北京航空航天大學
BEIHANG UNIVERSITY

Exploring the meson light-cone distribution amplitudes from lattice QCD

Based on arXiv:2403.17492, PRL129(2022)132001, PRL127(2021)062002, PRD99(2019)094036, et al.

In collaboration with LPC members and C.D. Lü, J. Xu, W. Wang, S. Zhao, et al.

Qi-An Zhang

Beihang University (BUAA)

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Outline

- **Motivation**
- **Light meson LCDAs**
  - **Recent progresses and challenges**
  - **Lattice QCD and Large-momentum effective theory**
  - **Pseudoscalar and vector meson LCDAs**
- **Heavy meson LCDAs**
  - **Challenges and recent progresses**
  - **Two-step factorization for heavy meson LCDAs**
  - **LCDAs in QCD and HQET**
- **Summary and outlook**

Motivation: Why LCDAs are important?

- Weak decays of B meson are critical for:
  - **Precise tests of SM**
  - **Searching for NP**
  - **Understanding the origins of CPV**
  - **……**

- $B \to \pi\pi$: *Beneke, Buchalla, Neubert, Sachrajda, 1999; 1422 citations*
- $B \to \pi K$: *Beneke, Buchalla, Neubert, Sachrajda, 2001; 1177 citations*
- $B \to \pi \ell \nu$: *Becher, Hill, 2005; 215 citations*
  Khodjamirian, Mannel, Offen, Wang, 2011; 192 citations
- $B \to K^{(*)} \ell \ell$: *Khodjamirian, Mannel, Pivavorov, Wang, 2010; 486 citations*
- $B \to D \ell \nu$: *HPQCD Collaboration, 2015; 387 citations*

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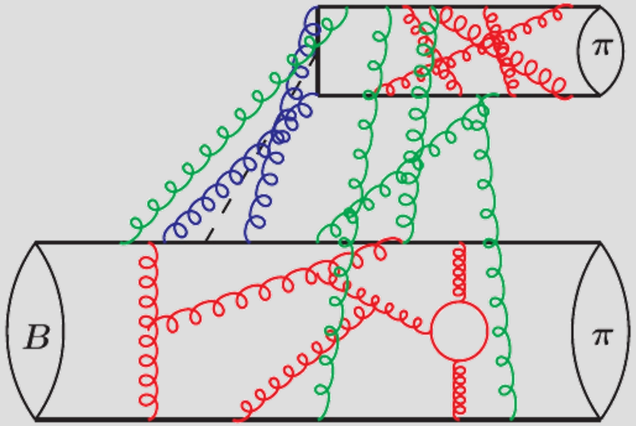
- Factorization: categories by different characteristic scales

$$\langle \pi(p')\pi(q)|Q_i|\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

Form factor = Hard kernel + LCDAs

Hard kernel (Perturbative)

Meson LCDAs (Nonperturbative)



Light meson LCDAs

Light Meson LCDAs: Research Progresses and Challenges

➢ Light meson LCDAs have been extensively pursued: (1970s - now)

- **Asymptotic LCDAs**
  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; Efremov, Radyushkin, 1980
- **Dyson-Schwinger Equation**
  Chang, Cloet, Cobos-Martinez, Roberts, Schmidt, 2013; Gao, Chang, Liu, Roberts, Schmidt, 2014; Roberts, Richards, Chang, 2021
- **Sum rules**
  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; Ball, Braun, Koike, Tanaka, 1998; Ball, Braun, 1998; Khodjamirian, Mannel, Melcher, 2004; Ball,, Lenz, 2007
- **Global Fits**
  Stefanis, 2020; Cheng, Khodjamirian, Rusov, 2020; Hua, Li, Lu, Wang, Xing, 2021
- **Models**
  Arriola, Broniowski, 2002, 2006; Zhong, Zhu, Fu, Wu, Huang, 2021;
- **Lattice with OPE**
  Martinelli, Sachrajda, 1987; Braun, Bruns, et al., 2016; RQCD collaboration, 2019, 2020
- **Lattice with current-current correlation**
  Bali, Braun, Gläßle, Göckeler, Gruber, 2017, 2018;
- **Lattice with LaMET**
  Zhang, Chen, Ji, Jin, Lin, 2017; LP3 Collaboration, 2019; Zhang, Honkala, Lin, Chen, 2020; Lin, Chen, Fan, Zhang², 2021; LPC Collaboration, 2021, 2022
- **Quantum Computing**
  QuNu Collaboration, 2023, 2024

Lattice QCD in a nutshell

Lattice v.s. Continuum	
We simulate:	**We want:**
😀 At finite lattice spacing a	🤔 $a \to 0$
😀 In finite volume L^3	🤔 $L \to \infty$
😀 Euclidean space	🤔 Minkowski space
😀 Lattice regularization	🤔 Some continuum scheme
😀 Some bare input quark masses: am_l, am_s, am_c, am_b In general, $m_\pi^{\mathrm{lat}} \neq m_\pi^{\mathrm{phy}}$	🤔 $m_q^{\mathrm{lat}} = m_q^{\mathrm{phy}}$

Lost the real time information!

Light-like nonlocal correlators cannot be simulated on Euclidean lattice directly!

Lattice with OPE: the low order moments

- **Light-like correlators cannot be simulated on Euclidean lattice directly $\Rightarrow$ Local correlators can!**

Lattice with OPE: the low order moments

➢ **Light-like correlators cannot be simulated on Euclidean lattice directly** $\Rightarrow$ **Local correlators can!**

- Lattice with OPE: **OPE moments ⇒ Gegenbauer moments**

$$\langle \xi^n \rangle = \int_0^1 dx (2x-1)^n \phi_\pi(x)$$

- The nonlocal operator can be defined as a generating function for renormalized local operators:

$$\bar{d}(z_2 n)\ \not{n}\gamma_5\, [z_2 n, z_1 n]\, u(z_1 n) = \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k! l!} n^\rho n^{\mu_1} \ldots n^{\mu_{k+l}} \mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}}$$

$$\mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} = \bar{d}(0) \overleftarrow{D}_{(\mu_1} \ldots \overleftarrow{D}_{\mu_k} \vec{D}_{\mu_{k+1}} \ldots \vec{D}_{\mu_{k+l}} \gamma_{\rho)}\, \gamma_5 u(0)$$

- Moments of the pion DA are given by matrix elements of local operators:

$$i^{k+l} \left\langle 0 \left| \mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} \right| \pi(p) \right\rangle = i f_\pi p_{(\rho} p_{\mu_1} \ldots p_{\mu_{k+l}} \left\langle x^l (1-x)^k \right\rangle$$

Lattice with OPE: the low order moments

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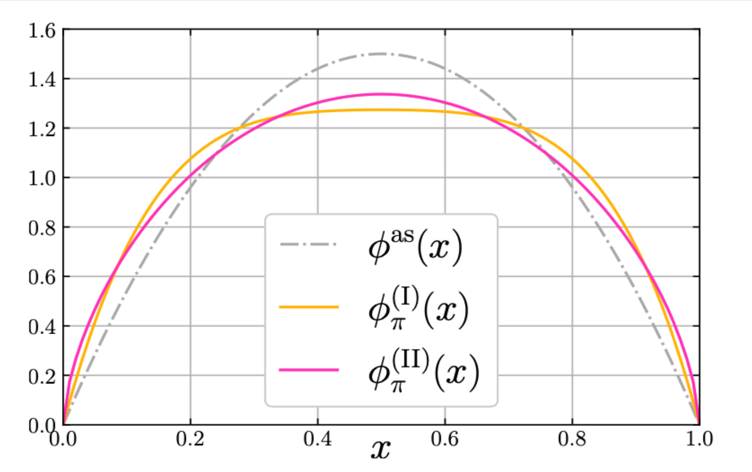
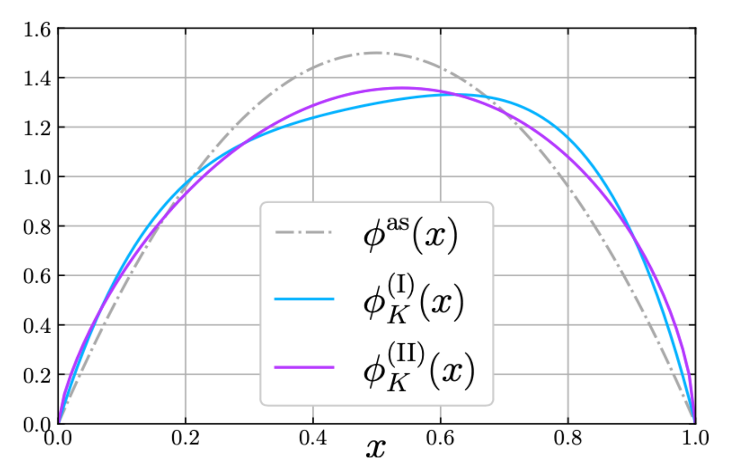
[RQCD collaboration, 2019]





$a_2^\pi = 0.101^{+24}_{-24}$ $a_4^\pi = 0.002^{+71}_{-71}$ $a_1^K = 0.0533^{+34}_{-35}$ $a_2^K = 0.090^{+19}_{-20}$

😀 **Precise at low order moments**

🤔 **Operator mixing**

Convergence problem

Computational complexity

Large-momentum effective theory

- **Light-like correlators cannot be simulated on Euclidean lattice directly** $\Rightarrow$ **Equal-time correlators can!**

Large-momentum effective theory

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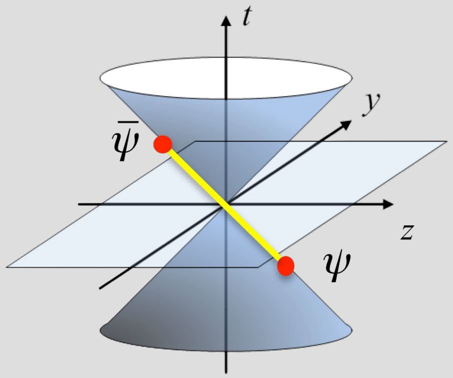
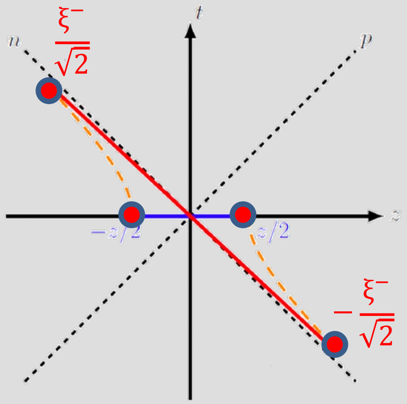
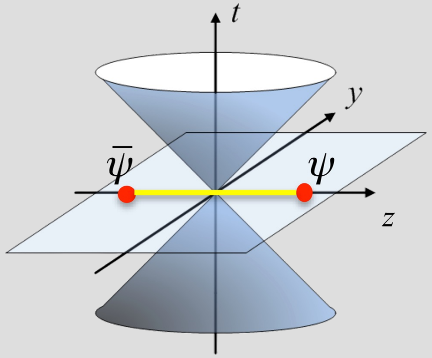






$z + ct = 0,\ z - ct \neq 0$

$t = 0,\ z \neq 0$

Light-like correlators:

PDFs, LCDAs, ……

Related by Lorentz boost

[Ji, 2013; Ji, Liu², Zhang, Zhao, 2021]

Quasi correlator:

Directly calculable on the lattice!

Large-momentum effective theory

➢ **Large-momentum expansion of quasi DA:**

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right),$$

Power corrections

Matching coefficient @ 1-loop: [Xu, QAZ, Zhao, 2018; Liu, Wang, Xu, QAZ, Zhao, 2019]

$$C_B^{(1)}\left(x, y, \frac{\mu}{P^z}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} [H_1(x,y)]_+ & x < 0 < y \\ [H_2(x, y, P^z/\mu)]_+ & 0 < x < y \\ [H_2(1-x, 1-y, P^z/\mu)]_+ & y < x < 1 \\ [H_1(1-x, 1-y)]_+ & y < 1 < x \end{cases}$$

$$H_1(x,y) = \frac{1+x-y}{y-x}\frac{1-x}{1-y}\ln\frac{y-x}{1-x} + \frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{y-x}{-x}$$

$$H_2(x, y, P^z/\mu) = \frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{4x(y-x)(P^z)^2}{\mu^2} + \frac{1+x-y}{y-x}\left(\frac{1-x}{1-y}\ln\frac{y-x}{1-x} - \frac{x}{y}\right)$$

Related to ERBL kernel

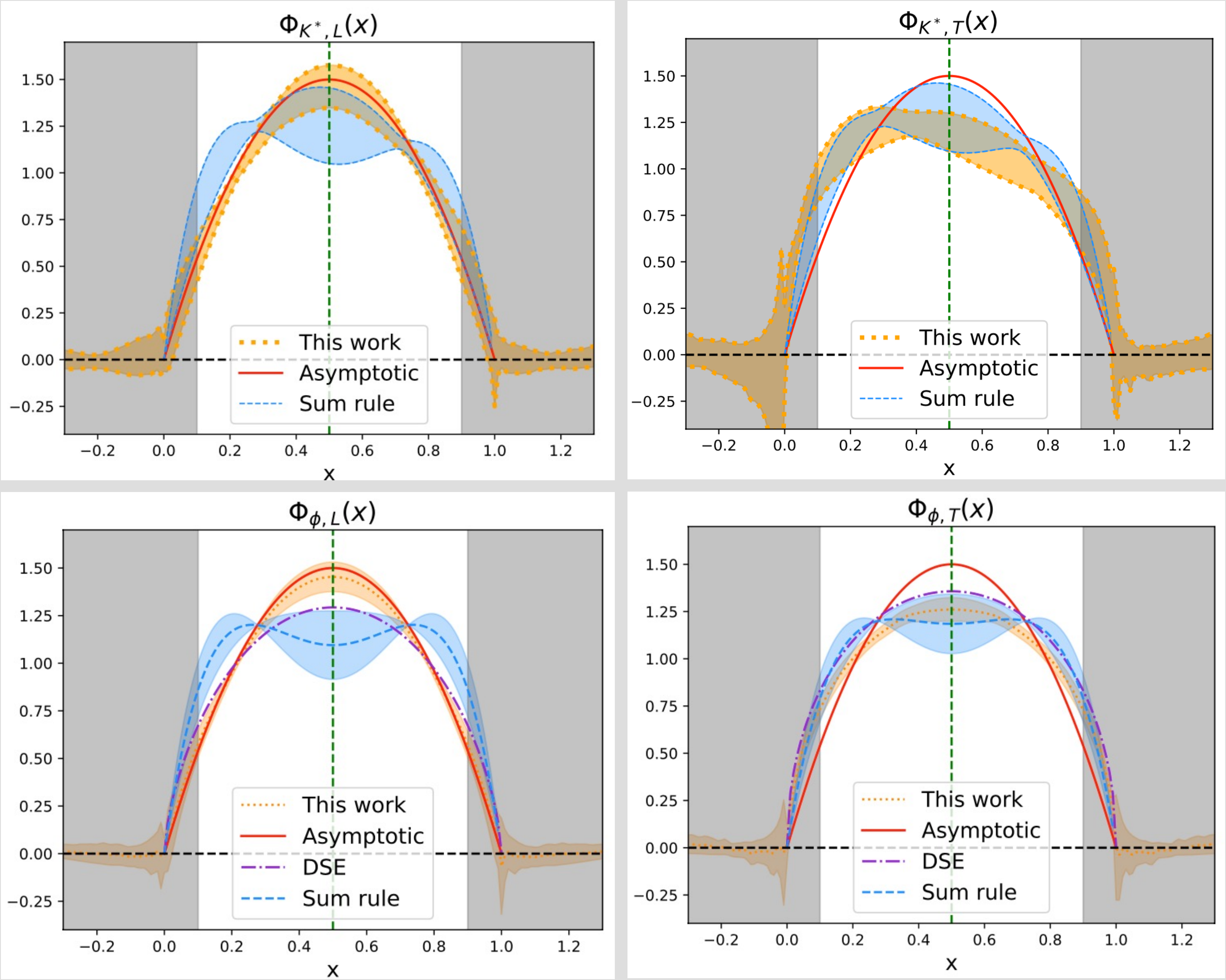
Count-term $C_{CT}^{(1)}$ in:

- UV cut-off and DR
- RI/MOM

Light vector meson LCDAs from lattice QCD

K^*

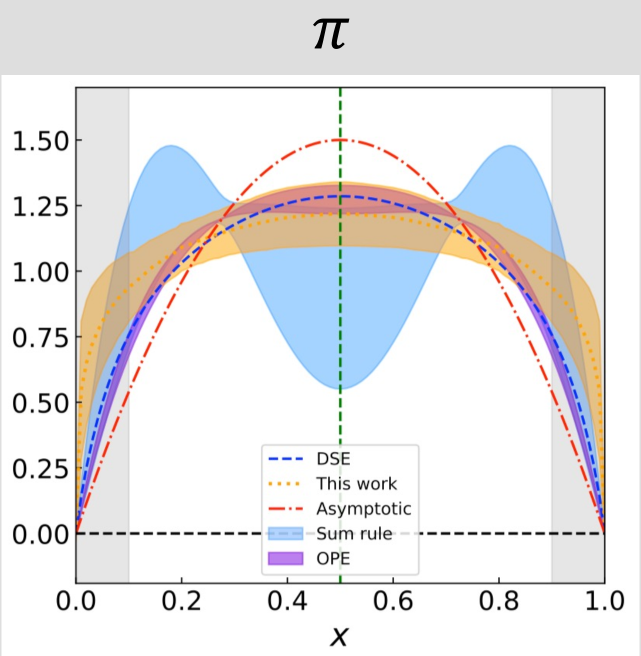
ϕ

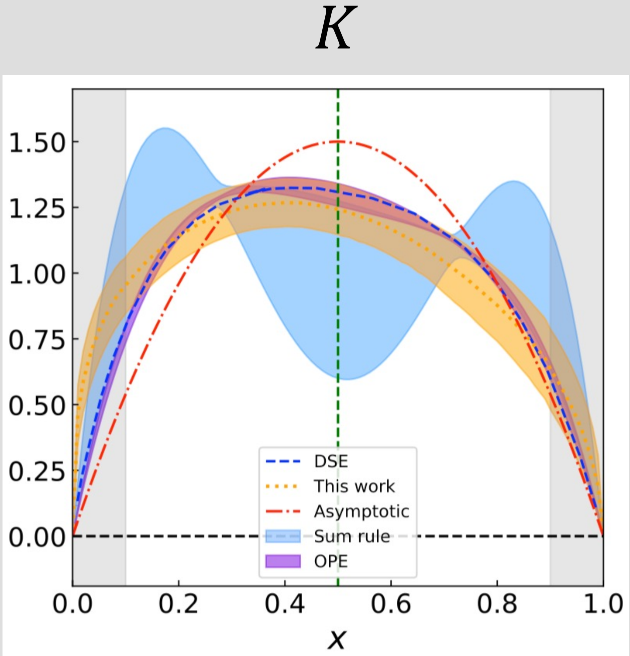


- Physical pion mass;
- Continuum extrapolation with 3 lattice spacings;
- Large-momentum extrapolation with 3 momenta;
- Nonperturbatively renormalized in hybrid scheme.

[Lattice Parton Collaboration (LPC), 2021]

Light pseudoscalar meson LCDAs from lattice QCD





- Physical pion mass;
- Continuum extrapolation with 3 lattice spacings;
- Large-momentum extrapolation with 3 momenta;
- The state-of-the-art hybrid scheme with self renormalization.

[Lattice Parton Collaboration (LPC), 2022]

Heavy meson LCDAs

Heavy Meson LCDAs: Research Progresses and Challenges

- **The HQET matrix element of heavy meson** [Grozin, Neubert, 1997; Beneke, Feldmann, 2000]

$$\langle 0\,|\bar{q}_\beta(\xi)[\xi,0]h_{v\alpha}(0)|\,\bar{B}(v)\rangle = -\frac{i\tilde{f}_B m_B}{8}\left\{\left[\tilde{\phi}_B^+(t,\mu)v_+\gamma_- + \tilde{\phi}_B^-(t,\mu)v_-\gamma_+\right]\gamma_5\right\}_{\alpha\beta}$$

Leading twist ($\tilde{\phi}_B^+$) — Sub-leading twist ($\tilde{\phi}_B^-$)

Heavy Meson LCDAs: Research Progresses and Challenges

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Leading twist — Sub-leading twist

- Evolution of φ_B^+ and φ_B^-: [Lange, Neubert, 2003; Bell, Feldmann, 2008]

$$\frac{d}{d\ln\mu}\varphi_B^+(\omega,\mu) = -\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\gamma_+^{(1)}(\omega,\omega',\mu)\,\varphi_B^+(\omega',\mu) + \mathcal{O}\left(\alpha_s^2\right)$$

$$\frac{d}{d\ln\mu}\varphi_B^-(\omega,\mu) = -\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\gamma_-^{(1)}(\omega,\omega',\mu)\,\varphi_B^-(\omega',\mu) + \mathcal{O}\left(\alpha_s^2\right)$$

with

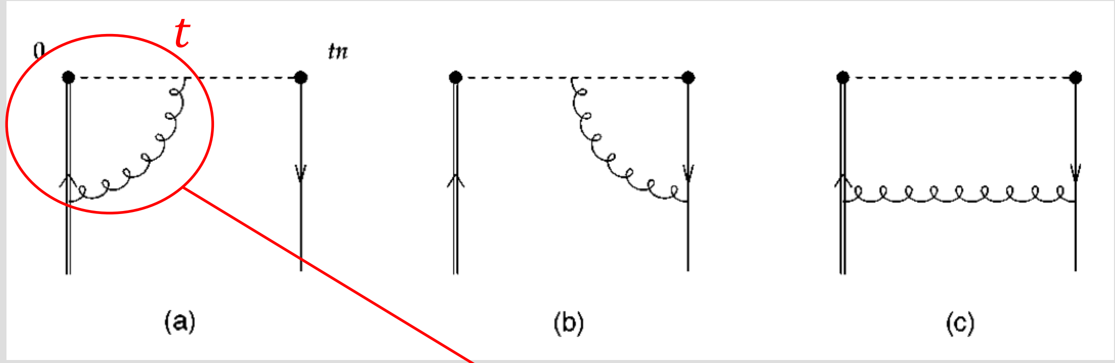
$$\gamma_+^{(1)}(\omega,\omega',\mu) = \left(\Gamma_{\rm cusp}^{(1)}\ln\frac{\mu}{\omega} - 2\right)\delta(\omega-\omega') - \Gamma_{\rm cusp}^{(1)}\,\omega\left[\frac{\theta(\omega'-\omega)}{\omega'(\omega'-\omega)} + \frac{\theta(\omega-\omega')}{\omega(\omega-\omega')}\right]_+,$$

$$\gamma_-^{(1)}(\omega,\omega';\mu) = \gamma_+^{(1)}(\omega,\omega';\mu) - \Gamma_{\rm cusp}^{(1)}\frac{\theta(\omega'-\omega)}{\omega'}.$$

- Solution of evolution equations: [Bell, Feldmann, Wang and Yip, 2013; Braun, Manashov, 2014]
- RG equations of $\varphi_B^+(\omega,\mu)$ at two-loops: [Braun, Ji, Manashov, 2019; Liu, Neubert, 2020]
- RG equations of the higher-twist B-meson distribution amplitudes: [Braun, Ji, Manashov, 2017]
- NNLO QCD correction to relevant hadronic B-meson decays: [Bell, Beneke, Huber, Li, 2020]

Heavy Meson LCDAs: Research Progresses and Challenges

➢ **Difficulties in calculating the moments of heavy meson LCDA:**



Diverge at $t \to 0$!

$$O_+^{\rm ren}(t,\mu) = O_+^{\rm bare}(t) + \frac{\alpha_s C_F}{4\pi}\left\{\left(\frac{4}{\hat{\epsilon}^2} + \frac{4}{\hat{\epsilon}}\ln(it\mu)\right)O_+^{\rm bare}(t) - \frac{4}{\hat{\epsilon}}\int_0^1 du \frac{u}{1-u}\left[O_+^{\rm bare}(ut) - O_+^{\rm bare}(t)\right]\right.$$

[Braun, Ivanov, Korchemsky, 2004]

- Diverge at $t \to 0 \Leftrightarrow$ No local limit
- Non-negative moments $\int dk\, k^n \varphi_+(k)$ for n=0,1,2,… are not related to OPE, and actually they diverge
- Cannot obtain φ_B from lattice QCD through their moments.

Heavy Meson LCDAs: Research Progresses and Challenges

- A theoretical attempt to extract the heavy meson LCDAs in the framework of LaMET:

$$\tilde{\varphi}_B^+(\xi,\mu) \propto \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{in_z\cdot v\xi\tau} \langle 0 \left| \boxed{(\bar{q}W_c)(\tau n_z)\,\eta_z\gamma_5\,(W_c^\dagger h_v)(0)} \right| \bar{B}(v)\rangle \quad \Rightarrow \quad \varphi_B^+(\omega,\mu)$$

Connecting the between **equal-time HQET correlator** and **light-cone one**.

Heavy Meson LCDAs: Research Progresses and Challenges

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Connecting the between **equal-time HQET correlator** and **light-cone one**.

- Leading twist matching @ 1-loop: *[Wang², Xu, Zhao, 2020; Xu, Zhang, 2022]*
- Sub-leading twist matching: *[Hu, Wang, Xu, Zhao, 2024]*
- Inverse moment and log moments: *[Xu, Zhang, Zhao, 2022; Hu, Xu, Zhao, 2024]*
- Ioffe-time distributions: *[Zhao, Radyushkin, 2021]*

🤔 Difficult to realize the boosted HQET field (time-like Wilson link) on lattice QCD.

Two-step factorization to access heavy meson LCDA

- Back to the beginning: Quasi DA, which calculated from LQCD

  - Light meson (2 characteristic scales: $P^z, \Lambda_{\text{QCD}}$) → Heavy meson (3 scales: $P^z, m_H, \Lambda_{\text{QCD}}$)

Two-step factorization to access heavy meson LCDA

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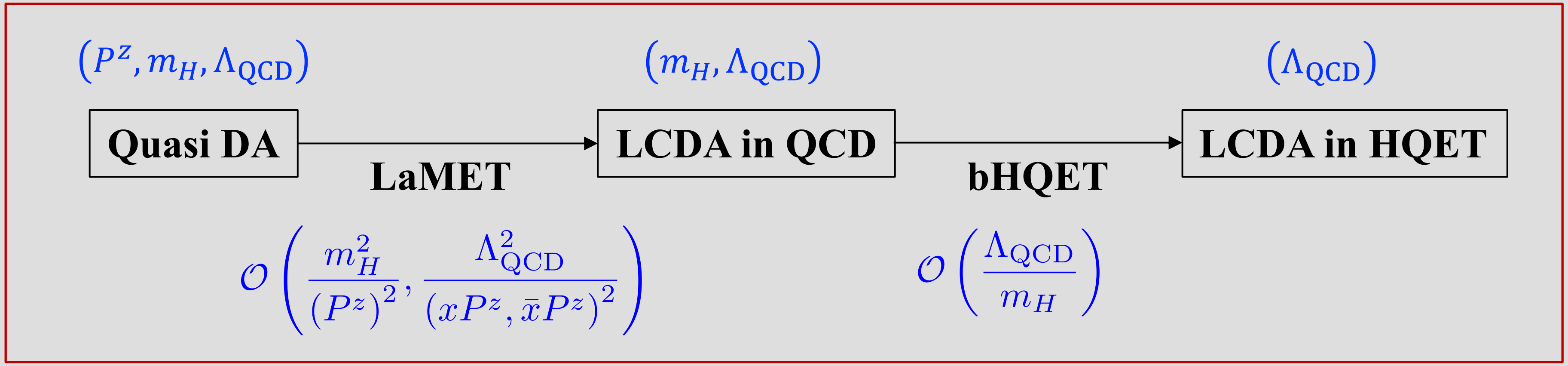


- A multi-scale processes:
  1. LaMET requires $\Lambda_{\rm QCD}, m_H \ll P^z$ and finally integrate out P^z;
  2. bHQET requires $\Lambda_{\rm QCD} \ll m_H$ and integrate out m_H;

⇒ **Hierarchy $\boldsymbol{\Lambda_{\rm QCD} \ll m_H \ll P^z}$.**

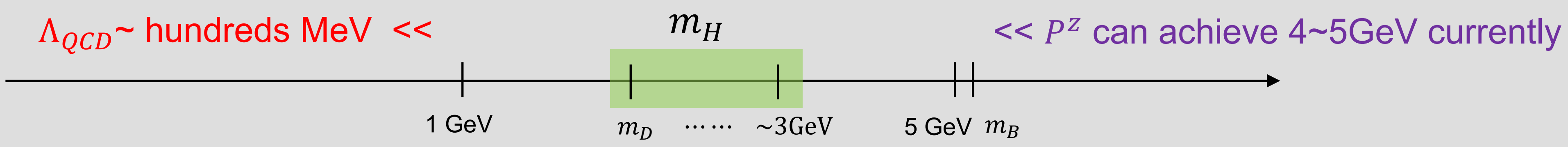
Two-step factorization to access heavy meson LCDA

$(P^z, m_H, \Lambda_{\rm QCD})$ **Quasi DA** → **LaMET** → $(m_H, \Lambda_{\rm QCD})$ **LCDA in QCD** → **bHQET** → $(\Lambda_{\rm QCD})$ **LCDA in HQET**

LaMET: $\mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$

bHQET: $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_H}\right)$

⇒ **Hierarchy $\boldsymbol{\Lambda_{\rm QCD} \ll m_H \ll P^z}$:** A big challenge for lattice simulation



Window for lattice simulation

<u>At this stage, the heavy meson could be D, but by no means be the B meson!</u>

Matching I: from quasi DAs to LCDAs in QCD

- Quasi DA $\tilde{\phi}(x, P^z)$, include the scales $\Lambda_{\rm QCD} \ll m_H \ll P^z$

$$\tilde{\phi}(x, P^z) = \int \frac{dz}{2\pi} e^{-ixP^z z} \tilde{M}(z, P^z)$$

- Matching formula in LaMET:

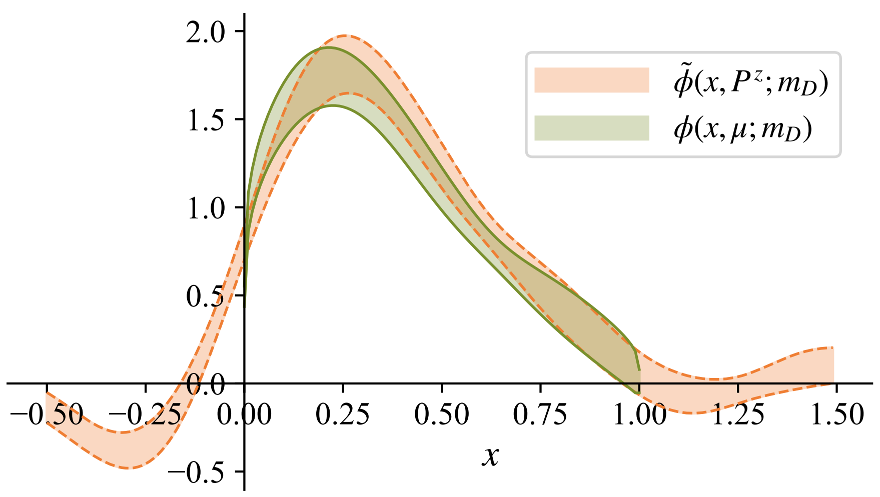
$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

Liu, Wang, Xu, **QAZ**, Zhao, 2019;
Han, Hua, Ji, Lu, Wang, Xu, **QAZ**, Zhao, 2024

This matching integrate out P^z, obtain the LCDAs in QCD.

Quasi DA vs QCD LCDA

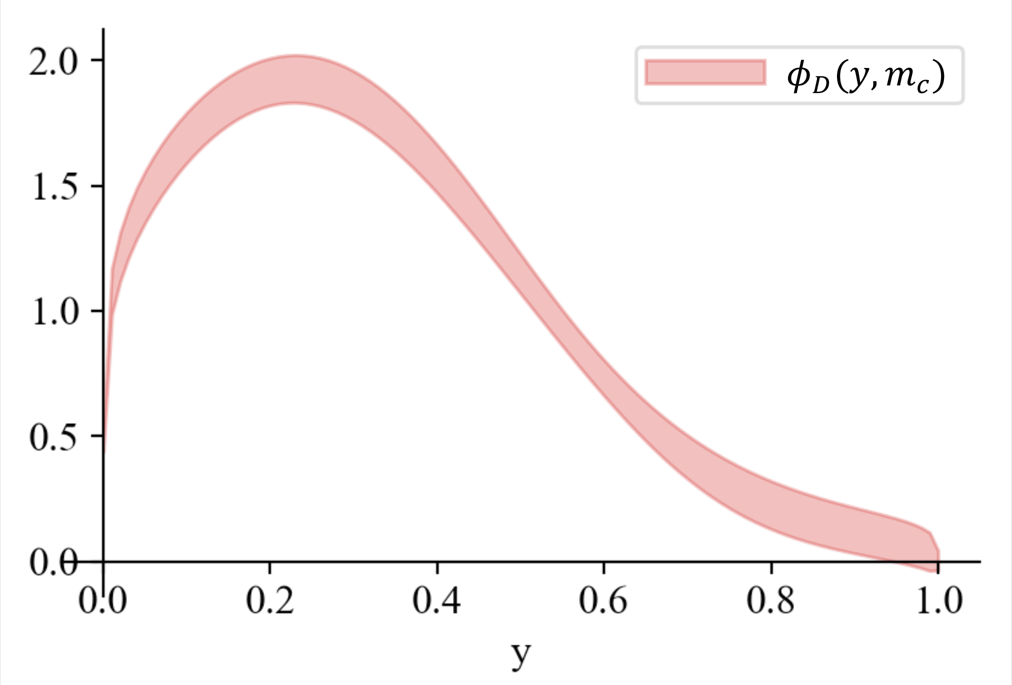
at $P^z = 3.98$GeV, $\mu = 2$GeV



LCDAs in QCD

- Heavy meson LCDAs in QCD

$$\phi(y,\mu) = \frac{1}{if_H}\int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}e^{iyP_H\tau n_+} \times \langle 0\,|\bar{q}(\tau n_+)\not{n}_+\gamma_5 W_c(\tau n_+,0)Q(0)|\,H(P_H)\rangle$$



- The peak position dominated by m_H and μ;
- At very large scale $\mu \gg m_H$, asymptotic form;

LCDAs in QCD

- Heavy meson LCDAs in QCD

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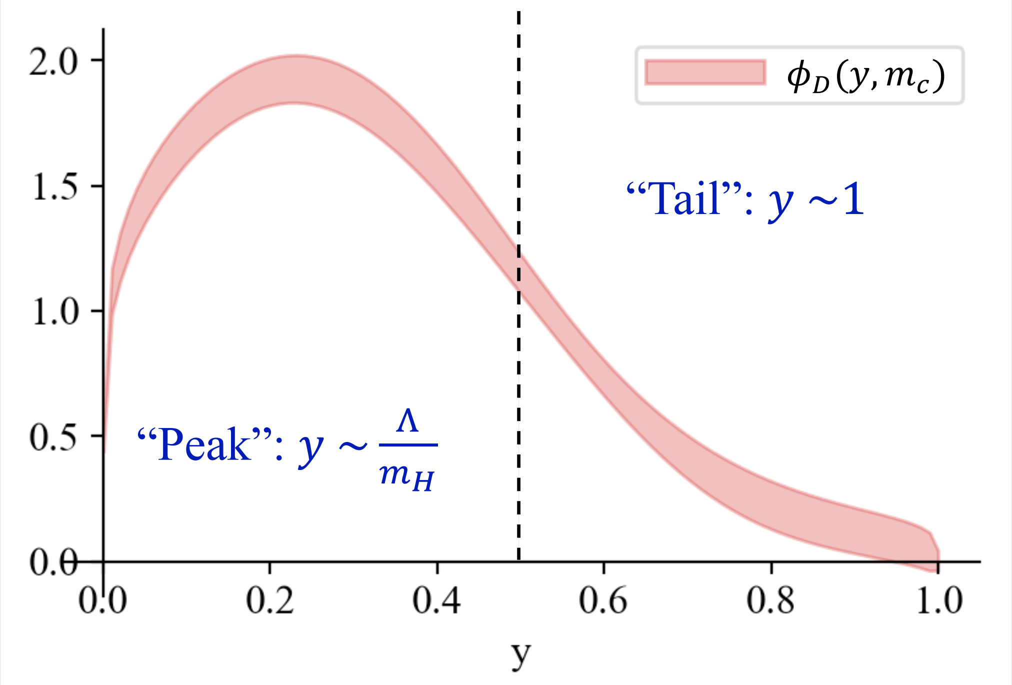


- The peak position dominated by m_H and μ;
- At very large scale $\mu \gg m_H$, asymptotic form;

- For the scale $\mu \lesssim m_Q$,
  - Light quark carries small momentum fraction $y \sim \Lambda/m_H$ ⇒ peak region, related to the HQET LCDA;

    [Ishaq, Jia, Xiong, Yang, 2020; Beneke, Finauri, Vos, Wei, 2023]
  - $y \sim O(1)$ region be suppressed in LCDA:

    SCET renormalized matrix element in this region contain only hard-collinear physics, and starts at the one-loop level.

Matching II: connecting LCDAs in QCD and HQET

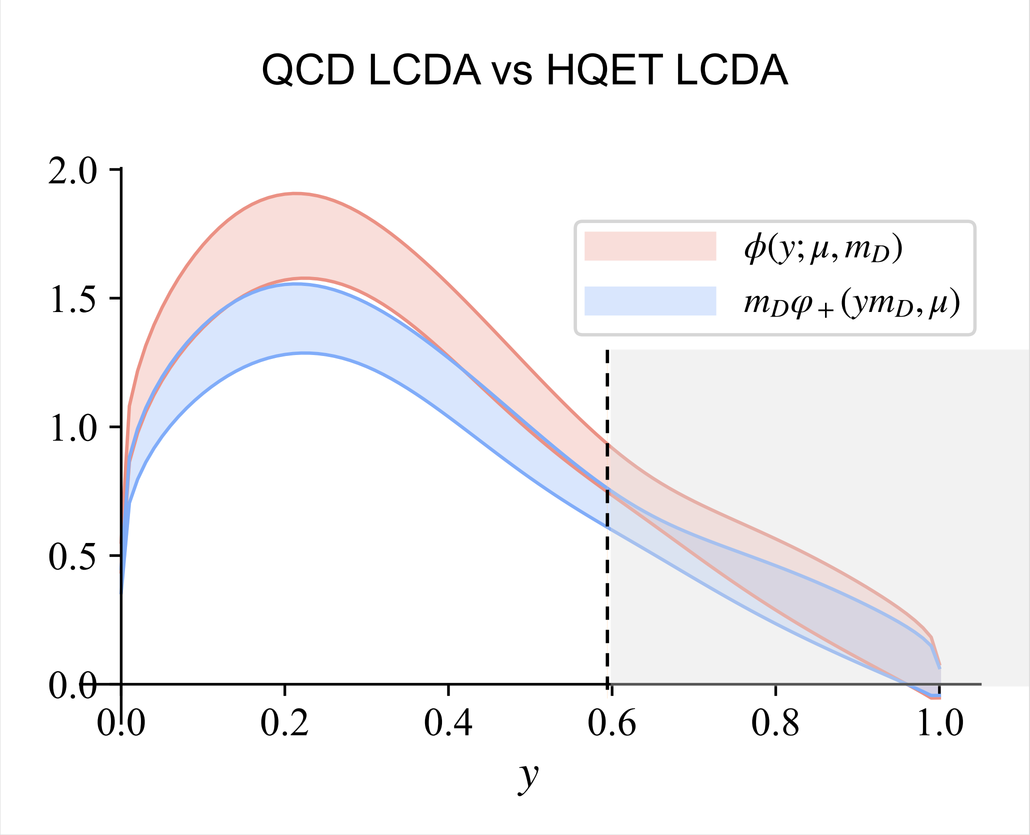
- Leading twist heavy meson LCDA in HQET

$$\varphi^+(\omega,\mu) = \frac{1}{i\tilde{f}_H(\mu) m_H} \int_{-\infty}^{+\infty} \frac{d\eta}{2\pi} e^{i\omega n_+\cdot v\eta} \times \langle 0 | \bar{q}(\eta n_+) \not{n}_+ \gamma_5 W_c(\eta n_+, 0) h_v(0) | H(v) \rangle$$

connected with the QCD LCDA through a multiplicative factorization in the peak region:

[Beneke, Finauri, Vos, Wei, 2023]

$$\phi(y,\mu;m_H) = \frac{\tilde{f}_H}{f_H} J_{\text{peak}} m_H \varphi^+(\omega,\mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_H}\right)$$



Tails of HQET LCDA

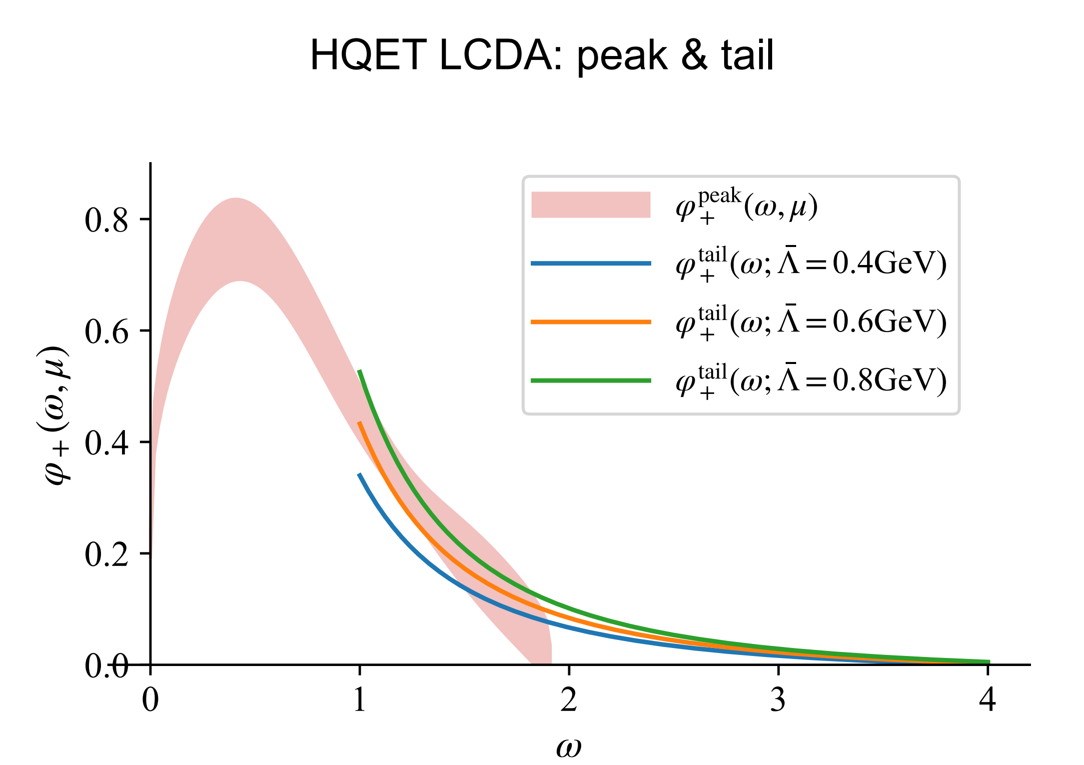
➢ The tail region of HQET LCDA is perturbative: *[Lee, Neubert, 2005]*

$$\varphi^{+}_{\rm tail}(\omega,\mu) = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right]$$

where $\bar{\Lambda} \equiv m_H - m_Q^{\rm pole}$ reflect the power correction, and usually be chosen as 400~600MeV.

- We use the difference between the lines to estimate the power correction.

The final results of HQET LCDA will merge the peak (from LQCD) and tail region (from 1-loop calculation).



Comparison with phenomenological models

➤ Several commonly used models: [Wang, Shen, 2015; Beneke, Braun, Ji, Wei, 2018; Gao, Huber, Ji, Wang², 2022]

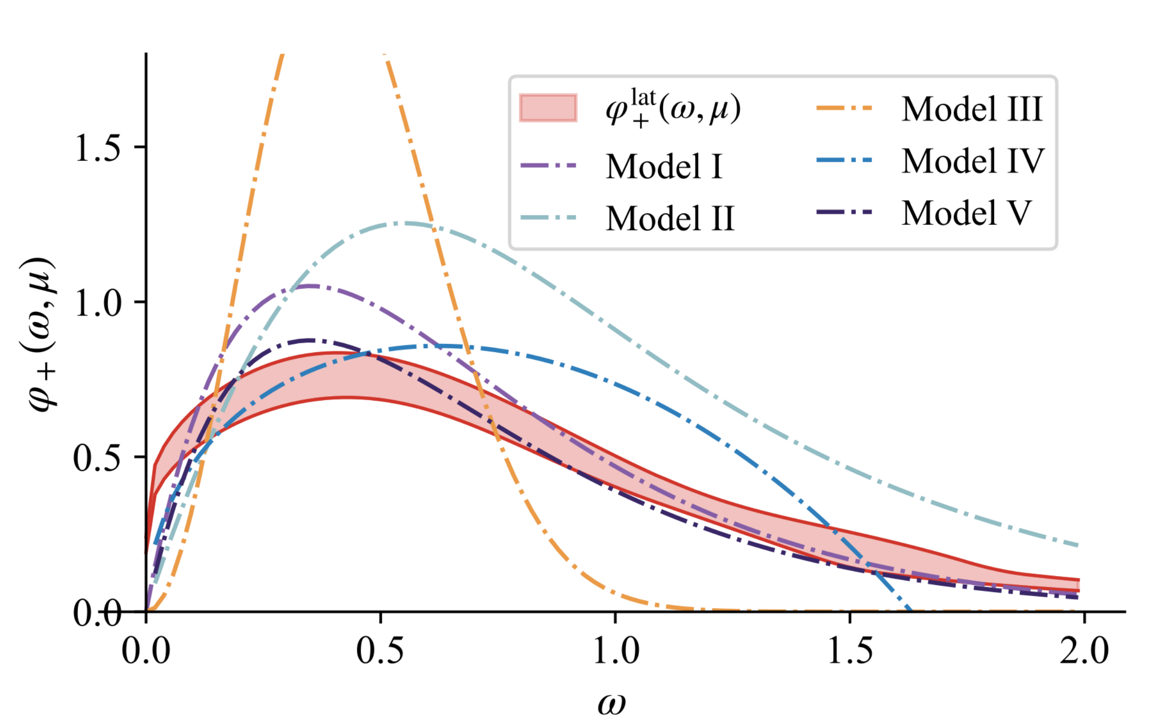
$$\varphi_{\mathrm{I}}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}\,,$$

$$\varphi_{\mathrm{II}}^{+}(\omega,\mu_0)=\frac{4}{\pi\omega_0}\frac{k}{k^2+1}\left[\frac{1}{k^2+1}-\frac{2\left(\sigma_B^{(1)}-1\right)}{\pi^2}\ln k\right]\,,$$

$$\varphi_{\mathrm{III}}^{+}(\omega,\mu_0)=\frac{2\omega^2}{\omega_0\omega_1^2}e^{-(\omega/\omega_1)^2}\,,$$

$$\varphi_{\mathrm{IV}}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0\omega_2}\frac{\omega_2-\omega}{\sqrt{\omega\left(2\omega_2-\omega\right)}}\theta\left(\omega_2-\omega\right)\,,$$

$$\varphi_{\mathrm{V}}^{+}(\omega,\mu_0)=\frac{\Gamma(\beta)}{\Gamma(\alpha)}\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}U(\beta-\alpha,3-\alpha,\omega/\omega_0)\,,$$



Summary and outlook

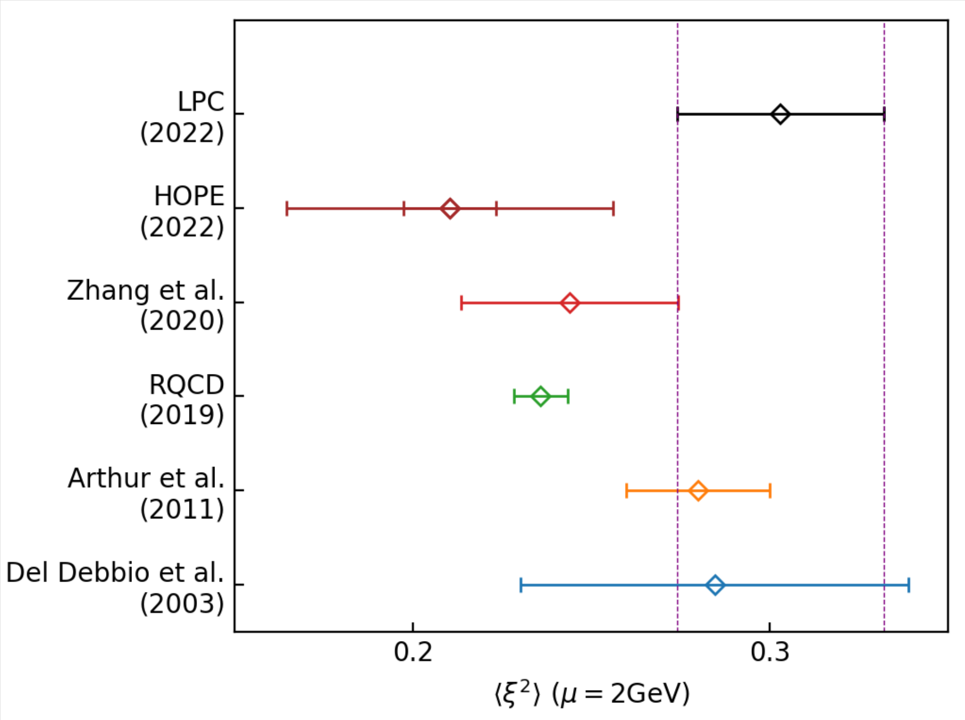
- ✓ Lattice QCD + LaMET provides a powerful tool for exploring the light-like correlations, as LCDAs, PDFs, TMDWFs, TMDPDFs, ……
- ✓ Light meson LCDAs: many research approaches available, LQCD+LaMET can provide a reliable prediction from the first principles;
- ✓ Heavy meson LCDAs: the first lattice-implementable scheme, which can continually be improved.

More importantly, improving the reliability of our results for the next stage……

Summary and outlook

Light meson LCDAs:

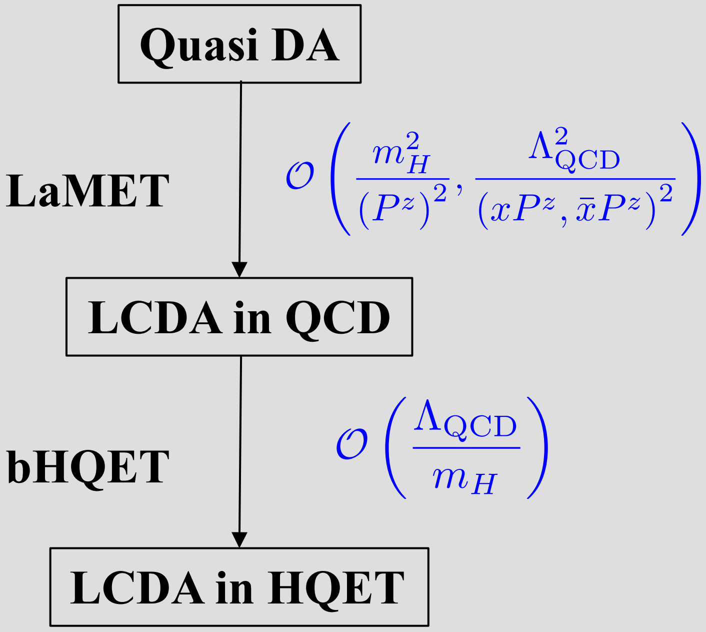
- The disagree between lattice OPE and LaMET ?
- More reliable predictions for end-point region of x:
  - Large P^z to suppress the power corrections;
  - Resummation, renormalon effects, ……
- More generalized distributions, as TMD-DA, ……



Summary and outlook

Heavy meson LCDAs:

- The most urgent is to properly control the power corrections within two step factorization:
  - Larger P^z to increase the window for lattice calculations;
  - Consider high power terms within matching?
- More systematic lattice QCD calculations:
  - Nonperturbative renormalization, continuum and physical mass extrapolation, operator mixing effects, ……
  - More reliable method to merge the peak and tail regions
- Realize the HQET quasi DA on lattice QCD directly?



Thanks

Backup slides

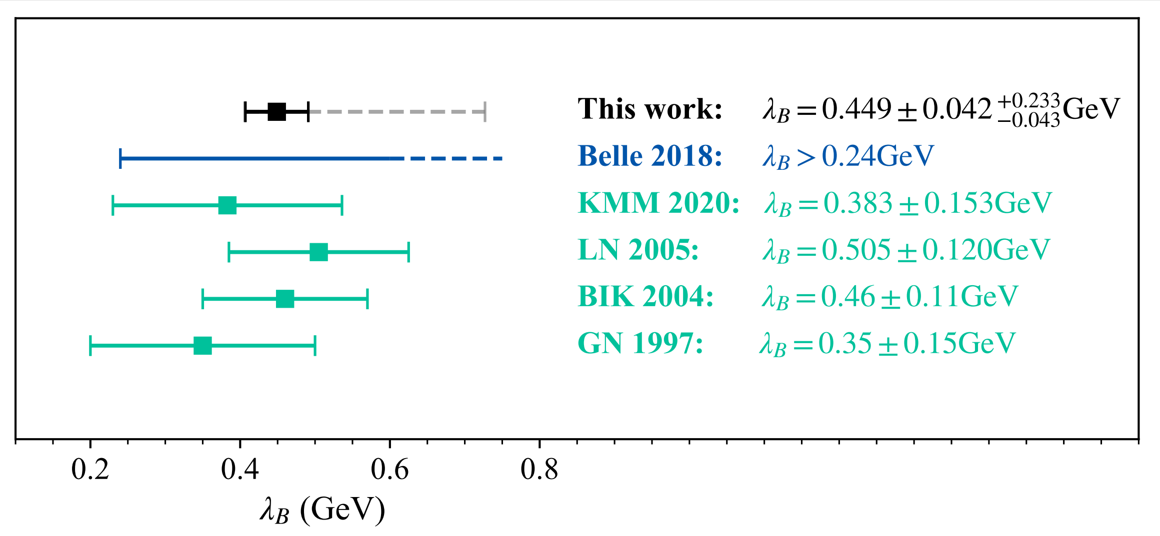
First inverse moment

➢ The first inverse moment

$$\lambda_B^{-1}(\mu) = \int_{-\infty}^{\infty} d\omega \frac{\varphi^{+}(\omega, \mu)}{\omega}$$

Models	I	II	III	IV	V
Parameters	$\omega_0 = 0.433(23)\text{GeV}$	$\omega_0 = 0.682(45)\text{GeV}$ $\sigma_B^{(1)} = 2.78(48)$	——	$\omega_0 = 0.427(21)\text{GeV}$	$\omega_0 = 0.449(42)\text{GeV}$
fit range	$\omega \in [0.2, 1.4]\text{GeV}$	$\omega \in [0.2, 1.4]\text{GeV}$		$\omega \in [0.4, 0.8]\text{GeV}$	$\omega \in [0.2, 1.4]\text{GeV}$
χ^2/d.o.f	1.4	1.2		2.1	1.0

- The current numerical results are unable to accomplish the integration over full-ω range;
- We determine the λ_B^{-1} by fitting the parameterization forms of different model.



PRD98,112016(2018), JHEP10,043(2020), PRD72,094028(2005), PRD69,034014(2004), PRD55,272(1997)