Global coupled-channel analysis of $e^+e^- \to c\bar{c}$ processes in $\sqrt{s} = 3.75 - 4.7$ GeV

arXiv: 2312.17658 + updates

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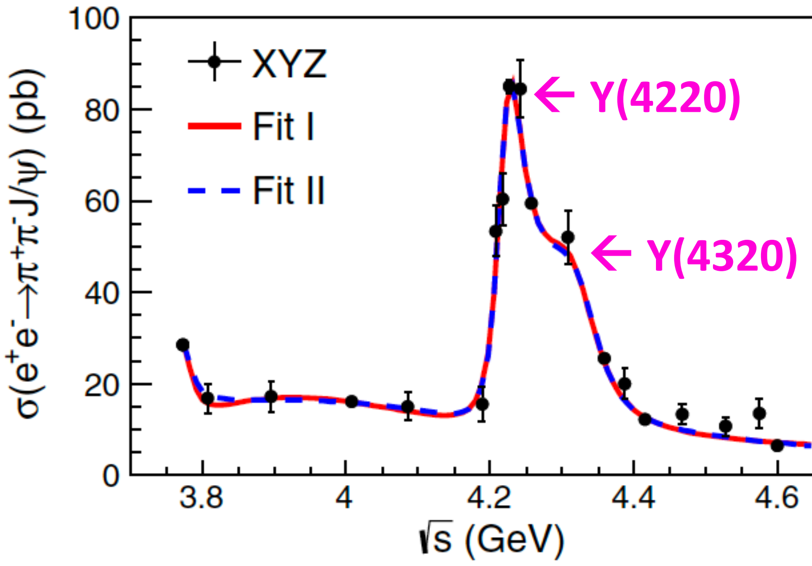
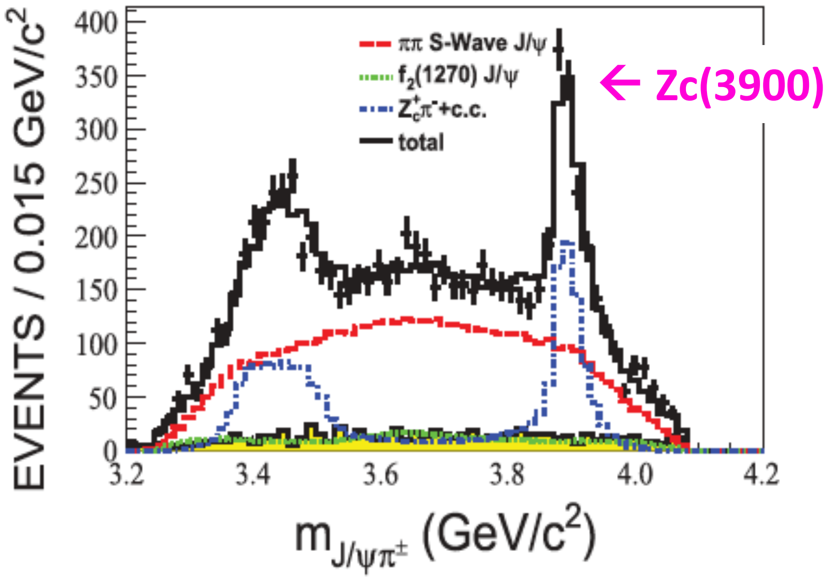
Introduction

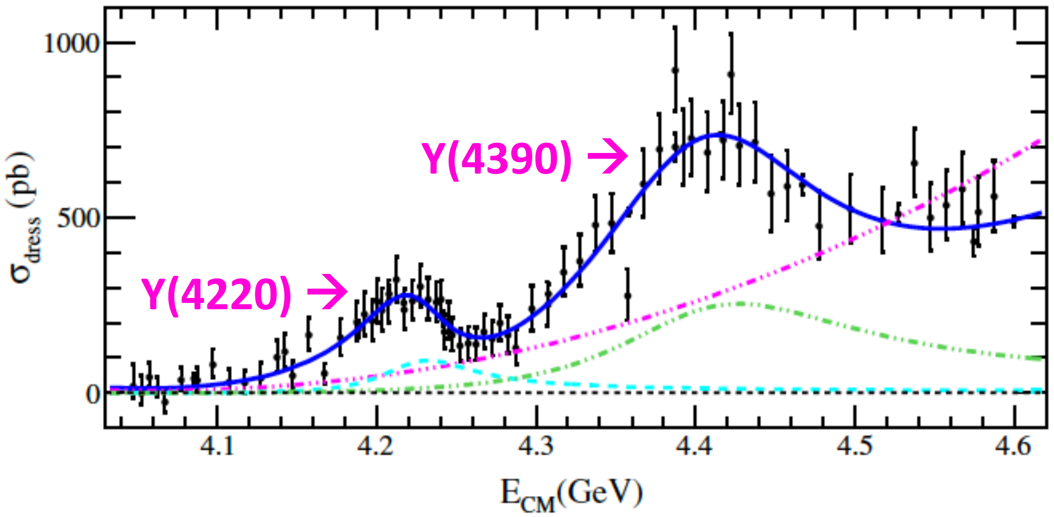
BESIII data for XYZ physics (only a few from many)

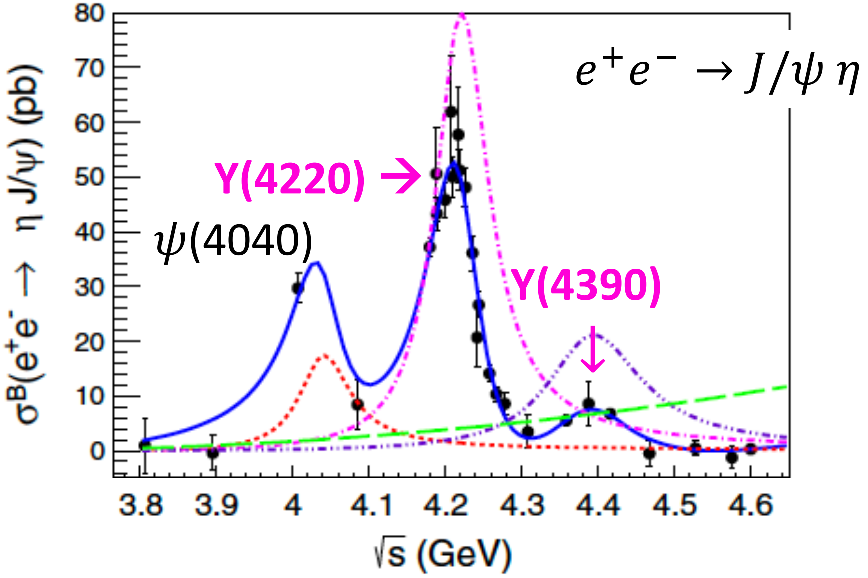
$e^+e^- \to J/\psi\,\pi^+\pi^-$ →





$e^+e^- \to \pi^+ D^0 D^{*-}$

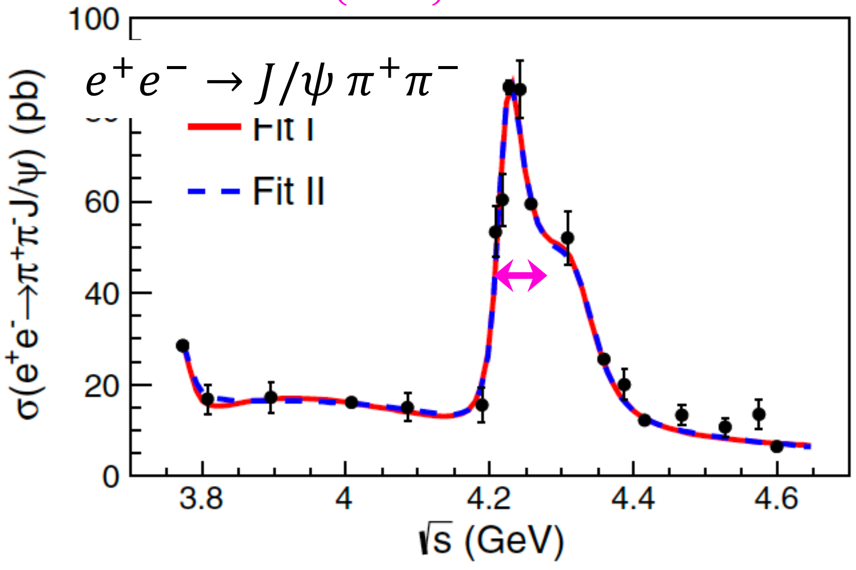




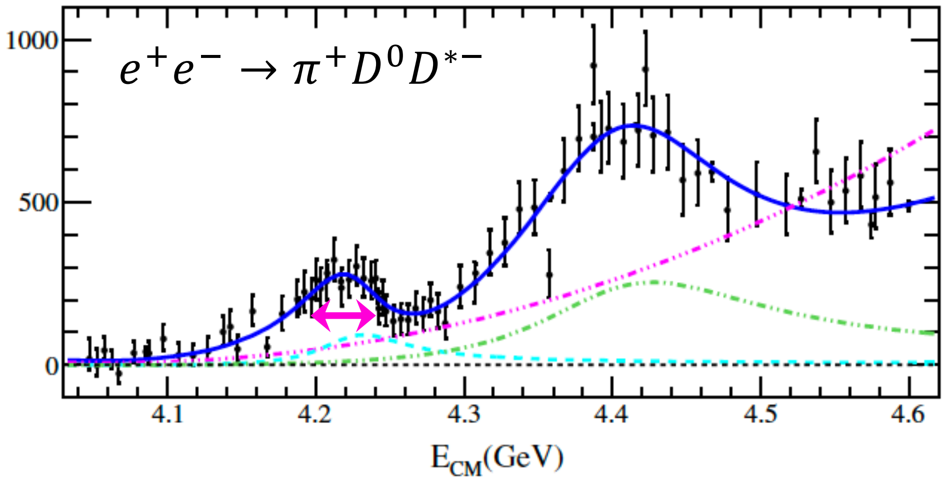
Outstanding question in XYZ physics : Y width problem

Why Y states seem to have different widths for different final states ?

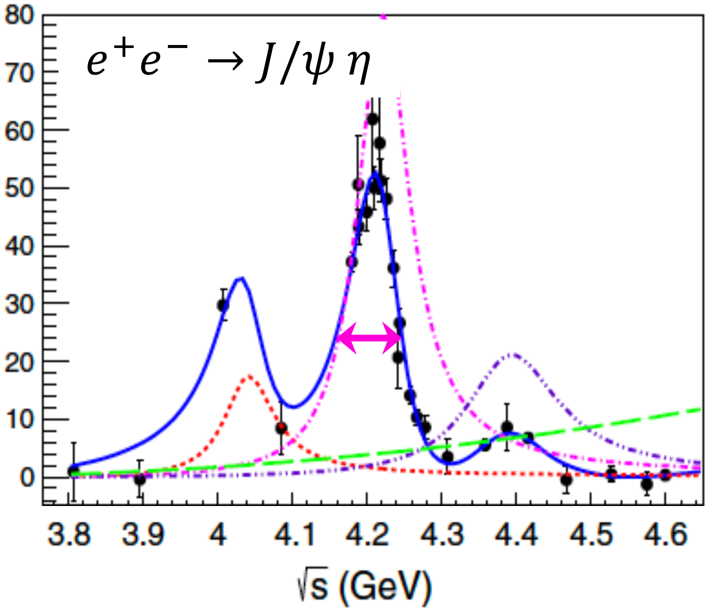
$\Gamma_{Y(4220)} \sim 44 \pm 4$ MeV



$\Gamma_{Y(4220)} \sim 77 \pm 7$ MeV



$\Gamma_{Y(4220)} \sim 82 \pm 6$ MeV



How to find solution to Y width problem ?

- Analyze different final states with different models (usual experimental method)

  → no simple relation between resonance parameters from different models → Y width problem created

- Analyze different final states simultaneously with a unified model (global analysis)

  * how various charmonia interfere to create different lineshapes in different final states
  * kinematical effects (threshold opening, triangle singularity) change lineshapes in some processes

  → Solution of the Y width problem

At the same time, global analysis determines:

(i) vector charmonium pole structure (pole locations)

(ii) couplings of the poles with decay channels (residues)

Now is the time to conduct global analysis of $e^+e^- \to c\bar{c}$ data, and determine vector charmonium poles and residues

BESIII accumulated high-quality data for various $e^+e^- \to c\bar{c}$ processes over wide energy region covering *Y*

$e^+e^- \to D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi\,\eta^{(\prime)}, \chi_{c0}\omega$ (two-body final states)

$e^+e^- \to \pi D^{(*)}\bar{D}^{(*)},\ J/\psi\pi\pi,\ \psi'\pi\pi, h_c\pi\pi, J/\psi K\bar{K}$ (three-body final states)

$e^+e^- \to \eta_c\rho\pi\ (\rho \to \pi\pi)$ (four-body final states)

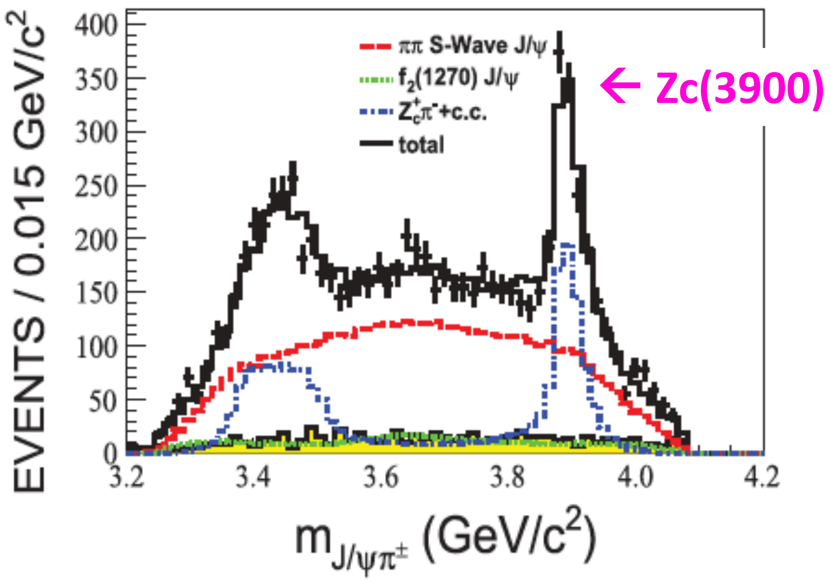
The global analysis is important not only for *Y* but also for well-established ψ(4040), ψ(4160), ψ(4415) because:

- Their properties were previously determined by simple Breit-Wigner fit to inclusive (e^+e^- → hadrons) *R* values
- Analyzing precise exclusive data → More detailed and precise information

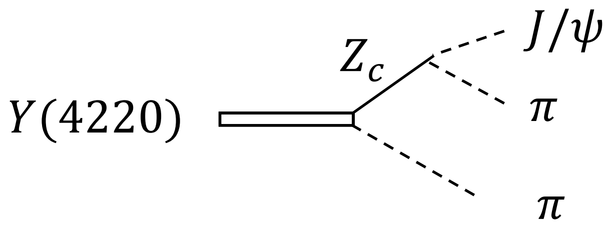
Understanding Y inevitably involves understanding Zc

Zc(3900), Zc(4020) : outstanding exotic candidates including $c\bar{c}u\bar{d}$

$e^+e^- \to J/\psi\, \pi^+\pi^-$ at Y(4220) region →



Zc appears as:



→ Y and Zc properties should be highly correlated

Global $e^+e^- \to c\bar{c}$ analysis consider Zc signals → address Y and Zc properties simultaneously

This work

- Global analysis of BESIII and Belle data in $3.75 \leq \sqrt{s} \leq 4.7$ GeV with a unified coupled-channel model

  $e^+e^- \to D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi\, \eta^{(\prime)}, \chi_{c0}\omega$ (9 two-body final states)

  $e^+e^- \to \pi D^{(*)}\bar{D}^{(*)},\ J/\psi\pi\pi,\ \psi'\pi\pi, h_c\pi\pi, J/\psi K\bar{K}$ (7 three-body final states)

  $e^+e^- \to \eta_c\rho\pi\ (\rho \to \pi\pi)$ (1 four-body final states)

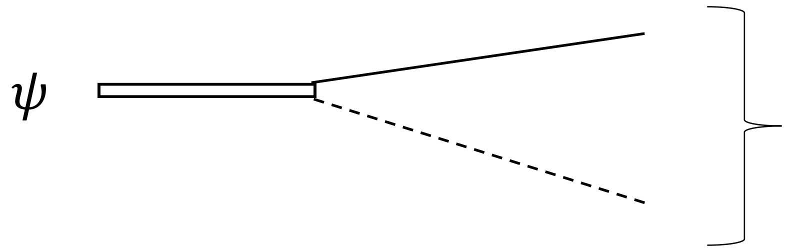
- Fit both total cross sections and invariant mass distributions
- Extract vector charmonium (ψ, Y) and Zc poles (mass, width)

Near-future work → Extraction of residues (branching fractions) and solution of Y width problem

MODEL

Coupled-channels

(quasi) two-body channels included; $J^{PC} = 1^{--}$



(A) $D_1(2420)\bar{D}^{(*)},\ D_1(2430)^0\bar{D}^{(*)},\ D_2^*(2460)\bar{D}^{(*)},\ D^{(*)}\bar{D}^{(*)}$

(B) $D_0^*(2300)\bar{D}^*,\ f_0 J/\psi,\ f_2 J/\psi,\ f_0\psi',\ f_0 h_c,\ Z_c\pi,\ Z_{cs}\bar{K}$

(C) $D_s^{(*)}\bar{D}_s^{(*)},\ J/\psi\eta,\ J/\psi\eta',\ \omega\chi_{c0}$

Group (B) $D_0^*(2300), f_0, f_2, Z_c$ as poles in two-body scattering amplitudes

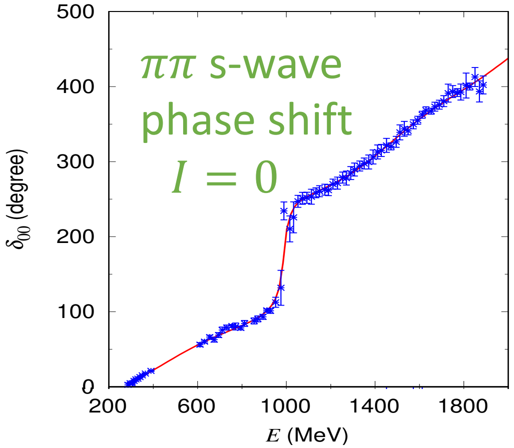
$D\pi$ s-wave amplitude fitting LQCD-based amplitude

Albaladejo et al. PLB 767, 465 (2017)

D_0^* pole : 2104 – i 100 MeV (ours)

$2105^{+6}_{-8} - i\ 102^{+10}_{-12}$ MeV (Albaladejo et al.)

$\pi\pi$ s[d]-wave amplitude fitting empirical amplitude



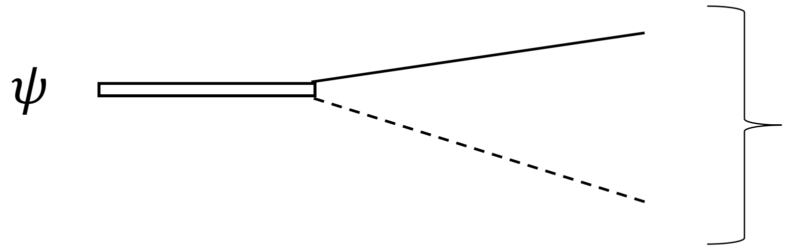
$f_0(500), f_0(980),$

$f_0(1370), f_2(1270)$ poles

→ consistent with PDG

Coupled-channels

(quasi) two-body channels included; $J^{PC} = 1^{--}$



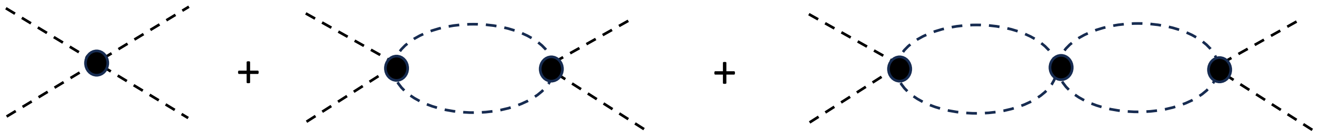
(A) $D_1(2420)\bar{D}^{(*)},\ D_1(2430)^0\bar{D}^{(*)},\ D_2^*(2460)\bar{D}^{(*)},\ D^{(*)}\bar{D}^{(*)}$

(B) $D_0^*(2300)\bar{D}^*,\ f_0 J/\psi,\ f_2 J/\psi,\ f_0\psi',\ f_0 h_c,\ Z_c\pi,\ Z_{cs}\bar{K}$

(C) $D_s^{(*)}\bar{D}_s^{(*)},\ J/\psi\eta,\ J/\psi\eta',\ \omega\chi_{c0}$

Group (B) $Z_c : J^{PC} = 1^{+-}$ $D^*\overline{D} - D^*\overline{D}^* - J/\psi\pi - \psi'\pi - h_c\pi - \eta_c\rho$ couple—channel scattering amplitude

driven by contact interactions; s-wave interactions except $h_c\pi$ p-wave interaction

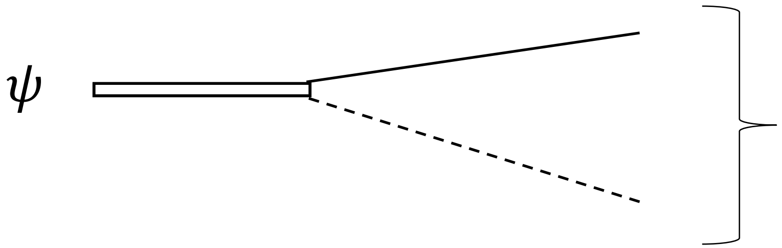
Z_c amplitude =  + …

intermediate loops include all possible coupled-channels

$v_{D^*\overline{D},D^*\overline{D}} = v_{D^*\overline{D}^*,D^*\overline{D}^*}$ (HQSS), no coupling between hidden-charm channels (e.g. $v_{J/\psi\pi,J/\psi\pi} = v_{J/\psi\pi,\psi'\pi} = 0$)

Nonzero couplings are determined by the global fit → poles may be generated if needed by data

Coupled-channels



(quasi) two-body channels included; $J^{PC} = 1^{--}$

(A) $D_1(2420)\bar{D}^{(*)},\ D_1(2430)^0\bar{D}^{(*)},\ D_2^*(2460)\bar{D}^{(*)},\ D^{(*)}\bar{D}^{(*)}$

(B) $D_0^*(2300)\bar{D}^*,\ f_0 J/\psi,\ f_2 J/\psi,\ f_0\psi',\ f_0 h_c,\ Z_c\pi,\ Z_{cs}\bar{K}$

(C) $D_s^{(*)}\bar{D}_s^{(*)},\ J/\psi\eta,\ J/\psi\eta',\ \omega\chi_{c0}$

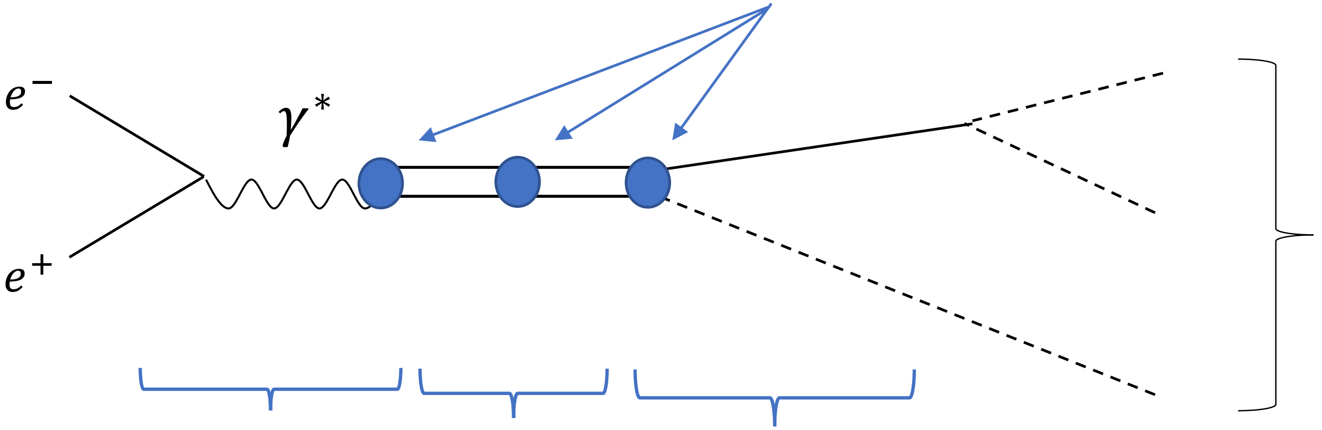
Group (A) $D_1(2420), D_1(2430)$, $D_2^*(2460), D^*$ → Breit-Wigner (BW) propagators; mass and width from PDG

Group (C) treated as stable particles

Full amplitude for three-body final states

Dressed vertices (propagator) : bare vertices (propagator) dressed by hadron scattering

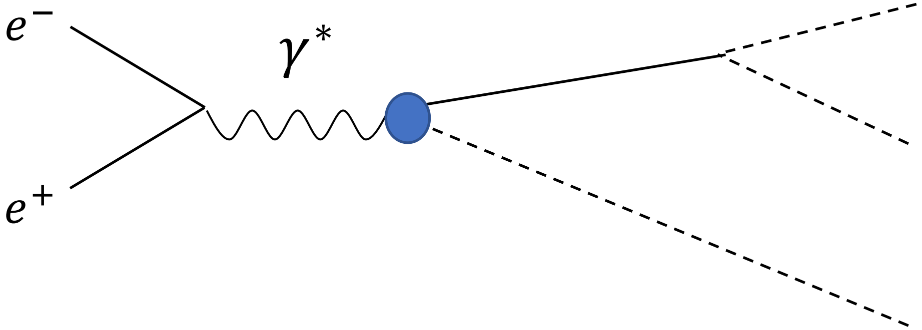
Unitarity requirement



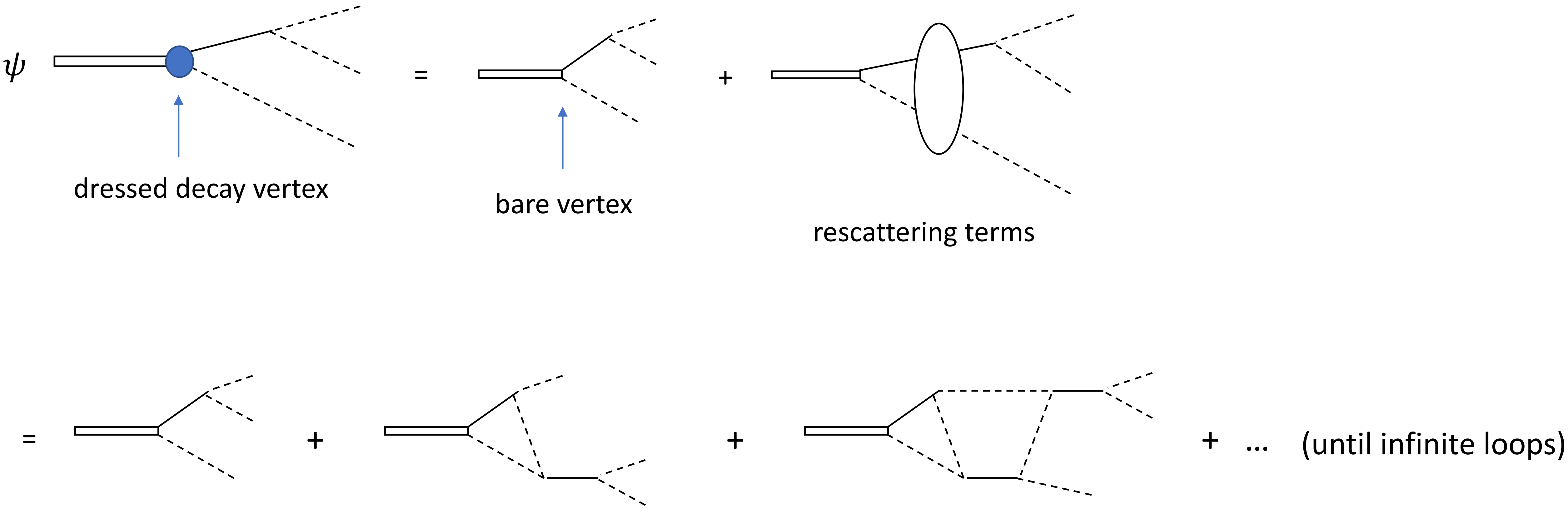
$\pi D^{(*)}\bar{D}^{(*)}$, $J/\psi\pi\pi$, $\psi'\pi\pi$, $h_c\pi\pi$, $\eta_c\rho\pi$, $J/\psi K\bar{K}$

ψ production,   propagation,   decay

Non-resonant mechanisms are also included



Three-body decays of ψ

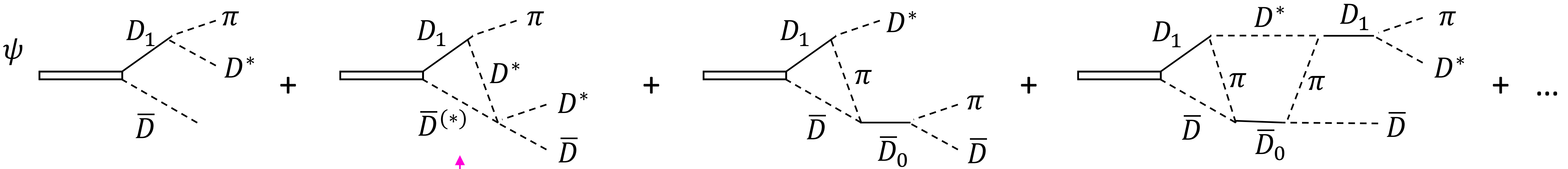


Final state interactions described by solution of Faddeev equation → Coupled-channels taken into account

Rescattering mechanisms (particle exchange) required by three-body unitarity are considered

Three-body decays of ψ

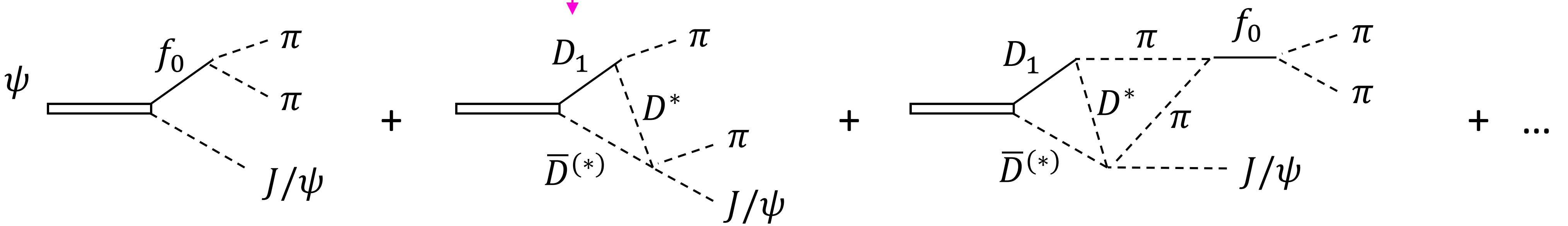
$e^+e^- \to \pi D^* \bar{D}$



(until infinite loops)

Triangle singularity loop

$e^+e^- \to J/\psi\pi\pi$



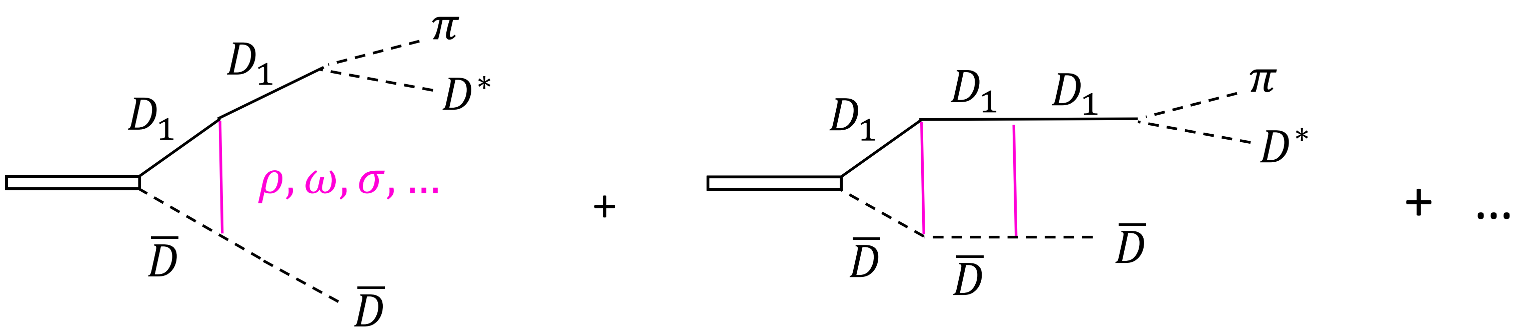
(until infinite loops)

Selected important diagrams; diagrams with more loops are usually more suppressed

Different processes share the same interactions ← unitarity requirement

In addition, more mechanisms (off-shell, short-range) should exist → absorbed in bare couplings and masses of ψ

example



Poorly understood mechanisms → fitting unknown coupling constants to data is computationally too expensive

→ Some bare ψ states could be hadron-molecules

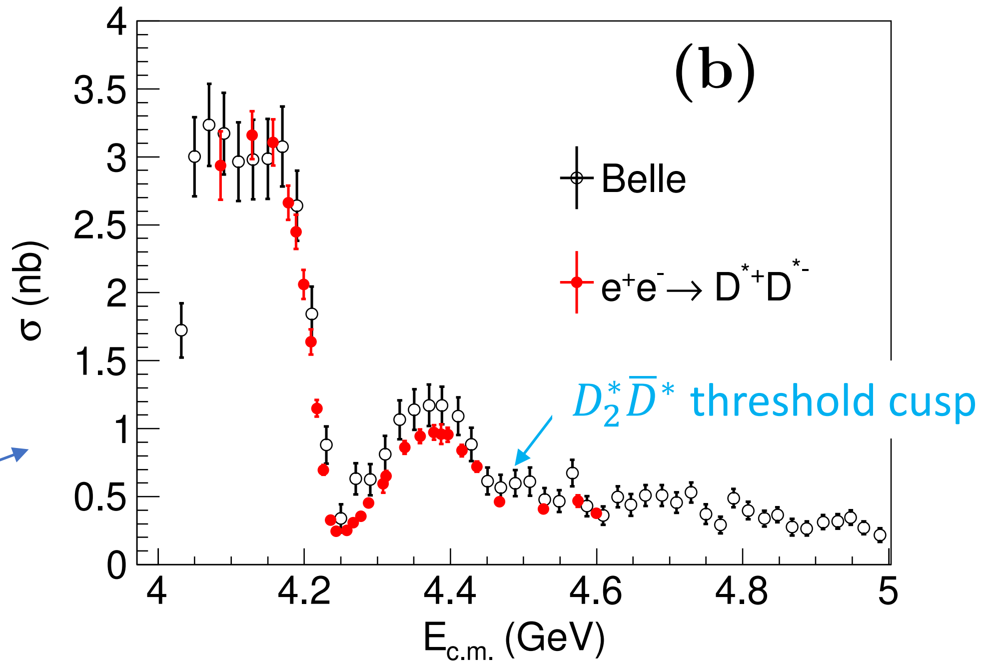
Ongoing update:

Contact interactions among $D_1\bar{D}^{(*)}, D_2^*\bar{D}^*, D^{(*)}\bar{D}^{(*)}$ channels

→ fitted to data (advantage of separable interactions)

High-precision BESIII data require these contributions (cusp)

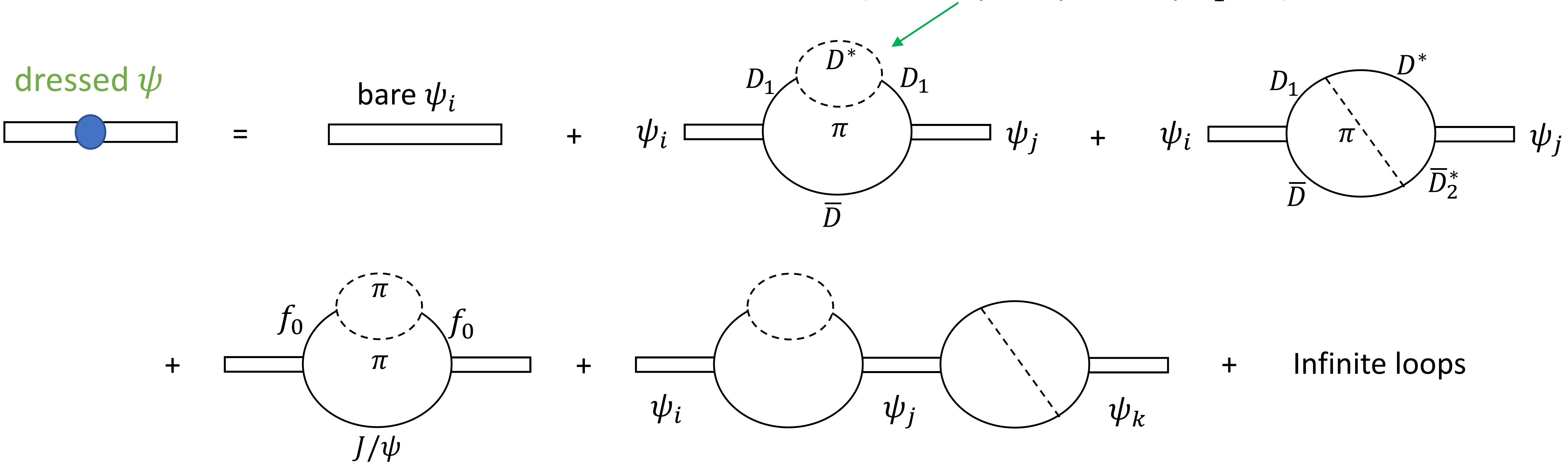
Non-perturbative treatment → three-body unitarity requirement



ψ propagator

(we do not use BW)

($D^*\pi$-loop is replaced by D_1 BW)



Charmonium poles are formed by non-perturbative couplings between bare ψ and $D_1\bar{D}$, $f_0\, J/\psi$, ...

(= poles of dressed ψ propagator)

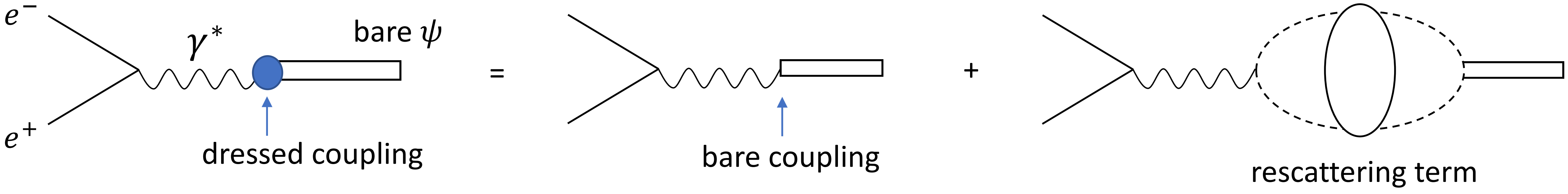
Unitary coupled-channel model : resonance pole (mass, width) and decay dynamics are explicitly related.

(unitarity requirement)

Breit-Wigner model : decay dynamics are simulated by BW mass and width parameters

ψ production mechanisms

$e^+e^- \to c\bar{c}$ data in $3.75 \leq \sqrt{s} \leq 4.7$ GeV region → Charmonium excitations are important mechanism



Data determine how many bare states to be included (5 bare states) and which charmonium states exist

Expected states $\psi(3770),\ \psi(4040),\ \psi(4160),\ \psi(4415),\ Y(4220),\ Y(4360)$

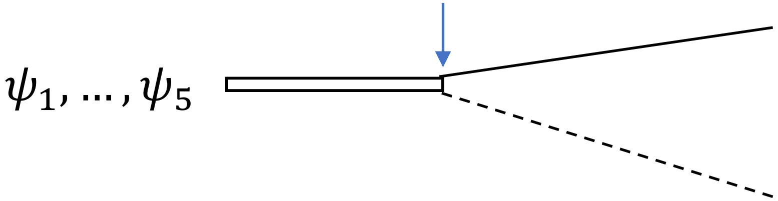
Data is not sufficient for coupled-channel analysis in $\sqrt{s} > 4.6$ GeV (three-body final states including $c\bar{c}s\bar{s}$)

→ $Y(4660)$ is not included in coupled-channel amplitude → included as a Breit-Wigner amplitude

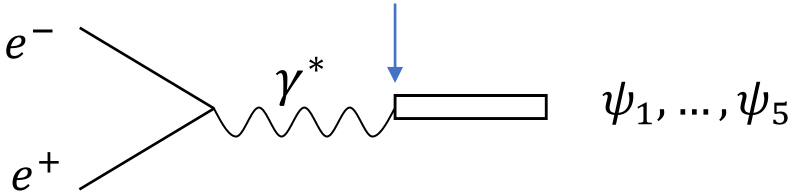
Fitting parameters in global analysis

* bare ψ masses (5 bare states)

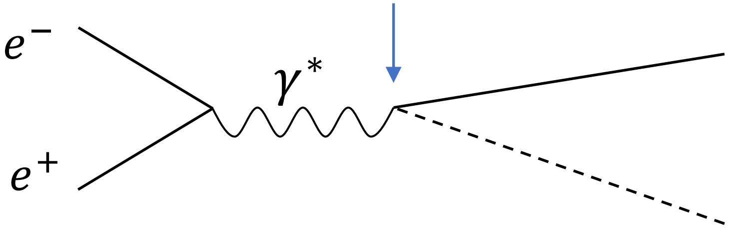
* bare ψ coupling constants (real)



* bare photon-ψ coupling constants (real)



* non-resonant photon coupling constants (real)



* $\psi(4660)$ Breit-Wigner mass, width, vertices

* coupling constants in Z_c amplitude :

  $v_{D^*\bar{D},D^*\bar{D}}$, $v_{D^*\bar{D},J/\psi\pi}$, $v_{D^*\bar{D},\psi'\pi}$ etc.

* $D^{(*)}\bar{D}^{(*)}$, $D_s^{(*)}\bar{D}_s^{(*)}$ contact interaction strengths

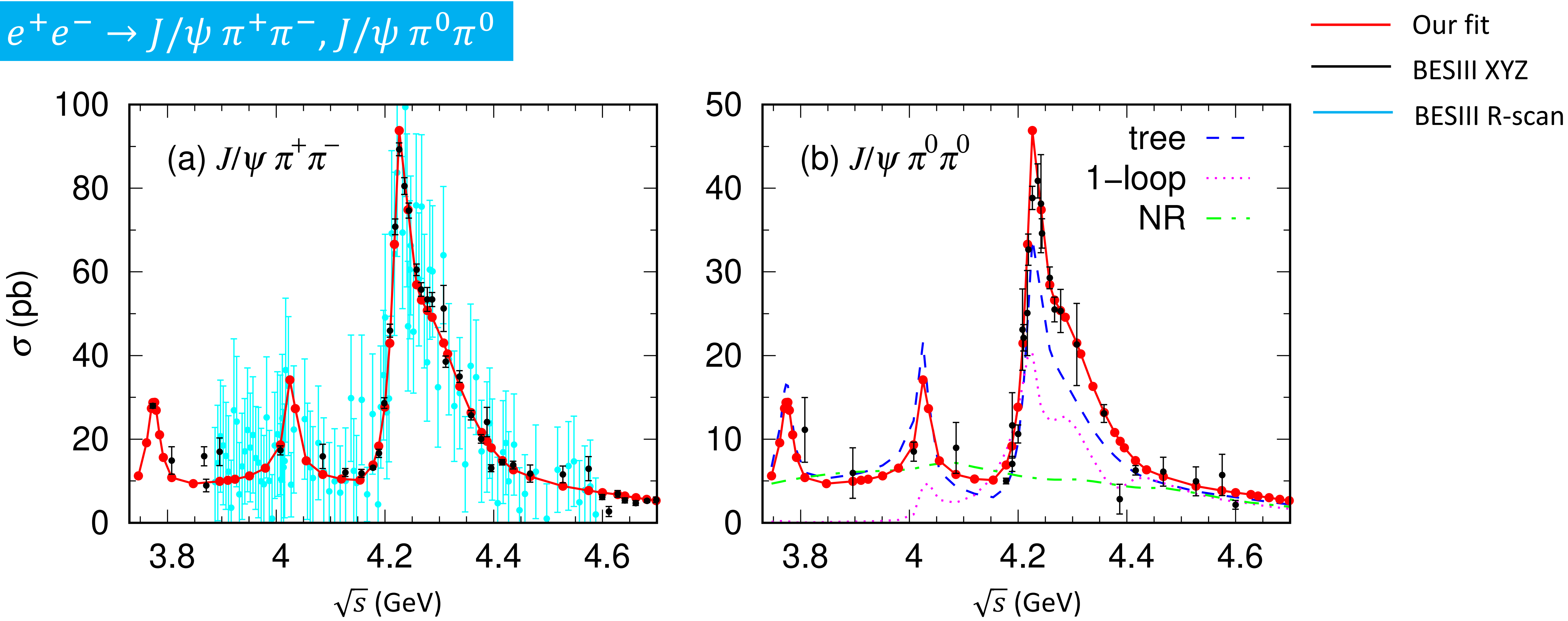
* Cutoffs in non-resonant $\gamma^* \to D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}$ vertices

In total, 177 fitting parameters

Selected fit results

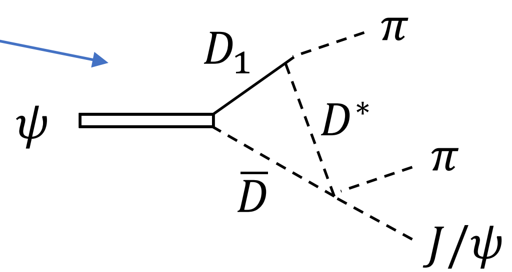
(ongoing update not included, unless otherwise stated)

$e^+e^- \to J/\psi\,\pi^+\pi^-, J/\psi\,\pi^0\pi^0$



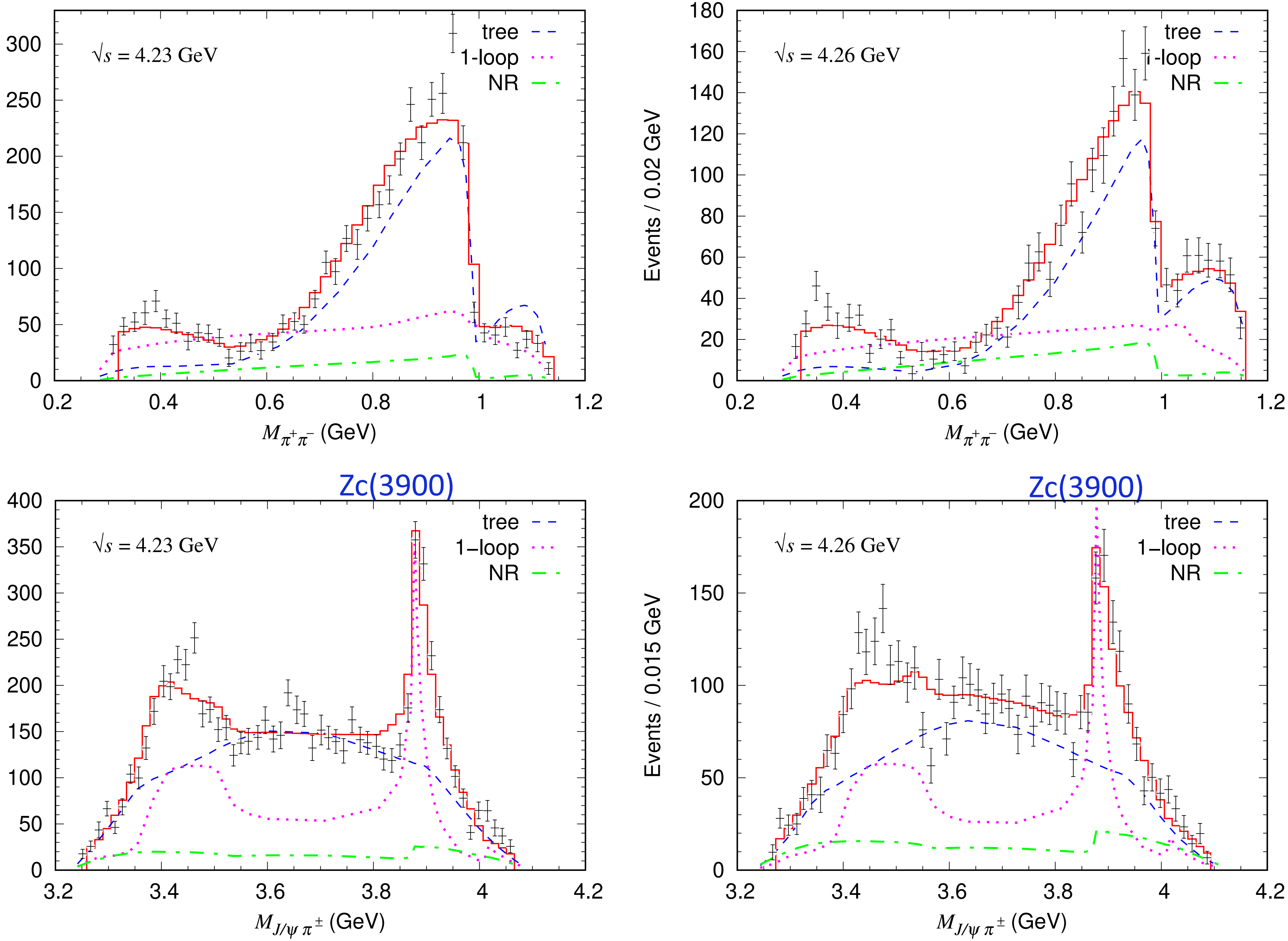
- Overall good agreement with data (our model is isospin symmetric, $\sigma(J/\psi\pi^+\pi^-) = 2\times\sigma(J/\psi\pi^0\pi^0)$)
- Triangle singularity effect is seen in 1-loop contribution at $\sqrt{s} \sim 4.28$ GeV

  → Y(4320)-like enhancement in full calculation → Y(4320) is TS in our analysis



- Peaking structure at $\sqrt{s} \sim 4$ GeV is a consequence of the combined fit ($\psi(4040)$)

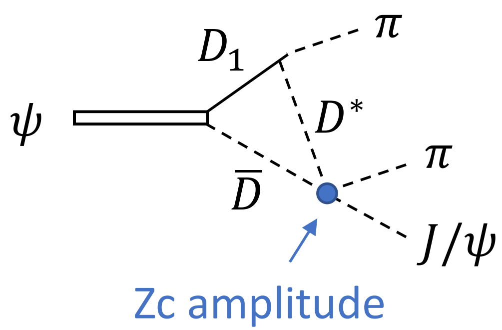
$e^+e^- \to J/\psi\,\pi^+\pi^-$



Fit to invariant masses

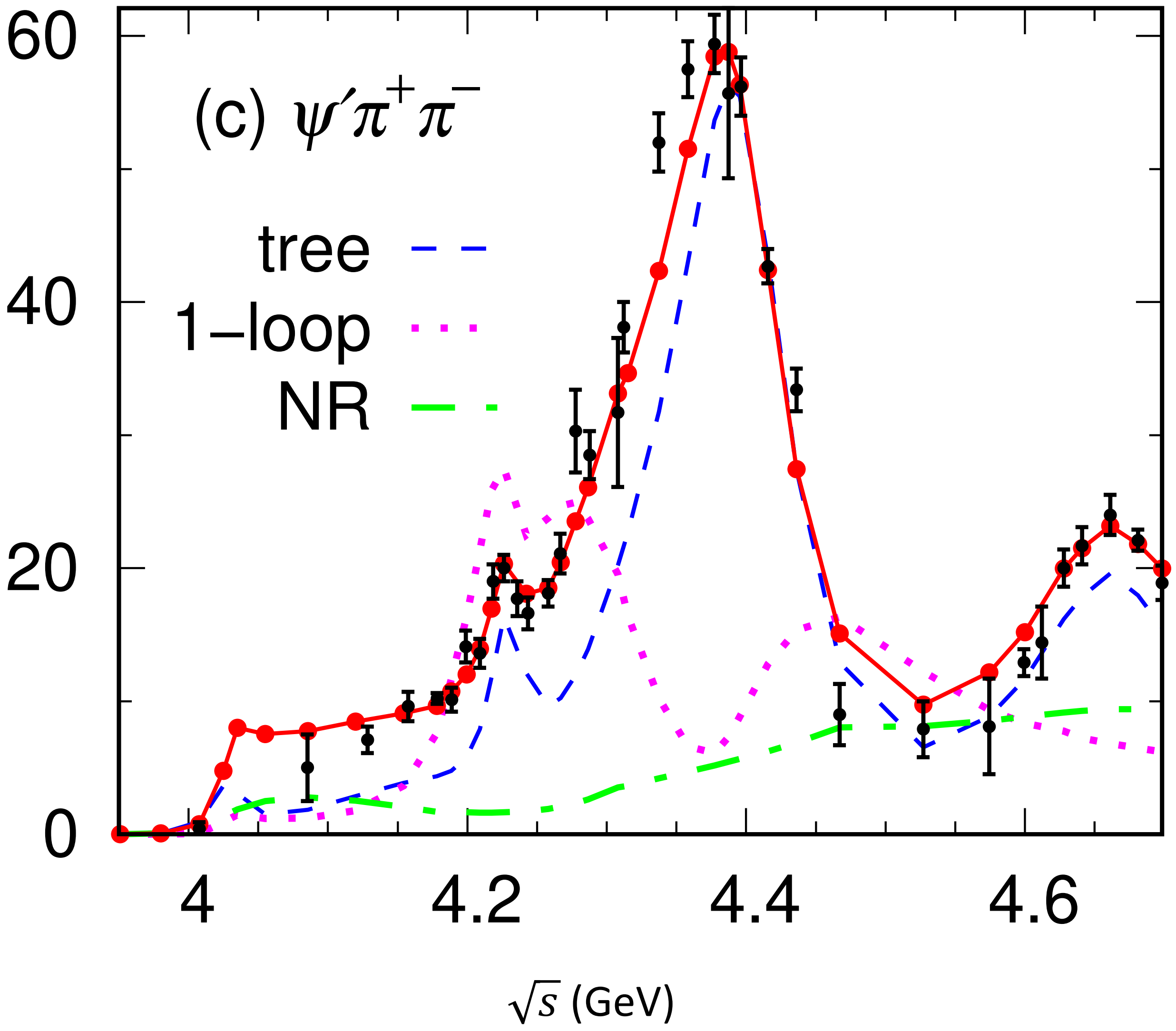
Zc(3900) peaks are well fitted

1-loop causes $D^*\bar{D}$ thres. cusp enhanced by a possible pole (a bit off TS condition)



We will examine Zc(3900) pole

$e^+e^- \to \psi' \pi^+\pi^-$



- Overall good fit
- Enhancement at ~ 4.03 GeV is from ψ(4040)

  ← consequence of coupled-channel fit
- 1-loop contribution is enhanced at

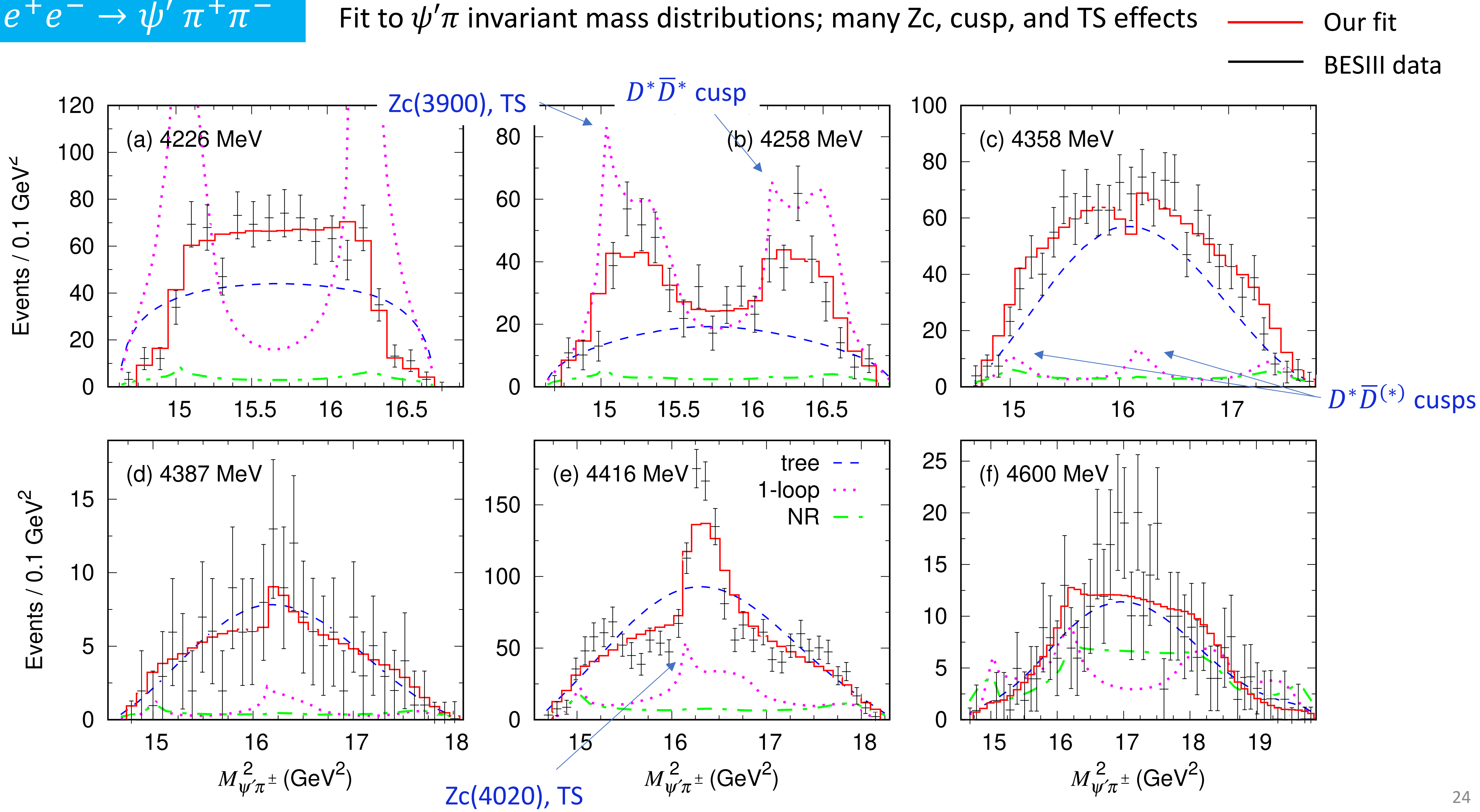
  ~ 4.28 GeV → $D_1(2420)\bar{D}$ threshold

  ~ 4.45 GeV → $D_1(2420)\bar{D}^*$, $D_2^*(2460)\bar{D}^*$ thresholds

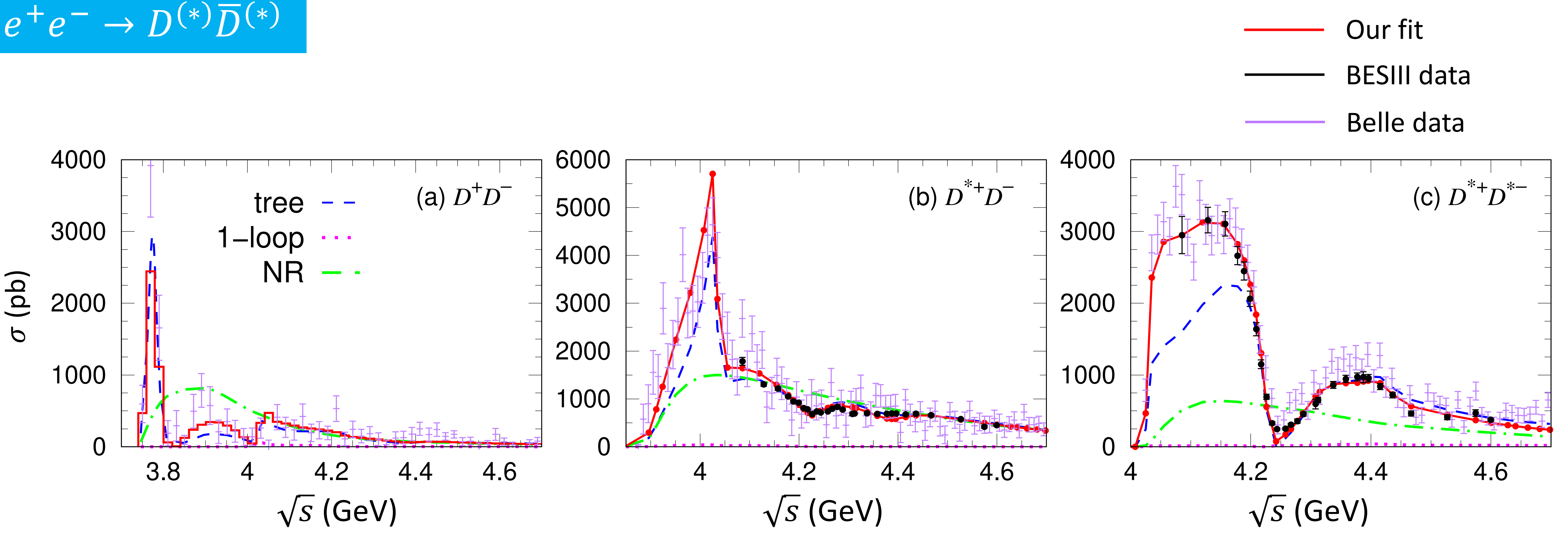
  due to opening the thresholds, triangle singularity

$e^+e^- \to \psi' \pi^+\pi^-$

Fit to $\psi'\pi$ invariant mass distributions; many Zc, cusp, and TS effects

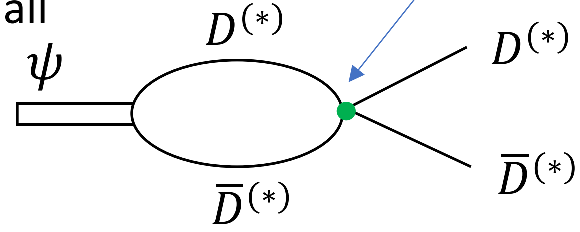


$e^+e^- \to D^{(*)}\bar{D}^{(*)}$



Tree dominates, 1-loop (particle exchange) is small

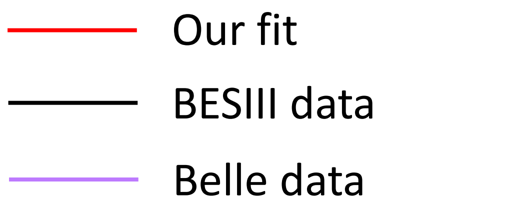
For $e^+e^- \to D^{(*)}\bar{D}^{(*)}$, $D_s^{(*)}\bar{D}_s^{(*)}$, we add

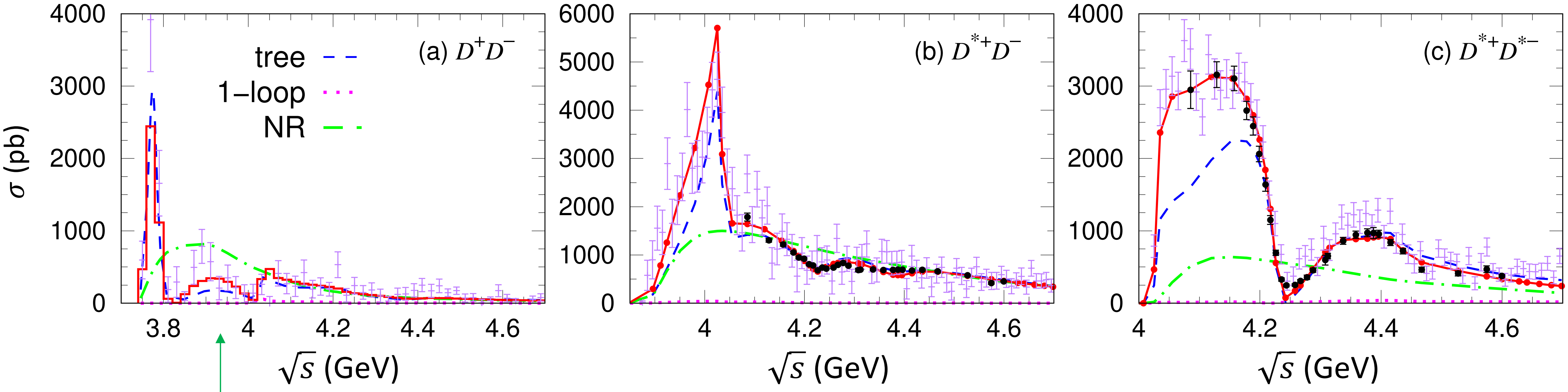


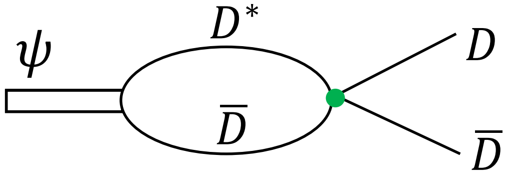
for threshold enhancement → better fit

(difference between blue and red curves above)

$e^+e^- \to D^{(*)}\bar{D}^{(*)}$





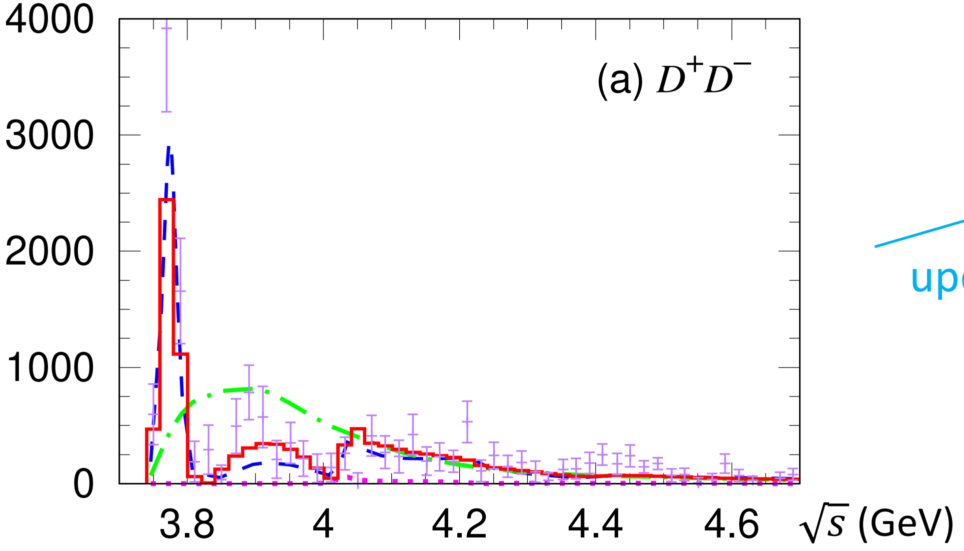


Sharp dip and resonance-like peak at 3.9 GeV (not G(3900) resonance)

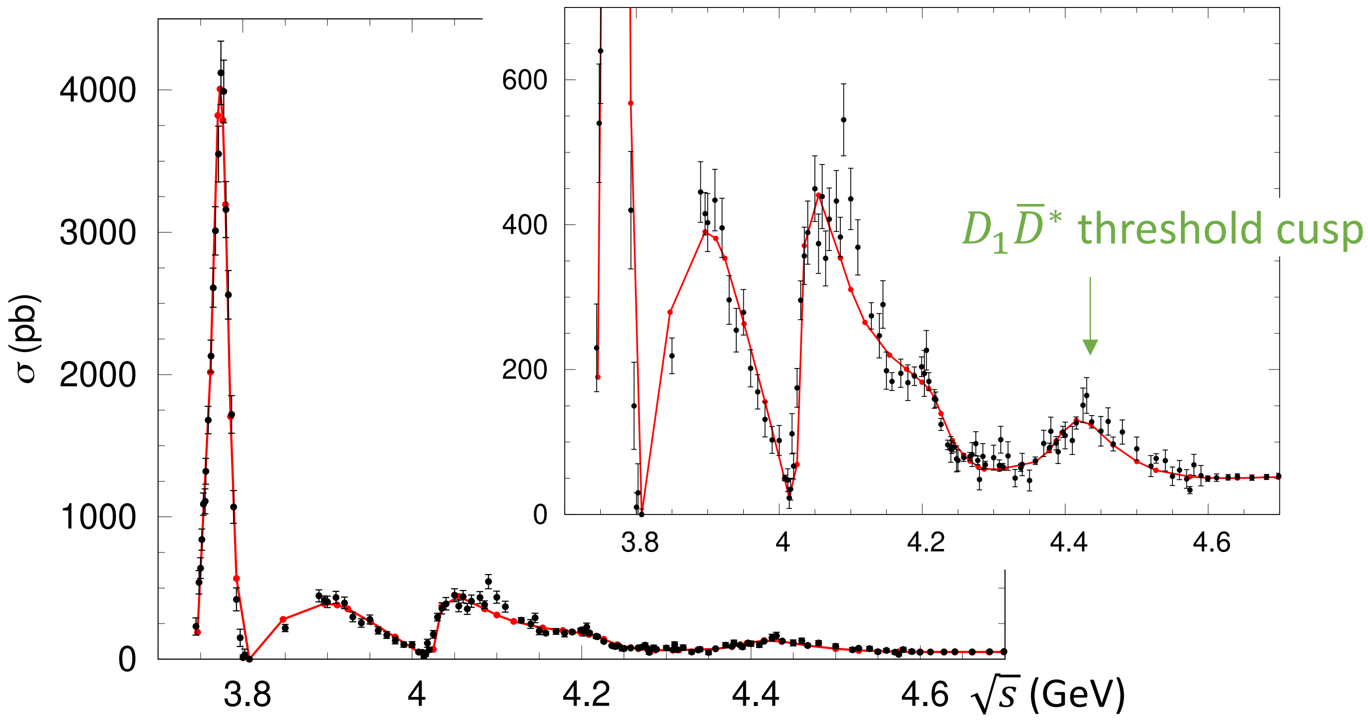
← Interference between ψ(3770), ψ(4040) and non-resonant amplitudes + $D^*\bar{D}$ threshold cusp

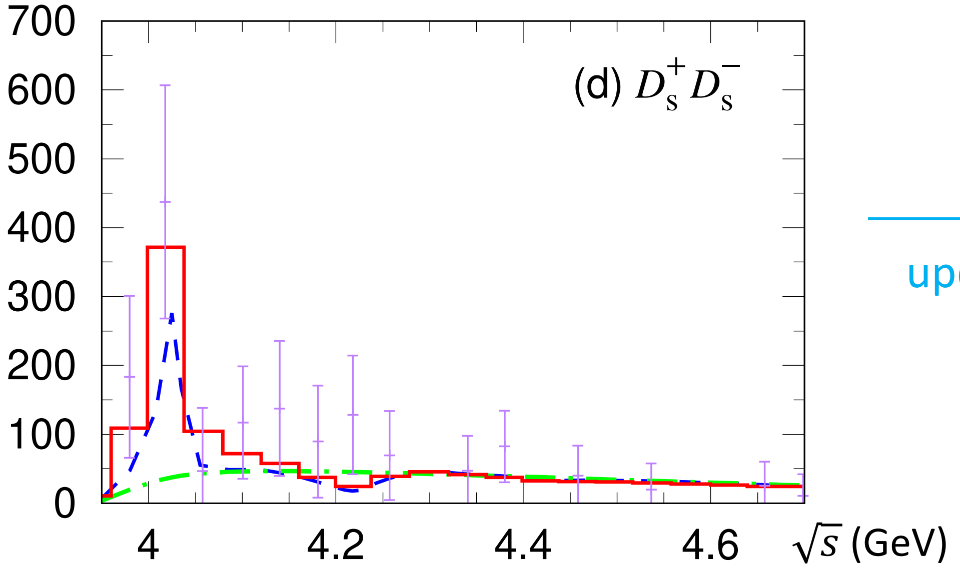
Ongoing update: New BESIII data included

$e^+e^- \to D\bar{D},\ D_s\bar{D}_s$ (preliminary fits)

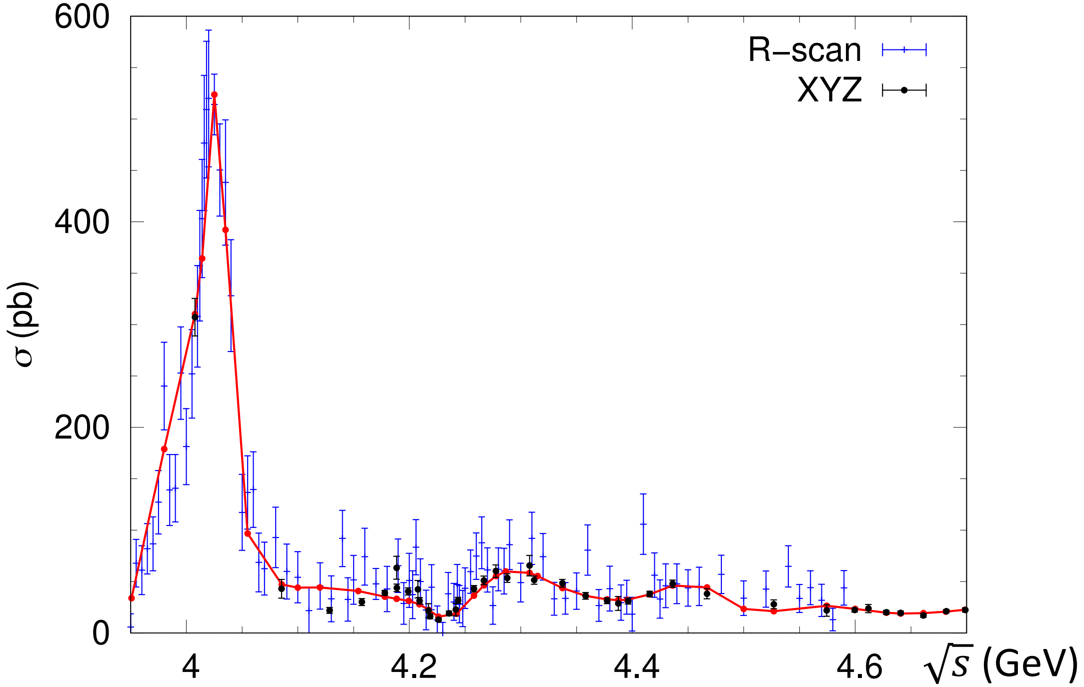


→ update





→ update

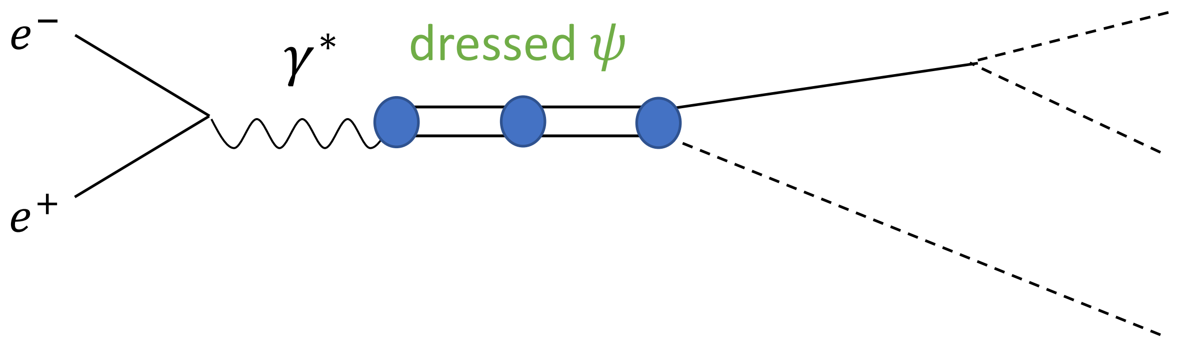


Poles and resonance properties

ψ poles from their dressed propagator

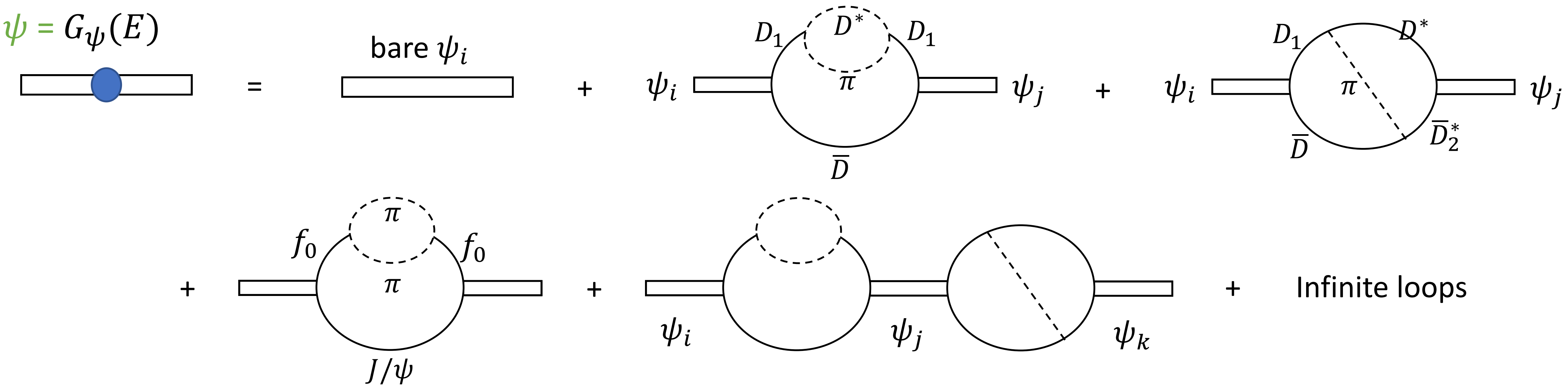
(we are not using BW)

Full amplitude



+ non-resonant

dressed ψ = $G_\psi(E)$



Search complex energy E_ψ where $G_\psi(E_\psi) = \infty$ (E_ψ: pole energy, pole position) by analytical continuation of $G_\psi(E)$

Resonance parameters

$M = \mathrm{Re}[E_\psi]$
$\Gamma = -2\times\mathrm{Im}[E_\psi]$

	This work		PDG [4]		
	M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	
	3775 ± 2.0	28 ± 1.0	3778.1 ± 0.7	27.5 ± 0.9	$\psi(3770)$
	4026 ± 0.1	25 ± 0.3	4039 ± 1	80 ± 10	$\psi(4040)$
	4232 ± 1.0	114 ± 1.7	4191 ± 5	70 ± 10	$\psi(4160)$
	4226 ± 0.4	36 ± 0.8	4222.5 ± 2.4	48 ± 8	$\psi(4230)$
	4309 ± 0.6	328 ± 0.9	–	–	–
	4369 ± 0.1	183 ± 0.2	4374 ± 7	118 ± 12	$\psi(4360)$
	4394 ± 0.7	93 ± 0.9	4421 ± 4	62 ± 20	$\psi(4415)$
BW fit →	4690 ± 7.3	106 ± 8.8	4630 ± 6	72^{+14}_{-12}	$\psi(4660)$

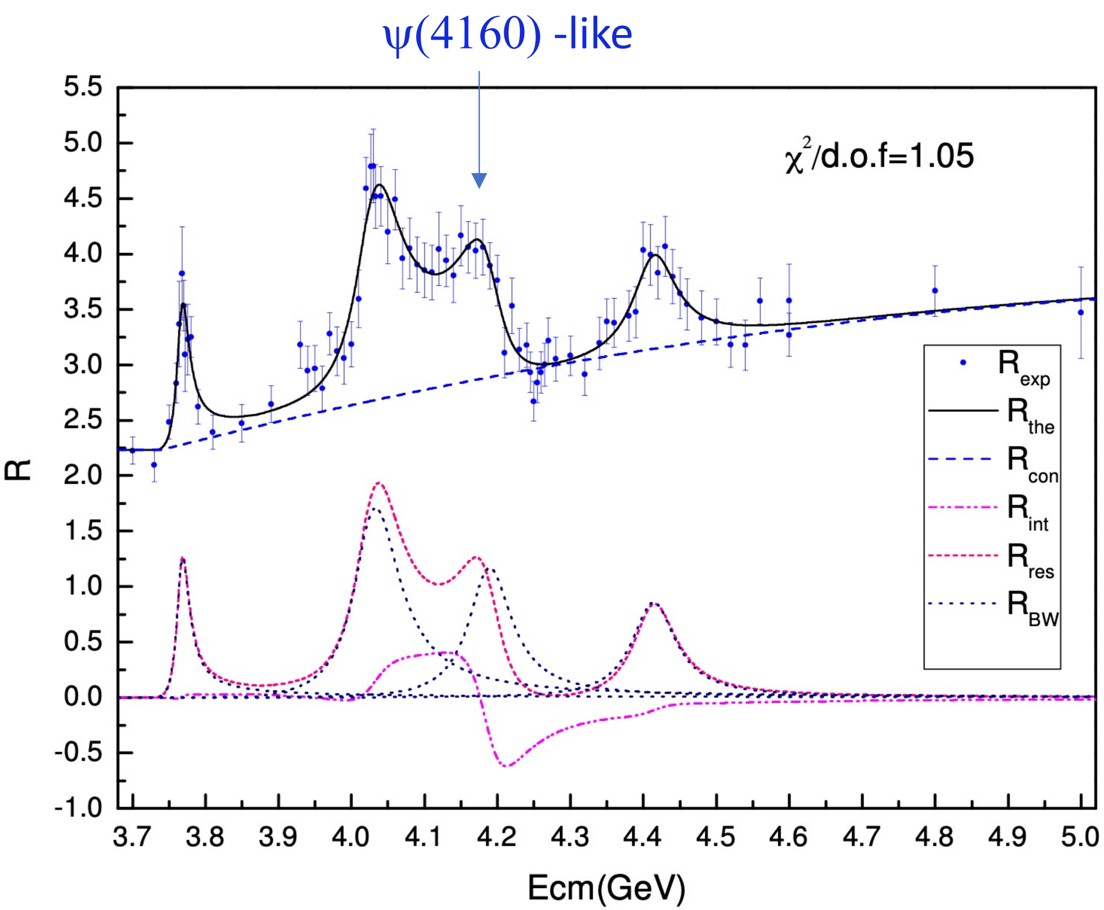
Noticeable differences from PDG

Two states at ~ 4230 MeV (narrow and broader states) rather than ψ(4160) and ψ(4230)

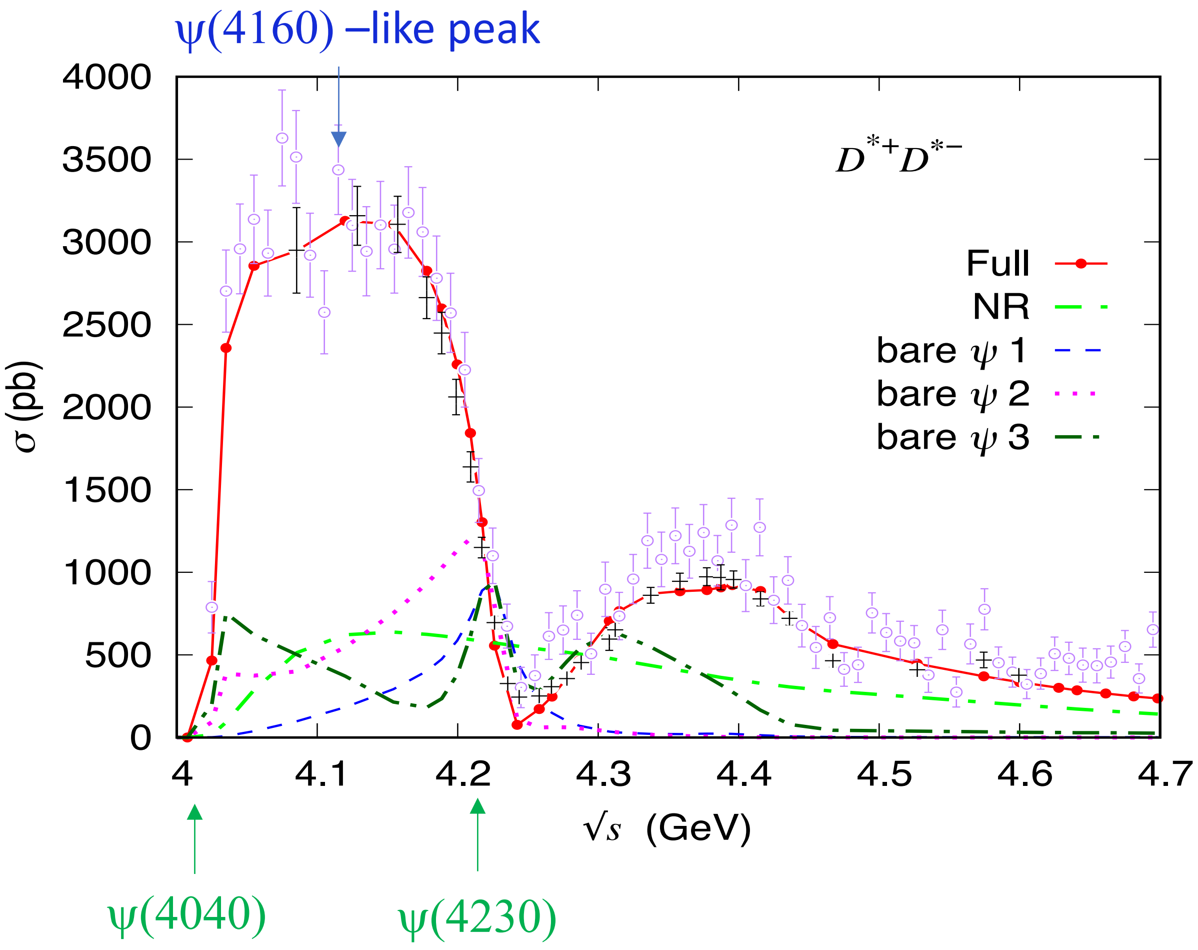
The pole uncertainty seems too small → will be improved in ongoing update

ψ(4160) from our analysis ?

ψ(4160) signal is seen in the R-scan data (BES2)



BES, PLB 660, 315 (2008)



ψ(4160) -like signals in exclusive processes can be described with interfering ψ(4040), ψ(4230), and non-resonant

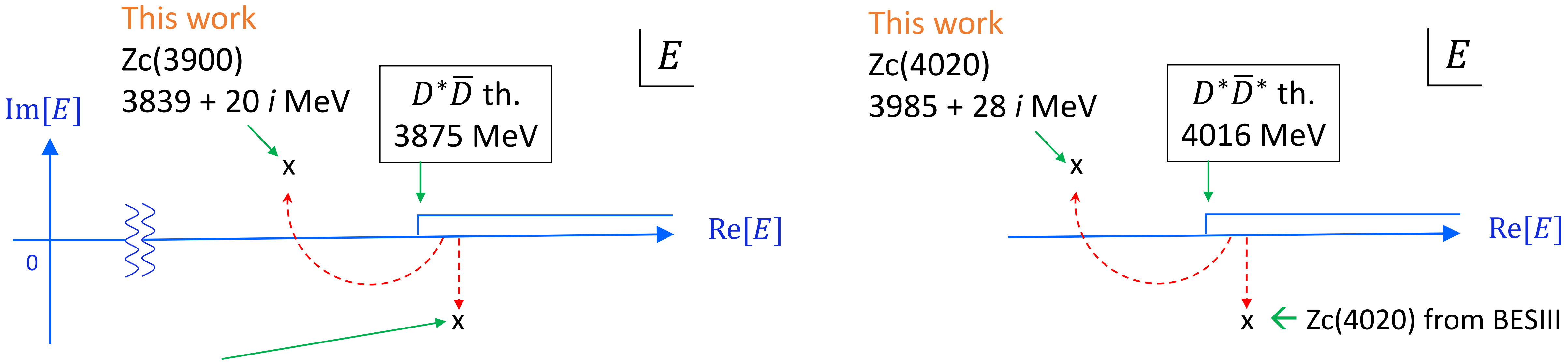
ψ(4160) properties in PDG are from a simple BW fit to *R* value → artifacts may happen

Zc poles

from $J^{PC}=1^{+-}$ $D^*\bar{D}-D^*\bar{D}^*-J/\psi\pi-\psi'\pi-h_c\pi-\eta_c\rho$ couple—channel amplitude

This work		PDG [4]		
$E_{\rm pole}$ (MeV)	RS	M (MeV)	Γ (MeV)	
$(3839\pm11)+(20\pm3)i$	up	3887.1 ± 2.6	28.4 ± 2.6	$Z_c(3900)$
$(3985\pm7)+(28\pm5)i$	pu	4024.1 ± 1.9	13 ± 5	$Z_c(4020)$

RS=$(s_{D^*\bar{D}}, s_{D^*\bar{D}^*})$



Zc(3900) from BESIII, Belle, D0 (BW fit), most previous theoretical models

Zc from our analysis are virtual states, different from Breit-Wigner fit and most of previous theoretical analyses

Present analysis result is consistent with lattice QCD

Previous LQCD analyses on $Z_c(3900)$ in:

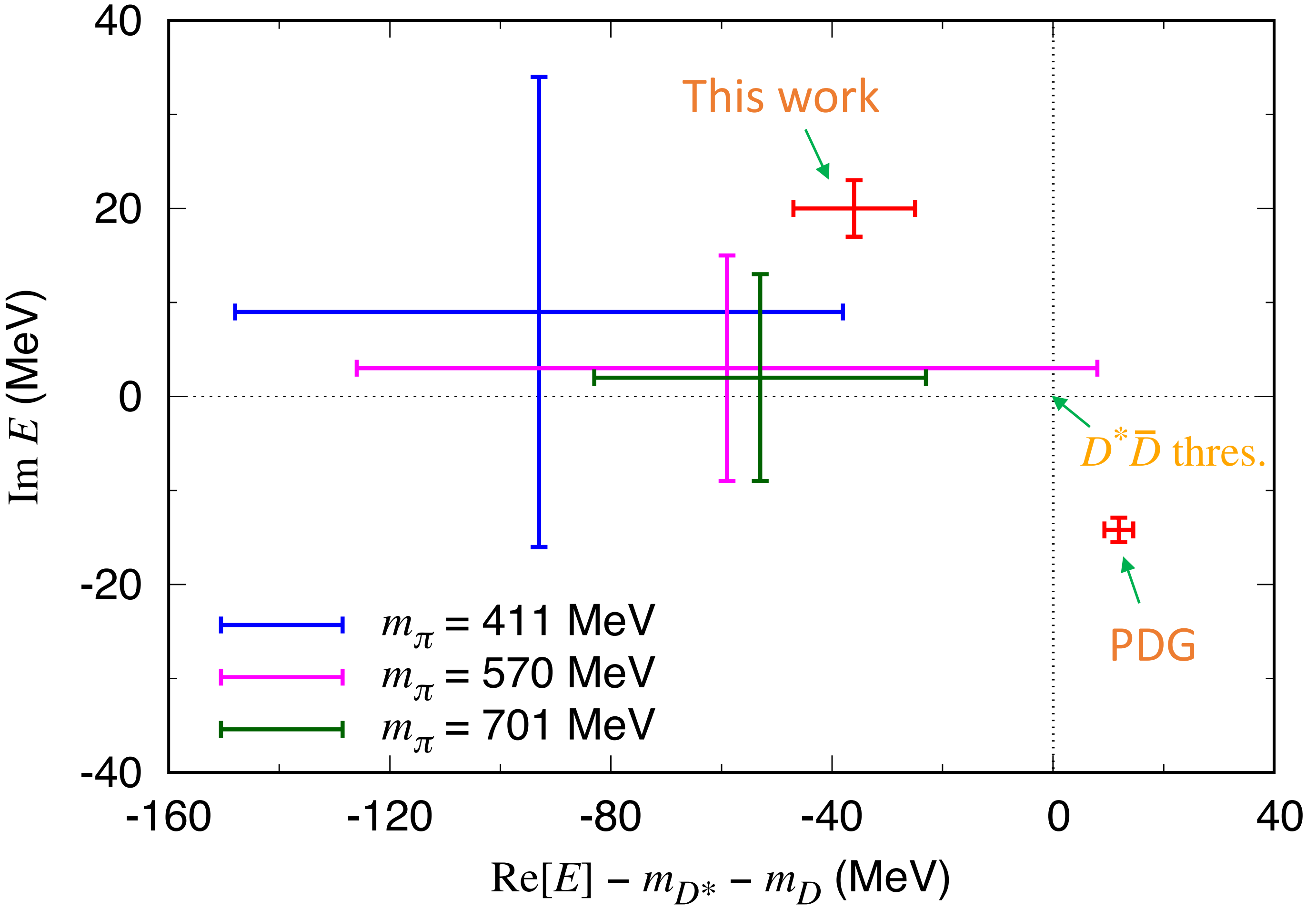
Prelovsek et al. PLB 727, 172 (2013), PRD 91, 014504 (2015)
Chen et al. PRD 89 , 094506 (2014)
Ikeda et al. (HAL QCD) PRL 117, 242001 (2016)
Cheung et al. (Hadron spectrum Collab.) JHEP 11, 033 (2017)

LQCD conclusion : $I=1,\ J^{PC}=1^{+-}\ D^*\bar{D}$ s-wave interaction is very weak,

disfavoring narrow $Z_c(3900)$ pole near $D^*\bar{D}$ threshold

Most of previous determinations of Zc(3900) pole are not consistent with LQCD

Zc(3900) pole: comparison with LQCD result



Zc(3900) pole positions in $D^*\overline{D}$ unphysical sheet

LQCD ($m_\pi = 411$ MeV)
HAL QCD, J. Phys. G 45, 024002 (2018)

$m_{D^*} + m_D - (93 \pm 55 \pm 21) + (9 \pm 25 \pm 7)i$ MeV

$S(\{-k_i^*\}) = S^*(\{k_i\})$ applied; PRD 105, 014034 (2022)

This work

$m_{D^*} + m_D - (36 \pm 11) + (20 \pm 3)i$ MeV

PDG

$m_{D^*} + m_D + (11.9 \pm 2.6) - (14.2 \pm 1.3)i$ MeV

LQCD and this work are fairly consistent (virtual poles)

Q. Can the global analysis tell Zc(3900) is resonance or virtual state ?

The presented analysis employed energy independent interactions for Zc amplitude

→ Only virtual or bound states are examined → virtual state works fine

Ongoing update

Zc amplitude with resonant Zc(3900) state is implemented in the three-body coupled-channel model

→ Its performance in the global fit will be examined

Summary and perspective

Summary

- Conducted global coupled-channel analysis of most of available $e^+e^- \to c\bar{c}$ data in $\sqrt{s} = 3.75 - 4.7$ GeV

  Global coupled-channel analysis is common for N*. The $e^+e^- \to c\bar{c}$ analysis now gets closer to the standard !
- Reasonable fits are obtained overall
- Vector charmonium and Zc poles extracted

  -- Two poles at ~ 4230 MeV with different widths

  -- Zc poles are virtual poles at ~ 40 MeV below $D^*\overline{D}^{(*)}$ thresholds, consistent with LQCD results

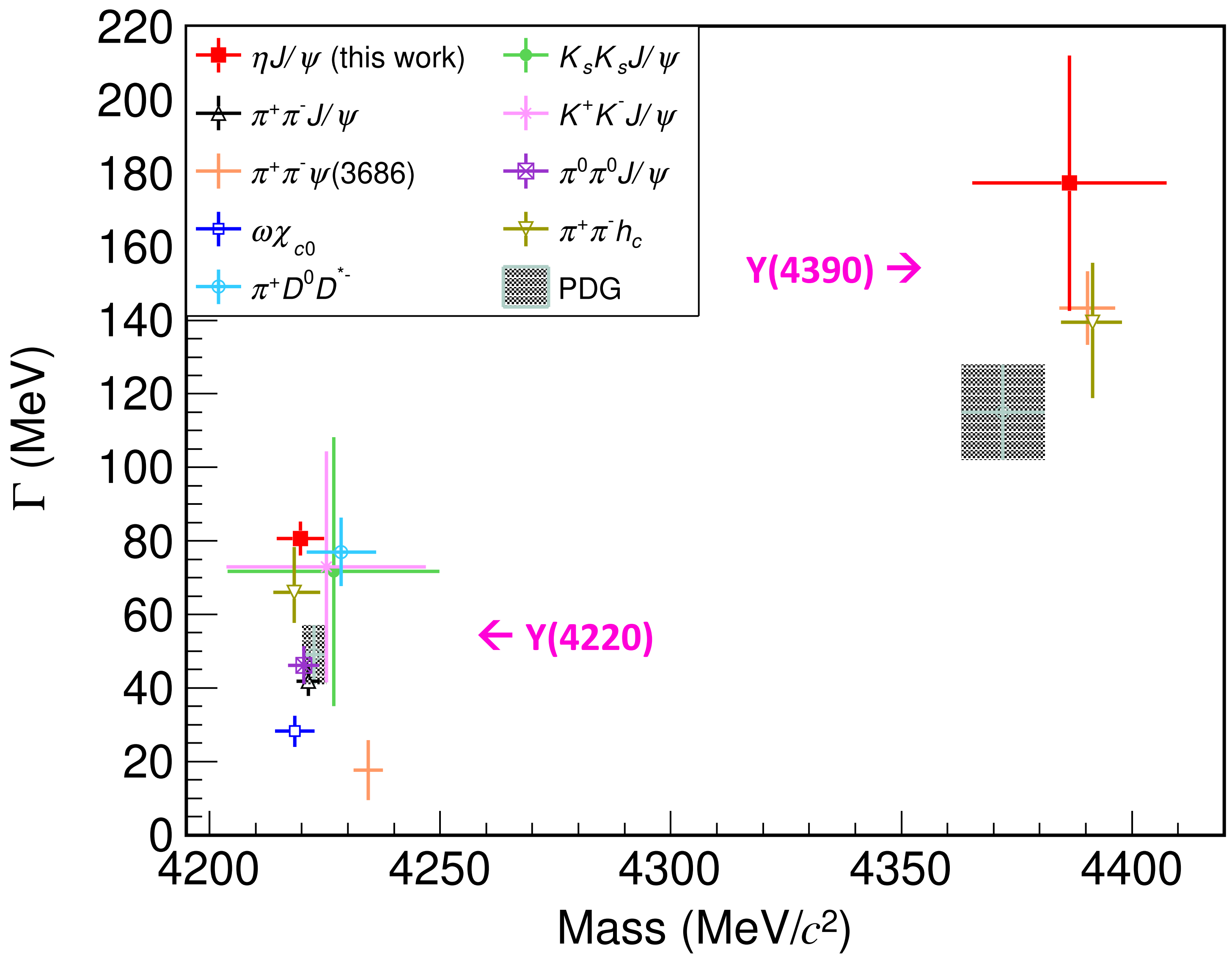
Future

- Pole residues will be extracted → address Y width problem, structure of exotic candidates *Y*
- Fit efficiency-corrected, background-free Dalitz plots (not 1D fit) to fully consider experimental constraints on charmonium and Zc properties
- Include $e^+e^- \to K\overline{D}_s^{(*)}D^{(*)}$ cross sections when available → include higher charmonium states

  → address Zcs(3985) from global analysis

Backup

Outstanding question in XYZ physics : Y width problem



BESIII, arXiv:2310.03361

Related works previously done

Three-body model

* M. Cleven, Q. Wang, F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Zhao, PRD 90, 074039 (2014)

  Analysis of $e^+e^- \to \pi D\bar{D}^*$, $J/\psi\pi\pi$, $h_c\pi\pi$ cross section and invariant mass in $4.1 \lesssim \sqrt{s} \lesssim 4.3$ GeV [Y(4230) region]

  Pioneering works, but the data were very limited → limited conclusions on Y(4230) properties

* L. Detten, C. Hanhart, V. Baru, Q. Wang, D. Winney, Q.Zhao, arXiv:2402.03057

  Fitting data in Y(4230) region; more final states than the above

Our analysis includes significantly more complete dataset

→ More reliable conclusion

Breit-Wigner fit to cross section data

* D.-Y. Chen, X. Liu, T. Matsuki, Eur. Phys. J. C 78, 136 (2018)

  Fitting of $e^+e^- \to \pi D\bar{D}^*$, $J/\psi\pi\pi$, $h_c\pi\pi$ cross sections → Y(4320) and Y(4390) not necessary

Two-body unitary model fitted to cross section data

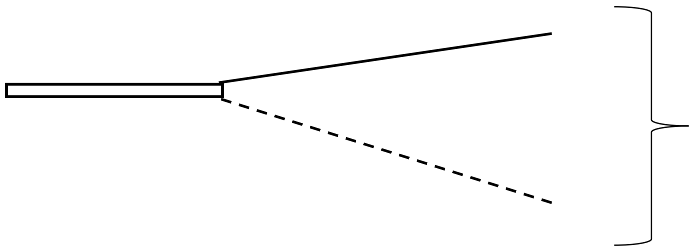
* Z.-Y. Zhou, C.-Y. Li, Z. Xiao, arXiv:2304.07052

  Fitting of $e^+e^- \to D^{(*)}\bar{D}^{(*)}$, $\pi D\bar{D}$ cross sections → ψ(4160) is Y(4230)

ψ decays (bare vertices)

(quasi) two-body channels included; $J^{PC} = 1^{--}$

bare ψ



(A) $D_1(2420)\bar{D}^{(*)}$, $D_1(2430)^0\bar{D}^{(*)}$, $D_2^*(2460)\bar{D}^{(*)}$, $D^{(*)}\bar{D}^{(*)}$

(B) $D_0^*(2300)\bar{D}^*$, $f_0 J/\psi$, $f_2 J/\psi$, $f_0\psi'$, $f_0 h_c$, $Z_c\pi$, $Z_{cs}\bar{K}$

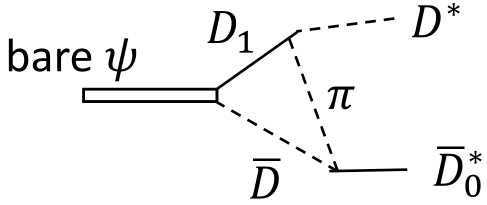
(C) $D_s^{(*)}\bar{D}_s^{(*)}$, $J/\psi\eta$, $J/\psi\eta'$, $\omega\chi_{c0}$

Group (B)

We do not include " bare $\psi \to D_0^*\bar{D}^*$, $Z_c\pi$ "

bare ψ dominantly decays to two-body states; D_0^* and Z_c are probably not compact states

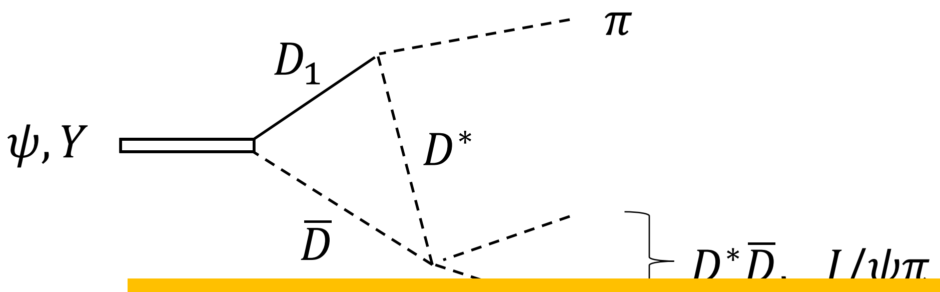
$D_0^*\bar{D}^*$ and $Z_c\pi$ channels are generated by coupled-channel effect like



$D_0^*(2300)$ → $D\pi$ s-wave amplitude fitted to LQCD-based amplitude   Albaladejo et al. PLB 767, 465 (2017)

D_0^* pole : 2104 – i 100 MeV (ours) , $2105^{+6}_{-8} - i\ 102^{+10}_{-12}$ MeV (Albaladejo et al.)

Triangle singularity (TS) from our model



Kinematical condition for TS

Energy-momentum is conserved everywhere as classical process

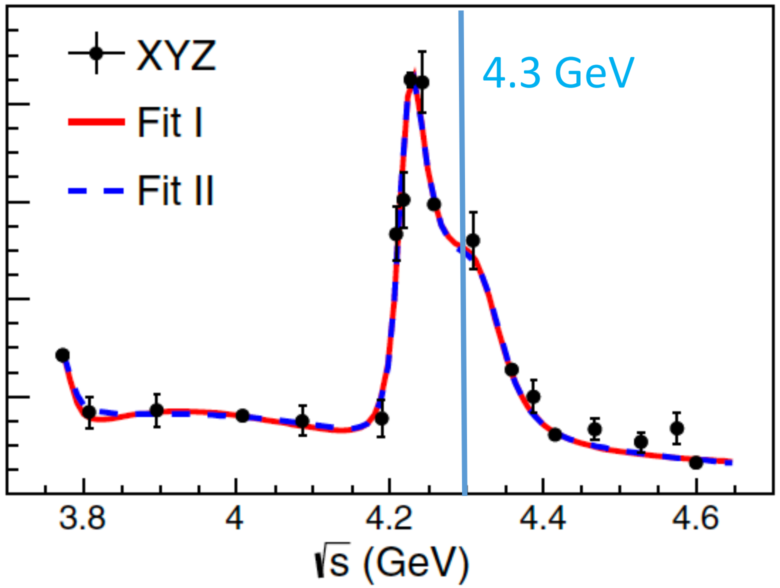
→ amplitude is significantly enhanced at

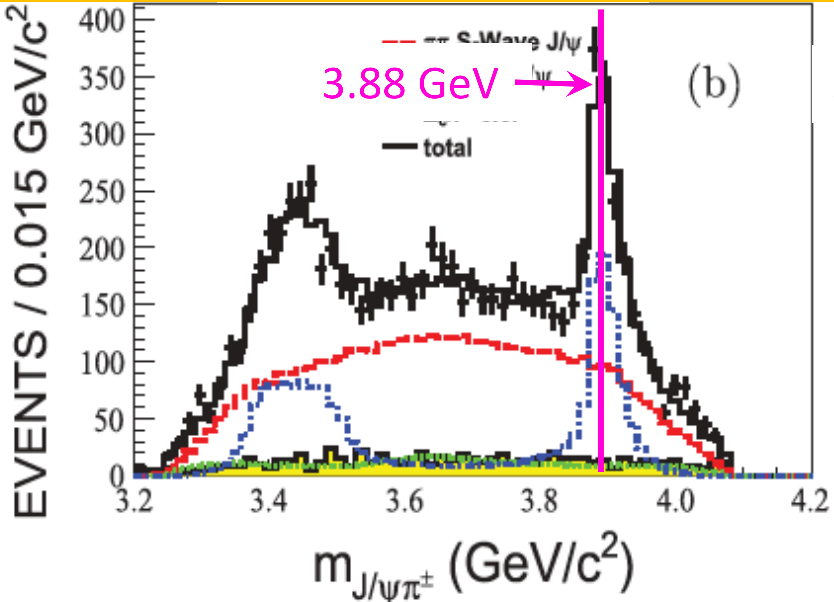
.88 GeV)

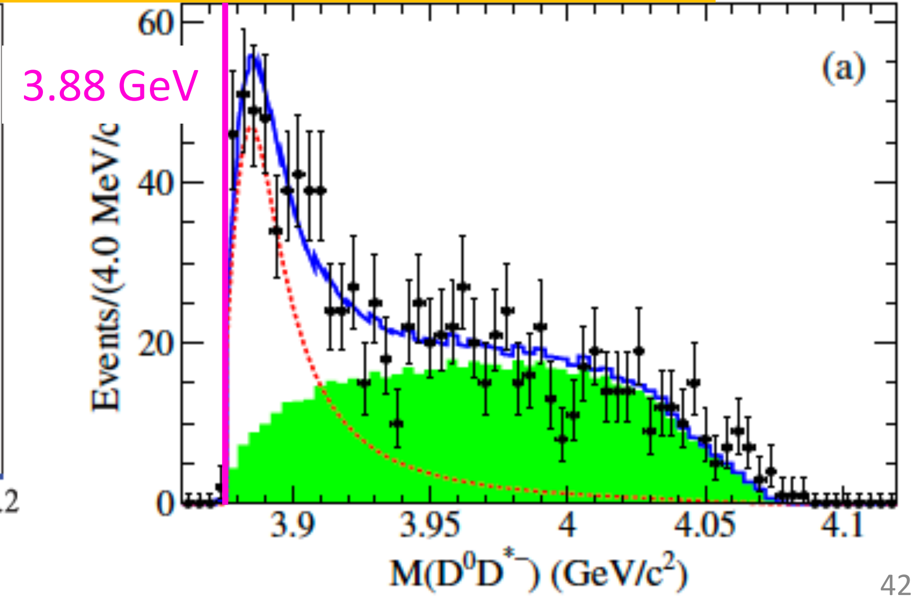
e

Data might indicate that Y(4320) and Zc peaks are TS effect

→ to be examined in our analysis

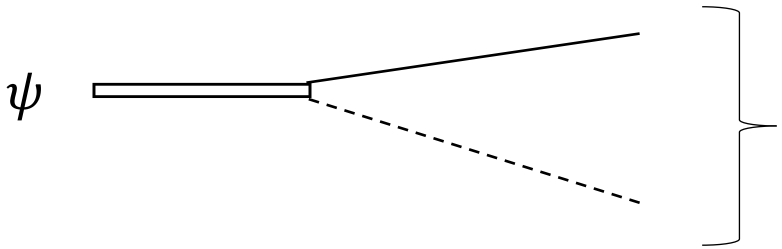






Coupled-channels

(quasi) two-body channels included; $J^{PC} = 1^{--}$



(A) $D_1(2420)\bar{D}^{(*)},\ D_1(2430)^0\bar{D}^{(*)},\ D_2^*(2460)\bar{D}^{(*)},\ D^{(*)}\bar{D}^{(*)}$

(B) $D_0^*(2300)\bar{D}^*,\ f_0 J/\psi,\ f_2 J/\psi,\ f_0\psi',\ f_0 h_c,\ Z_c\pi,\ Z_{cs}\bar{K}$

(C) $D_s^{(*)}\bar{D}_s^{(*)},\ J/\psi\eta,\ J/\psi\eta',\ \omega\chi_{c0}$

Group (A)

$D_1(2420), D_1(2430)$, $D_2^*(2460), D^*$ → Breit-Wigner (BW) propagators; mass and width from PDG

$D_J^{(*)} \to D^{(*)}\pi$ coupling strength is determined, assuming the following decays saturate the width

$D_1(2420) \to D^*\pi$ (mainly d-wave decay); small s–wave coupling fixed by helicity angle distribution data

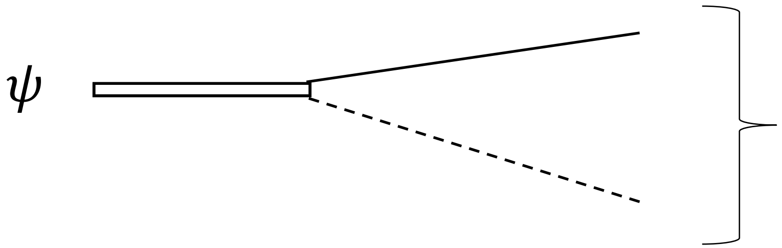
Babar, PRD 82, 111101 (2010)

$D_1(2430) \to D^*\pi$ (s−wave decay)

$D_2^*(2460) \to D^*\pi + D\pi;\ \Gamma(D\pi)/\Gamma(D^*\pi)\sim 1.5$

$D^{*+} \to D\pi$

Coupled-channels



(quasi) two-body channels included; $J^{PC} = 1^{--}$

(A) $D_1(2420)\bar{D}^{(*)},\ D_1(2430)^0\bar{D}^{(*)},\ D_2^*(2460)\bar{D}^{(*)},\ D^{(*)}\bar{D}^{(*)}$

(B) $D_0^*(2300)\bar{D}^*,\ f_0 J/\psi,\ f_2 J/\psi,\ f_0\psi',\ f_0 h_c,\ Z_c\pi,\ Z_{cs}\bar{K}$

(C) $D_s^{(*)}\bar{D}_s^{(*)},\ J/\psi\eta,\ J/\psi\eta',\ \omega\chi_{c0}$

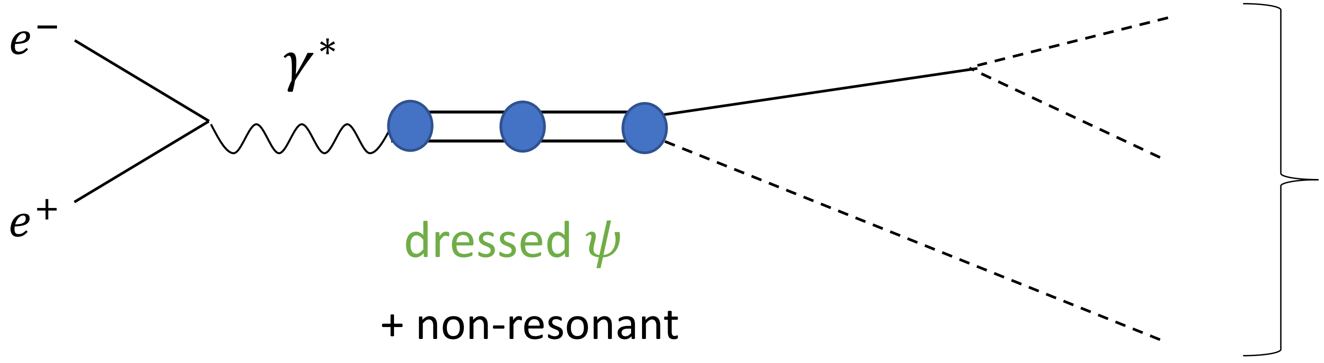
Group (C) treated as stable particles

Because of using BW for Group (A), three-body unitarity is not fully satisfied

Main mechanisms required by three-body unitarity are considered (particle-exchange mechanisms)

Full amplitude for three-body final states

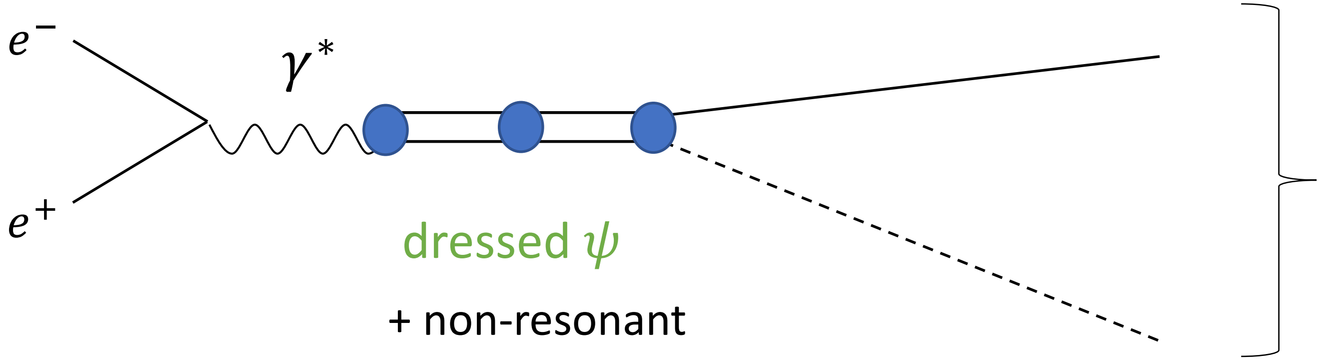
$e^+e^- \to \pi D^{(*)}\bar{D}^{(*)},\ J/\psi\pi\pi,\ \psi'\pi\pi, h_c\pi\pi, \eta_c\rho\pi, J/\psi K\bar{K}$



$\pi D^{(*)}\bar{D}^{(*)},\ J/\psi\pi\pi,\ \psi'\pi\pi, h_c\pi\pi, \eta_c\rho\pi, J/\psi K\bar{K}$

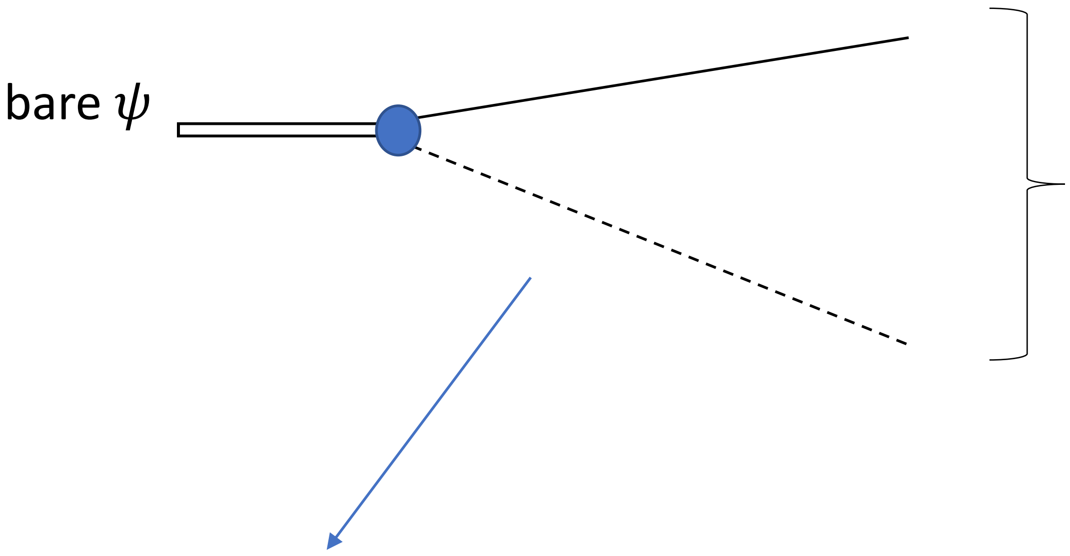
Full amplitude for two-body final states

$e^+e^- \to D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi\,\eta^{(\prime)}, \chi_{c0}\omega$



$D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi\,\eta^{(\prime)}, \chi_{c0}\omega$

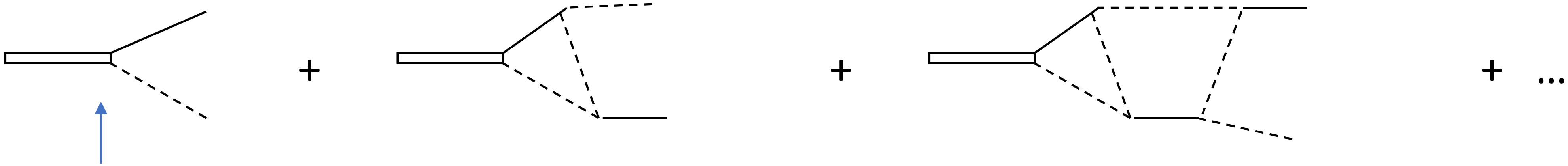
Two-body decay processes of ψ and Y



$D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi\, \eta^{(\prime)}, \chi_{c0}\omega$

bare decay mechanism only

(long-range particle-exchange unavailable)



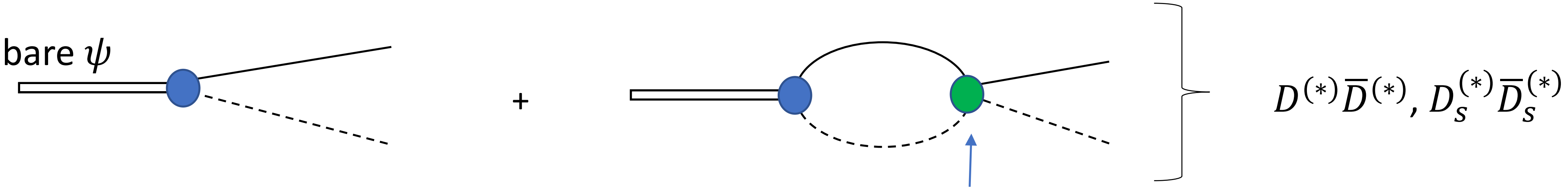
(until infinite loops)

bare decay vertex

Final state interactions described by solution of Faddeev equation

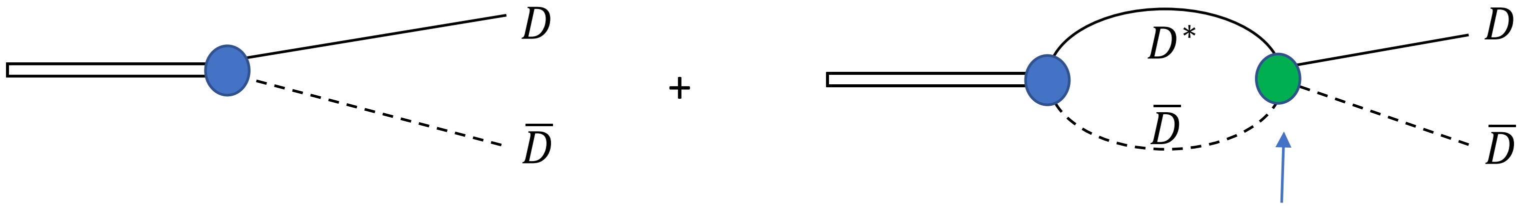
Two-body decays of ψ

For $D^{(*)}\bar{D}^{(*)}$, $D_s^{(*)}\bar{D}_s^{(*)}$, moderately attractive interactions added → threshold enhancements → better fits



elastic scattering amplitudes from contact interactions

Exception: $D\bar{D}$ final state ($D^*\bar{D}$ threshold enhancement needed to fit data)



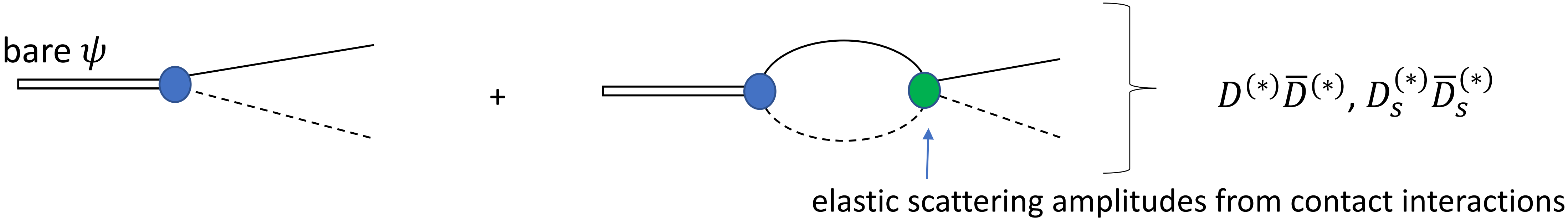
$D^*\bar{D}$ elastic scattering + perturbative transition to $D\bar{D}$

This (semi-perturbative) contact interactions violates the unitarity

→ Improved in ongoing update: Contact interactions are included in coupled-channel scattering equation

Two-body decays of ψ

For $D^{(*)}\overline{D}^{(*)}$, $D_s^{(*)}\overline{D}_s^{(*)}$, moderately attractive interactions added → threshold enhancements → better fits



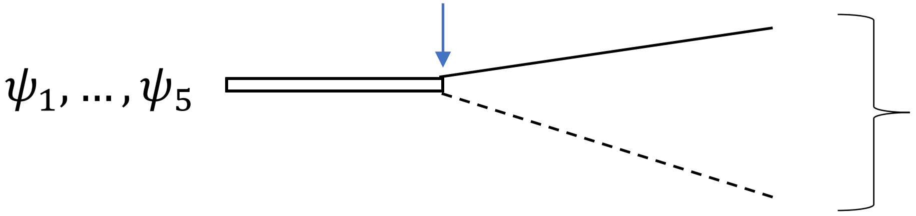
This (semi-perturbative) contact interactions violates the unitarity

→ Improved in ongoing update: Contact interactions are fully included in coupled-channel scattering equation

Fitting parameters in global analysis

* bare ψ masses (5 bare states)

* bare ψ coupling constants (real)

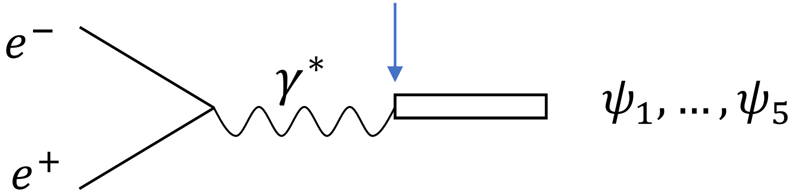


(A) $D_1(2420)\bar{D}^{(*)}$, $D_1(2430)^0\bar{D}^{(*)}$, $D_2^*(2460)\bar{D}^{(*)}$, $D^{(*)}\bar{D}^{(*)}$

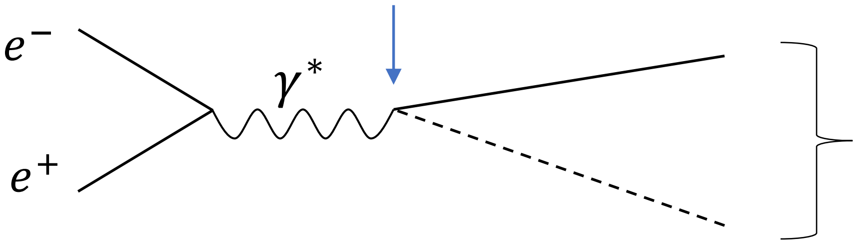
(B) ~~$D_0^*(2300)\bar{D}^*$~~, $f_0 J/\psi$, $f_2 J/\psi$, $f_0\psi'$, $f_0 h_c$, ~~$Z_c\pi$~~, $Z_{cs}\bar{K}$

(C) $D_s^{(*)}\bar{D}_s^{(*)}$, $J/\psi\eta$, $J/\psi\eta'$, $\omega\chi_{c0}$

* bare photon-ψ coupling constants (real)



* non-resonant photon coupling constants (real)



(A) $D_1(2420)\bar{D}^{(*)}$, $D_1(2430)^0\bar{D}^{(*)}$, $D_2^*(2460)\bar{D}^{(*)}$, $D^{(*)}\bar{D}^{(*)}$

(B) ~~$D_0^*(2300)\bar{D}^*$~~, $f_0 J/\psi$, $f_2 J/\psi$, $f_0\psi'$, $f_0 h_c$, ~~$Z_c\pi$~~, $Z_{cs}\bar{K}$

(C) $D_s^{(*)}\bar{D}_s^{(*)}$, $J/\psi\eta$, $J/\psi\eta'$, $\omega\chi_{c0}$

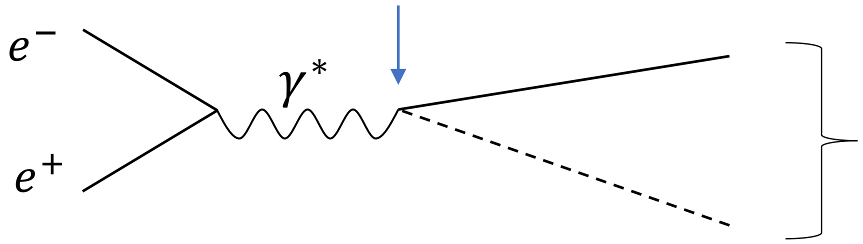
Fitting parameters in global analysis

* $\psi(4660)$ Breit-Wigner mass, width, $\psi(4660) \rightarrow f_0\psi'$ complex vertices

* In $J^{PC} = 1^{+-}$ $D^*\bar{D} - D^*\bar{D}^* - J/\psi\pi - \psi'\pi - h_c\pi - \eta_c\rho$ couple—channel scattering amplitude (Z_c amplitude)
  coupling constants: $v_{D^*\bar{D},D^*\bar{D}}$, $v_{D^*\bar{D},J/\psi\pi}$, $v_{D^*\bar{D},\psi'\pi}$ etc.

* Additional $D^{(*)}\bar{D}^{(*)}$, $D_s^{(*)}\bar{D}_s^{(*)}$ elastic contact interactions → coupling constants

* Cutoffs (dipole form factors) to adjust energy dependence of nonresonant amplitudes

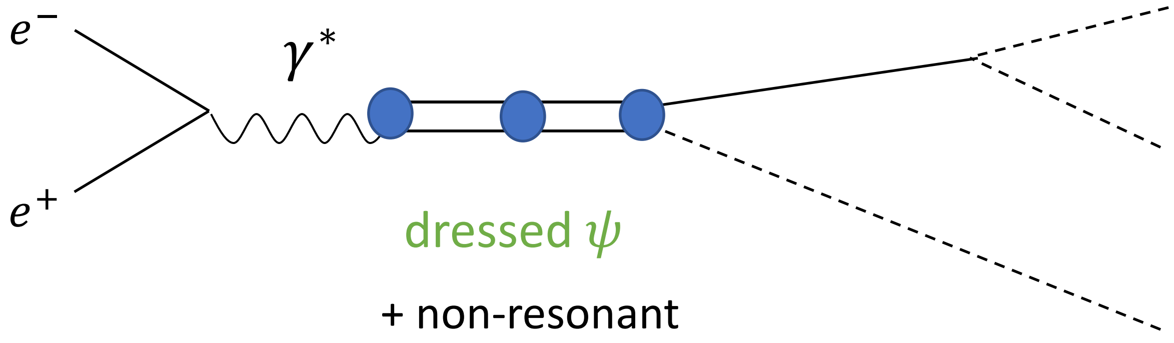


$D^{(*)}\bar{D}^{(*)}$, $D_s^{(*)}\bar{D}_s^{(*)}$

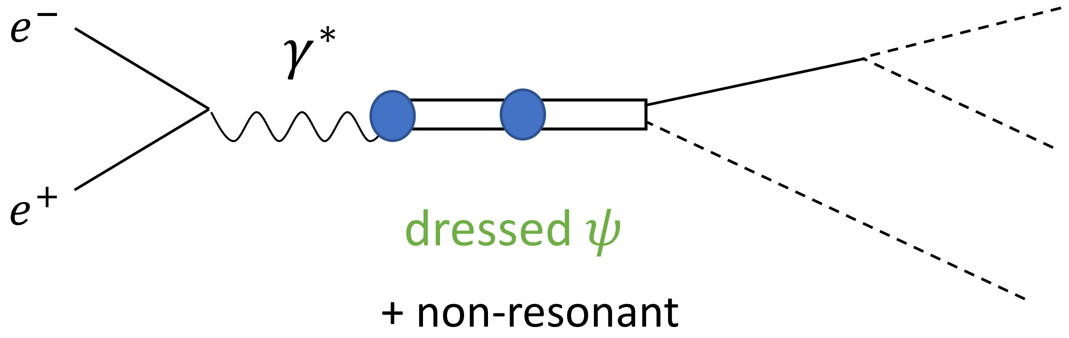
Other cutoffs → 1 GeV

In total, 177 fitting parameters

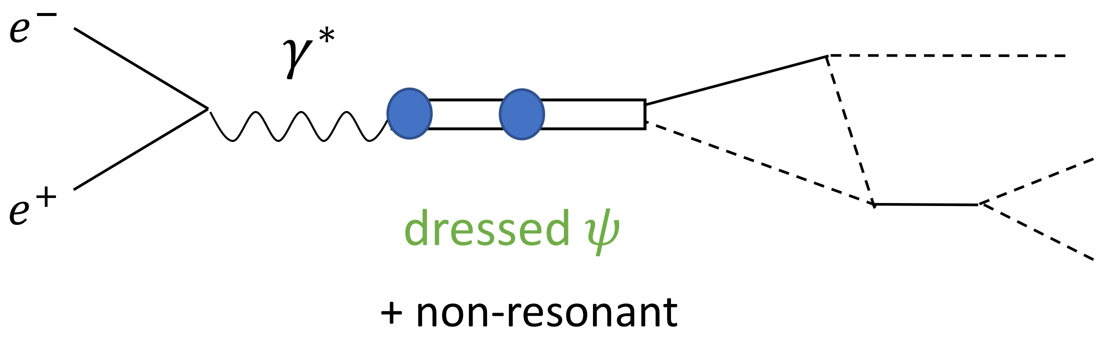
Full amplitude



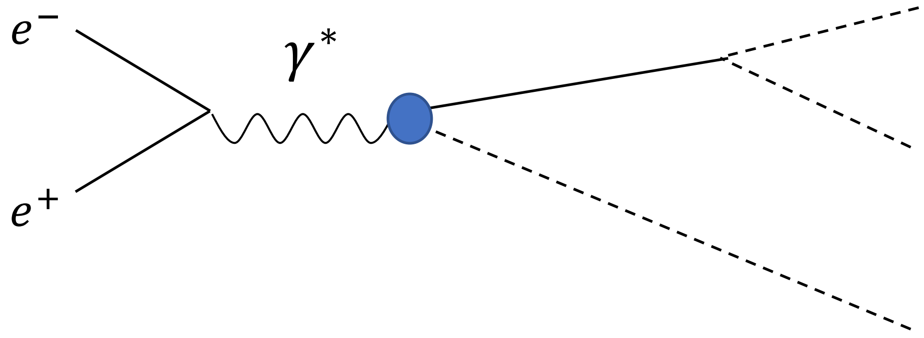
tree

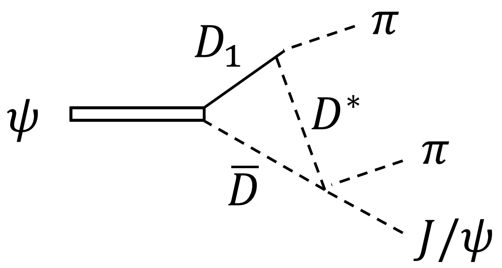


1-loop



NR (non-resonant)

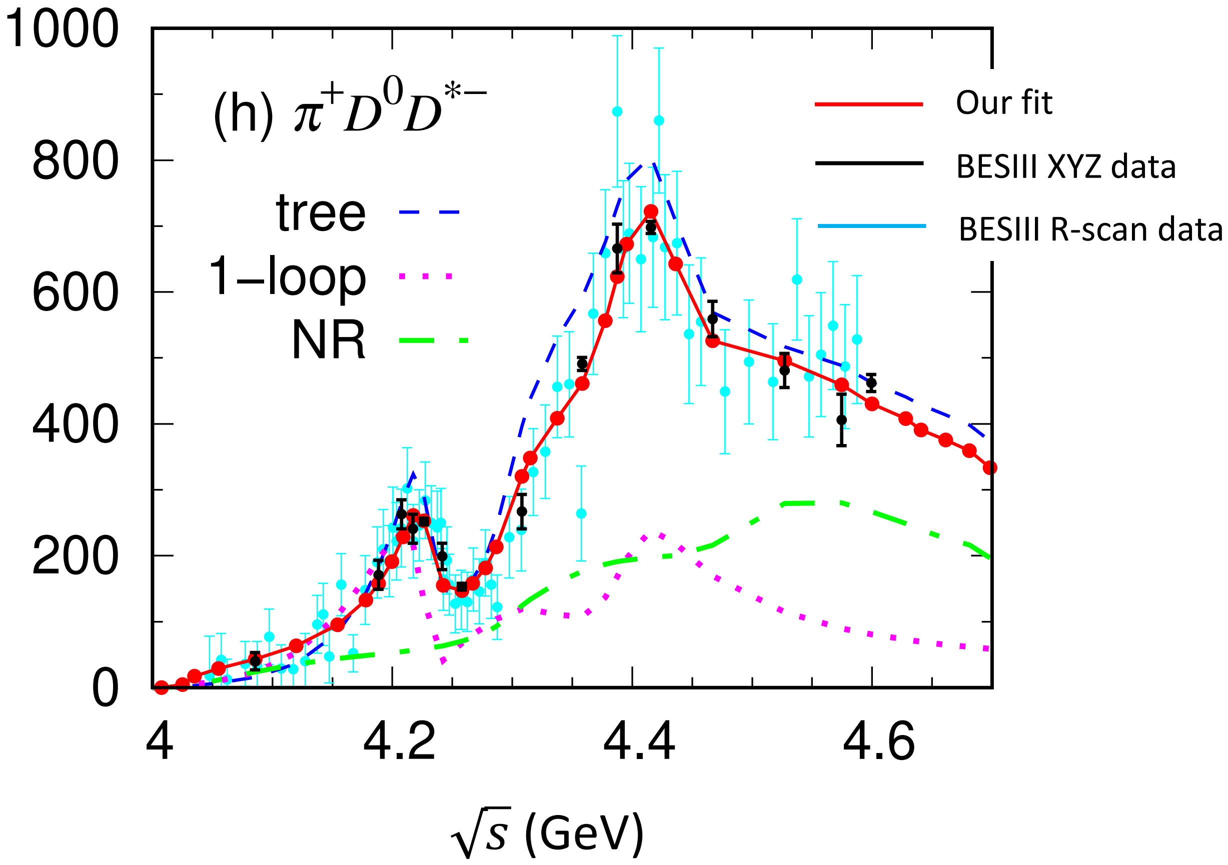




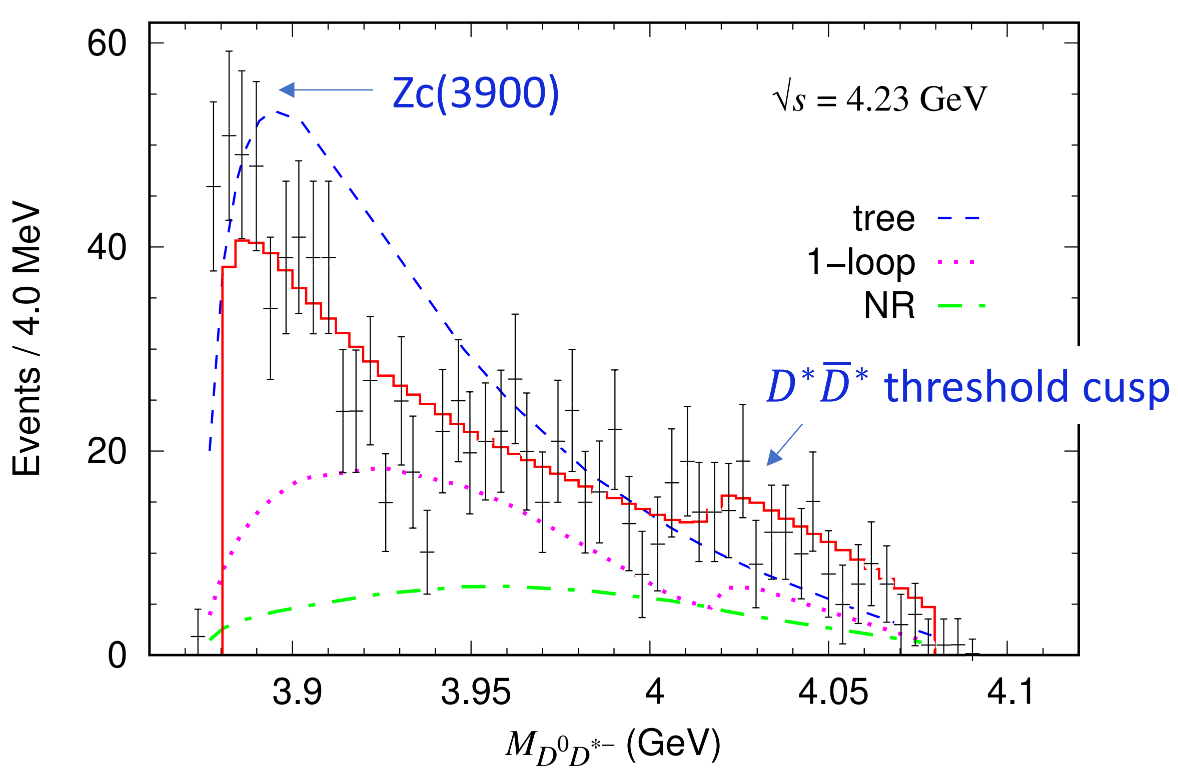
$D^*\overline{D}$ threshold cusp and/or

TS occurs from 1-loop

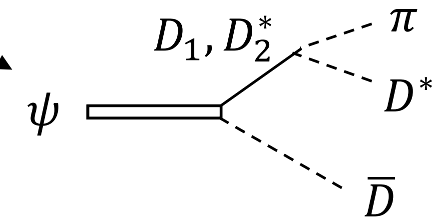
$e^+e^- \to \pi^+ D^0 D^{*-}$

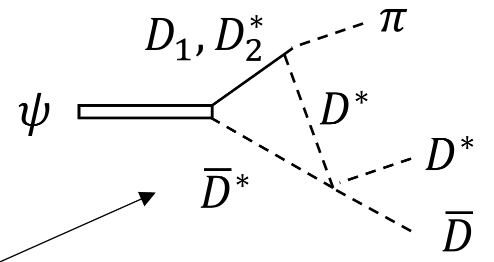


$D^0 D^{*-}$ invariant mass distributions



Tree contribution is dominant





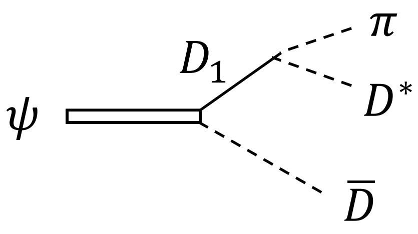
- $D^*\bar{D}^*$ threshold cusp is caused by
- $D^*\bar{D}$ threshold enhancement is mostly from tree; $\psi \to D_1\bar{D}$

$e^+e^- \to \pi^+ D^0 D^{*-}$

Conflict with BESIII analysis result

Conclusion from BESIII PRD 92, 092006 (2015)

we conclude that the $D\bar{D}_1(2420)$ contribution to our observed Born cross section is smaller than its relative systematic uncertainty.

→ 

is very small !

Difficult to make our model consistent with this BESIII conclusion. Why ? Insufficient information !!

Hope BESIII to conduct amplitude analysis on this process, and present detailed results and/or Dalitz plots.

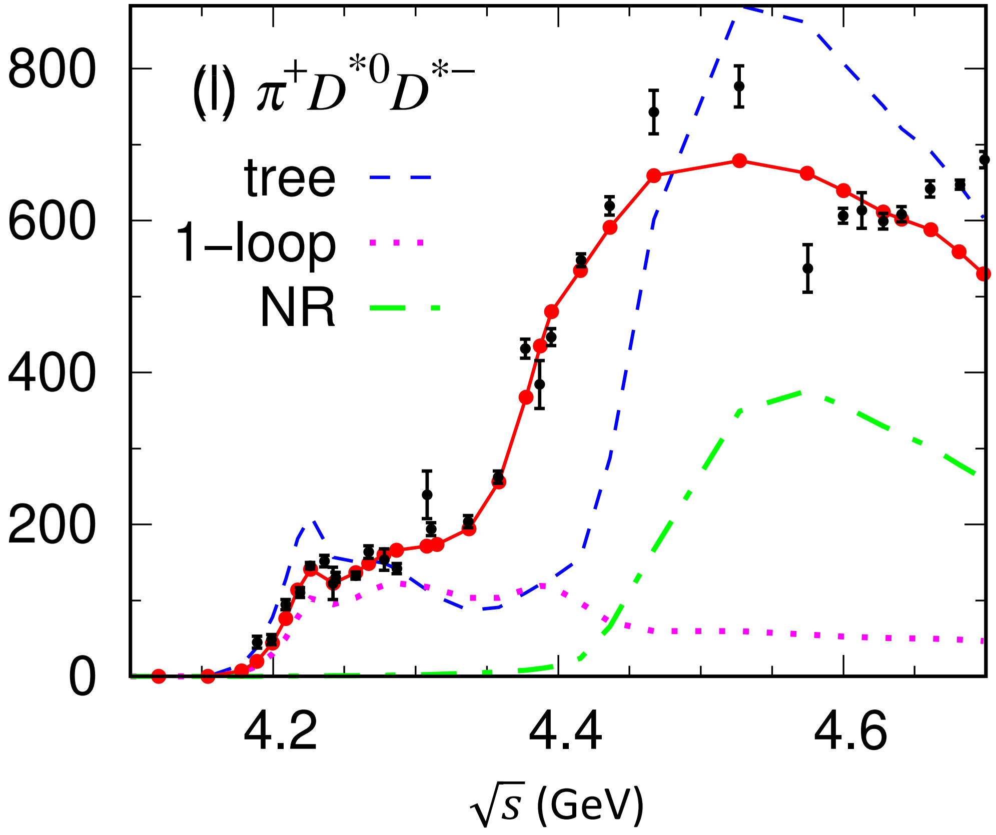
Without this information, $e^+e^- \to \pi^+ D^0 D^{*-}$ data cannot be well fitted, giving bad influence on the global fit overall

Most of previous theoretical models share the same problem

$e^+e^- \to \pi^+(D^*\bar{D}^*)^-$

Our fit

BESIII data



Good fit in < 4.45 GeV

Not good in higher energy region

BESIII reported a new charmonium at

M = 4675.3 ± 29.5 ± 3.5 MeV, Γ= 218.3 ± 72.9 ± 9.3MeV

This state seems important to describe higher energy region

Inclusion of BW amplitude does not improve the fit

→ This state needs to be included in coupled-channel amplitude

The data (other final states) are not enough

for coupled-channel fit in > 4.6 GeV

→ We wait for more data including $c\bar{c}s\bar{s}$ channels

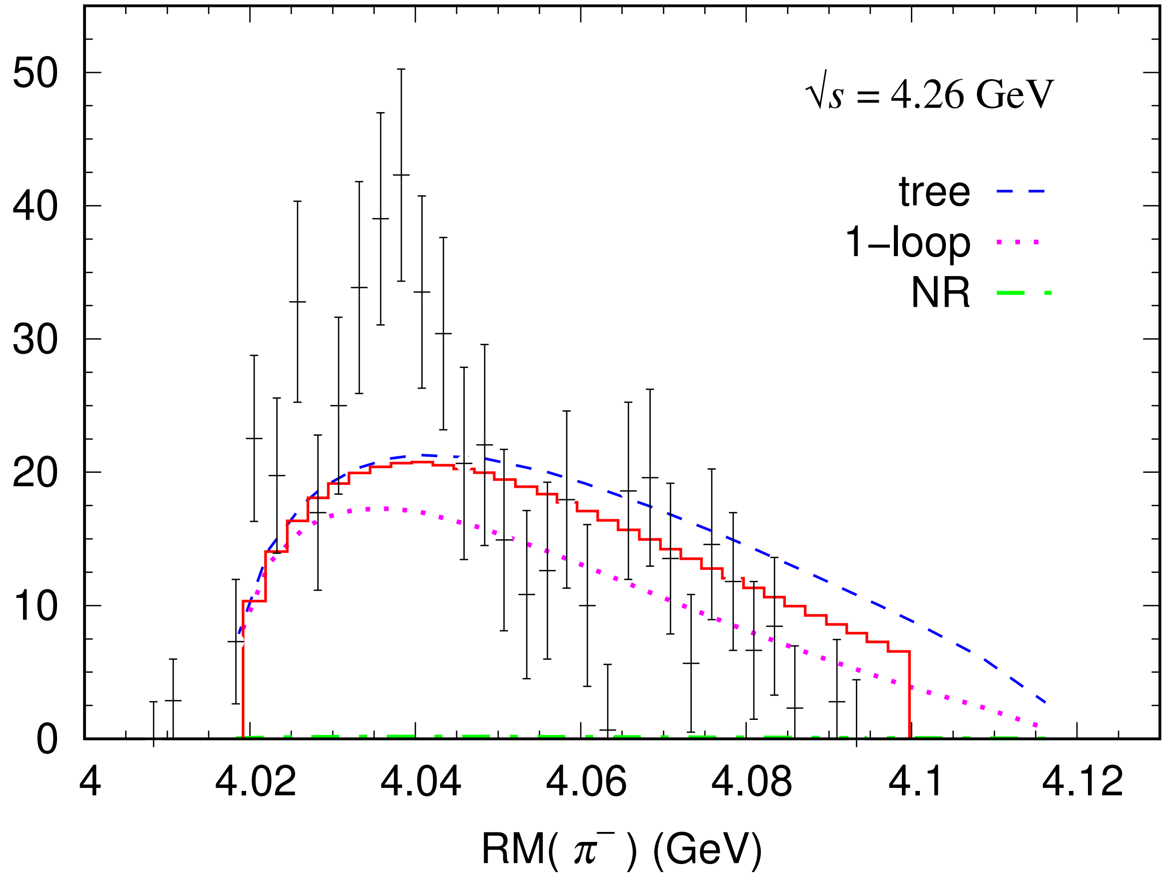
such as $D_{sJ}^{(*)}\bar{D}_s^{(*)}$, $KD^{(*)}\bar{D}_s^{(*)}$

$e^+e^- \to \pi^-(D^*\bar{D}^*)^+$

$D^*\bar{D}^*$ invariant mass distributions (pion recoil mass)

Our fit

BESIII data



Fit does not seem good, however

Kinematical end of the data ~ 4.09 GeV

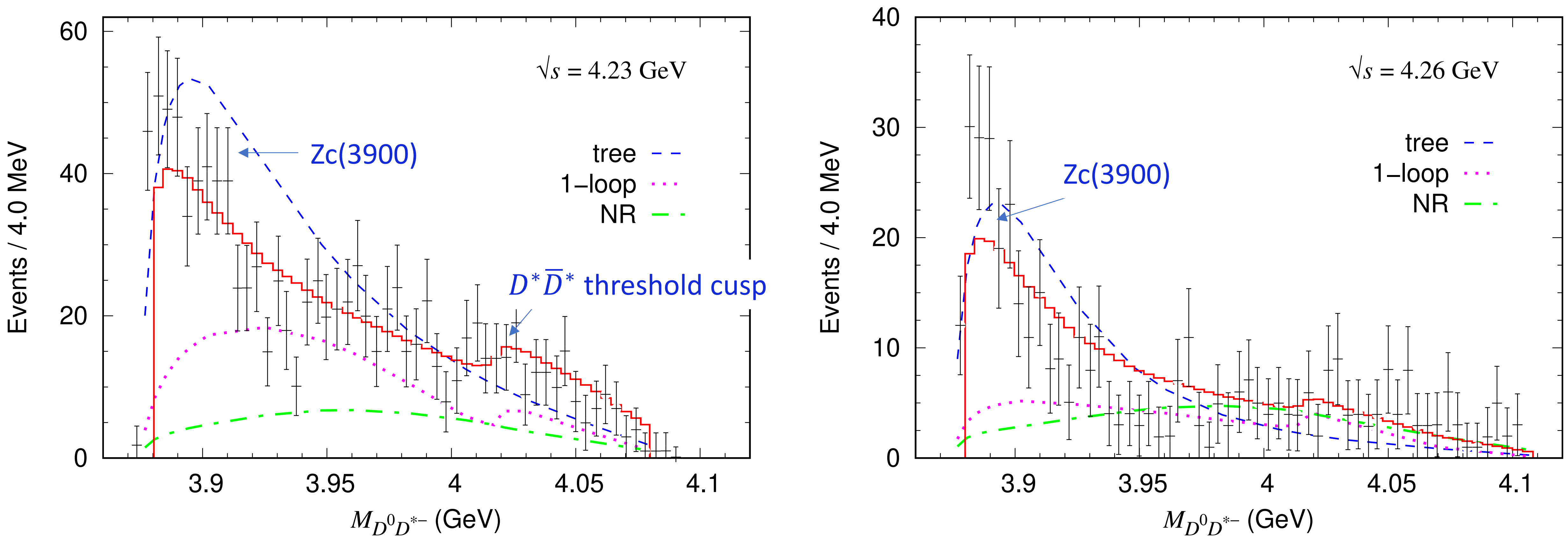
Actual kinematical end ~ 4.12 GeV

→ Efficiency correction seems significant for this lineshape data

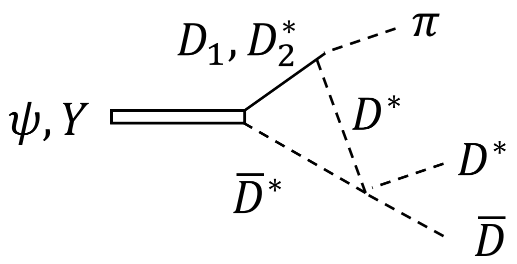
Wait for efficiency corrected data (or MC output) for future improvement of coupled-channel model

$e^+e^- \to \pi^+ D^0 D^{*-}$

$D^0 D^{*-}$ invariant mass distributions

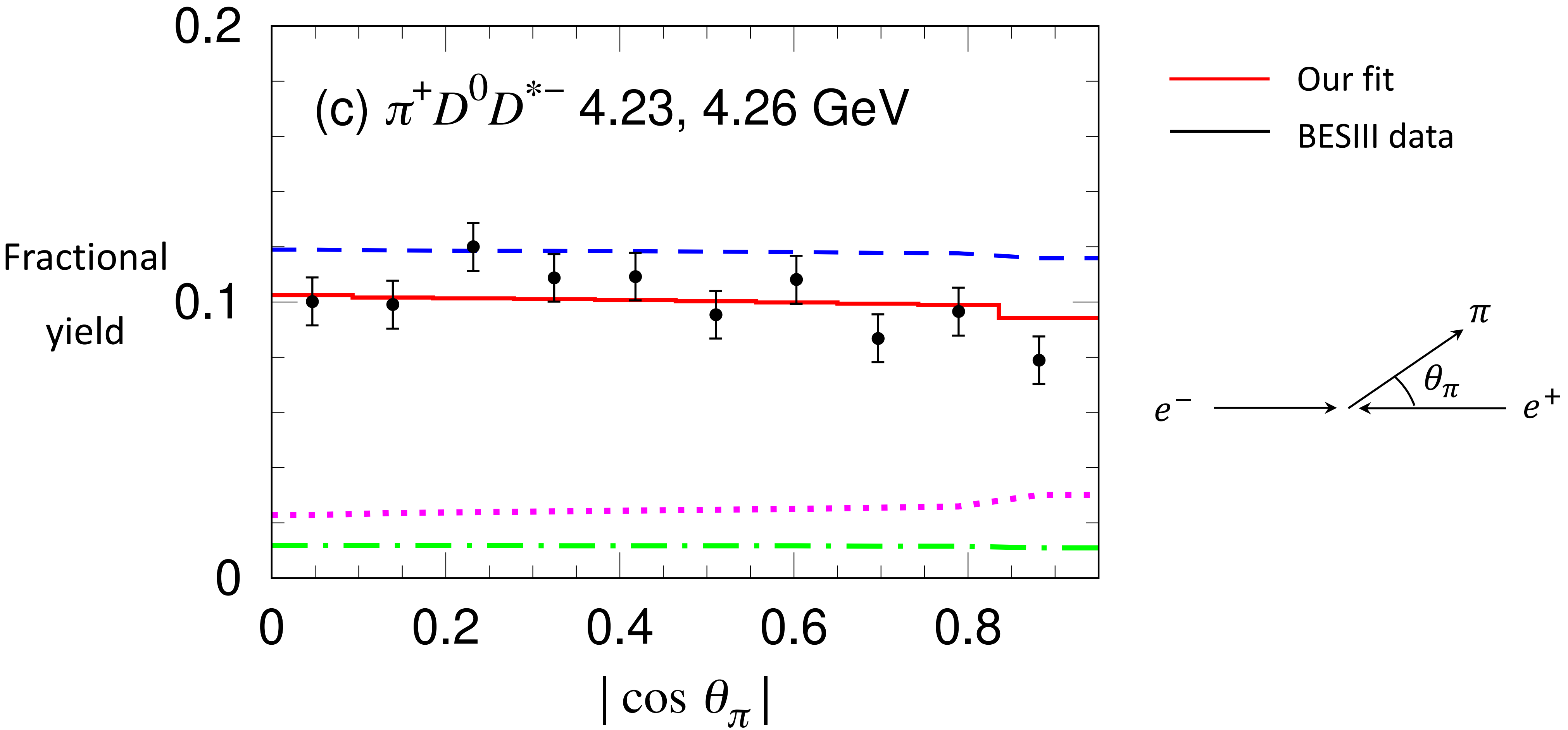


- Threshold enhancement (or Zc(3900) contribution) is fitted by the model
- $D^*\bar{D}^*$ threshold cusps are caused by →
- $D^*\bar{D}$ threshold enhancement is mostly from tree; $\psi \to D_1 \bar{D}$



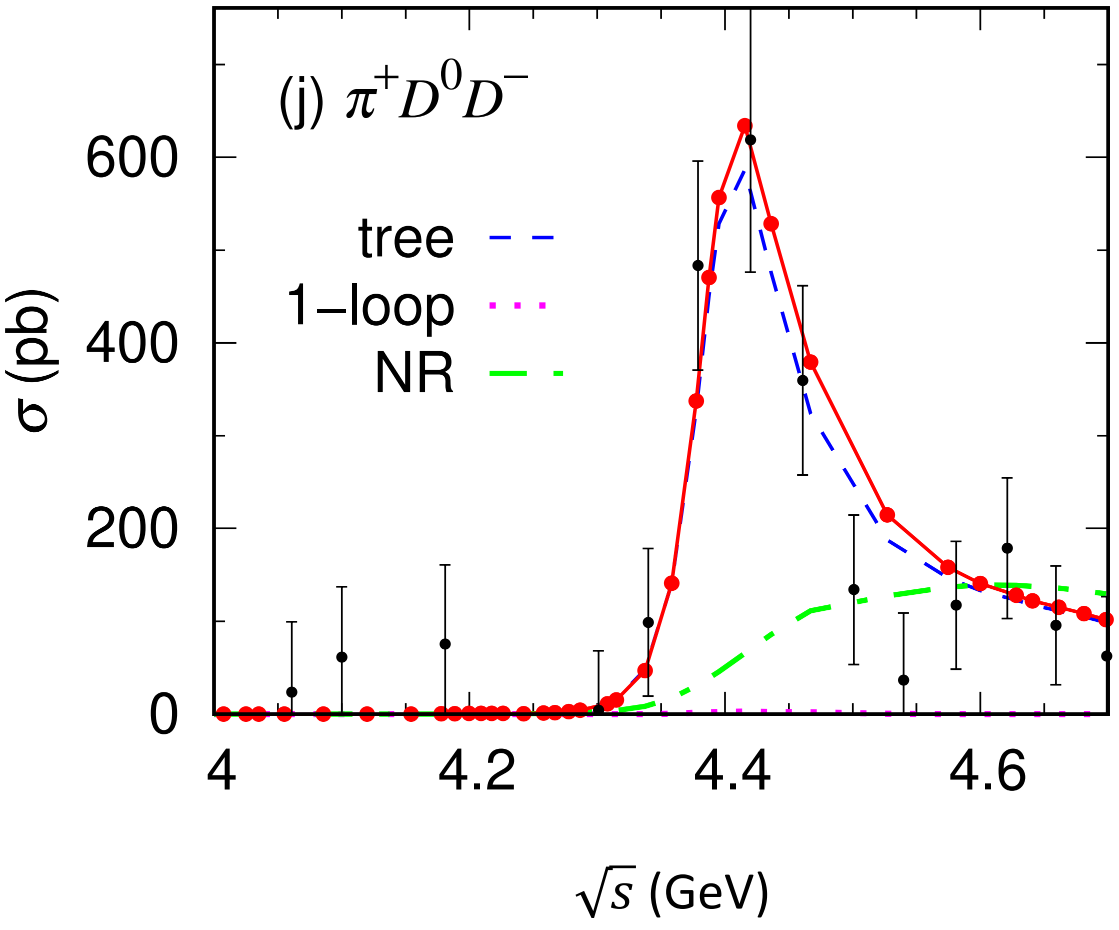
$e^+e^- \to \pi^+ D^0 D^{*-}$

Pion angle distributions from e^+e^- beam direction in total CM frame

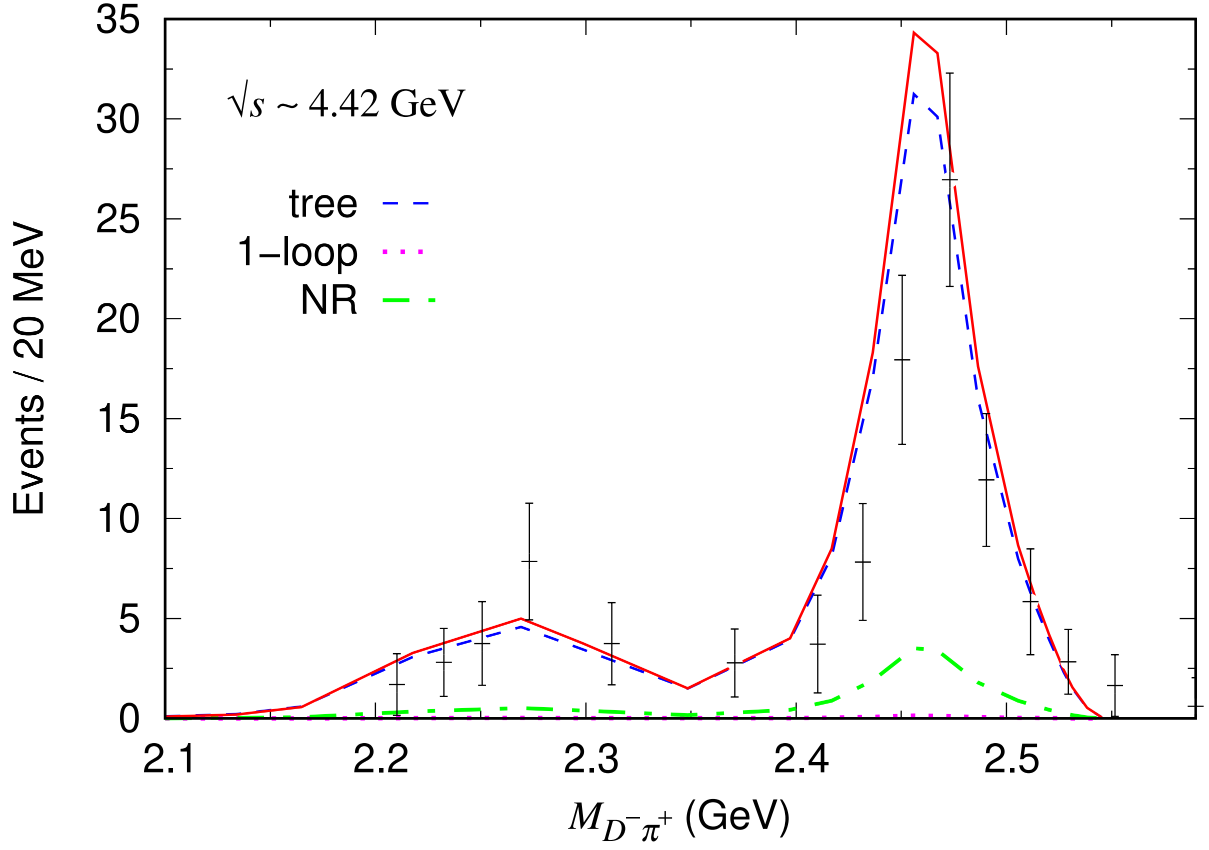


Data are average of 4.23 GeV ($N = 418$) and 4.26 GeV ($N = 239$) data

$e^+e^- \to \pi^+ D^0 D^-$



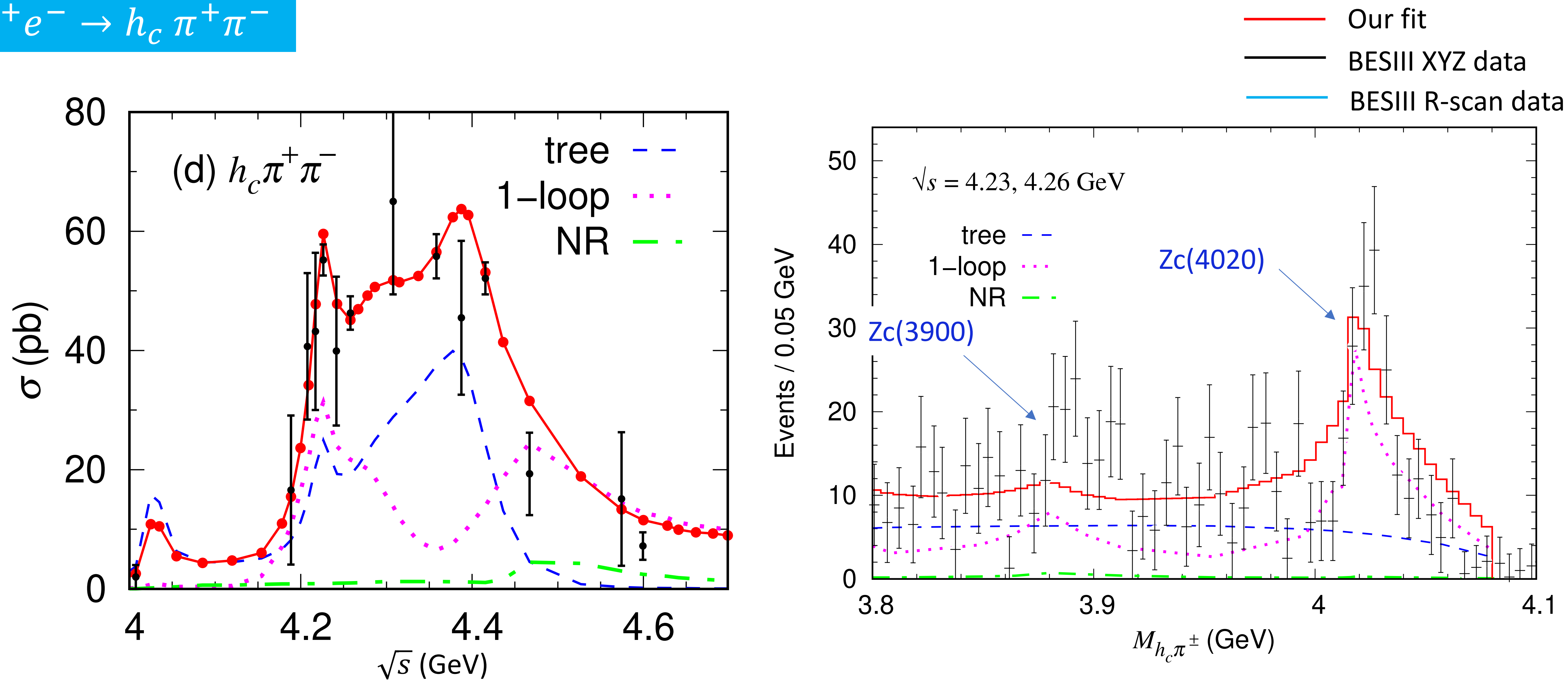
Clear $\psi(4420)$ peak is well fitted



Dominat $D_2^*(2460)$ contribution

Hope to have a better quality data from BESIII ! → important for coupled-channel analysis

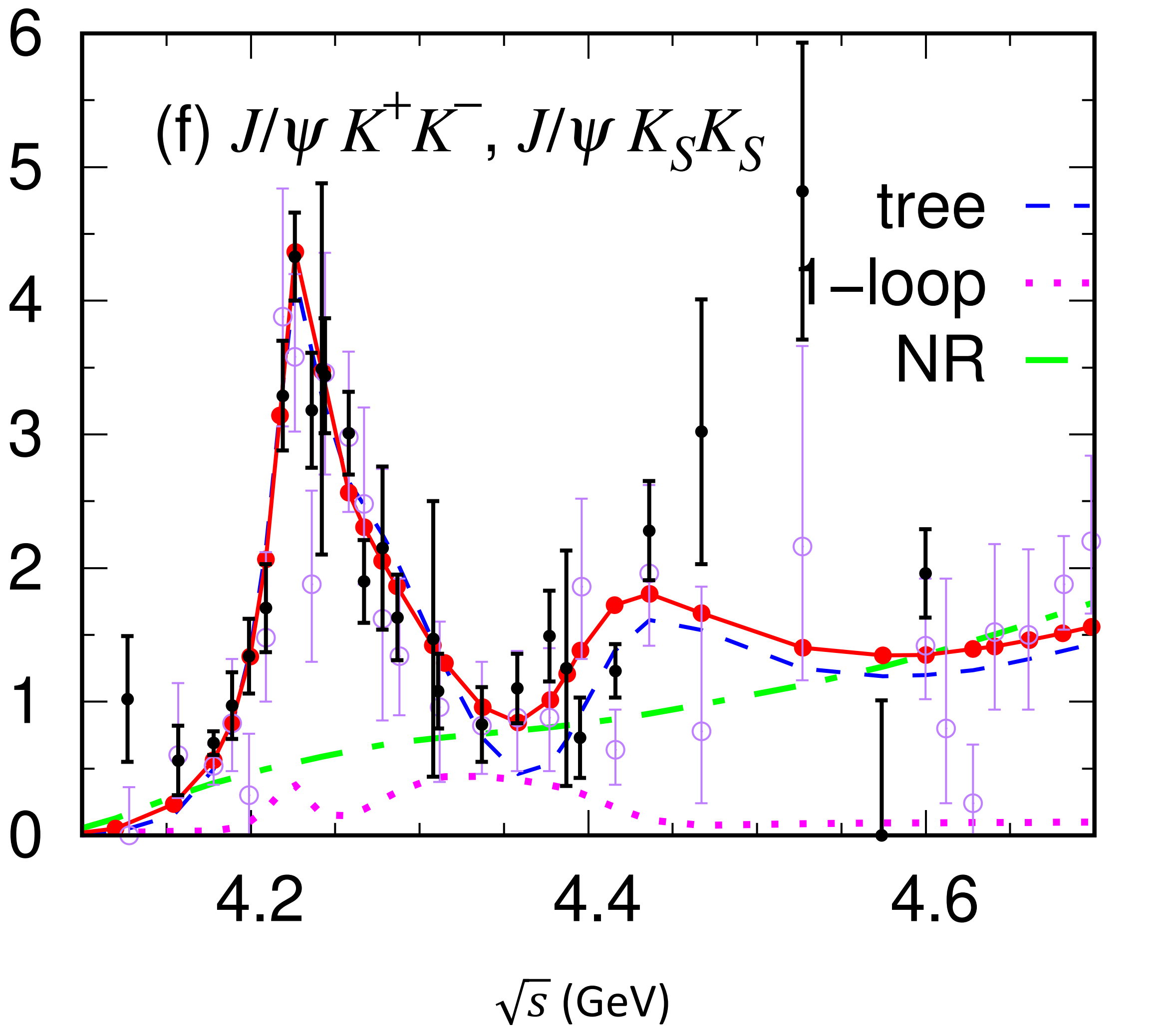
$e^+e^- \to h_c\pi^+\pi^-$



- Enhancement at ~ 4.03 GeV is from ψ(4040) ← consequence of coupled-channel fit
- 1-loop contribution is enhanced due to opening the thresholds, triangle singularity

~ 4.28 GeV → $D_1(2420)\bar{D}$ threshold ~ 4.45 GeV → $D_1(2420)\bar{D}^*$, $D_2^*(2460)\bar{D}^*$ thresholds

$e^+e^- \to J/\psi\, K^+K^-, J/\psi K_S K_S$



- Overall good agreement with data

  (our model is isospin symmetric

  → $\sigma(J/\psi K^+K^-) = 2\times\sigma(J/\psi K_S K_S)$)

- Model does not fit bump at ~4.5 GeV in $J/\psi\ K^+K^-$ data

  * $J/\psi K_S K_S$ data do not show the same bump

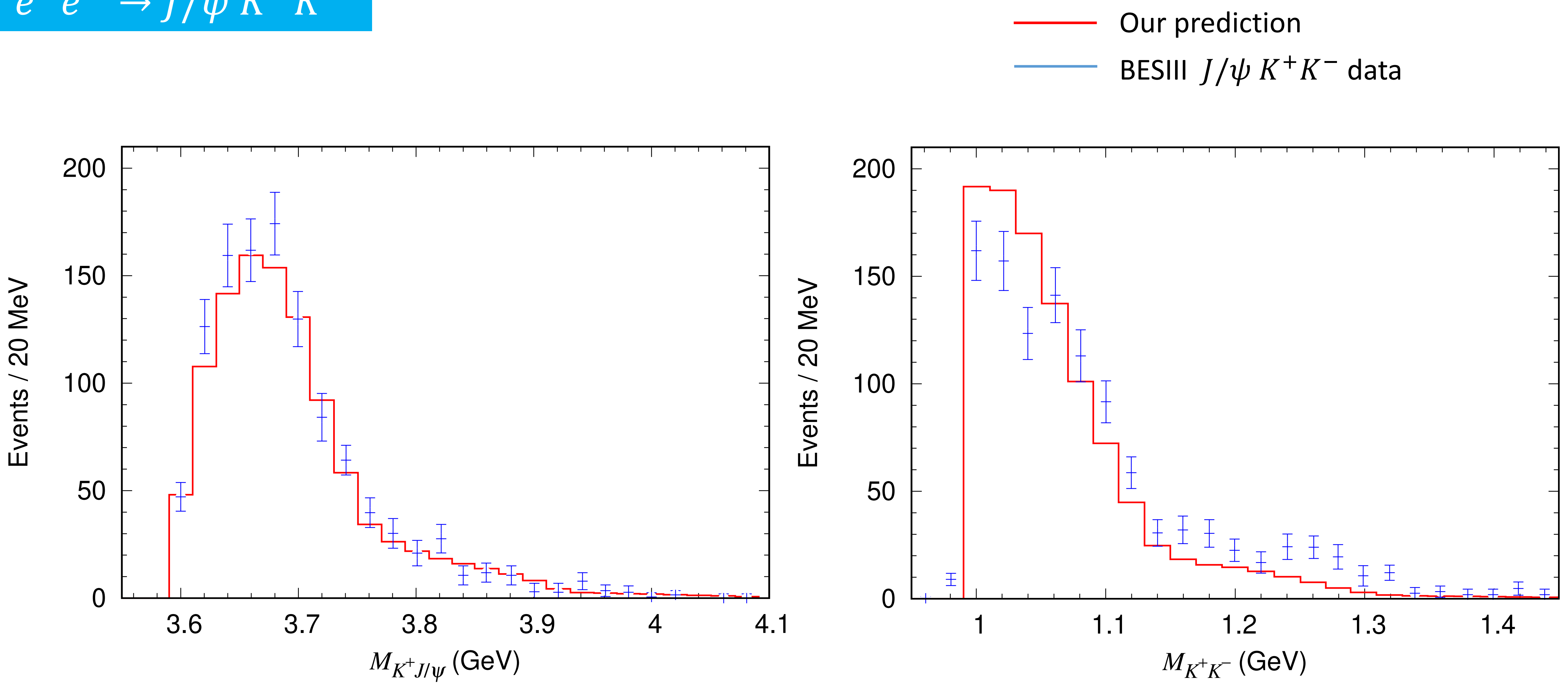
  * data largely fluctuate and error is large

→ our model does not have Y(4500)

more precise data is important to pin-down

the existence of Y(4500)

$e^+e^- \to J/\psi\, K^+K^-$

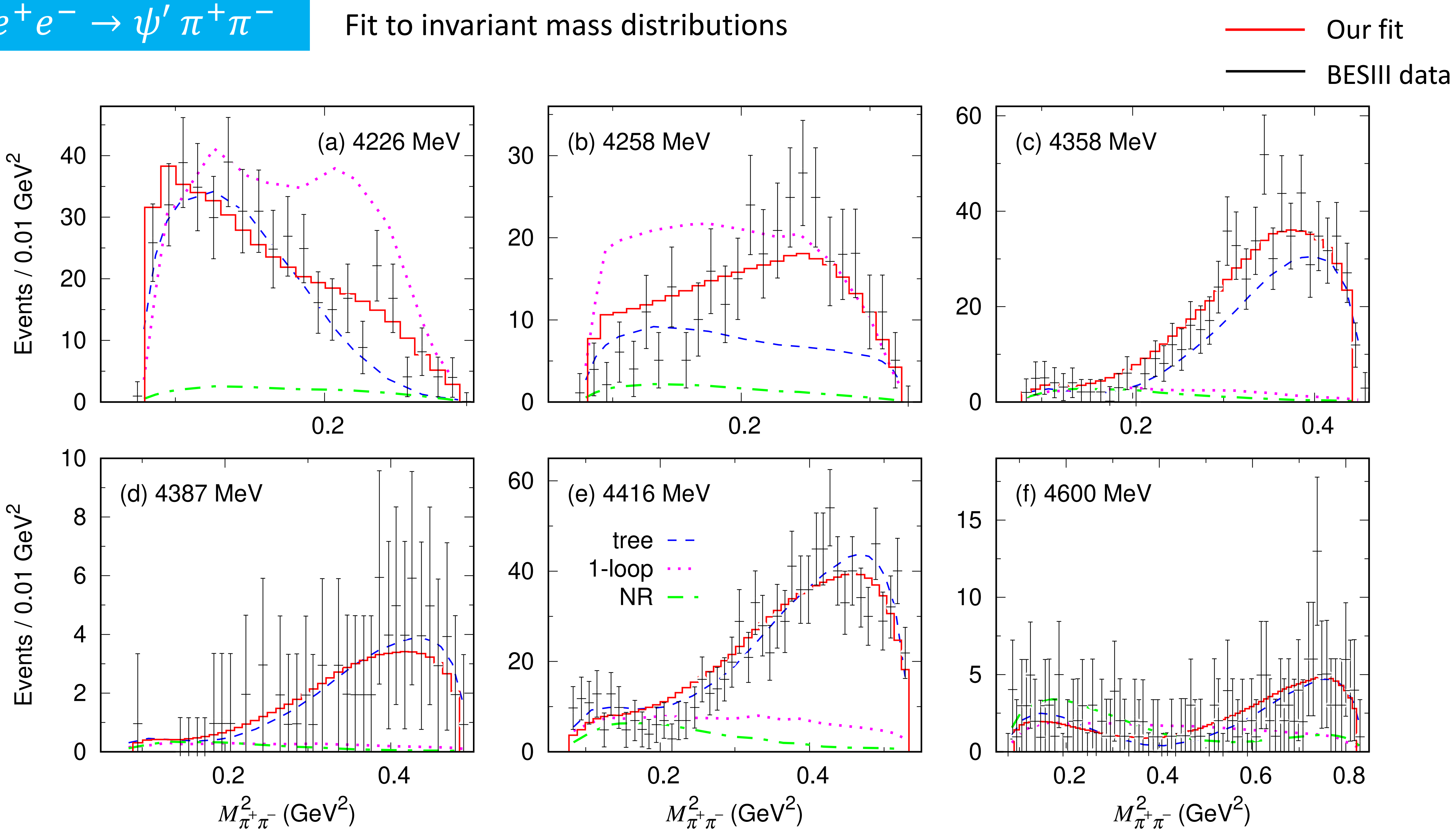


Data are sum of $\sqrt{s}$ = 4.1–4.6 GeV data

Good agreement (this data is not included in our fit)

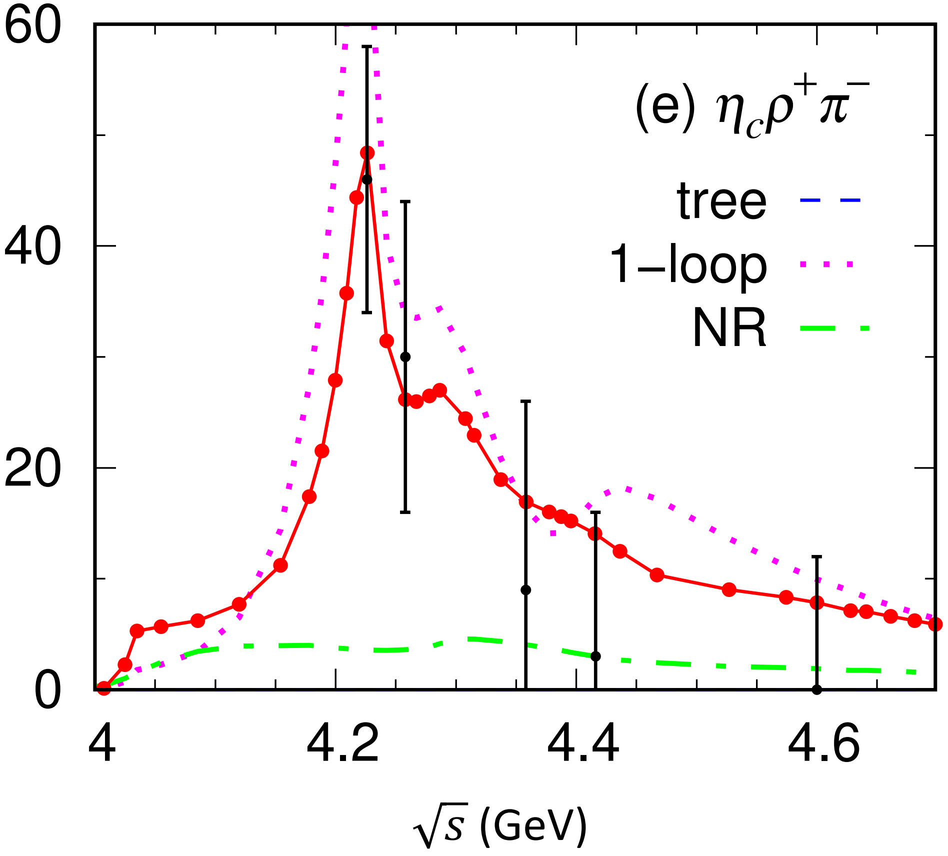
$e^+e^- \to \psi' \pi^+\pi^-$

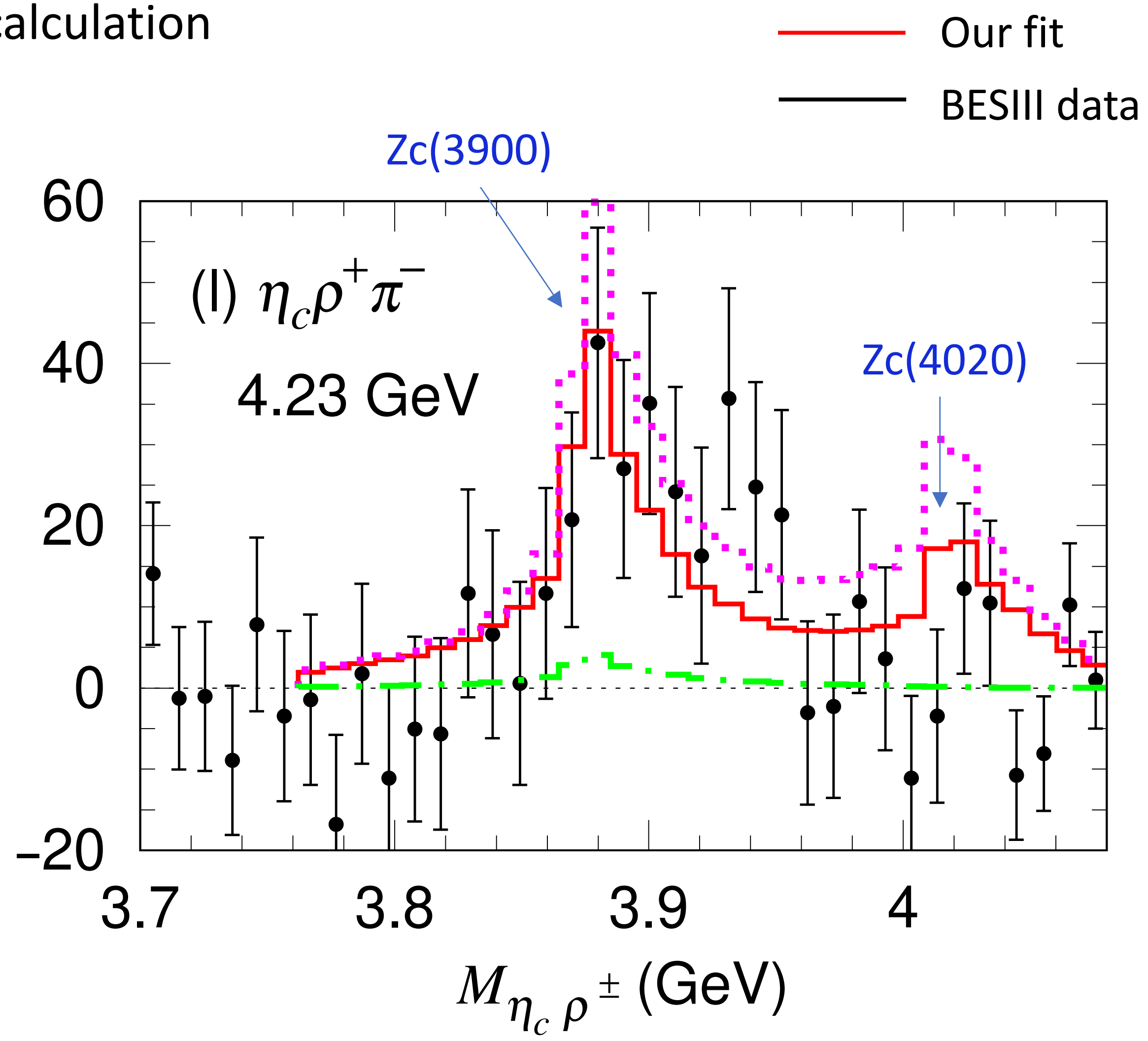
Fit to invariant mass distributions



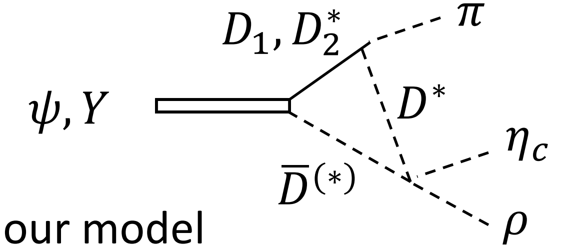
$e^+e^- \to \eta_c\,\rho^+\pi^-$

$\rho \to \pi\pi$ taken into account in calculation





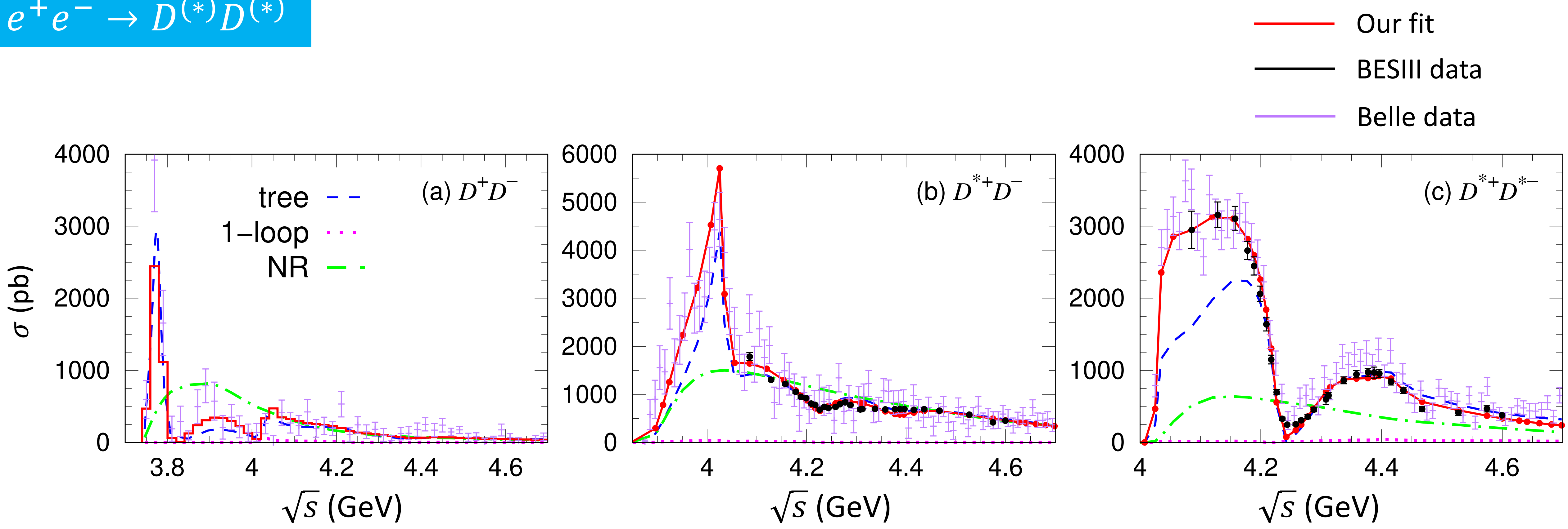
Mostly from 1-loop →



No tree mechanism for $\eta_c\rho\pi$ in our model

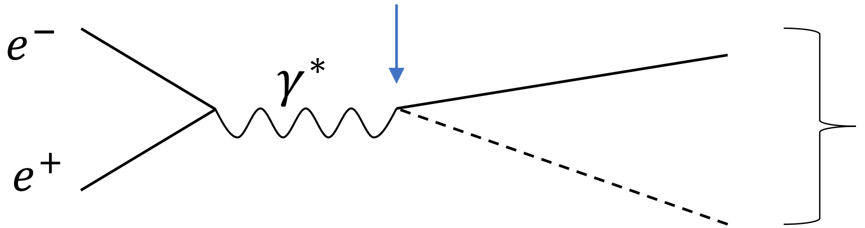
Zc(3900) peak is fitted

$e^+e^- \to D^{(*)}\bar{D}^{(*)}$



Energy dependence of NR contribution is important to fit data at higher energies

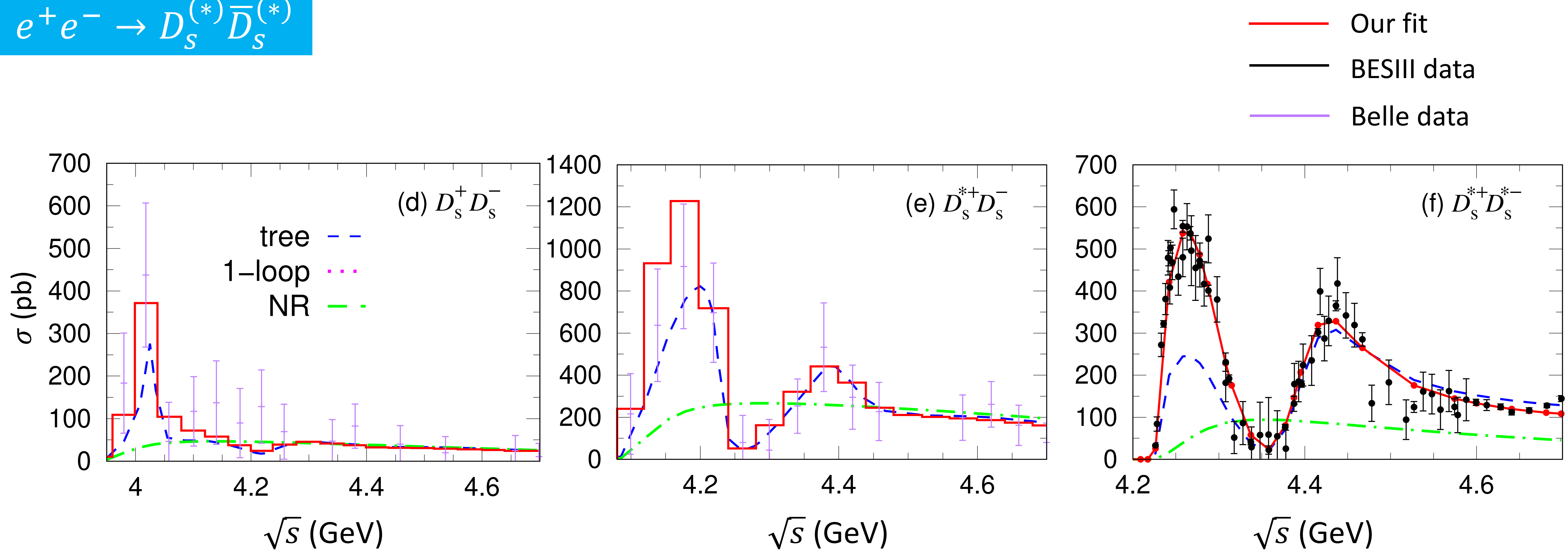
→ * Cutoffs (dipole form factors) to adjust energy dependence of nonresonant amplitudes



$D^{(*)}\bar{D}^{(*)}$, $D_s^{(*)}\bar{D}_s^{(*)}$

Other cutoffs → 1 GeV

$e^+e^- \to D_s^{(*)}\overline{D}_s^{(*)}$

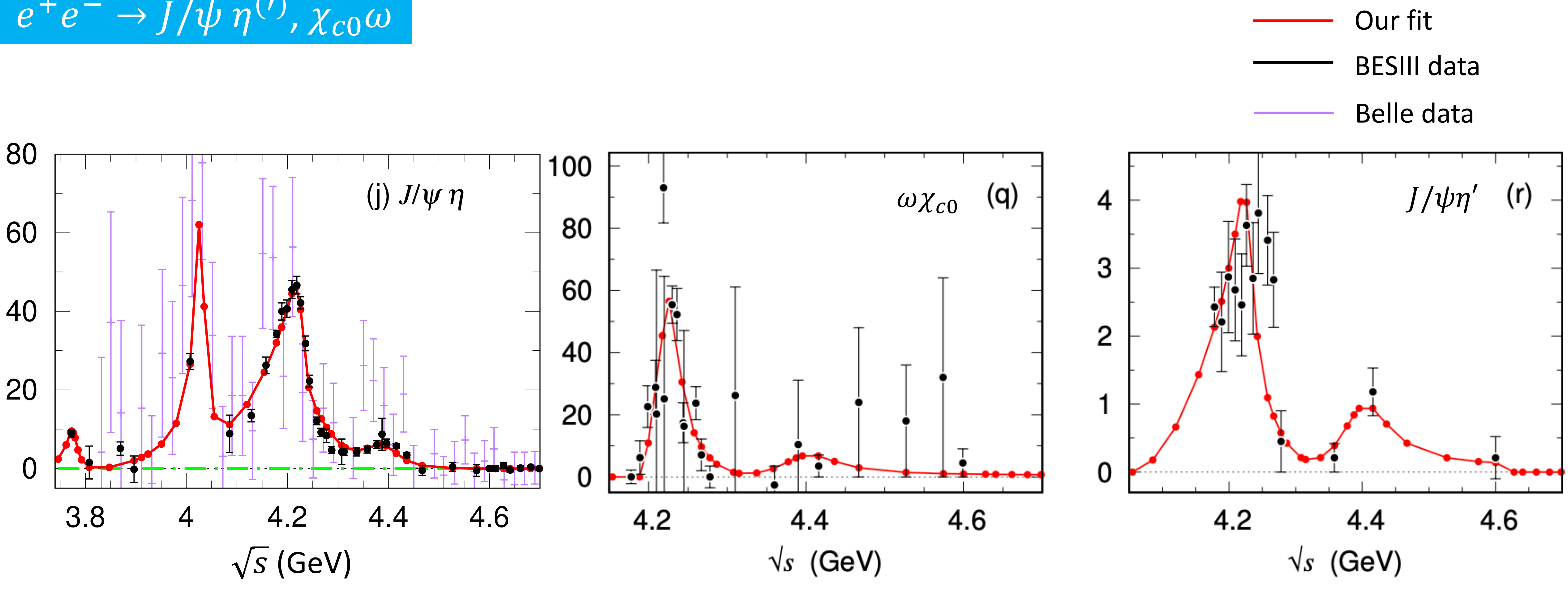


Similar observations as $e^+e^- \to D^{(*)}\overline{D}^{(*)}$

- Threshold enhancements needed for good fits (contact interactions added to bring blue to red curves)
- Energy dependence of NR contribution is important to fit data at higher energies (cutoff adjusted)

Ongoing update: New BESIII data on $e^+e^- \to D_s\overline{D}_s$ included

$e^+e^- \to J/\psi\,\eta^{(\prime)},\, \chi_{c0}\omega$



For $J/\psi\eta$, a sharp peak appears at 4.02 GeV, as a consequence of coupled-channel fit

← BESIII does not have data point, but Belle data seems to favor this result

Resonance parameters

$M = \mathrm{Re}[E_\psi]$
$\Gamma = -2\times\mathrm{Im}[E_\psi]$

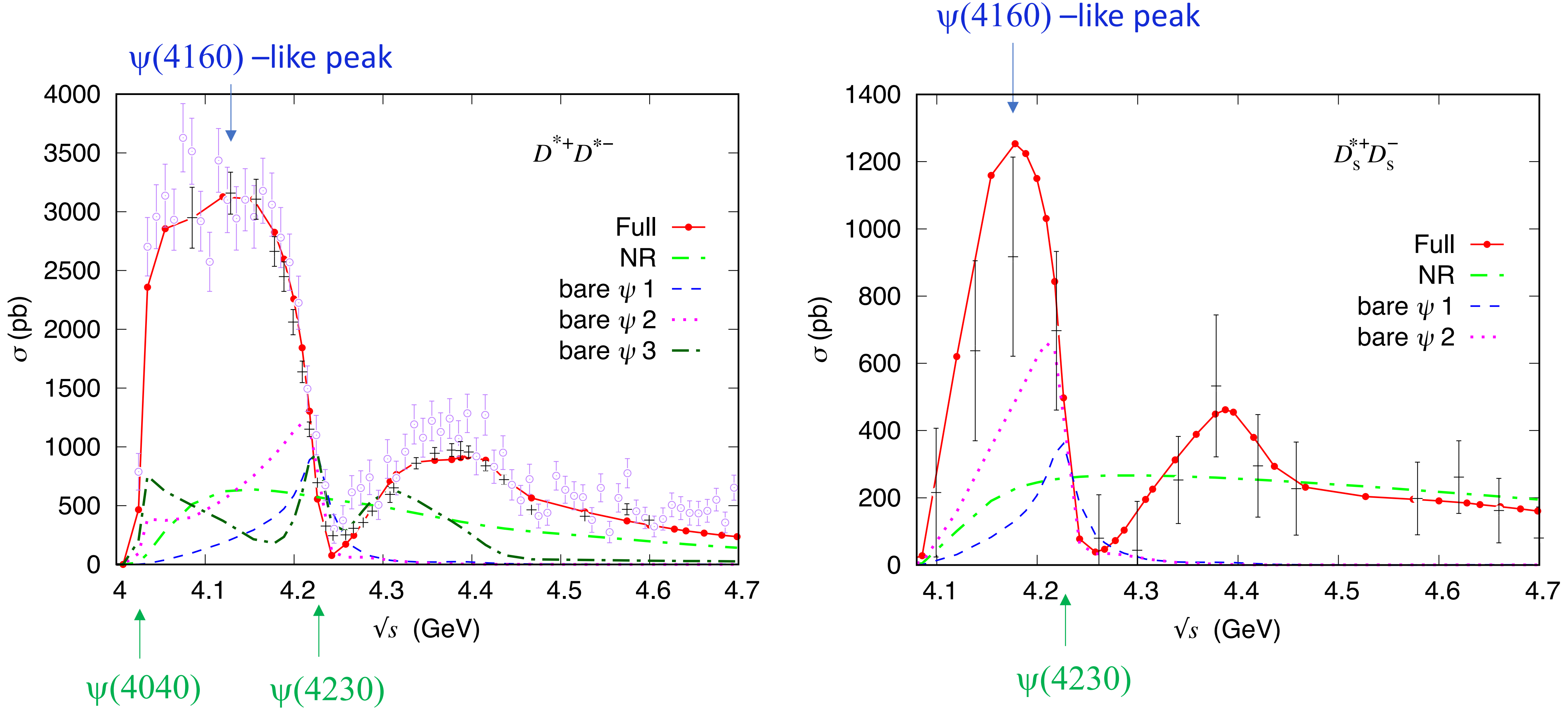
This work		PDG [4]		
M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	
3775 ± 2.0	28 ± 1.0	3778.1 ± 0.7	27.5 ± 0.9	$\psi(3770)$
4026 ± 0.1	25 ± 0.3	4039 ± 1	80 ± 10	$\psi(4040)$
4232 ± 1.0	114 ± 1.7	4191 ± 5	70 ± 10	$\psi(4160)$
4226 ± 0.4	36 ± 0.8	4222.5 ± 2.4	48 ± 8	$\psi(4230)$
4309 ± 0.6	328 ± 0.9	–	–	–
4369 ± 0.1	183 ± 0.2	4374 ± 7	118 ± 12	$\psi(4360)$
4394 ± 0.7	93 ± 0.9	4421 ± 4	62 ± 20	$\psi(4415)$
BW fit → 4690 ± 7.3	106 ± 8.8	4630 ± 6	72^{+14}_{-12}	$\psi(4660)$

When several poles are found nearby but on different Riemann sheets,

they correspond to the same state and only the one closest to the physical real energy is listed

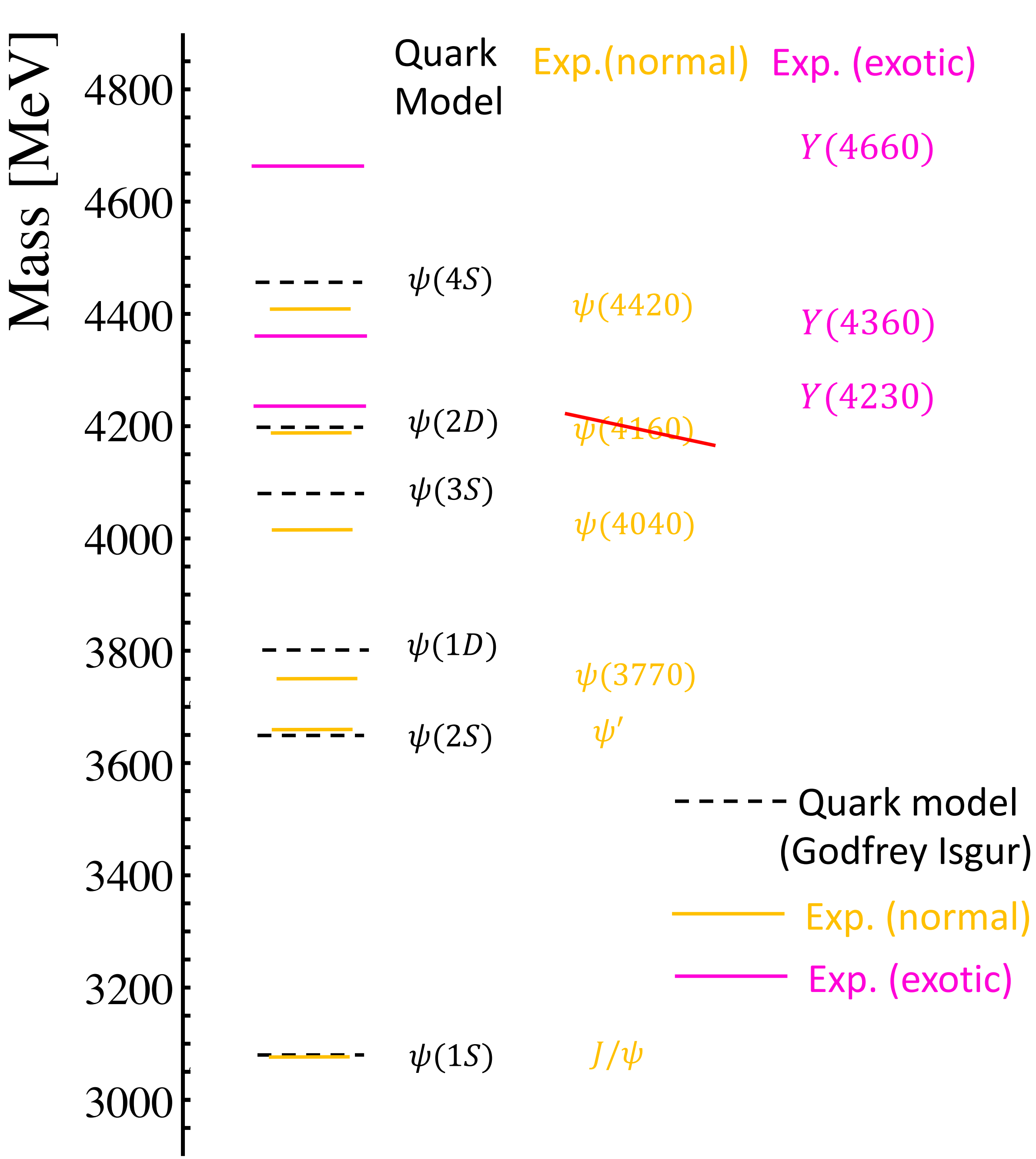
The pole uncertainty seems too small → will be improved in ongoing update

No ψ(4160) from our analysis, why ?



In the above processes, ψ(4160) -like peaks are from interfering ψ(4040), ψ(4230), and NR

Charmonium spectrum ($J^{PC} = 1^{--}$)



No ψ(4160) → impact on Y

Y are considered exotic since not predicted by quark model

If ψ(4160) does not exist, a natural assignment is

$Y(4230) = \psi(2D)$ (conventional $c\bar{c}$)

Several theory papers proposed Y(4230) as $D_1\bar{D}$ molecule

→ $Y(4230) \to D_1\bar{D}$ is main Y(4230) decay mode

By examining the $Y(4230)$ pole residues,

we can support/disfavor this scenario (future work)

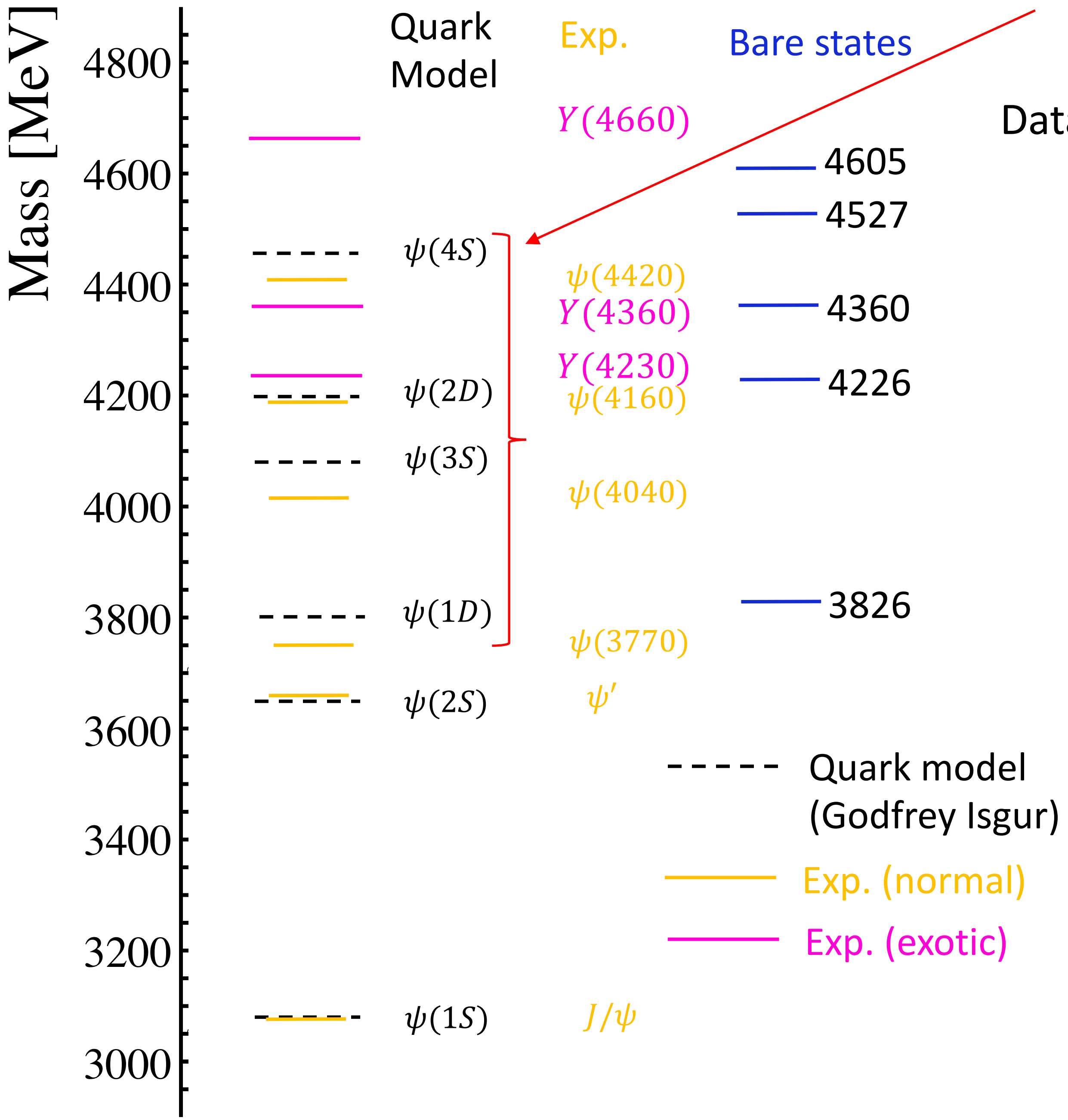
(From previous slide)

Y(4230) → $D^{(*)}\bar{D}^{(*)}$, $D_s^{(*)}\bar{D}_s^{(*)}$ might occur more often

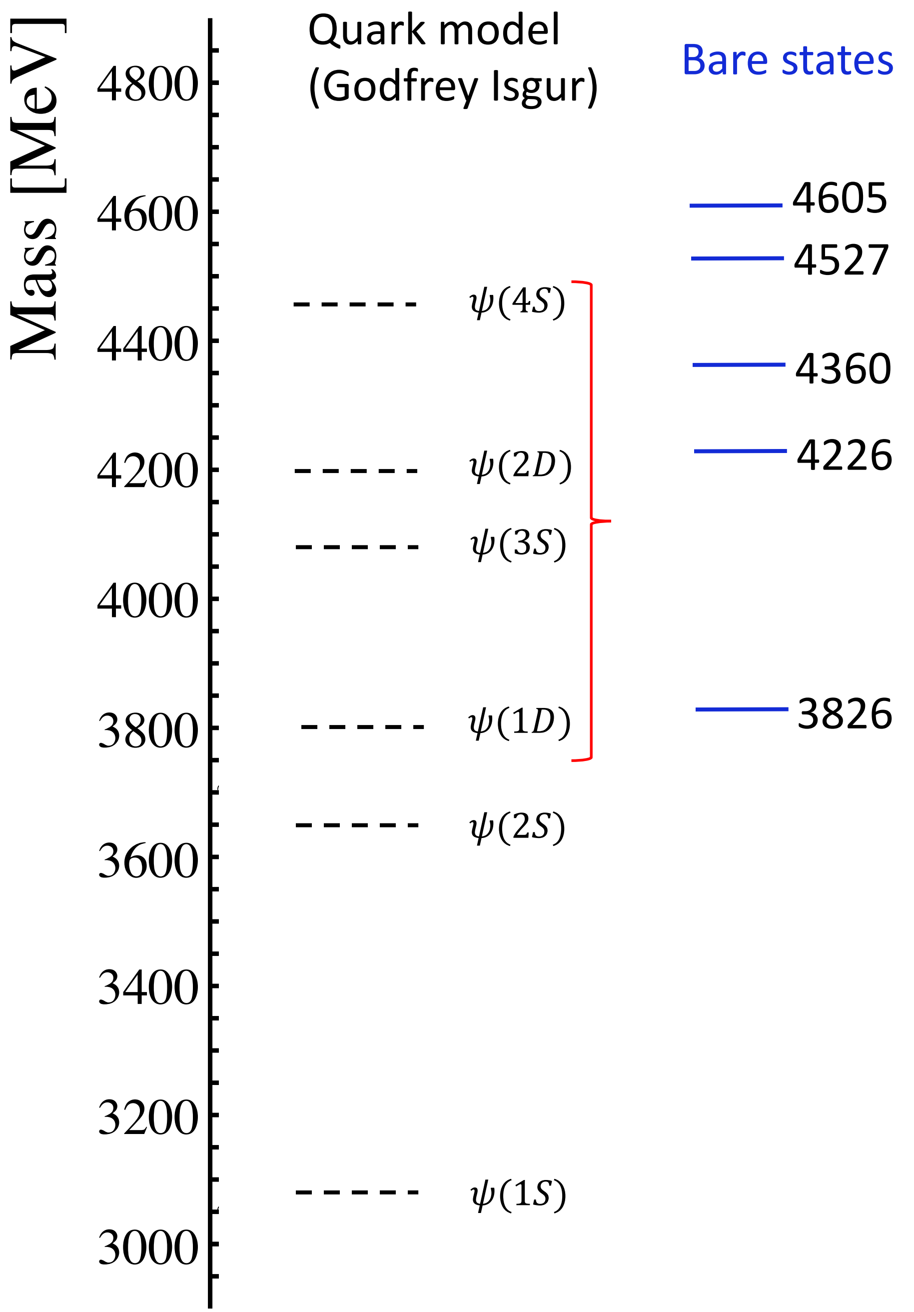
Charmonium spectrum ($J^{PC} = 1^{--}$)

Quark model predicts four states in the relevant energy region

Data require five bare states for achieving reasonable fit



Charmonium spectrum ($J^{PC} = 1^{--}$)



Quark model predicts four states in the relevant energy region

Data require five bare states for achieving reasonable fit

Conceptually, quark-model-state and our bare state is similar

→ Resonance without hadron-hadron continuum components

Very model-dependent argument/questions

One bare state is not accommodated in the quark model

→ Is it exotic bare state ?

Does it generate Y(4230) and Y(4360) after being dressed ?

Does it correspond to hybrid state predicted by LQCD ?

Liu et al., JHEP 07 (2012) 126

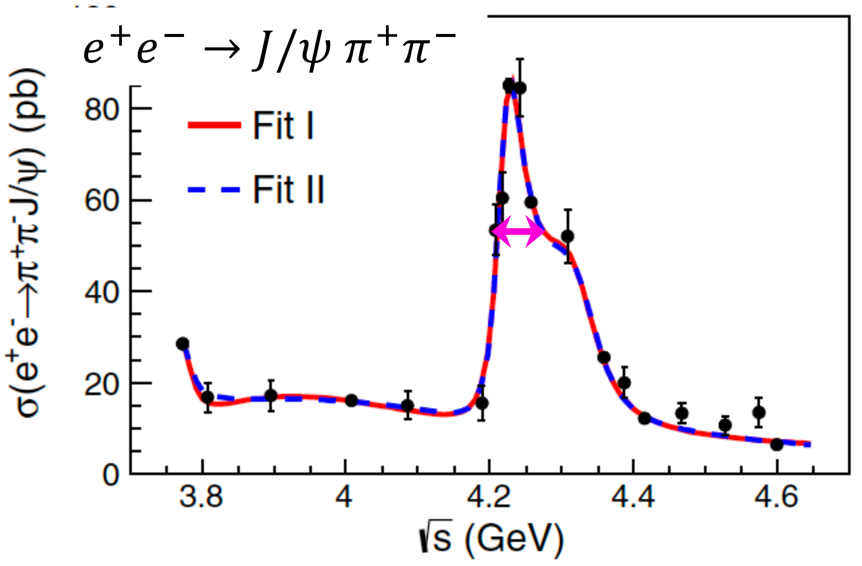
Our model alone cannot answer these interesting questions

Maybe possible by combining with structure model (quark model, etc.)

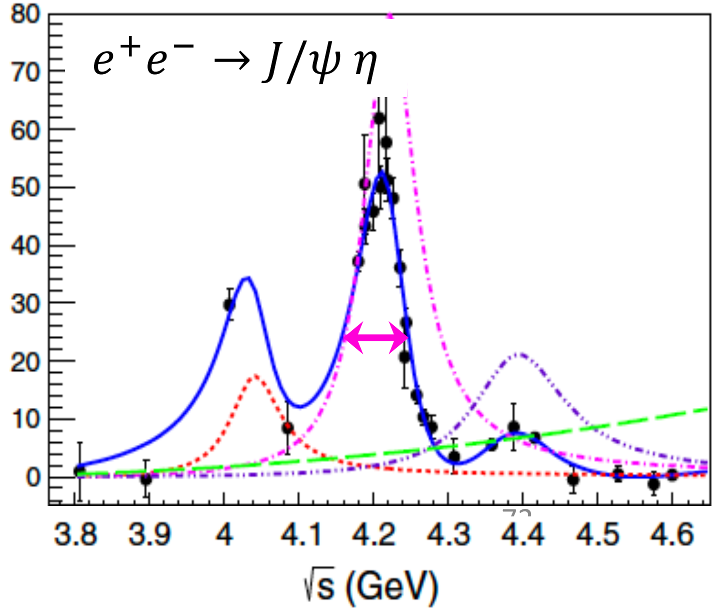
(speculation) Possible solution to Y width problem

	M (MeV)	Γ (MeV)
	3775	28
	4026	25
$\psi_{\rm wid}$	4232	114
$\psi_{\rm nar}$	4226	36
	4309	328
	4369	183
	4394	93

$\Gamma_{Y(4220)} \sim 44 \pm 4$ MeV



$\Gamma_{Y(4220)} \sim 82 \pm 6$ MeV



Two poles at $M \sim$ 4230 (4380) MeV with narrow ($\psi_{\rm nar}$) and wide ($\psi_{\rm wid}$) widths. We can explain Y widths if:

For $e^+e^- \to J/\psi\, \pi^+\pi^-$ $\quad |g_{\psi_{\rm nar}\to J/\psi\pi\pi}| \gg |g_{\psi_{\rm wid}\to J/\psi\pi\pi}|$

For $e^+e^- \to J/\psi\, \eta$ $\quad |g_{\psi_{\rm nar}\to J/\psi\eta}| \ll |g_{\psi_{\rm wid}\to J/\psi\eta}|$

$g_{\psi_{\rm nar}\to J/\psi\pi\pi}$: pole residue

Residues will be extracted in near future, and address the Y width problem

Relation between bare state and pole

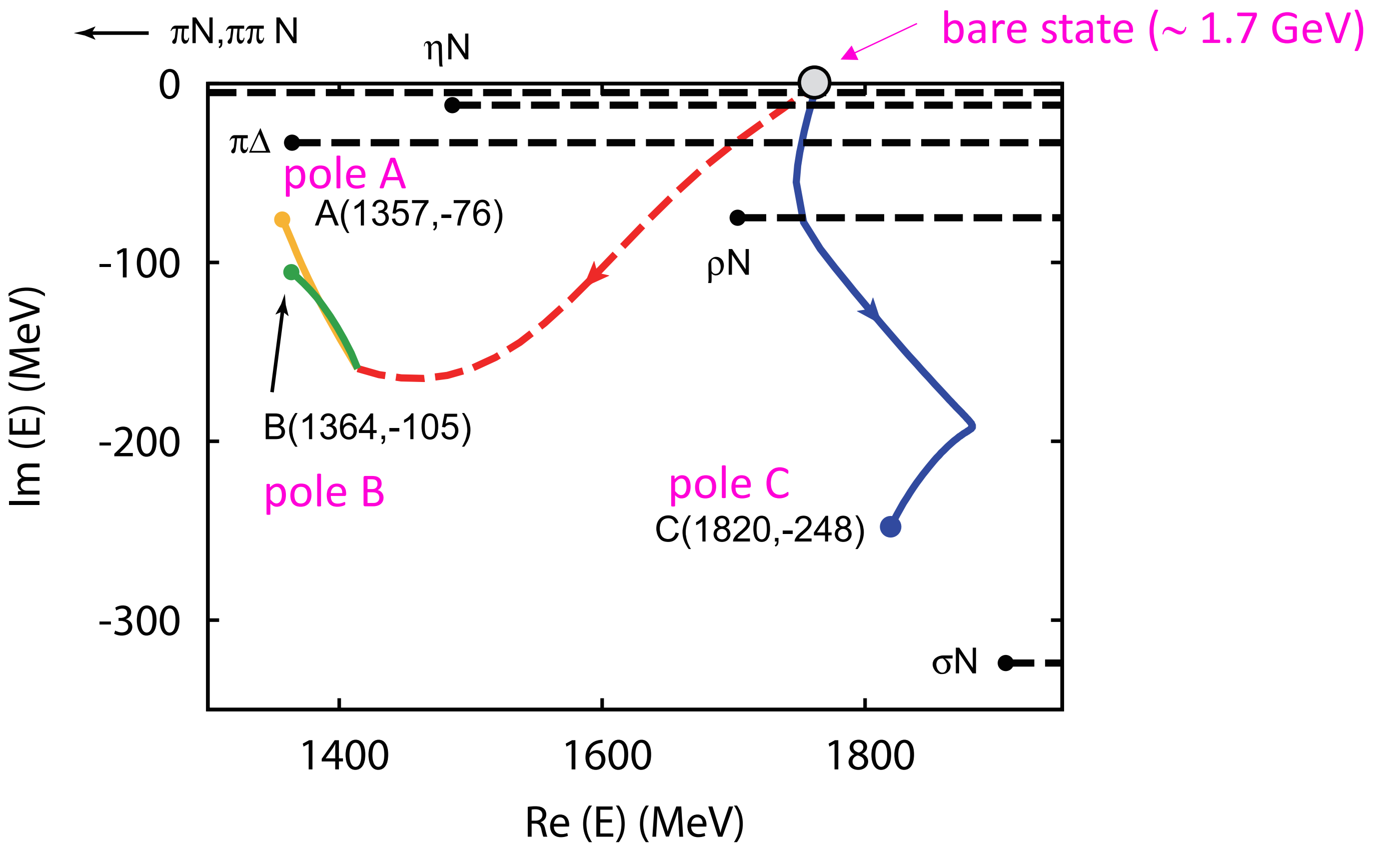
Data require five bare states

→ dressed by hadron continuum

→ seven poles

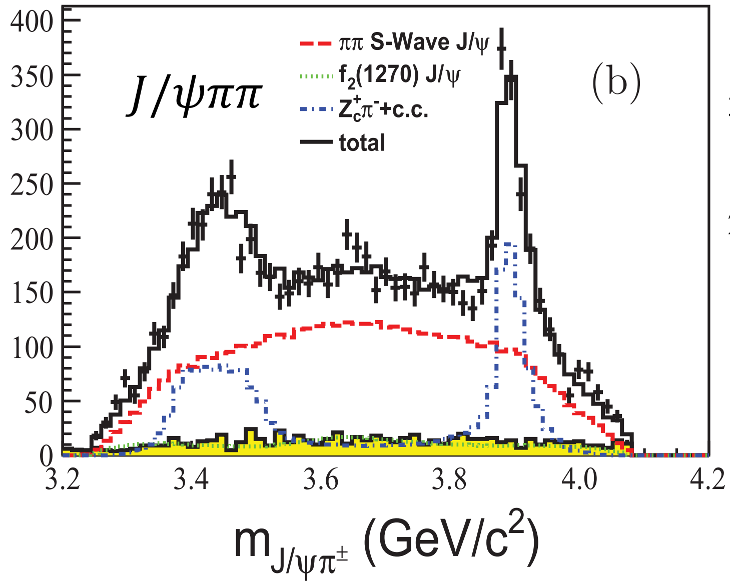
M (MeV)	Γ (MeV)
3775	28
4026	25
4232	114
4226	36
4309	328
4369	183
4394	93

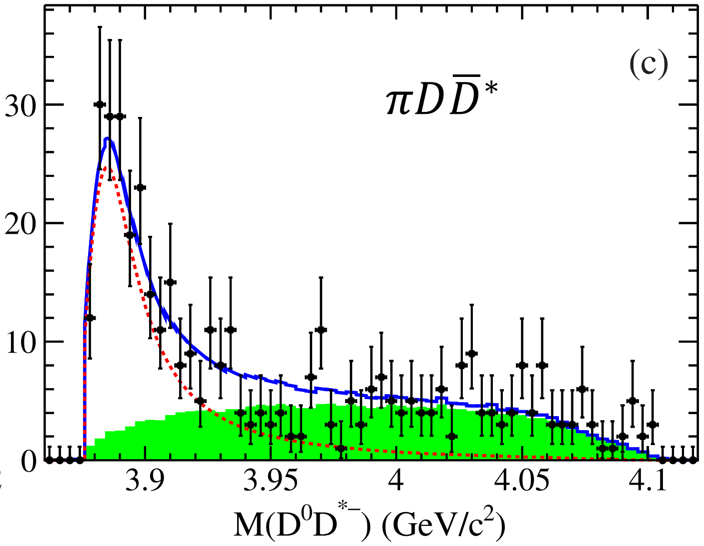
Similar finding in nucleon resonances   Suzuki et al. (EBAC) PRL 104, 042302 (2010)



Future work : Which pair of poles come from the same bare state (mainly) ?

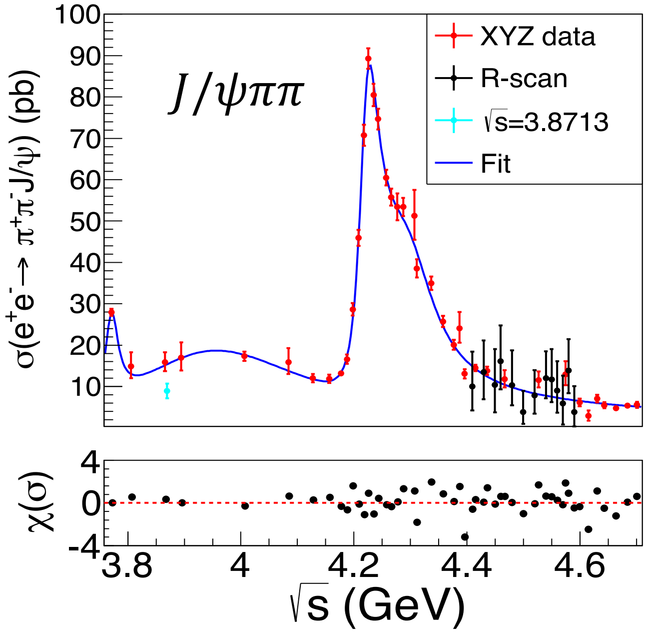
Common problem in previous theoretical analyses on Zc(3900)

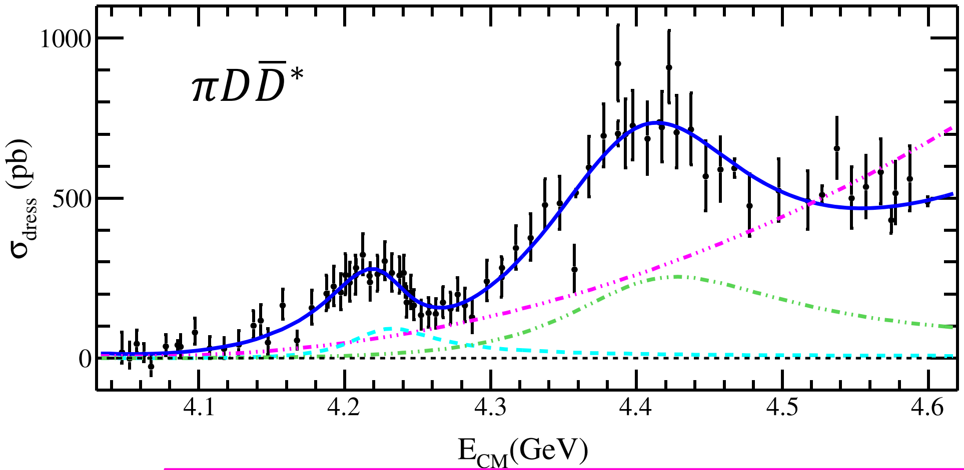




Invariant mass distributions (left, event numbers) are fitted to determine Zc(3900) pole

→ model's overall normalization is arbitrary but model has $\sigma(e^+e^- \to J/\psi\pi\pi)/\sigma(e^+e^- \to \pi D\bar{D}^*)$





In previous theoretical analyses, cross sections (left) were not considered

→ $\sigma(e^+e^- \to J/\psi\pi\pi)/\sigma(e^+e^- \to \pi D\bar{D}^*)$ from model is unchecked

Cross section data can test Zc production mechanism, Zc decay residues

$\frac{\sigma(e^+e^-\to J/\psi\,\pi\pi)}{\sigma(e^+e^-\to \pi D\bar{D}^*)}$ should be checked to see if models are reasonable

Our analysis cleared this problem

Present analysis result is consistent with lattice QCD

Previous LQCD analyses on $Z_c(3900)$ in:

Prelovsek et al. PLB 727, 172 (2013), PRD 91, 014504 (2015)
Chen et al. PRD 89 , 094506 (2014)
Ikeda et al. (HAL QCD) PRL 117, 242001 (2016)
Cheung et al. (Hadron spectrum Collab.) JHEP 11, 033 (2017)

LQCD conclusion : $I=1,\ J^{PC}=1^{+-}\ D^*\bar{D}$ s-wave interaction is very weak,

disfavoring narrow $Z_c(3900)$ pole near $D^*\bar{D}$ threshold

Most of previous determinations of Zc(3900) pole are not consistent with LQCD

Possible $Z_c(3900)$ poles from LQCD: HAL QCD Collaboration (Ikeda et al.) J. Phys. G 45, 024002 (2018)

Yamada et al. [PRD 105, 014034 (2022)]:

Combining the HAL QCD result with symmetry of S-matrix due to unitarity, $S(\{-k_i^*\}) = S^*(\{k_i\})$,

$Z_c(3900)$ pole nearest to the physical region is obtained

$\{k_i\}$: set of channel momenta