Simple Hadron vs Complex QCD

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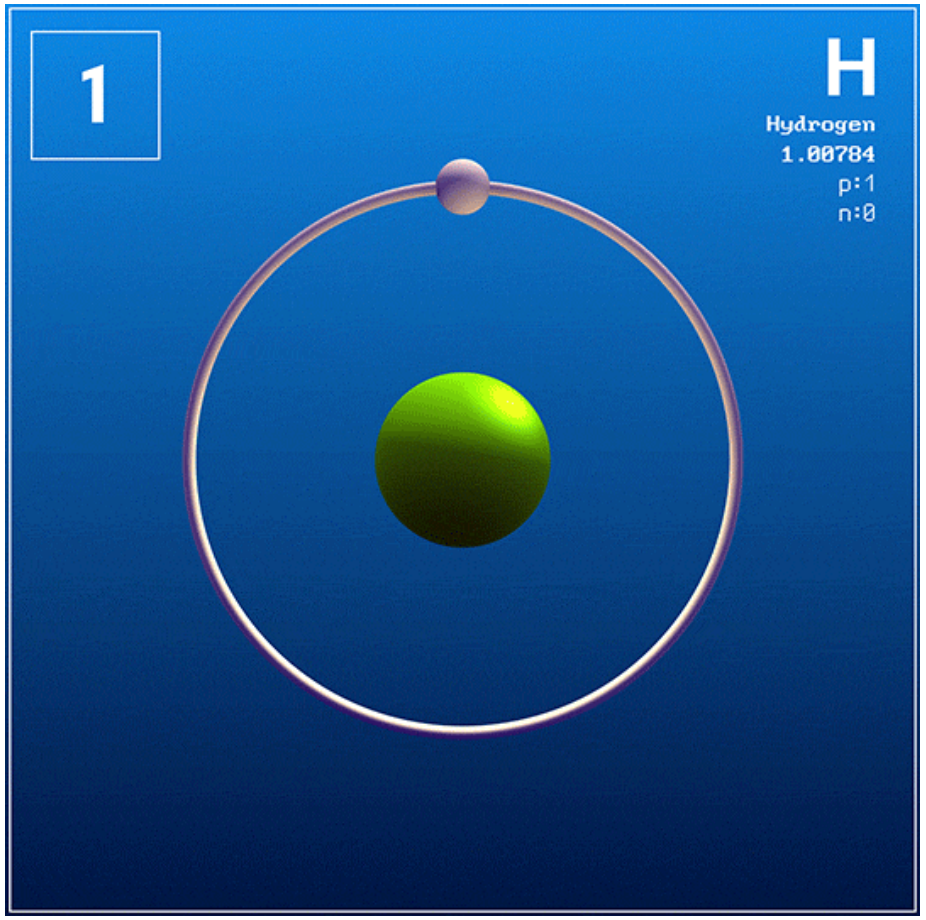
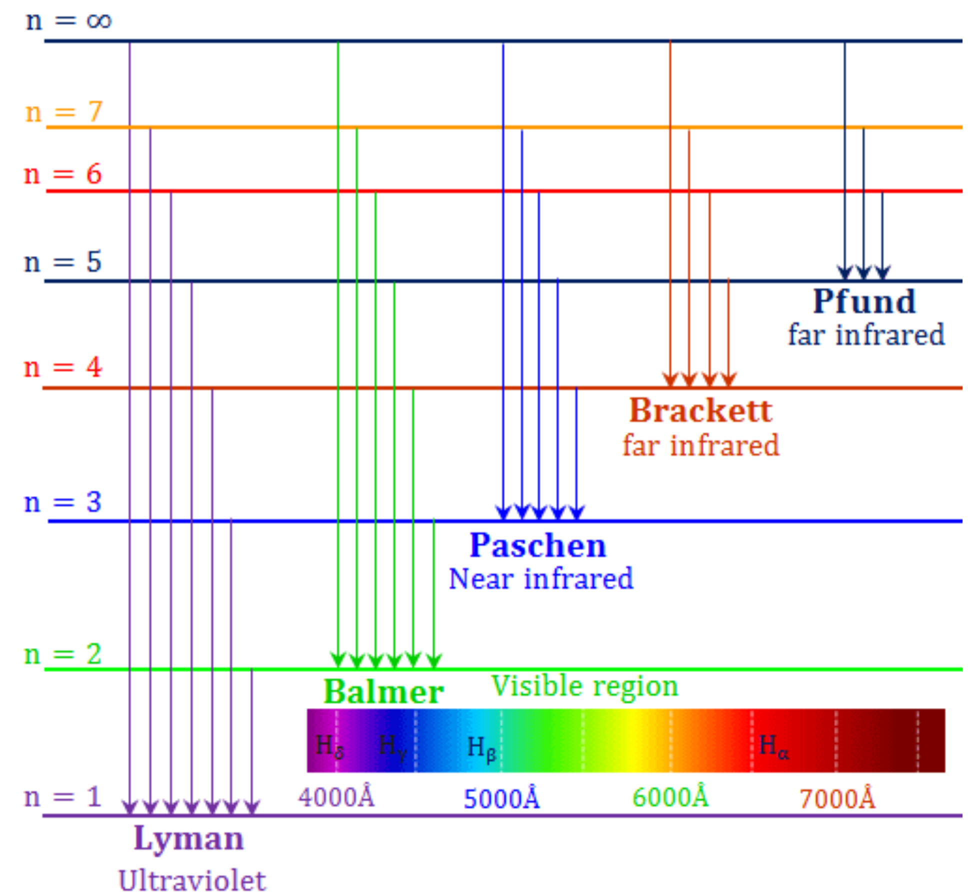
History: Hydrogen atom — The simplest element





$$H\,|\psi_n\rangle = E_n\,|\psi_n\rangle$$

$$H = H_{\text{kinetic}} + H_{\text{Coulomb}}$$

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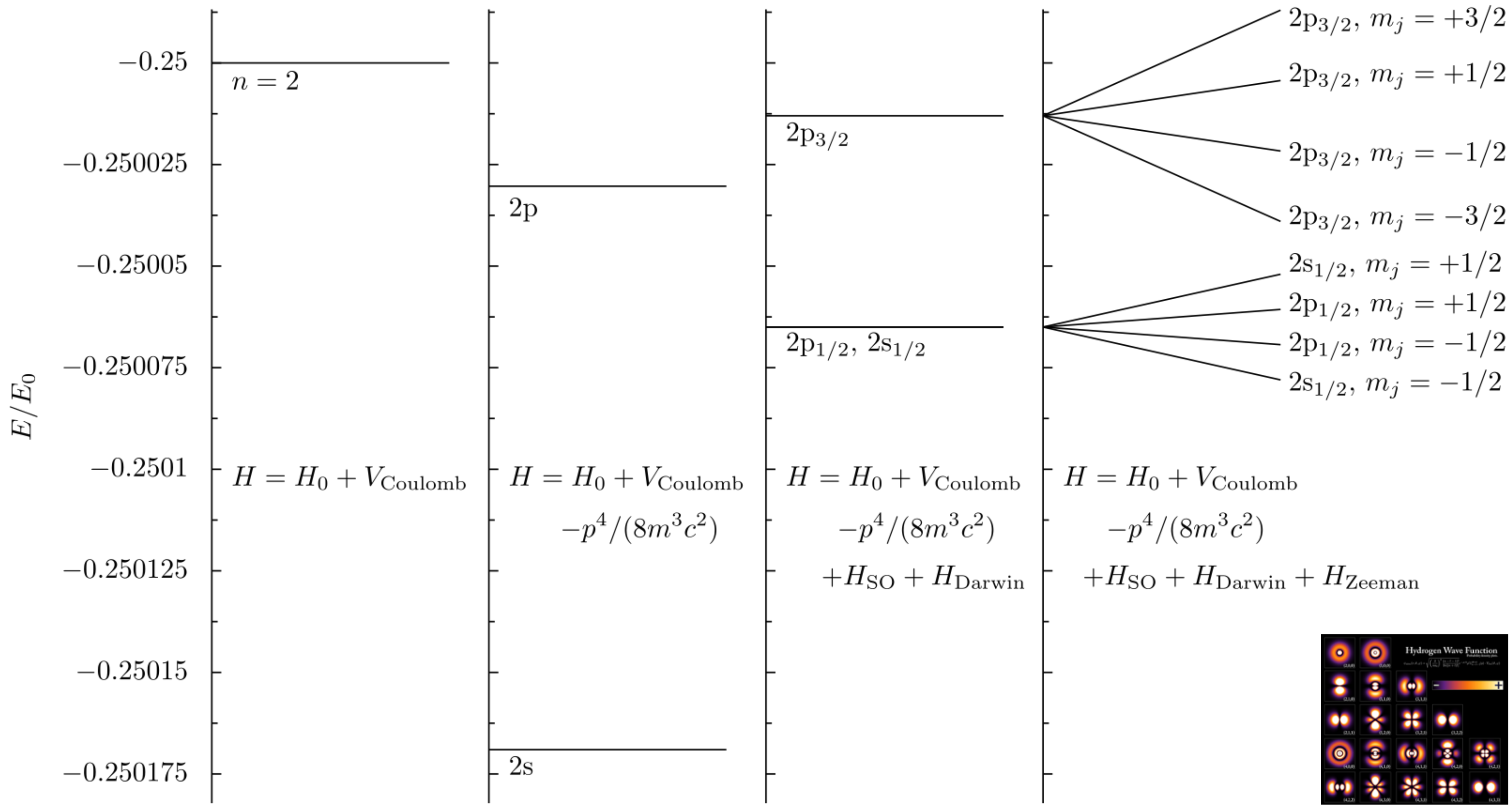
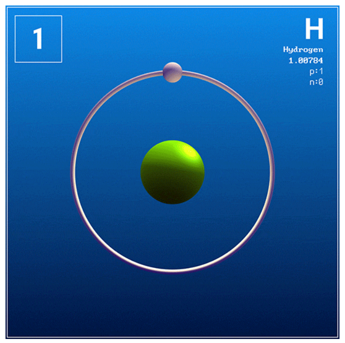
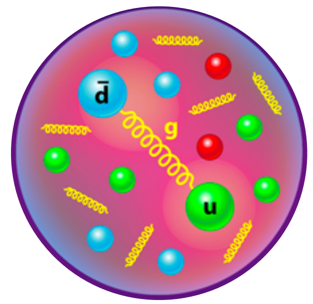
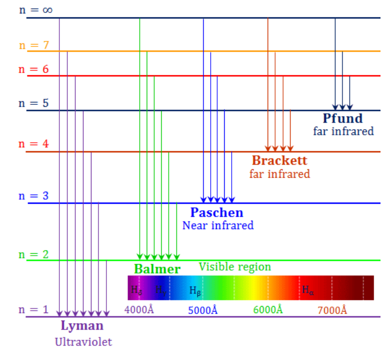


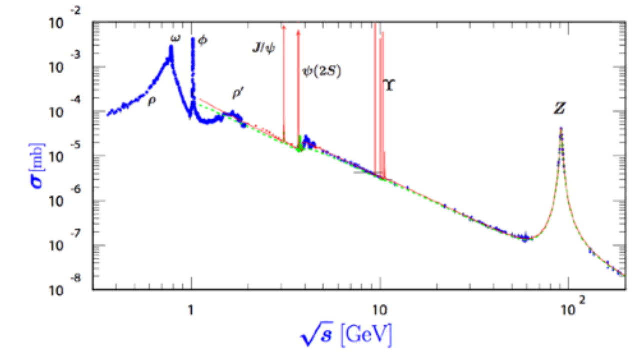
$$\boxed{H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{QED}}}$$

Background: Hadron — The QCD “atom”









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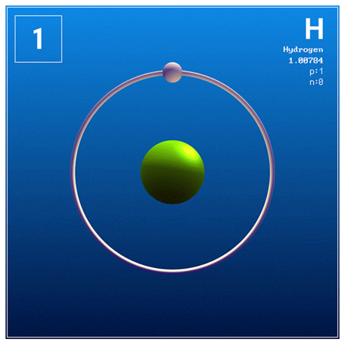
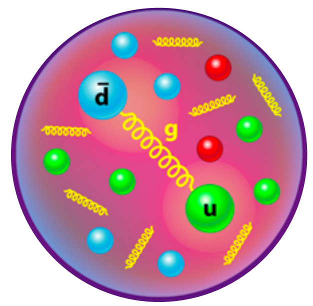
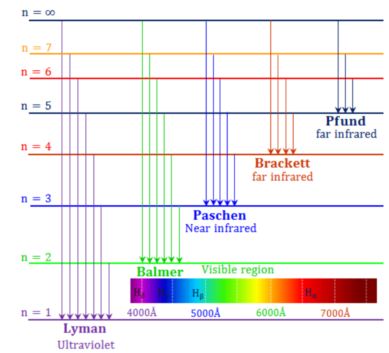
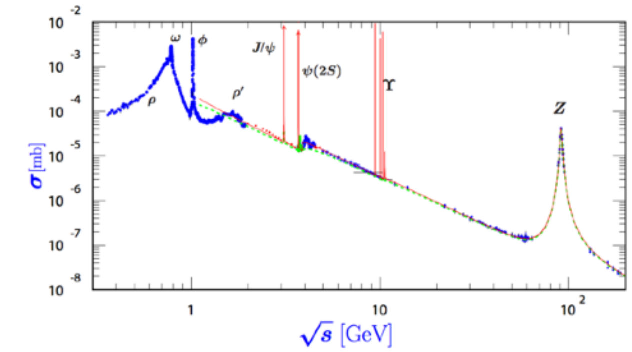




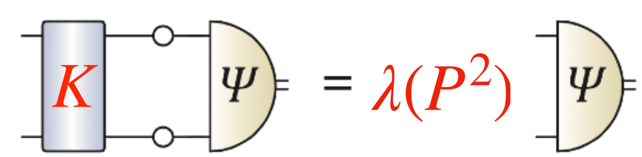




QM: Schrödinger equation

$$H\,|\psi\rangle = E\,|\psi\rangle$$

QFT: Bound-state equation



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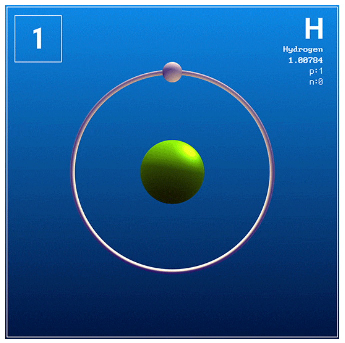

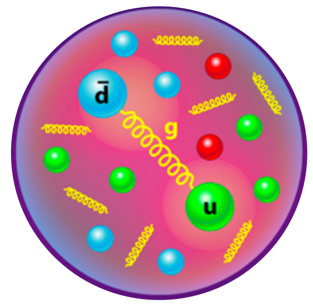

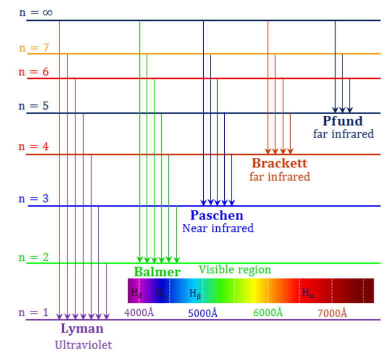

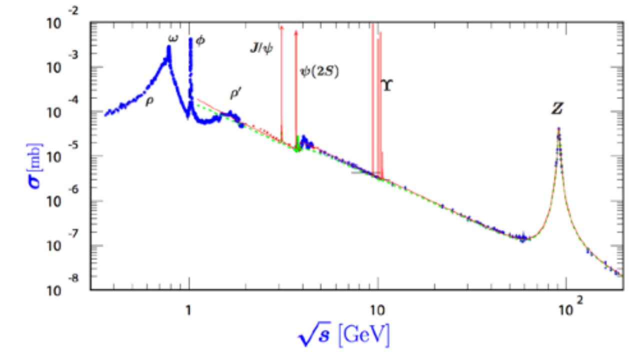

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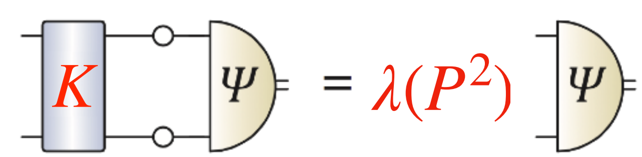


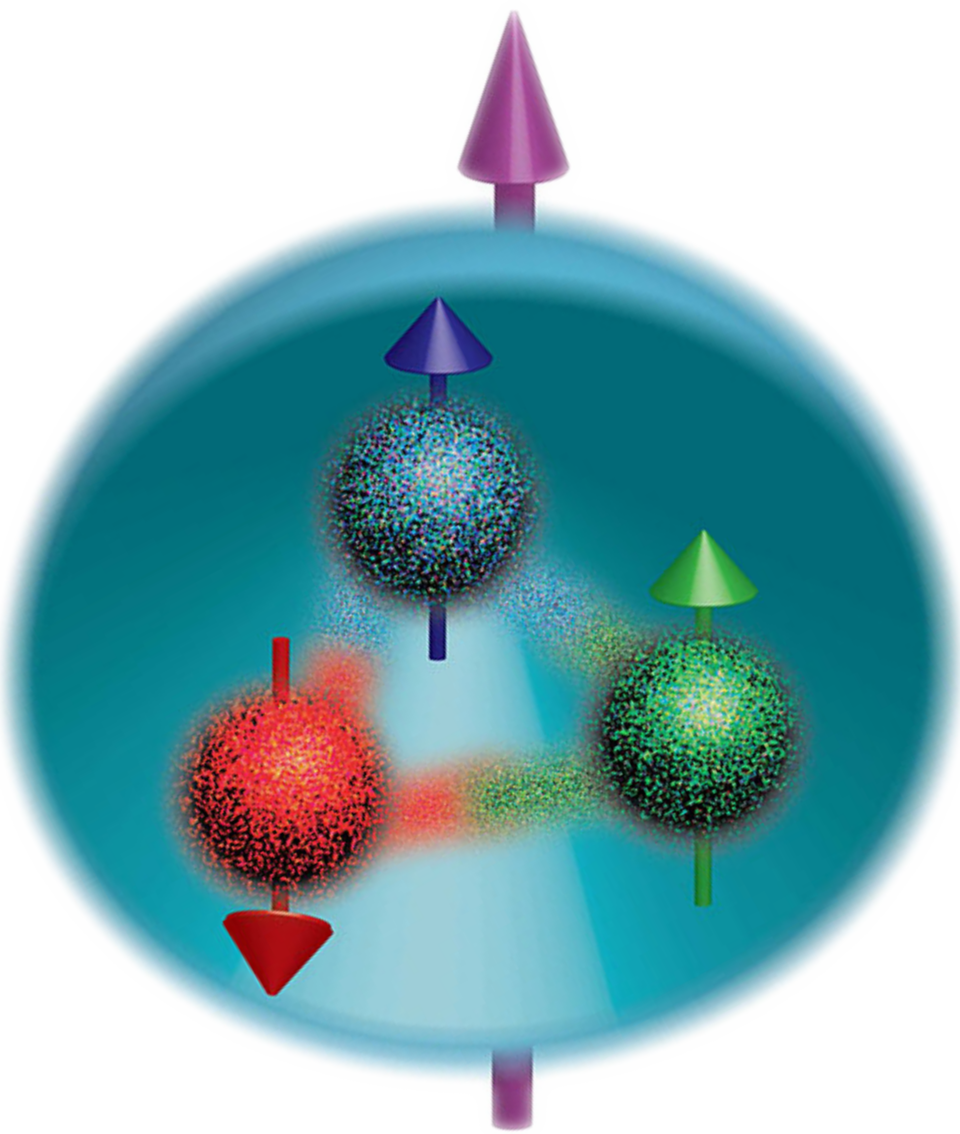
✦ **Theories**: Simple (few-body) objects could involve surprisingly rich physics.

✦ **Experiments**: High-precision measurements could make the story very different.

Chapter I: Theory

Physics of quark, gluon, vertex, and kernel

Continuum QCD: Interaction between quarks



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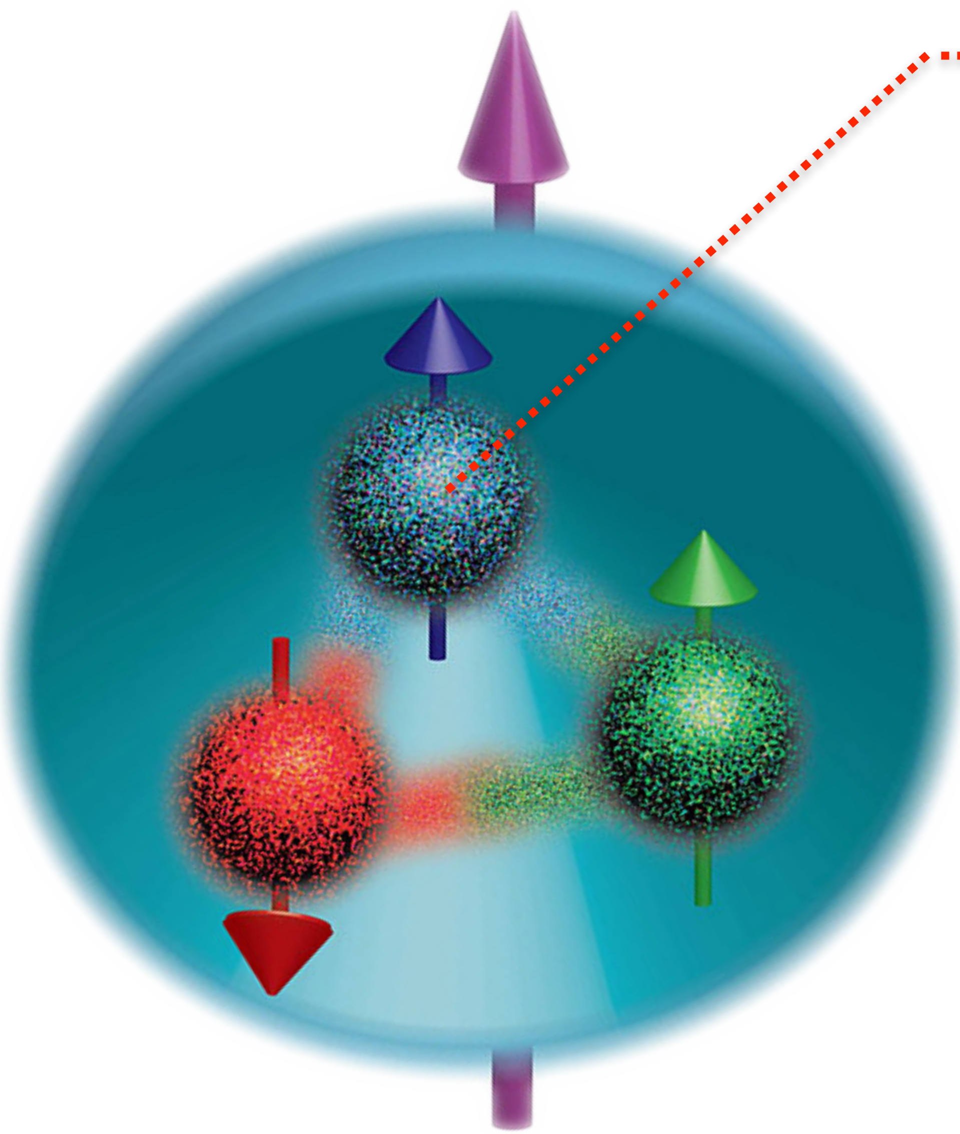

Quasi-particle quark

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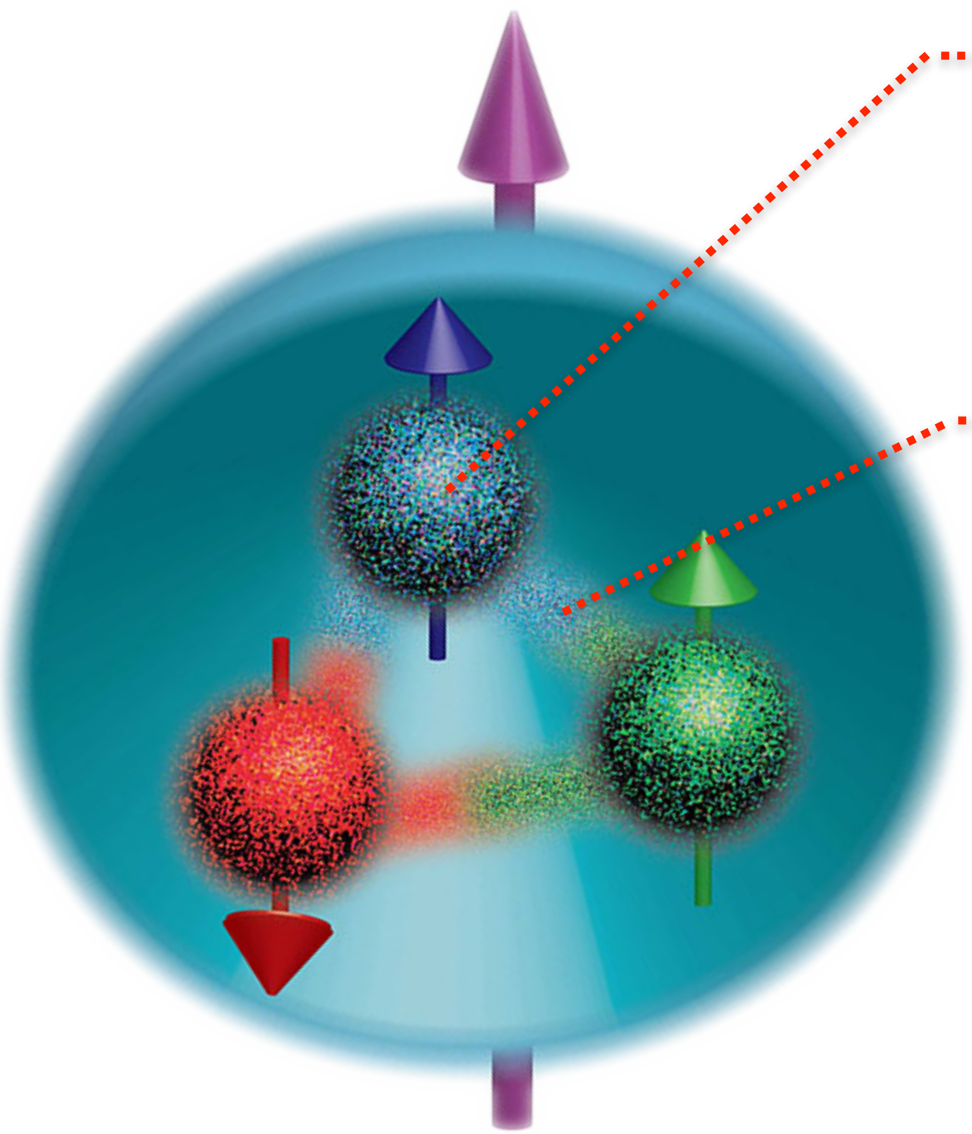

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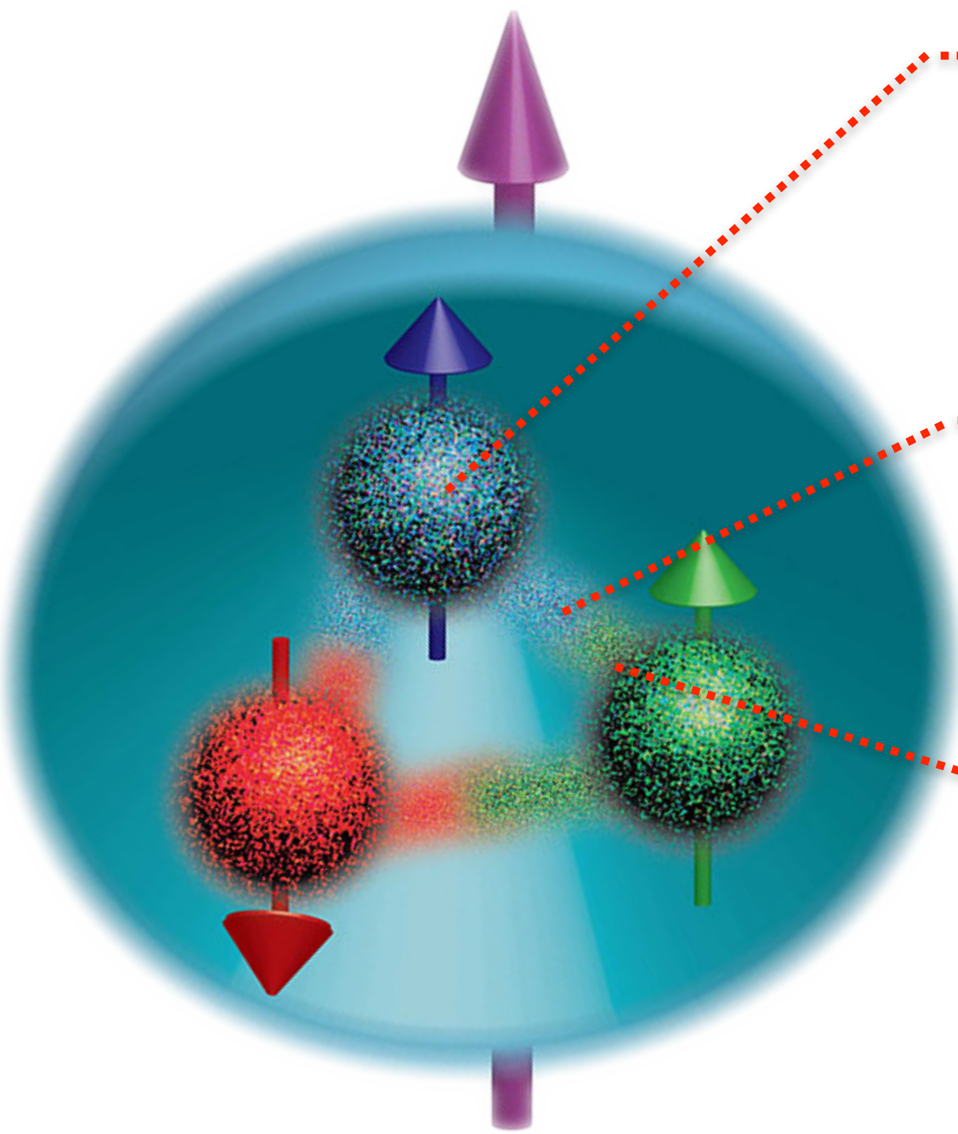

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Quasi-particle gluon

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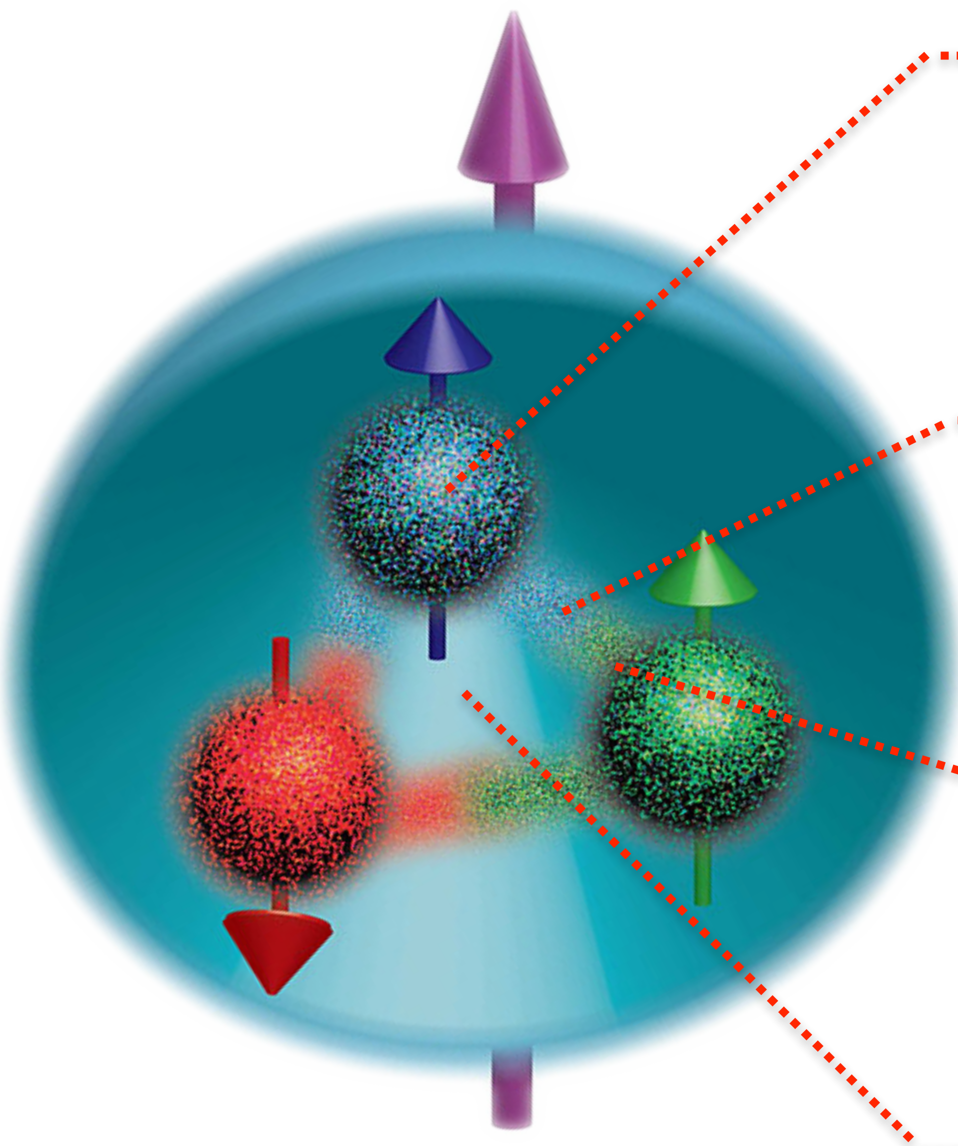

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Ways of gluon-exchange

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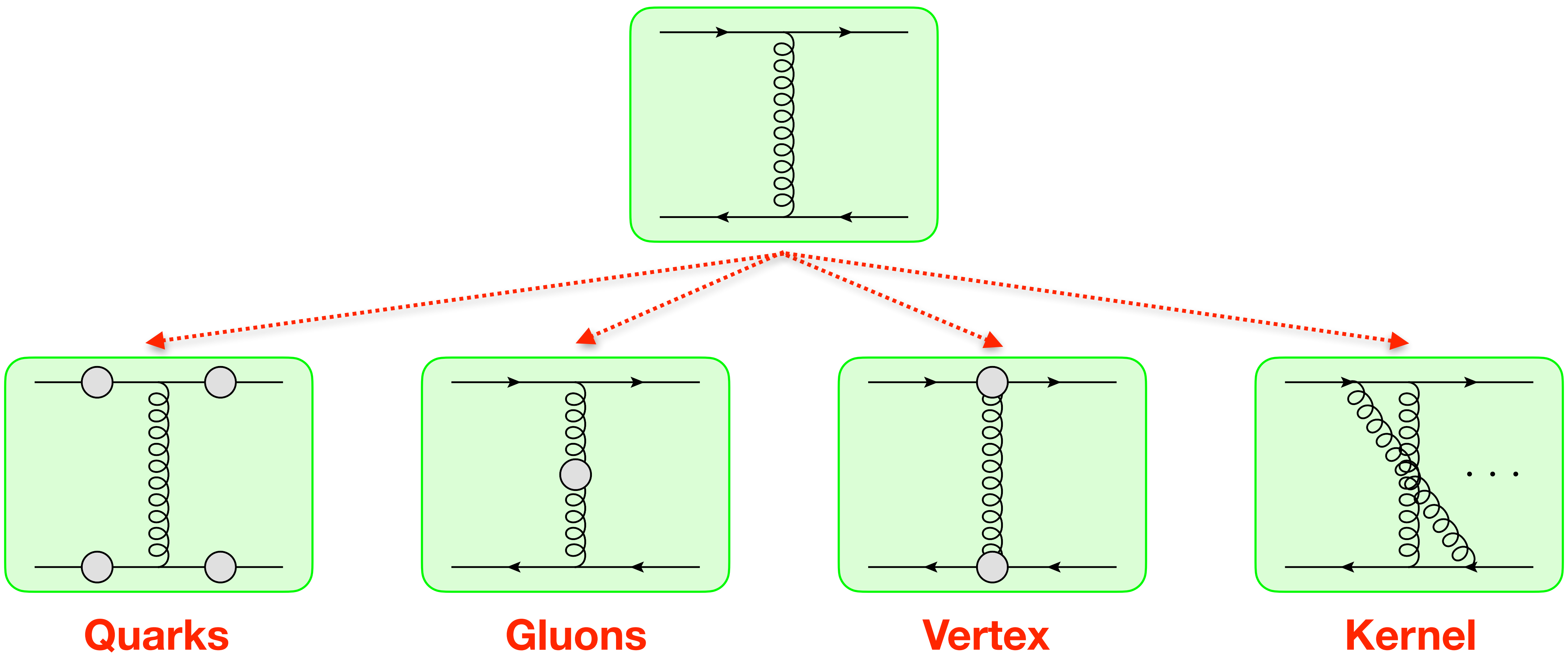

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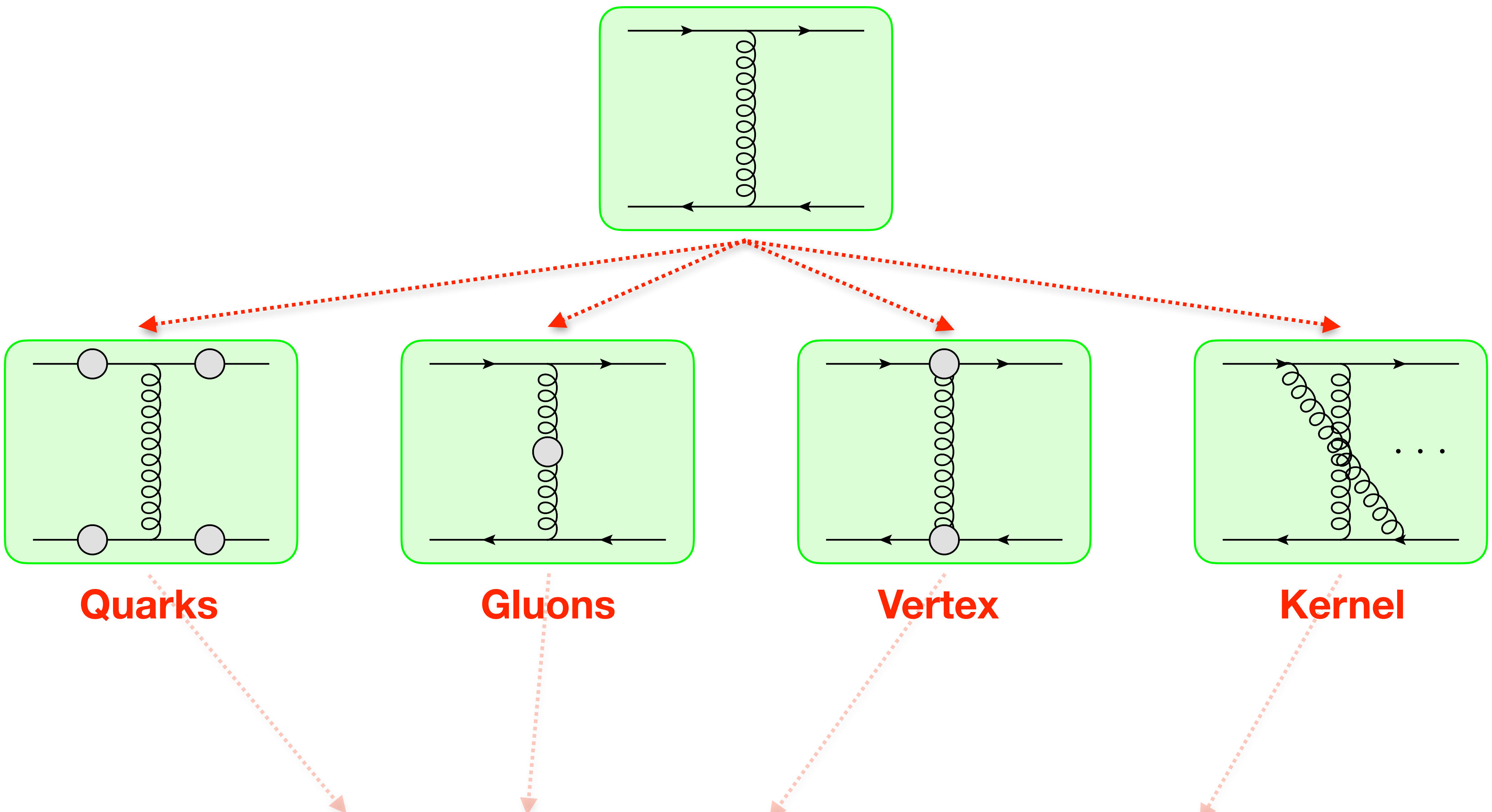

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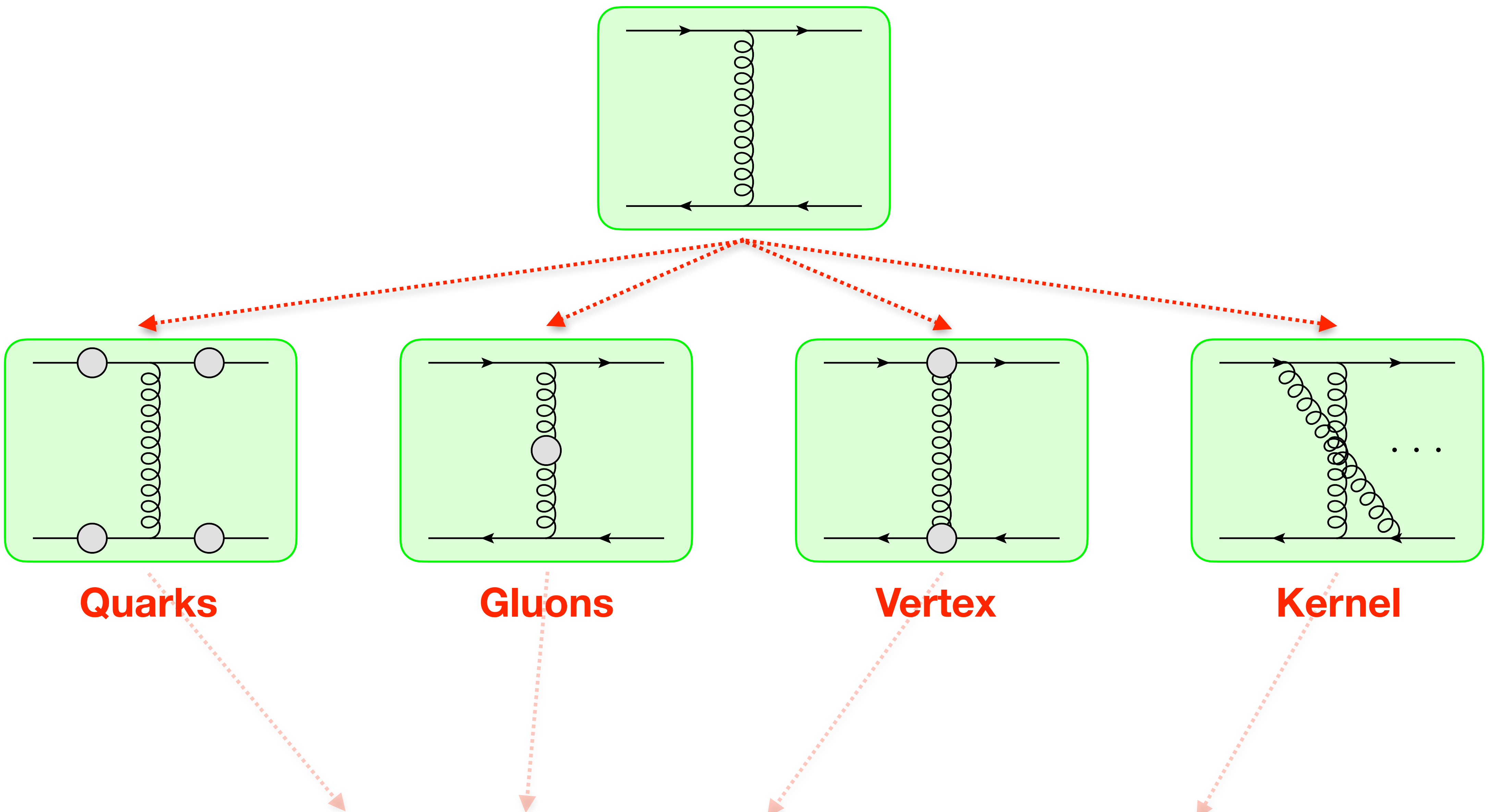

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Ground states — **Excited states** → M

Continuum QCD: Equations of motion

Principle of Least Action

$$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

Dyson-Schwinger Equations

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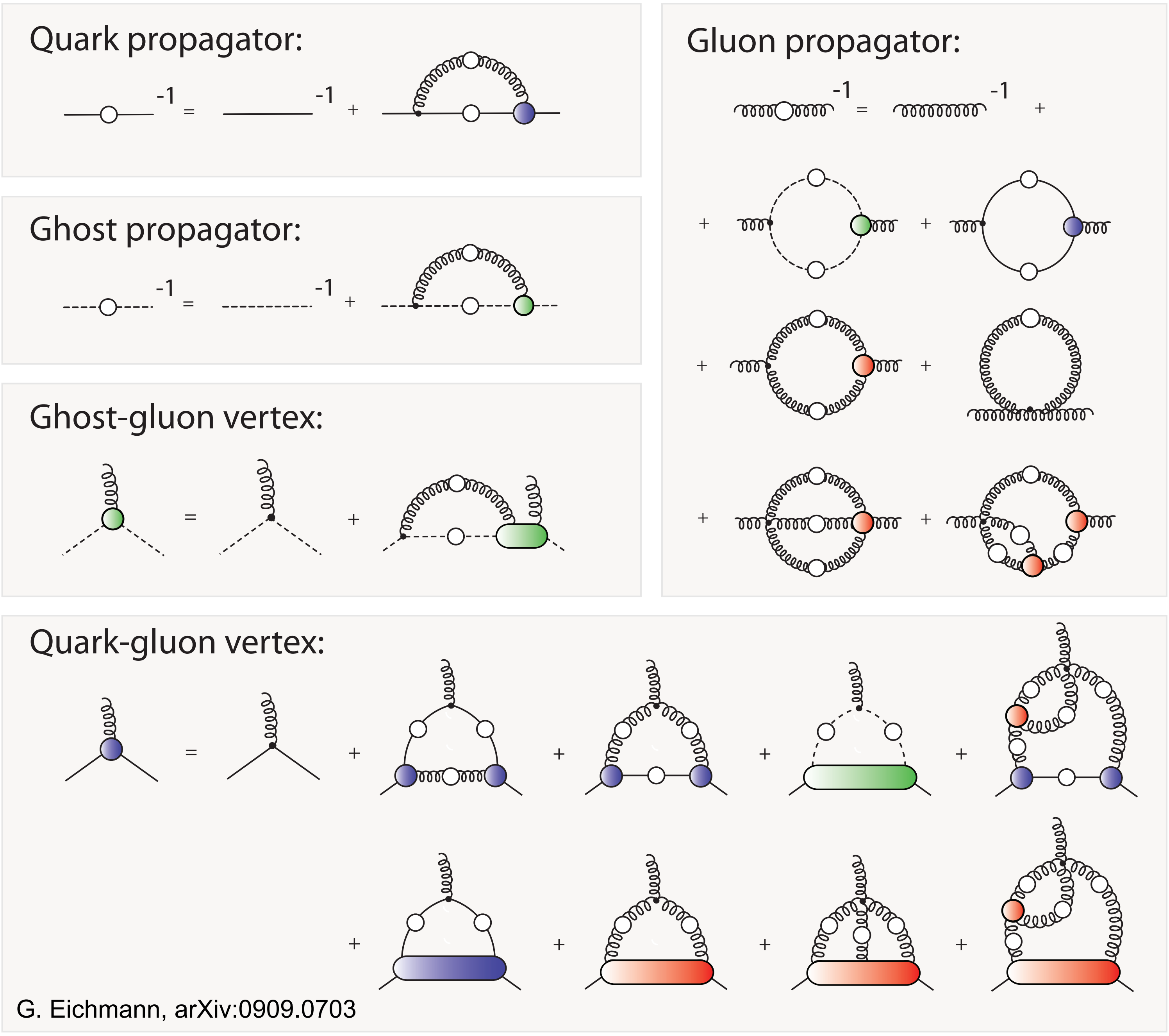

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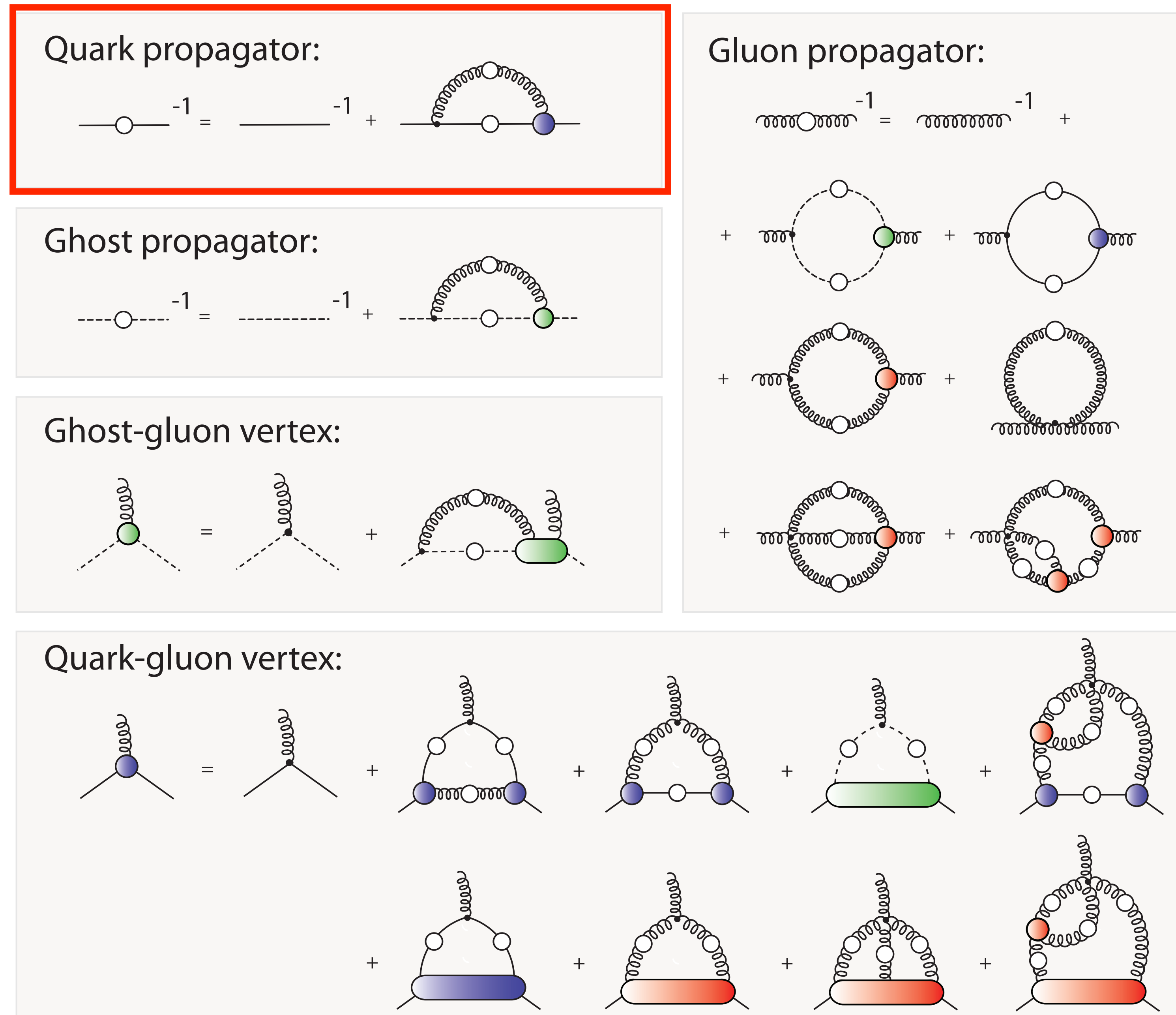
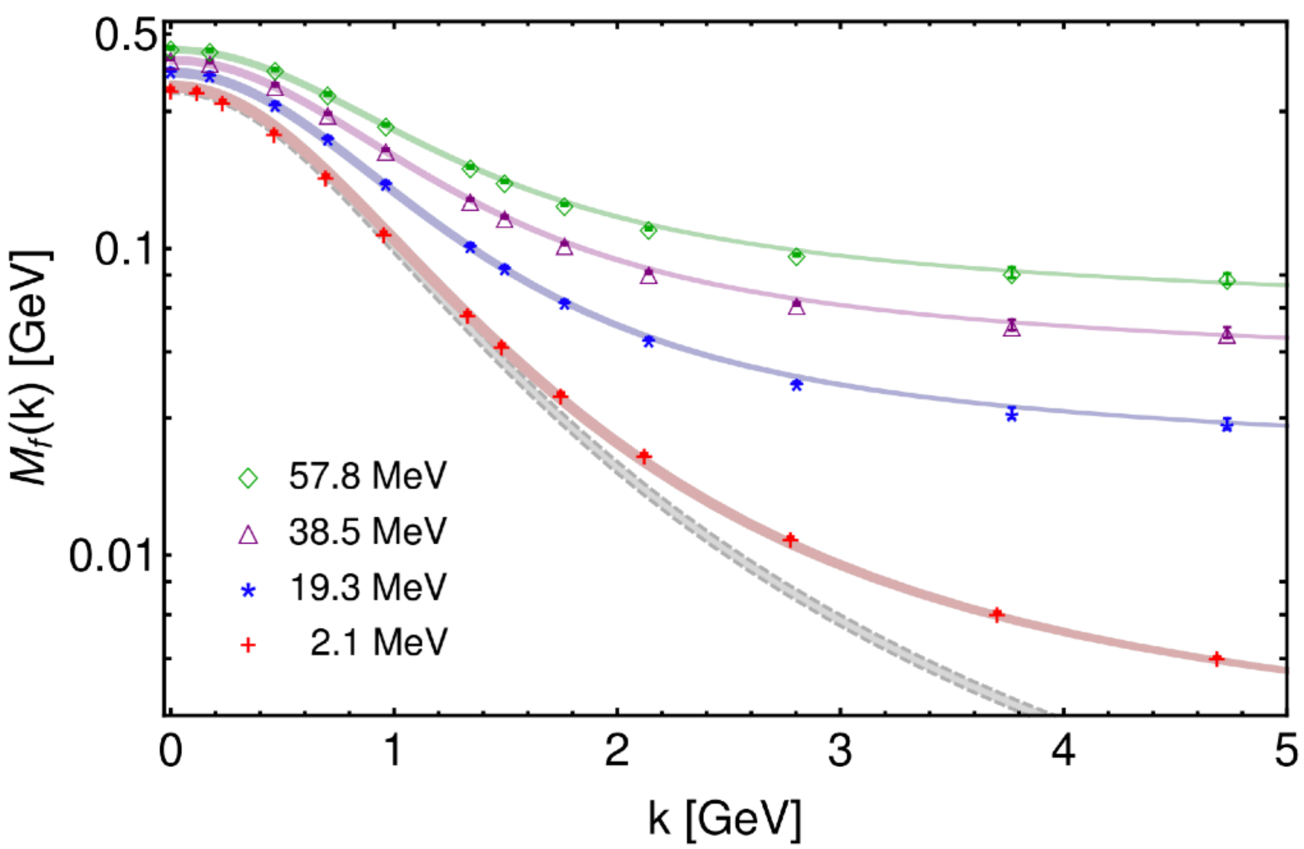


G. Eichmann, arXiv:0909.0703

Quark: Running mass function

$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Chang, Yang, et. al., PRD 104, 094509 (2021)



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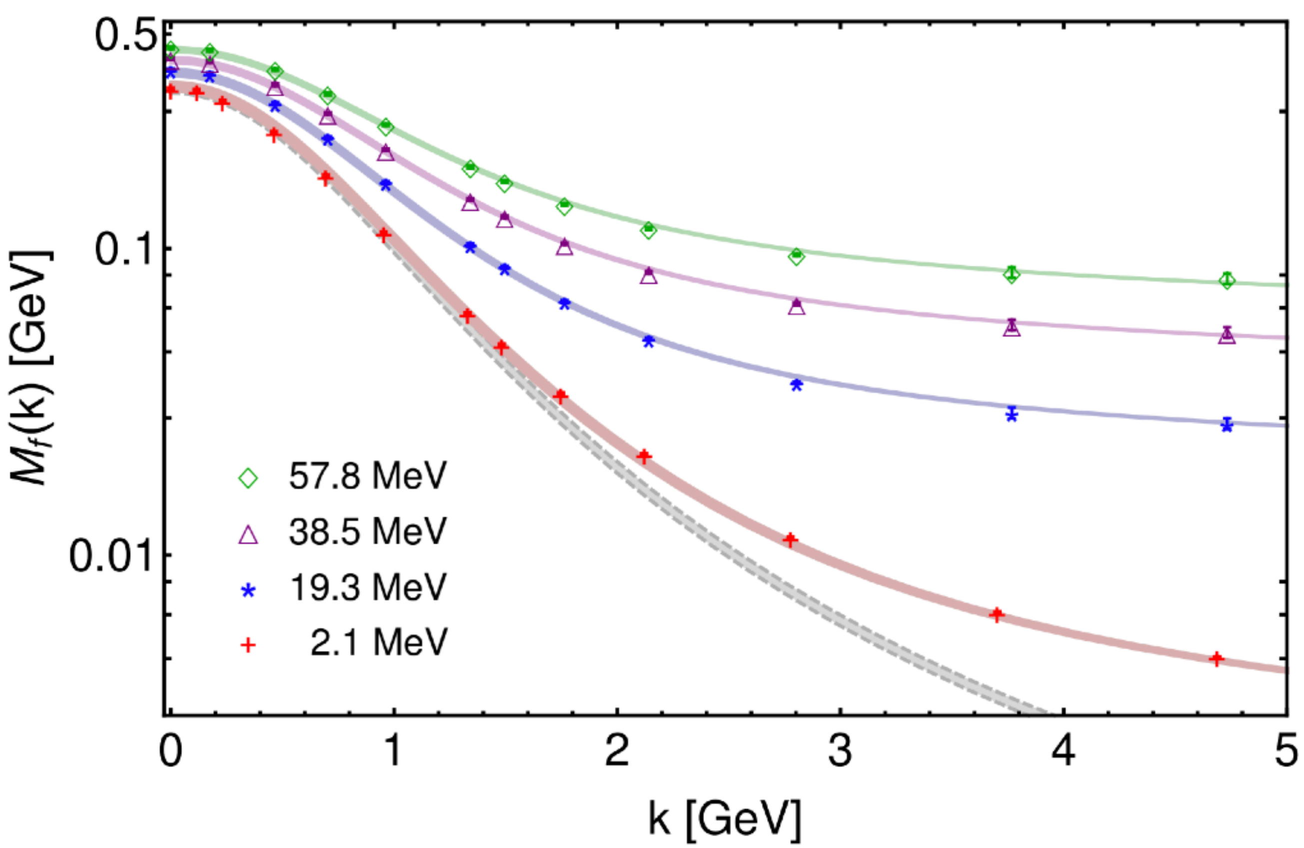

✦ **Now:**

1. The quark's **effective mass** runs with its momentum.

2. The most **constituent mass** of a light quark comes from a cloud of gluons.

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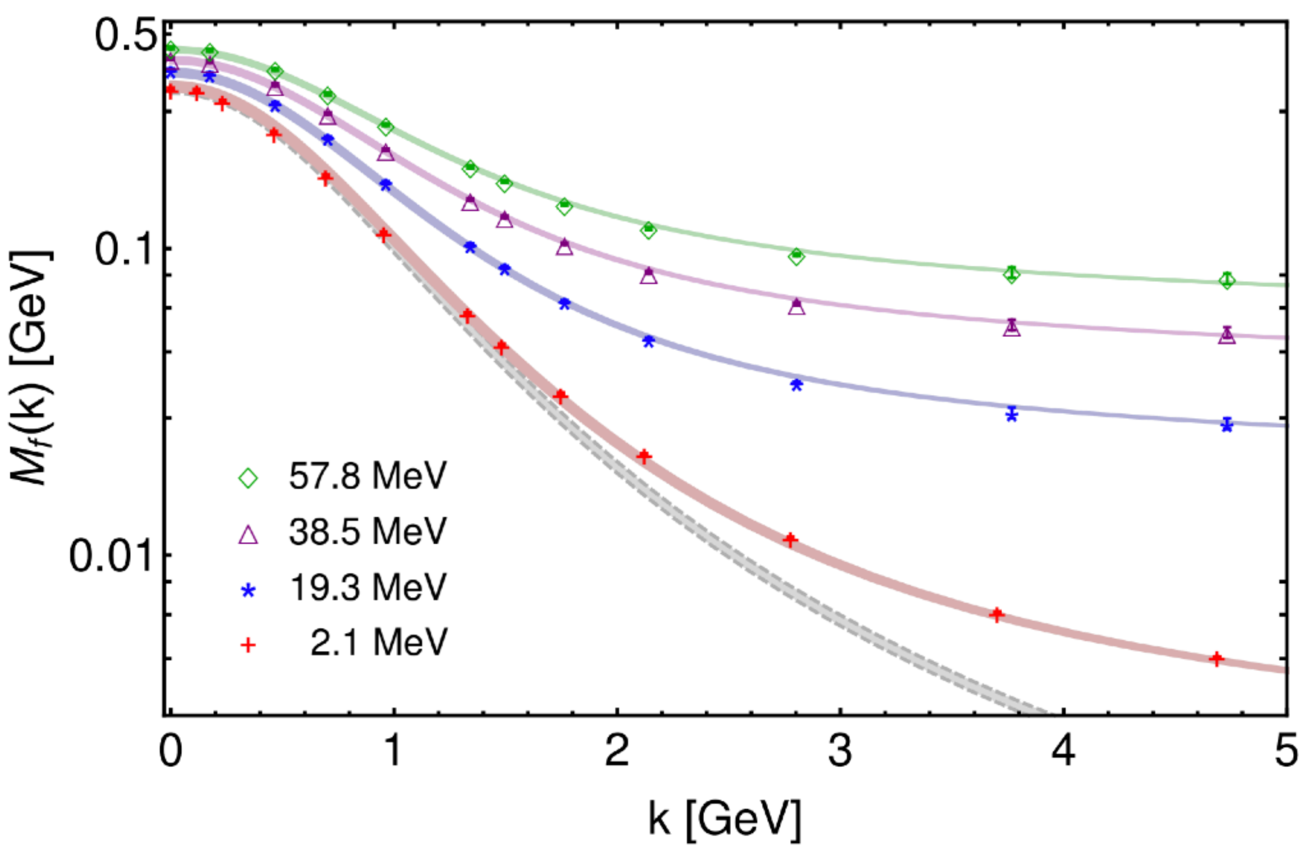

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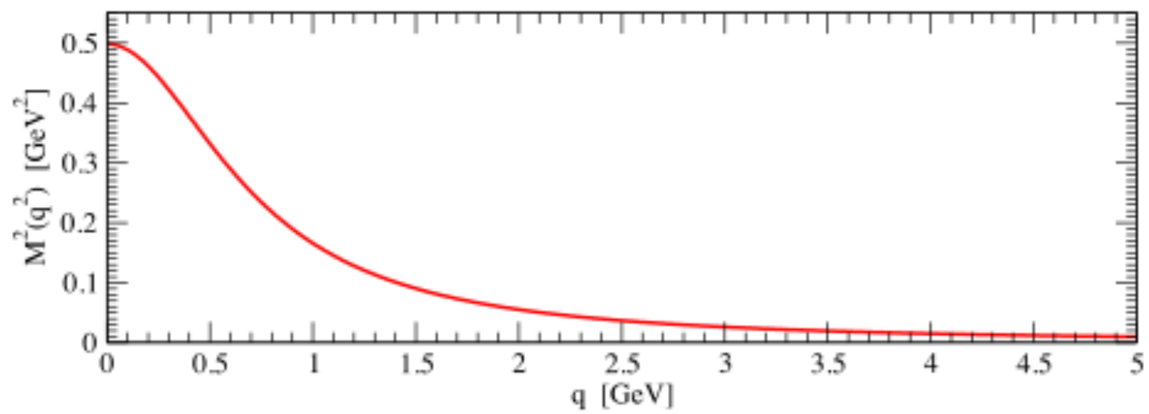
✦ **Next:**

1. What is the **infrared scale** of quark mass function?

2. How does the **transition** connect the non-perturbative and perturbative regions?

Gluon: Dynamical mass scale

Gluon mass function: O. Oliveira et. al., J.Phys. G38, 045003 (2011)



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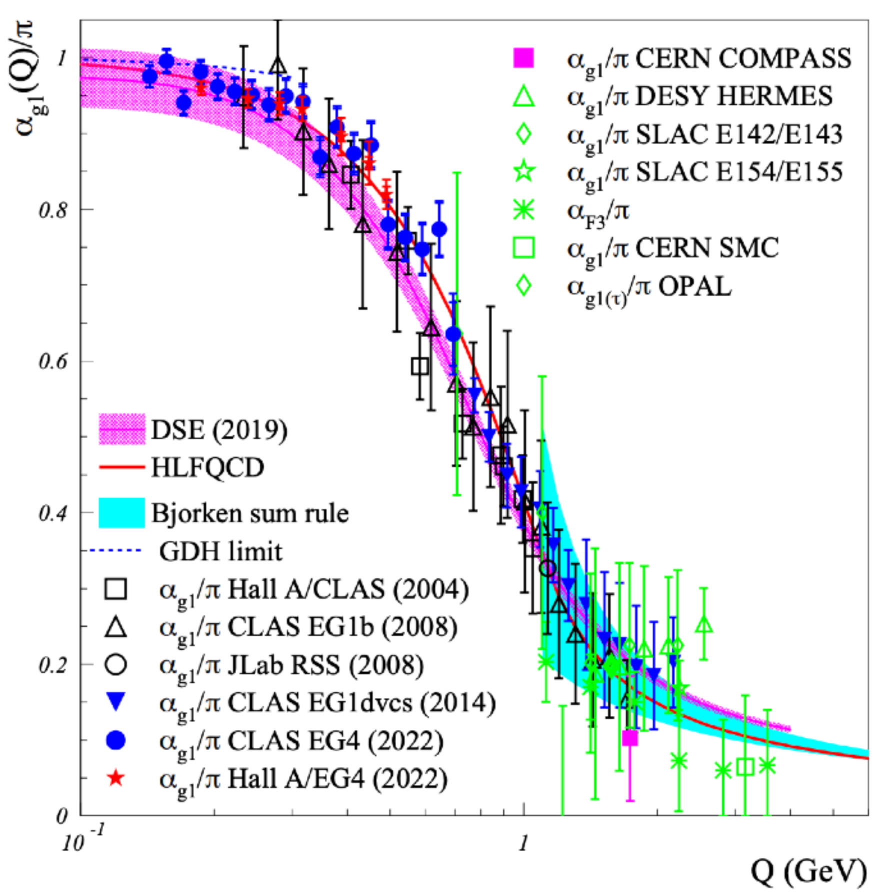

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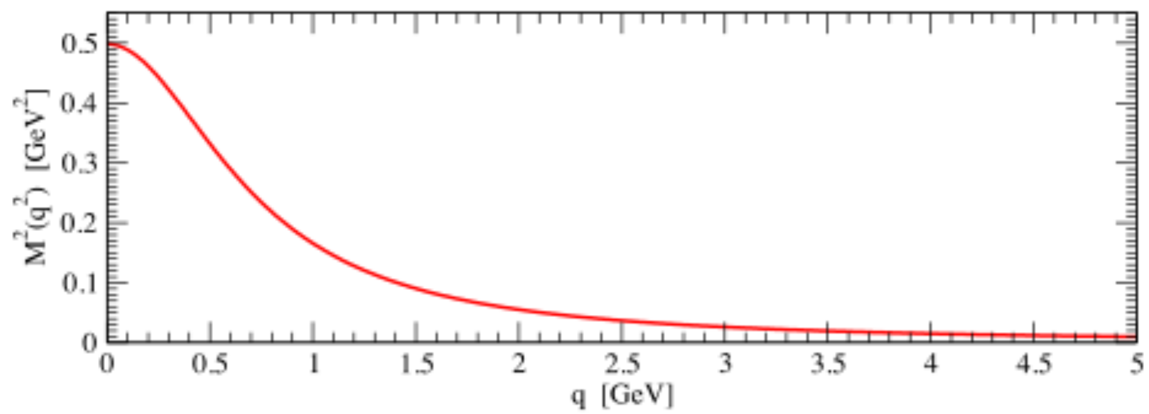

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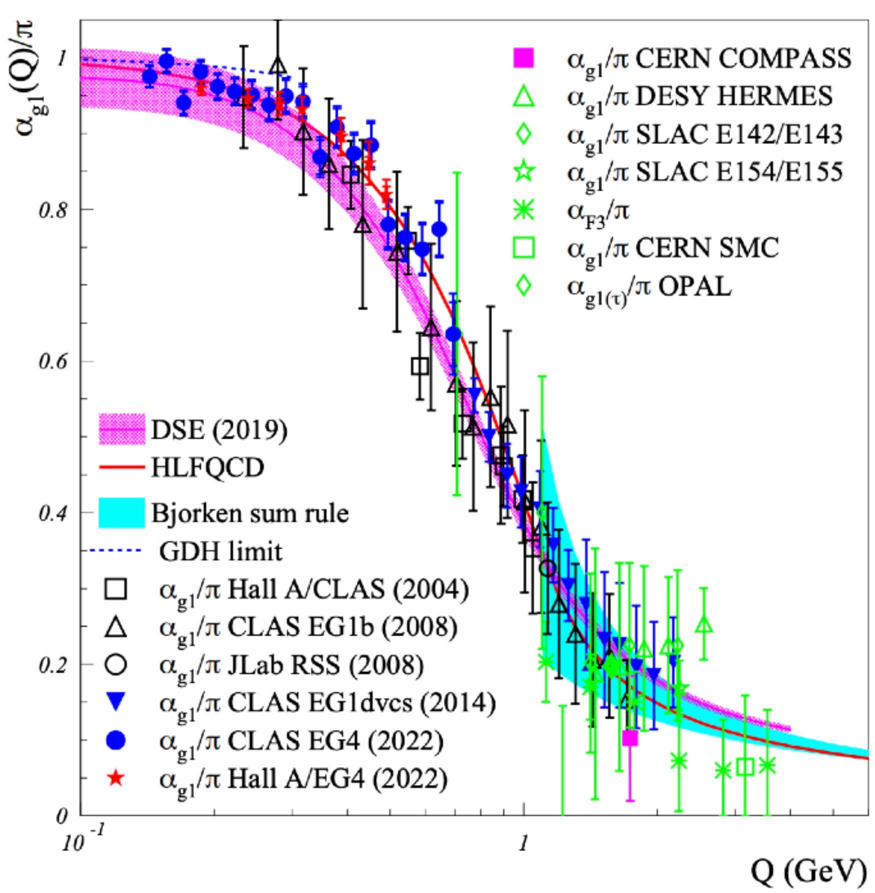

✦ **Now:**

1. The dressed gluon can be well parameterized by a **mass scale**

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2}$$

2. The effective running coupling **saturates** in the infrared limit.

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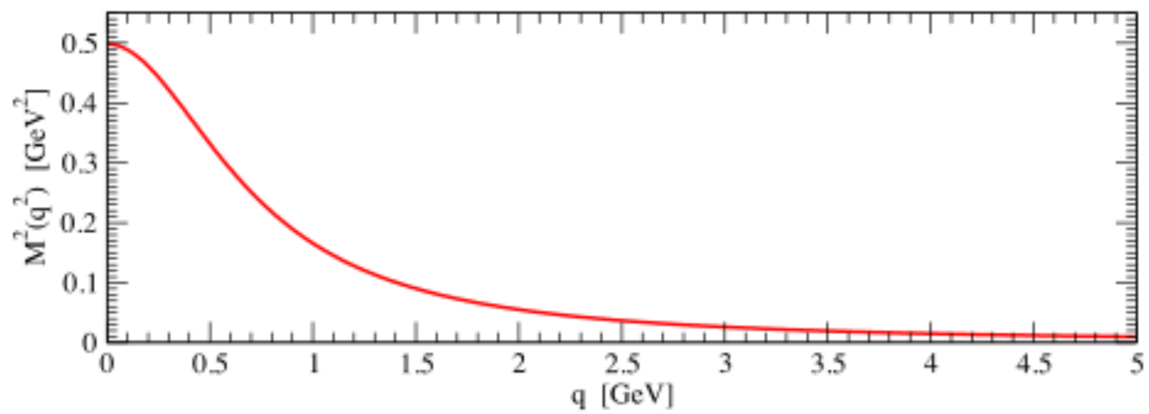

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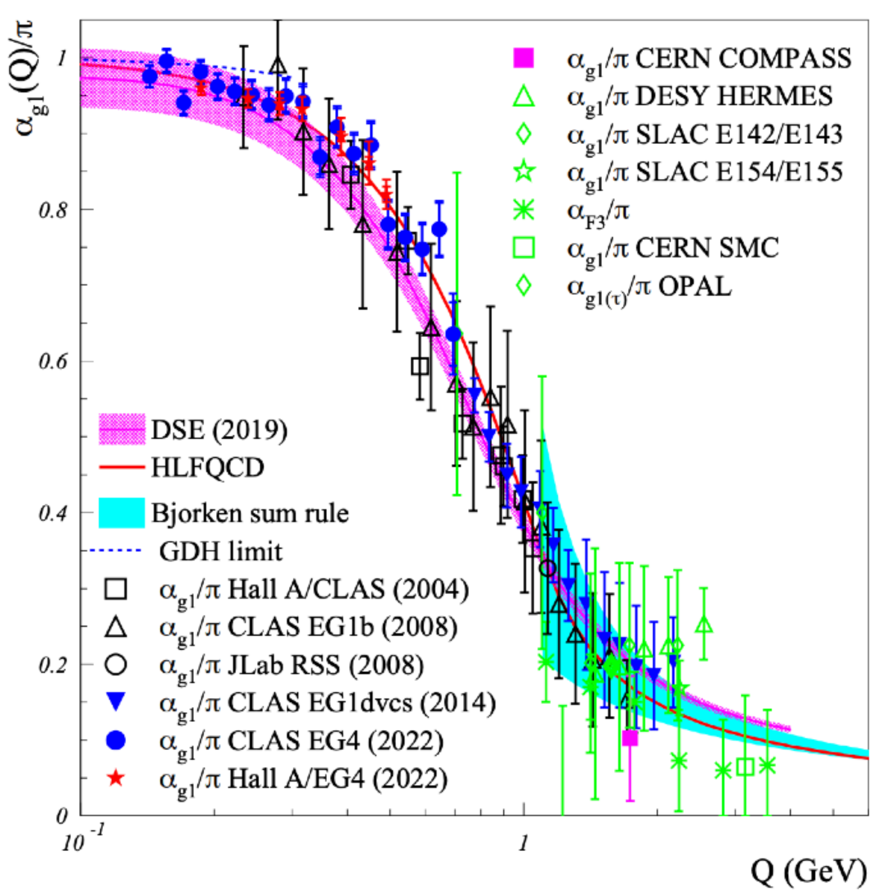

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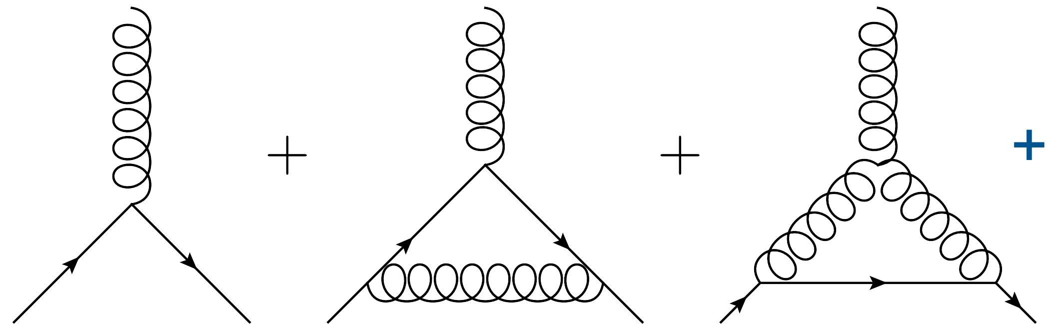
✦ **Next:**

1. What is the **mass scale** of gluon?
2. What is the **infrared magnitude** of running coupling?

Vertex: DCSB feedback

Quark-gluon vertex:



\+ … = ?

See, e.g., PLB722, 384 (2013)

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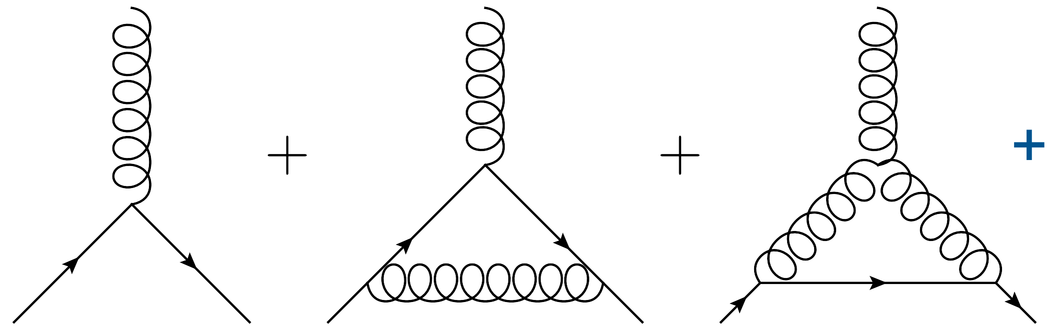

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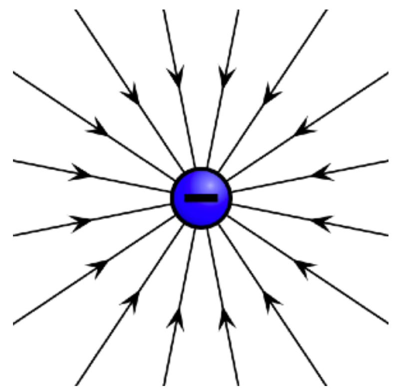
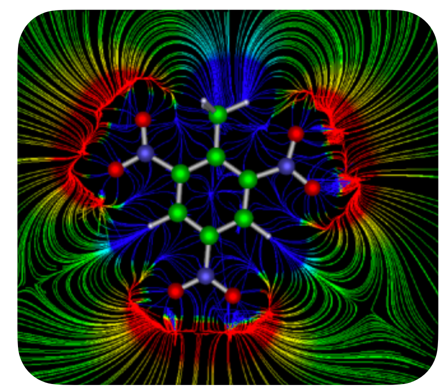
point charge





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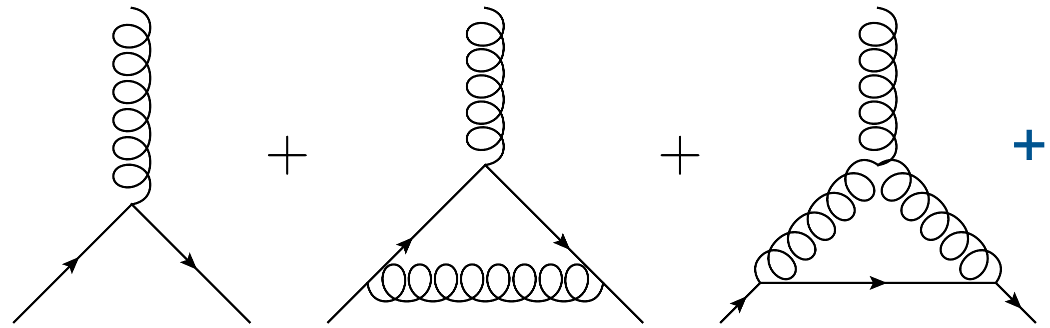

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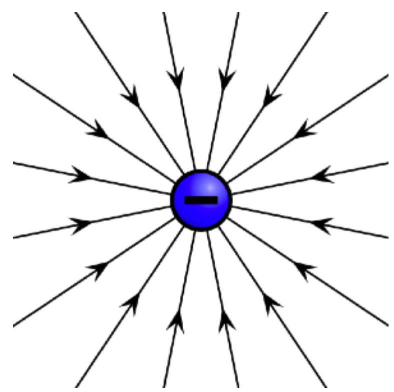

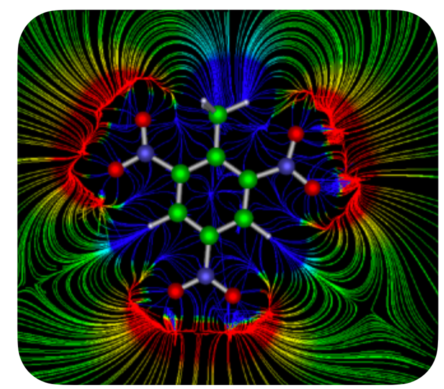

distributed charge

✦ The **Dirac** and **Pauli** terms: for an on-shell fermion, the vertex can be decomposed by two form factors:

$$\Gamma^{\mu}(P',P) = \gamma^{\mu}F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f}Q^{\nu}F_2(Q^2)$$

✦ The form factors express (color-)**charge** and (color-)**magnetization** densities. And the so-called **anomalous magnetic moment** is proportional to the Pauli term.

See, e.g., PLB722, 384 (2013)

Vertex: DCSB feedback

Equation of Motion (DSEs)

$$S = \frac{1}{i\gamma \cdot p A(p^2) \ + \ B(p^2)}$$

$$\gamma^\mu \Sigma_A \ + \ i\sigma_{\mu\nu} Q^\nu \Delta_B \ + \ \text{etc.} \sim \Gamma^\mu$$

Ward identities (Symmetries)

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✦ **Now:**

1. There is a dynamic chiral symmetry breaking (**DCSB**) **feedback**.

2. The **appearance** of the vertex is dramatically modified by the **dynamics**.

See, e.g., PLB722, 384 (2013)

Vertex: DCSB feedback

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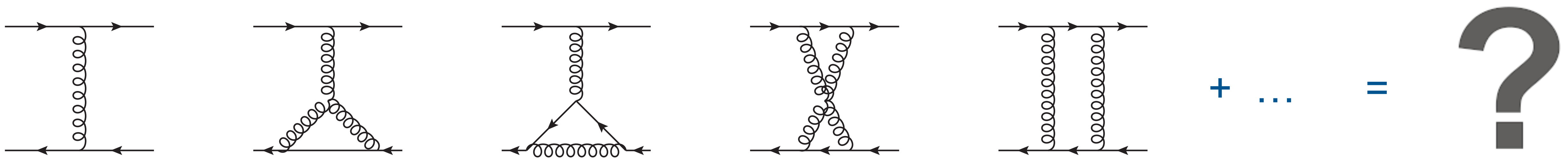
✦ Next:

1. What are the exact **strengths** of the terms in the vertex?

2. What the exact **behaviors** of the form factors in the vertex?

See, e.g., PLB722, 384 (2013)

Kernel: Twofold role of pion



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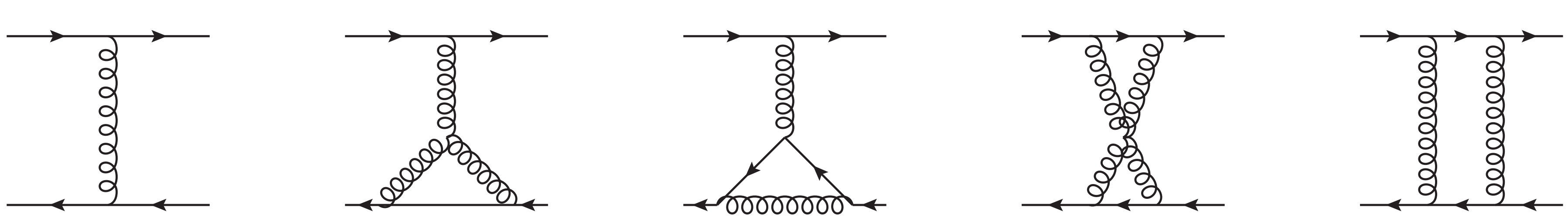
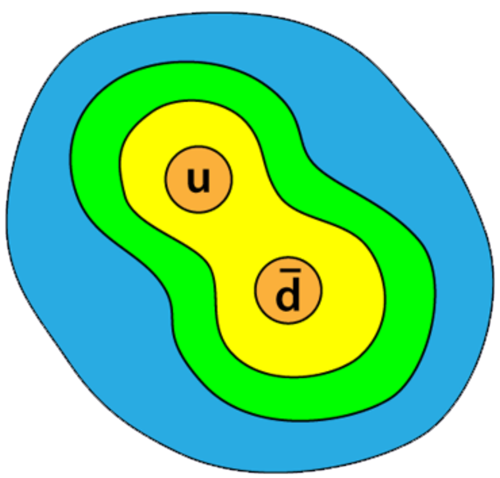


$+ \ \ldots \quad = \quad ?$



Pion

- **Bound state** of quark and anti-quark, but abnormally light:

$$M_\pi \ll M_u + M_{\bar{d}}$$

- **Goldstone's theorem:** If a generic continuous symmetry is spontaneously broken, then new massless scalar particles appear in the spectrum of possible excitations.

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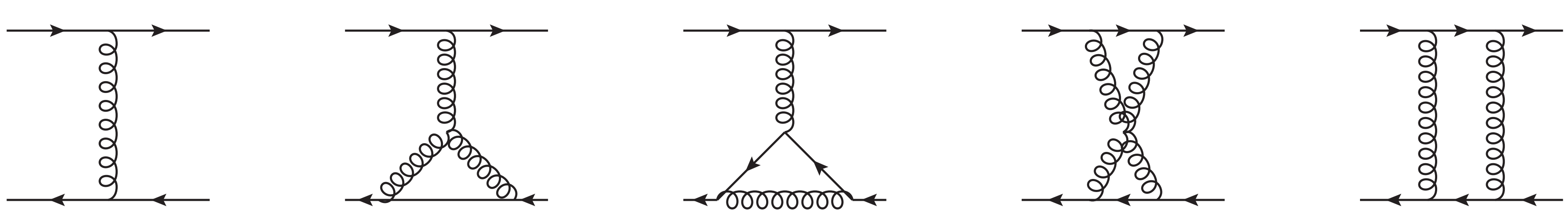

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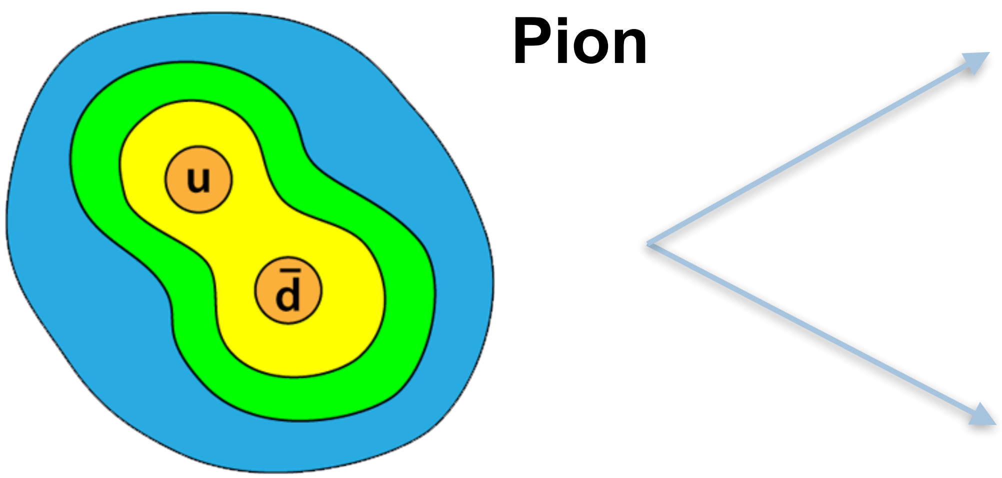

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- **Goldstone's theorem:** If a generic continuous symmetry is spontaneously broken, then new massless scalar particles appear in the spectrum of possible excitations.

- The **discrete** and **continuous** **symmetries** strongly constrain the kernel:

Poincaré symmetry **C-, P-, T-symmetry**	**Gauge symmetry** **Chiral symmetry**

Kernel: Twofold role of pion

✦ **Now:**

1. A deep connection between one-body and two-body problem:

$$\boxed{f_\pi E_\pi(k^2) = B(k^2)}$$

Pion exists if, and only if, the **quark mass** is dynamically generated.

Two-body problem solved, almost completely, once solution of **one-body** problem is known.

See, e.g., CPL 38 (2021) 7, 071201

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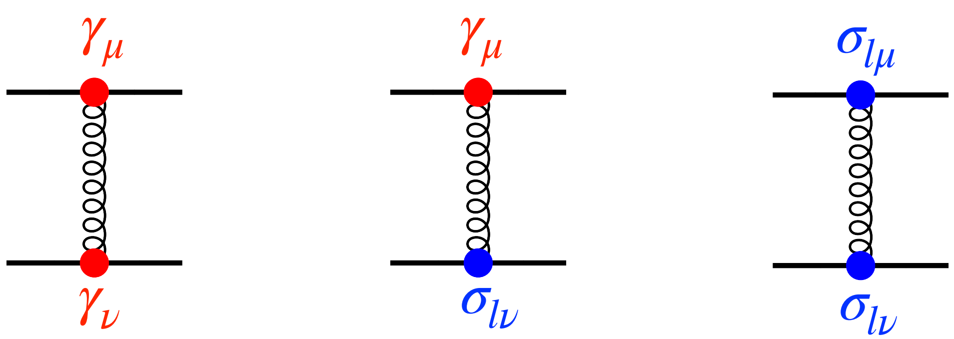

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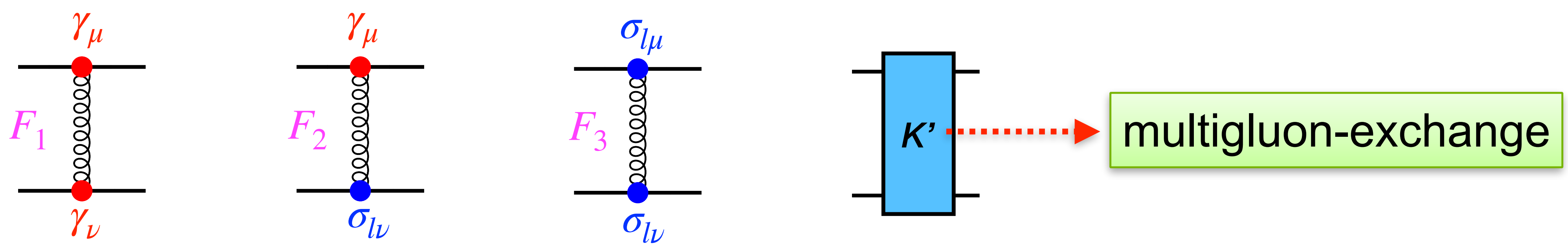

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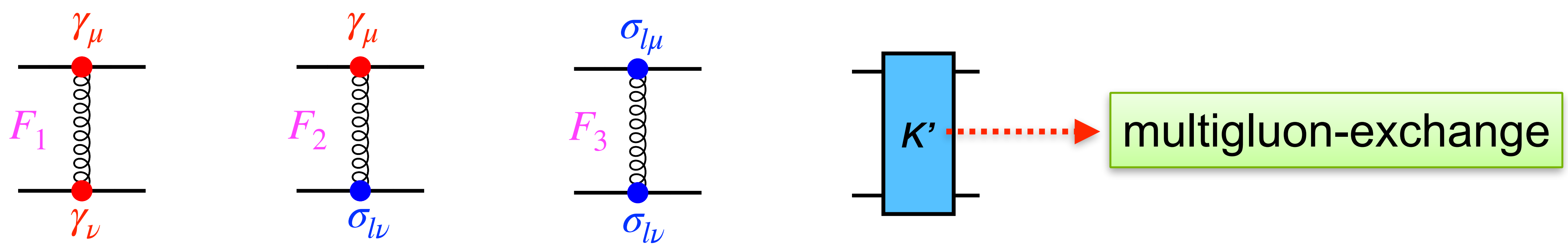

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1. How to further **pin down structures** of the kernel?

2. How to **simplify the kernel** for more practical applications?

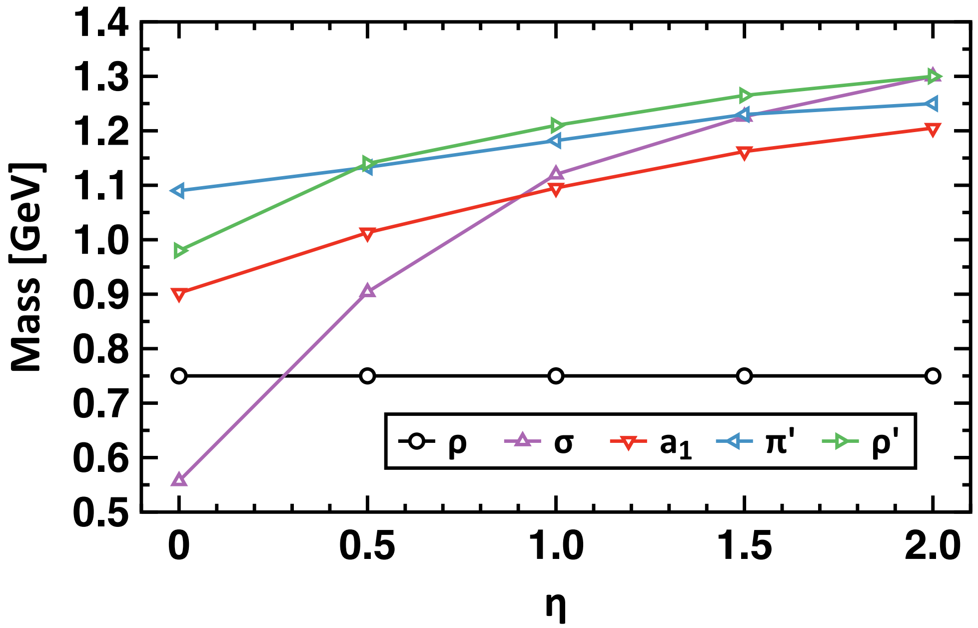
See, e.g., CPL 38 (2021) 7, 071201

Chapter II: Applications

Spectra of mesons and baryons with light and heavy flavors.

Application: Meson sector

✦ Impact of the Pauli term (AM):



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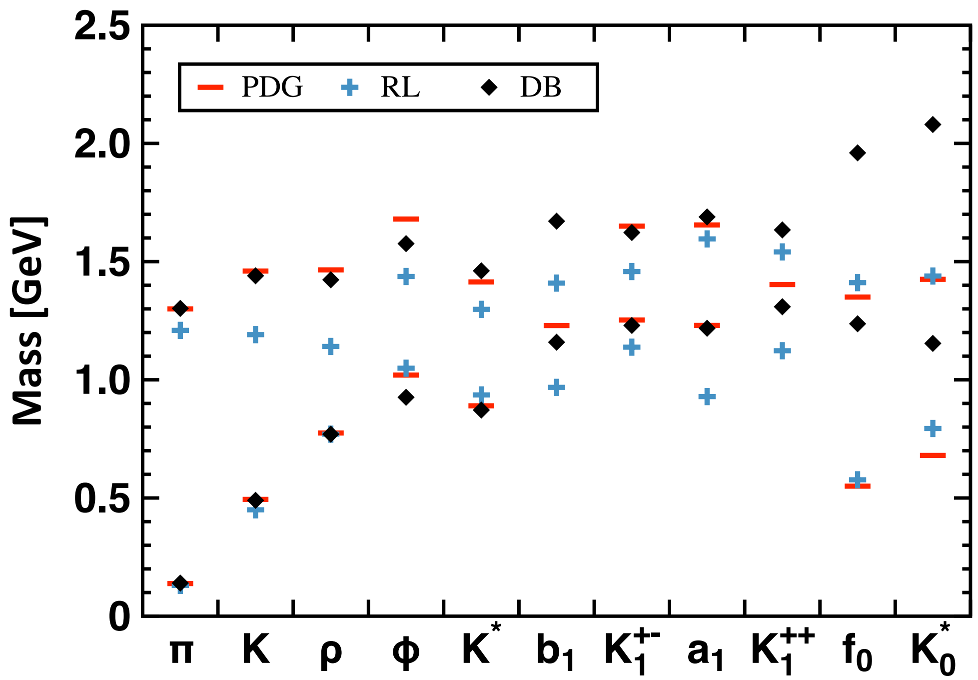

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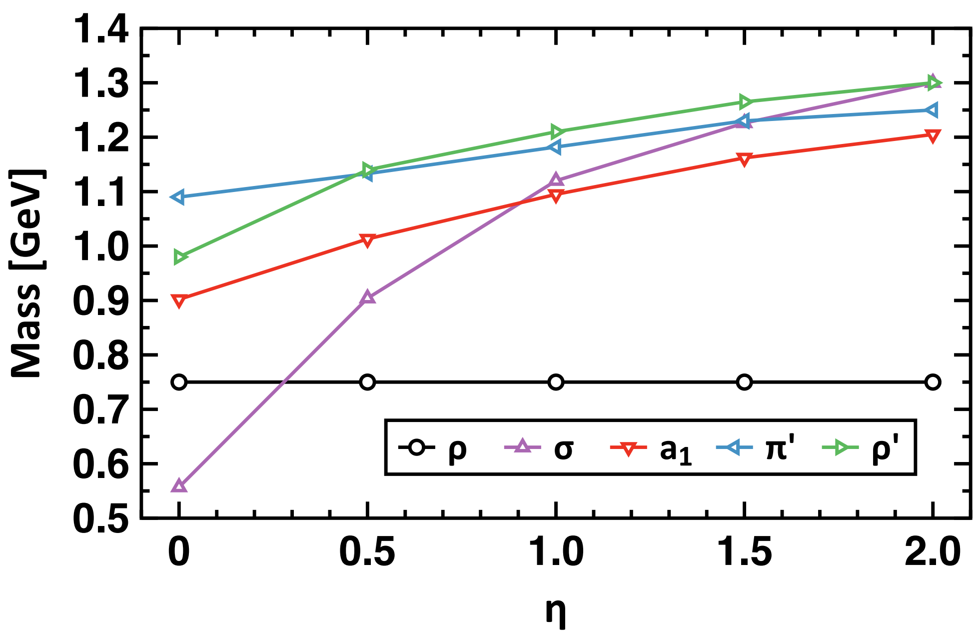

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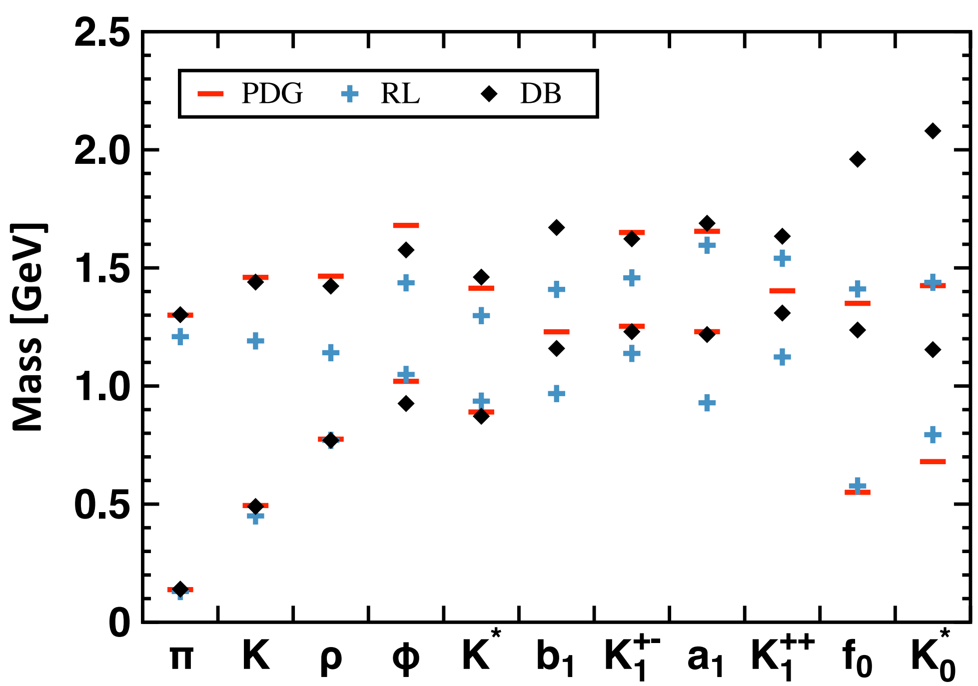

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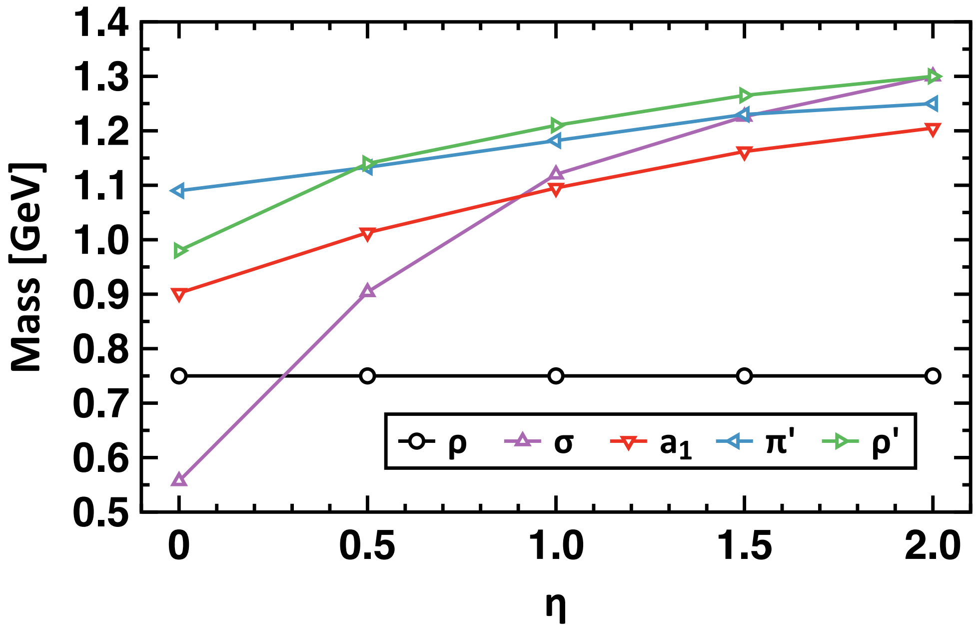

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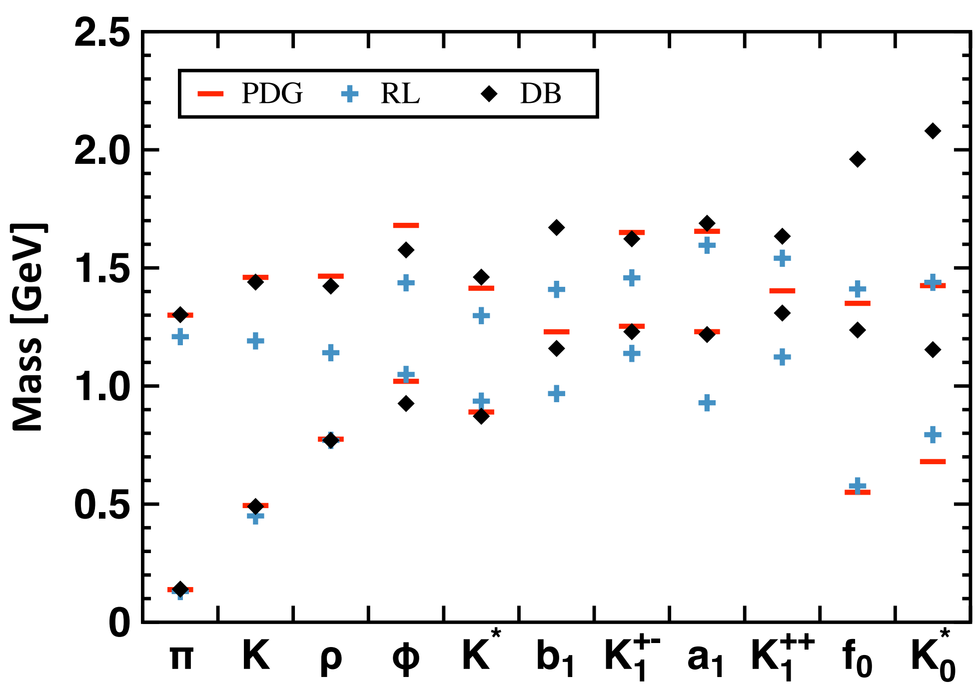

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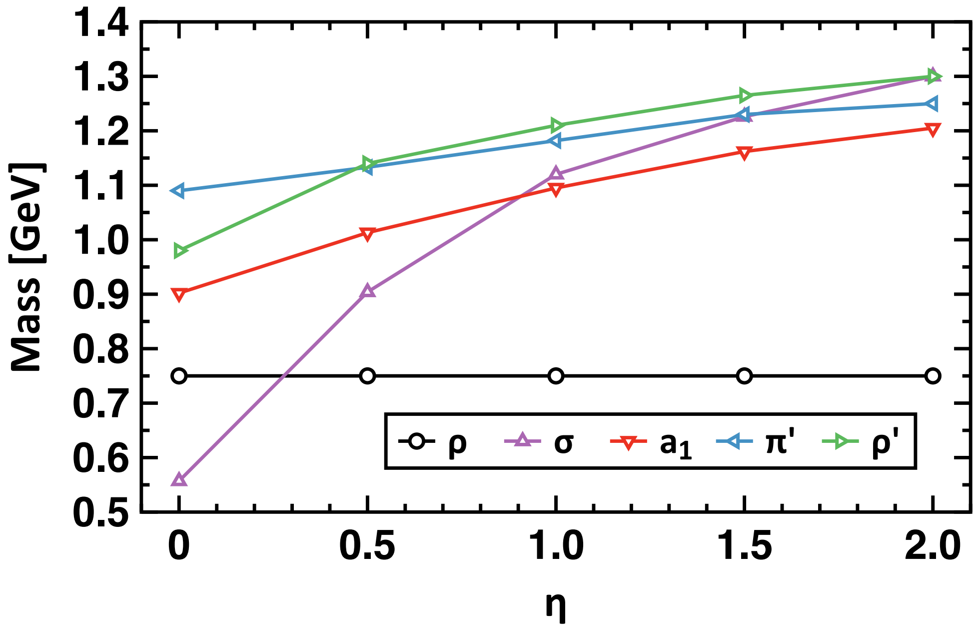

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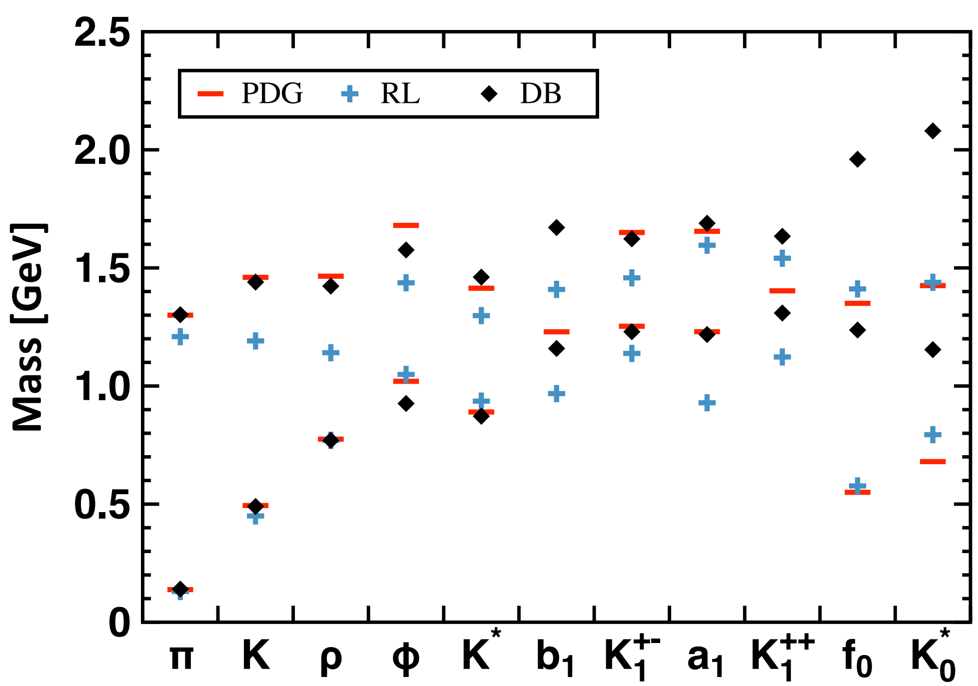

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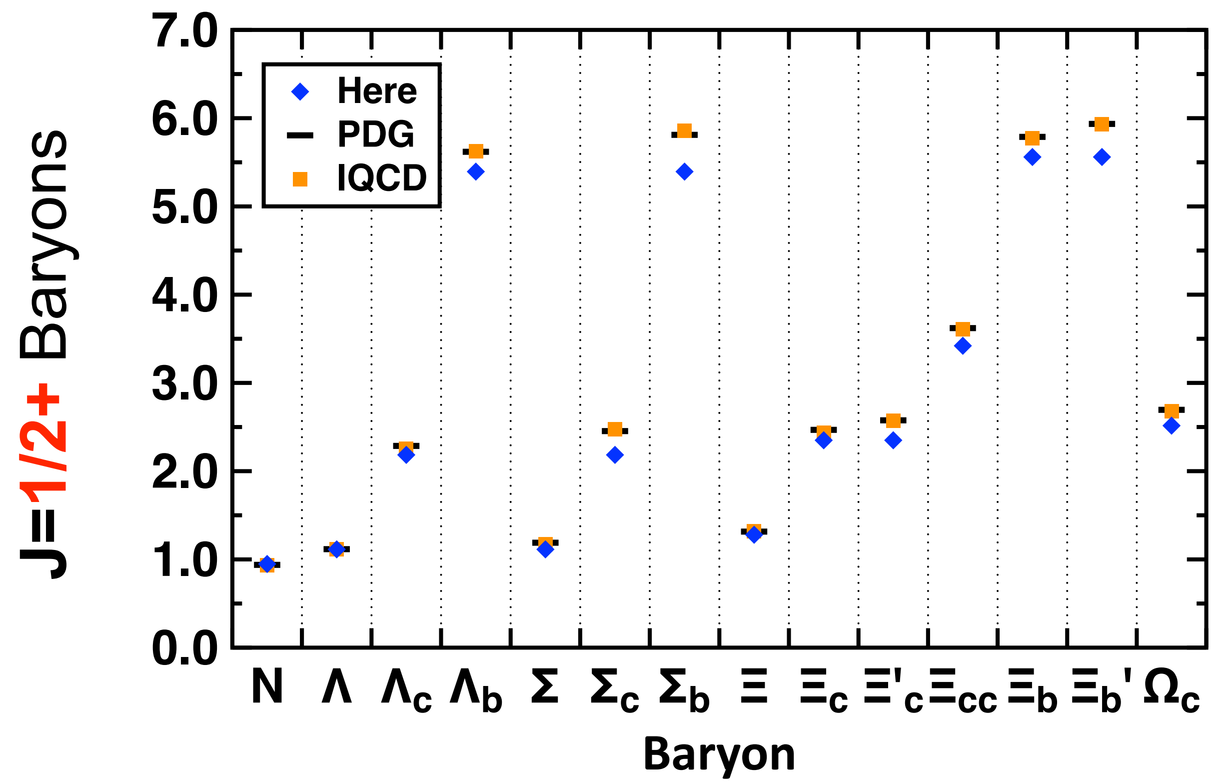

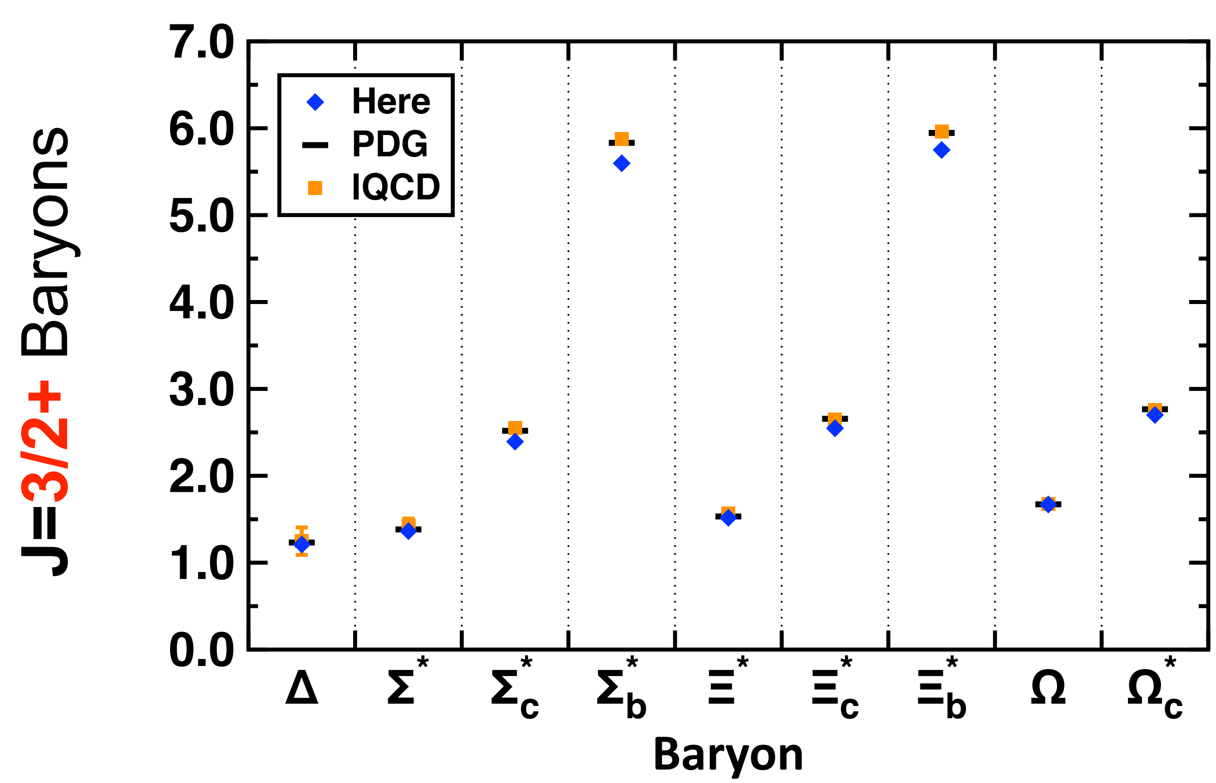

See, e.g., Few-Body Syst 60, 26 (2019)

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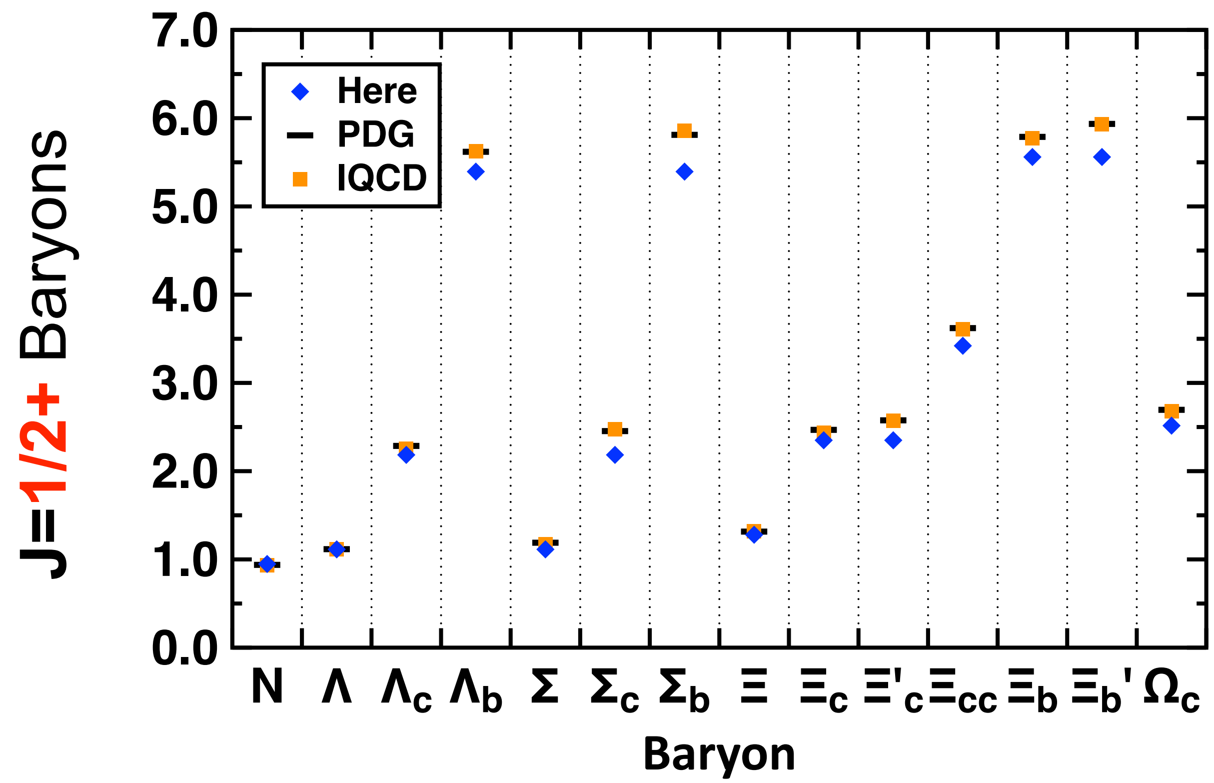

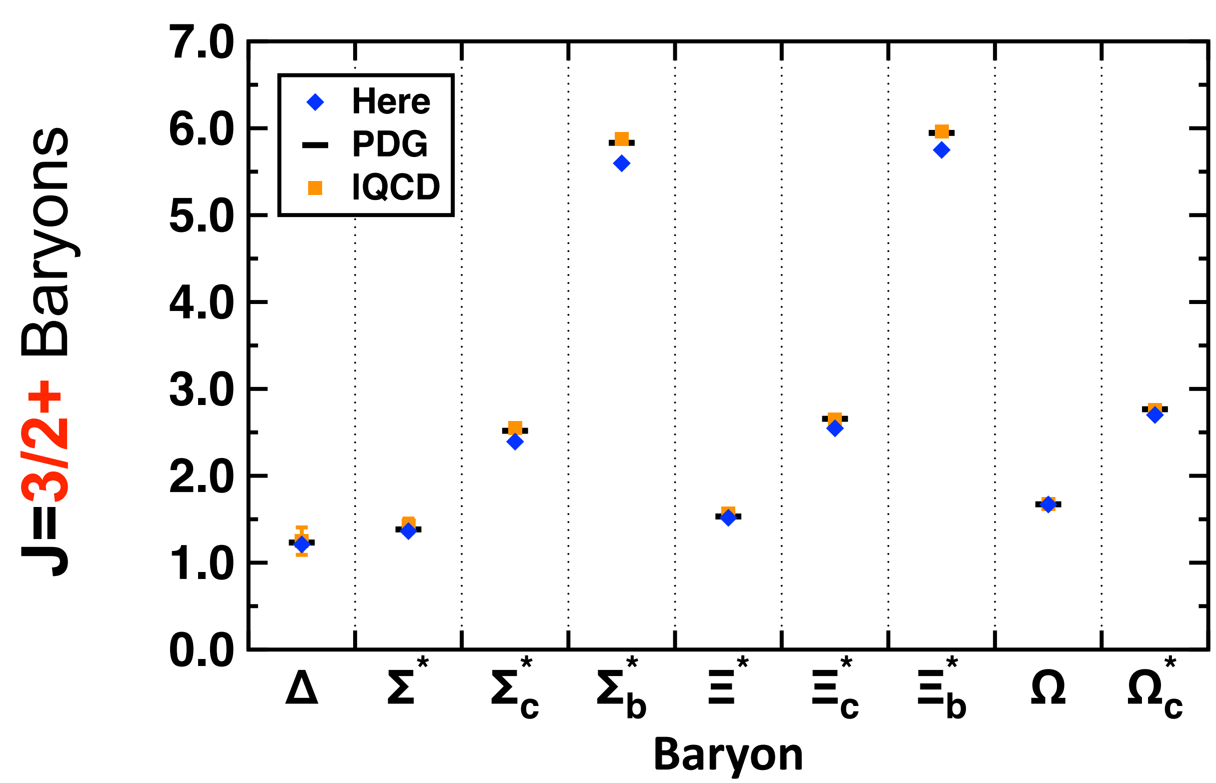

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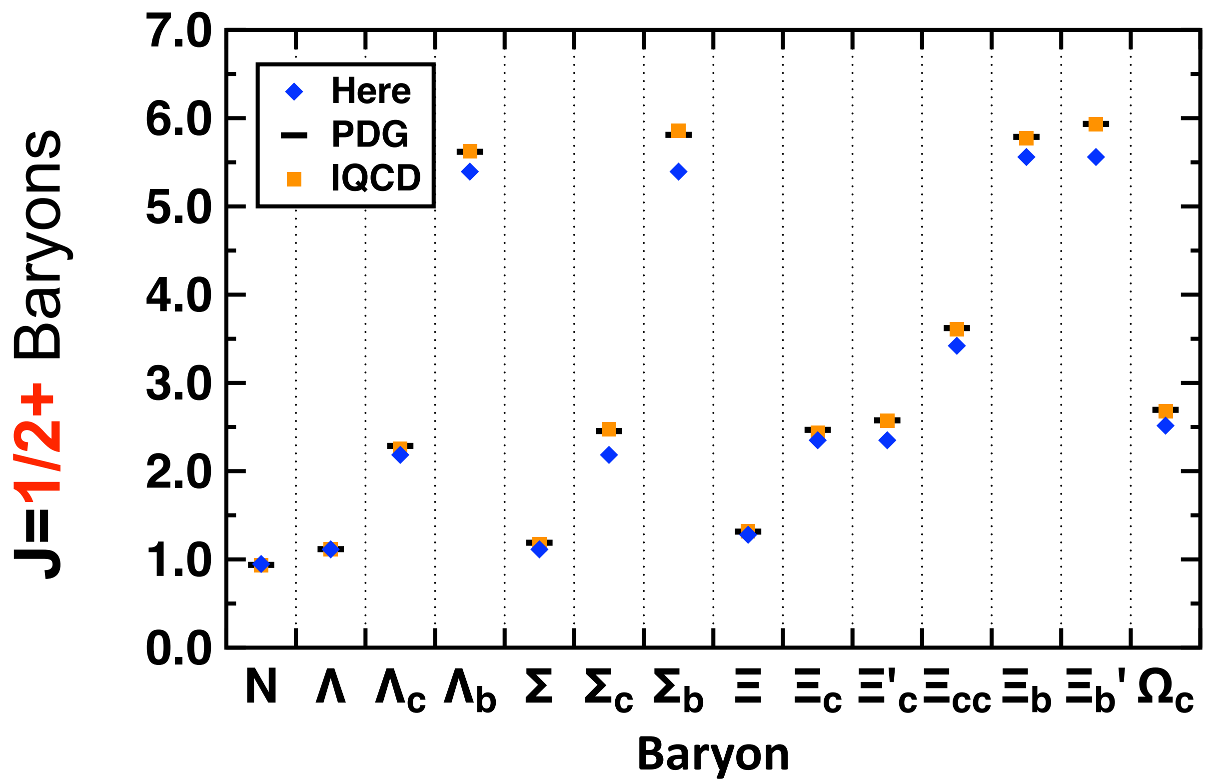

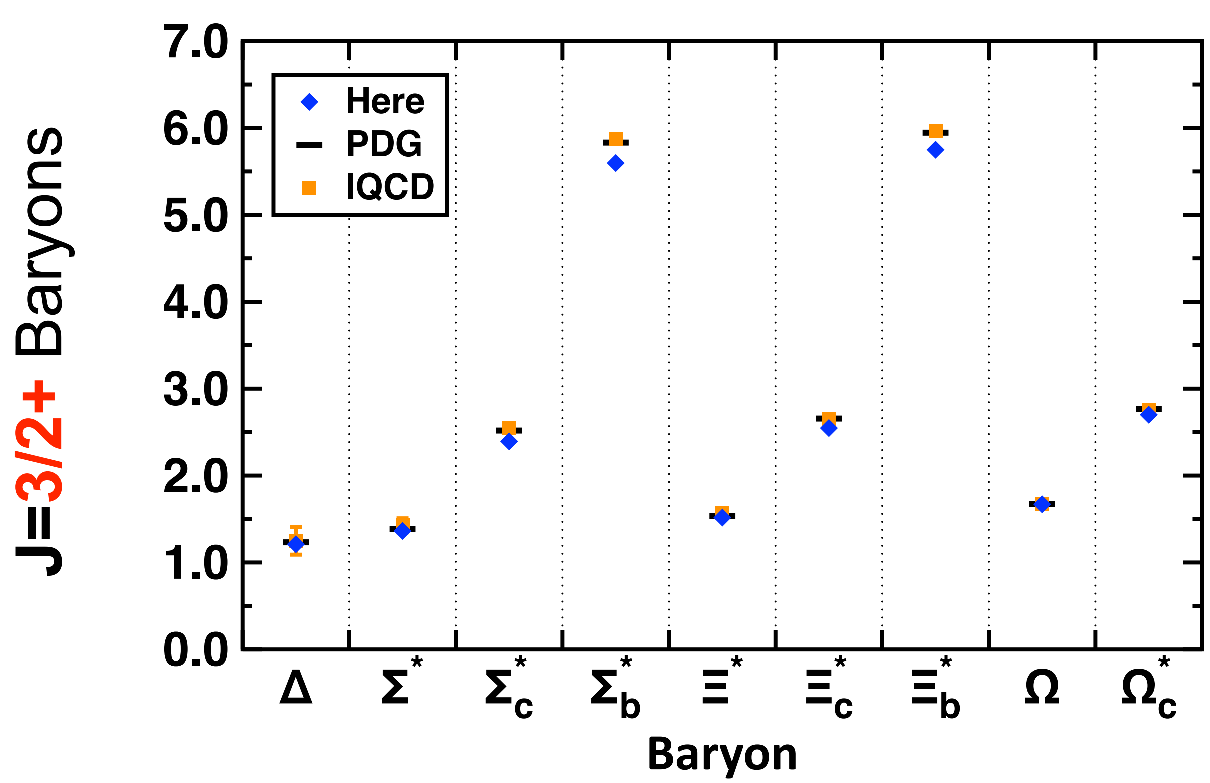

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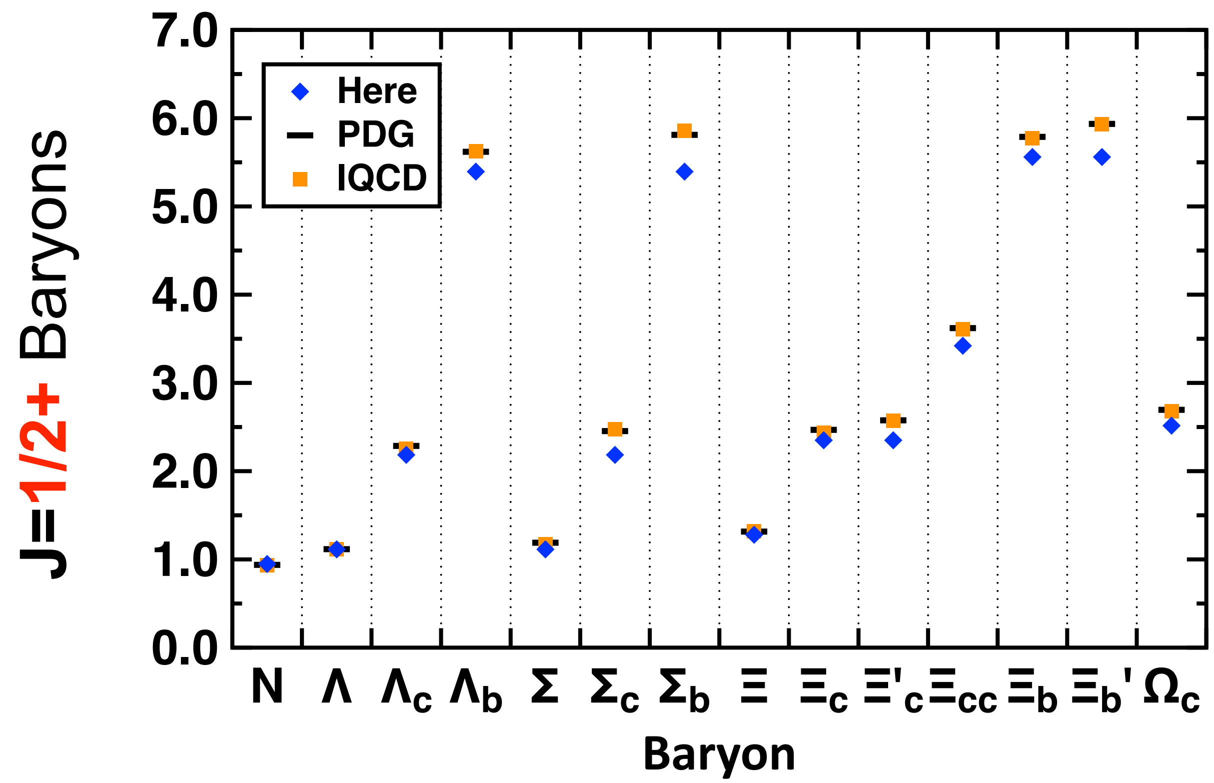

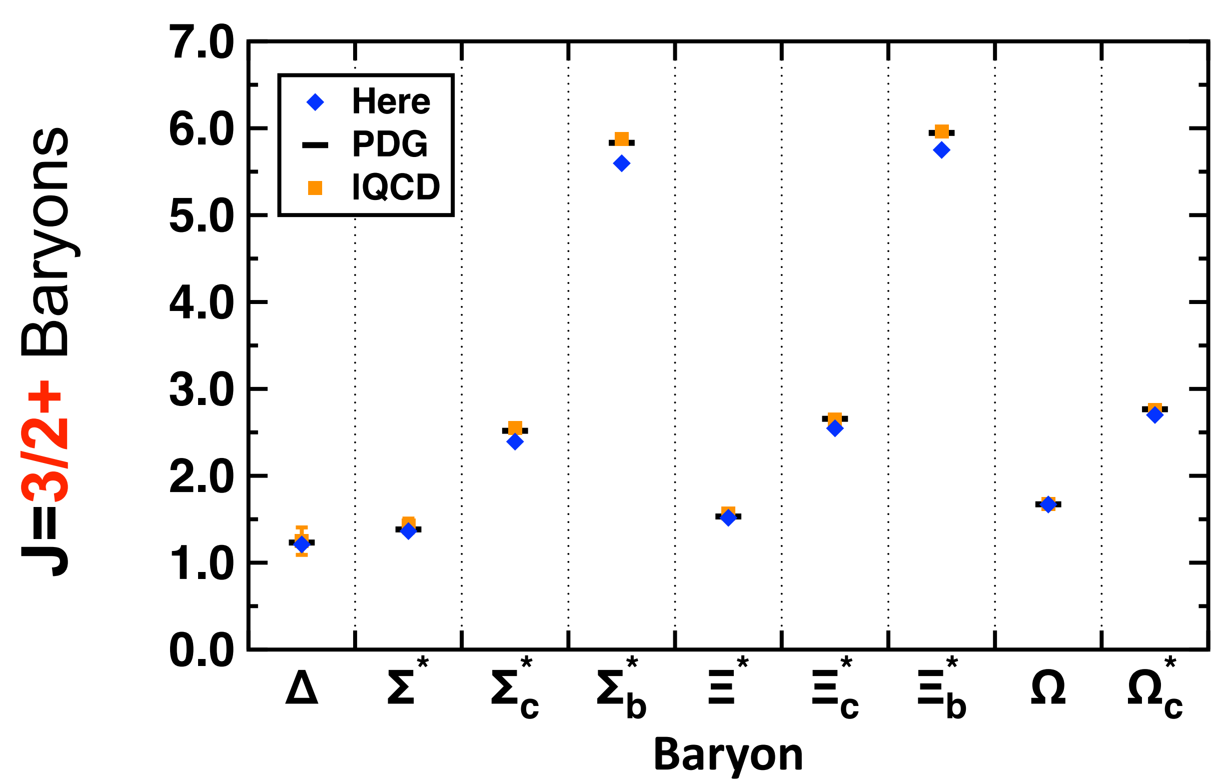

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Summary

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Outlook

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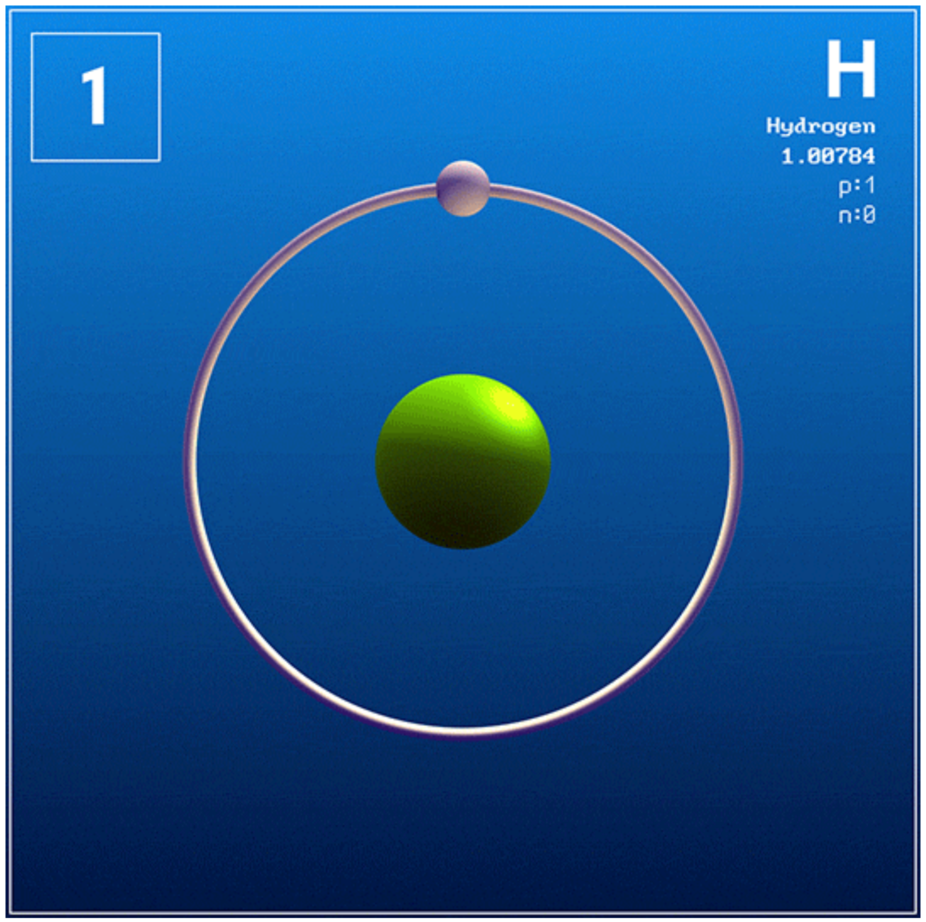
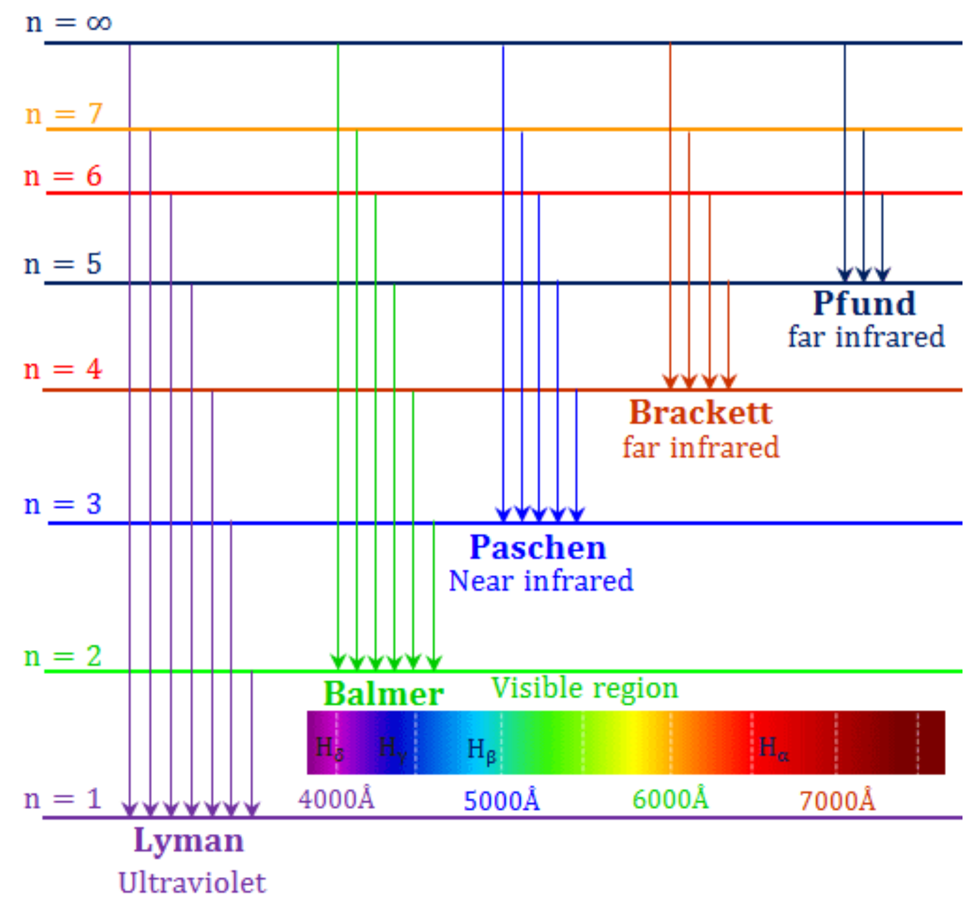
History: Hydrogen atom — The simplest element





$$H\,|\psi_n\rangle = E_n\,|\psi_n\rangle$$

$$H = H_{\text{kinetic}} + H_{\text{Coulomb}}$$

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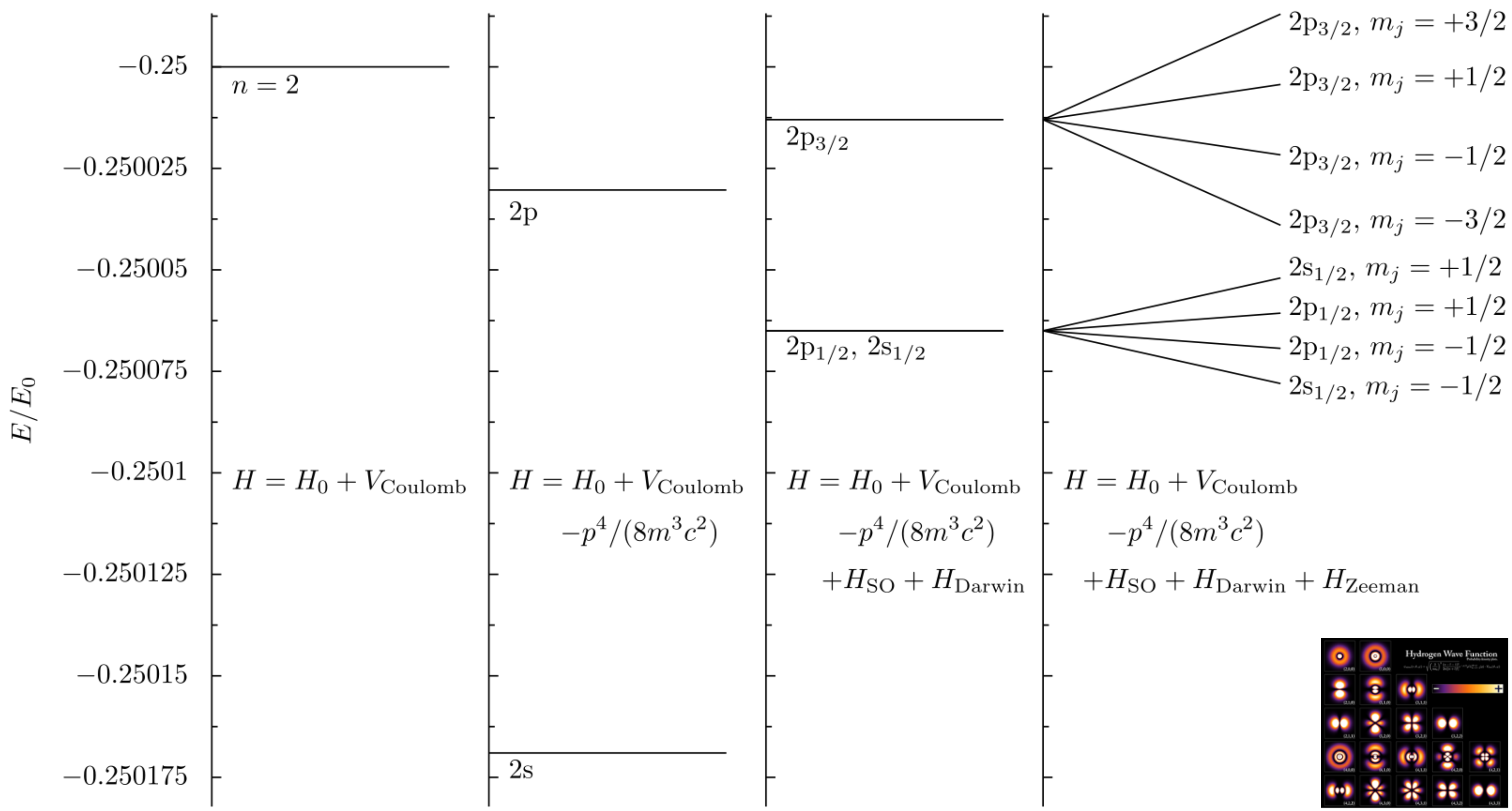

$$\boxed{H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{QED}}}$$

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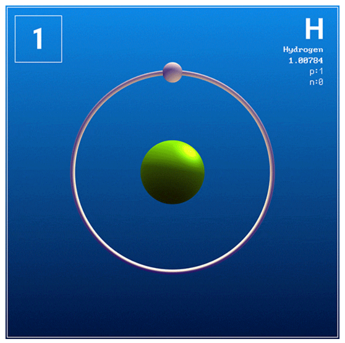
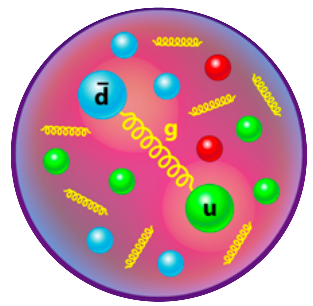
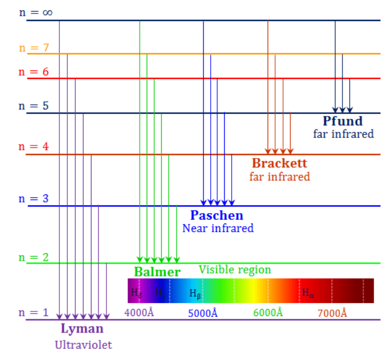
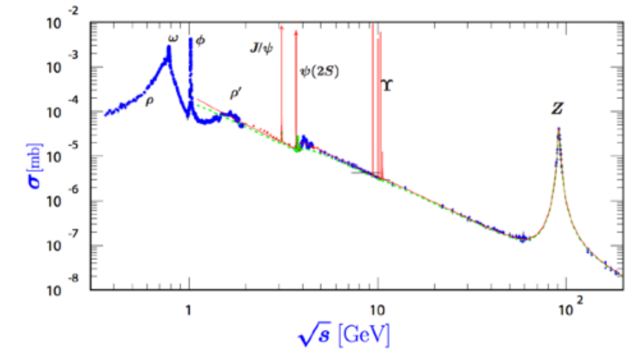




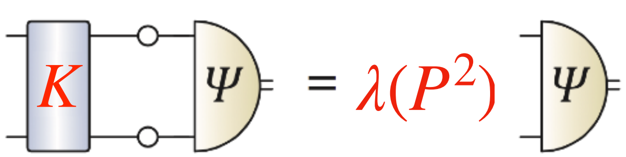




QM: Schrödinger equation

$$H|\psi\rangle = E|\psi\rangle$$

QFT: Bound-state equation



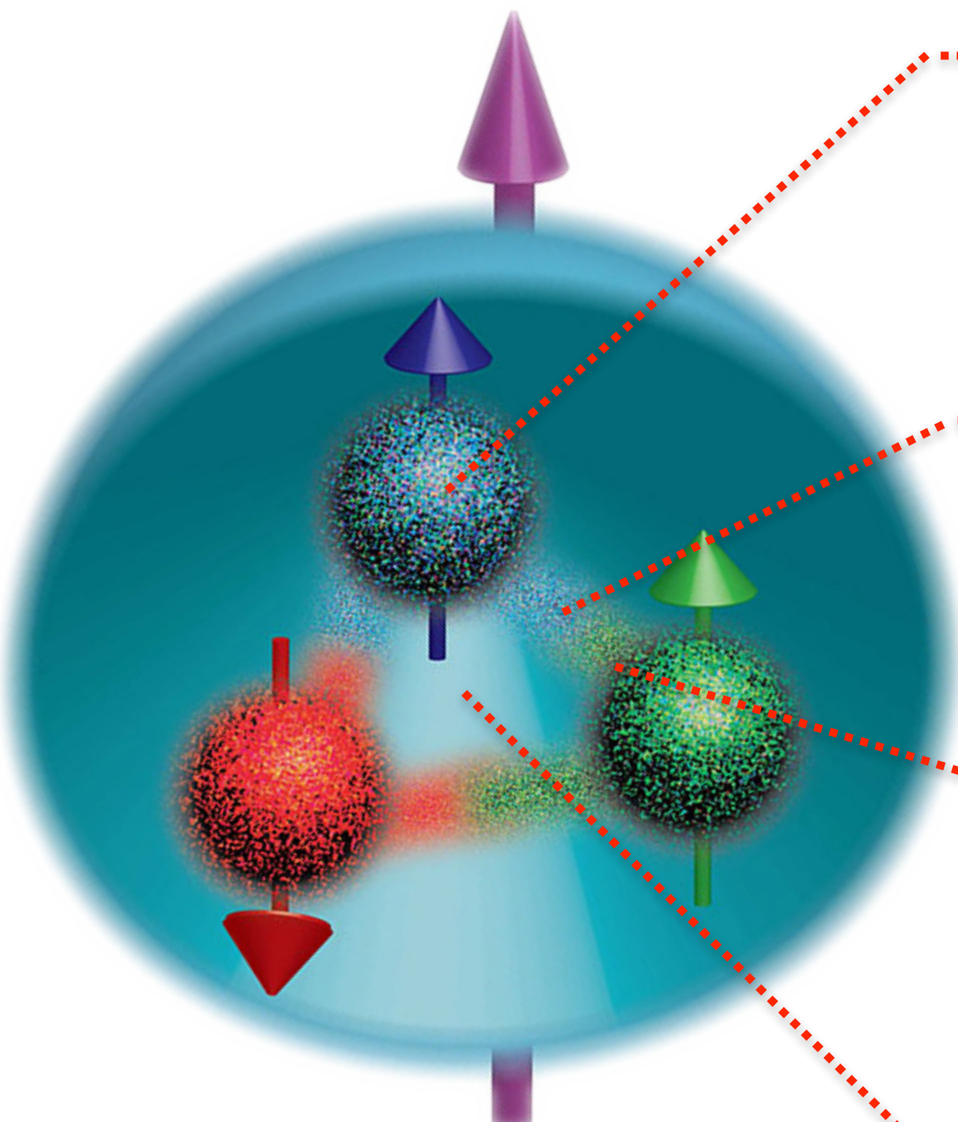
✦ **Theories**: Simple (few-body) objects could involve surprisingly rich physics.

✦ **Experiments**: High-precision measurements could make the story very different.

Chapter I: Theory

Physics of quark, gluon, vertex, and kernel

Continuum QCD: Interaction between quarks



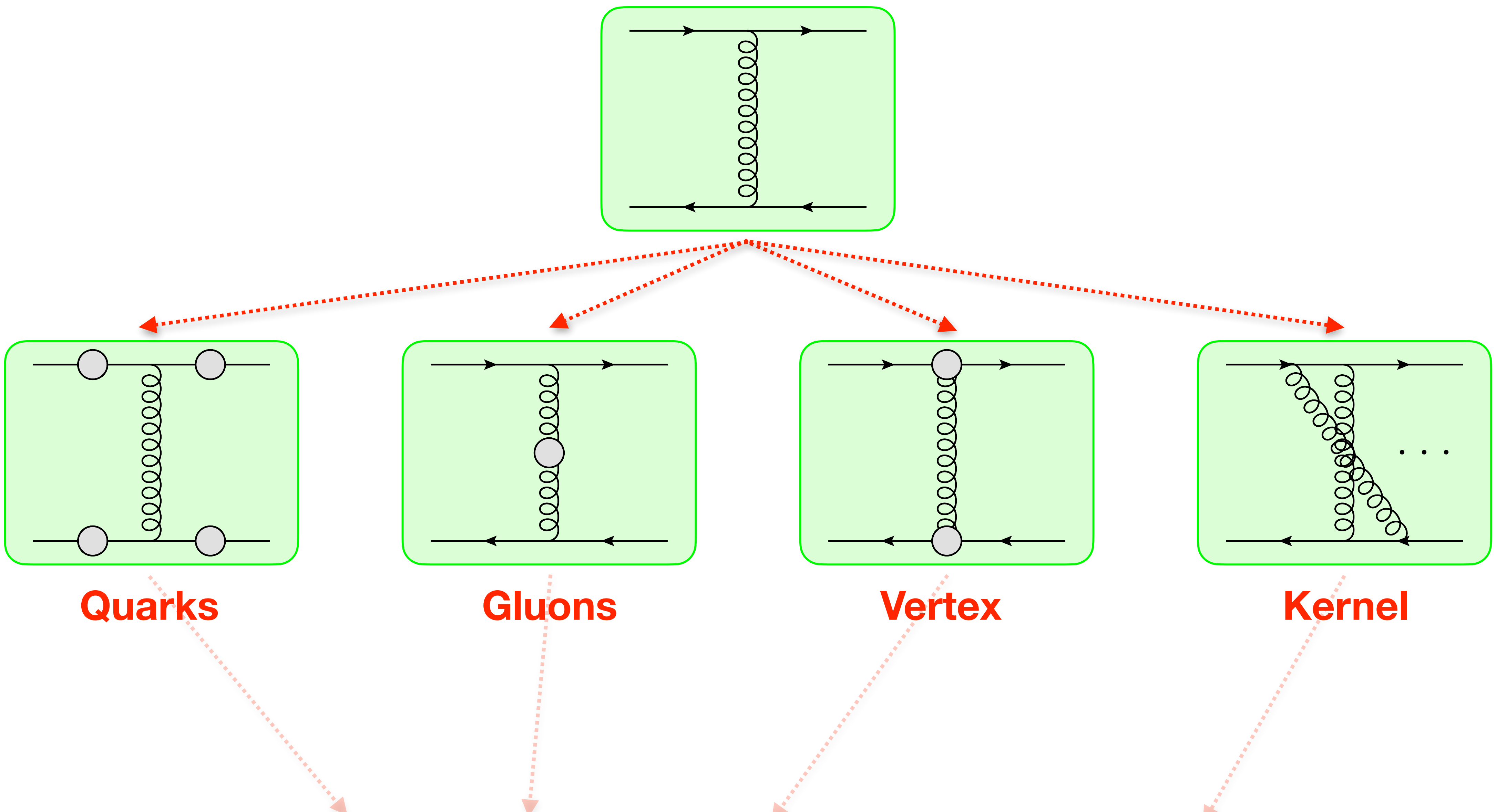
Quasi-particle quark

Quasi-particle gluon

Dressed coupling vertex

Ways of gluon-exchange

Continuum QCD: Interaction between quarks



$$H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{QED}}$$

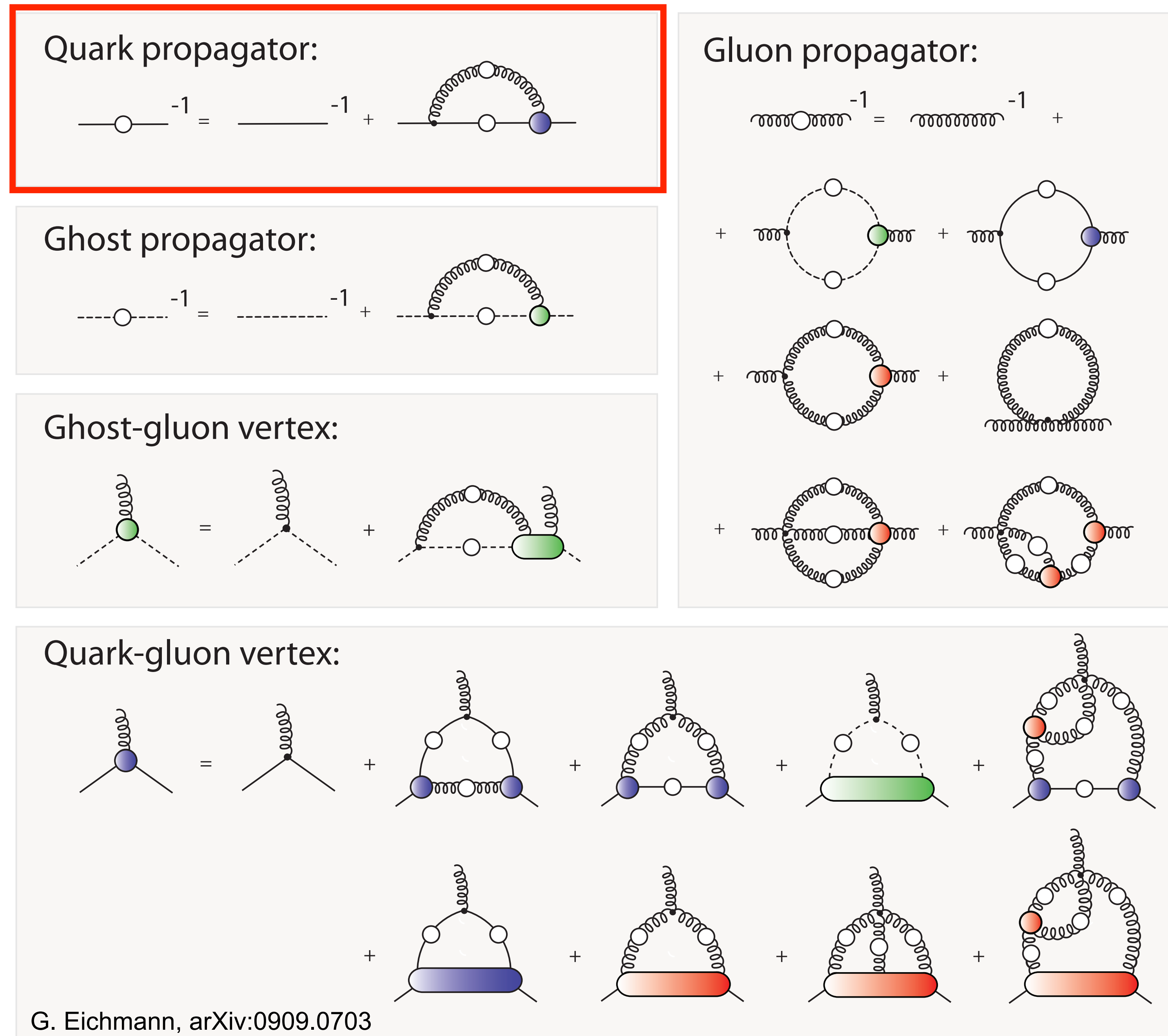
Ground states — **Excited states** → M

Continuum QCD: Equations of motion

Principle of Least Action

$$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

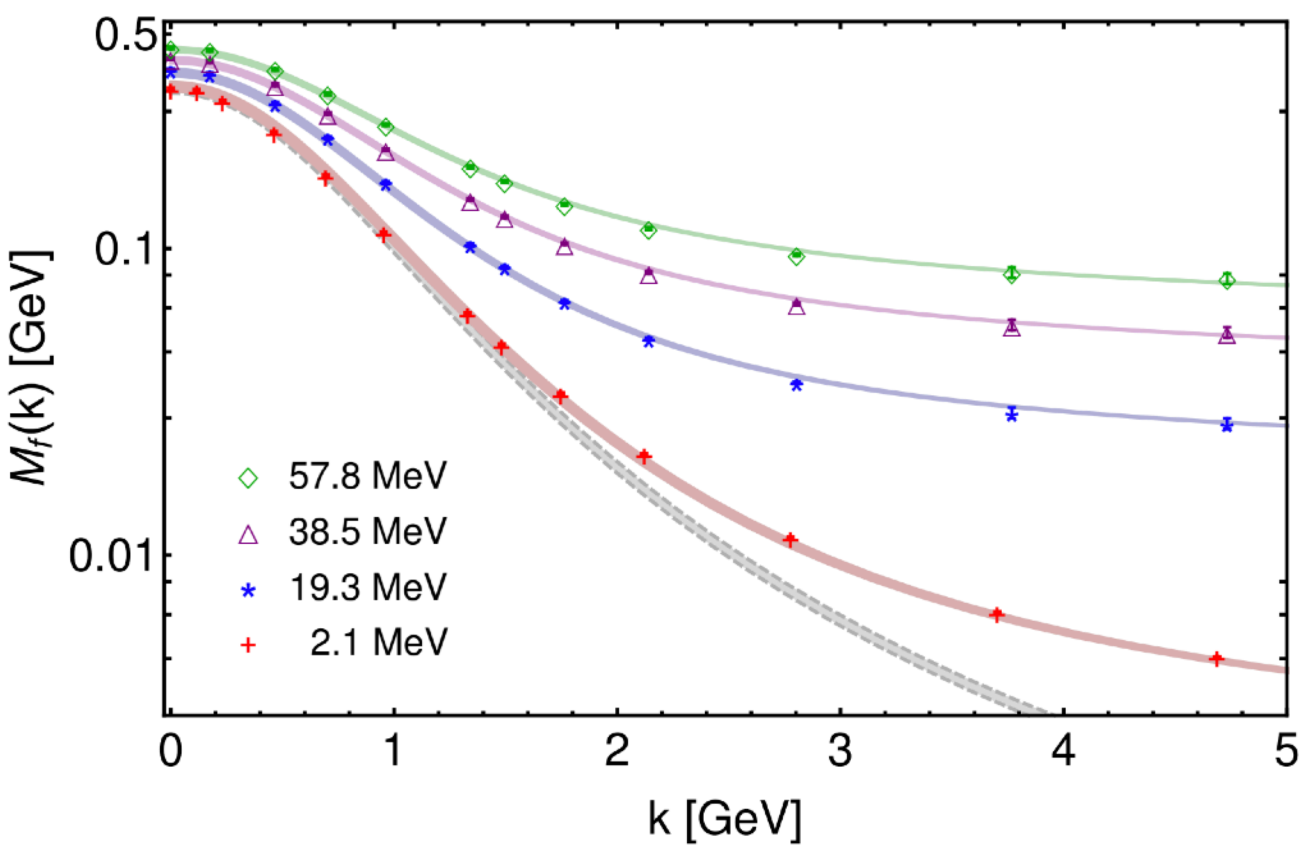
Dyson-Schwinger Equations



Quark: Running mass function

$$S(p) = \frac{1}{i\gamma \cdot pA(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Chang, Yang, et. al., PRD 104, 094509 (2021)



✦ **Now:**

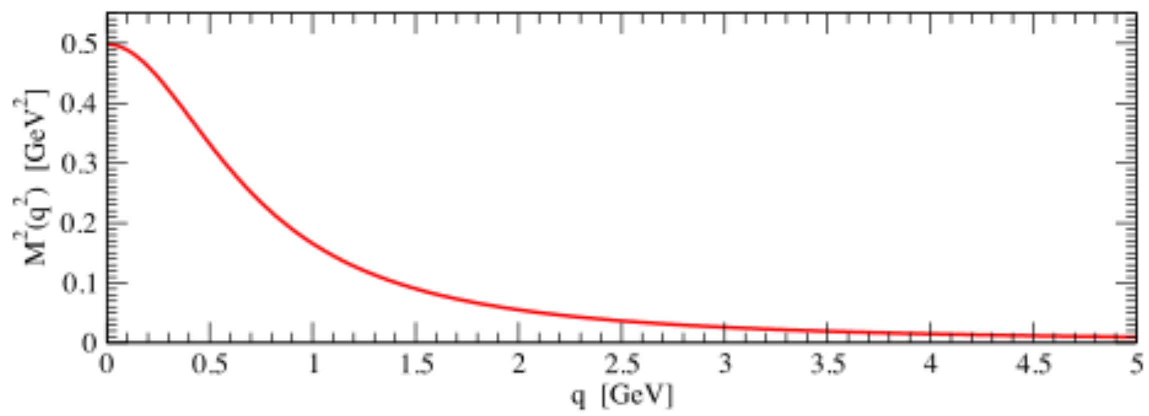
1. The quark's **effective mass** runs with its momentum.
2. The most **constituent mass** of a light quark comes from a cloud of gluons.

✦ **Next:**

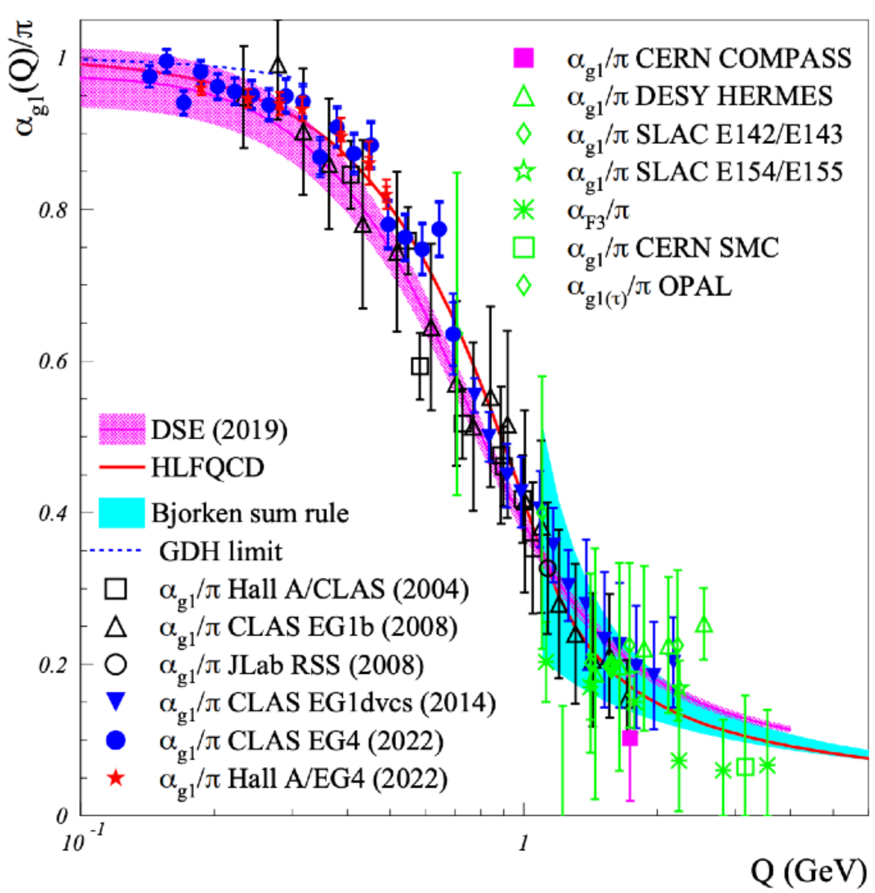
1. What is the **infrared scale** of quark mass function?
2. How does the **transition** connect the non-perturbative and perturbative regions?

Gluon: Dynamical mass scale

Gluon mass function: O. Oliveira et. al., J.Phys. G38, 045003 (2011)



Running coupling: Deur, Brodsky, Roberts, PPNP, 104081 (2024)



✦ **Now:**

1. The dressed gluon can be well parameterized by a **mass scale**

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2}$$

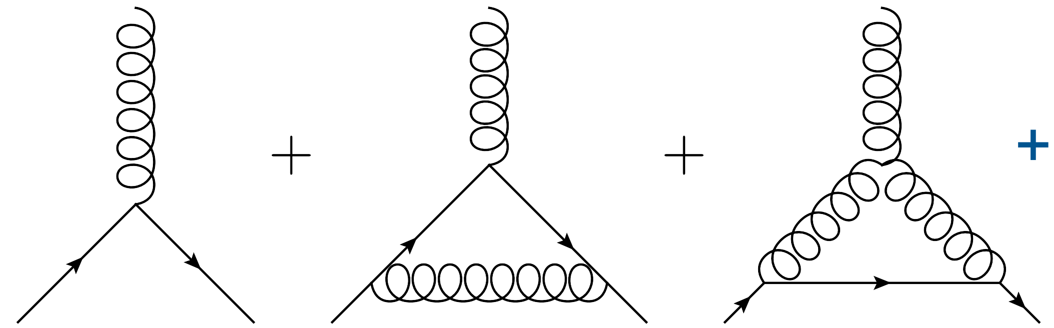
2. The effective running coupling **saturates** in the infrared limit.

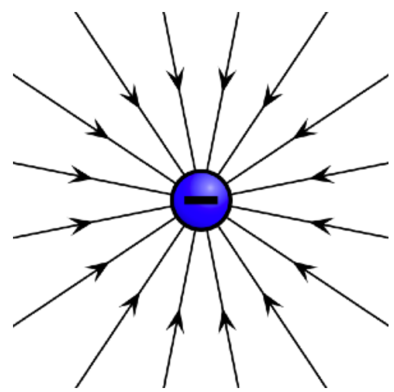
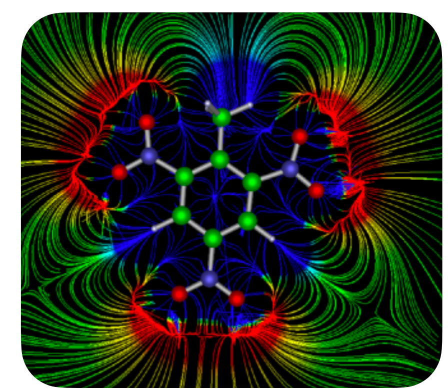
✦ **Next:**

1. What is the **mass scale** of gluon?

2. What is the **infrared magnitude** of running coupling?

Vertex: DCSB feedback

Quark-gluon vertex:  + … = ?

point charge   distributed charge

✦ The **Dirac** and **Pauli** terms: for an on-shell fermion, the vertex can be decomposed by two form factors:

$$\Gamma^{\mu}(P',P) = \gamma^{\mu} F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f} Q^{\nu} F_2(Q^2)$$

✦ The form factors express (color-)**charge** and (color-)**magnetization** densities. And the so-called **anomalous magnetic moment** is proportional to the Pauli term.

See, e.g., PLB722, 384 (2013)

Vertex: DCSB feedback

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Equation of Motion (DSEs)

$$S = \frac{1}{i\gamma\cdot p A(p^2) \; + \; B(p^2)}$$

$$\gamma^\mu \Sigma_A \; + \; i\sigma_{\mu\nu} Q^\nu \Delta_B \; + \; \text{etc.} \sim \Gamma^\mu$$

Ward identities (Symmetries)

✦ Now:

1. There is a dynamic chiral symmetry breaking (**DCSB**) **feedback**.

2. The **appearance** of the vertex is dramatically modified by the **dynamics**.

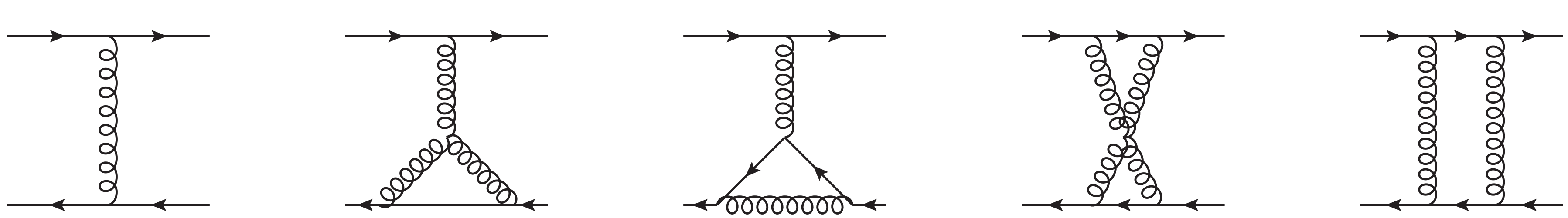
✦ Next:

1. What are the exact **strengths** of the terms in the vertex?

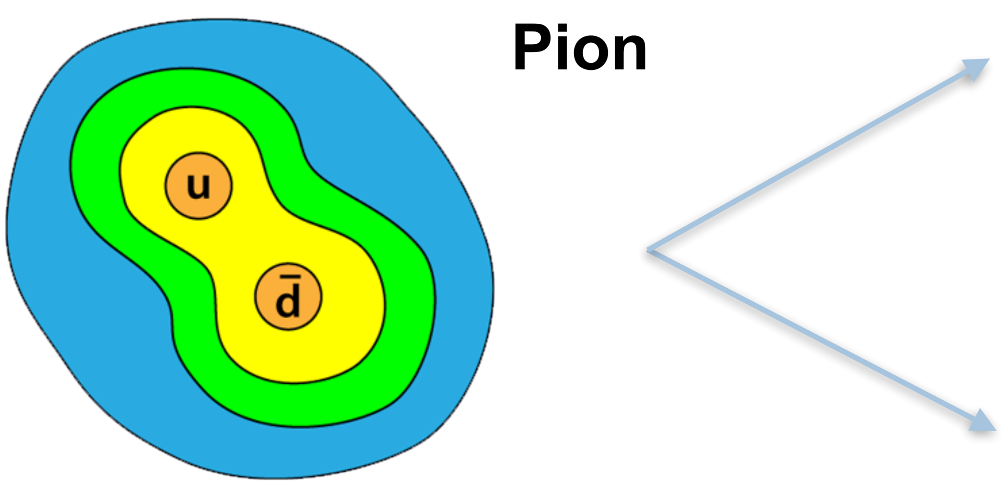
2. What the exact **behaviors** of the form factors in the vertex?

See, e.g., PLB722, 384 (2013)

Kernel: Twofold role of pion



$+ \ \dots \ = \ ?$



- ✦ **Bound state** of quark and anti-quark, but abnormally light:

$$M_\pi \ll M_u + M_{\bar{d}}$$

- ✦ **Goldstone's theorem:** If a generic continuous symmetry is spontaneously broken, then new massless scalar particles appear in the spectrum of possible excitations.

- ✦ The **discrete** and **continuous** **symmetries** strongly constrain the kernel:

Poincaré symmetry	Gauge symmetry
C-, P-, T-symmetry	Chiral symmetry

Kernel: Twofold role of pion

✦ **Now:**

1. A deep connection between one-body and two-body problem:

$$\boxed{f_\pi E_\pi(k^2) = B(k^2)}$$

Pion exists if, and only if, the **quark mass** is dynamically generated.

Two-body problem solved, almost completely, once solution of **one-body** problem is known.

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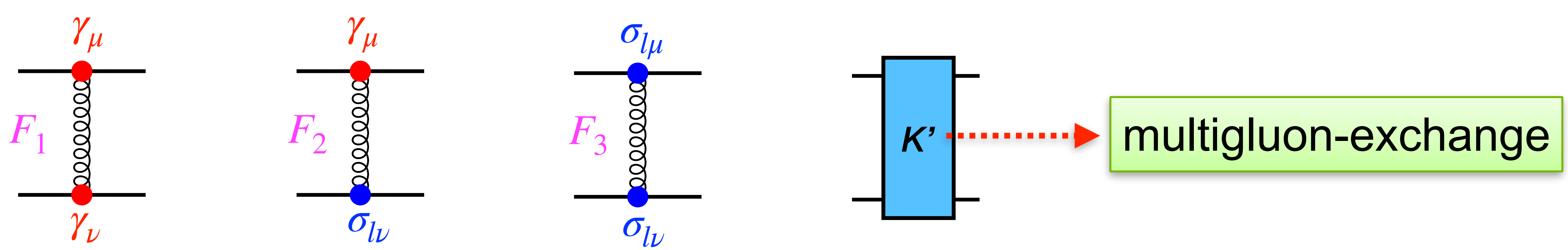

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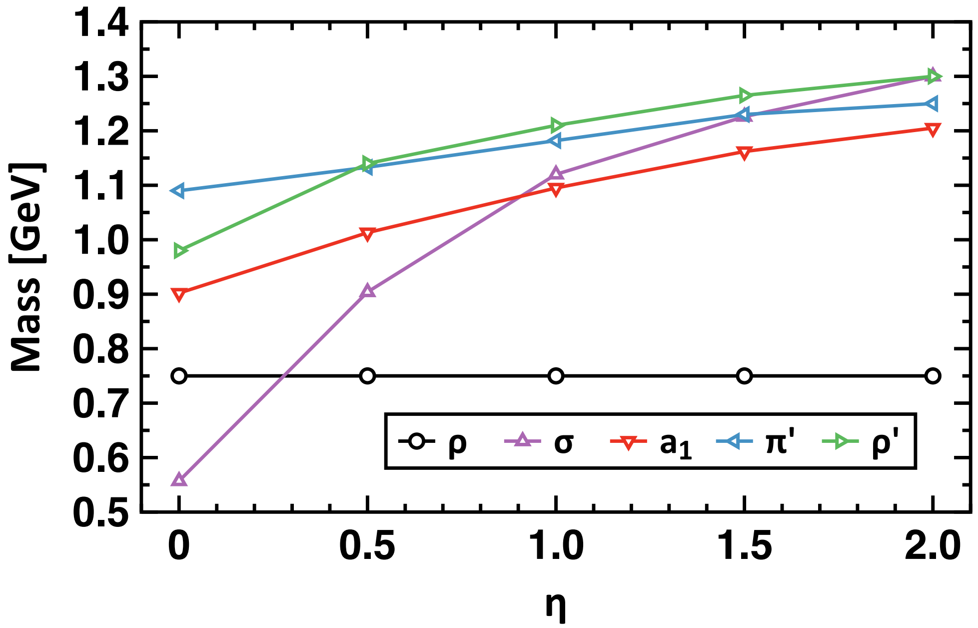

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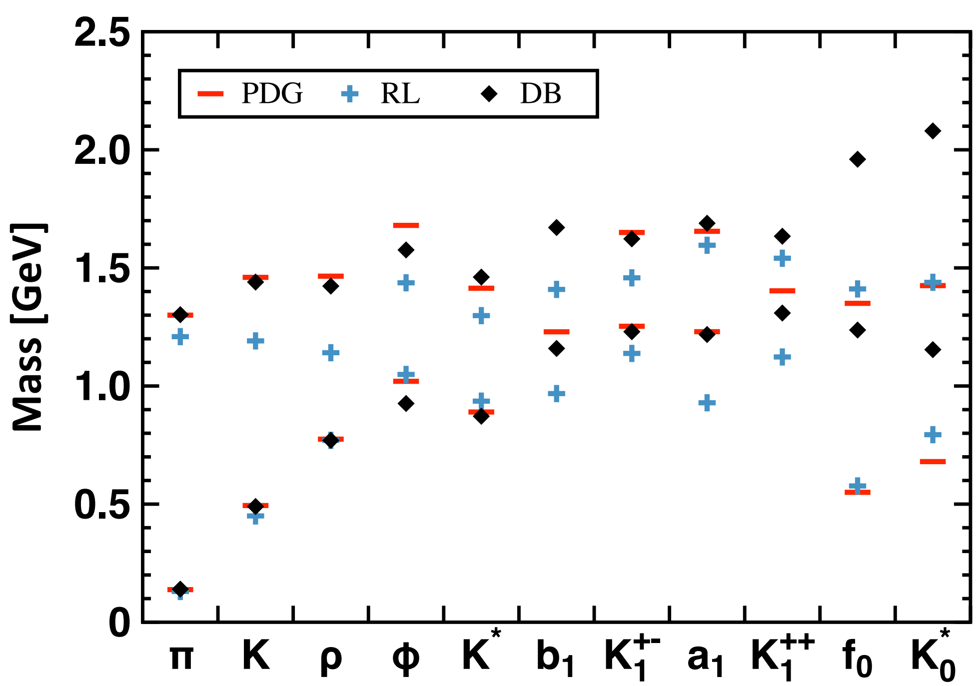

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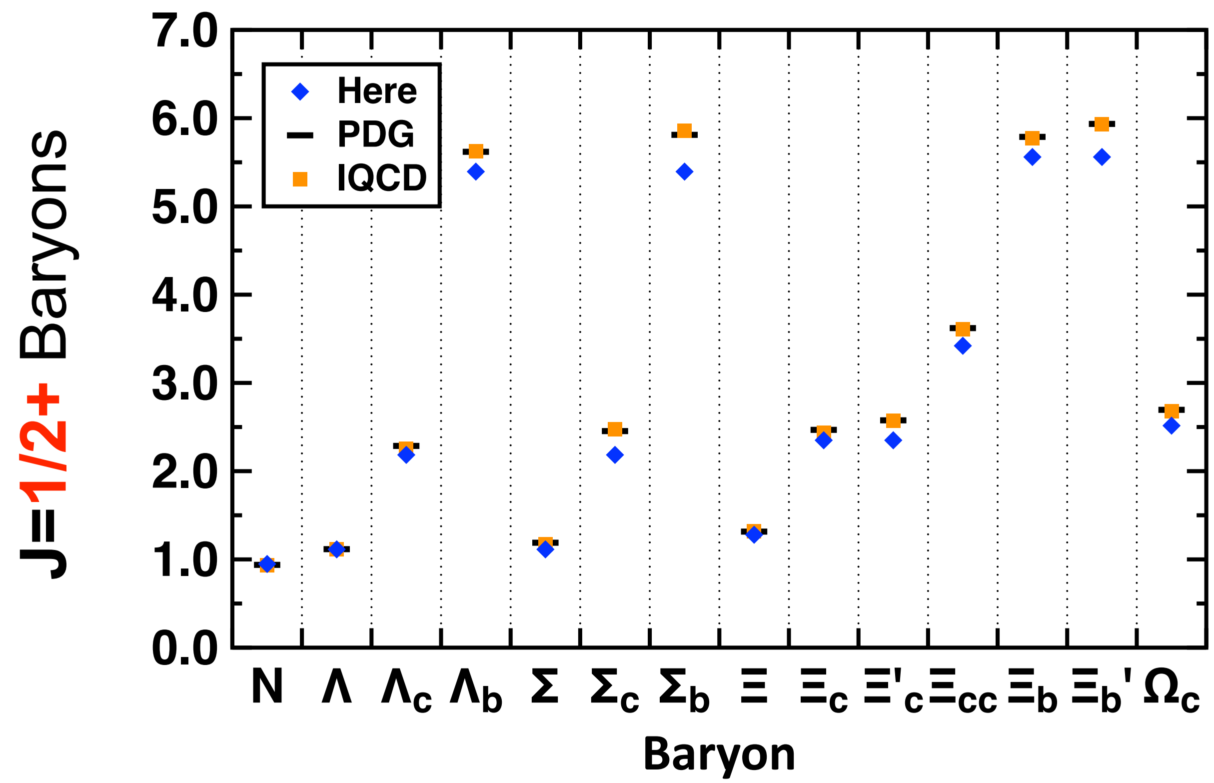

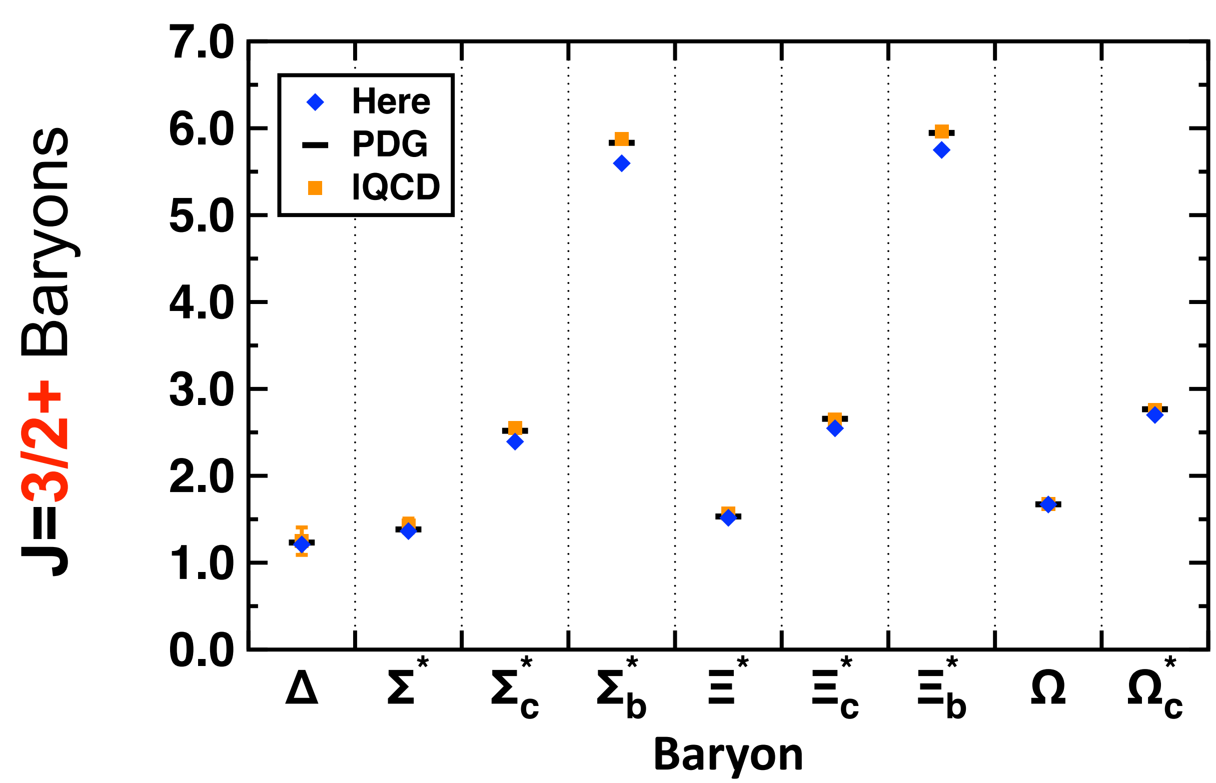

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