the QCD-axion case.

❑ Reliable $a\pi$ interaction is crucial to determine Γ_a for $T_D < T_c \approx 155$ MeV

- **For a long time, only the LO $a\pi \leftrightarrow \pi\pi$ amplitude is employed to calculate the axion thermalization rate Γ_a, e.g.,** [Chang, Choi, PLB'93] [Hannestad, et al., JCAP'05] [Hannestad, et al., JCAP'05] [D'Eramo, et al., PRL'22] ... ...

- **Recent NLO calculation of Γ_a shows that the reliable chiral expansion is only valid for $T_\chi \leq 70$ MeV for perturbative amplitudes at NLO** [Di Luzio, et al., PRL'21]

- **Chiral unitarization approach is adopted to extend the applicable ranges of energy and temperature.** [Di Luzio, et al., PRD'23]

- **However, to our knowledge, all the previous works have ignored the thermal corrections to the $a\pi \leftrightarrow \pi\pi$ amplitudes. We give the first estimation of such effects on the determination of axion parameters.**

[Wang, ZHG, Zhou, PRD'24]

$$\Gamma_a(T) = \frac{1}{n_a^{\rm eq}} \int d\tilde{\Gamma} \sum \boxed{|\mathcal{M}_{a-\rm SM}|^2} n_B(E_1) n_B(E_2) [1 + n_B(E_3)][1 + n_B(E_4)]$$

Calculation of thermal $a\pi \leftrightarrow \pi\pi$ amplitudes at one-loop level

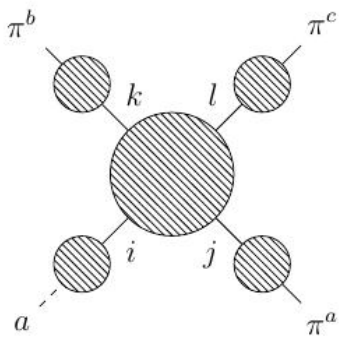
- Finite-temperature effects are included by imaginary time formalism (ITF), where [Kapusta and Gale, 2011, Bellac, 2011, Laine and Vuorinen, 2016]

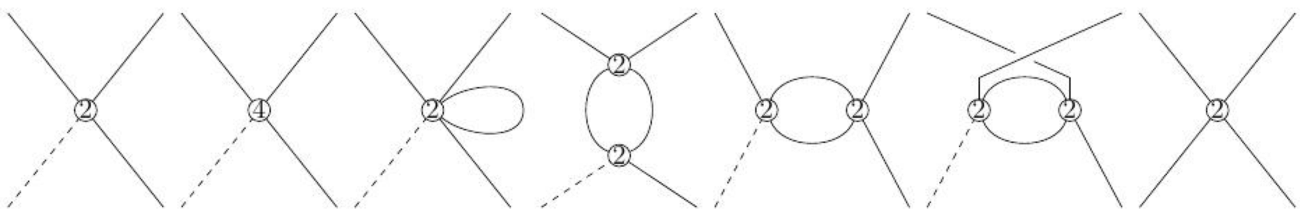
$$p^0 \to i\omega_n, \quad \text{with}\, \omega_n = 2\pi n T\,, n \in \mathbb{Z}\,,$$

$$-i\int \frac{\mathrm{d}^d q}{(2\pi)^d} \to -i\int_\beta \frac{\mathrm{d}^d q}{(2\pi)^d} \equiv T\sum_n \int \frac{\mathrm{d}^{d-1} q}{(2\pi)^{d-1}}\,.$$

- Compute the thermal Green functions in ITF

$$G^T_{a\pi^a;\pi^b\pi^c}(p_1,p_2;p_3,p_4) = \sum_{i,j,k,l} G_{ai}(p_1^2)G_{\pi^a j}(p_2^2)G_{k\pi^b}(p_3^2)G_{l\pi^c}(p_4^2)A_{ij;kl}(p_1,p_2;p_3,p_4)\,.$$





Feynman diagrams for amputated functions up to NLO.

- The effective Lagrangian at $\mathcal{O}(p^4)$

$$\begin{aligned}\mathcal{L}_4 \supset & \frac{l_3+l_4}{16}\left\langle \chi_a U^\dagger + U\chi_a^\dagger\right\rangle\left\langle \chi_a U^\dagger + U\chi_a^\dagger\right\rangle + \frac{l_4}{8}\left\langle \partial_\mu U \partial^\mu U^\dagger\right\rangle\left\langle \chi_a U^\dagger + U\chi_a^\dagger\right\rangle \\ & - \frac{l_7}{16}\left\langle \chi_a U^\dagger - U\chi_a^\dagger\right\rangle\left\langle \chi_a U^\dagger - U\chi_a^\dagger\right\rangle + \frac{h_1-h_3-l_4}{16}\left[\left(\left\langle \chi_a U^\dagger + U\chi_a^\dagger\right\rangle\right)^2 \right.\\ & \left. + \left(\left\langle \chi_a U^\dagger - U\chi_a^\dagger\right\rangle\right)^2 - 2\left\langle \chi_a U^\dagger \chi_a U^\dagger + U\chi_a^\dagger U\chi_a^\dagger\right\rangle\right] + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\mathrm{NLO}}\,,\end{aligned}$$

$$\begin{aligned}J_A^\mu|_{\mathrm{NLO}} \supset & -il_1\left\langle Q_a\left\{\partial^\mu U, U^\dagger\right\}\right\rangle\left\langle \partial_\nu U \partial^\nu U^\dagger\right\rangle \\ & - i\frac{l_2}{2}\left\langle Q_a\left\{\partial_\nu U, U^\dagger\right\}\right\rangle\left\langle \partial^\mu U \partial^\nu U^\dagger + \partial^\nu U \partial^\mu U^\dagger\right\rangle \\ & - i\frac{l_4}{4}\left\langle Q_a\left\{\partial^\mu U, U^\dagger\right\}\right\rangle\left\langle \chi_a U^\dagger + U\chi_a^\dagger\right\rangle\,.\end{aligned}$$

Unitarization of the partial-wave $a\pi \leftrightarrow \pi\pi$ amplitude

Inverse amplitude method (IAM)

$$\mathcal{M}^{\rm IAM}_{a\pi;IJ} = \frac{\left(\mathcal{M}^{(2)}_{a\pi;IJ}\right)^2}{\mathcal{M}^{(2)}_{a\pi;IJ} - \mathcal{M}^{(4)}_{a\pi;IJ}}$$

$$\mathcal{M}_{a\pi;IJ}(E_{cm}) = \frac{1}{2}\int_{-1}^{+1} \mathrm{d}\cos\theta\, \mathcal{M}_{a\pi;I}(E_{cm}, \cos\theta) P_J(\cos\theta)$$

$$\mathrm{Im}\mathcal{M}_{a\pi;IJ}(E_{cm}) \overset{s}{=} \frac{1}{2}\rho^T_{\pi\pi}(E_{cm})\mathcal{M}^{IJ*}_{\pi\pi;\pi\pi}\mathcal{M}_{a\pi;IJ}\,, \quad (E_{cm} > 2m_\pi)$$

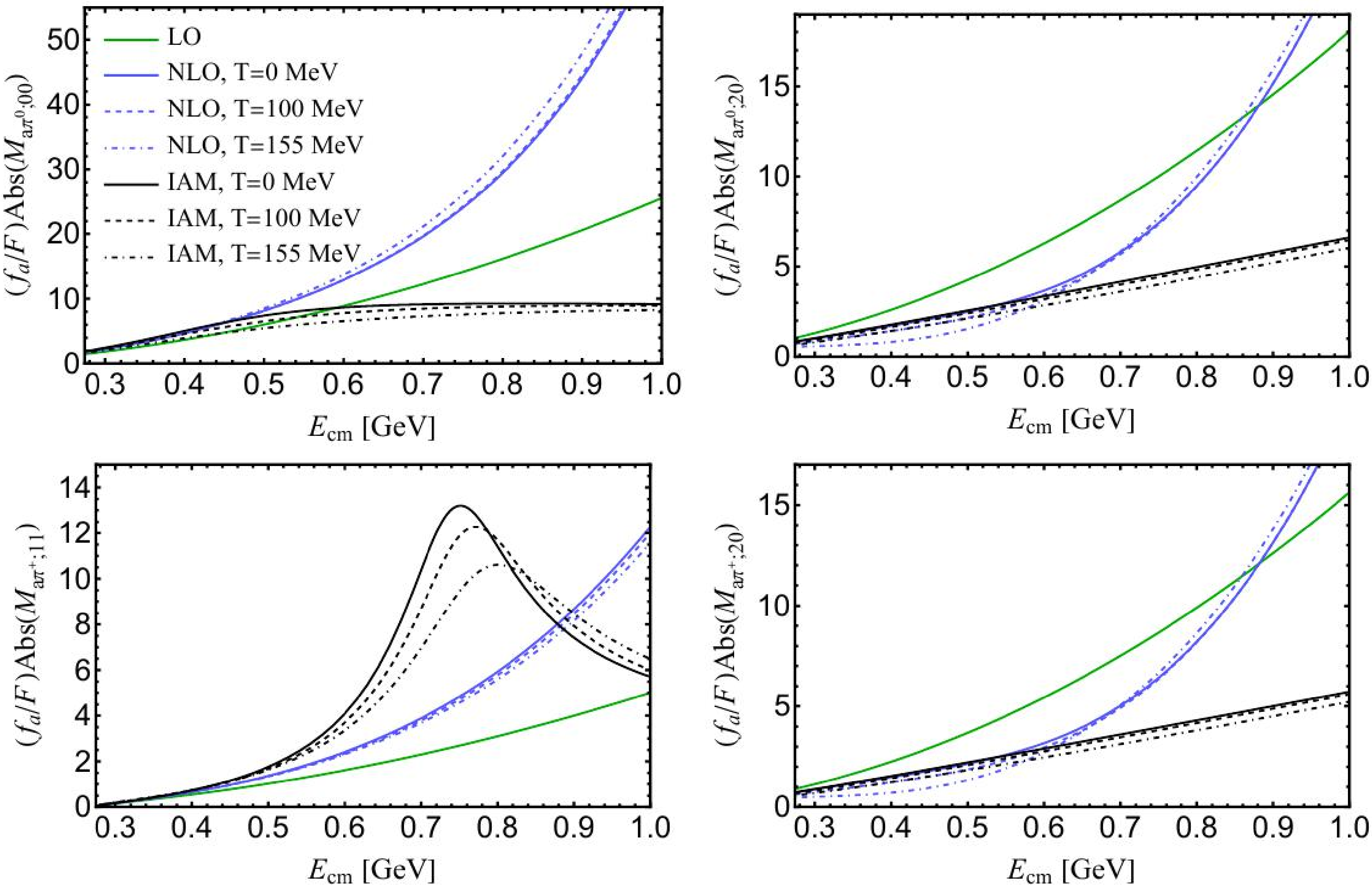
$$\rho^T_{\pi\pi}(E_{cm}) = \frac{\sigma_\pi(E^2_{cm})}{16\pi}\left[1 + 2n_B(\frac{E_{cm}}{2})\right]\,, \qquad \sigma_\pi(s) = \sqrt{1 - \frac{4m^2_\pi}{s}}\,, \quad n_B(E) = \frac{1}{e^{E/T} - 1}$$

- Resonances poles on the second Riemann sheet

	$f_0(500)/\sigma$		$\rho(770)$	
	$M_\sigma \pm i\frac{\Gamma_\sigma}{2}$	$\lvert f_a g_{\sigma a\pi}\rvert$	$M_\rho \pm i\frac{\Gamma_\rho}{2}$	$\lvert f_a g_{\rho a\pi}\rvert$
$T = 0$ MeV	$422 \pm i240$ MeV	0.032 GeV^2	$739 \pm i72$ MeV	0.035 GeV^2
$T = 100$ MeV*	$368 \pm i310$ MeV	0.037 GeV^2	$744 \pm i77$ MeV	0.036 GeV^2

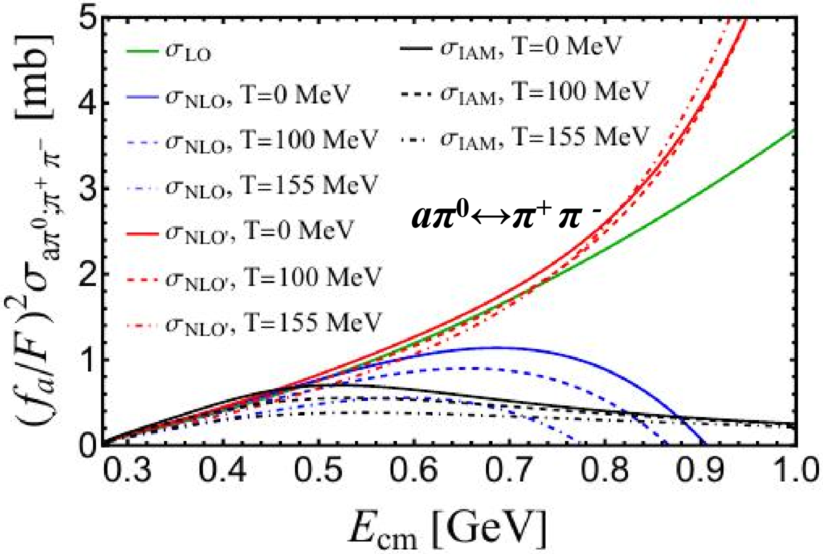
*Only include s-channel unitary thermal correction.

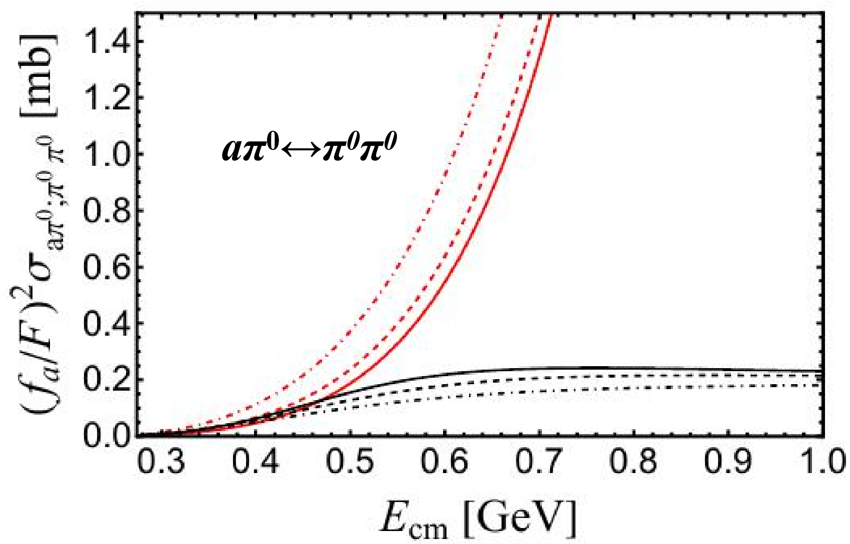
$a\pi \leftrightarrow \pi\pi$ amplitude at finite temperatures



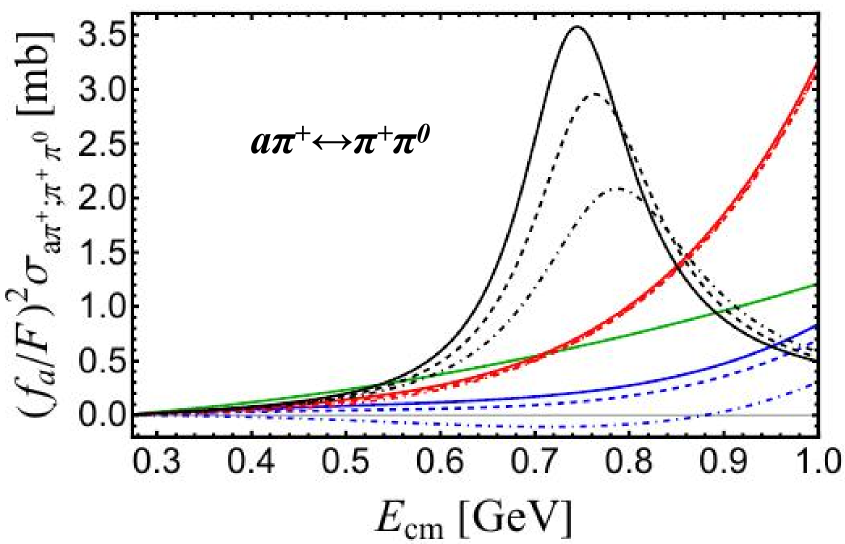
aπ ↔*ππ* cross-sections at finite temperatures

$$|\mathcal{M}^{(2)} + \mathcal{M}^{(4)}|^2 = \left(\mathcal{M}^{(2)}\right)^2 + 2\mathcal{M}^{(2)}\mathrm{Re}\left(\mathcal{M}^{(4)}\right) + |\mathcal{M}^{(4)}|^2 .$$





- The chiral perturbative amplitudes up to NLO begin to be unreliable around $E \simeq 500$ MeV.



Cosmology implication of thermal corrections to axion-pion scattering amplitudes

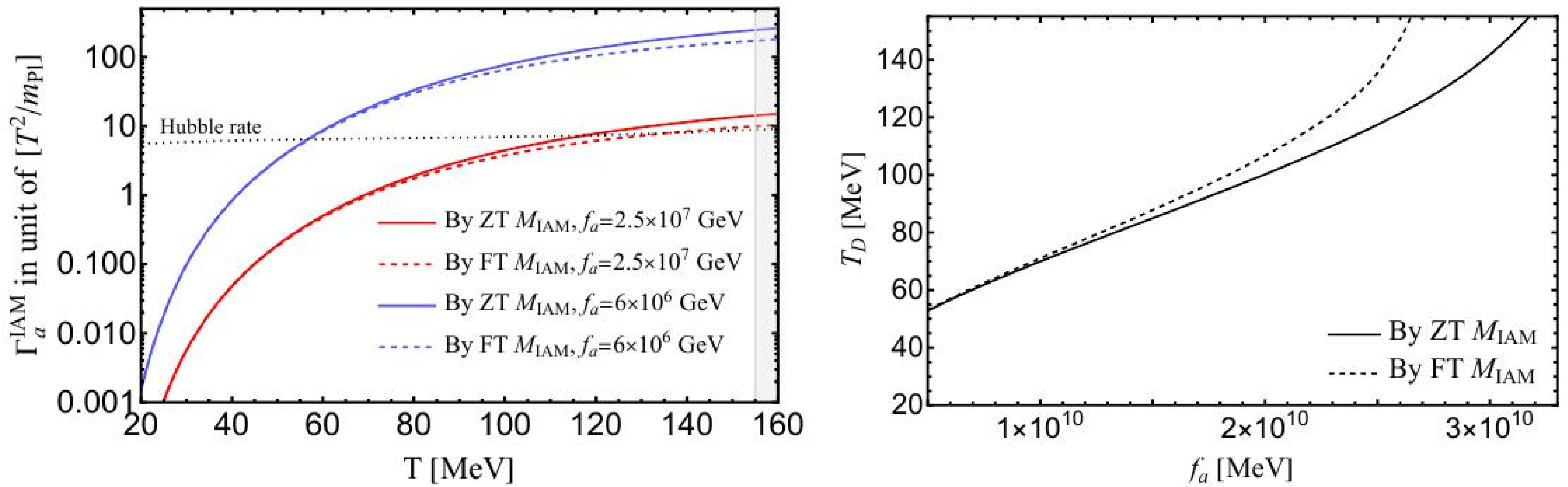
- We calculate the axion rate by the temperature dependent $a\pi \to \pi\pi$ scattering amplitudes [Chang and Choi, 1993, Hannestad et al., 2005]

$$\Gamma_a(T) = \frac{1}{n_a^{\text{eq}}} \int \mathrm{d}\widetilde{\Gamma} \sum |\mathcal{M}_{a\pi;\pi\pi}|^2 n_B(E_1) n_B(E_2)[1 + n_B(E_3)][1 + n_B(E_4)] \,,$$

where the phase space integral

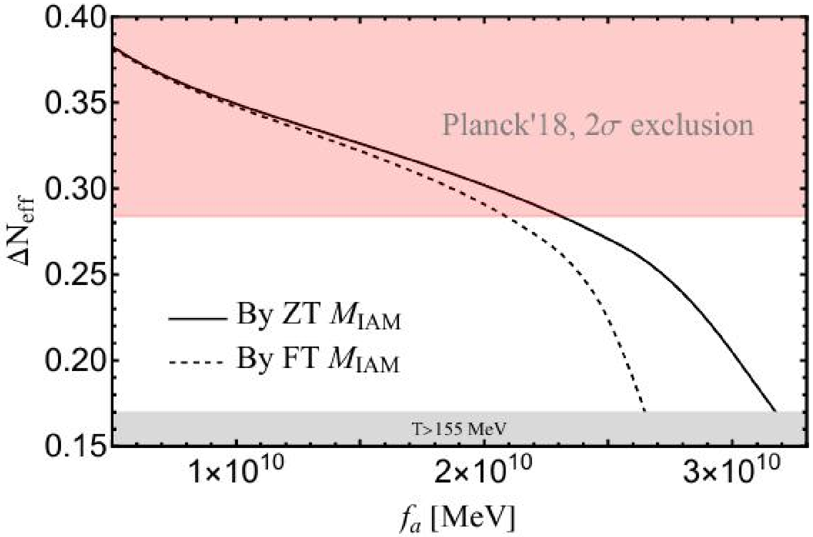
$$\int \mathrm{d}\widetilde{\Gamma} = \int \left(\prod_{i=1}^{4} \frac{\mathrm{d}^3 p_i}{(2\pi)^3} \frac{1}{2E_i} \right) (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \,.$$

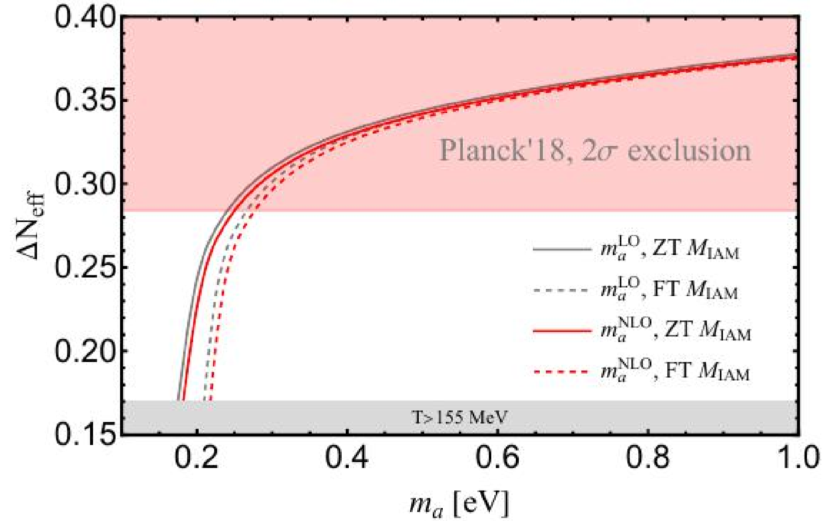
Reslust from the thermal-IAM improved thermliazation rates



Updated bounds on the axion parameters

[Wang,ZHG,Zhou,PRD'24]





- The constrians

10% corrections are observed

	lower limit of f_a	upper limit of m_a by m_a^{LO}	upper limit of m_a by m_a^{NLO}
ZT	2.3×10^7 GeV	0.24 eV	0.25 eV
FT	2.1×10^7 GeV	0.27 eV	0.28 eV

- The QCD axion mass up to LO & NLO

$$m_a^2|_{\mathrm{LO}} = \gamma_{ud}\, m_\pi^2 \frac{F^2}{f_a^2}\,, \qquad \text{where} \quad \gamma_{ud} = \frac{m_u m_d}{(m_u + m_d)^2}\,,$$

$$m_a^2|_{\mathrm{NLO}} = \gamma_{ud}\, m_\pi^2 \frac{F^2}{f_a^2} \left\{ 1 - 2\frac{m_\pi^2}{(4\pi F)^2} \log\frac{m_\pi^2}{\mu^2} + 2\left[h_1^r(\mu^2) - h_3\right]\frac{m_\pi^2}{F^2} - 8 l_7 \gamma_{ud} \frac{m_\pi^2}{F^2} \right\}$$

Summary

- **Chiral perturbation theory provides a systematical and useful framework to study the axion-meson interactions .**

- **π-η-η'-*a* mixing is worked out at NLO and lattice data are found to be very useful to predict the axion mixing pattern and axion mass.**

- **$g_{\pi\gamma\gamma}$, $g_{\eta\gamma\gamma}$, $g_{\eta'\gamma\gamma}$ and the π-η-η'-*a* mixing are used to predict $g_{a\gamma\gamma}$ within U(3) AχPT up to NLO.**

- **Thermal $a\pi \leftrightarrow \pi\pi$ amplitudes in SU(2) AχPT are worked out. Thermal corrections to amplitudes cause around 10% shift of the axion parameters.**

谢谢！