



Study the low-lying excited baryons in the decays of charmed hadrons

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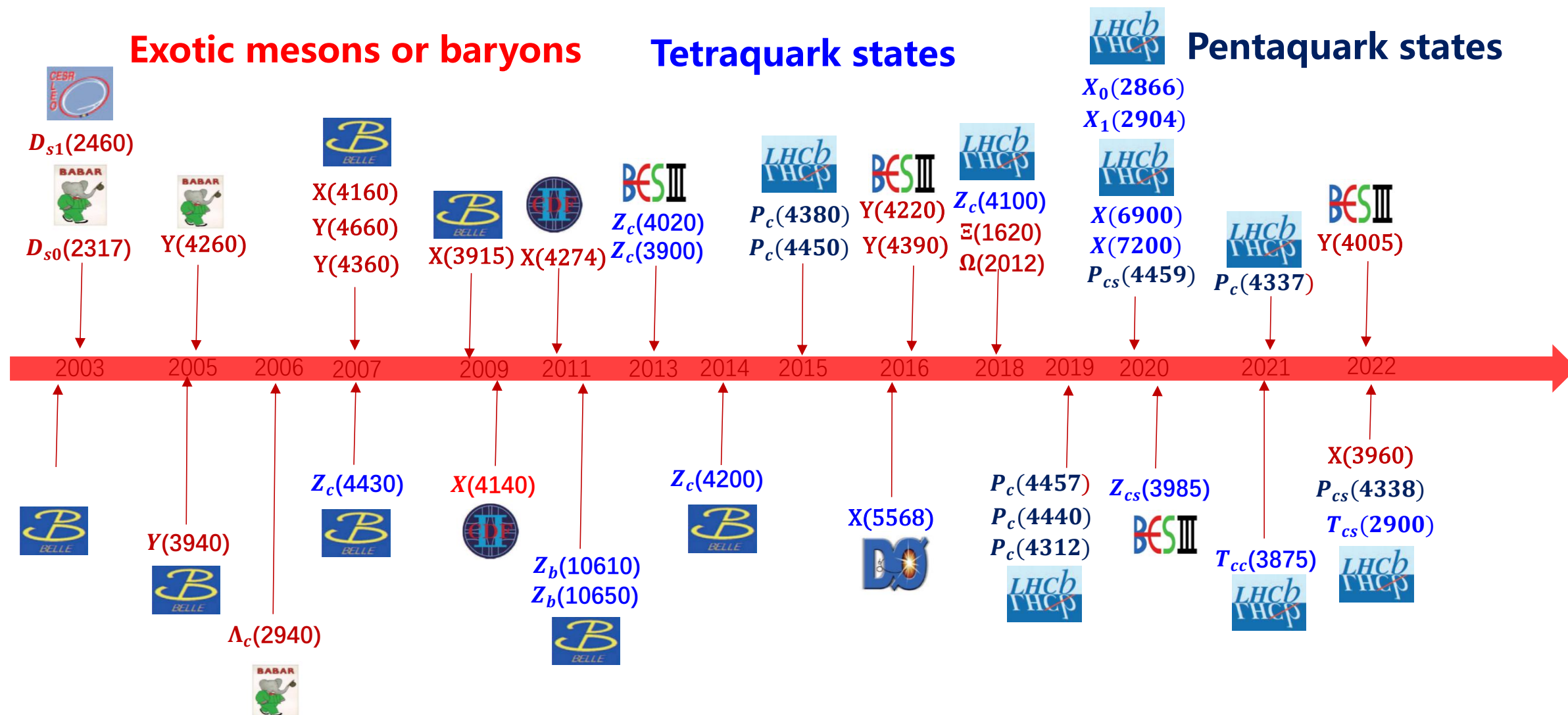
2024年6月30日-7月3日

第二届强子物理新发展研讨会
暨强子物理在线论坛100期特别活动@合肥

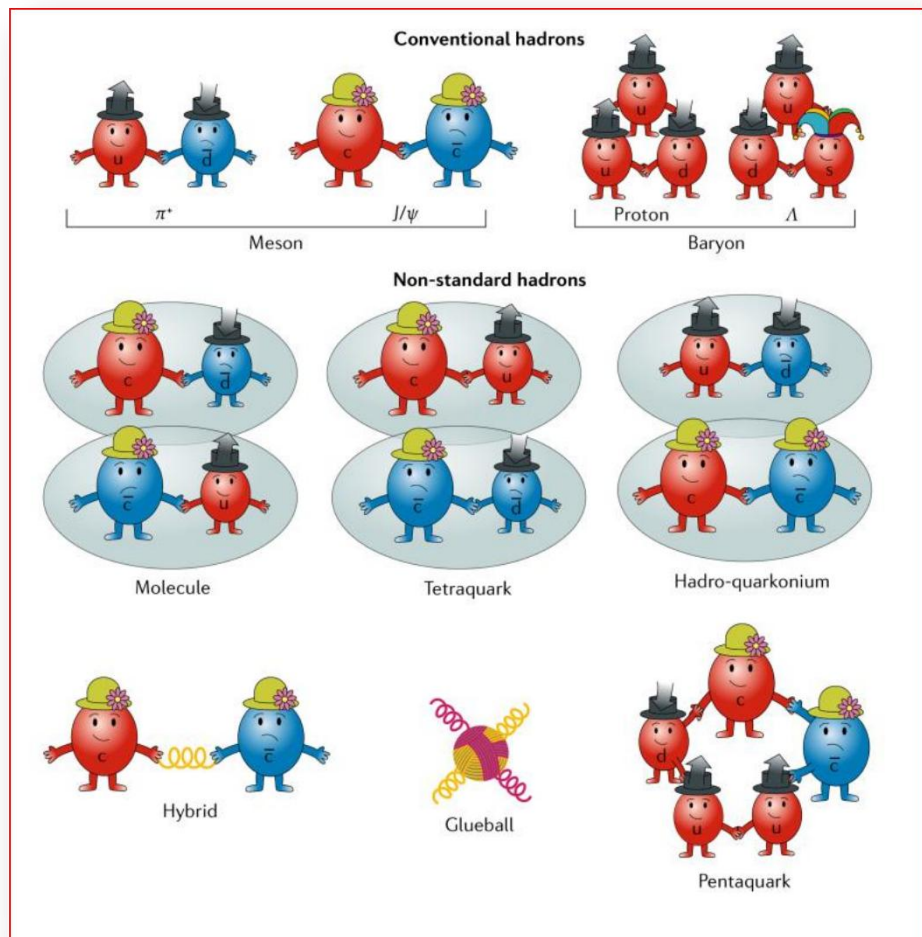


Exotic states

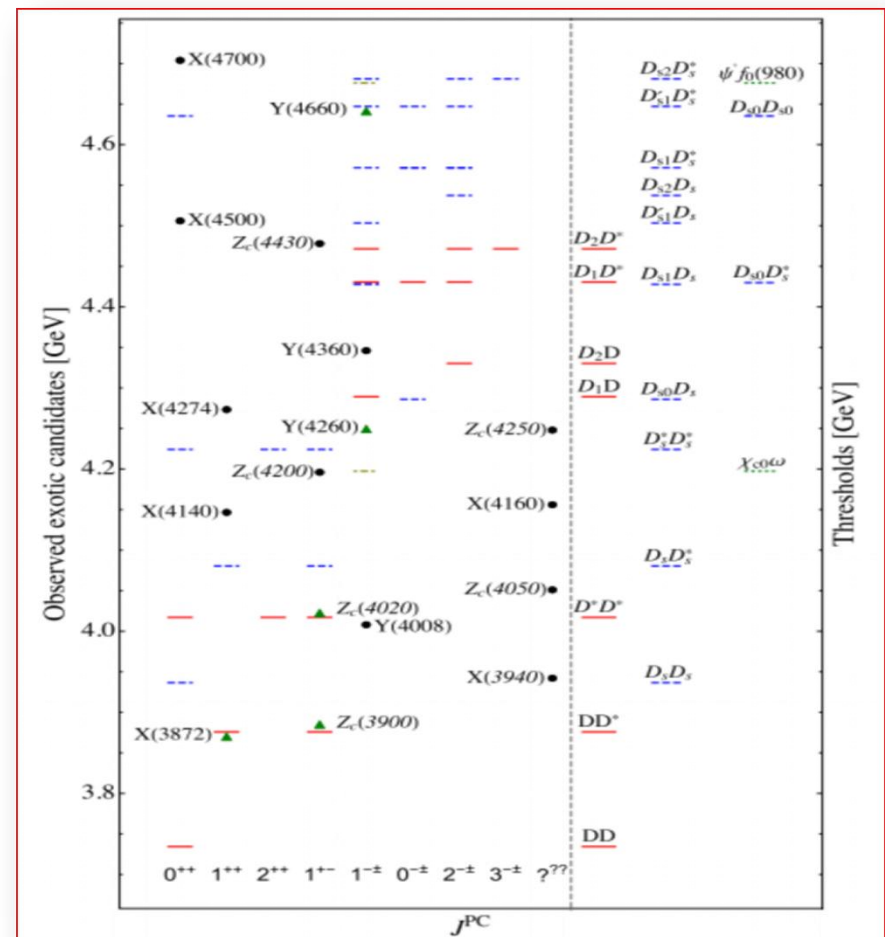
From Li-Sheng Geng



Hadrons



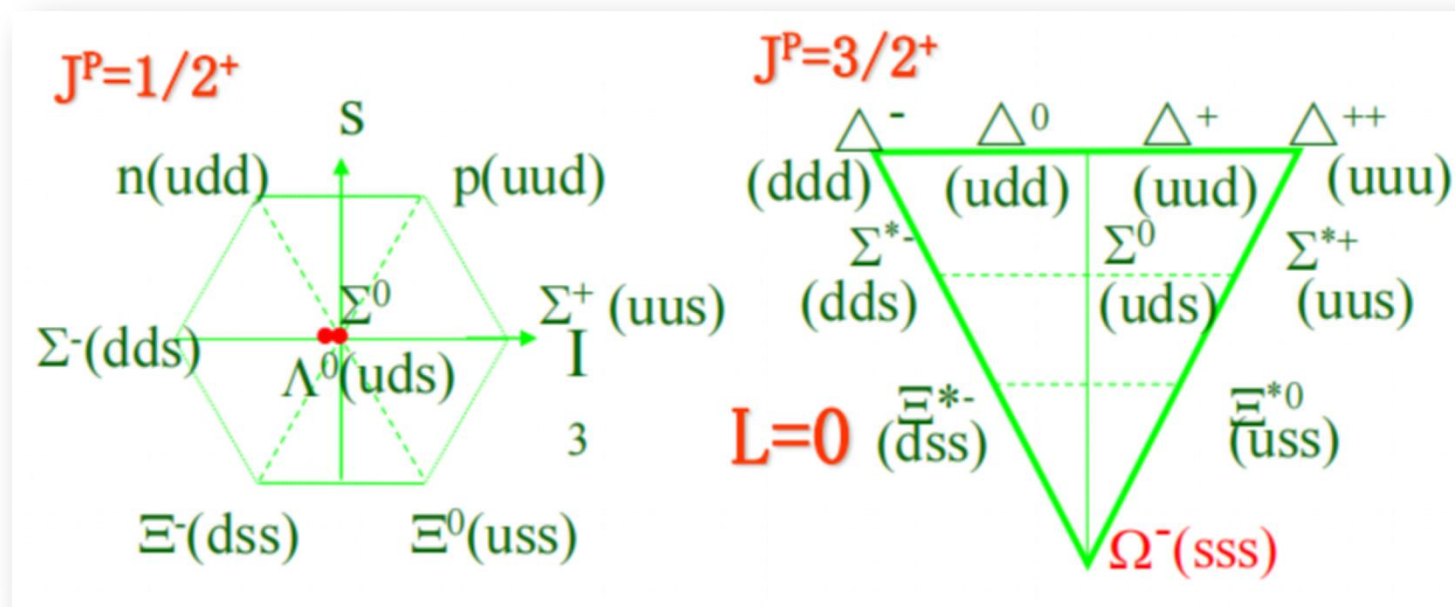
C.Z.Yuan, Nature Rev. Phys. 1 (2019) 480



FKGuo, et.al, Mod. Phys. 90 (2018) 015004

Ground light baryons

Ground baryons



盖尔曼-大久保质量:

$$M = a + bY + c \left[I(I + 1) - \frac{1}{4}Y^2 \right]$$

质量公式预言 $m_{\Omega} = 1670 \text{ MeV}$

实验: $m_{\Omega} = 1672.45 \pm 0.29 \text{ MeV}$



Low-lying baryons with $J^P=1/2^-$

$1/2^-$ baryon nonet with strangeness

Zou, EPJA 35 (2008) 325

- Mass pattern : quenched or unquenched ?

$$uds (L=1) 1/2^- \sim \Lambda^*(1670) \sim [us][ds] \bar{s}$$

$$uud (L=1) 1/2^- \sim N^*(1535) \sim [ud][us] \bar{s}$$

$$uds (L=1) 1/2^- \sim \Lambda^*(1405) \sim [ud][su] \bar{u}$$

$$uus (L=1) 1/2^- \sim \Sigma^*(1390) \sim [us][ud] \bar{d}$$

Zou et al, NPA835 (2010) 199 ; CLAS, PRC87(2013)035206

- Strange decays of $N^*(1535)$ and $\Lambda^*(1670)$:

$N^*(1535)$ large couplings $g_{N^*N\eta}$, $g_{N^*K\Lambda}$, $g_{N^*N\eta'}$, $g_{N^*N\phi}$

$\Lambda^*(1670)$ large coupling $g_{\Lambda^*\Lambda\eta}$

Citation: R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022)

$\Sigma(1620) 1/2^-$

$I(J^P) = 1(\frac{1}{2}^-)$ Status: *

OMITTED FROM SUMMARY TABLE

Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018) and 2019 update

$\Sigma(1480)$ Bumps

$I(J^P) = 1(?^-)$ Status: *

OMITTED FROM SUMMARY TABLE

These are peaks seen in $\Lambda\pi$ and $\Sigma\pi$ spectra in the reaction $\pi^+ p \rightarrow (Y\pi)K^+$ at 1.7 GeV/c. Also, the Y polarization oscillates in the same region.

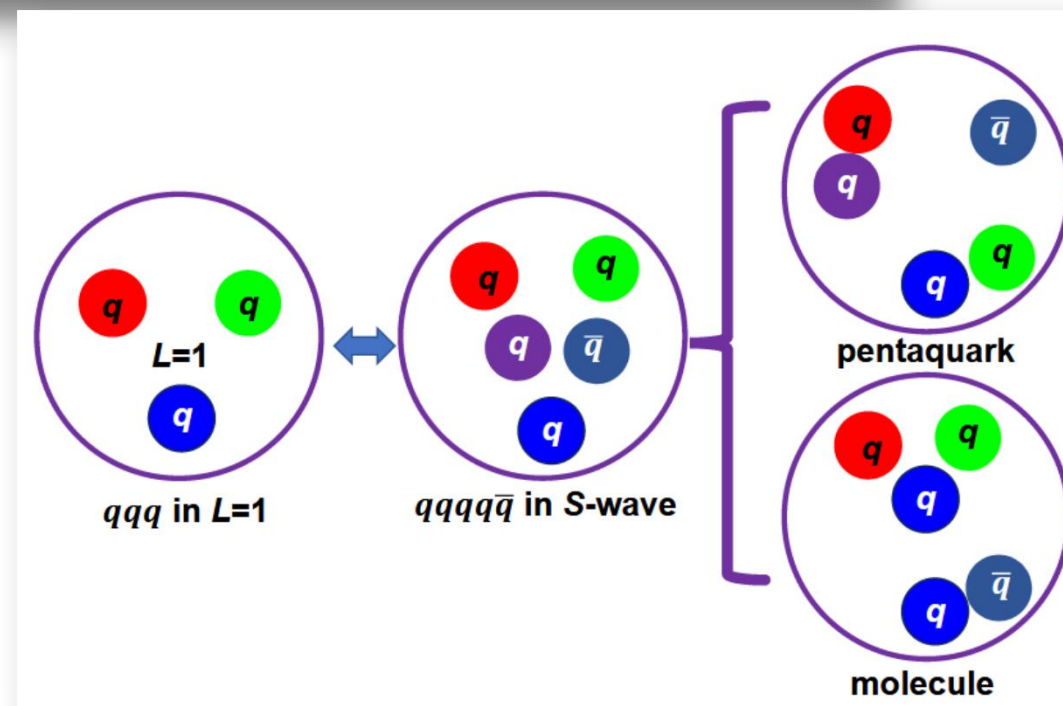
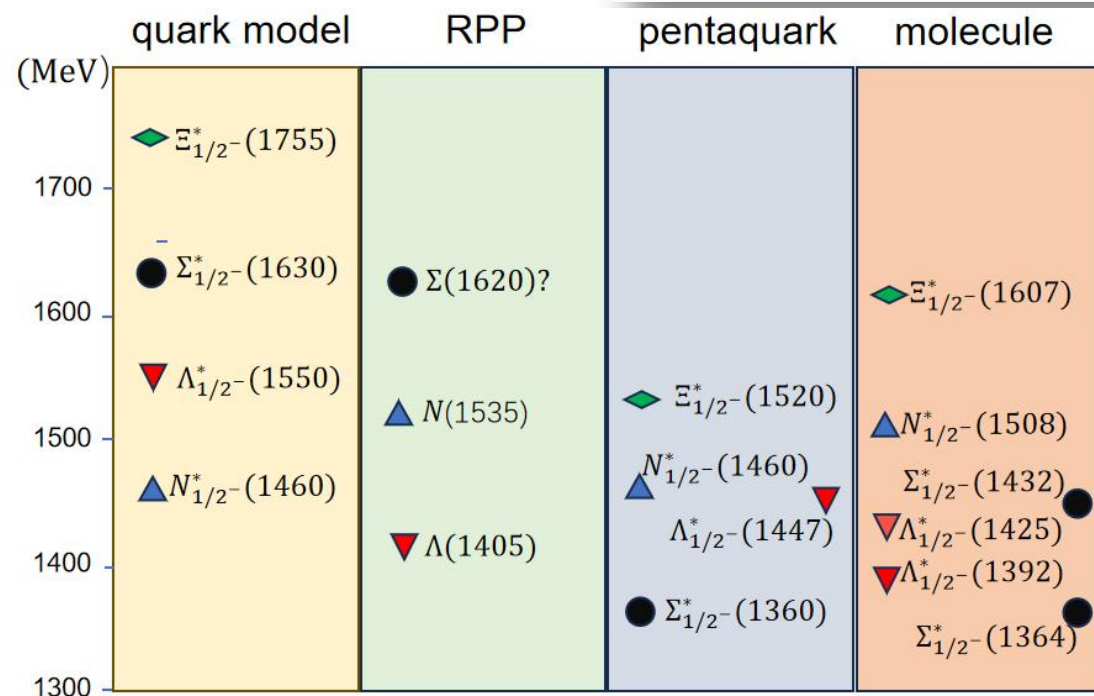
Review about $\Sigma^*(1/2^-)$

arXiv: 2406.07839

Review of the low-lying excited baryons $\Sigma^*(1/2^-)$

En Wang,^{1,2,*} Li-Sheng Geng,^{3,4,5,6,†} Jia-Jun Wu,^{7,6,‡} Ju-Jun Xie,^{6,8,9,§} and Bing-Song Zou^{10,11,7,6,¶}

Strong empirical and phenomenological indications exist for large sea-quark admixtures in the low-lying excited baryons. Investigating the low-lying excited baryon $\Sigma^*(1/2^-)$ is important to determine the nature of the low-lying excited baryons. We review the experimental and theoretical progress on the studies of the $\Sigma^*(1/2^-)$. Although several candidates have received intensive discussions, such as $\Sigma(1620)$ and $\Sigma(1480)$, their existence needs further confirmation. Following the prediction of the unquenched quark models for the $\Sigma^*(1/2^-)$, many theoretical works suggested the existence of these states in various processes. Future experimental measurements could shed light on the existence of the low-lying excited $\Sigma^*(1/2^-)$ state.



1-star state $\Sigma(1620)$

PDG2024

$\Sigma(1620)$ MASS

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
1600 to 1650 (≈ 1620) OUR ESTIMATE			
1681 ± 6	SARANTSEV	19	DPWA $\bar{K}N$ multichannel
1600 ± 15	ZHANG	13A	DPWA $\bar{K}N$ multichannel
1600 ± 6	¹ MORRIS	78	DPWA $K^- n \rightarrow \Lambda \pi^-$
1608 ± 5	² CARROLL	76	DPWA Isospin-1 total σ
1630 ± 10	LANGBEIN	72	IPWA $\bar{K}N$ multichannel
1620	KIM	71	DPWA K-matrix analysis
• • • We do not use the following data for averages, fits, limits, etc. • • •			
1633 ± 10	³ CARROLL	76	DPWA Isospin-1 total σ

$\Sigma(1620)$ WIDTH

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
40 to 100 (≈ 70) OUR ESTIMATE			
40 ± 12	SARANTSEV	19	DPWA $\bar{K}N$ multichannel
400 ± 152	ZHANG	13A	DPWA $\bar{K}N$ multichannel
87 ± 19	¹ MORRIS	78	DPWA $K^- n \rightarrow \Lambda \pi^-$
15	² CARROLL	76	DPWA Isospin-1 total σ
65 ± 20	LANGBEIN	72	IPWA $\bar{K}N$ multichannel
40	KIM	71	DPWA K-matrix analysis
• • • We do not use the following data for averages, fits, limits, etc. • • •			
10	³ CARROLL	76	DPWA Isospin-1 total σ

Eur. Phys. J. A (2019) 55: 180
DOI 10.1140/epja/i2019-12880-5

THE EUROPEAN
PHYSICAL JOURNAL A

Regular Article – Experimental Physics

Hyperon II: Properties of excited hyperons

A.V. Sarantsev^{1,2}, M. Matveev^{1,2}, V.A. Nikonov^{1,2}, A.V. Anisovich^{1,2}, U. Thoma¹, and E. Klempt^{1,a}

$\Sigma(1620)1/2^-$ and $\Sigma(1750)1/2^-$: The $\Sigma(1620)1/2^-$ to $\Sigma(1750)1/2^-$ region is problematic. If we assume no resonance, the fit is unacceptable. A fit with one $1/2^-$ resonance only returns a mass of $M = (1692 \pm 11)$ MeV and $\Gamma = (208 \pm 18)$ MeV. We tentatively identify this resonance with $\Sigma(1750)1/2^-$. The real part of our pole posi-

a mass of (1692 ± 11) MeV, which is below $M_\eta + M_\Sigma$. Our BRs add up to $(78 \pm 11)\%$. A fit with two resonances gives a small but significant improvement for a second narrow resonance, which is found only slightly below $\Sigma(1750)1/2^-$. We list this resonance under $\Sigma(1620)1/2^-$ even though these are likely different objects. We find a sum of branch-

Exp. signals of $\Sigma(1480)$

$$\pi^+ p \rightarrow \pi^+ K^+ \Lambda$$

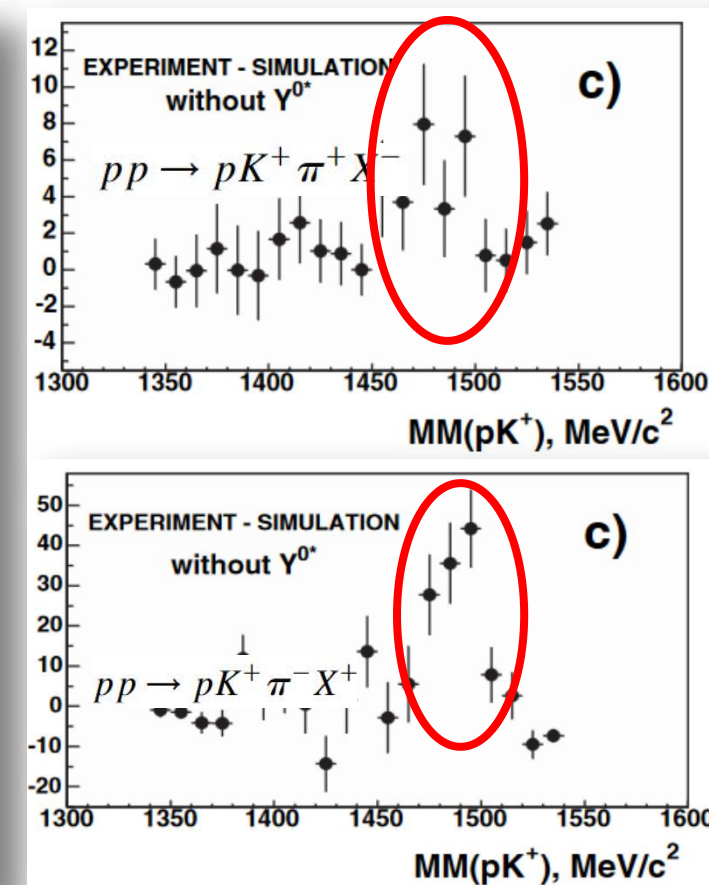
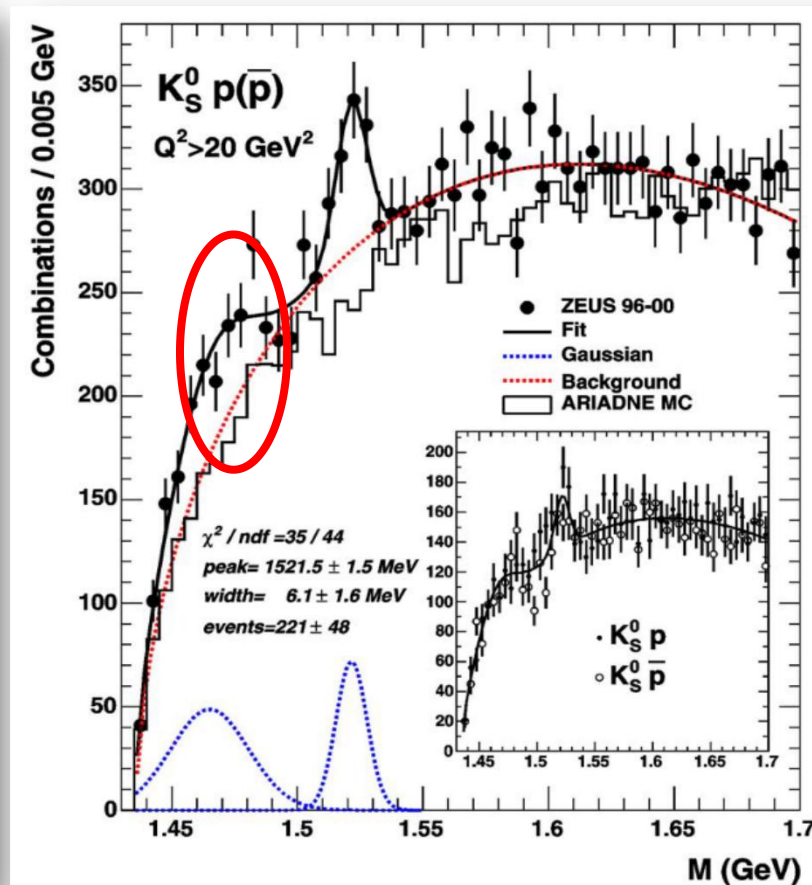
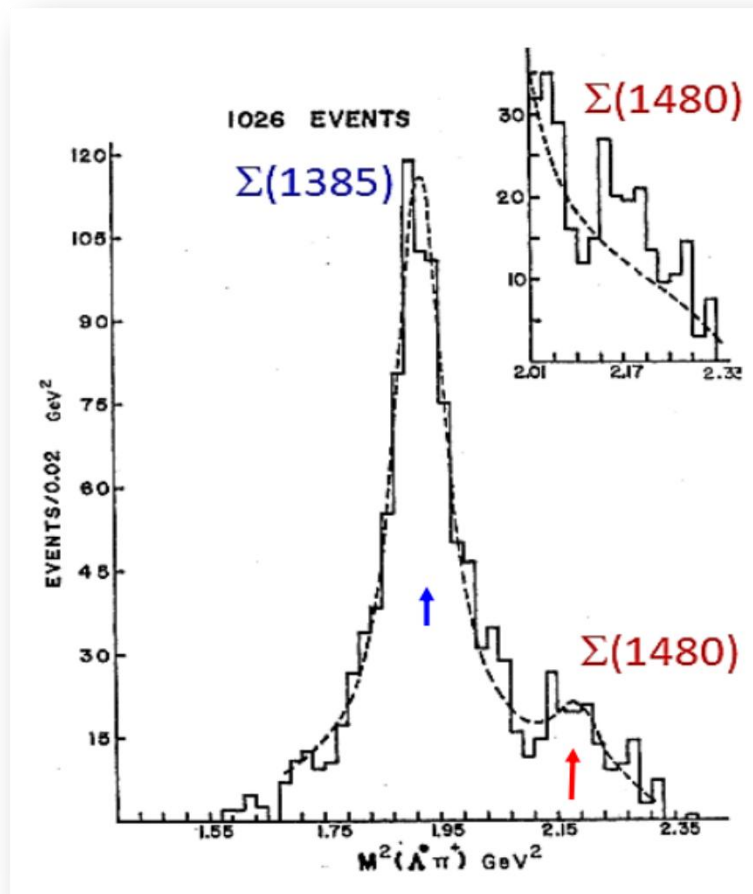
Yu-Li Pan et al, PRD2, 449 (1970)

$$e^+ p \rightarrow e^+ K^0 p X$$

ZEUS PLB591 (2004) 7-22

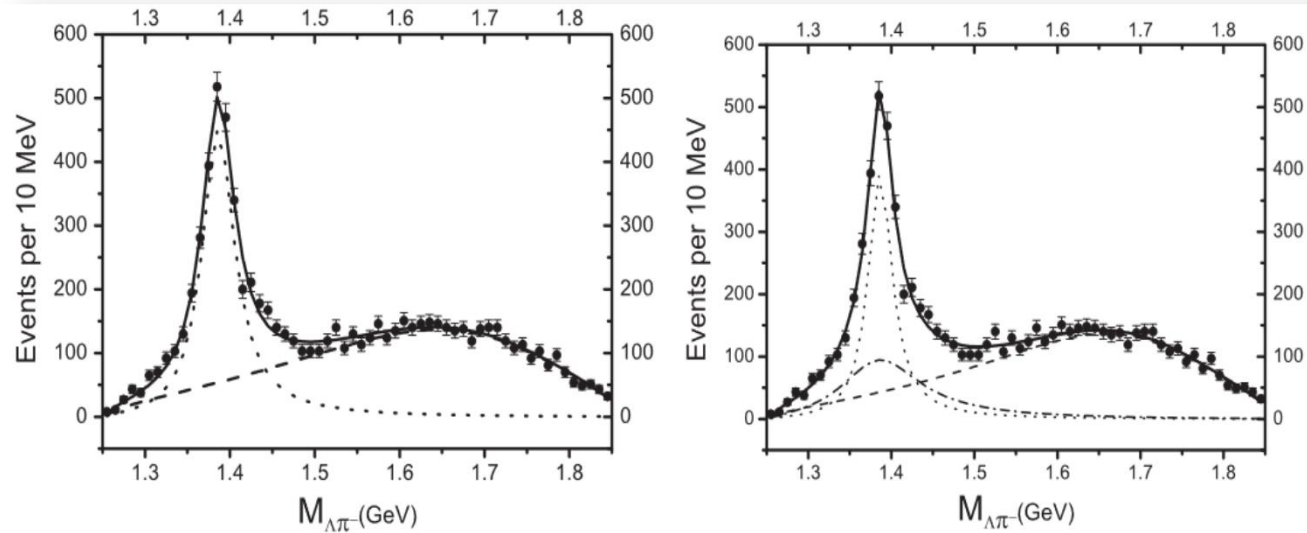
$$pp \rightarrow p K^+ Y^{0*}$$

COSY-Juich PRL 96, 012002 (2006)



Evidence of $\Sigma^*(1/2^-)$

□ $K^- p \rightarrow \Lambda \pi^+ \pi^-$, Wu-Dulat-Zou, PRD80(2009)017503

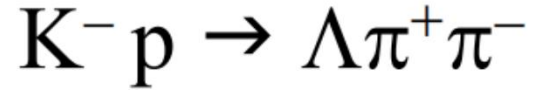


$$\frac{dN}{dm_{\Lambda\pi^-}} \propto p_1 \times p_2 \times \sum_{i=1}^3 \frac{|a_i|}{(m_{\Lambda\pi^-}^2 - m_i^2)^2 + m_i^2 \times \Gamma_i^2},$$

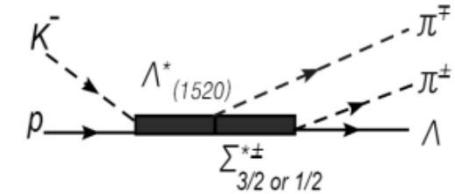
Here we reexamine some old data of the $K^- p \rightarrow \Lambda \pi^+ \pi^-$ reaction and find that besides the well-established $\Sigma^*(1385)$ with $J^P = 3/2^+$, there is indeed some evidence for the possible existence of a new Σ^* resonance with $J^P = 1/2^-$ around the same mass but with broader decay width. There are also indications for such a possibility in the $J/\psi \rightarrow \bar{\Sigma} \Lambda \pi$ and $\gamma n \rightarrow K^+ \Sigma^{*-}$ reactions. At present, the evidence is not strong. Therefore, high statistics studies

	$M_{\Sigma^*(3/2)}$	$\Gamma_{\Sigma^*(3/2)}$	$M_{\Sigma^*(1/2)}$	$\Gamma_{\Sigma^*(1/2)}$	χ^2/ndf (Fig. 1)	χ^2/ndf (Fig. 2)
Fit1	1385.3 ± 0.7	46.9 ± 2.5			68.5/54	10.1/9
Fit2	$1386.1^{+1.1}_{-0.9}$	$34.9^{+5.1}_{-4.9}$	$1381.3^{+4.9}_{-8.3}$	$118.6^{+55.2}_{-35.1}$	58.0/51	3.2/9

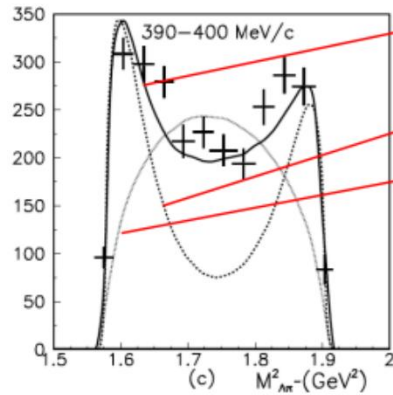
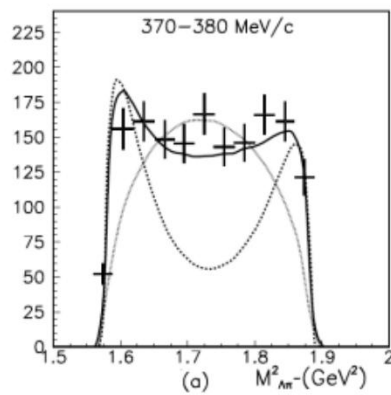
Evidence of $\Sigma^*(1/2^-)$



$P_K = 0.3-0.6$ GeV J. J. Wu, S. Dulat and B. S. Zou PRC 81,045210



J.J.Wu's slide

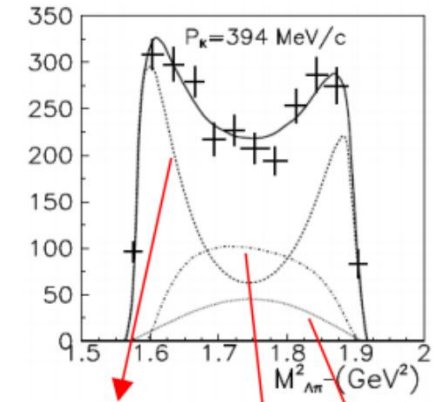


59% $\Sigma^*(3/2^+)$ + 41% $\Sigma^*(1/2^-)$

100% $\Sigma^*(3/2^+)$

Phase space

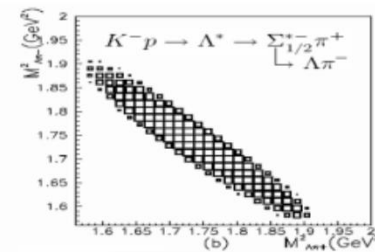
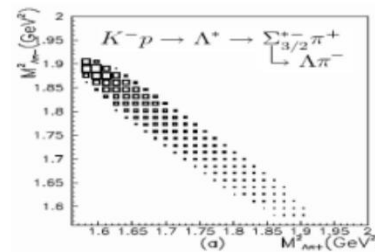
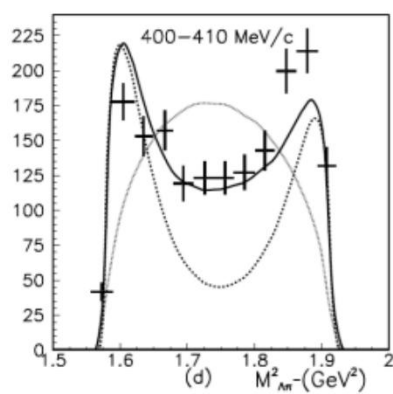
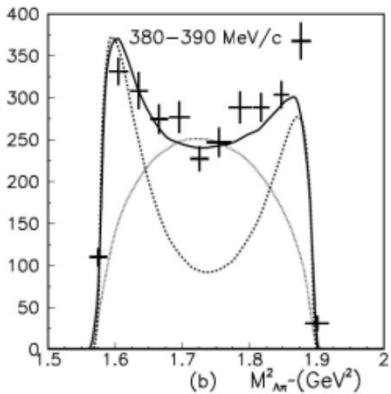
First reason: S-wave between the $\Sigma^*(3/2^+)$ and π^+ ; but P-wave between the $\Sigma^*(1/2^-)$ and π^+ .



59% $\Sigma^*(3/2^+)$

Interference

12.5% $\Sigma^*(1/2^-)$



Second reason: the width of $\Sigma^*(3/2^+)$ is 35.5 MeV; but that of $\Sigma^*(1/2^-)$ is 118.6 MeV from fit before.

Search for $\Sigma^*(1/2^-)$

- $\Lambda_c^+ \rightarrow \Lambda \eta \pi$, Xie-Geng, PRD95(2017) 074024, JJWu-**EW**-LSGeng-JJXie, 2405.09226
- $\gamma n \rightarrow K \Sigma(1/2^-)$, Lyu-**EW**-Xie-Wei, CPC47 (2023) 053108
- $\chi_{c0} \rightarrow \bar{\Sigma} \Sigma \pi$, **EW**-Xie-Oset, PLB753(2016)526
- $\chi_{c0} \rightarrow \bar{\Lambda} \Sigma \pi$, **EW**-Xie-Oset, PRD98(2018)114017
- $\Lambda_c \rightarrow \Sigma^+ \pi^+ \pi^0 \pi^-$, Xie-Oset, Phys.Lett.B 792 (2019) 450
- $\gamma N \rightarrow \Sigma(1/2^-) N$, Kim-Nam-Hosaka, PRD(2021)114017
- $\Lambda_c^+ \rightarrow \bar{K}^0 \eta p$, YLi-SWLiu-**EW**-DMLi-LSGeng-JJXie, 2406.01209
- Review of $\Sigma^*(1/2^-)$, **EW**-JJWu-JJXie-LSGeng-BSZou2406.07839
-

Low-lying baryons with $J^P=1/2^-$

□ Chiral Lagrangian

$$L_1^{(B)} = \langle \bar{B} i \gamma^\mu \nabla_\mu B \rangle - M_B \langle \bar{B} B \rangle \\ + \frac{1}{2} D \langle \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{1}{2} F \langle \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle$$

$$\nabla_\mu B = \partial_\mu B + [\Gamma_\mu, B],$$

$$\Gamma_\mu = \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger),$$

$$U = u^2 = \exp(i\sqrt{2}\Phi/f),$$

$$u_\mu = iu^\dagger \partial_\mu U u^\dagger.$$

**Oset Ramos,
NPA635(1998)99**

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \quad B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

At lowest order in momentum

$$L_1^{(B)} = \langle \bar{B} i \gamma^\mu \frac{1}{4f^2} [(\Phi \partial_\mu \Phi - \partial_\mu \Phi \Phi) B - B(\Phi \partial_\mu \Phi - \partial_\mu \Phi \Phi)] \rangle,$$

$$V_{ij} = -C_{ij} \frac{1}{4f^2} \bar{u}(p') \gamma^\mu u(p) (k_\mu + k'_\mu)$$



**Neglect the spatial components at
low energies**

$$V_{ij} = -C_{ij} \frac{1}{4f^2} (k^0 + k'^0)$$

Low-lying baryons with $J^P=1/2^-$

$$V_{ij} = -\textcircled{C_{ij}} \frac{1}{4f^2} (k^0 + k'^0)$$

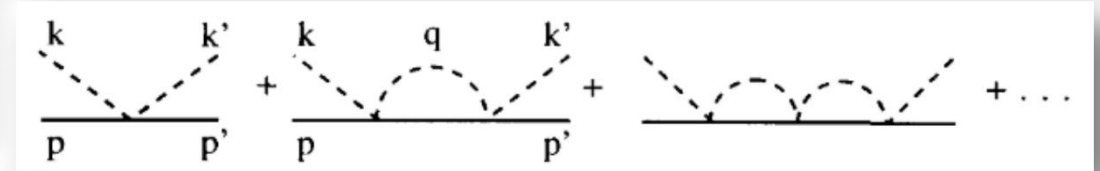
I=0	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$
$\bar{K}N$	3	$-\sqrt{\frac{3}{2}}$	$\frac{3}{\sqrt{2}}$	0
$\pi\Sigma$		4	0	$\sqrt{\frac{3}{2}}$
$\eta\Lambda$			0	$-\frac{3}{\sqrt{2}}$
$K\Xi$				3

I=1	$\bar{K}N$	$\pi\Sigma$	$\pi\Lambda$	$\eta\Sigma$	$K\Xi$
$\bar{K}N$	1	-1	$-\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{3}{2}}$	0
$\pi\Sigma$		2	0	0	1
$\pi\Lambda$			0	0	$-\sqrt{\frac{3}{2}}$
$\eta\Sigma$				0	$-\sqrt{\frac{3}{2}}$
$K\Xi$					1

Lippmann-Schwinger equations

$$t_{ij} = V_{ij} + V_{il} G_l T_{lj},$$

$$V_{il} G_l T_{lj} = i \int \frac{d^4 q}{(2\pi)^4} \frac{M_l}{E_l(\mathbf{q})} \frac{V_{il}(\mathbf{k}, \mathbf{q}) T_{lj}(\mathbf{q}, \mathbf{k}')}{k^0 + p^0 - q^0 - E_l(\mathbf{q}) + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}.$$



On-shell approximations

$$2iV_{\text{on}} \int \frac{d^3 q}{(2\pi)^3} \int \frac{dq^0}{2\pi} \frac{M}{E(q)} \frac{q^0 - k^0}{k^0 - q^0} \frac{1}{q^{02} - \omega(q)^2 + i\epsilon}$$

Low-lying baryons with $J^P=1/2^-$

□ Bethe-Salpter Equation

$$T = [1 - VG]^{-1}V$$

$$G_l = i \int \frac{d^4 q}{(2\pi)^4} \frac{M_l}{E_l(\mathbf{q})} \frac{1}{k^0 + p^0 - q^0 - E_l(\mathbf{q}) + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}$$

$$= \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2\omega_l(q)} \frac{M_l}{E_l(\mathbf{q})} \frac{1}{p^0 + k^0 - \omega_l(\mathbf{q}) - E_l(\mathbf{q}) + i\epsilon},$$

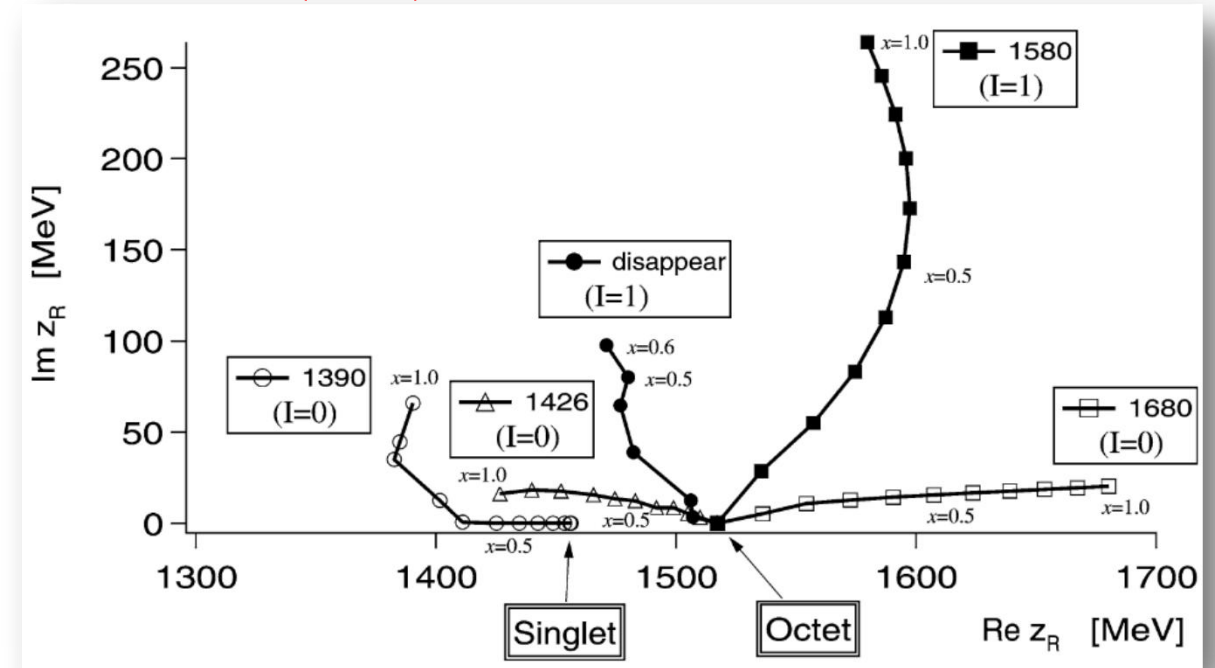
$$G_l = i2M_l \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(P-q)^2 - M_l^2 + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}$$

$$= \frac{2M_l}{16\pi^2} \left\{ a_l(\mu) + \ln \frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s} \ln \frac{m_l^2}{M_l^2} \right.$$

$$+ \frac{q_l}{\sqrt{s}} [\ln(s - (M_l^2 - m_l^2) + 2q_l\sqrt{s}) + \ln(s + (M_l^2 - m_l^2) + 2q_l\sqrt{s})$$

$$\left. - \ln(-s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) - \ln(-s - (M_l^2 - m_l^2) + 2q_l\sqrt{s})] \right\}$$

Jido Oller Oset Ramos Meissner
NPA725 (2003) 181

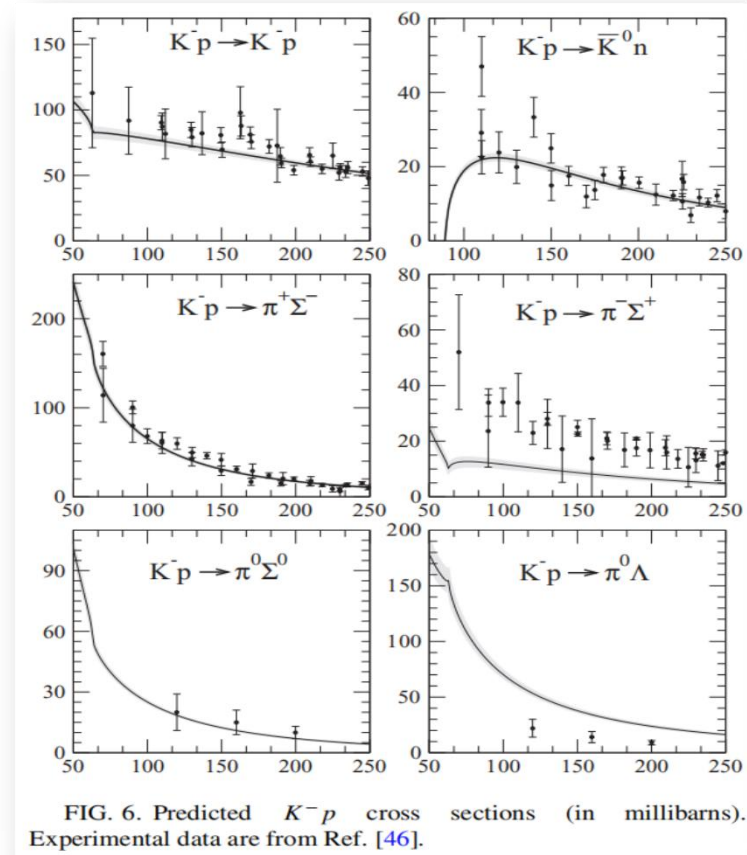
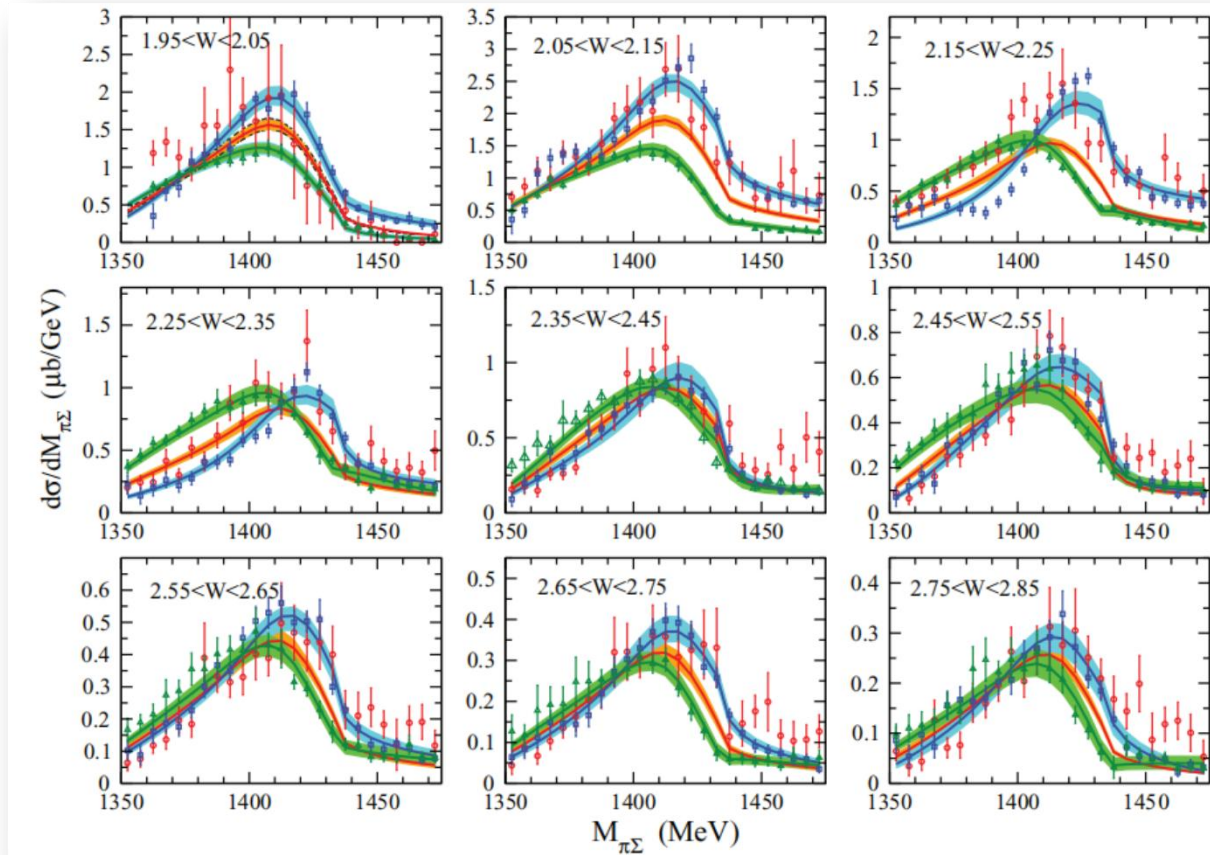


pole positions and couplings

$$T_{ij} = \frac{g_i g_j}{z - z_R}$$

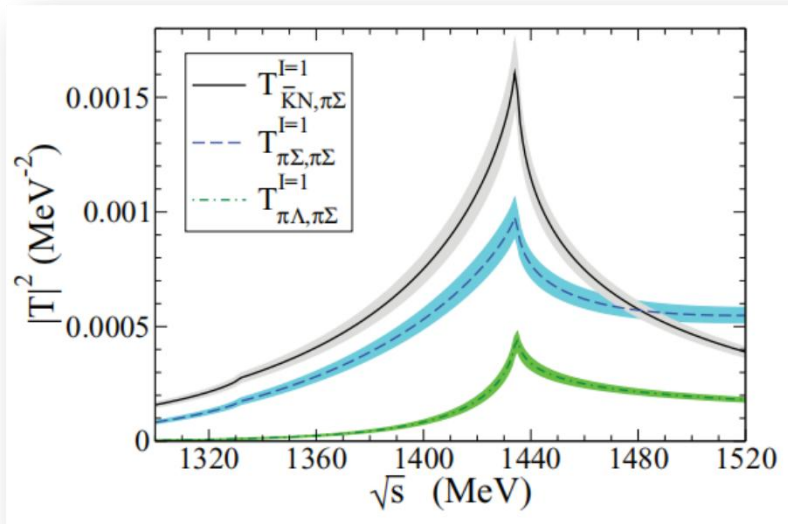
$\Sigma^*(1/2^-)$ in the $\pi\Sigma$ photoproduction

□ $\pi\Sigma$ photoproduction, Roca-Oset, PRC 88, 055206 (2013)



$\Sigma(1430)$

□ $\pi\Sigma$ photoproduction, **Roca-Oset, PRC 88, 055206 (2013)**



$$T = [1 - VG]^{-1} V$$

$$V_{ij} = -C_{ij} \frac{1}{4f^2} (k^0 + k'^0)$$

$$C_{ij}^1 = \begin{pmatrix} \alpha_{11}^1 & -\alpha_{12}^1 & -\sqrt{\frac{3}{2}}\alpha_{13}^1 \\ -\alpha_{12}^1 & 2\alpha_{22}^1 & 0 \\ -\sqrt{\frac{3}{2}}\alpha_{13}^1 & 0 & 0 \end{pmatrix}$$

α_{11}^0	α_{12}^0	α_{22}^0	α_{11}^1	α_{12}^1	α_{13}^1	α_{22}^1
1.037	1.466	1.668	0.85	0.93	1.056	0.77

□ Oset-Ramos, NPA635 (1998) 99 [nucl-th/9711022].

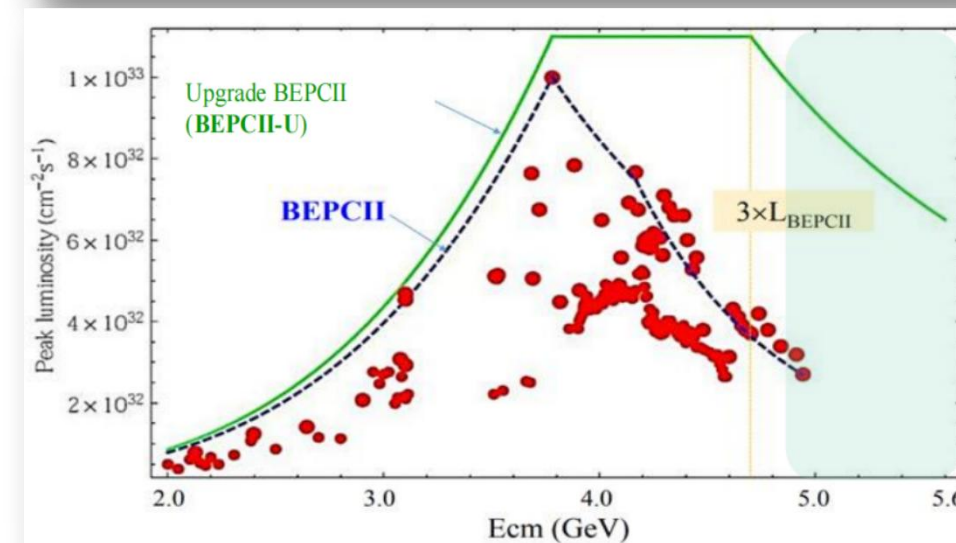
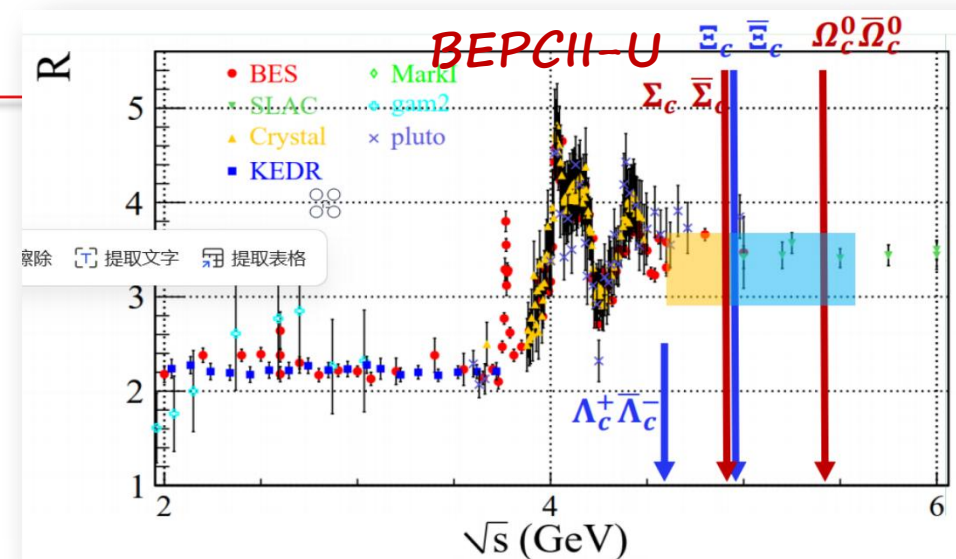
□ PB, VB, Hosaka, PRD 85, 114020 (2012)

□ Oller-Meißner, Phys. Lett. B 500 (2001) 263 [hep-ph/0011146]

Λ_c^+ at BESIII

Hadronic decays	
$\Lambda_c \rightarrow pK\pi + 11 \text{ CF modes}$	PRL 116, 052001 (2016)
$\Lambda_c \rightarrow pK^+K^-, p\pi^+\pi^-$	PRL 117, 232002 (2016)
$\Lambda_c \rightarrow nK_s\pi$	PRL 118, 112001 (2017)
$\Lambda_c \rightarrow p\eta, p\pi^0$	PRD 95, 111102(R) (2017)
$\Lambda_c \rightarrow \Sigma\pi^+\pi^-\pi^0$	PLB 772, 338 (2017)
$\Lambda_c \rightarrow \Xi^{0(*)}K$	PLB 783, 200 (2018)
$\Lambda_c \rightarrow \Lambda\eta\pi$	PRD 99, 032010 (2019)
$\Lambda_c \rightarrow pK_s\eta$	PLB 817 (2021) 136327

7月2日下午耿聪老师报告



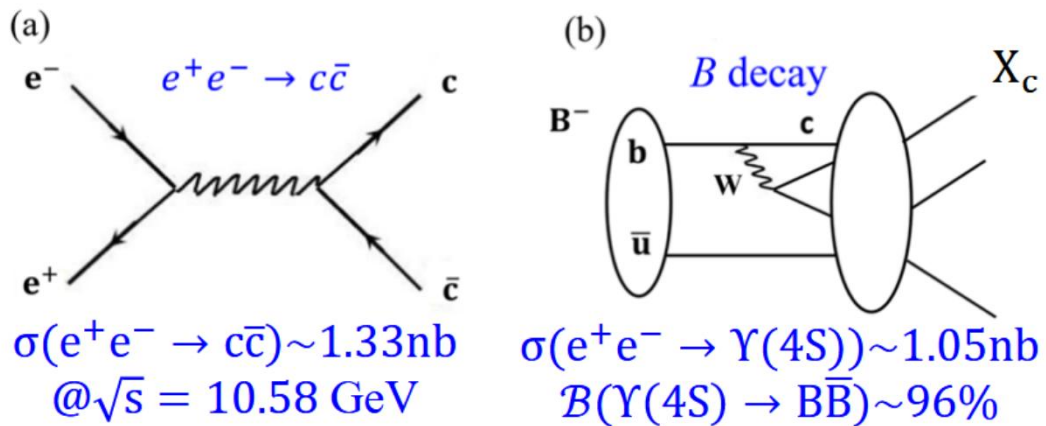
7月2日上午郑阳恒老师报告

Charm hadrons at Belle II/LHCb

Experiment	Machine	Operation	C.M.	Luminosity	N_{prod}	Efficiency	Characters
	BEPC-II (e^+e^-)	2010-2011 (2021-) 2016-2019 2014+2020	3.77 GeV 4.18-4.23 GeV 4.6-4.7 GeV	2.9 (8 → 20) fb^{-1} 7.3 fb^{-1} 4.5 fb^{-1}	$D^{0,+}$: $10^7 (\rightarrow 10^8)$ D_s^+ : 5×10^6 Λ_c^+ : 0.8×10^6 ★★☆	~ 10-30% ★★★	☺ extremely clean environment ☺ quantum coherence ☺ pure D-beam, almost no background ☹ no CM boost, no time-dept analyses
	SuperKEKB (e^+e^-)	2019-	10.58 GeV	0.4 (→ 50) ab^{-1}	D^0 : $6 \times 10^8 (\rightarrow 10^{11})$ $D_{(s)}^+$: $10^8 (\rightarrow 10^{10})$ Λ_c^+ : $10^7 (\rightarrow 10^9)$	$\mathcal{O}(1-10\%)$	☺ clear event environment ☺ high trigger efficiency ☺ good-efficiency detection of neutrals ☺ time-dependent analysis ☹ smaller cross-section than LHCb
	KEKB (e^+e^-)	1999-2010	10.58 GeV	1 ab^{-1}	D : 10^9 Λ_c^+ : 10^8 ★★☆	★★	

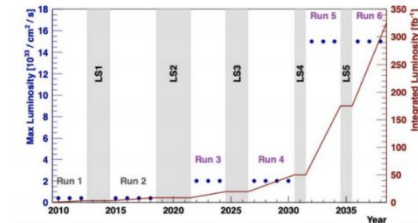
复旦大学 李阳报告

Two ways to produce charm samples at Belle (II)



Future opportunity at LHCb

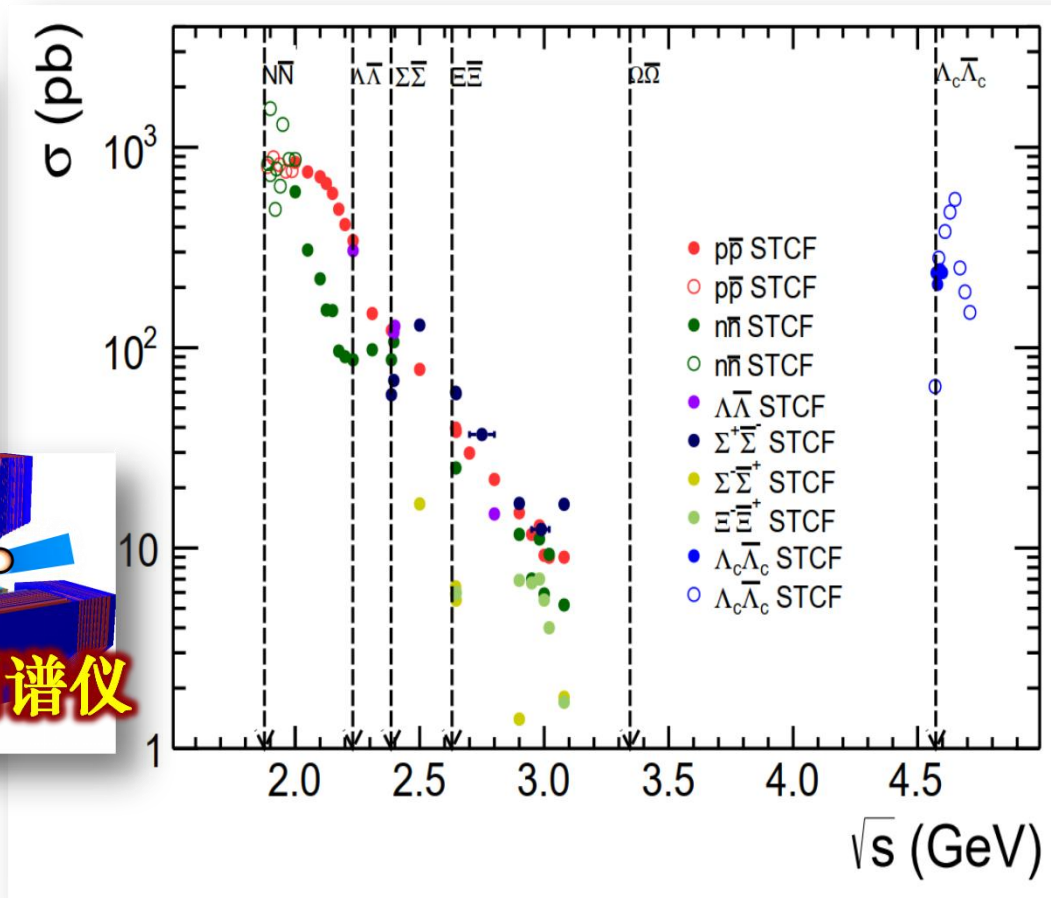
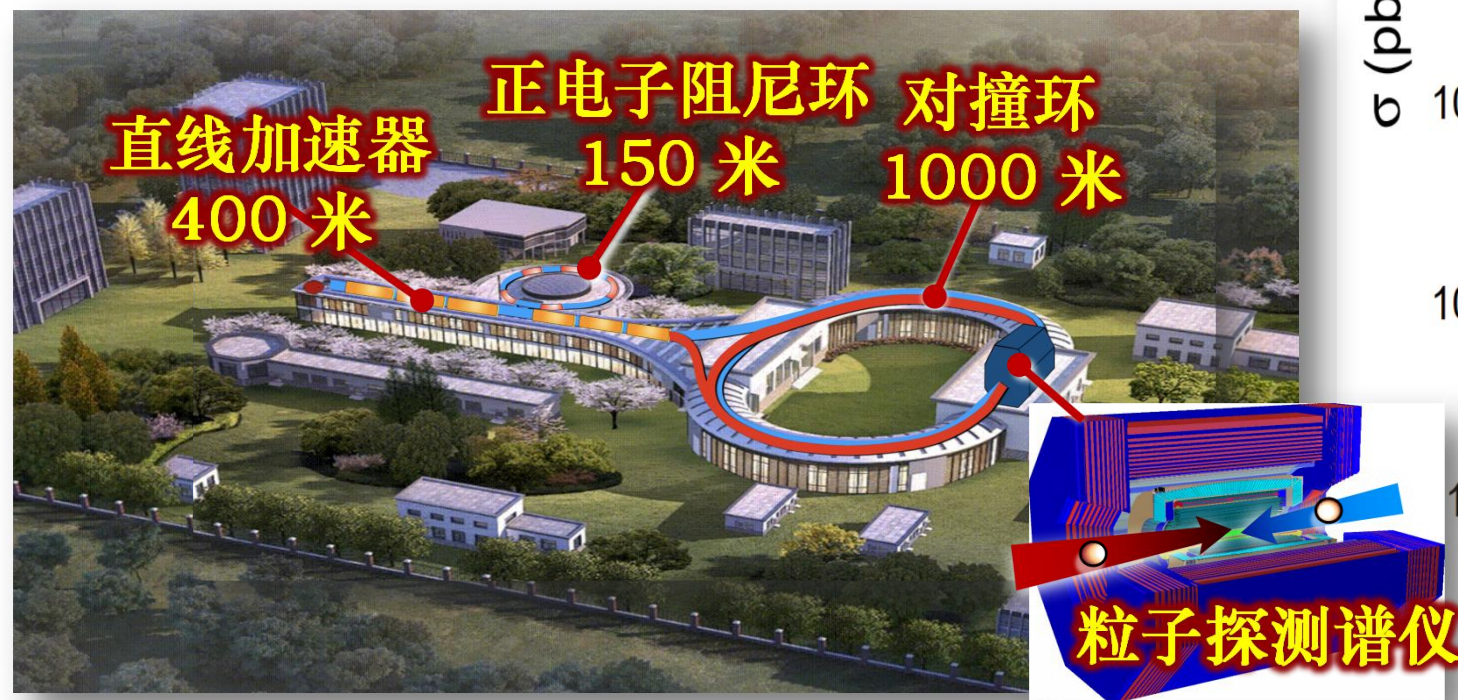
- RUN1&2: 9 fb^{-1}
- RUN3&4: 50 fb^{-1}
- x10 more statistics



- Further improvement on mass and lifetime measurement
- SCS and DCS hadronic decays
 - e.g. $\Xi_c^0 \rightarrow pK^-$, $\Xi_c^+ \rightarrow pK_S$, $\Omega_c^0 \rightarrow \Lambda K_S$, pK^-
- Semi-leptonic decays via b-baryon four-body decays
 - e.g. $\Lambda_c^+ \rightarrow pK^- \mu^+ \nu$, $p\pi^- \mu^+ \nu$; $\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu$; $\Xi_c^+ \rightarrow \Lambda \mu^+ \nu$; $\Omega_c^0 \rightarrow \Omega^- \mu^+ \nu$
- Decay asymmetries and CPV search via prompt production or b-baryon decays
 - e.g. $\Lambda_c^+ \rightarrow pK_S$, $\Lambda\pi^+$, ΛK^+ ; $\Xi_c^0 \rightarrow \Lambda K_S$, $\Xi^- \pi^+$, $\Xi^- K^+$; $\Omega_c^0 \rightarrow \Omega^- \pi^+$, $\Omega^- K^+$, $\Xi^- \pi^+$
- Amplitude analysis of multi-body hadronic decays

7月2日上午郑阳恒老师报告

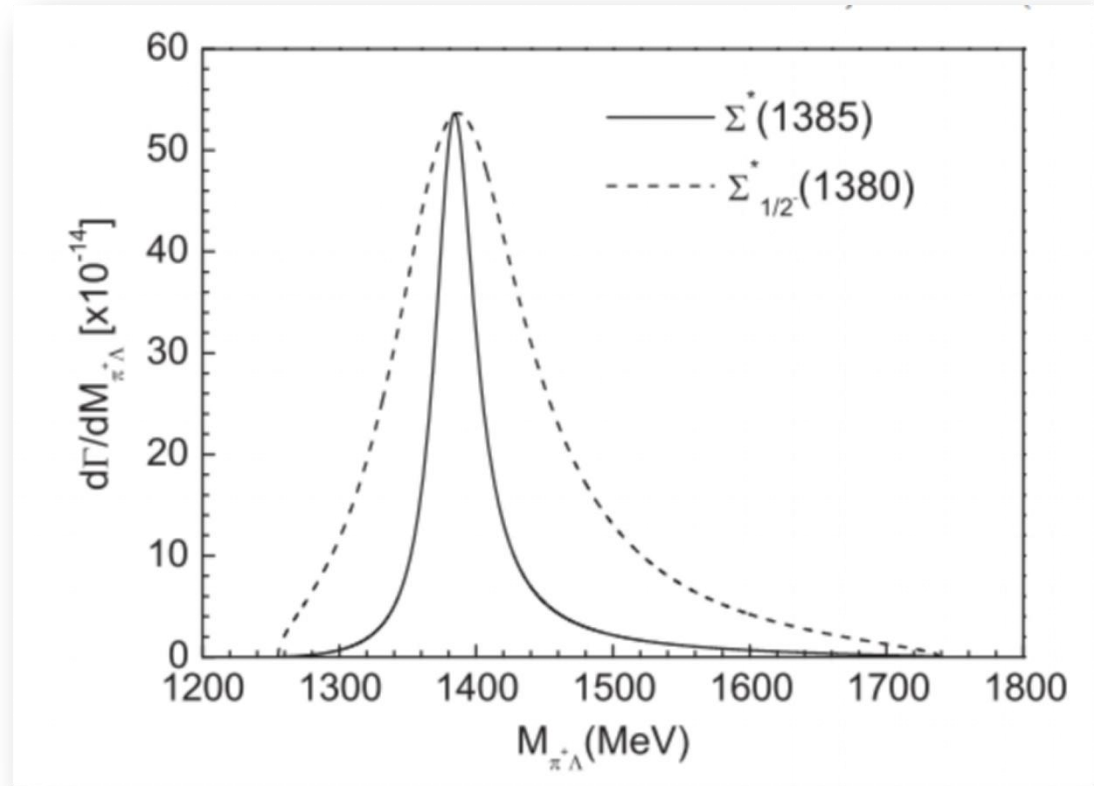
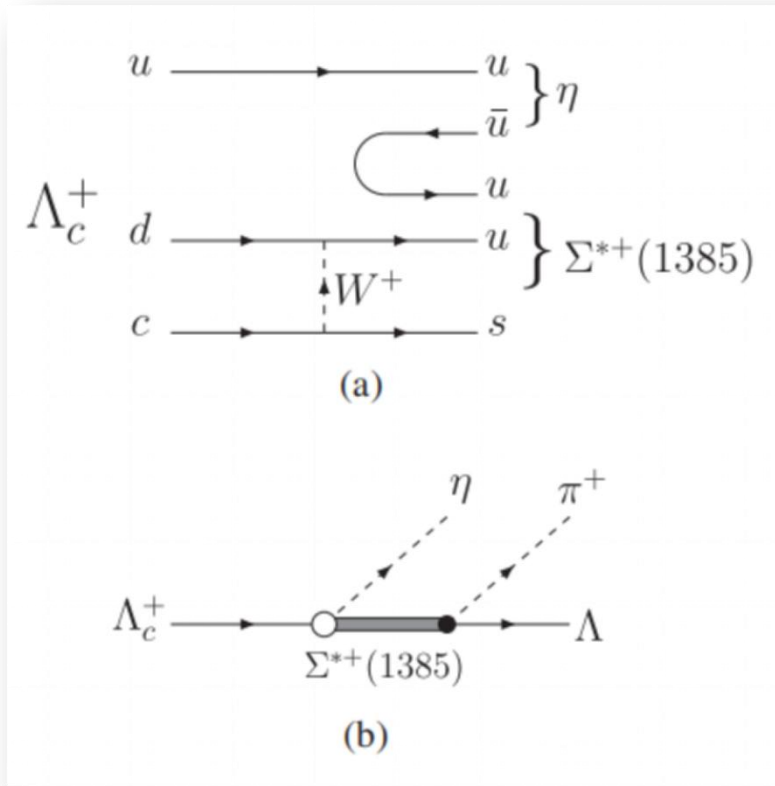
Charm hadrons at STCF



7月1日下午彭海平老师报告

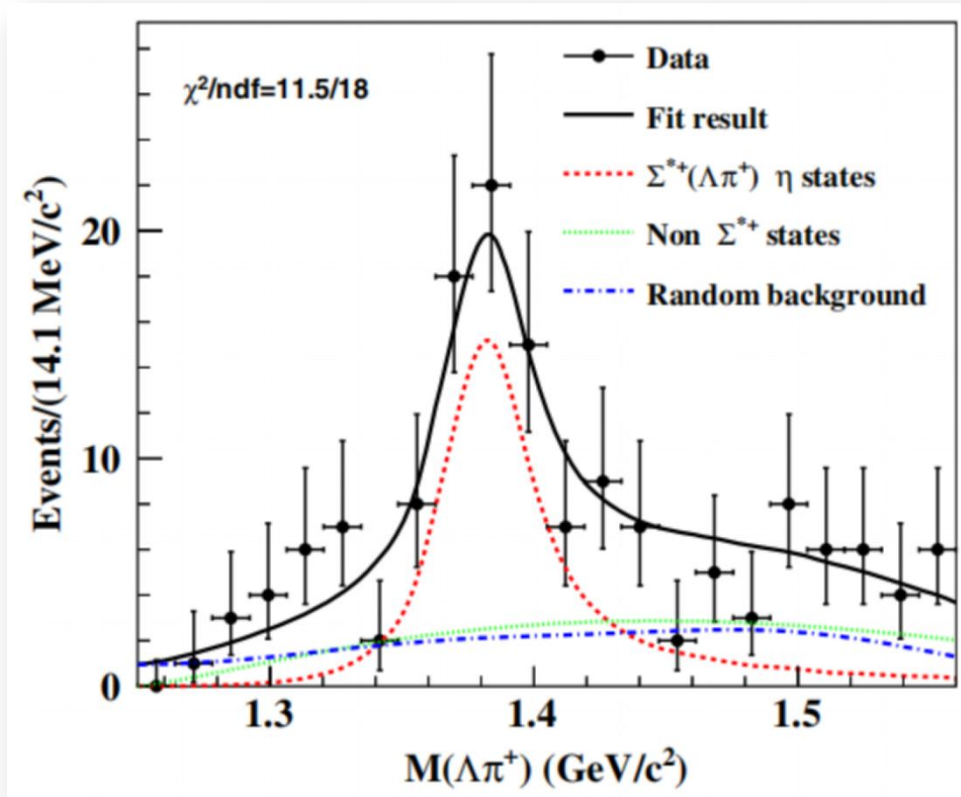
$\Sigma^*(1/2^-)$ in $\Lambda_c^+ \rightarrow \Lambda \eta \pi$

□ J.J.Xie, L.S.Geng, EPJC76(2016) 496, PRD95(2017) 074024

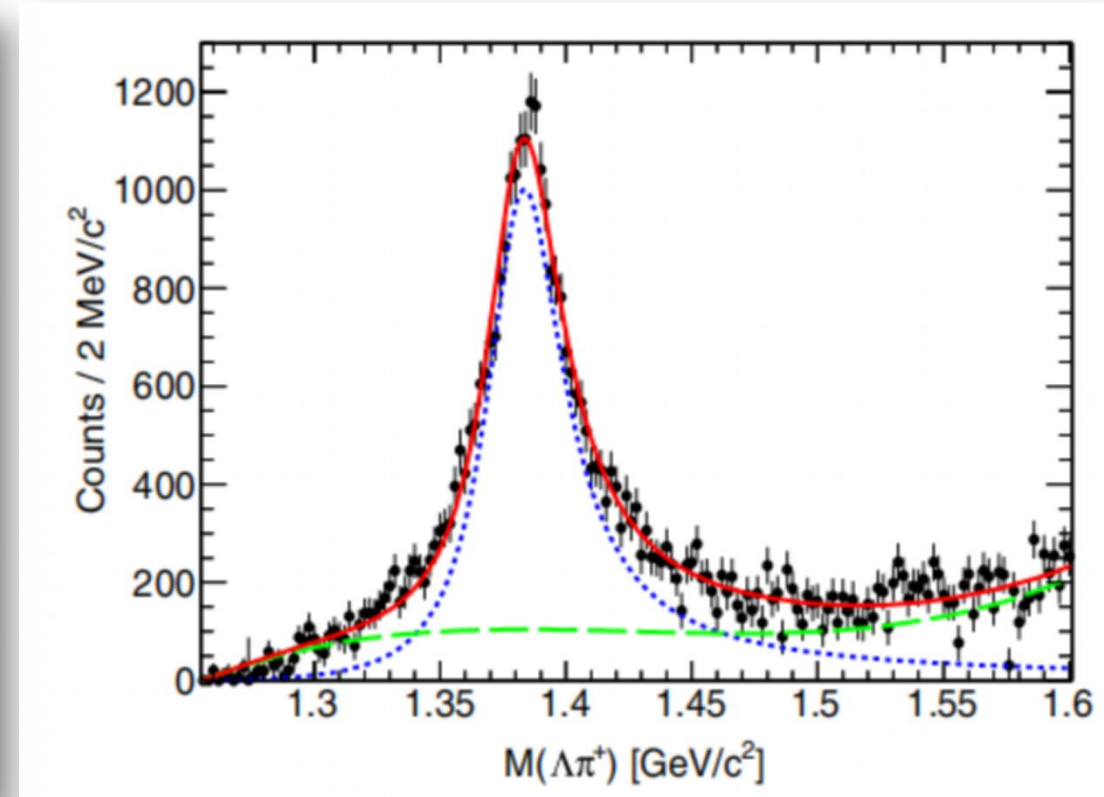


Belle and BESIII measurements

$$\Lambda_c^+ \rightarrow \Lambda \eta \pi$$



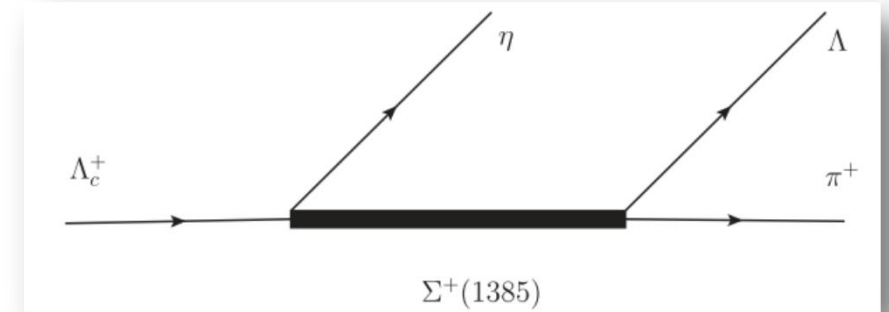
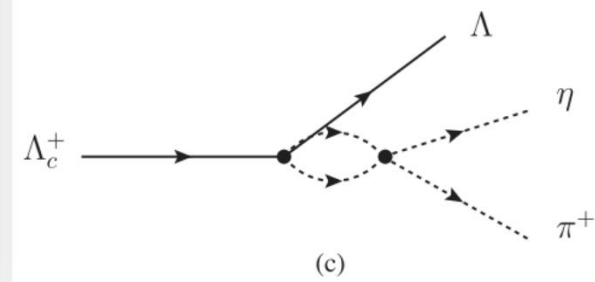
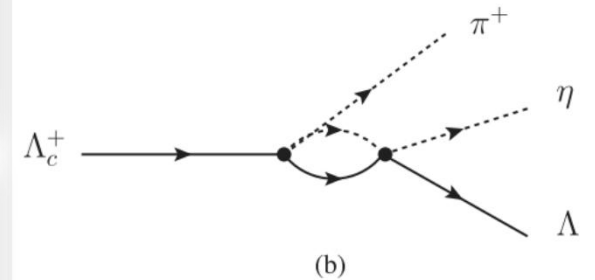
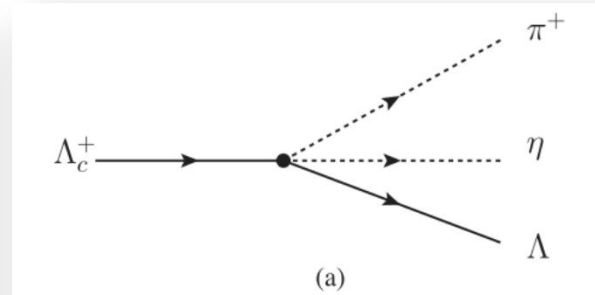
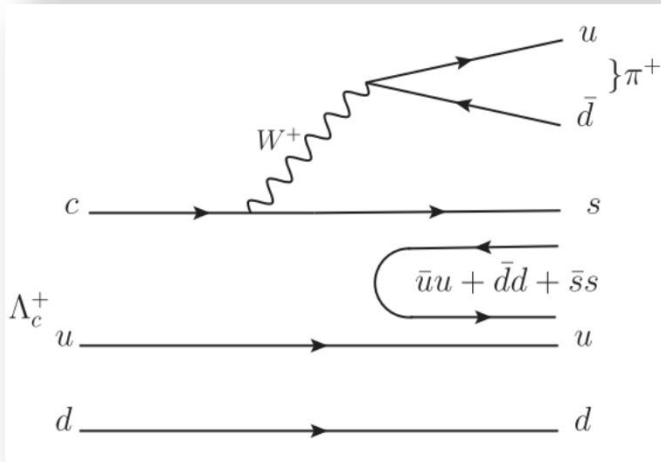
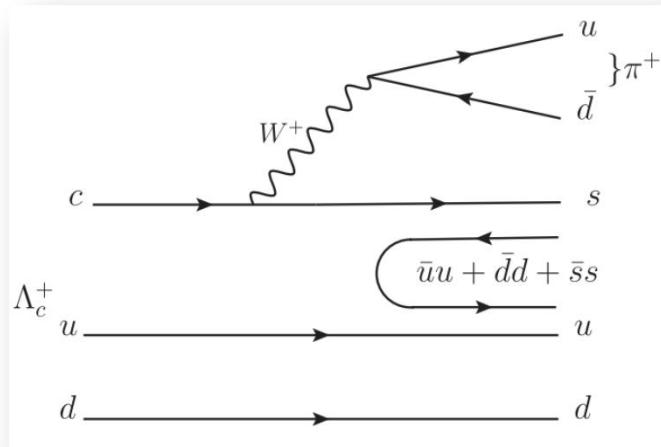
BESIII: PRD99, 032010 (2019)



Belle: PRD103(2021)052005

Mechanism of $\Lambda_c^+ \rightarrow \eta \Lambda \pi$

□ Theoretical model



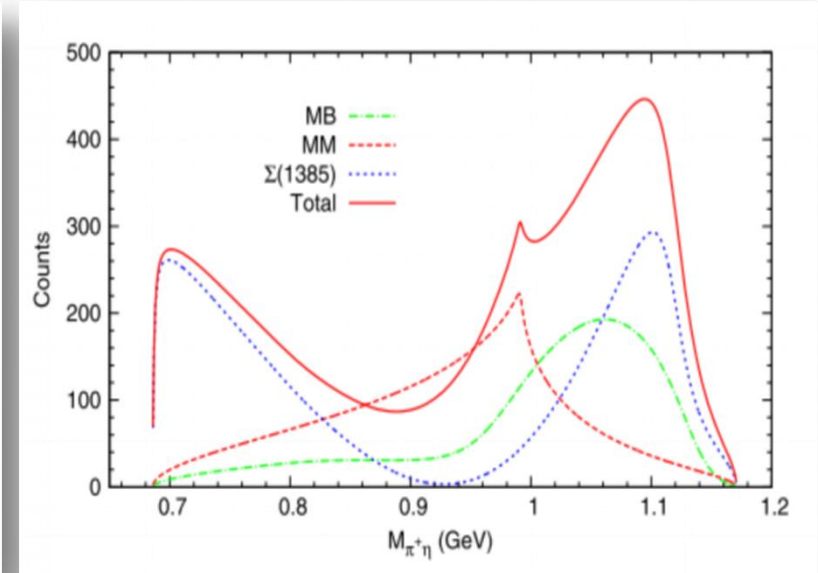
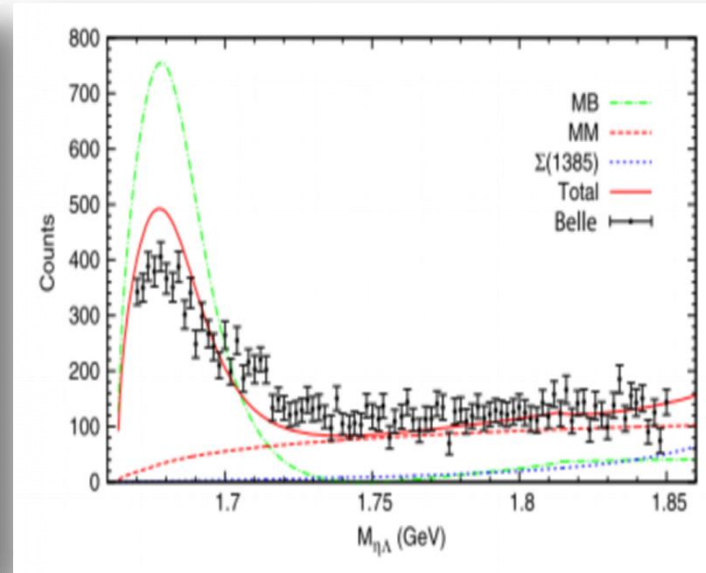
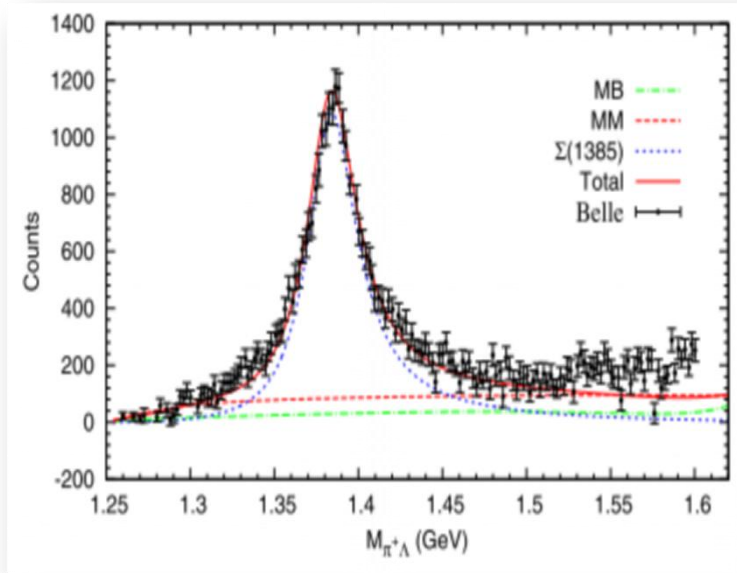
$$T^{\Sigma^*}(M_{\pi^+\Lambda}) = V_P'' \frac{|\vec{p}_\pi| \cdot |\vec{p}_\eta| \cdot \cos \theta}{M_{\pi^+\Lambda} - M_{\Sigma^*} + i \frac{\Gamma_{\Sigma^*}}{2}},$$

$$T^{\text{MB}}(M_{\eta\Lambda}) = V_P \left\{ -\frac{\sqrt{2}}{3} + G_{K^-p}(M_{\eta\Lambda}) t_{K^-p \rightarrow \eta\Lambda}(M_{\eta\Lambda}) \right. \\ \left. + G_{\bar{K}^0 n}(M_{\eta\Lambda}) t_{\bar{K}^0 n \rightarrow \eta\Lambda}(M_{\eta\Lambda}) \right. \\ \left. - \frac{\sqrt{2}}{3} G_{\eta\Lambda}(M_{\eta\Lambda}) t_{\eta\Lambda \rightarrow \eta\Lambda}(M_{\eta\Lambda}) \right\},$$

$$T^{\text{MM}}(M_{\pi^+\eta}) = V_P' \frac{2\sqrt{2}}{3} \left\{ 1 + G_{\pi^+\eta}(M_{\pi^+\eta}) t_{\pi^+\eta \rightarrow \pi^+\eta}(M_{\pi^+\eta}) \right. \\ \left. + \frac{\sqrt{3}}{2} G_{K^+\bar{K}^0}(M_{\pi^+\eta}) t_{K^+\bar{K}^0 \rightarrow \pi^+\eta}(M_{\pi^+\eta}) \right\}, \quad ($$

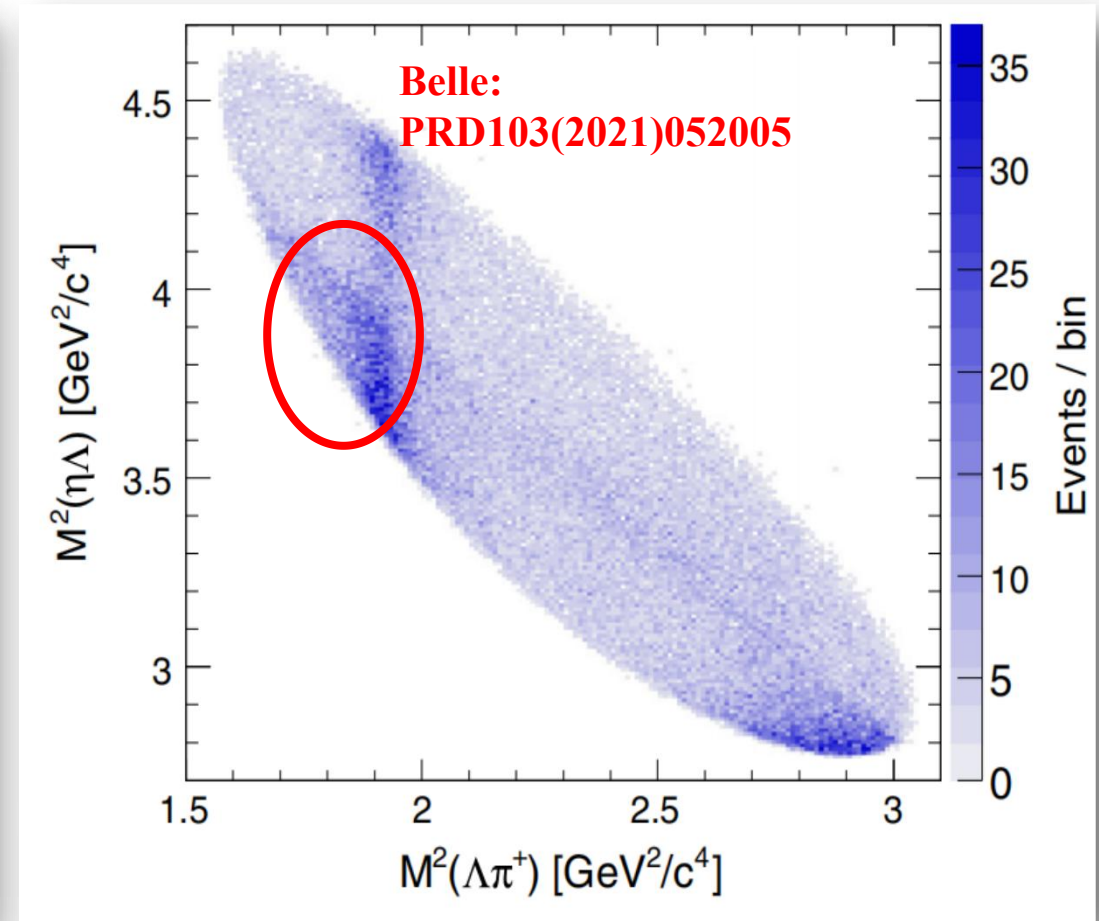
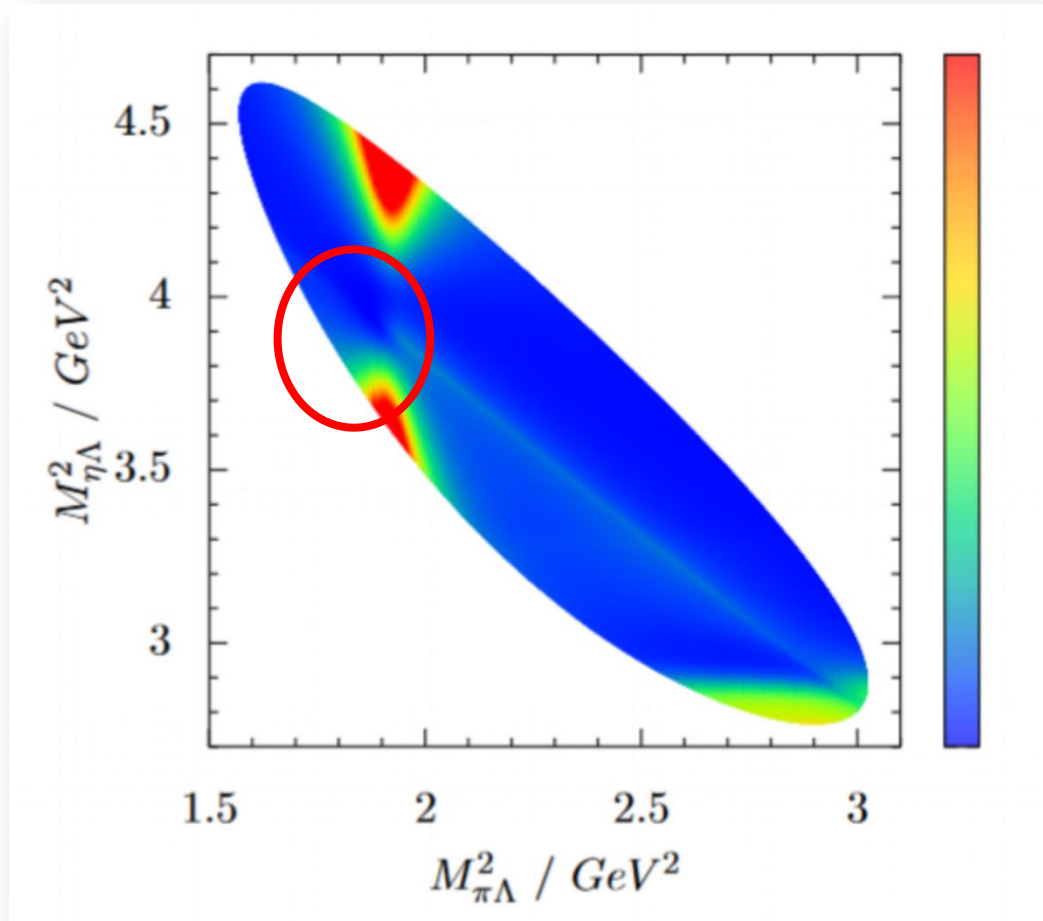
Analysis the Belle data

□ $\Lambda_c^+ \rightarrow \Lambda \eta \pi$, GYW-EW-Xie-Geng-Wei, PRD 106, 056001 (2022)



By regarding the $\Lambda(1670)$ as the molecule, we could well reproduce the Belle data of the mass distributions.

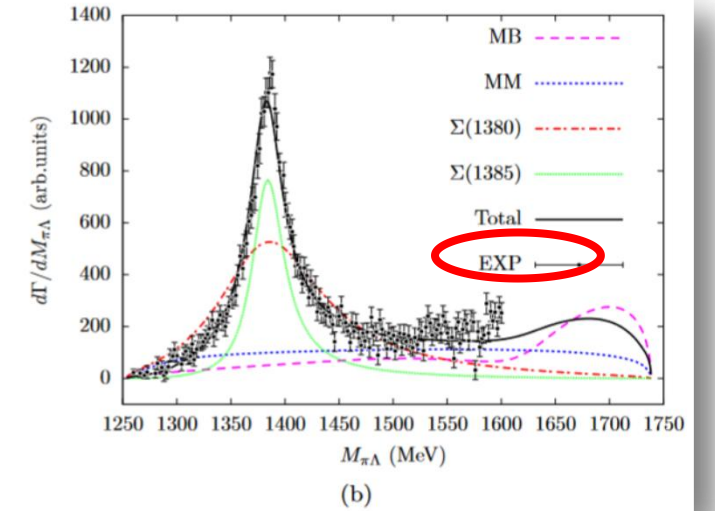
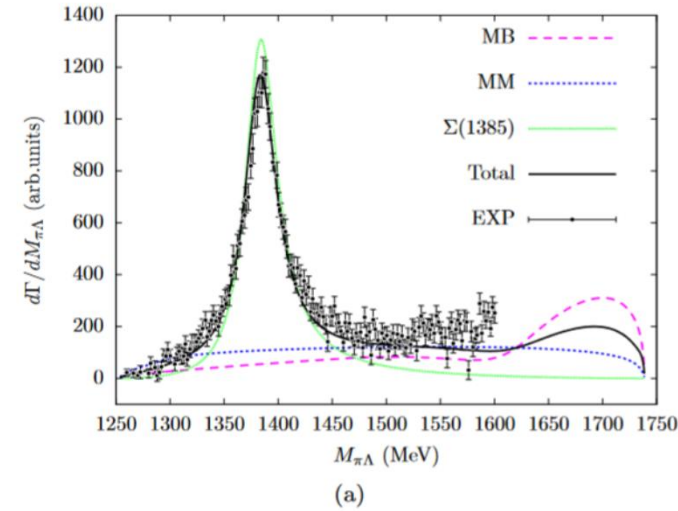
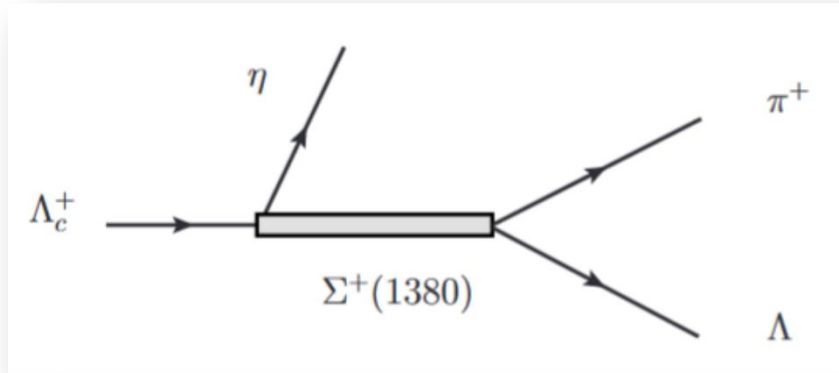
Dalitz plot of $\Lambda_c^+ \rightarrow \eta \Lambda \pi$



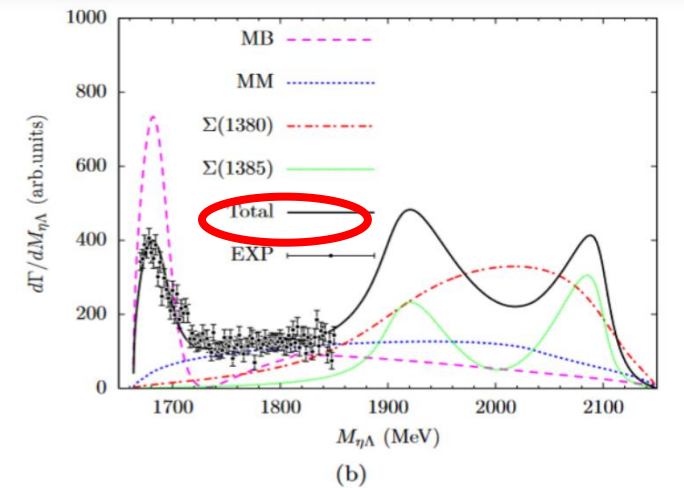
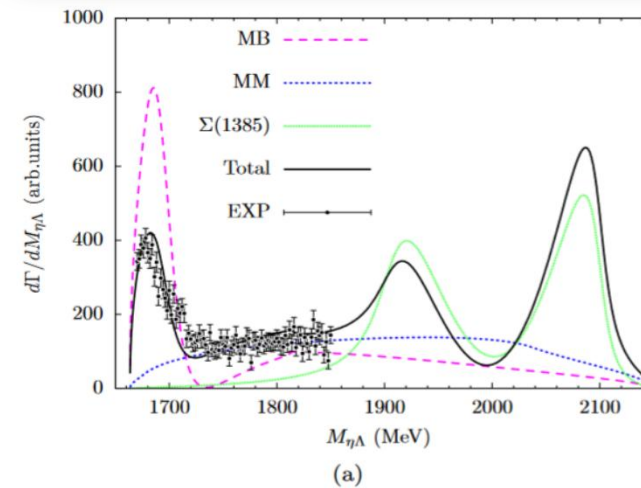
$\Sigma(1/2^-)$ in $\Lambda_c^+ \rightarrow \eta \Lambda \pi$

arXiv:2405.09226

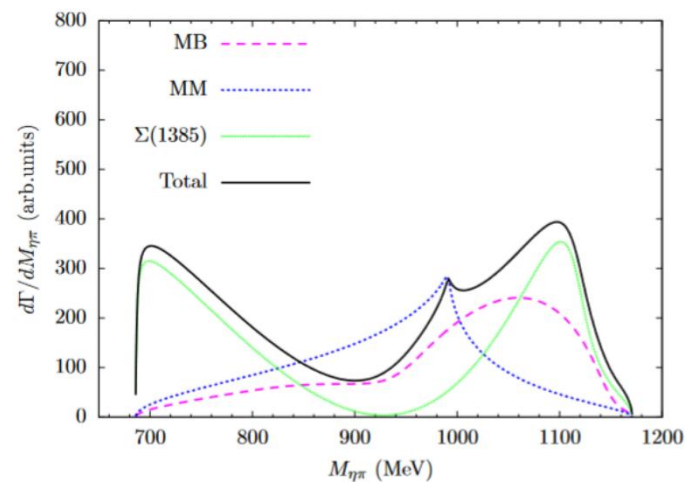
Intermediate of $\Sigma(1/2^-)$



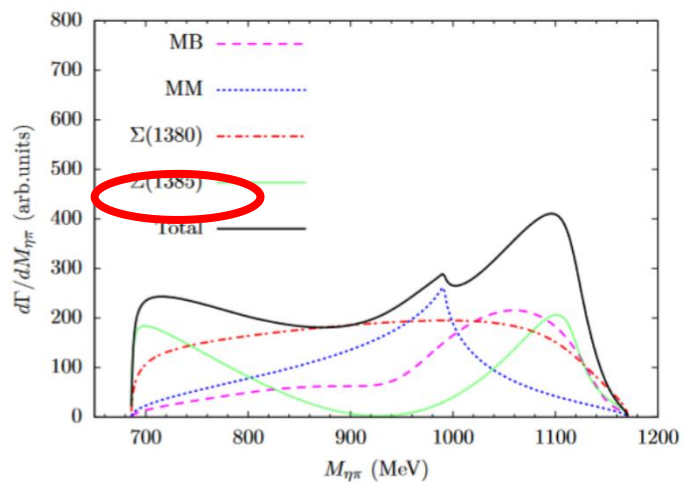
$$\mathcal{T}^{\Sigma(1/2^-)} = \frac{V^{\Sigma(1/2^-)} M_{\Sigma(1/2^-)} \Gamma_{\Sigma(1/2^-)}}{M_{\pi^+ \Lambda}^2 - M_{\Sigma(1/2^-)}^2 + i M_{\Sigma(1/2^-)} \Gamma_{\Sigma(1/2^-)}},$$



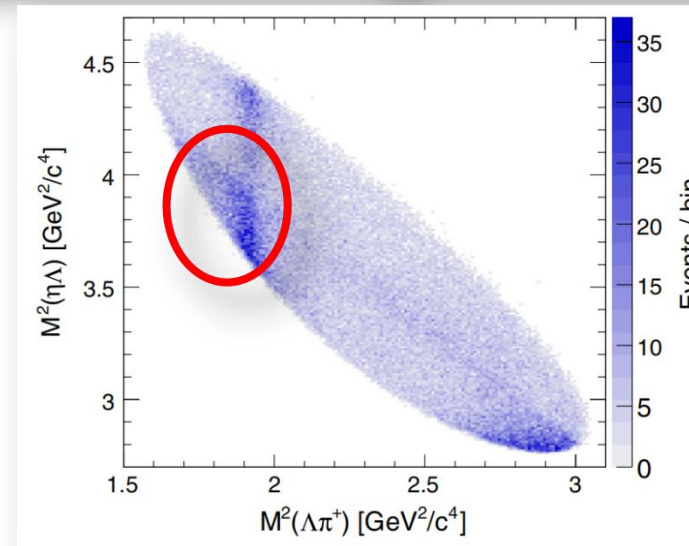
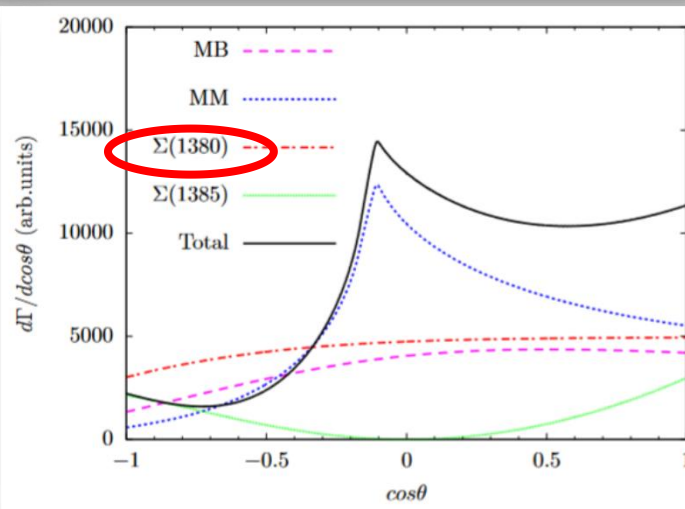
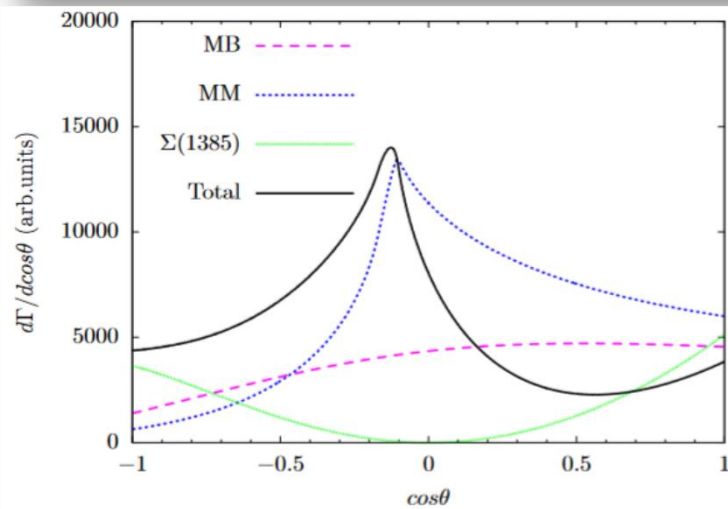
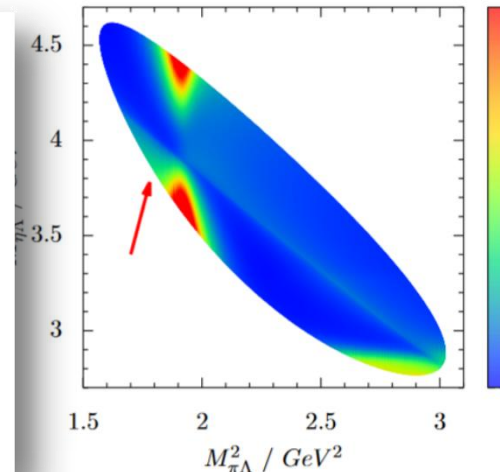
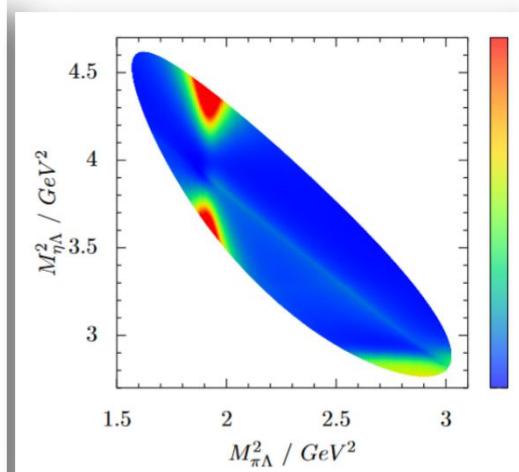
The results with/without $\Sigma(1380)$



(a)



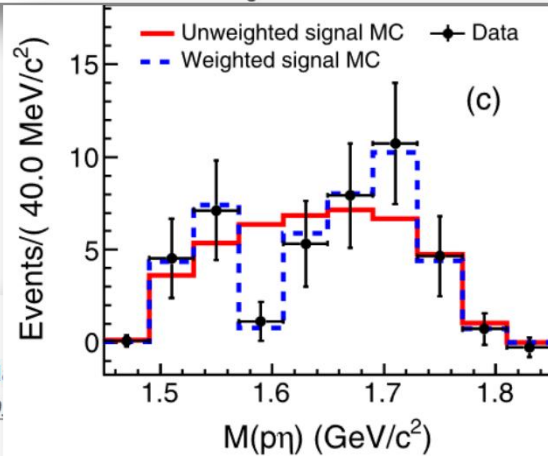
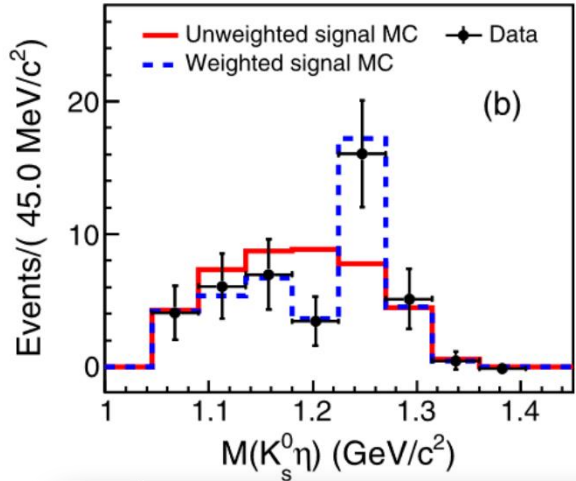
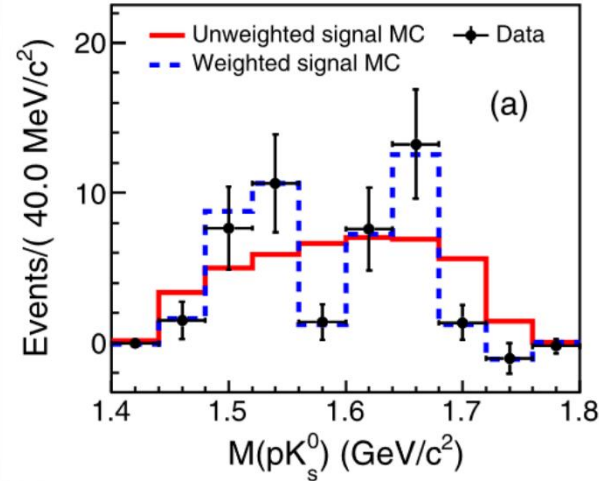
(b)



$M_{\pi\Lambda} \geq 1450 \text{ MeV}$ and $M_{\eta\Lambda} \geq 1760 \text{ MeV}$.

to be prepared

Exp of $\Lambda_c^+ \rightarrow \bar{K}^0 \eta p$



**BESIII: PLB817 (2021)
136327**

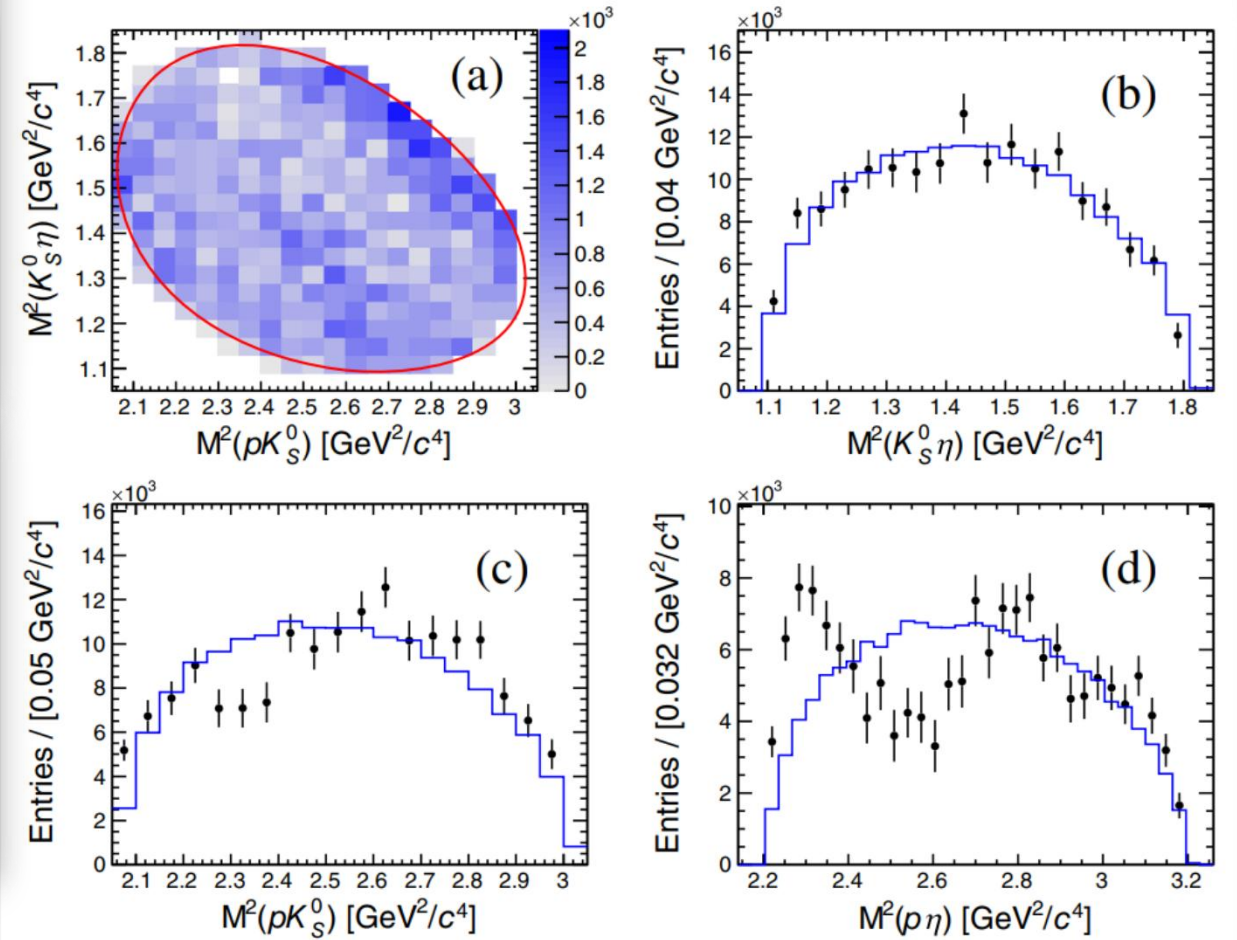
Role of the $N^*(1535)$ in the $\Lambda_c^+ \rightarrow \bar{K}^0 \eta p$ decay

Ju-Jun Xie (Lanzhou, Inst. Modern Phys.), Li-Sheng Geng (BeiH
Phys.Rev.D 96 (2017) 5, 054009 • e-Print: [1704.05714](https://arxiv.org/abs/1704.05714) • DOI: [10.1103/PhysRevD.96.054009](https://doi.org/10.1103/PhysRevD.96.054009)

Production of $N^*(1535)$ and $N^*(1650)$ in $\Lambda_c \rightarrow \bar{K}^0 \eta p (\pi N)$ decay

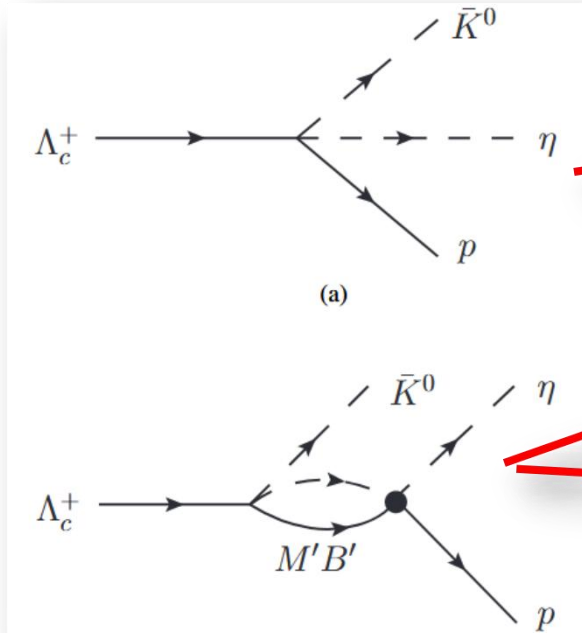
R. Pavao (Valencia U. and Valencia U., IFIC), S. Sakai (Valencia U. and Valencia U., IFIC), E. Oset
Phys.Rev.C 98 (2018) 1, 015201 • e-Print: [1802.07882](https://arxiv.org/abs/1802.07882) • DOI: [10.1103/PhysRevC.98.015201](https://doi.org/10.1103/PhysRevC.98.015201)

Belle: PRD107 (2023) 032004



$\Sigma(1/2^-)$ in $\Lambda_c^+ \rightarrow \bar{K}^0 \eta p$

arXiv:2406.01209



$$\mathcal{T}^{N(1535)} = V_1 (h_{\eta N} + h_{\eta N} G_{\eta N} t_{\eta N \rightarrow \eta N} + h_{\pi N} G_{\pi N} t_{\pi N \rightarrow \eta N} + h_{K\Lambda} G_{K\Lambda} t_{K\Lambda \rightarrow \eta N})$$

$$h_{\eta N} = \frac{1}{\sqrt{3}}, \quad h_{\pi N} = -\frac{\sqrt{6}}{2}, \quad h_{K\Lambda} = -\frac{\sqrt{6}}{3},$$

$$T = [1 - VG]^{-1} V, \quad \text{PhysRevC.65.035204}$$

$$\tilde{\mathcal{T}}^{N(1535)} = \frac{\tilde{V}_1 M_{N(1535)} \Gamma_{N(1535)}}{M_{\eta p}^2 - M_{N(1535)}^2 + i M_{N(1535)} \Gamma_{N(1535)}},$$

$$\mathcal{T}^{N(1650)} = \frac{V_2 M_{N^*} \Gamma_{N^*}}{M_{\eta p}^2 - M_{N^*}^2 + i M_{N^*} \Gamma_{N^*}},$$

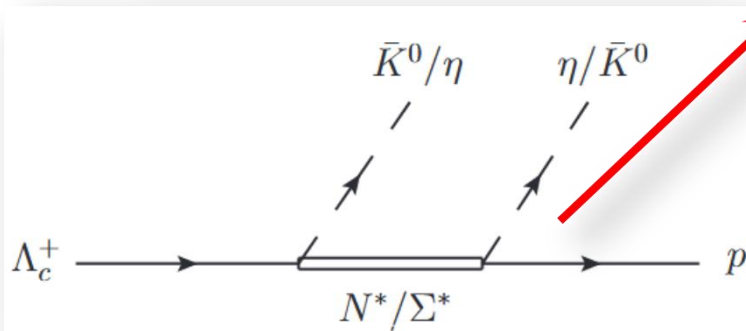
$$\mathcal{T}^{\Sigma^*(1/2^-)} = \frac{V_3 M_{\Sigma^*} \Gamma_{\Sigma^*}}{M_{p\bar{K}^0}^2 - M_{\Sigma^*}^2 + i M_{\Sigma^*} \Gamma_{\Sigma^*}},$$

$$|\mathcal{T}^A|^2 = |\mathcal{T}^{N(1535)} + \mathcal{T}^{N(1650)} e^{i\phi}|^2,$$

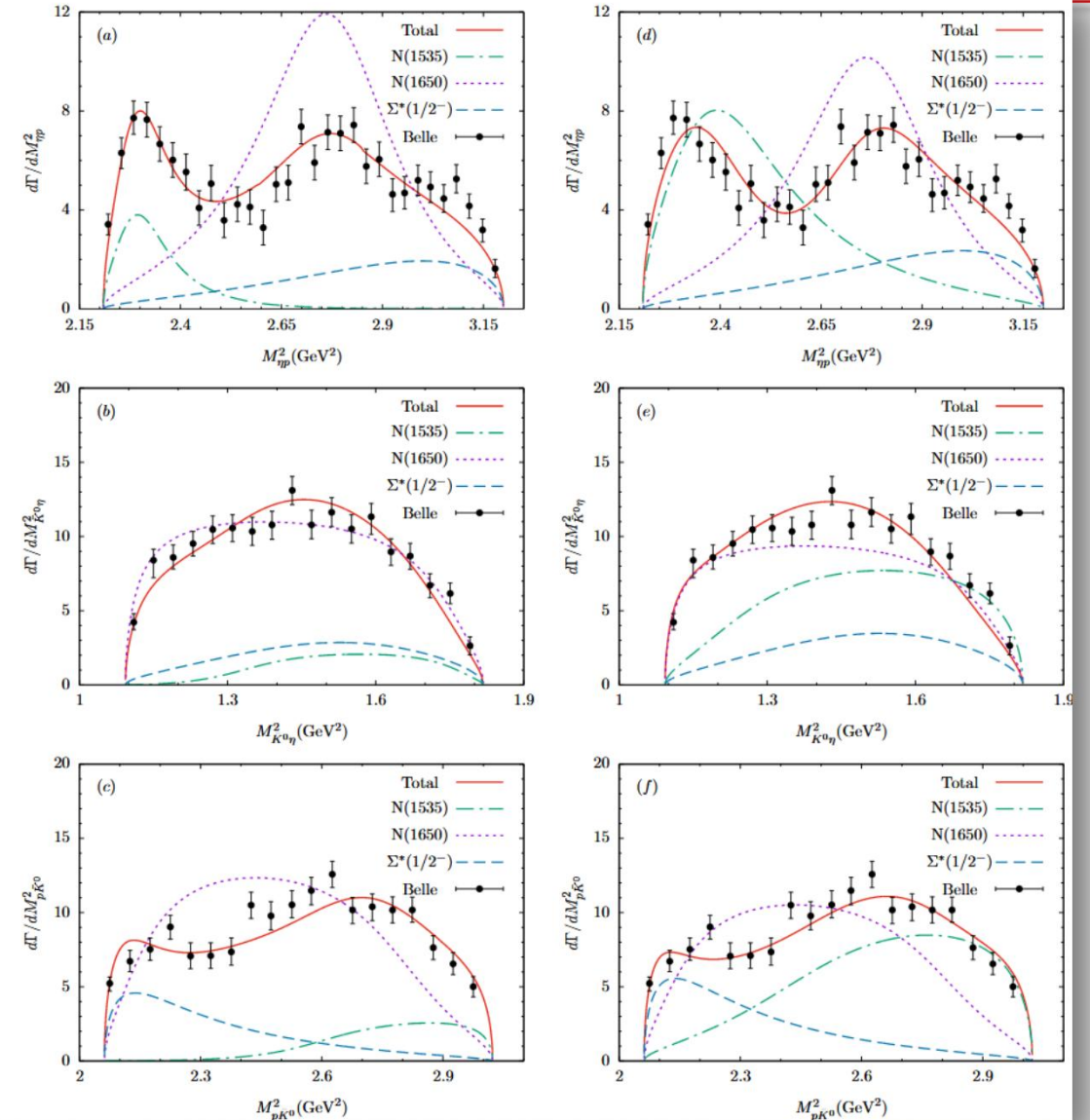
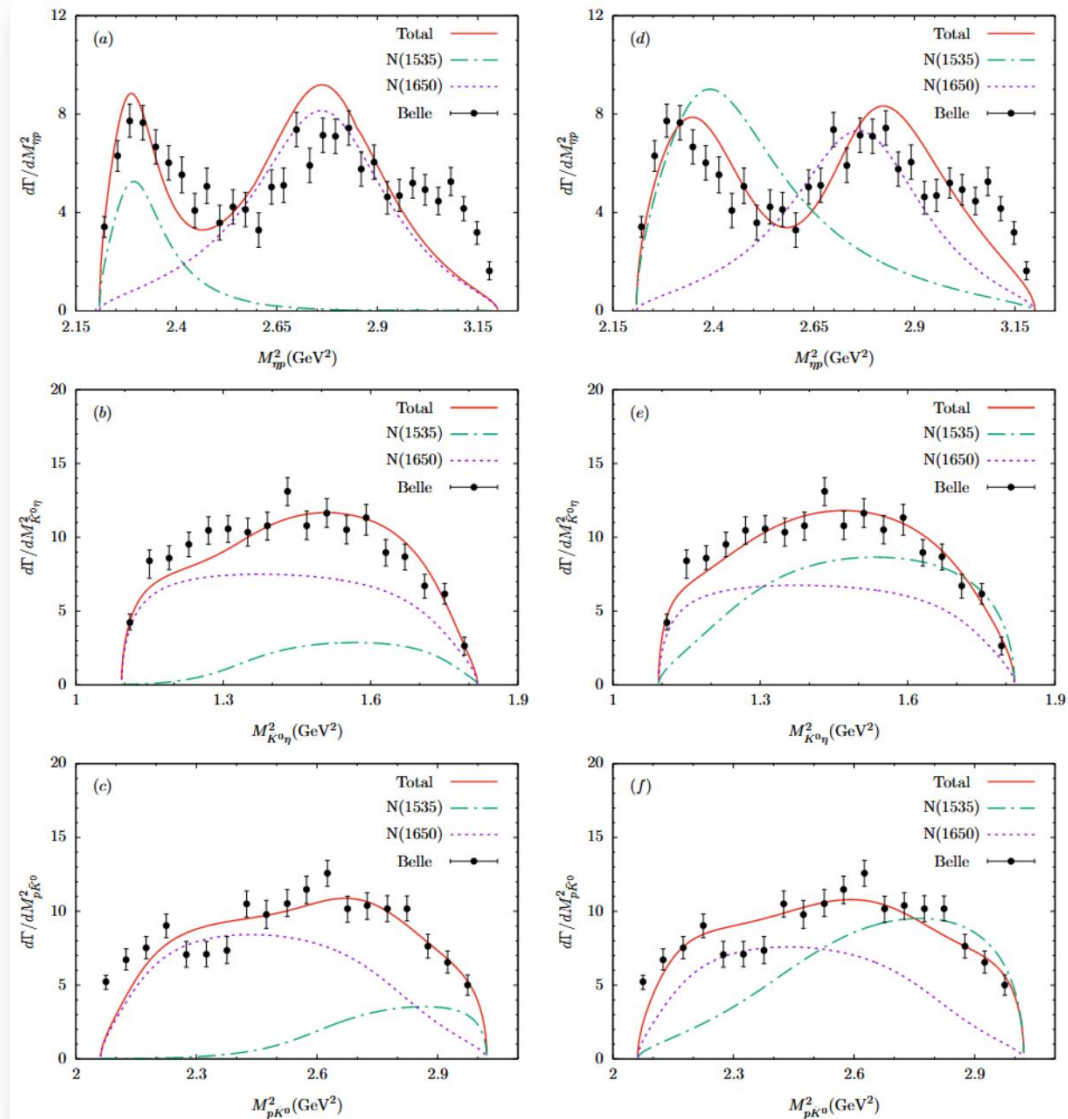
$$|\mathcal{T}^B|^2 = |\tilde{\mathcal{T}}^{N(1535)} + \mathcal{T}^{N(1650)} e^{i\phi}|^2,$$

$$|\mathcal{T}^C|^2 = |\mathcal{T}^{N(1535)} + \mathcal{T}^{N(1650)} e^{i\phi} + \mathcal{T}^{\Sigma^*(1/2^-)} e^{i\phi'}|^2,$$

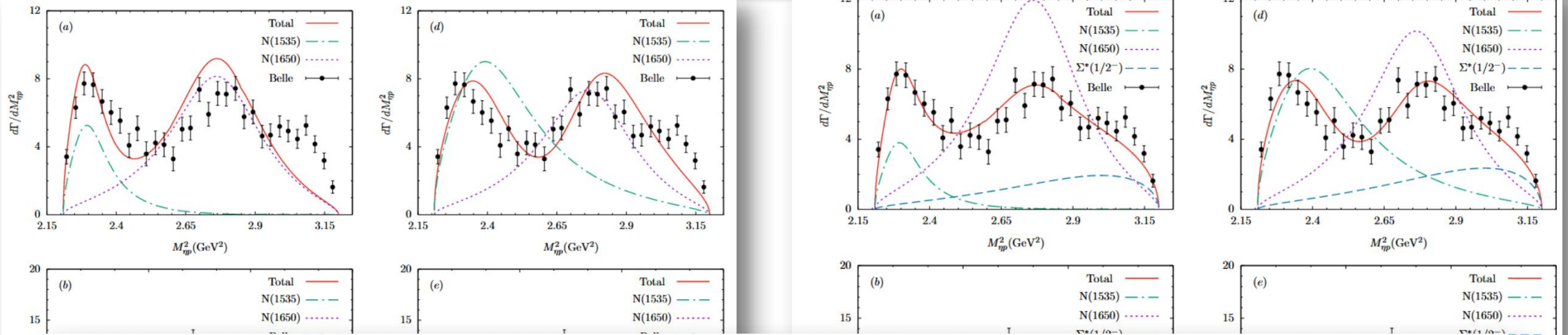
$$|\mathcal{T}^D|^2 = |\tilde{\mathcal{T}}^{N(1535)} + \mathcal{T}^{N(1650)} e^{i\phi} + \mathcal{T}^{\Sigma^*(1/2^-)} e^{i\phi'}|^2,$$



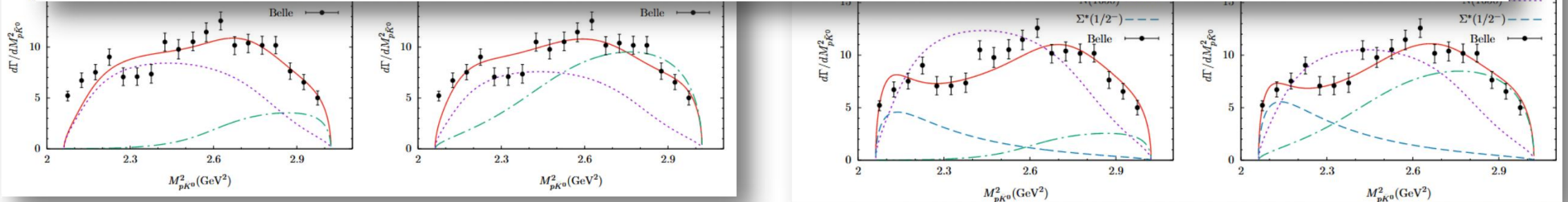
Results of $\Lambda_c^+ \rightarrow \bar{K}^0 \eta p$



Results of $\Lambda_c^+ \rightarrow \bar{K}^0 \eta p$

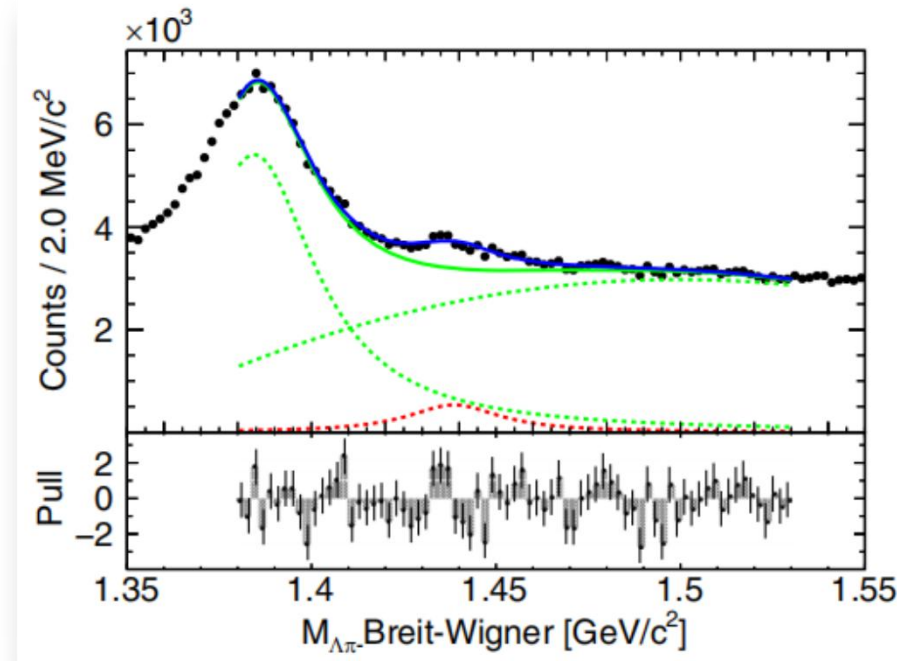
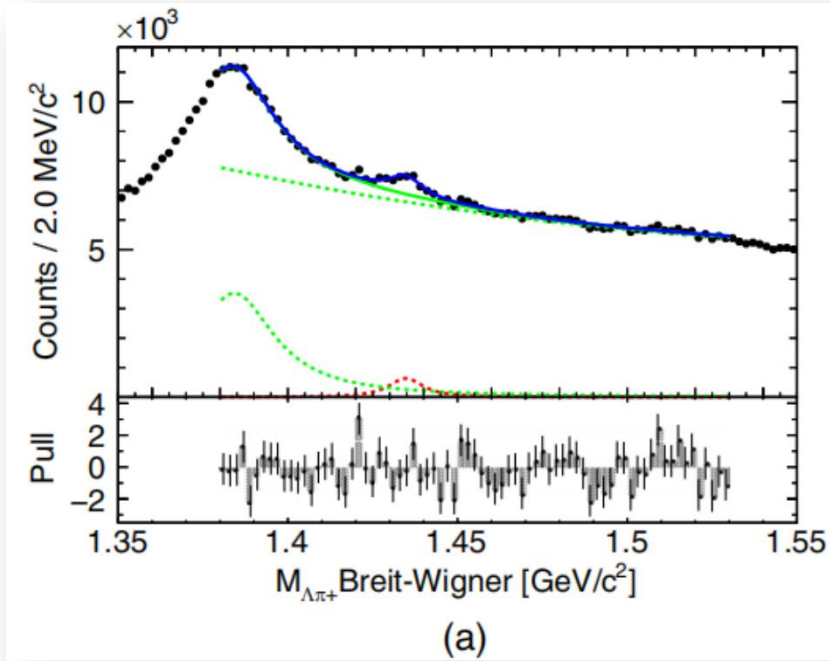


Parameters	$V_1(\tilde{V}_1)$	V_2	V_3	ϕ	ϕ'	$\chi^2/\text{d.o.f.}$
Model A	13939 ± 895	43369 ± 3227	—	$(0.98 \pm 0.21)\pi$	—	5.67
Model B	50151 ± 3730	41144 ± 3059	—	$(1.77 \pm 0.14)\pi$	—	3.20
Model C	11848 ± 813	52495 ± 3938	56230 ± 4019	$(1.35 \pm 0.43)\pi$	$(1.83 \pm 0.35)\pi$	1.55
Model D	47337 ± 2868	48454 ± 2457	61947 ± 4506	$(1.94 \pm 0.21)\pi$	$(0.62 \pm 0.16)\pi$	1.69



Belle measurements

□ $\Lambda_c^+ \rightarrow \Lambda \pi^+ \pi^+ \pi^-$, Belle, PRL130, 151903 (2023)



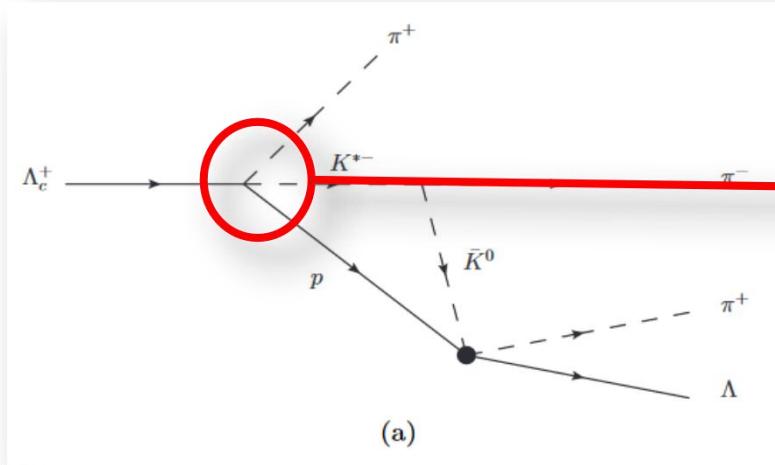
Mode	E_{BW} (MeV/ c^2)	Γ (MeV/ c^2)	χ^2/NDF
$\Lambda \pi^+$	1434.3 ± 0.6	11.5 ± 2.8	74.4/68
$\Lambda \pi^-$	1438.5 ± 0.9	33.0 ± 7.5	92.3/68

Evidence of $\Sigma(1430)$

$$\square \Lambda_c^+ \rightarrow \Lambda \pi^+ \pi^+ \pi^-$$

Dai-Pavao-Sakai-Oset, PRD 97, 116004 (2018)

Xie-Oset, PLB 792, 450-453 (2019)



$$t_{\Lambda_c^+ \rightarrow \pi^+ K^{*-} p} = A \vec{\sigma} \cdot \vec{\epsilon},$$

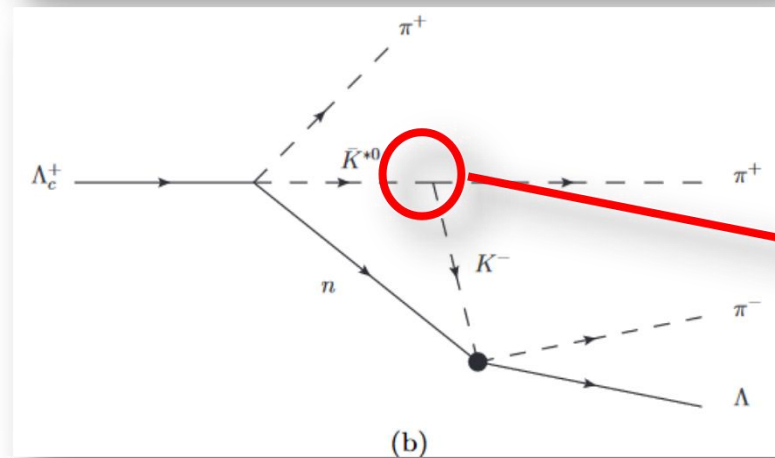
$$\frac{d\Gamma_{\Lambda_c^+ \rightarrow \pi^+ K^{*-} p}}{dM_{\text{inv}}(K^{*-} p)} = \frac{1}{(2\pi)^3} \frac{2M_{\Lambda_c^+} 2M_p}{4M_{\Lambda_c^+}^2} p_{\pi^+} \tilde{p}_{K^{*-}} \times \sum \sum |t_{\Lambda_c^+ \rightarrow \pi^+ K^{*-} p}|^2,$$

$$\mathcal{B}(\Lambda_c^+ \rightarrow \pi^+ \bar{K}^{*0} p) = (1.4 \pm 0.5) \times 10^{-2}$$

$$|A|^2 = (3.9 \pm 1.4) \times 10^{-16} \text{ MeV}^{-2}$$

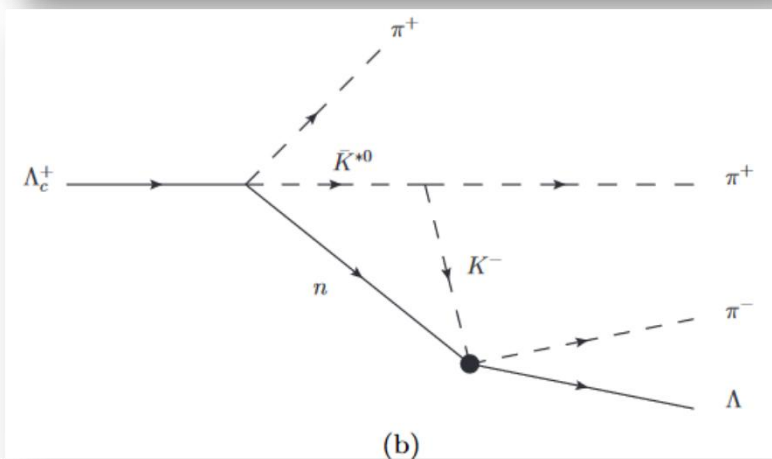
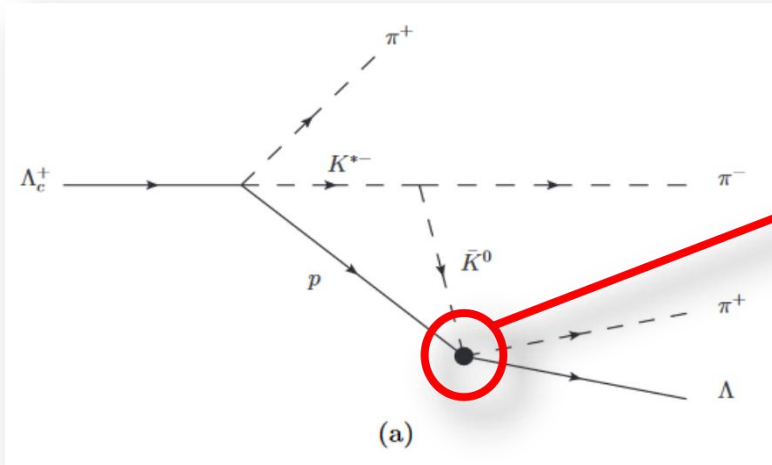
$$\mathcal{L}_{VPP} = -ig \langle V^\mu [P, \partial P] \rangle$$

$$\mathcal{L}_{\bar{K}^* \rightarrow \pi \bar{K}} = -ig (K^{*-})^\mu (\pi^- \partial_\mu \bar{K}^0 - \partial_\mu \pi^- \bar{K}^0).$$



Evidence of $\Sigma(1430)$

$$\square \Lambda_c^+ \rightarrow \Lambda \pi^+ \pi^+ \pi^-$$



$$\mathcal{T}^{\text{TS}} = -Ag(\vec{\sigma} \cdot \vec{k}_a t_T^a \mathcal{M}^a + \vec{\sigma} \cdot \vec{k}_b t_T^b \mathcal{M}^b),$$

$$\mathcal{M}^a = t_{K^- n \rightarrow \pi^- \Lambda}$$

$$T = [1 - VG]^{-1}V,$$

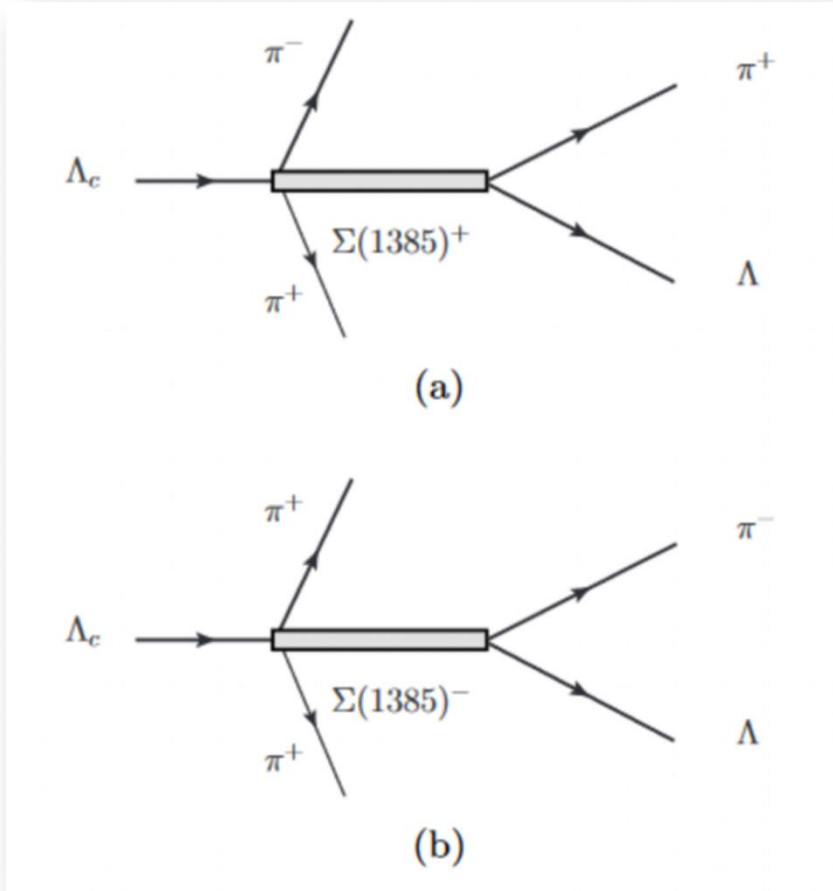
$$\mathcal{M}^b = t_{\bar{K}^0 p \rightarrow \pi^+ \Lambda}$$

E. Oset, A. Ramos, NPA 635, 99

$$\begin{aligned} t_T^a = & \int \frac{d^3q}{(2\pi)^3} \frac{2M_p}{8\omega_p \omega_{K^{*-}} \omega_{\bar{K}^0}} \frac{1}{k_a^0 - \omega_{K^{*-}} - \omega_{\bar{K}^0} + i\frac{\Gamma_{K^{*-}}}{2}} \\ & \times \frac{1}{P^0 + \omega_p + \omega_{\bar{K}^0} - k_a^0} \left(2 + \frac{\vec{q} \cdot \vec{k}}{|\vec{k}|^2}\right) \\ & \times \frac{2P^0 \omega_p + 2k_a^0 \omega_{\bar{K}^0} - 2(\omega_p + \omega_{\bar{K}^0})(\omega_p + \omega_{\bar{K}^0} + \omega_{K^{*-}})}{P^0 - \omega_{K^{*-}} - \omega_p + i\frac{\Gamma_{K^{*-}}}{2}} \\ & \times \frac{1}{P^0 - \omega_p - \omega_{\bar{K}^0} - k_a^0 + i\varepsilon}, \end{aligned} \quad (19)$$

Evidence of $\Sigma(1430)$

$$\square \Lambda_c^+ \rightarrow \Lambda \pi^+ \pi^+ \pi^-$$



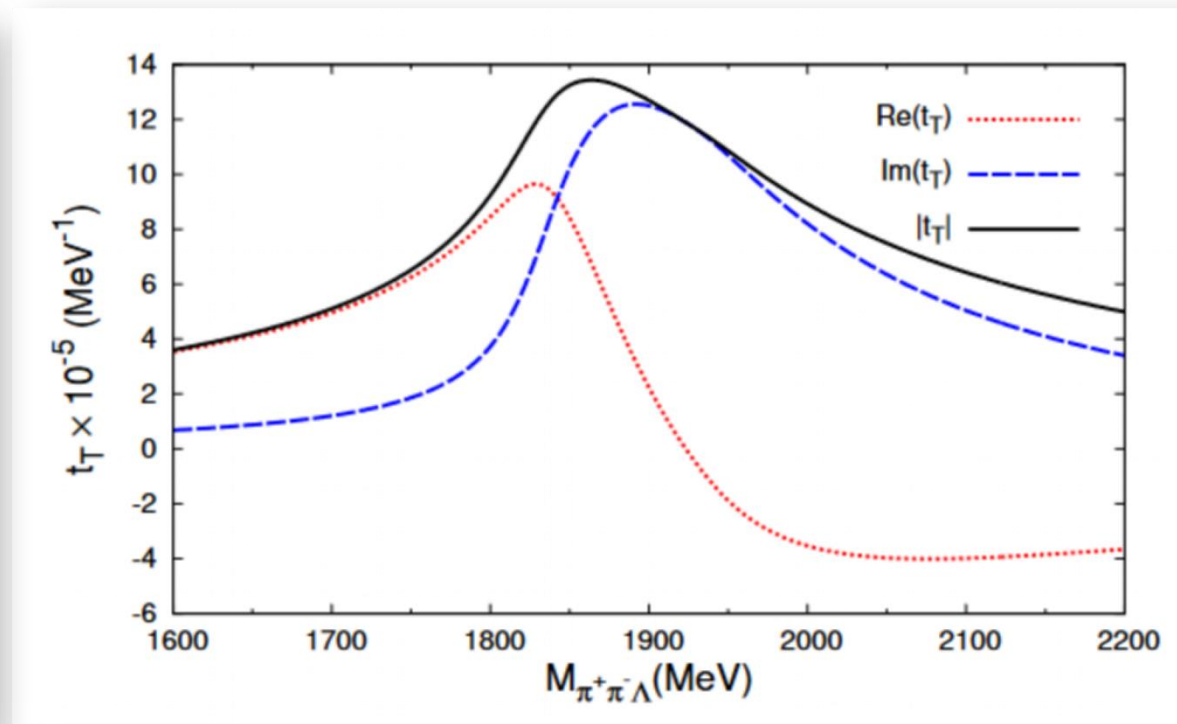
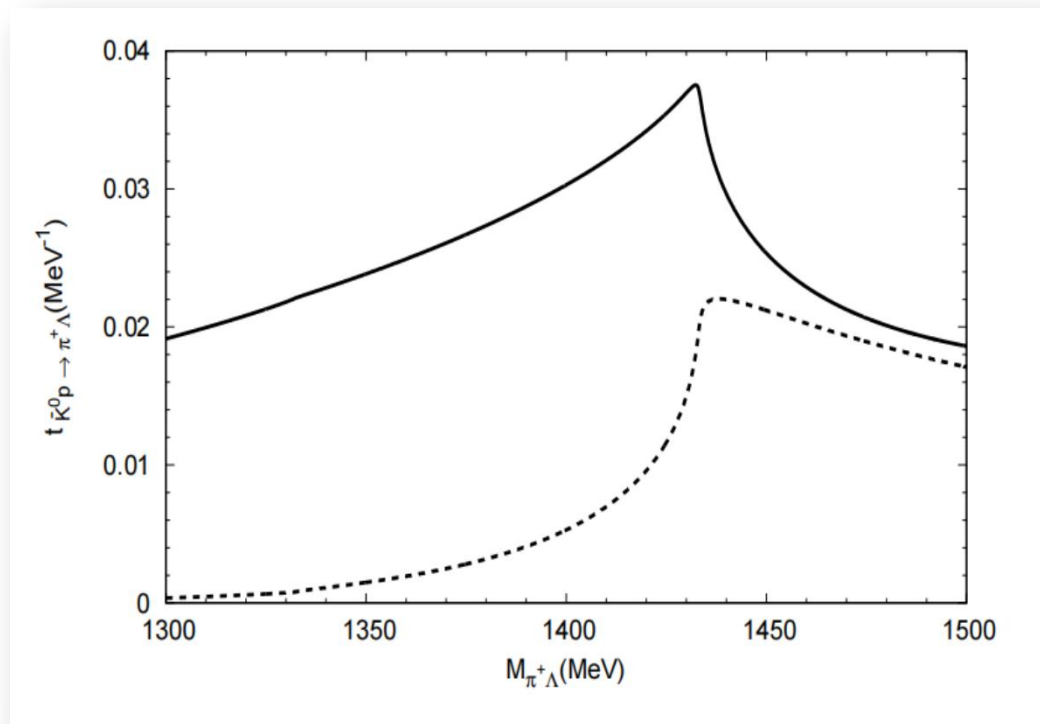
$$T^{\Sigma^{*+}(1385)} = \frac{V_p |p_{\pi^+}|}{M_{\pi^+\Lambda} - M_{\Sigma^{*+}} + i \frac{\Gamma_{\Sigma^{*+}}}{2}},$$

$$T^{\Sigma^{*-}(1385)} = \frac{V_p |p_{\pi^-}|}{M_{\pi^-\Lambda} - M_{\Sigma^{*-}} + i \frac{\Gamma_{\Sigma^{*-}}}{2}},$$

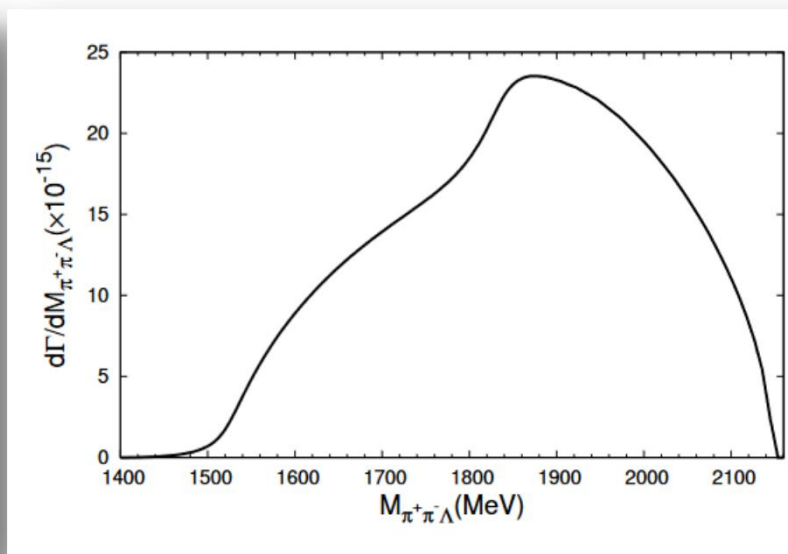
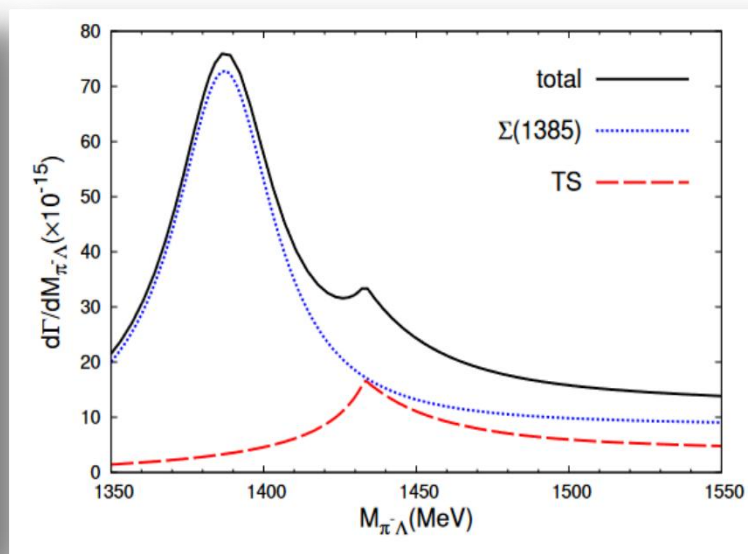
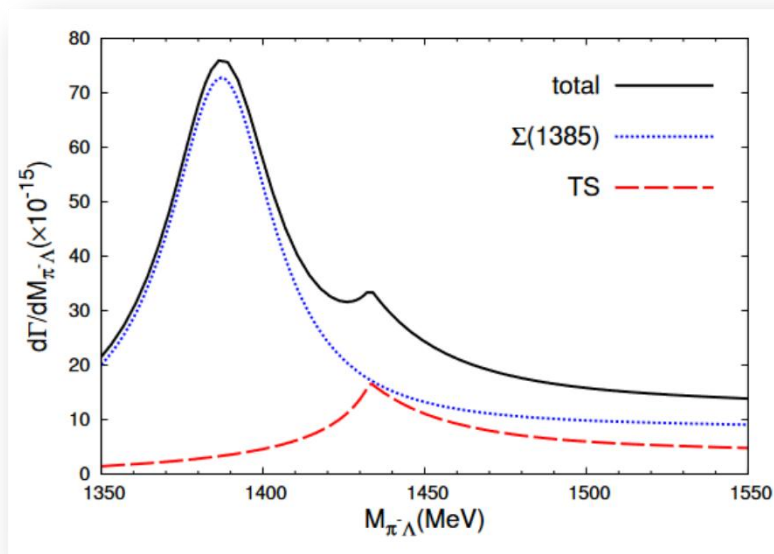
$$\begin{aligned} \frac{d^3\Gamma}{dM_{\pi^+\pi^-\Lambda} dM_{\pi^+\Lambda} dM_{\pi^-\Lambda}} &= \frac{g^2 |A|^2}{64\pi^5} \frac{M_\Lambda}{M_{\Lambda_c^+}} \tilde{p}_{\pi^+} \frac{M_{\pi^+\Lambda} M_{\pi^-\Lambda}}{M_{\pi^+\pi^-\Lambda}} \\ &\left\{ |\vec{k}_a|^2 |t_T^a \mathcal{M}^a|^2 + |\vec{k}_b|^2 |t_T^b \mathcal{M}^b|^2 + 2\text{Re}[t_T^a \mathcal{M}^a (t_T^b \mathcal{M}^b)^*] \right. \\ &\left. \times \vec{k}_a \cdot \vec{k}_b + |T^{\Sigma^{*+}(1385)}|^2 + |T^{\Sigma^{*-}(1385)}|^2 \right\}, \end{aligned} \quad (29)$$

Evidence of $\Sigma(1430)$

□ $\Lambda_c^+ \rightarrow \Lambda \pi^+ \pi^+ \pi^-$, Lyu-GYW-EW-Xie-Geng, to prepare

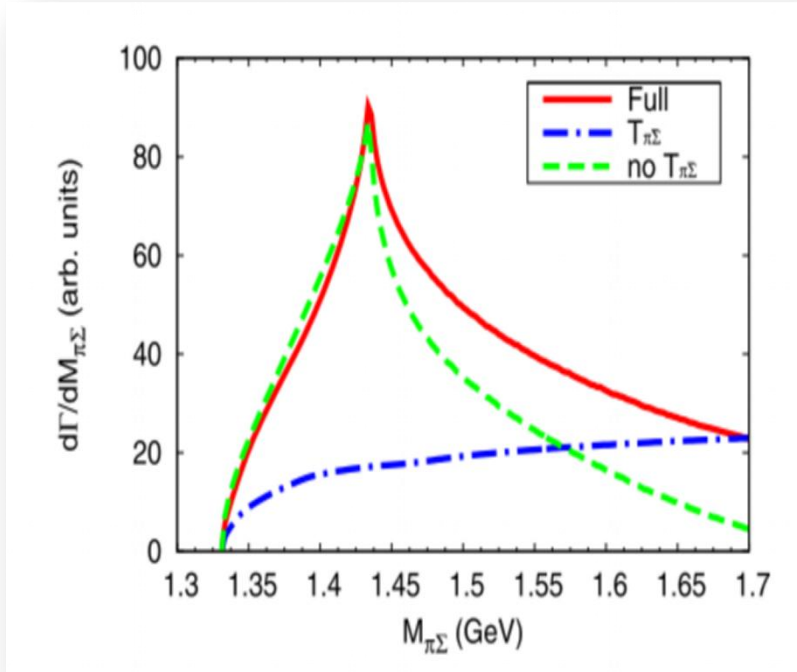


Results of $\Lambda_c^+ \rightarrow \Lambda \pi^+ \pi^+ \pi^-$



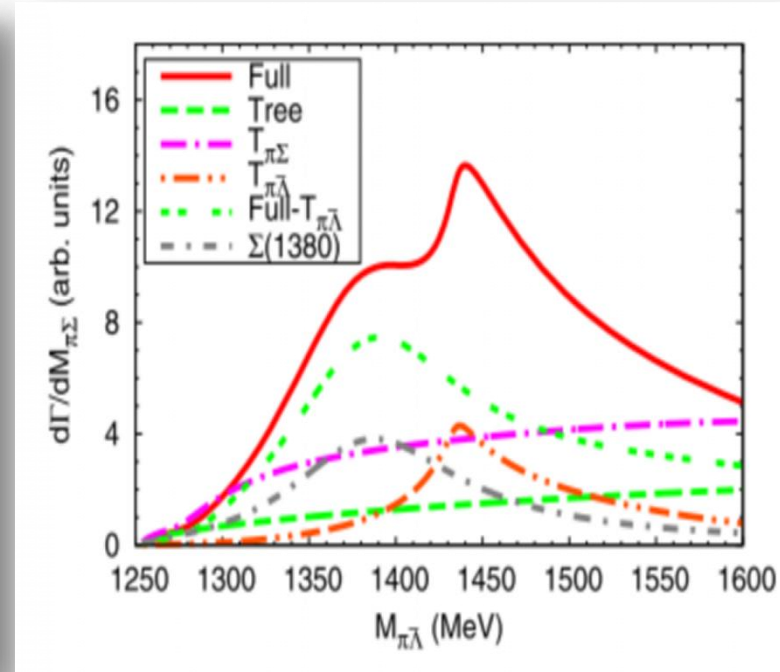
Cusp signal of $\Sigma(1/2^-)$ around $\bar{K}N$ threshold!

Search for $\Sigma^*(1/2^-)$ in other processes



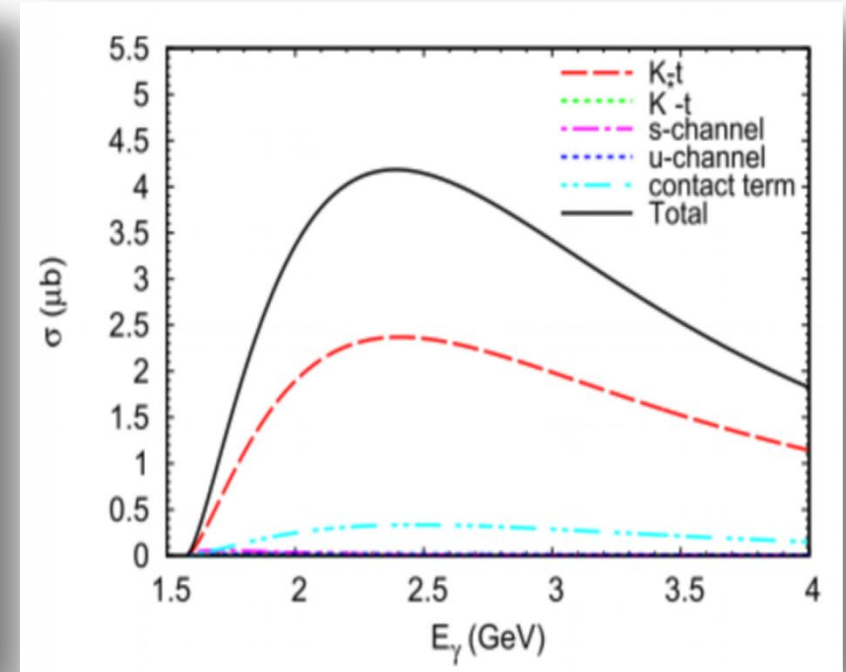
$$\chi_{c0} \rightarrow \bar{\Sigma}\Sigma\pi$$

PLB753(2016)526



$$\chi_{c0} \rightarrow \bar{\Lambda}\Sigma\pi$$

PRD98(2018)114017



$$\gamma n \rightarrow K\Sigma(1/2^-)$$

CPC47 (2023) 053108

Two poles of $\Sigma^*(1/2^-)$

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Cross-Channel Constraints on Resonant Antikaon-Nucleon Scattering

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It is interesting to note that in our NNLO fit there exist two $I = 1$ states around the $\bar{K}N$ threshold located at (1435, −39) MeV and (1440, −135) MeV on the (− − + + +) sheet, the order of which corresponds to $\pi\Lambda$, $\pi\Sigma$, $\bar{K}N$, $\eta\Lambda$, $\eta\Sigma$, $K\Xi$ respectively. Both states are well above the K^-p threshold and appear as cusps on the real axis. In the Fit “NNLO*” in which the constraints from baryon masses are omitted, the two $I = 1$ states are located at (1364, −110) MeV and (1432, −18) MeV also on the (− − + + +) sheet. In this case, the narrower state still shows up as a cusp but the broader one becomes a broad enhancement on the $I = 1$ amplitude on the real axis. We note that the existence of a $\Sigma^*(\frac{1}{2}^-)$ state has been predicted in a number of UChPT

Are there two poles of $\Sigma(1/2^-)$?

Summary

- Belle/BESIII measurements of $\Lambda_c^+ \rightarrow \eta \Lambda \pi / \bar{K}^0 \eta p$ show some hints of the $\Sigma^*(1/2^-)$, and the more precise measurements could be used to test the existence of $\Sigma^*(1/2^-)$.
- The cusp structure around 1430 MeV in $\Lambda_c^+ \rightarrow \Lambda \pi \pi \pi$ could be associated with the $\Sigma(1430)$.
- Some processes could be used to search for $\Sigma^*(1/2^-)$, such as $\chi_{c0} \rightarrow \bar{\Sigma} \Sigma \pi$, $\chi_{c0} \rightarrow \bar{\Lambda} \Sigma \pi$, $\gamma n \rightarrow K \Sigma(1/2^-)$.

Thank you very much!