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正德厚生 笃学敏行

重味介子衰变过程中的极化与量子自旋纠缠效应

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2404.06221; 2310.03425

2024年6月30日-7月4日 @中国科学技术大学

第二届强子物理新发展研讨会

Background

- **Heavy Flavor hadron physics**

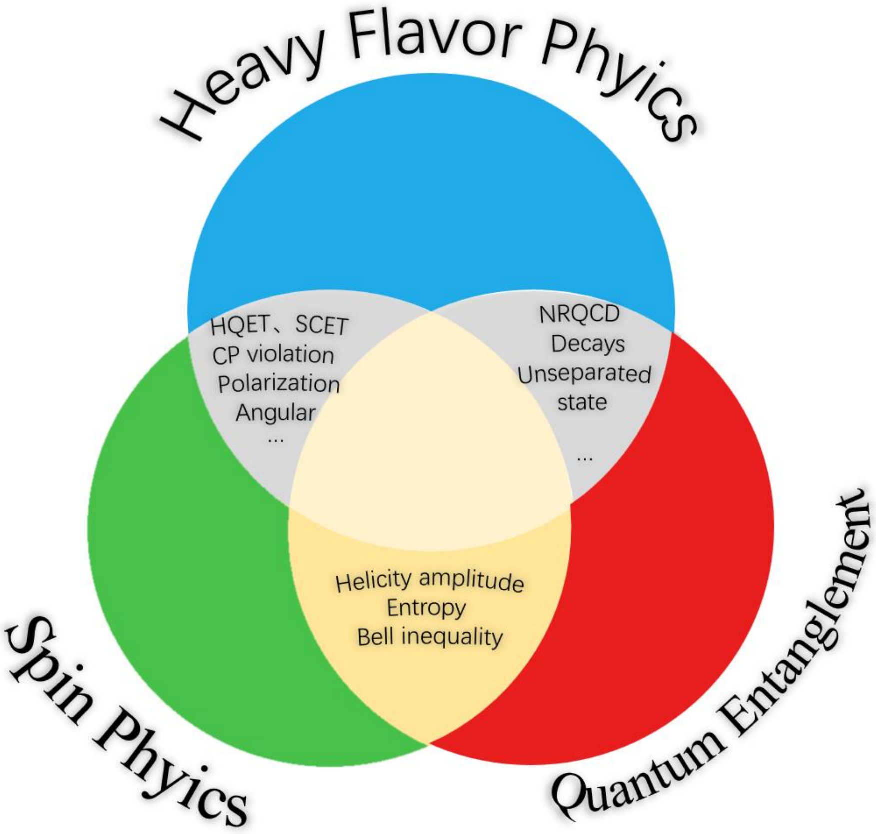
  CP violation; New physics；
  Heavy quark symmetry;
  nonrelativistic at Fermi-scale;
  test effect theory of QCD;

- **Spin Physics**

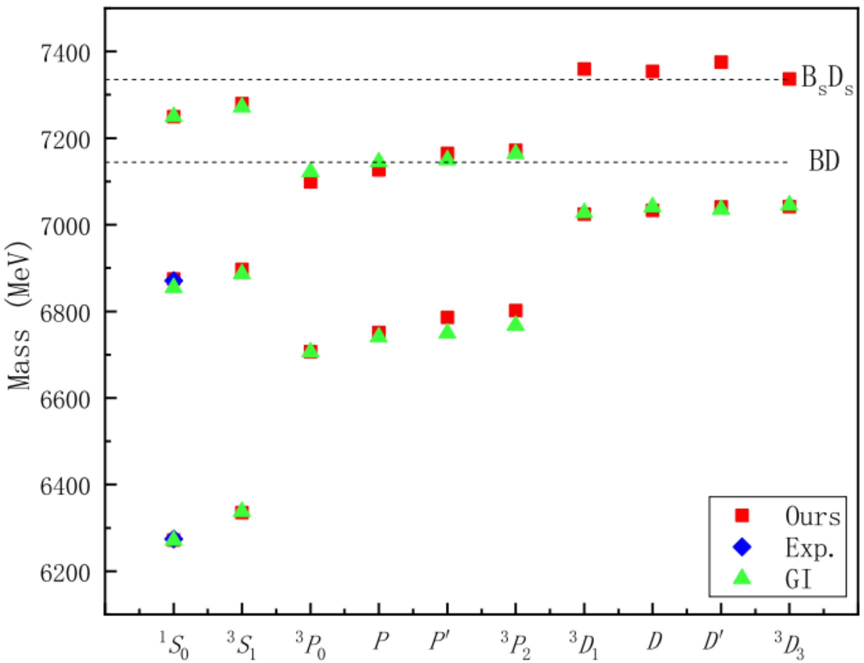
  provide more information
  in angular distributions of
  polarized particles

- **Quantum entanglement**

  Verify entanglement and
  violation of Bell inequality
  in strong and weak interactions

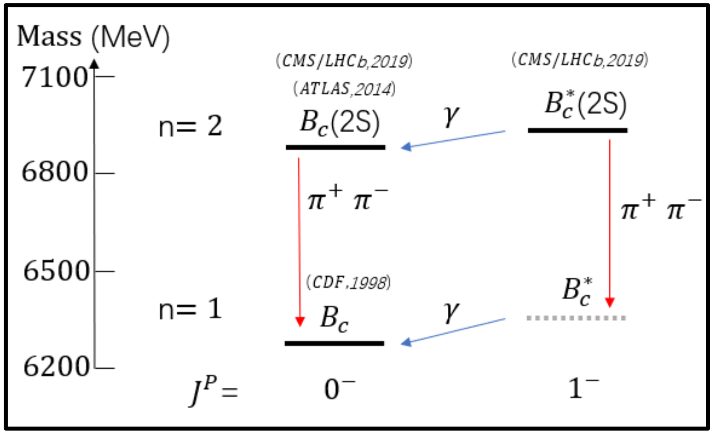


Bc meson family spectrum

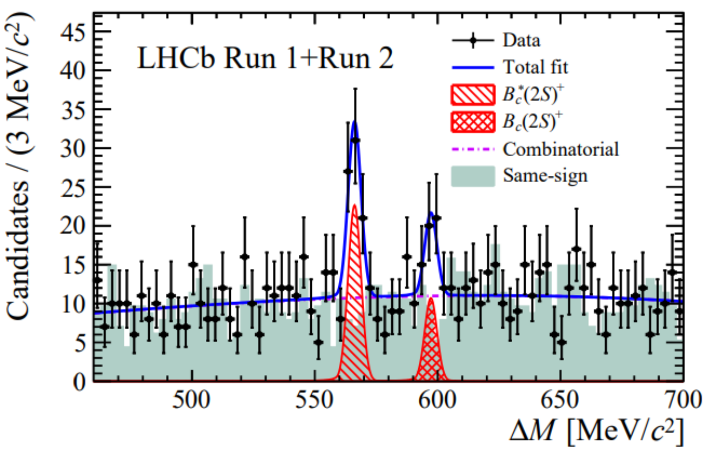


nonrelativistic potential model
+ Coupled channel analysis

Hao-Zhu, 2402.18898

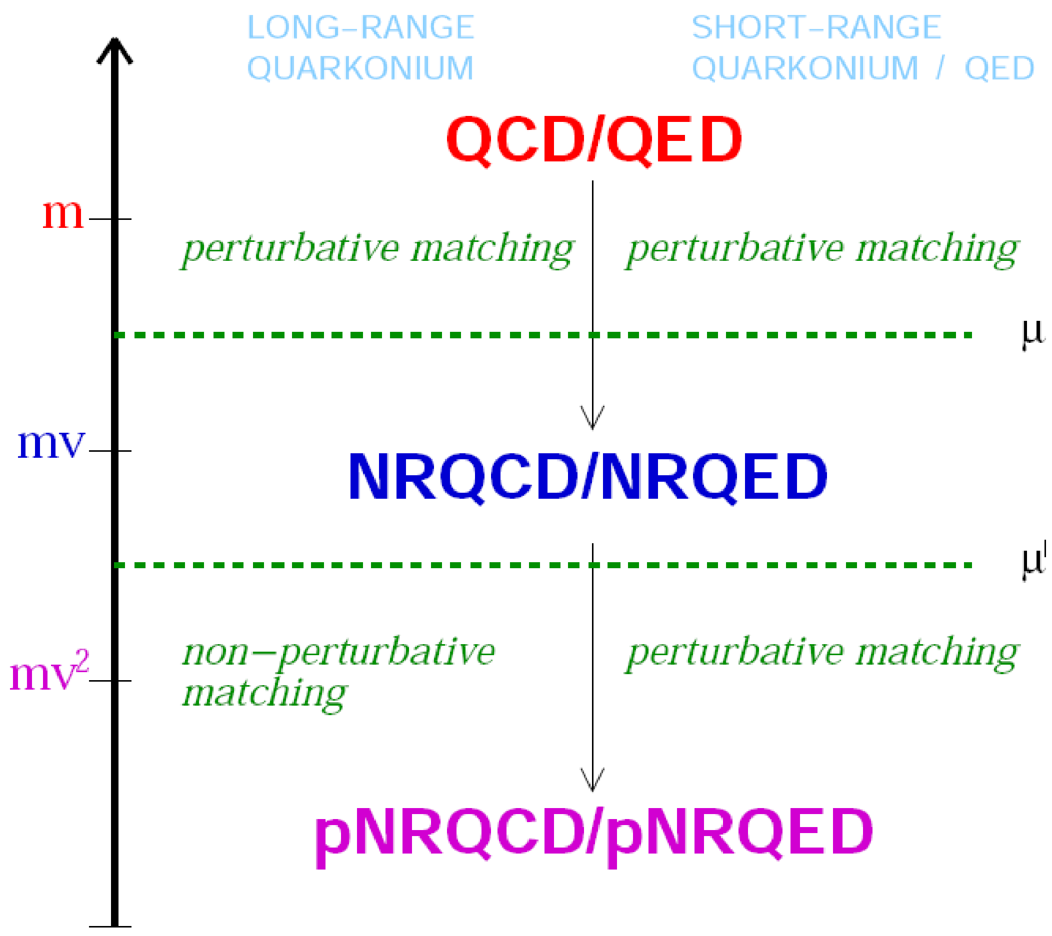


Bc meson family



$$\Delta M = M(B_c^+\pi^+\pi^-) - M(B_c^+)$$

Nonrelativistic QCD/QED



$\alpha_s(mv) \sim v$

$v^2 \approx 0.1$ for the Υ

Bodwin-Braaten-Lapage
1995

Pineda-Soto-Brambilla-Vairo
2000

NRQCD/pNRQCD/γpNRQCD in unequal mass case

- **QCD Lagrangian**

$$\mathcal{L}_{\rm QCD} = \sum_{q=u,d,s,\ldots} \overline{\psi}_{qi}(x)\left[(i\gamma_\mu D^\mu)_{ij} - m_q\delta_{ij}\right]\Psi_{qj}(x) - \frac{1}{4}F^a_{\mu\nu}(x)F^{\mu\nu\,a}(x) \ ,$$

- **Rewrite heavy quark field and do the NR expansion**

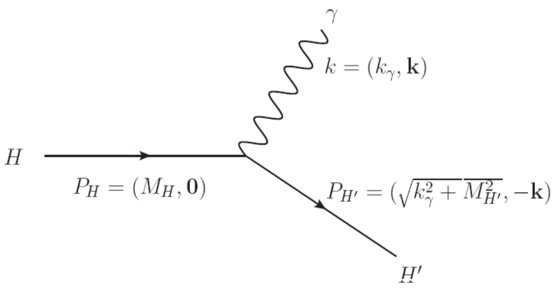
$$\Psi = e^{-iMt}\tilde{\Psi} = e^{-iMt}\begin{pmatrix}\psi\\ \chi\end{pmatrix}, \qquad \Psi' = e^{iM't}\tilde{\Psi} = e^{iM't}\begin{pmatrix}\psi'\\ \chi'\end{pmatrix},$$

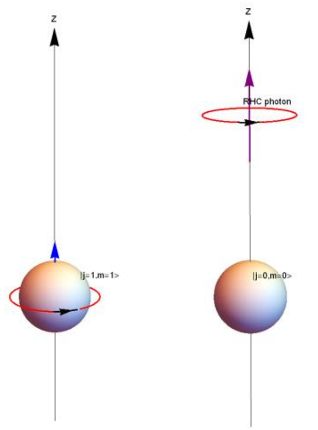
- **Obtain NRQCD Lagrangian**

$$\begin{aligned}\mathcal{L}_{NRQCD} =& \psi^\dagger\left(iD_t - \frac{1}{2M}(i\mathbf{D})^2\right)\psi + \frac{c_F}{2M}\psi^\dagger\boldsymbol{\sigma}\cdot g\mathbf{B}\psi \\ &+ \frac{c_D}{8M^2}\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi^\dagger + \frac{c_S}{8M^2}\psi^\dagger(i\boldsymbol{\sigma}\cdot\mathbf{D}\times g\mathbf{E} - i\boldsymbol{\sigma}\cdot g\mathbf{E}\times\mathbf{D})\psi^\dagger \\ &+ \frac{c_4}{8M^3}\psi'^\dagger\left(\mathbf{D}^2\right)^2\psi' + \mathcal{O}\left(1/M^3\right) \\ &+ [\psi \to i\sigma^2\chi'*, A_\mu \to -A^T_\mu, M \to M'] + \mathcal{L}_{light}].\end{aligned}$$

Vector Bc* meson decay width

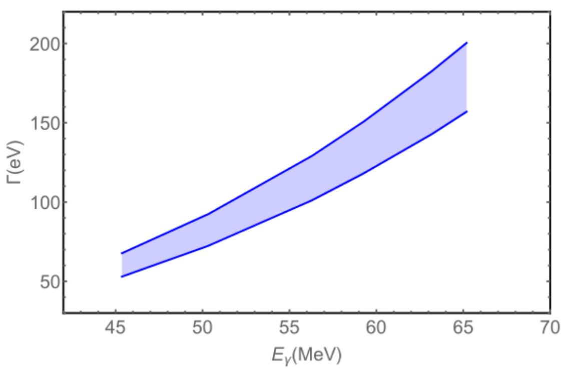
- **Bc* (1S) major (99.99%) electromagnetic decays to Bc(1S): M1 transition**





$$
\begin{aligned}
\mathcal{L}_{\gamma\mathrm{pNRQCD}} = \int d^3r \,\mathrm{Tr}\Big[& e\frac{e_Q - e'_Q}{2} V_A^{\mathrm{em}} \mathrm{S}^\dagger \mathbf{r}\cdot\mathbf{E}^{\mathrm{em}} \mathrm{S} \\
& + e\Big(\frac{e_Q m'_Q - e'_Q m_Q}{4 m_Q m'_Q}\Big) \Big[V_S^{\frac{\sigma\cdot B}{m}} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}} \right\} \mathrm{S} \\
& + \frac{1}{8} V_S^{(r\cdot\nabla)^2 \frac{\sigma\cdot B}{m}} \left\{ \mathrm{S}^\dagger, \mathbf{r}^i \mathbf{r}^j \left(\boldsymbol{\nabla}^i \nabla^j \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}} \right) \right\} \mathrm{S} \\
& + V_O^{\frac{\sigma\cdot B}{m}} \left\{ \mathrm{O}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}} \right\} \mathrm{O} \Big] \\
& + e\Big(\frac{e_Q m_{Q'}^2 - e'_Q m_Q^2}{32 m_Q^2 m_{Q'}^2}\Big) \Big[4\frac{V_S^{\frac{\sigma\cdot B}{m^2}}}{r} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}} \right\} \mathrm{S} \\
& + 4\frac{V_S^{\frac{\sigma\cdot(r\times r\times B)}{m^2}}}{r} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot[\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\mathbf{B}^{\mathrm{em}})] \right\} \mathrm{S} \\
& - V_S^{\frac{\sigma\cdot\nabla\times E}{m^2}} \left[\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot[-i\boldsymbol{\nabla}\times, \mathbf{E}^{\mathrm{em}}] \right] \mathrm{S} \\
& - V_S^{\frac{\sigma\cdot\nabla_r\times r\cdot\nabla E}{m^2}} \left[\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\left[-i\boldsymbol{\nabla}_r\times, \mathbf{r}^i \left(\boldsymbol{\nabla}^i \mathbf{E}^{\mathrm{em}}\right)\right] \right] \mathrm{S} \Big] \\
& + e\Big(\frac{e_Q m_{Q'}^3 - e'_Q m_Q^3}{8 m_Q^3 m_{Q'}^3}\Big) \Big[V_S^{\frac{\nabla_r^2 \sigma\cdot B}{m^3}} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}} \right\} \nabla_r^2 \,\mathrm{S} \\
& + V_S^{\frac{(\nabla r\cdot\sigma)(\nabla r\cdot B)}{m^3}} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}^i \mathbf{B}^{\mathrm{em}j} \right\} \boldsymbol{\nabla}_r^i \nabla_r^j \,\mathrm{S} \Big]\Big],
\end{aligned}
$$

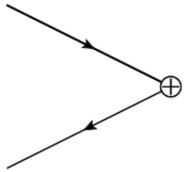
- **We generalize the pNRQCD to unequal mass case and obtain the effective Lagrangian**



Decay constant

- **Bc* decay constants in QCD**

$$\left\langle 0 \left| \bar{b}\gamma^{\mu} c \right| B_c^*(P,\varepsilon)\right\rangle = f_{B_c^*}^{v} m_{B_c^*}\varepsilon^{\mu},$$


(a) LO

- **Bc* decay constants in NRQCD**

mathing coefficients NRQCD LDMEs

$$f_{B_c^*}^{v} = \sqrt{\frac{2}{m_{B_c^*}}}\,\underline{C_v(m_b, m_c, \mu_f)}\,\underline{\left\langle 0 \left| \chi_b^{\dagger}\sigma\cdot\varepsilon\psi_c \right| B_c^*(\mathbf{P})\right\rangle}(\mu_f) + O(v^2)$$

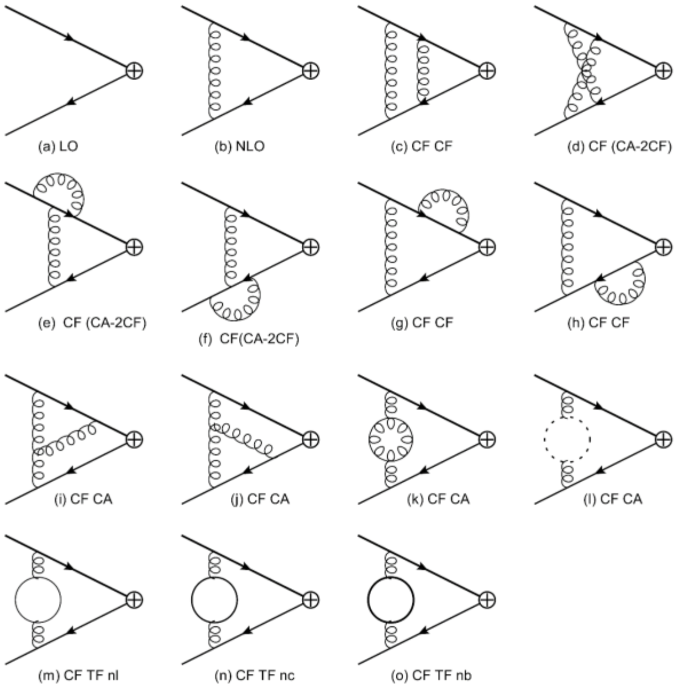
- **Matching Formulae**

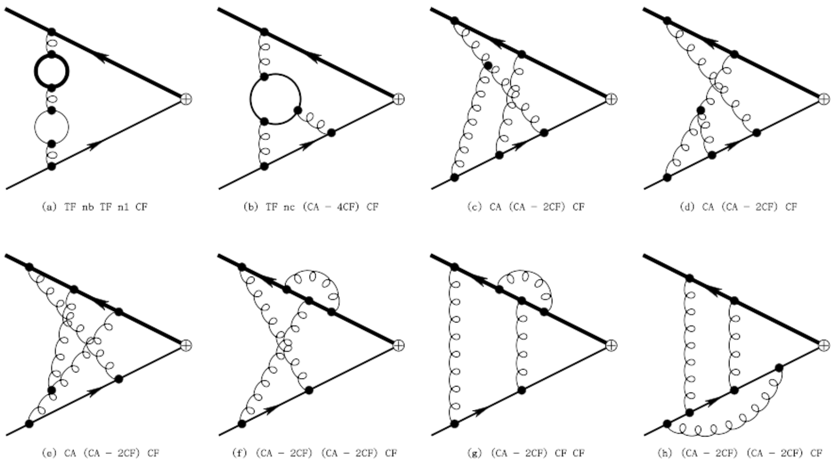
Braaten-Fleming,PRD52,181(1995);
Lee-Sang-Kim,JHEP01,113(2011)

$$Z_J Z_{2,b}^{\frac{1}{2}} Z_{2,c}^{\frac{1}{2}} \Gamma_J = C_J \tilde{Z}_J^{-1} \tilde{Z}_{2,b}^{\frac{1}{2}} \tilde{Z}_{2,c}^{\frac{1}{2}} \tilde{\Gamma}_J$$

$\tilde{Z}_J$:NRQCD $\overline{\text{MS}}$ current renormalization constants

Typical diagrams up to three-loop





LO:1, NLO:1, NNLO:13, N^3LO:268

Matching coefficients up to three loops

➢ for vector current

$$\mathcal{C} = 1 - 2.29\left(\frac{\alpha_s^{(n_l)}}{\pi}\right) - 35.44\left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^2 - 1686.27\left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4),$$

for $n_l = 3, n_c = 1, n_b = 0,$

Sang-Zhang-Zhou, arXiv:2210.02979

➢ for pseudoscalar current

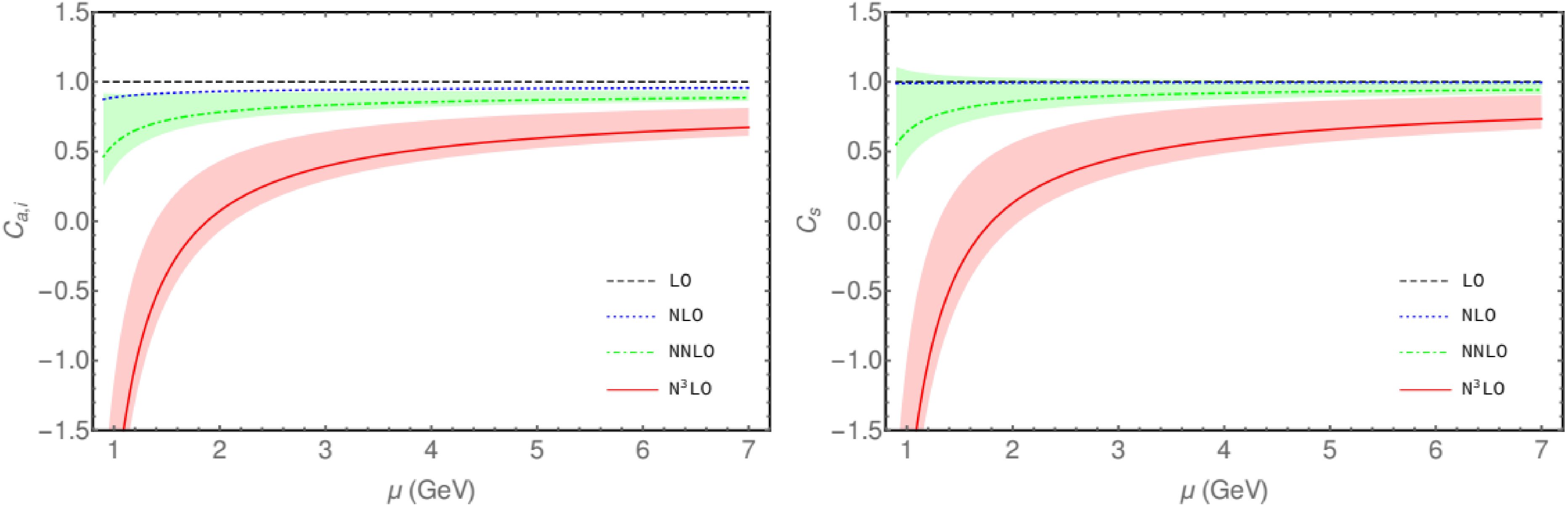
$$\mathcal{C}(x_{\text{phys}}) = 1 - 1.62623\left(\frac{\alpha_s^{(n_l)}(m_r)}{\pi}\right) - 6.51043\left(\frac{\alpha_s^{(n_l)}(m_r)}{\pi}\right)^2 - 1520.59\left(\frac{\alpha_s^{(n_l)}(m_r)}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4)$$

Feng-Jia-Mo-Pan-Sang-Zhang, arXiv:2208.04302

for axial-vector and scalar currents

- **Matching coefficients for axial-vector and scalar up to three loops**

Tao-Xiao-Zhu: 2303.07220



Nonconvergence behaviors also in other two currents

Multi-loop integral calculation performed by AMFlow (Ma et al)

Sub-leading Contribution

➢ **Relativistic corrections**

$$\langle 0|\overline{Q_1}\gamma^5 Q_2 \,|Q_2\overline{Q_1}\rangle_{\rm QCD}$$

$$= \sqrt{2M_H}\left[C_0^P \left\langle 0\left|\chi_1^\dagger\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle_{\rm NRQCD} + C_2^P\left\langle 0\left|(\mathbf{D}\chi_1)^\dagger\cdot\mathbf{D}\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle_{\rm NRQCD}+\cdots\right]$$

Employing EOM: $\left\langle 0\left|(\mathbf{D}\chi_1)^\dagger\mathbf{D}\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle = -2m_r E\left\langle 0\left|\chi_1^\dagger\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle.$

$$f_{B_c^*} = 2\sqrt{\frac{N_c}{m_{B_c^*}}}\left[\mathcal{C}_v + \frac{d_v E_{B_c^*}}{12}\left(\frac{8}{M}-\frac{3}{m_r}\right)\right]|\Psi_{B_c^*}(0)|,$$

$$f_{B_c} = 2\sqrt{\frac{N_c}{m_{B_c}}}\left[\mathcal{C}_p - \frac{d_p E_{B_c}}{4m_r}\right]|\Psi_{B_c}(0)|,$$

Wave function scale dependence

- Wave function at origin

For Power-law potential $$V(r) = Ar^a + C$$

Exact solution $$|\psi_\mu^n(0)|^2 = f(n,a)(A\mu)^{3/(2+a)}$$

Scale relation $$|\Psi_{B_c^*}(0)| = |\Psi_{J/\psi}(0)|^{1-y}\ |\Psi_\Upsilon(0)|^y,$$

$$y = y_c = \ln((1 + m_c/m_b)/2)/\ln(m_c/m_b)$$

Collins-Imbo-King-Martell, PLB 393 (1997) 155–160

$$|\psi_1(0)|^2 = |\psi_1^{(0)}(0)|^2 \left(1 + \sum_{k=1}^{n} f_k a_s^k\right).$$

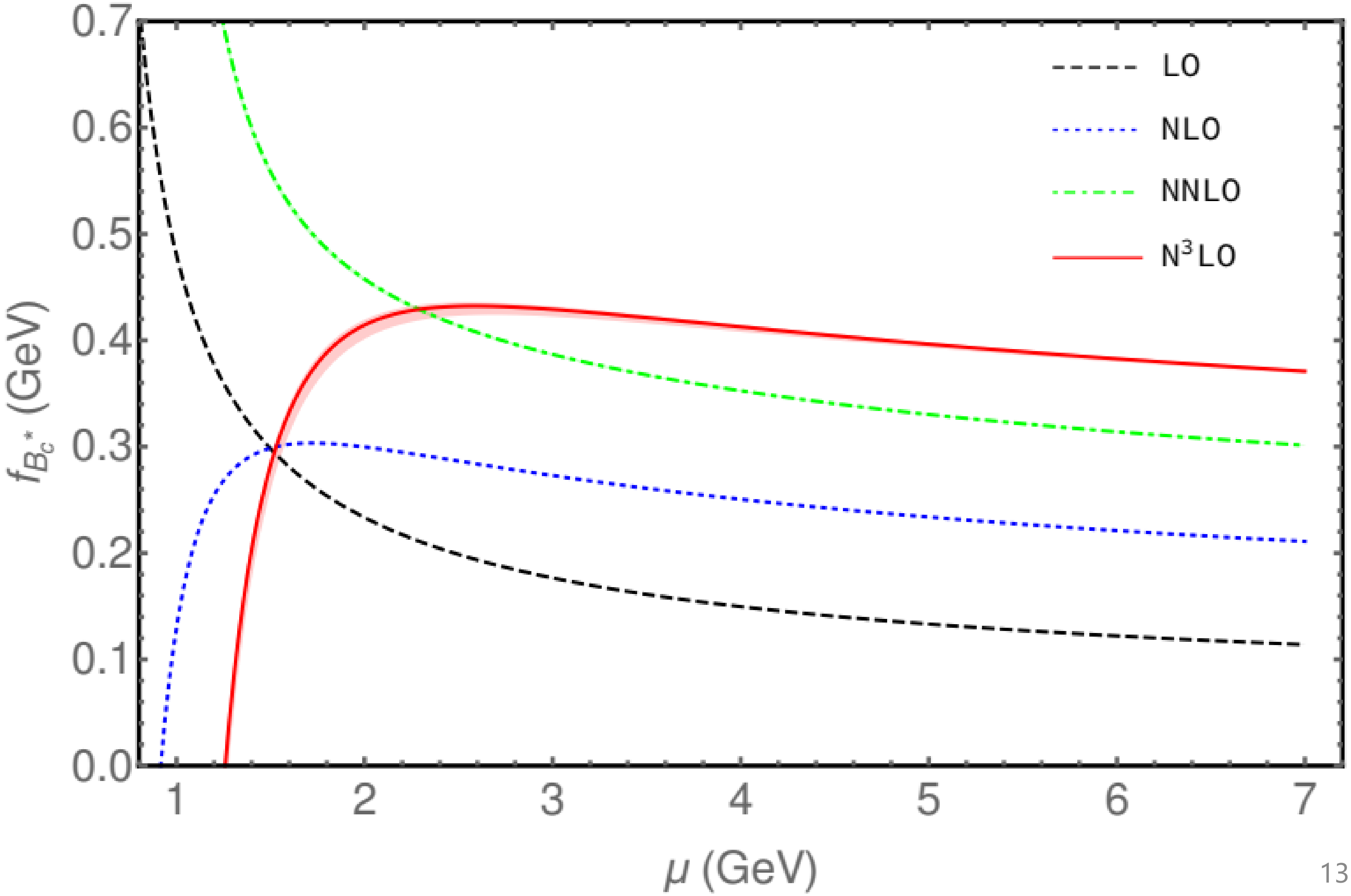
$$\left|\psi_1^{(0)}(0)\right|^2 = \frac{(m_b C_F \alpha_s)^3}{8\pi},$$

$$E_1^{(0)} = -\frac{1}{4} m_b (C_F \alpha_s)^2,$$

Beneke et al., PRL. 112, 151801 (2014)

Convergent vector Bc* decay constant

Tao-Xiao-Zhu: 2303.07220



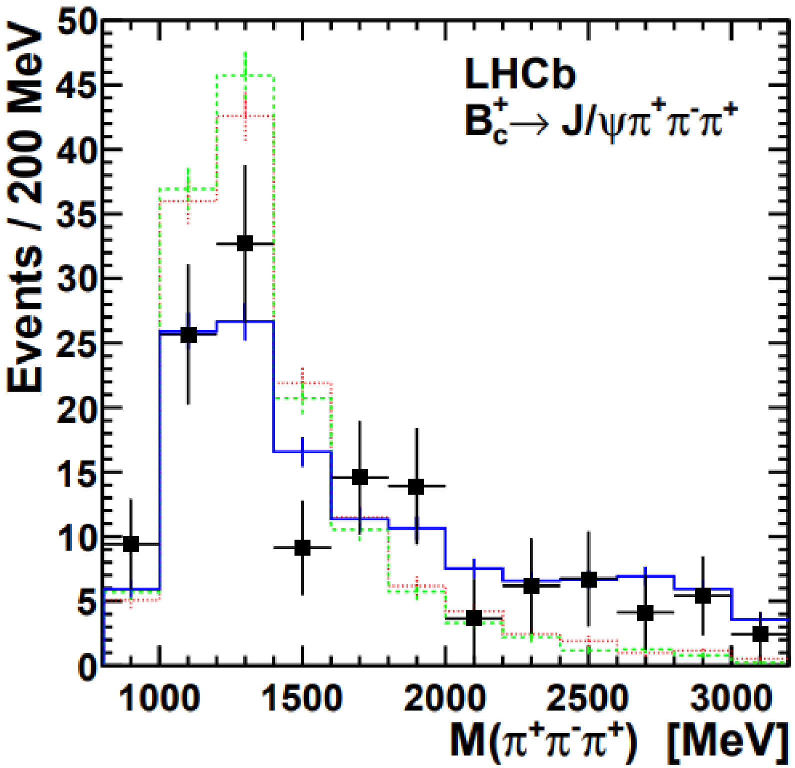
Leptonic decay branching ratios

Branching ratios	N^{3}LO
$\mathcal{B}(B_c^{*+} \to e^+\nu_e)$	$(3.85^{+0.29-0.07-1.35}_{-0.46+0.03+0.37}) \times 10^{-6}$
$\mathcal{B}(B_c^{*+} \to \mu^+\nu_\mu)$	$(3.85^{+0.29-0.07-1.35}_{-0.46+0.03+0.37}) \times 10^{-6}$
$\mathcal{B}(B_c^{*+} \to \tau^+\nu_\tau)$	$(3.40^{+0.25-0.06-1.19}_{-0.41+0.03+0.33}) \times 10^{-6}$
$\mathcal{B}(B_c^{+} \to e^+\nu_e)$	$(1.91^{+0.15-0.19-0.70}_{-0.23+0.12+0.22}) \times 10^{-9}$
$\mathcal{B}(B_c^{+} \to \mu^+\nu_\mu)$	$(8.18^{+0.63-0.83-2.99}_{-1.00+0.52+0.94}) \times 10^{-5}$
$\mathcal{B}(B_c^{+} \to \tau^+\nu_\tau)$	$(1.96^{+0.15-0.20-0.72}_{-0.24+0.12+0.23}) \times 10^{-2}$

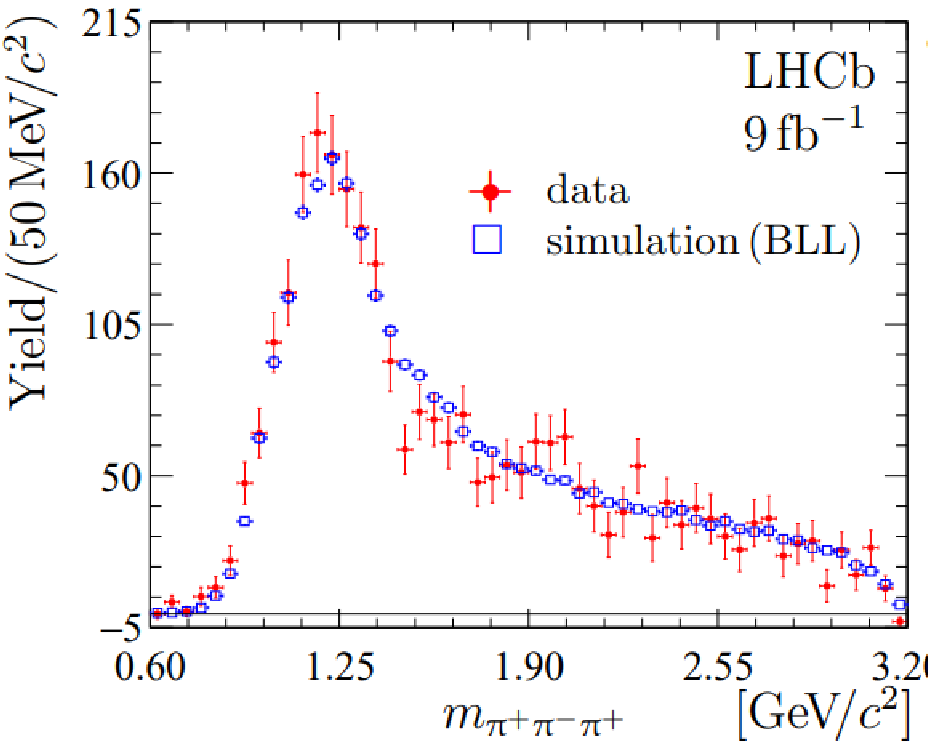
$$\Gamma(B_c^*(\lambda=\pm1)\to\ell\nu_\ell) = \frac{|V_{cb}|^2}{12\pi} G_F^2 f_{B_c^*}^2 \left(1-\frac{m_\ell^2}{m_{B_c^*}^2}\right)^2 \times m_{B_c^*}^3,$$

$$\Gamma(B_c^*(\lambda=0)\to\ell\nu_\ell) = \frac{m_\ell^2 \Gamma(B_c^{*+}(\lambda=\pm1)\to\ell\nu_\ell)}{2m_{B_c^*}^2},$$

Bc and Bc* decays along with 3 pions



LHCb, arXiv:1204.0079



LHCb, arXiv:2111.03001

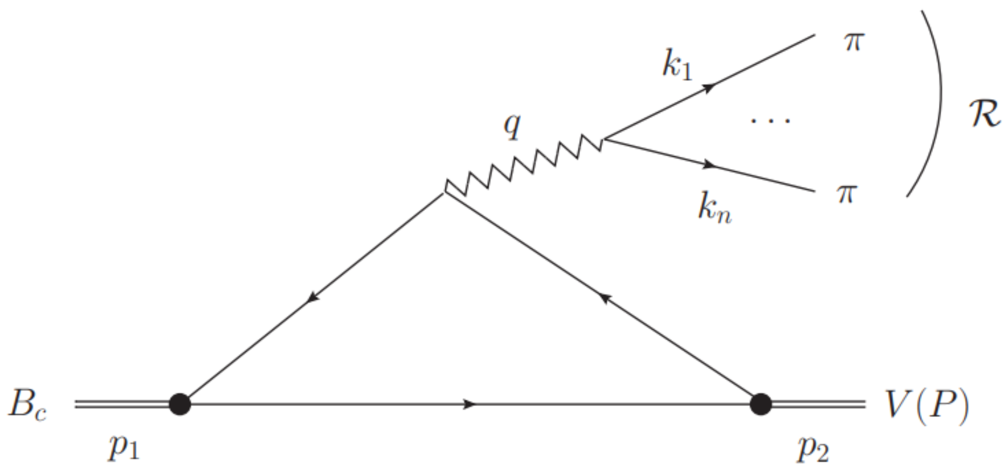
Around 10^5 Bc to Jpsi+X events

Invariant mass distribution in Bc* decays to J/psi +n h

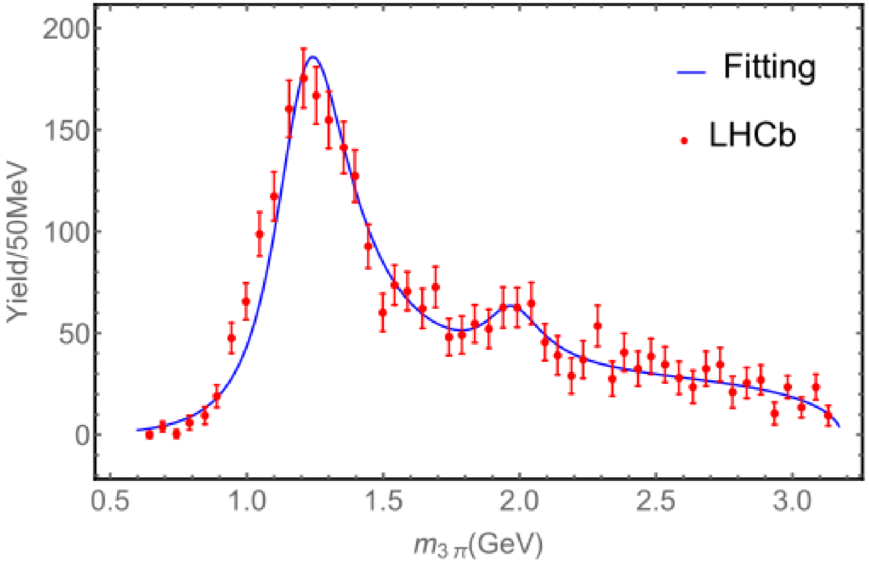
- **Helicity decomposition of weak decay width**

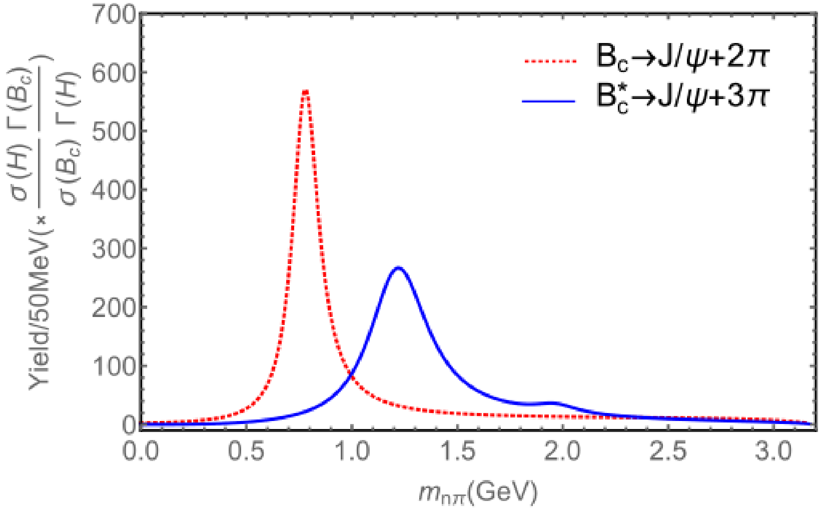
$$\frac{d\Gamma(B_c^{(*)} \to J/\psi + nh)}{dq^2} = \sum_{\lambda_i} \frac{|V_{cb}|^2 G_F^2 a_1^2 |\mathbf{p}'|}{32\pi M^2} \Gamma_{J_1\lambda_1 J_2\lambda_2\lambda_{nh}},$$

$$\Gamma_{11110} = 2\left[V_1^2\left((M-M')^2-q^2\right)\left((M'+M)^2-q^2\right) + \left(A_1\left(M^2-M'^2\right)+A_2q^2\right)^2\right]\rho_T^{nh}(q^2),$$



Results of invariant mass distribution

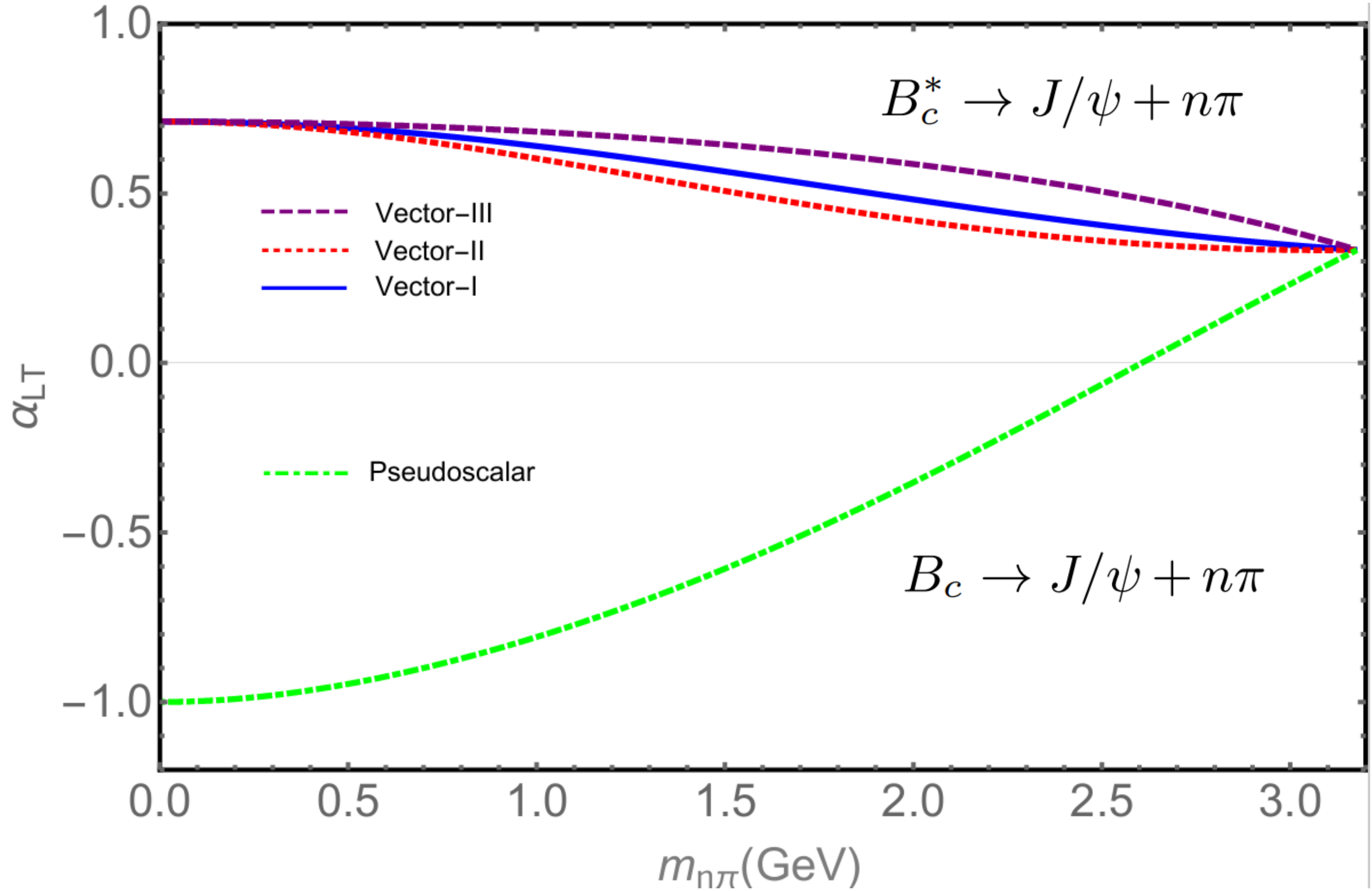




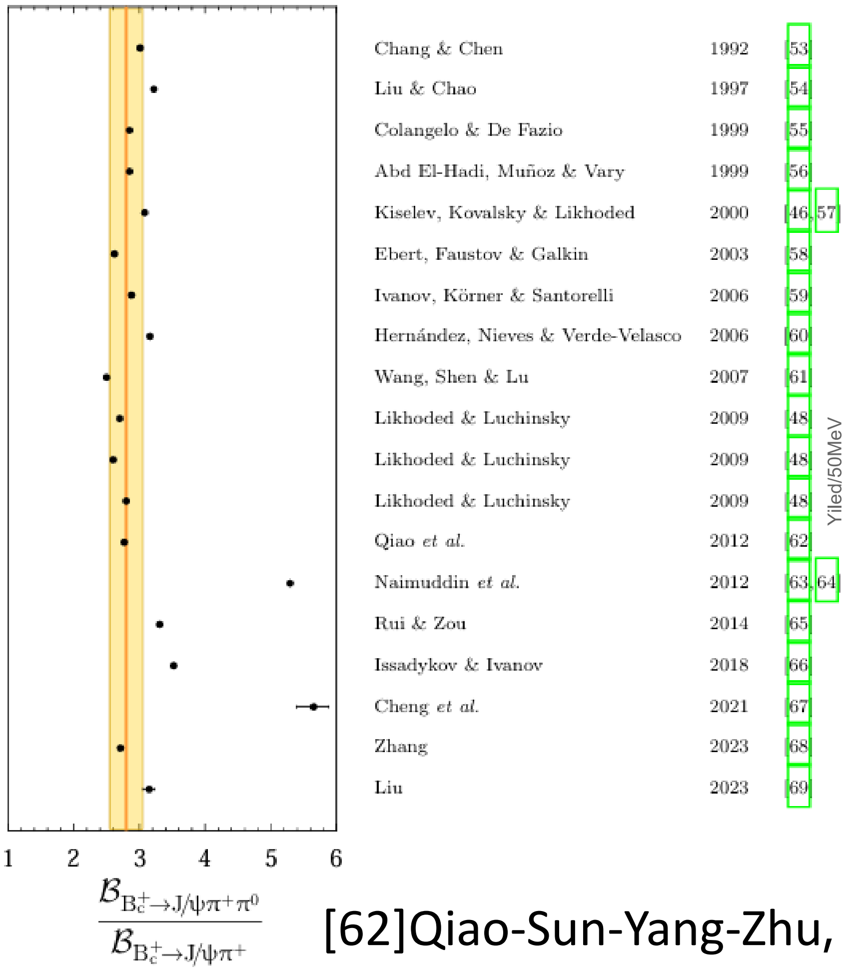
LHCb, arXiv:2111.03001

Polarization Asymmetry(A general law in V(P) to V transitions)

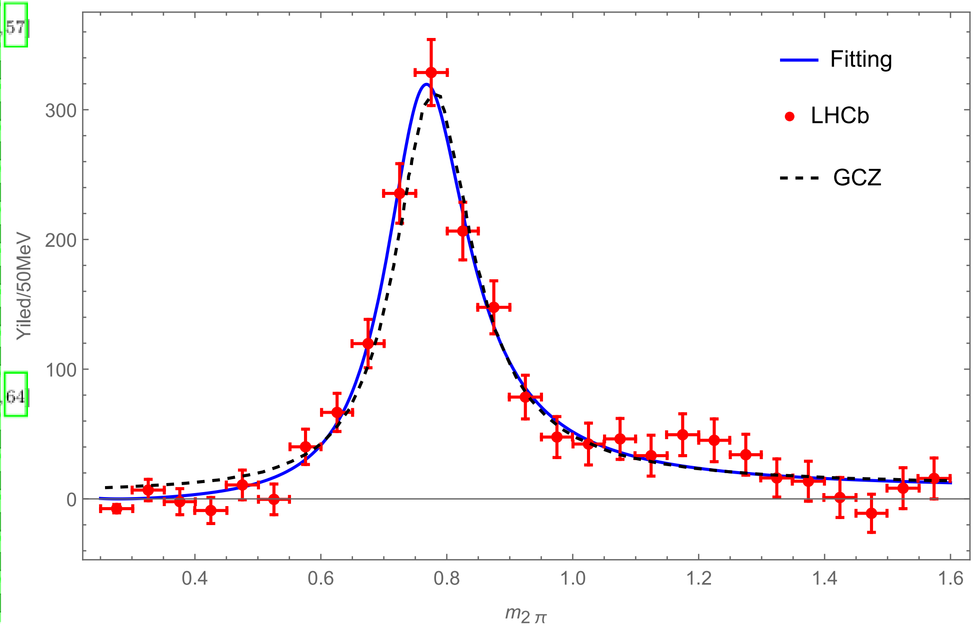
$$\alpha_{LT} = \sum_{\lambda_1, \lambda_{nh}} \frac{\Gamma_{J_1 \lambda_1 1 1 \lambda_{nh}} - \Gamma_{J_1 \lambda_1 1 0 \lambda_{nh}}}{\Gamma_{J_1 \lambda_1 1 1 \lambda_{nh}} + \Gamma_{J_1 \lambda_1 1 0 \lambda_{nh}}},$$



Bc decays along with 2 pions



$$\mathcal{R} = \frac{\mathcal{B}_{B_c^+\to J/\psi\pi^+\pi^0}}{\mathcal{B}_{B_c^+\to J/\psi\pi^+}} = 2.80 \pm 0.15 \pm 0.11 \pm 0.16\,,$$



[62]Qiao-Sun-Yang-Zhu, 1209.5859

LHCb, arXiv: 2402.05523

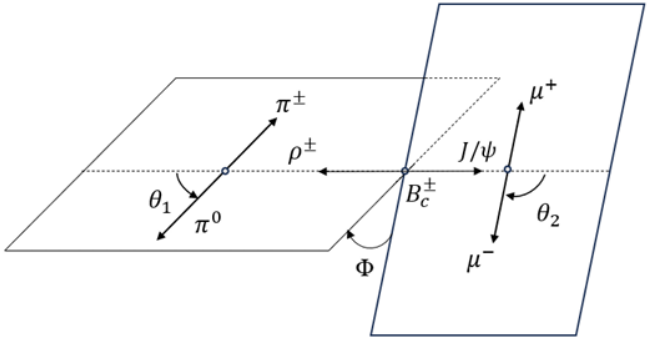
2404.06221, 2310.03425

Helicity angular distributions

➢ weak decay amplitude

$$h_\lambda \equiv \langle \rho(p_1,\lambda) J/\psi(p_2,\lambda)|\mathcal{H}_{eff}|B_c(p)\rangle$$
$$= \epsilon_{1\mu}(\lambda)^* \epsilon_{2\nu}(\lambda)^* \left(a g^{\mu\nu} + \frac{b p^\mu p^\nu}{m_1 m_2} + \frac{i c \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_\beta}{m_1 m_2} \right),$$

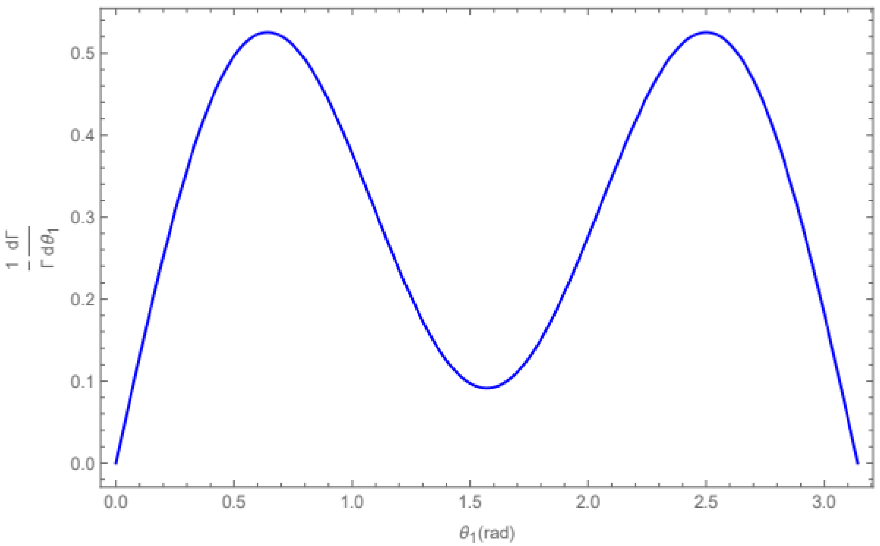
➢ Helicity angular distribution

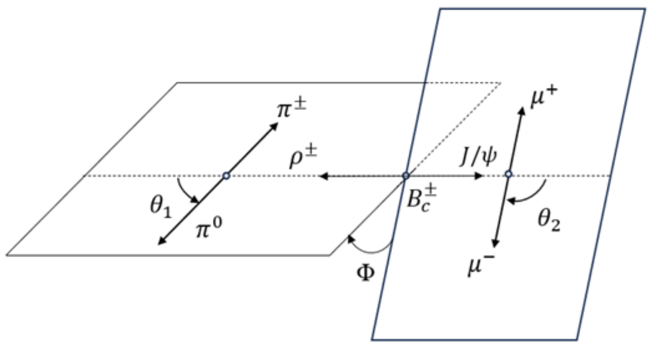


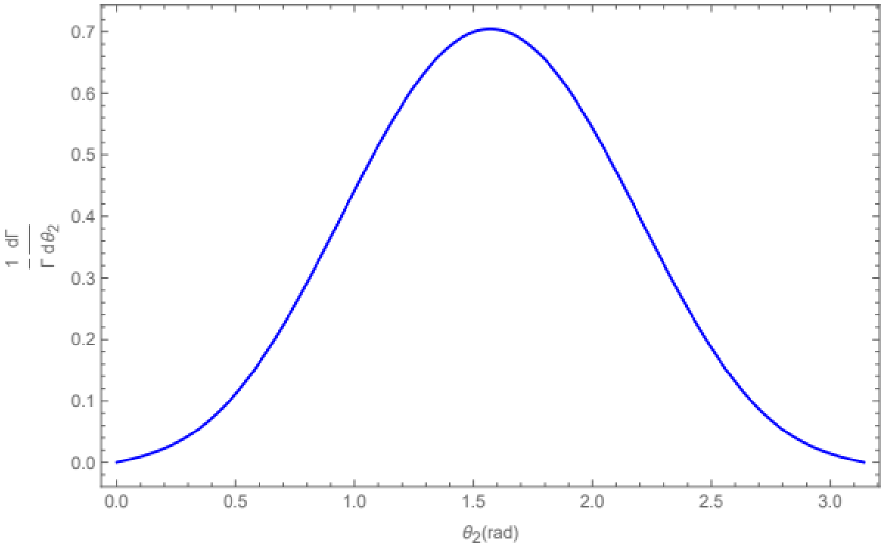
$$\frac{d^3\Gamma\left(B_c \to J/\psi(\mu^+\mu^-) + \rho(\pi\pi)\right)}{d\cos\theta_1 d\cos\theta_2 d\phi} =$$
$$\frac{9p_m}{128\pi^2 M^2}\Big\{ \cos^2\theta_1 \sin^2\theta_2 H_{00} + \frac{1}{4}\sin^2\theta_1\left(1+\cos^2\theta_2\right)\left(H_{11}+H_{-1-1}\right)$$
$$-\frac{1}{2}\sin^2\theta_1 \sin^2\theta_2 \left[\cos 2\phi \,\mathrm{Re}\left(H_{1-1}\right) - \sin 2\phi \,\mathrm{Im}\left(H_{1-1}\right)\right]$$
$$-\frac{1}{4}\sin 2\theta_1 \sin 2\theta_2 \left[\cos\phi \,\mathrm{Re}\left(H_{10}+H_{-10}\right) - \sin\phi \,\mathrm{Im}\left(H_{10}-H_{-10}\right)\right]\Big\},$$

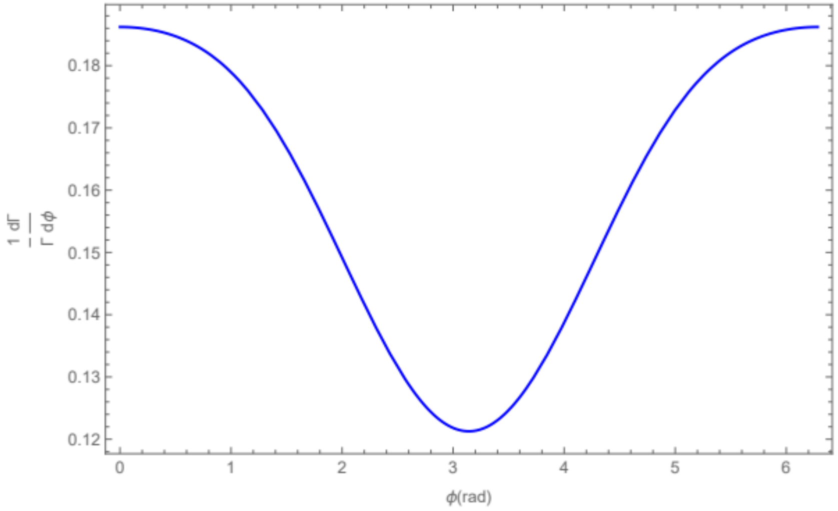
$$f_L = \frac{|h_0|^2}{|h_{+1}|^2 + |h_{-1}|^2 + |h_0|^2}. \qquad \alpha_{LT} = 1 - 2 f_L. \qquad f_L(J/\psi) = f_L(\rho) \simeq 0.877,$$

Helicity angular distributions









Quantum spin entanglement and Von Neumann entropy

- **Quantum spin entanglement state: two qutrits**

$$|\Psi\rangle = \frac{1}{\sqrt{|H|^2}} \left[h_{+1} |J/\psi(+1)\rho(+1)\rangle + h_0 |J/\psi(0)\rho(0)\rangle + h_{-1} |J/\psi(-1)\rho(-1)\rangle \right],$$

- **Von Neumann entropy**

$$\varrho = \frac{1}{|H|^2} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{+1}h_{+1}^* & 0 & h_{+1}h_0^* & 0 & h_{+1}h_{-1}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_{+1}^* & 0 & h_0h_0^* & 0 & h_0h_{-1}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{-1}h_{+1}^* & 0 & h_{-}h_0^* & 0 & h_{-1}h_{-1}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\varrho_{J/\psi} = \varrho_\rho = \frac{1}{|H|^2} \begin{pmatrix} h_{+1}h_{+1}^* & 0 & 0 \\ 0 & h_0h_0^* & 0 \\ 0 & 0 & h_{-1}h_{-1}^* \end{pmatrix}.$$

$$\varepsilon = -Tr[\varrho_A \ln \varrho_A] = -Tr[\varrho_B \ln \varrho_B]. \qquad \varepsilon = 0.405.$$

Quantum spin entanglement and Bell inquality

- **Bell inquality**

$$|s\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow> - |\downarrow\uparrow> \right).$$

$$P(\vec{a},\vec{b}) = \int d\lambda \rho(\lambda) A(a,\lambda) B(b,\lambda).$$

$$|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \le \int d\lambda \rho(\lambda) \left(1 - A(b,\lambda) A(c,\lambda)\right)$$

$$= \int d\lambda \rho(\lambda) - \int d\lambda \rho(\lambda) A(b,\lambda) A(c,\lambda) = 1 + P(\vec{b},\vec{c}).$$

- **Collins-Gisin-Linden-Massar-Popescu qutrits inquality**

$$A_1, A_2, B_1, B_2 = 0, \ldots, d-1.$$

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$$\begin{aligned} I_3 = & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ & - P(A_1 = B_1 - 1) - P(B_1 = A_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1) \le 2. \end{aligned}$$

$$I_3 = tr(\rho B) = tr(|\psi\rangle\langle\psi| B) = \langle\psi|B|\psi\rangle.$$

$$I_3 = |-2.91| = 2.91 > 2$$

CGLMP inquality breaks down in Bc to Jpsi+rho process.

Summary

- ✓ Bc family spectrum and Bc* decay width are studied in QCD effective approaches
- ✓ Convergent Bc* decay constant up to three-loop accuracy is obtained
- ✓ Distinguishing vector Bc* meson at LHC is possible by helicity decomposition
- ✓ Angular distribution and quantum spin entanglement are studied in Bc decays along with two pions.

Thank you a lot!