Two PMC applications at the e+e- colliders

Xing-Gang Wu

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OUTLINE

- QCD scale-setting problem
- Solutions of scale-setting problem
- Two PMC applications at the e+e- colliders
  - Top-pair production near the threshold region
  - The ratio Ruds
- Summary and outlook

➢ QCD scale-setting problem

Fixed-order predictions such as those at the hadronic colliders

General factorization picture

$$d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$$

parton distribution functions

Perturbative partonic cross section (virtual & real radiation)

Fragmentation models

$$\hat{\sigma} = \sigma_0[1 + \alpha_s + \alpha_s^2 + \cdots]$$

How to set the renormalization scale ?

Contains α_s^n of tree level process

NLO

NNLO

Any physical quantity ρ could be expanded in following form (In perturbative region)

$$\rho = \underline{r_0 \alpha_s^{p} (\mu_R)} + \underline{r_1 \alpha_s^{p+1} (\mu_R)} + \underline{r_2 \alpha_s^{p+2} (\mu_R)} + \ldots$$

Renormalization scheme and scale ensure the reliability of pQCD prediction

Regularization
Renormalization

Scale-settiing

FaS-separation of 微扰/非微扰
ReS-accuracy

Reliability and accuracy are of same importance

Physical observable = Up to infinite order, any choice of scheme/scale should result in the same prediction.

$$\frac{\partial \rho}{\partial \mu_R} \equiv 0; \frac{\partial \rho}{\partial R} \equiv 0$$

Renormalization Group Invariance

(Standard RGI)

Standard RGI

Equivalence to: $\frac{\partial \rho_n}{\partial \mu_R} \neq 0; \frac{\partial \rho_n}{\partial \mathrm{R}} \neq 0;$ $n-$perturbative order, R-scheme

At any fixed-order, the QCD series is non-conformal, the prediction shall be scale and scheme dependent due to mismatching of α_s with its coefficients for an arbitrary choice of scale.

Initial perturbative series 原始微扰序列

Improved series ?

➢ Solutions of scale-setting problem

First round: BLM/PMS/FAC

$\partial \varrho_n / \partial(\mathrm{RS}) = \mathcal{O}(a_s^{p+n+1}) \sim 0$

PMC=> β-term

Corrected

BLM=> nf-term

Fix α_s-value, the new series naturally satisfy RGI

两大假设（有效处理）
未知高阶项贡献为零；QCD理论是共形理论

Optimized perturbation theory –
minimize the higher-order contributions – **PMS**

Directly require it to satisfy the RG invariance

寻找微扰序列的最优能标和最优方案

Two typical ideas

1980's attempts to answer why

Volume 100B, number 1 PHYSICS LETTERS 19 March 1981

PMS-scheme

RESOLUTION OF THE RENORMALISATION-SCHEME AMBIGUITY IN PERTURBATIVE QCD

P.M. STEVENSON
Physics Department, University of Wisconsin-Madison, Madison, WI 53706, USA

Received 24 November 1980

Nex-to-leading-order QCD predictions depend on the arbitrary choice of renormalisation scheme. I resolve this ambiguity and show how to find the unique, optimum prediction, given the result in some arbitrary, initial scheme.

PHYSICAL REVIEW D VOLUME 28, NUMBER 1 1 JULY 1983

On the elimination of scale ambiguities in perturbative quantum chromodynamics

Stanley J. Brodsky
Institute for Advanced Study, Princeton, New Jersey 08540
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G. Peter Lepage
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Paul B. Mackenzie
Fermilab, Batavia, Illinois 60510

(Received 23 November 1982)

BLM-scheme

PMS 最小敏感度方案

Forces fixed-order prediction to be minimum over the choices of scheme/scale

Any even higher-orders give zero contributions

P.M. Stevenson, Phys. Rev. D23 (1981) 2916

Using the fact: $\{\beta_{i>=2}\}$-functions are different for different schemes

PMS Equations at n-th order

$$\frac{\partial \varrho_n}{\partial \tau} = 0 \quad \frac{\partial \varrho_n}{\partial \beta_m} = 0, (m \geq 2)$$

$$\varrho_n(Q) = a_s(\mu)\left(1 + \sum_{i=1}^{n} C_i(\mu, Q) a_s^i(\mu)\right),$$

where Q is the scale at which ϱ is measured and $a_s = \alpha_s/\pi$. The local RGI indicates that

$$\frac{\partial \varrho_n}{\partial \tau} = \left(\frac{\partial}{\partial \tau}\bigg|_{a_s} + \beta(a_s)\frac{\partial}{\partial(a_s/4)}\right)\varrho_n = 0,$$

$$\frac{\partial \varrho_n}{\partial \beta_m} = \left(\frac{\partial}{\partial \beta_m}\bigg|_{a_s} - \beta(a_s)\int_0^{a_s/4} \mathrm{d}\left(\frac{a_s'}{4}\right)\frac{(a_s'/4)^{m+2}}{[\beta(a_s')]^2}\frac{\partial}{\partial(a_s/4)}\right)\varrho_n = 0, \ (m = 2, 3, \ldots)$$

To compare with standard RGI, we call it Local RGI

At n-th order, **2n+1 parameters** $\tilde{a}_s, \tilde{\tau}, \tilde{\beta}_2, \ldots, \tilde{\beta}_n, \tilde{\mathcal{C}}_1, \ldots, \tilde{\mathcal{C}}_n$

n PMS Equations $\dfrac{\partial \varrho_n}{\partial \tau} = 0 \quad \dfrac{\partial \varrho_n}{\partial \beta_m} = 0, (m \geq 2)$

One Basic RGE (β-function)

n RG-invariant coefficients

$$\beta(a_s) = \mu^2 \frac{\partial}{\partial \mu^2}\left(\frac{\alpha_s}{4\,\pi}\right) = -\sum_{i=0} \beta_i \left(\frac{\alpha_s}{4\,\pi}\right)^{i+2}$$

$$\rho_1 = \frac{1}{4}\, p\beta_0\tau - \mathcal{C}_1$$

$$\rho_2 = \mathcal{C}_2 - \frac{(1+p)\mathcal{C}_1^2}{2p} - \frac{\beta_1\mathcal{C}_1}{4\beta_0} + \frac{p\beta_2}{16\beta_0}$$

$$\rho_3 = 2\mathcal{C}_3 + \frac{\mathcal{C}_1^2\beta_1}{4p\beta_0} - \frac{\mathcal{C}_1\beta_2}{8\beta_0} + \frac{p\beta_3}{64\beta_0} + \frac{2(1+p)(2+p)\mathcal{C}_1^3}{3p^2} - \frac{2(2+p)\mathcal{C}_1\mathcal{C}_2}{p}$$

$\Rightarrow$ **2n+1 equations to solve 2n+1 parameters, could be solved numerically**

Good features

determining effective coupling without Λ_{QCD} uncertainty

PMS prediction（局限）

Does suppress Renormalon divergence, but cannot satisfy the normal pQCD convergence==main contribution lies in LO

Lower-order predictions are generally incredible

$R_{e^+e^-}$ four-loop level

	n_f=3	n_f=4	n_f=5
C_1	1.6401	1.5249	1.4097
C_2	-10.284	-11.6857	-12.8047
C_3	-106.896	-92.9124	-80.0075
$C_1^{\rm PMS}$	-0.458	-0.1105	0.0479
$C_2^{\rm PMS}$	-1.1361	0.2103	1.3075
$C_3^{\rm PMS}$	22.2133	24.9881	16.4108

TABLE I. Coefficients for the perturbative expansion of $R_3(Q)$ before and after the PMS scale setting, where we have set Q = 1.2 GeV for n_f=3, Q = 3 GeV for n_f=4, and Q = 31.6 GeV for n_f=5.

Factorial divergence suppressed !

R_τ (four loop)

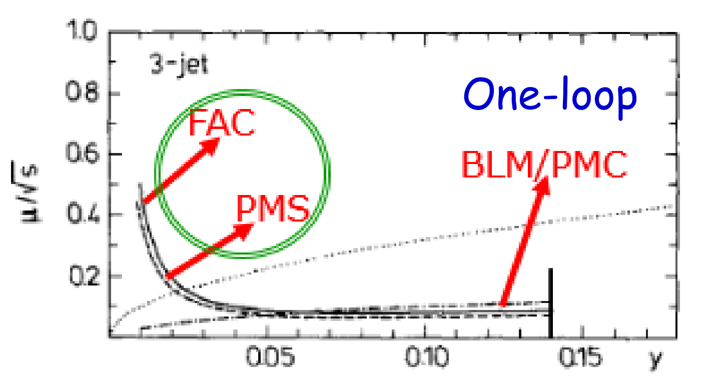
	C'_1	C'_2	C'_3
Conv.	5.2023	26.3659	127.079
PMS	0.3906	1.2380	-6.1747

TABLE V. Coefficients for the perturbative expansion of r_3^τ and after the PMS scale setting. $\mu_0 = M_\tau$.

$\Gamma(H \to b\bar{b})$ (four loop)

	C''_1	C''_2	C''_3
Conv.	29.145	41.765	-825.598
PMS	0.34376	21.2286	-142.849

TABLE IX. Coefficients for the perturbative expansion of $\tilde{R}_3$ before and after the PMS scale setting. $\mu_0 = M_H$.



G. Kramer and B. Lampe, Z. Phys. C 39, 101 (1988).

$R_{e^+e^-}$ four-loop level

	LO	NLO	N^{2}LO	N^{3}LO	*total*
Conv.	0.04499	0.00285	-0.00117	-0.00033	0.04635
PMS	0.04608	0.00010	0.00013	0.00007	0.04638

Accidental series convergence

R_τ (four loop)

	LO	NLO	N^{2}LO	N^{3}LO	*total*
Conv.	0.10320	0.05541	0.02898	0.01441	0.20200
PMS	0.19935	0.01552	0.00981	-0.00975	0.21493

$\varrho^{LO}_{3,PMS} \gg \varrho^{NLO}_{3,PMS} \sim \varrho^{N^2LO}_{3,PMS} \sim \varrho^{N^3LO}_{3,PMS}$

	LO	NLO	N^{2}LO	N^{3}LO	*total*
Conv.	0.20371	0.03767	0.00194	-0.00138	0.24194
PMS	0.23967	0.00061	0.00161	-0.00046	0.24144

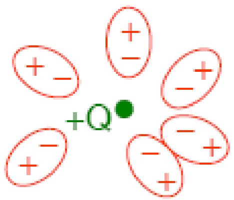
$\varrho^{NLO}_{3,PMS} < \varrho^{N^2LO}_{3,PMS}$

$\Gamma(H \to b\bar{b})$ (four loop)

BLM

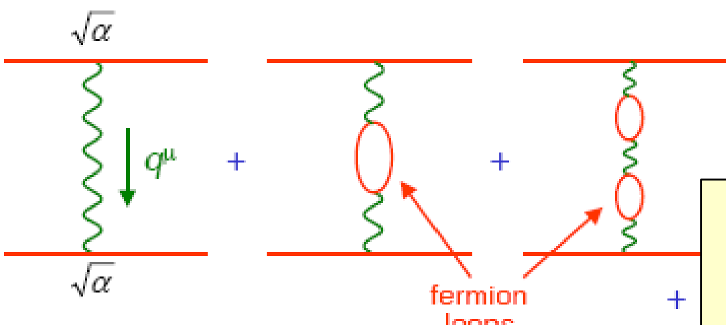
Running Couplings

- QED:



"Bare" electric charge is screened by "halo" of e^+e^- pairs

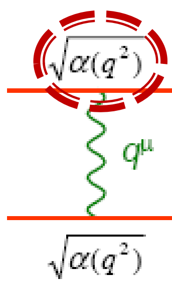
In terms of Feynman diagrams:



→ infinite series of diagrams which can be (approximately) summed:

$$\alpha(q^2)=\alpha(q_0^2)\Big/\left[1-\frac{\alpha(q_0^2)}{3\pi}\ln\left(\frac{q^2}{q_0^2}\right)\right]$$

→ replace by single diagram with running coupling



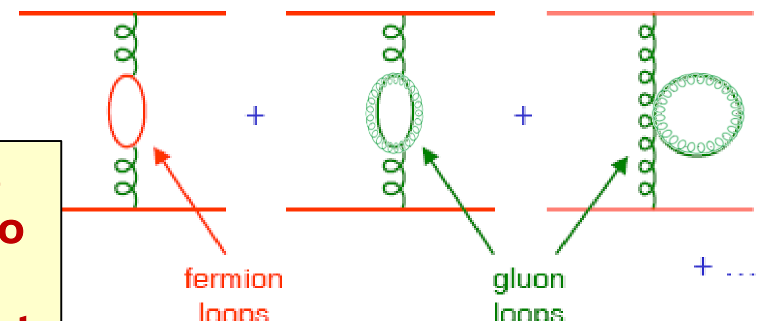
Q_0 – Initial scale

Resummed to achieve a scale-invariant prediction independent to Q_0

链圈图近似

类比找思路

- QCD: similar, but now have:



tra diagrams produce anti-screening

$$\alpha_s(q^2)=\alpha_s(q_0^2)\Big/\left[1+B\alpha_s(q_0^2)\ln\left(\frac{q^2}{q_0^2}\right)\right]$$

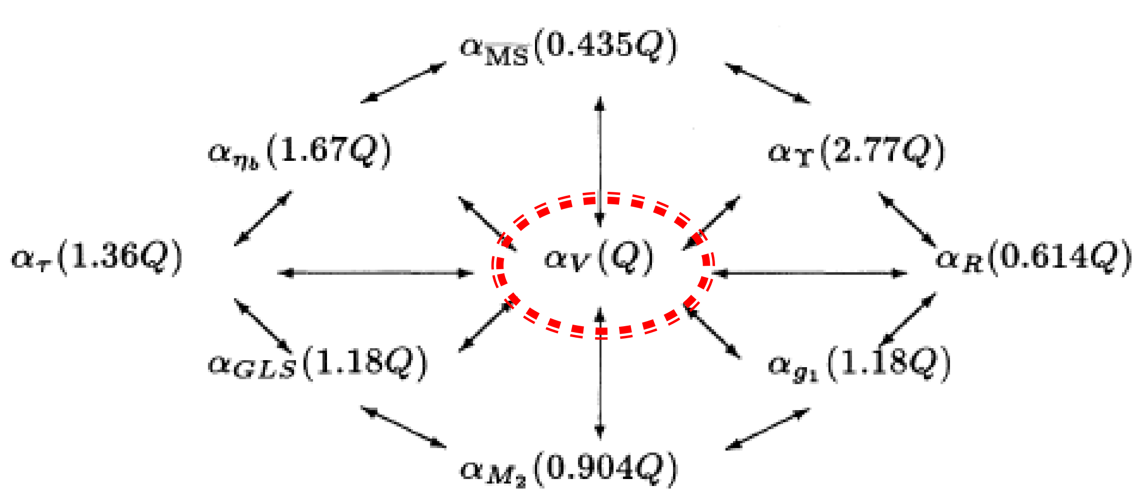
where $B=\dfrac{11N_c-2N_f}{12\pi}$ with $N_c=3$ $N_f=6$

GM-L：重求和泡泡图可获得准确的α值 → **BLM：重求和泡泡图可获得准确的α_s值**

Commensurate relation among different α_s

S.J. Brodsky and H.J. Lu, Phys.Rev. D**51**, 3652(1995);

TABLE I. Leading order commensurate scale relations.



$$R_{e^+e^-}(Q) \equiv R^0_{e^+e^-}(Q)\left[1+\frac{\alpha_s^R(Q)}{\pi}\right]$$

Transitivity

deduce

Second round: works around BLM
And the development of PMC

Initial

1981， BLM-scheme
1992， Wrong attempt of BLM extension, leading to wrong explanation
1995， Commensurate Scale Relation (CSR), BLM up to two-loop level
1997-2010， realizing the need of using β-function to replace nf-term
2011， BLM transforms to PMC at the one-loop level (nf-> β_0)
PMC-I, PMC-BLM correspondence principal, BLM up to all orders
2012， Application of PMC to top-pair production
2013， PPNP review, general arguments on renormalization scale-setting
2014， PMC-II, demonstrate scheme-independence for any R_δ-scheme
2015， The equivalence of PMC I and PMC II
A through comparison of PMC and PMS
RPP review, based on RGI
2017， Extend CSR up to any order, General Crewther Relation
Demonstrate scheme-independence for any scheme
2019， PPNP review
2024， PPNP review

developing

“Controversy drives forward”

Progress in Particle and Nuclear Physics 72 (2013) 44–98

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ELSEVIER

Review

The renormalization scale-setting problem in QCD

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Review

Renormalization group invariance and optimal QCD renormalization scale-setting: a key issues review

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ELSEVIER

Review

The QCD renormalization group equation and the elimination of fixed-order scheme-and-scale ambiguities using the principle of maximum conformality

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Renormalization
Principle of maximum conformality

ABSTRACT

The conventional scale se tions is based on a guess the large log-terms of the uncertainty. This ad hoc a of the QCD running cou the choices of both the leads to conventional re

2019

Progress in Particle and Nuclear Physics 135 (2024) 104092

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ELSEVIER

Review

High precision tests of QCD without scale or scheme ambiguities

The 40th anniversary of the Brodsky–Lepage–Mackenzie method

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2024

QCD scale-setting problem PMC

Suspected Battle ? the PMS itself has no any improvement since its invention
In fact, PMC and PMS are not at the same level

Phys. Lett. B 847 (2023) 138288

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ELSEVIER

Letter

‘Maximal conformality’ does not work

P.M. Stevenson

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ARTICLE INFO

Editor: B. Grinstein

ABSTRACT

The so-called “principle of maximal conformality” is ineffective and does nothing to resolve the renormalization-scheme-dependence problem. Some essential facts about that problem are summarized. It is stressed that RG invariance is a symmetry and that any viable method for resolving the scheme-dependence problem should be formulatable in terms of the invariants of that symmetry.

The Principal of Maximal Conformality does work and resolve the renormalization-scheme-dependence problem

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Still in progress

Basis—renormalization group equation (RGE)

Phys.Rev.D86,054018 (2012)

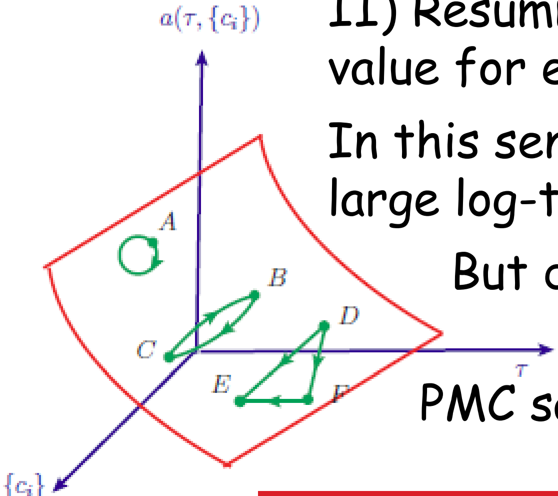
$\frac{\partial \rho_n}{\partial \mu_R} \neq 0; \frac{\partial \rho_n}{\partial \mathrm{R}} \neq 0;$ We cannot get exact constraints from those inequalities

Key idea of PMC: we can only get the answer from RGE itself, which can be used to determine the running behavior of coupling constant, thus fixing scale ambiguity.

I) Using RGE to determine the beta-terms at each order.

II) Resumming all the same type beta-terms, determining the exact value for exch perturbative order.

In this sense, PMC is similar to resummation, which resum a kind of large log-terms to form a steady prediction.



But different to a pure resummation to improve the reliability, PMC tends to solve the scale-setting problem.

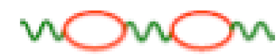
PMC satisfies RGE-properties: symmetry, reflecxity,transitivity

PMC tries to solve scheme-and-scale ambiguities simultaneously

PMC

Initial pQCD series --- First step

$$\rho(Q)=\sum_{i=1}^{n}\left(\sum_{j=0}^{i-1} c_{i,j}\, n_f^{j}\right) a_s^{p+i-1}(\mu_r)+\mathcal{O}(a_s^{n+1}),$$



n_f

Different to seBLM

Transformation nf-series into RGE-involved β-series ---Second step

$$\begin{aligned}\rho(Q) = {} & r_{1,0}a_s(\mu_r) + (r_{2,0}+\beta_0 r_{2,1})a_s^2(\mu_r) \\ & +(r_{3,0}+\beta_1 r_{2,1}+2\beta_0 r_{3,1}+\beta_0^2 r_{3,2})a_s^3(\mu_r) \\ & +(r_{4,0}+\beta_2 r_{2,1}+2\beta_1 r_{3,1}+\frac{5}{2}\beta_1\beta_0 r_{3,2} \\ & +3\beta_0 r_{4,1}+3\beta_0^2 r_{4,2}+\beta_0^3 r_{4,3})a_s^4(\mu_r)+\mathcal{O}(a_s^5)\end{aligned}$$

There are also β–terms that are pertained to Msbar mass and etc., which should be treated separately

PMC single-scale effective procedure ---Third step

$$\rho(Q) = \sum_{n\geq 1} r_{n,0}\alpha(\mu)^{n+p-1} + \sum_{n\geq 1}\left[(n+p-1)\alpha(\mu)^{n+p-2}\beta\right]\sum_{j\geq 1}(-1)^j \Delta_n^{(j-1)} r_{n+j,j}$$

Basing on RGI: $\boxed{r_{i,j} = \sum_{k=0}^{j} C_j^k \tilde{r}_{i-k,j-k} L^k}$ $\quad \tilde{r}_{i,j} = r_{i,j}|_{\mu=Q} \quad L = \ln\mu^2/Q^2$

$$\rho(Q) = \sum_{n\geq 1} \tilde{r}_{n,0}\alpha(\mu)^{n+p-1} + \sum_{n\geq 1}\left[(n+p-1)\alpha(\mu)^{n+p-2}\beta\right]\sum_{j\geq 1}(-1)^j \Delta_n^{(j-1)} \tilde{r}_{n+j,j}$$
$$+ \sum_{k\geq 1} L^k \sum_{n\geq 1}\left[(n+p-1)\alpha(\mu)^{n+p-2}\beta\right]\sum_{j\geq k}(-1)^j C_j^k \Delta_n^{(j-1)} \tilde{r}_{n+j-k,j-k},$$

Eliminate all β-terms

$$\rho(Q) = \sum_{n\geq 1} \tilde{r}_{n,0}\alpha(Q_\star)^{n+p-1}$$

The overall PMC scale Q* is also in perturbative form

Scheme independence for any renormalization scheme (Using the C-scheme coupling with single parameter C to characterize the scheme)

$$\frac{1}{a_\mu} + \frac{\beta_1}{\beta_0}\ln a_\mu = \beta_0\left(\ln\frac{\mu^2}{\Lambda^2} - \int_0^{a_\mu}\frac{da}{\tilde{\beta}(a)}\right).$$

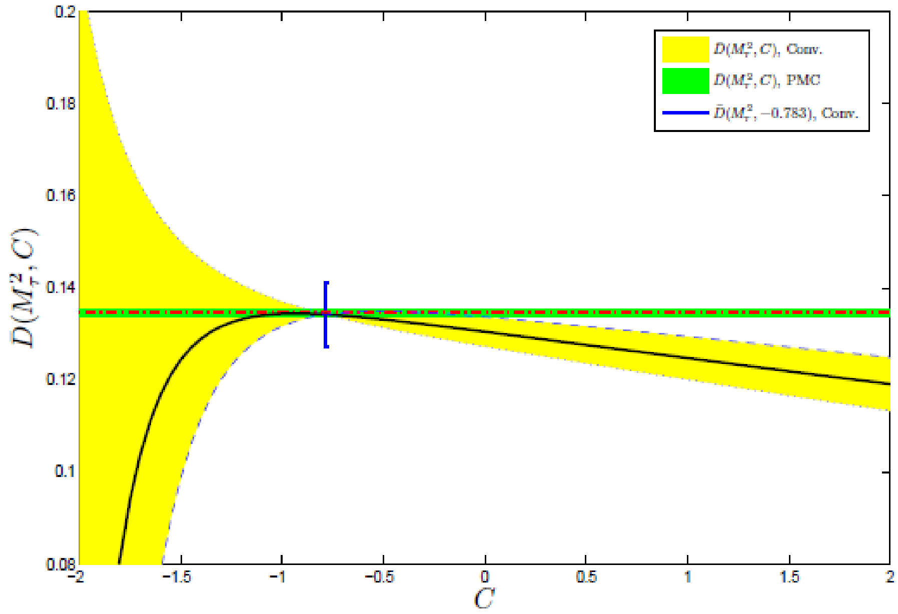
a new coupling $\hat{a}_\mu = \hat{\alpha}_s(\mu)/\pi$ via the following way

$$\frac{1}{\hat{a}_\mu} + \frac{\beta_1}{\beta_0}\ln\hat{a}_\mu \;=\; \beta_0\left(\ln\frac{\mu^2}{\Lambda^2} + C\right),$$

New coupling constant, all scheme-dependence are introduced into C-parameter; its scale and scheme running satisfies the same RGE, which is scheme-independent

$$\hat{\beta}(\hat{a}_\mu) = \mu^2\frac{\partial\hat{a}_\mu}{\partial\mu^2} = -\frac{\beta_0\hat{a}_\mu^2}{1-\frac{\beta_1}{\beta_0}\hat{a}_\mu} = -\beta_0\hat{a}_\mu^2$$

$$\frac{\partial\hat{a}_\mu}{\partial C} = \hat{\beta}(\hat{a}_\mu).$$



After fixing the scale ambiguity,
what we still have to do for perturbative theory ?
It is not the end of the story, but a new beginning

The reliability of perturbative series (feasible，reliability, precision、predictive)

Any perturbative series cannot solve all things, after removing scale and scheme ambiguity, there is still residual scale dependence due to unknown higher-order (UHO) terms !

Thus we need to have ways to estimate UHO contribution

$$\rho_n^{[N/M]}(Q) = a^p \times \frac{b_0 + b_1 a + \cdots + b_N a^N}{1 + c_1 a + \cdots + c_M a^M}$$
$$= \sum_{i=1}^{n} C_i a^{p+i-1} + C_{n+1}\, a^{p+n} + \ldots$$

Several ways to estimate higher-order contributions

- **Conventional:Varying scale — Rough order estimation and cannot estimate conformal contribution**
- **Convervative: The one-order higher shall always be smaller than the given order**
- **Resummation: Find a proper generating function, such as fractional function – Pade approximation**
- **Probability analysis：Bayesian analysis**

P(A∩B) = P(A)*P(B|A)=P(B)*P(A|B)

Give the probability of the higher-order magnitude

Comparing with initial series, the PMC series has advantages:

Better convergence; More accurate without scheme-and-scale dependence; The coefficients have no RGE-relations; ...

Thus it has good potential to do the estimation; especially it can achieve more precise prediction with less given orders.

Example 1: top-pair production near the threshold

Letter

Reanalysis of the top-quark pair production via the e^+e^- annihilation near the threshold region up to N^{3}LO QCD corrections

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Revisiting the top-quark pair production at future e^+e^- colliders*

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Chinese Physics C Vol. 48, No. 5 (2024) 053113

Precise determination of the top-quark on-shell mass M_t via its scale-invariant perturbative relation to the top-quark $\overline{\mathrm{MS}}$ mass $\overline{m}_t(\overline{m}_t)$*

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Chinese Physics C Vol. 45, No. 11 (2021) 113102

Reanalysis of the top-quark pair hadroproduction and a precise determination of the top-quark pole mass at the LHC*

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CPC (2021)
CPC (2024)
CPC (2024)
PLB (2024)

NLO QCD correction

J. Jersak, E. Laermann, and P. M. Zerwas, Phys. Rev. D 25, 1218 (1982)

N^{2}LO QCD correction

J. Gao and H. X. Zhu, Phys. Rev. Lett. 113, 262001 (2014)

L. Chen, O. Dekkers, D. Heisler, W. Bernreuther, and Z. G. Si, J. High Energy Phys. 12 (2016) 098

N^{3}LO QCD correction

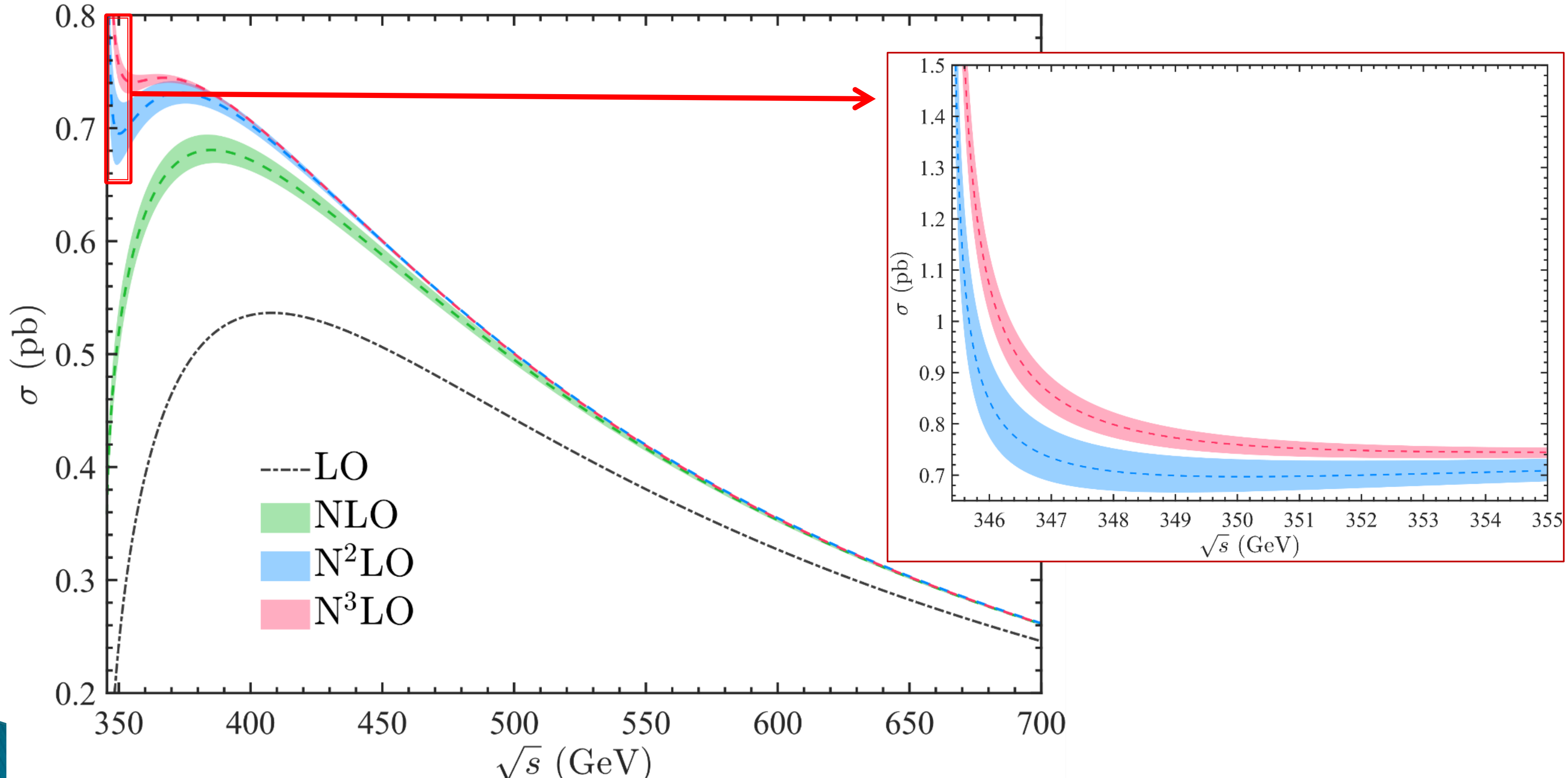
M. Fael, F. Lange, K. Schönwald, and M. Steinhauser, Phys. Rev. Lett. 128, 172003 (2022)

M. Fael, F. Lange, K. Schönwald, and M. Steinhauser, Phys. Rev. D 106, 034029 (2022)

$\boxed{\textbf{L. Chen, X. Chen, X. Guan and Y. Q. Ma, Phys. Rev. Lett. 132, 10 (2024)}}$

$$e^+e^- \to \gamma^* \to t\bar{t}$$

$\sqrt{s} \approx 2m_t$



Up to N^{3}LO, total cross section of $e^+e^- \to \gamma^* \to t\bar{t}$ can be written as

$$\sigma = \sigma_0\left(1 + r_1\alpha_s + r_2\alpha_s^2 + r_3\alpha_s^3 + \cdots\right)$$

$$\sigma_0 = N_C \frac{4\pi\alpha^2}{3s}\frac{v(3-v^2)}{2}e_t^2$$

$$r_1 = \frac{1}{v}r_{1,v} + r_{1,+}$$

$$r_2 = \frac{1}{v^2}r_{2,v^2} + \frac{1}{v}r_{2,v} + r_{2,+}$$

$$r_3 = \frac{1}{v^2}r_{3,v^2} + \frac{1}{v}r_{3,v} + r_{3,+}$$

$\boldsymbol{v = \sqrt{1 - \frac{4m_t^2}{s}}}$ **the velocity of produced quarks**

only numerical results !

we schematically factorize total cross section as the product of Coulomb and non-Coulomb parts

PSLQ algorithm

$$\sigma = \sigma_0 \times \mathcal{R}_{\mathrm{NC}} \times \mathcal{R}_{\mathrm{C}}$$

Coulomb part

reconstructing analytic form !

Non-Coulomb part

H. R. P. Ferguson, and D. H. Bailey, RNR Technical Report RNR-91-032 (1992)
H. R. P. Ferguson, D. H. Bailey, and S. Arno, Math. Comp. 68, 351 (1999)

The QCD coupling $\alpha_s^{\mathrm{V}}(\mathbf{q}^2)$ has been introduced for describing the interaction of the non-relativistic heavy quark-antiquark pair, which is defined as the effective charge in the following Coulomb-like potential:

$$V(\mathbf{q}^2) = -4\pi C_F \frac{\alpha_s^{\mathrm{V}}(\mathbf{q}^2)}{\mathbf{q}^2},$$

where $\alpha_s^{\mathrm{V}}(\mathbf{q}^2)$ absorbs all the higher-order QCD corrections, which is related to the $\overline{\mathrm{MS}}$-scheme coupling via the following way

$$\alpha_s^{\mathrm{V}}(\mathbf{q}^2) = \alpha_s(\mu^2) + \left(a_1 - \beta_0 \ln\frac{\mathbf{q}^2}{\mu^2}\right)\alpha_s^2(\mu^2) + \left(a_2 - (2a_1\beta_0 + \beta_1)\ln\frac{\mathbf{q}^2}{\mu^2} + \beta_0^2 \ln^2\frac{\mathbf{q}^2}{\mu^2}\right)\alpha_s^3(\mu^2) + \cdots$$

$$a_1 = \frac{1}{4\pi}\left(\frac{31}{9}C_A - \frac{20}{9}T_F n_l\right)$$

$$a_2 = \frac{1}{(4\pi)^2}\left[\left(\frac{4343}{162} + 4\pi^2 - \frac{\pi^2}{4} + \frac{22}{3}\zeta_3\right)C_A^2 - \left(\frac{1798}{81} + \frac{56}{3}\zeta_3\right)C_A T_F n_l - \left(\frac{55}{3} - 16\zeta_3\right)C_F T_F n_l + \left(\frac{20}{9}T_F n_l\right)^2\right]$$

T. Appelquist, M. Dine and I. J. Muzinich, Phys. Lett. B 69, 231 (1977)
W. Fischler, Nucl. Phys. B 129, 157 (1977)

M. Peter, Phys. Rev. Lett. 78, 602 (1997)
Y. Schroder, Phys. Lett. B 447, 321 (1999)

$$\sigma = \sigma_0 \times \mathcal{R}_{\rm NC} \times \mathcal{R}_{\rm C}$$

exactly non-conformal term

$$\mathcal{R}_{\rm C} = 1 + C_F \frac{\pi}{2v} \alpha_s^{\rm V}(sv^2) + C_F^2 \frac{\pi^2}{12v^2} \alpha_s^{\rm V,2}(sv^2) + \boxed{C_F \left(\frac{\pi^3}{3v} \beta_0^2 - C_F \frac{2\zeta_3}{v^2} \beta_0 \right)} \alpha_s^{\rm V,3}(sv^2) + \cdots$$

PMC

$$\mathcal{R}_{\rm C}\Big|_{\rm PMC} = 1 + C_F \frac{\pi}{2v} \alpha_s^{\rm V}(Q_{*,\rm C}^2) + C_F^2 \frac{\pi^2}{12v^2} \alpha_s^{\rm V,2}(Q_{*,\rm C}^2) + \boxed{0 \times} \alpha_s^{\rm V,3}(Q_{*,\rm C}^2) + \cdots$$

resum $\frac{X}{1-\exp(-X)}$

Sommerfeld-Gamow-Sakharov factor

$$X = \pi\, C_F \frac{\alpha_s^{\rm V}(Q_{*,\rm C}^2)}{v}$$

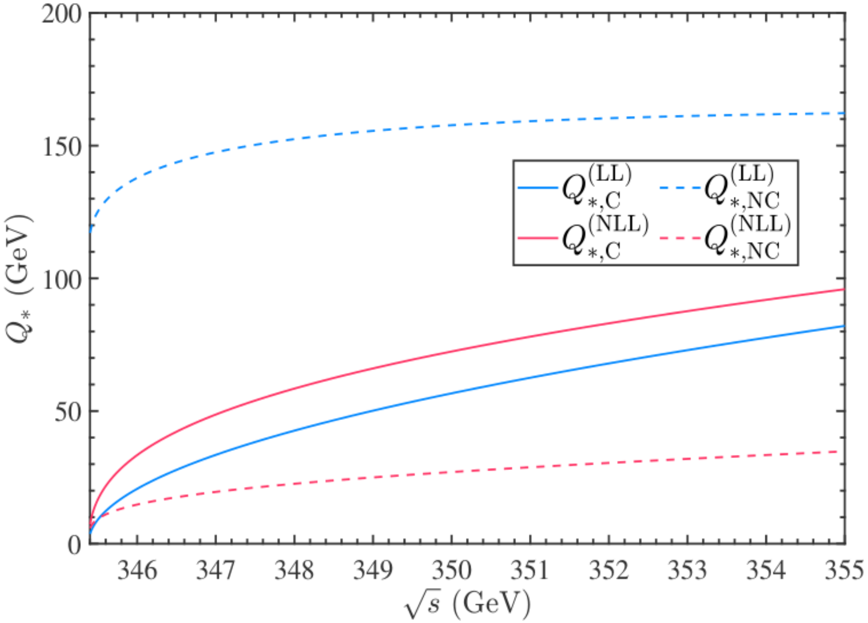
Obtained by solving NR Schrödinger equation

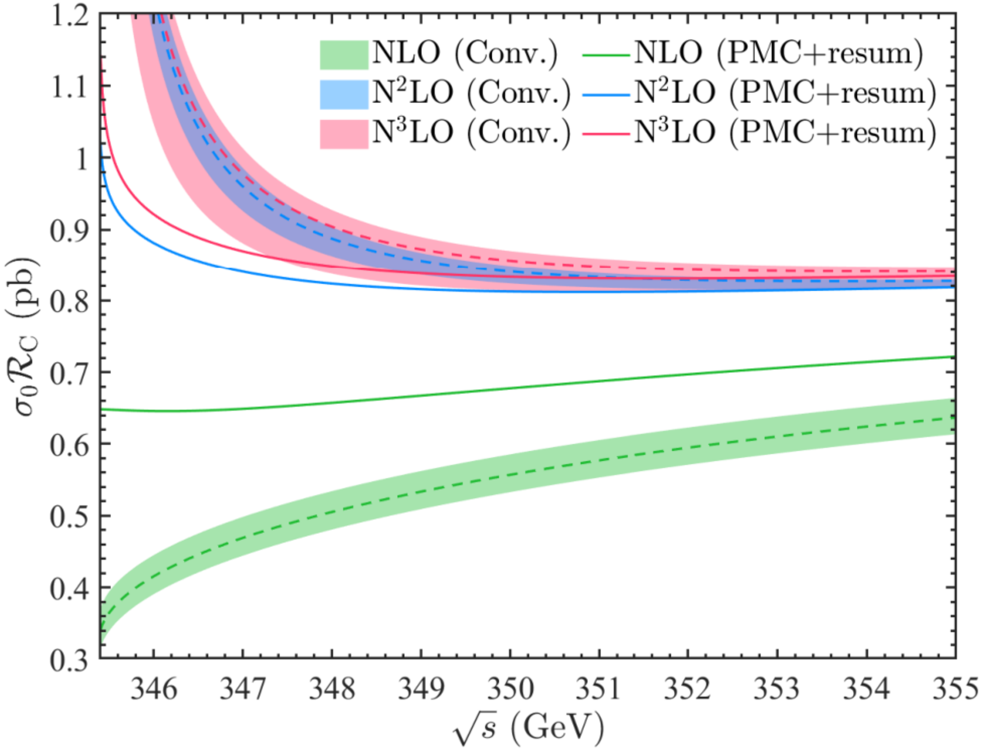
A. D. Sakharov, Zh. Eksp. Teor. Fiz. 18, 631 (1948)

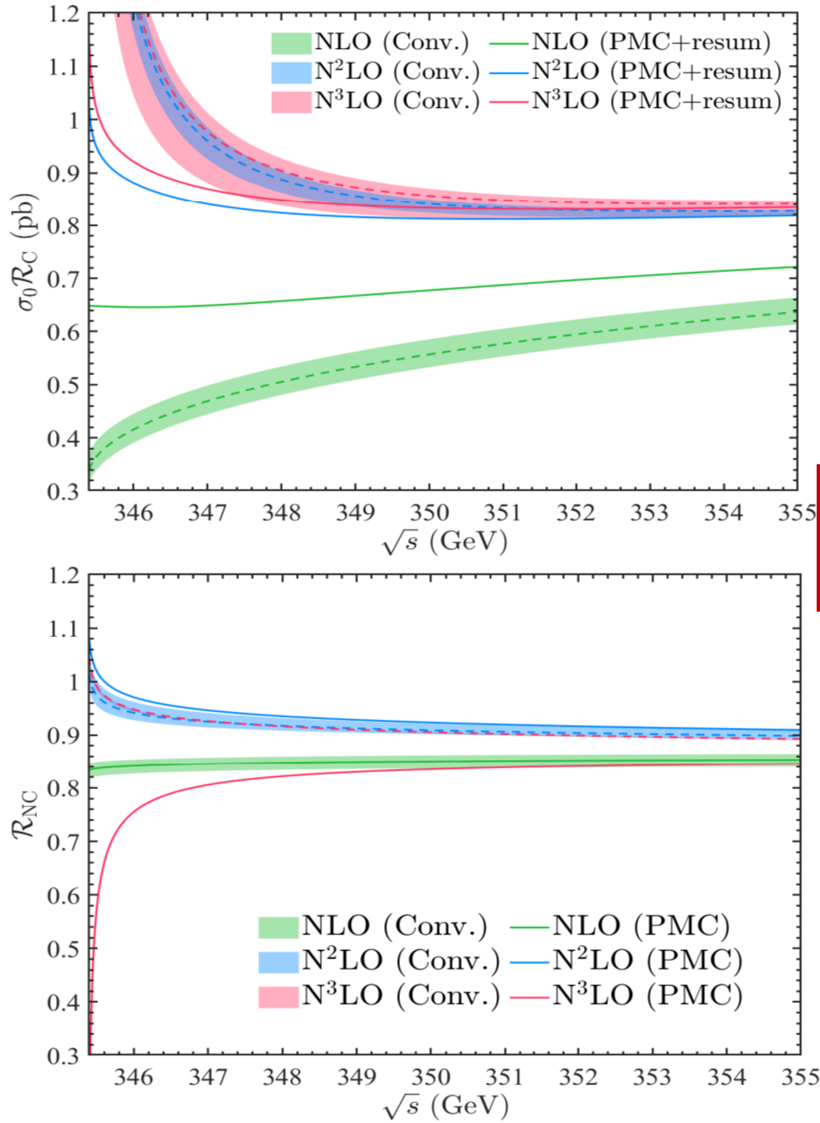
$$\frac{X}{1-\exp(-X)} = 1 + \frac{X}{2} + \frac{X^2}{12} - \frac{X^4}{720} + \cdots$$

The X^3-coefficient is exactly zero!

$$\lim_{v\to 0^+} v \frac{\pi C_F \alpha_s^V / v}{1-\exp(-\pi C_F \alpha_s^V / v)} = \pi C_F \alpha_s^V \text{ is a finite value}$$







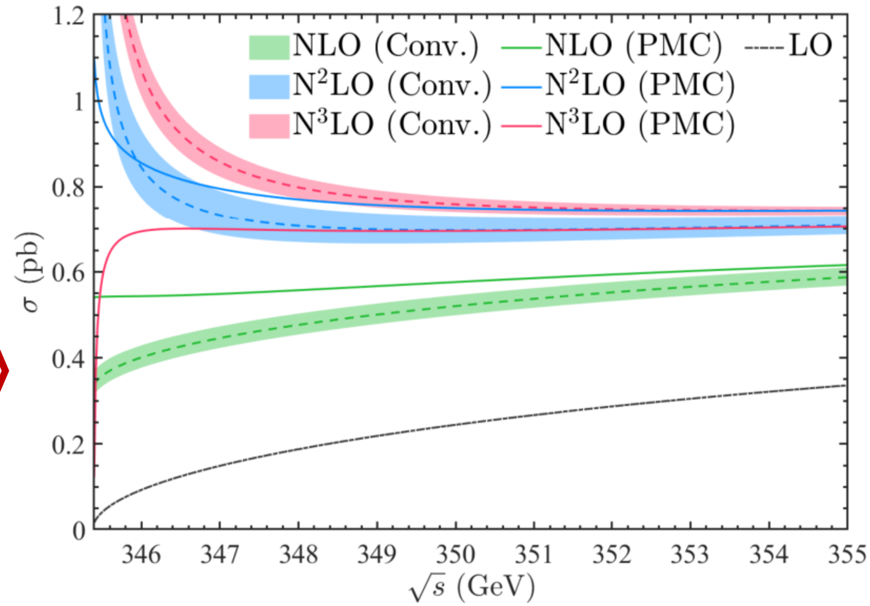
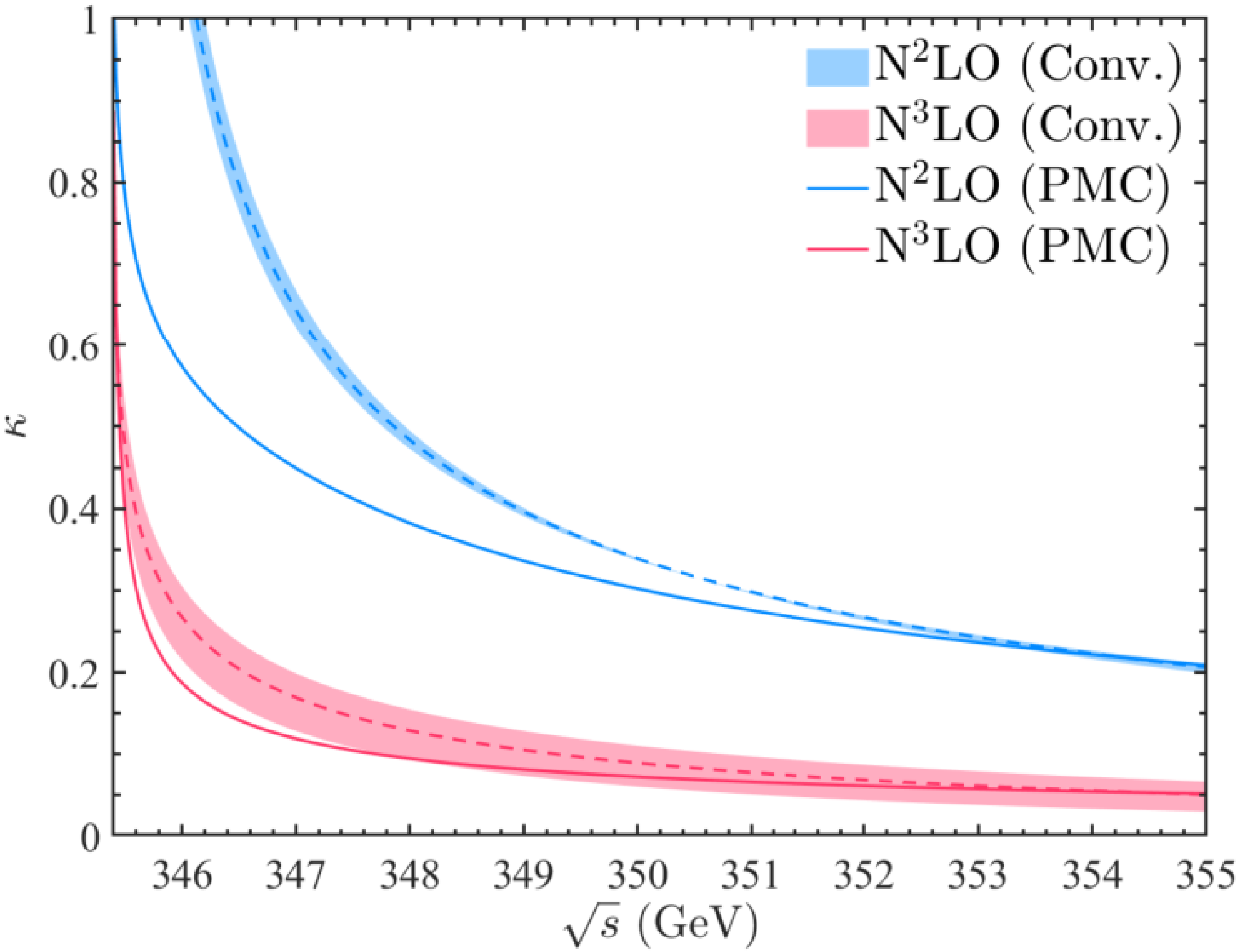
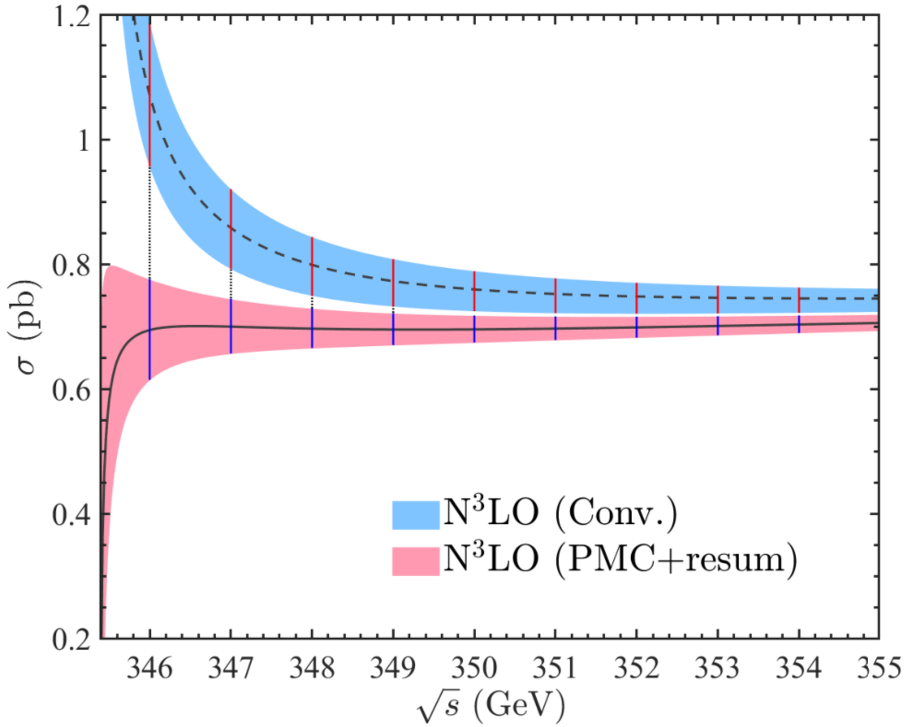


FIG. 3. (Color online) Total cross section $\sigma_{t\bar{t}}$ with different QCD corrections under conventional (dashed line) and PMCs (solid line) scale-setting approaches, respectively.

$$\kappa(\mathrm{N^iLO}) = \left|\frac{\sigma(\mathrm{N^iLO}) - \sigma(\mathrm{N^{i-1}LO})}{\sigma(\mathrm{N^{i-1}LO})}\right|$$

After applying the PMC, the scale dependence is removed and the pQCD convergence near the threshold region is improved.



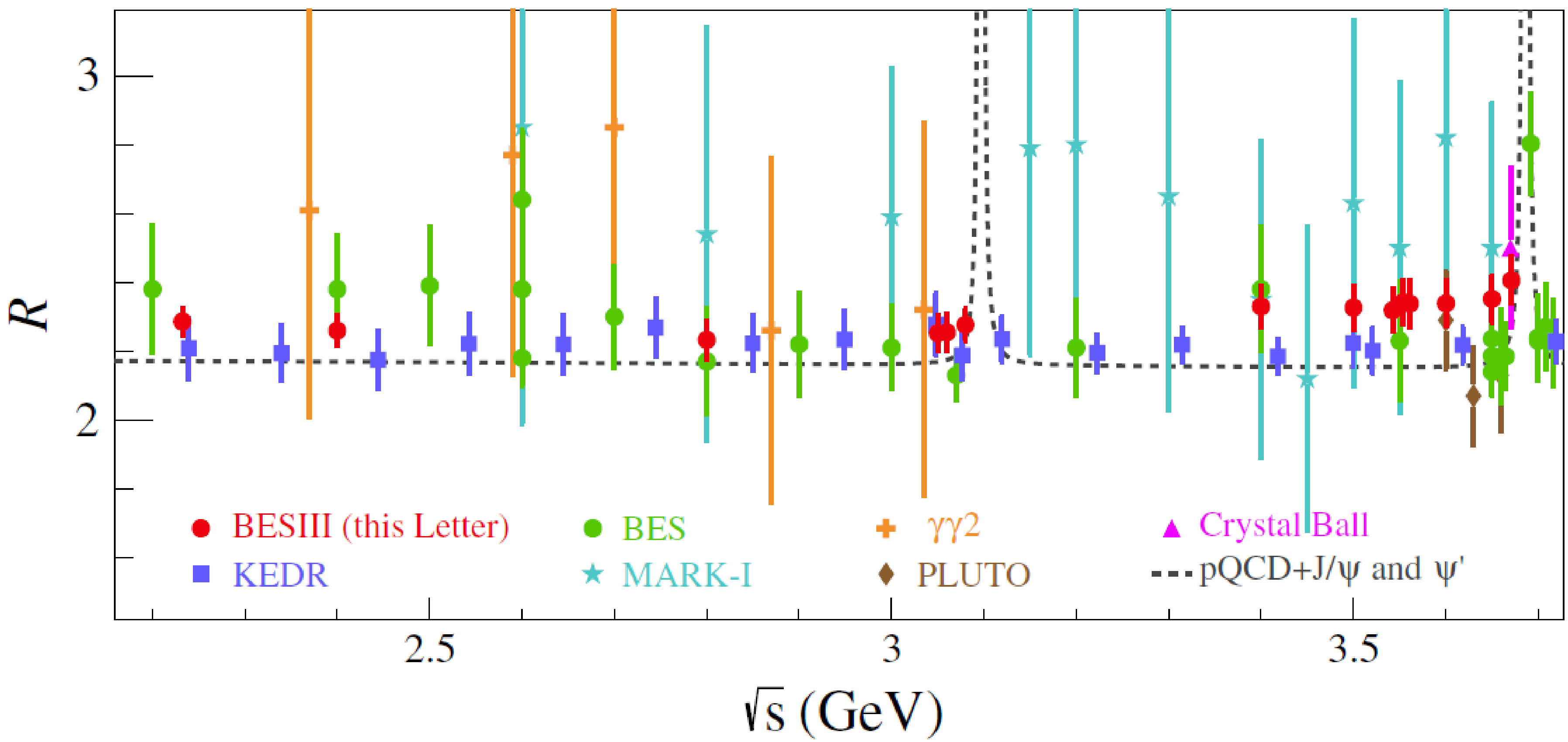


Providing more reliable foundation for constraining predictions of UHO contributions

By applying PMC, uncertainties caused by the UHO-terms become smaller. These results confirm the importance of the PMC scale-setting approach.

Example 2: Ruds

$$R(Q) = \frac{\sigma(e^+e^- \to hadrons, Q)}{\sigma(e^+e^- \to \mu^+\mu^-, Q)} = R_{\mathrm{EW}}(Q)\left(1 + \delta_{\mathrm{QCD}}(Q)\right)$$



BES III, Phys. Rev. Lett. 128, 062004 (2022)

Massless cases

2-loop, [Phys. Lett. B 85 (1979) 277-279]
3-loop, [Phys. Lett. B 259 (1991) 144-150]
3-loop, [Phys. Rev. Lett. 66 (1991) 560-563]
4-loop, [Phys. Rev. Lett. 101 (2008) 012002]
4-loop, [Phys. Rev. Lett. 104 (2010) 132004]
4-loop, [Phys. Rev. Lett. 108 (2012) 222003]
4-loop, [JHEP 07 (2012) 017]
4-loop, [Phys. Lett. B 714 (2012) 62-65]

$$\frac{1}{2}R^{(4)}_{\rm uds}(s)|_{\rm Conv.} = 1+\sum_{i=1}^{\ell} r_i \left(\frac{\alpha_s(s)}{\pi}\right)^i = 1+\frac{\alpha_s(s)}{\pi}+0.1661\alpha_s^2(s)-0.3317\alpha_s^3(s)-1.0972\alpha_s^4(s)$$

$$\frac{1}{2}R^{(4)}_{\rm uds}(s)|_{\rm PMCs} = 1+\sum_{i=1}^{\ell} r_{i,0} \left(\frac{\alpha_s(Q_*^2)}{\pi}\right)^i = 1+\frac{\alpha_s(Q_*^2)}{\pi}+0.2174\alpha_s^2(Q_*^2)+0.1108\alpha_s^3(Q_*^2)+0.0698\alpha_s^4(Q_*^2),$$

$$\ln\frac{Q_*^2}{s} = \sum_{i=0}^{2} S_i \left(\frac{\alpha_s(Q_*^2)}{\pi}\right)^i = 0.2249+1.5427\alpha_s(Q_*^2)+2.4933\alpha_s^2(Q_*^2)$$

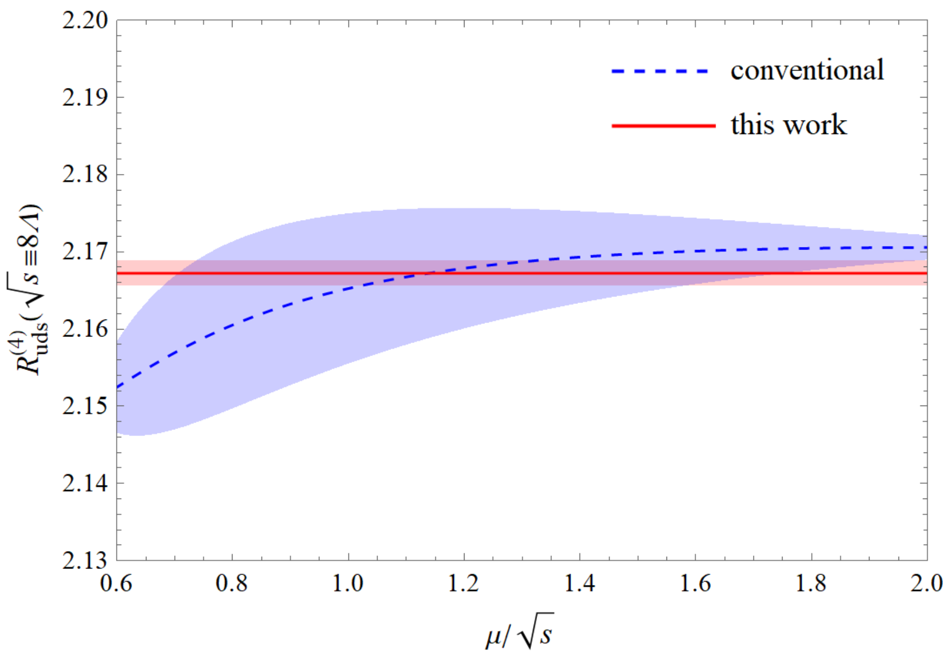
基于初始序列估算

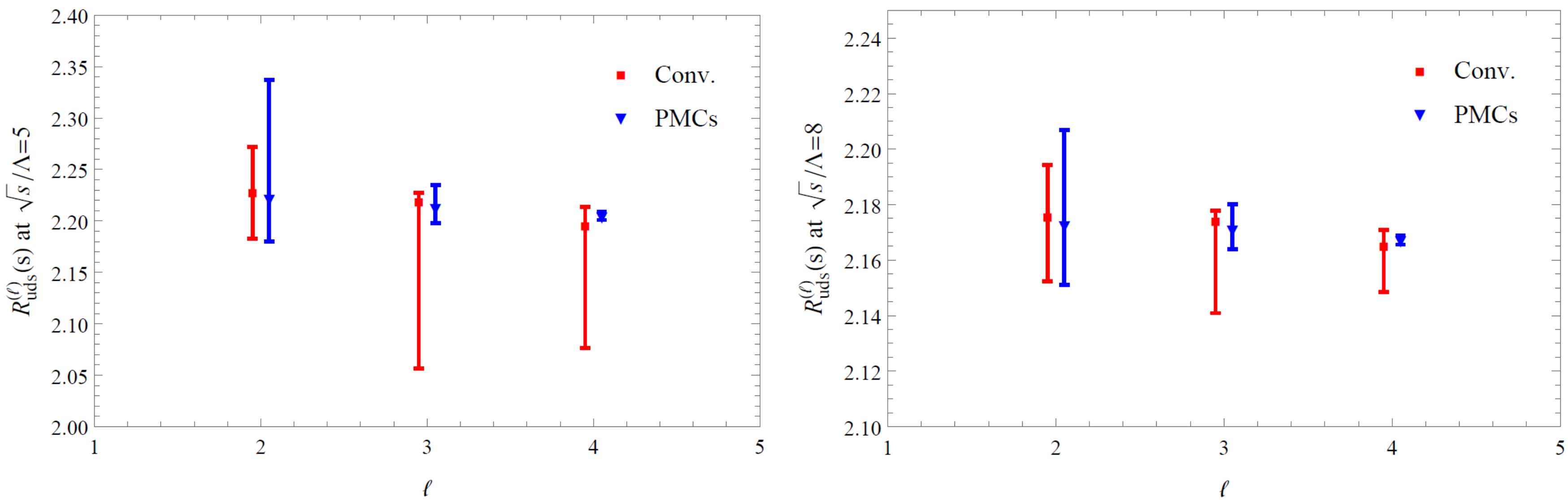
计算中心值：$\mu_r = Q$

计算理论误差：

① $\pm$ |最后一阶大小|

② $Q/2 < \mu_r < 2Q$





Based on the Bayes analysis

随着阶数增加，理论预言快速收敛到准确值（**蓝色**）

Summary and Outlook

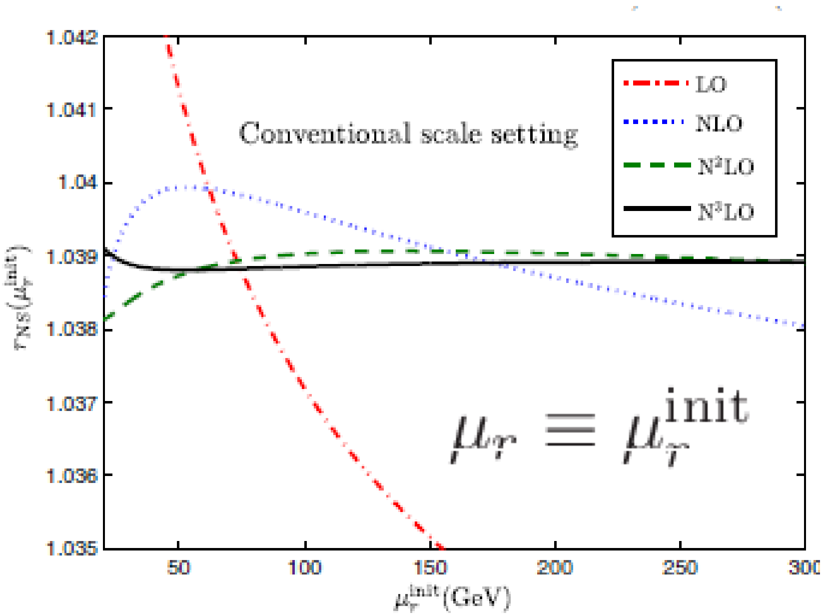
Up to infinite order, the predictions are scheme and scale independent, there is no scale ambiguity

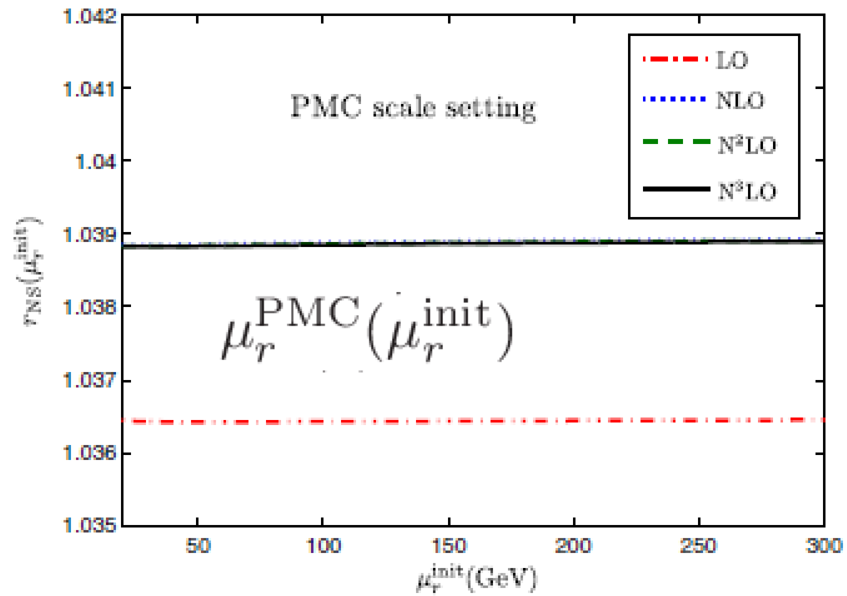
At fixed-order, guessing/using typical momentum flow as the scale, one cannot get precise value for all-orders, and also for each order, becoming an important systematic error

PMC is not simply chosen "special/effective scale", but basing on RGE and standard RGI and using general way to set the optimal scale such that to achieve precise prediction for any fixed order

更收敛、更精确的序列是估算未知高阶项贡献的基石

Before and after applying the PMC, the issues are always like this

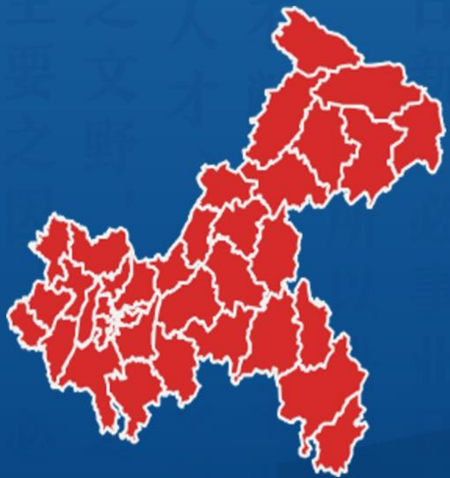




PMC predictions:
Quickly approaches its “true” value
More accurate predictions for low orders without initial scale dependence
Residual scale uncertainty is small

Great thanks !

魅力“山城” - 重庆



- 总面积:
- 8.24 万平方千米

- 常住人口:
- 3213.3万人

- GDP:
- 2023度3万亿元（GDP第五，综合第六）

中国，重庆
Chongqing, China

魅力"山城" - 重庆

重庆 — 中国十大宜居城市（2022中科院发布）

重庆房价合理，是一座民众幸福指数很高的城市。

2023年10月全国城市住宅房价排行榜（部分）

排名	城市	房价平均价格
1	上海	71,255元/m²
2	北京	70,729元/m²
3	深圳	67,579元/m²
4	厦门	52,020元/m²
5	广州	45,265元/m²
……	成都	18,828元/m²
……	西安	17,845元/m²
47	**重庆**	**12,378元/m²**

重庆的房价均价

≈ 1/6的 **上海** 房价均价

≈ 1/6的 **北京** 房价均价

≈ 1/5的 **深圳** 房价均价

≈ 1/4的 **厦门** 房价均价

≈ 1/3的 **广州** 房价均价

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弘深优秀学者	45周岁	税前年薪：60万元	160万元 （含地方资助）	100-300万元
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