



# 粒子自旋和宇称量子数的测量

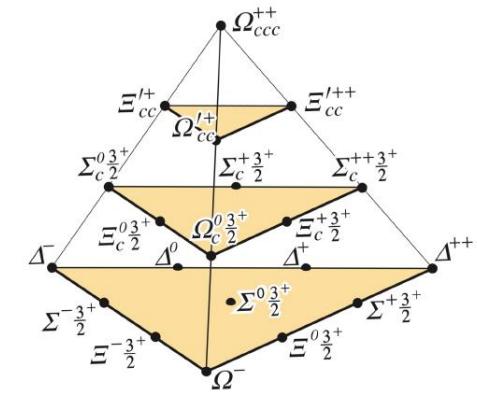
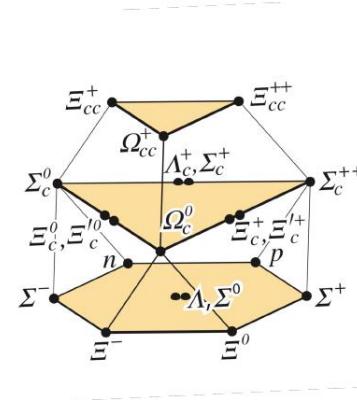
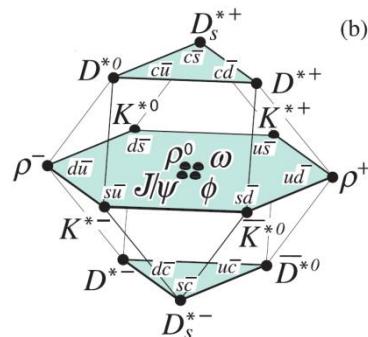
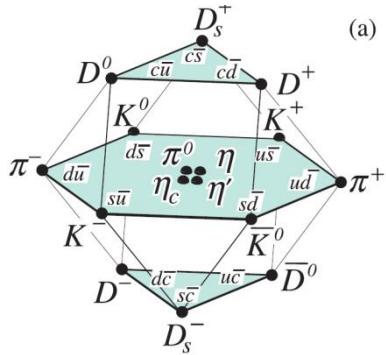
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第二届强子物理新发展研讨会 暨 强子物理在线论坛**100**期特别活动，  
合肥，中国科学技术大学，30 June 2024 to 4 July 2024

# 常规强子和奇异态物质

- ✓ Conventional hadrons: meson  $|q\bar{q}\rangle$ , baryon:  $|qqq\rangle$  or  $|\bar{q}\bar{q}\bar{q}\rangle$



- ✓ Conventional Spin-parity quantum numbers

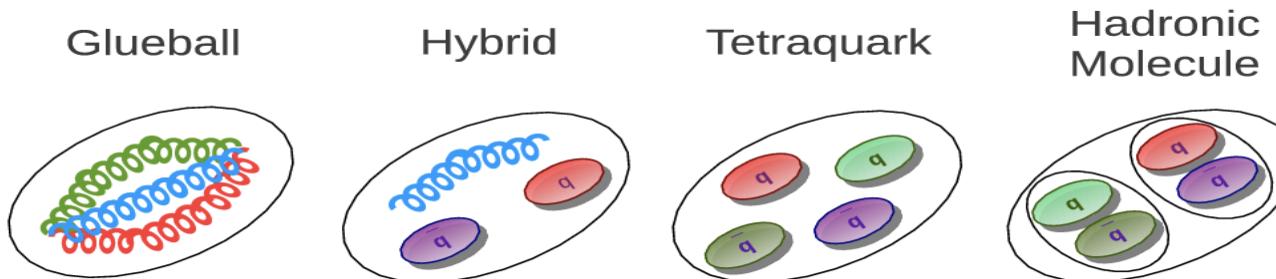
Mesons:  $n^{2S+1} L_J$  ,  $P = (-1)^{L+1}$  ;  $\exists C, C = (-1)^{L+S}$   
 e.g.  $0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 2^{++}, 2^{--}, \dots$

# 常规强子和奇特态物质

- ✓ Conventional Spin-parity quantum numbers (continued)

Baryons: classified by  $(D, L_N^P)$ , e.g.  $N = 0$ : nucleon and  $\Delta$ ,  $P = +$   
or classified by  $|X^{2S+1} L_\pi J^P|$  with  $X = N, \Delta, \dots, \pi = M$  or  $A$  (**symmetry**)

- ✓ QCD allows for “exotics”

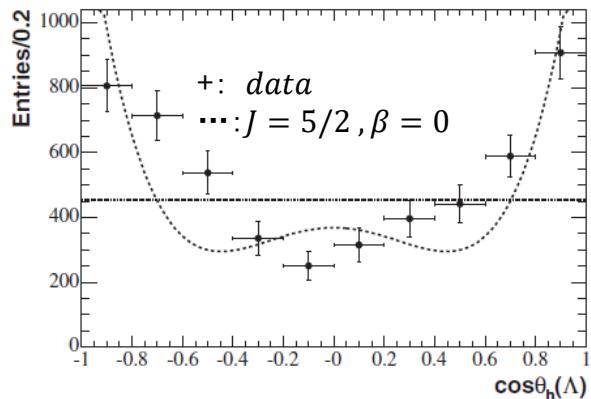
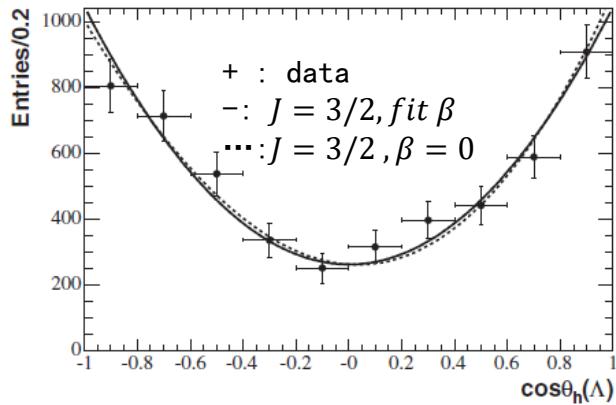
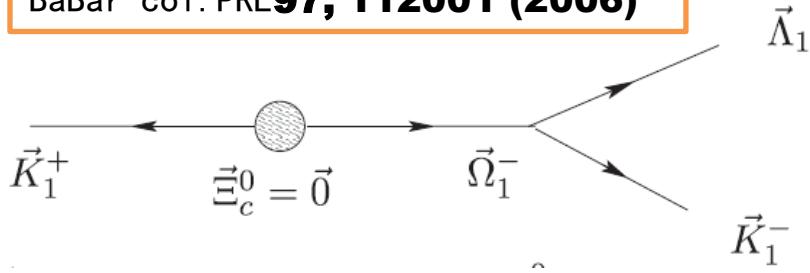


Exotic quantum numbers:  $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \dots$

# 自旋-宇称量子数的测量方法

- ✓ Angular distribution analysis:  $\Omega^-$ , :  $J^P = 3/2^+$

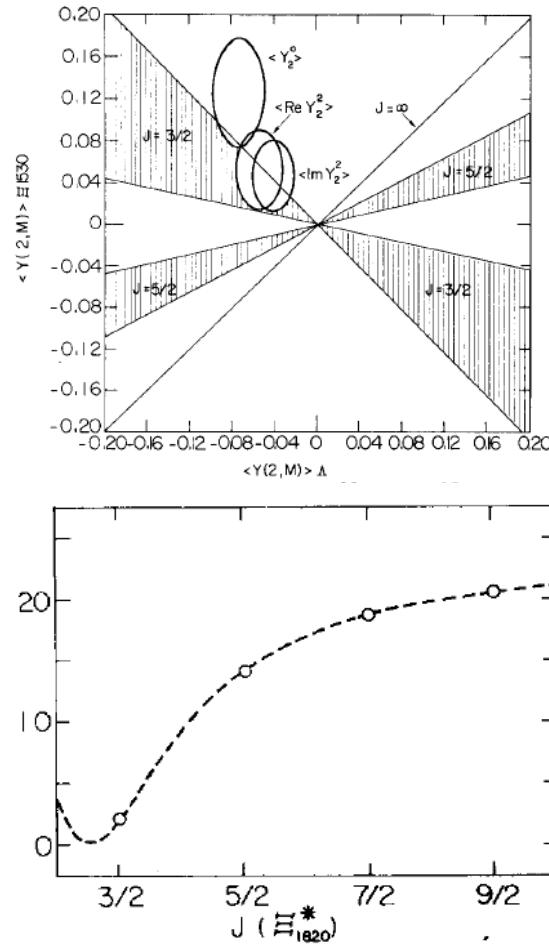
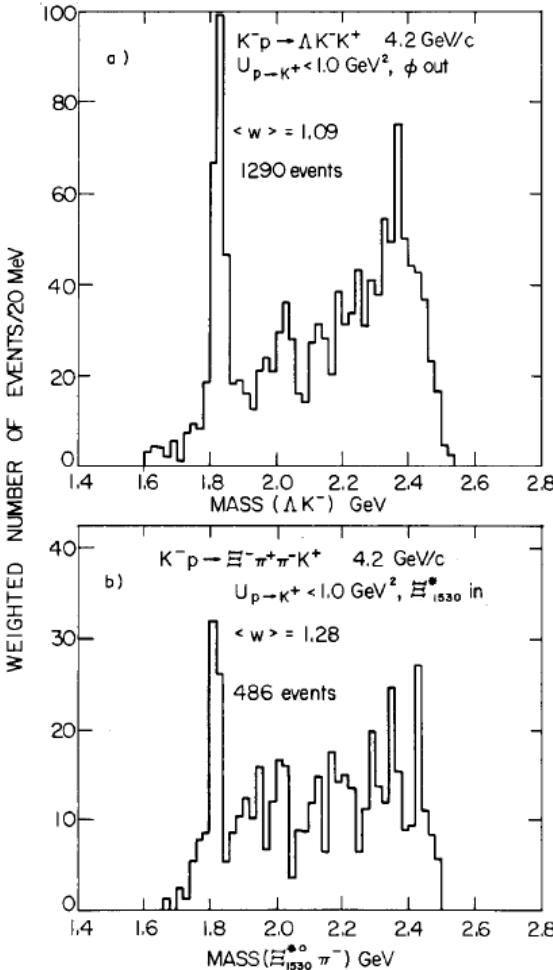
BaBar col. PRL **97**, 112001 (2006)



- ✓ 
$$\frac{dN}{d \cos \theta_h} \propto \begin{cases} 1 + \beta \cos \theta_h & (J = 1/2) \\ 1 + 3 \cos^2 \theta_h + \beta \cos \theta_h (5 - 9 \cos^2 \theta_h) & (J = 3/2) \\ 1 - 2 \cos^2 \theta_h + 5 \cos^4 \theta_h + \beta \cos \theta_h * \\ (5 - 26 \cos^2 \theta_h + 25 \cos^4 \theta_h) & (J = 5/2) \end{cases}$$

- ✓  $\chi^2$  test

✓ Moment analysis:  $\Xi(1820) : J = 3/2$  Phys.Lett., 77B, 451



- ✓ @4.2GeV :  $K^- p \rightarrow \Lambda K^+ K^-$ ,  $\Xi^- K^+ \pi^+ \pi^-$ 
  - $\Xi^{*-} \rightarrow \Lambda K^-$
  - $\Xi^{*-} \rightarrow \Xi^{*0}(1530)\pi^-$

Moment relation:

$$\checkmark \quad \langle Y_L^M \rangle_{\Xi^{*-}(1530)} = \left[ 1 - \frac{2L(L+1)(1-\gamma')}{(2J+3)(2J-1)} \right] \langle Y_L^M \rangle_\Lambda$$

✓ Data:  
CERN  $K^-$  beam  
to 2m hydrogen  
bubble chamber

# $\Lambda_c$ 自旋宇称量子数的测量

✓ PDG 2018



$$I(J^P) = 0(\frac{1}{2}^+) \text{ Status: } ****$$

The parity of the  $\Lambda_c^+$  is defined to be positive (as are the parities of the proton, neutron, and  $\Lambda$ ). The quark content is *udc*. Results of an analysis of  $pK^-\pi^+$  decays (JEZABEK 92) are consistent with  $J = 1/2$ . Nobody doubts that the spin is indeed  $1/2$ .

We have omitted some results that have been superseded by later experiments. The omitted results may be found in earlier editions.

✓ PDG 2022



$$I(J^P) = 0(\frac{1}{2}^+) \text{ Status: } ****$$

The parity of the  $\Lambda_c^+$  is defined to be positive (as are the parities of the proton, neutron, and  $\Lambda$ ). The quark content is *udc*. Results of an analysis of  $pK^-\pi^+$  decays (JEZABEK 92) are consistent with

$J = 1/2$ . ABLIKIM 21N determines the  $\Lambda_c^+$  spin to be  $J = 1/2$ , from an angular analysis of various 2-body  $\Lambda_c^+$  decays in  $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$ .

We have omitted some results that have been superseded by later experiments. The omitted results may be found in earlier editions.

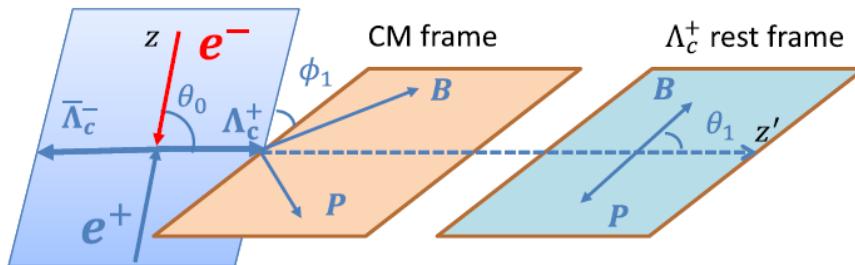
1. M. Ablikim et al. (BESIII Collaboration), Phys. Rev., D103, L091101 (2021);

- 587 pb<sup>-1</sup> at  $\sqrt{s} = 4.6$  GeV
- likelihood test using angular distribution for  $e^+e^- \rightarrow \Lambda_c\bar{\Lambda}_c$ ,  $\Lambda_c \rightarrow pK_S^0, \Lambda\pi^+, \Sigma^0\pi^+$ , and  $\Sigma^+\pi^0$ .
- Two spin hypotheses,  $J = \frac{1}{2}, \frac{3}{2}$  are tested.
- likelihood function:
- Angular distribution:

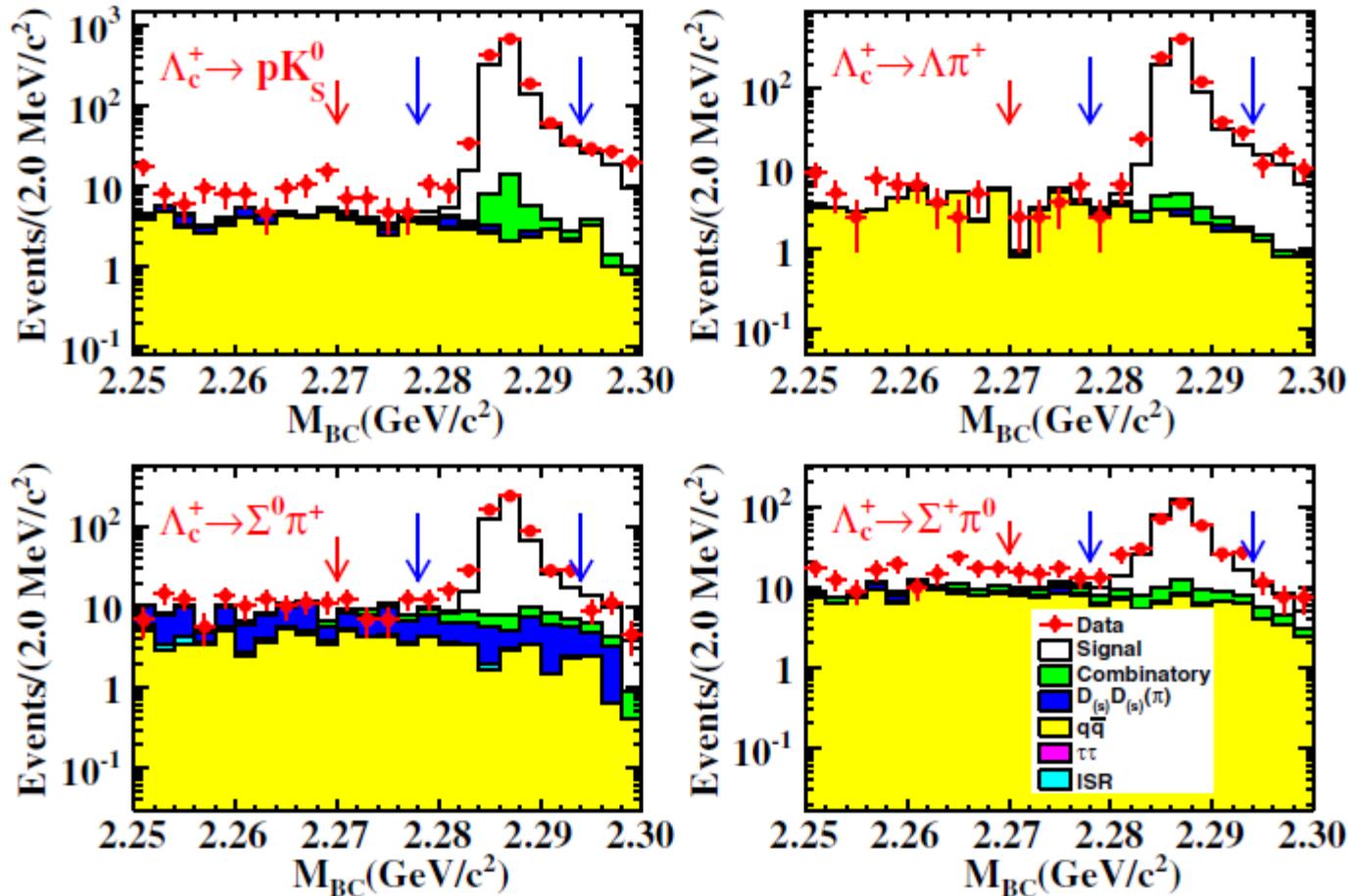
$$\mathcal{L}_i^J(N^i) = \prod_{k=1}^{N^i} \frac{1}{C^i} \mathcal{W}^J(\theta_0^k, \theta_1^k, \phi_1^k, \dots, \theta_n^k, \phi_n^k),$$

$$\mathcal{W}^{J=\frac{1}{2}}(\theta_0, \theta_1, \phi_1) \propto 1 + \alpha \cos^2 \theta_0 + \mathcal{P}_T \sin \theta_1 \sin \phi_1,$$

with  $\mathcal{P}_T = \alpha_{[pK_S^0]} \sqrt{1 - \alpha^2} \cos \theta_0 \sin \theta_0 \sin \xi$ ,

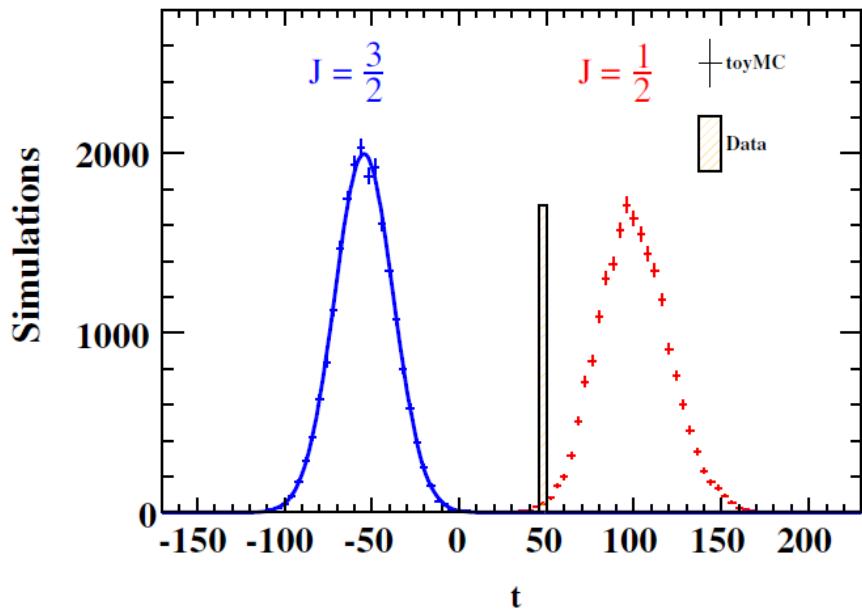
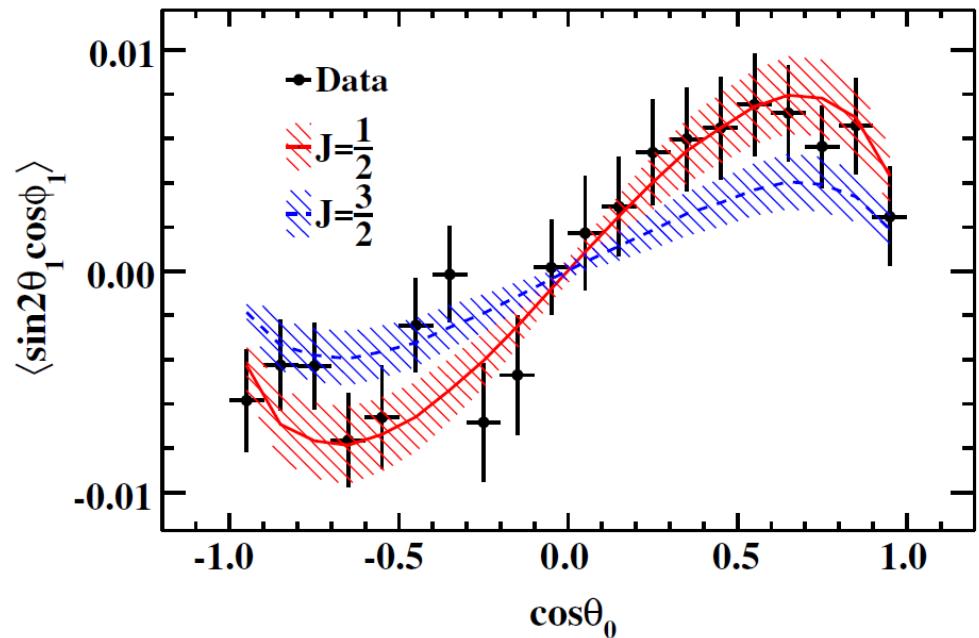


## ■ Survived events: 2829 events, 242 bkg. events



✓  $-\ln \sum L_i = -45.2 \left( J = \frac{1}{2} \right),$   
 $= -21.5 \left( J = \frac{3}{2} \right)$

✓  $t = -2 \ln(L^{3/2}/L^{1/2})$   
✓ 6 $\sigma$  significance for spin  
 $J = 1/2$  over 3/2

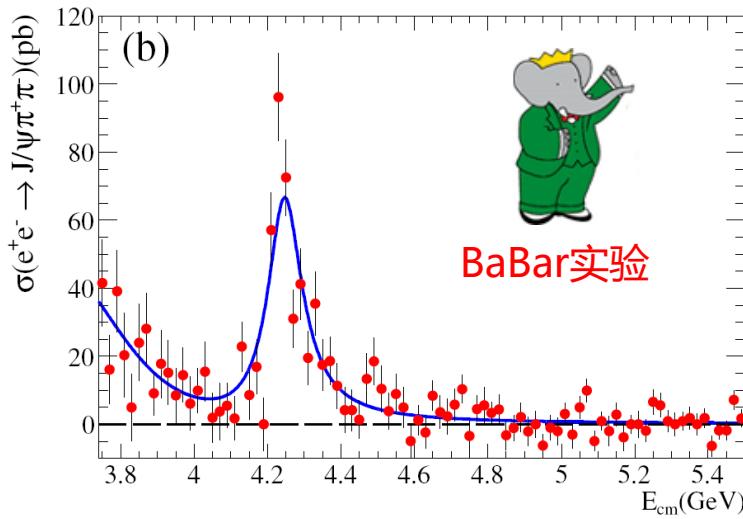


# $Z_c(3900)$ 自旋和宇称的测量

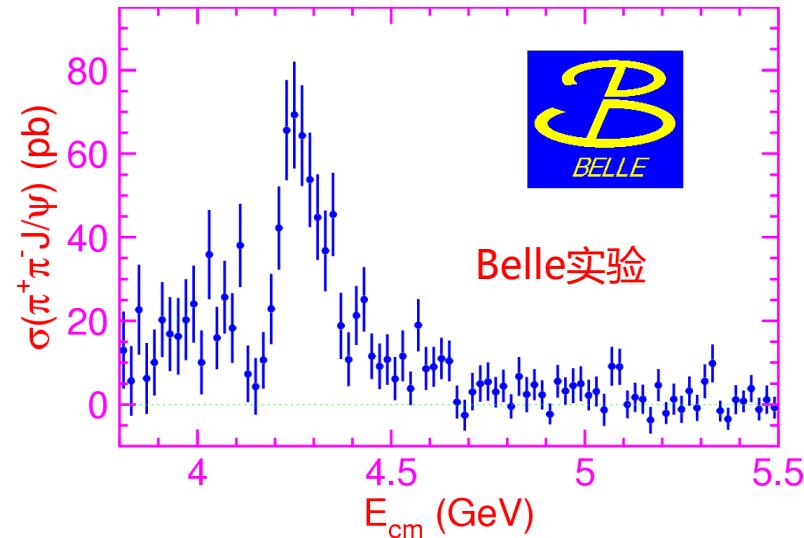
$Z_c(3900)$	$I^G(J^{PC}) = 1^+(1^{+-})$
$Z_c(3900)$ MASS	$3887.1 \pm 2.6$ MeV ( $S = 1.7$ )
$Z_c(3900)$ WIDTH	$28.4 \pm 2.6$ MeV

1. Ablikim, M. et.al, (BESIII) Phys.Rev.Lett., 110, 252001 (2013).
2. Ablikim, M. et.al, (BESIII), Phys.Rev.Lett., 119, 072001 (2017).

# $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ 事例



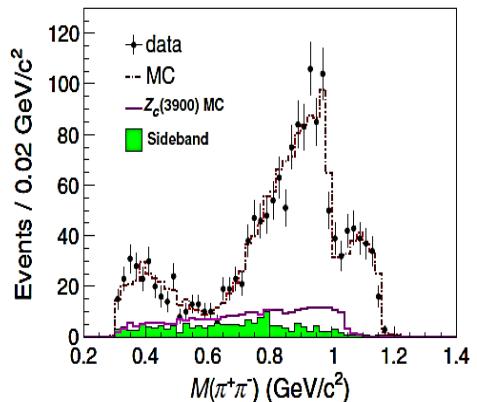
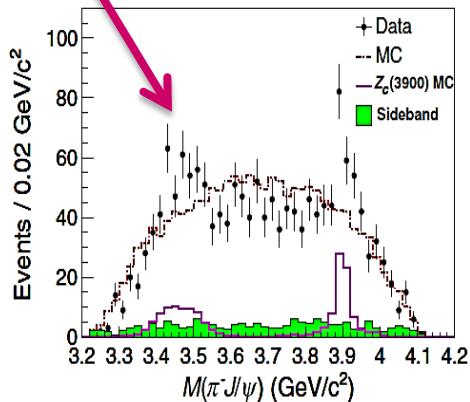
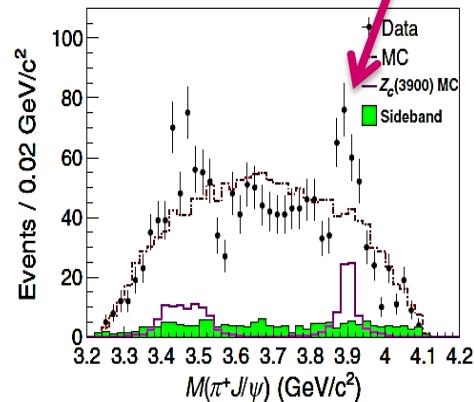
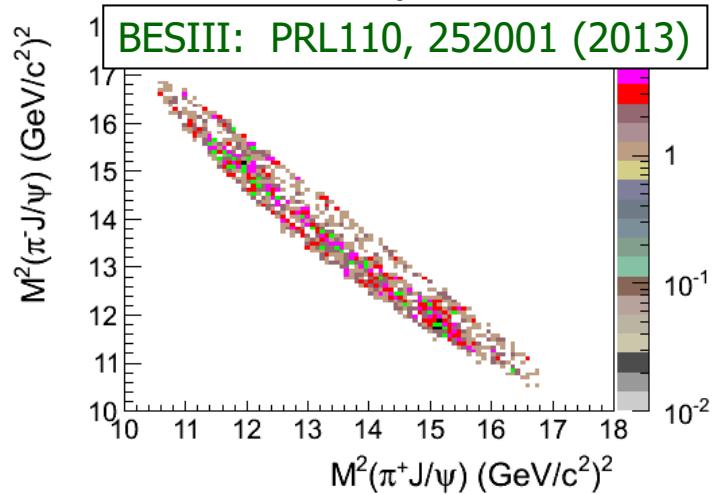
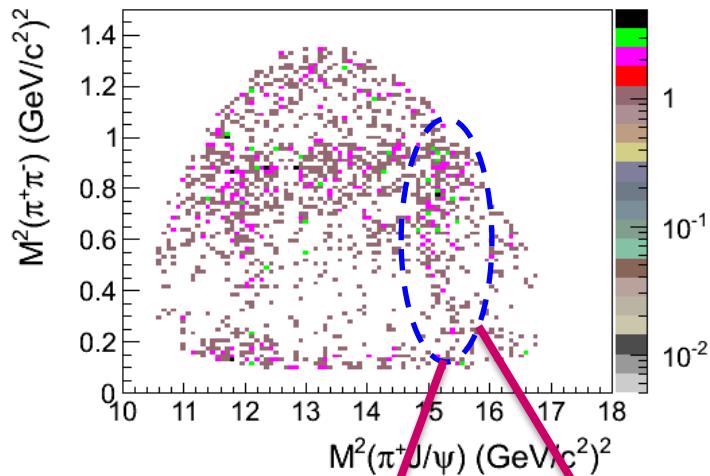
BaBar实验



Belle实验

- ✓  $\sigma(e^+e^- \rightarrow \pi^+\pi^-J/\psi) \sim 4.26 \text{ GeV}$  截面最大
- ✓ 建议在4.26GeV采集45天数据，可积累 $500 \text{ pb}^{-1}$  数据
- ✓ 在30天内积累了  $525 \text{ pb}^{-1}$  !是 CLEO-c 数据的40倍!

# Dalitz plot and mass spectrum



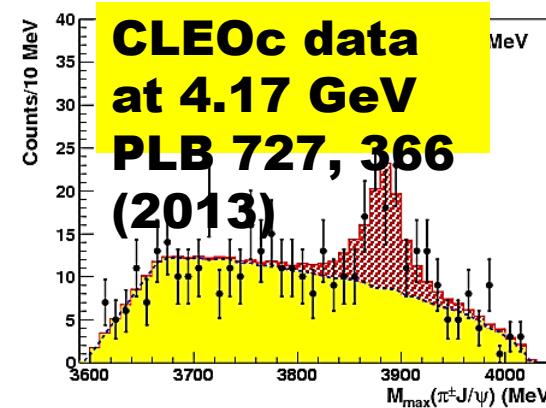
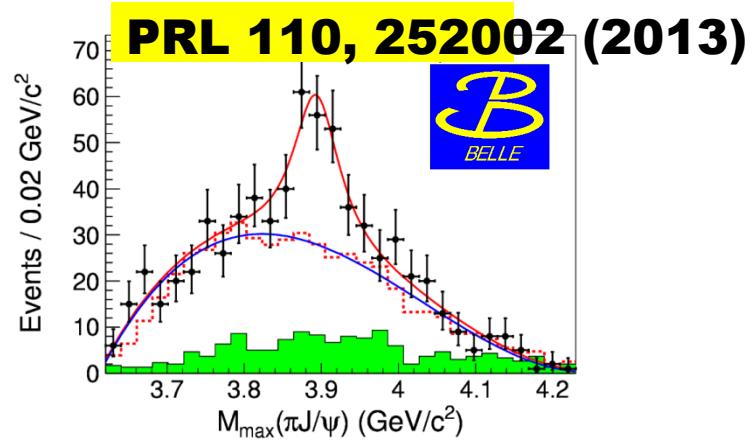
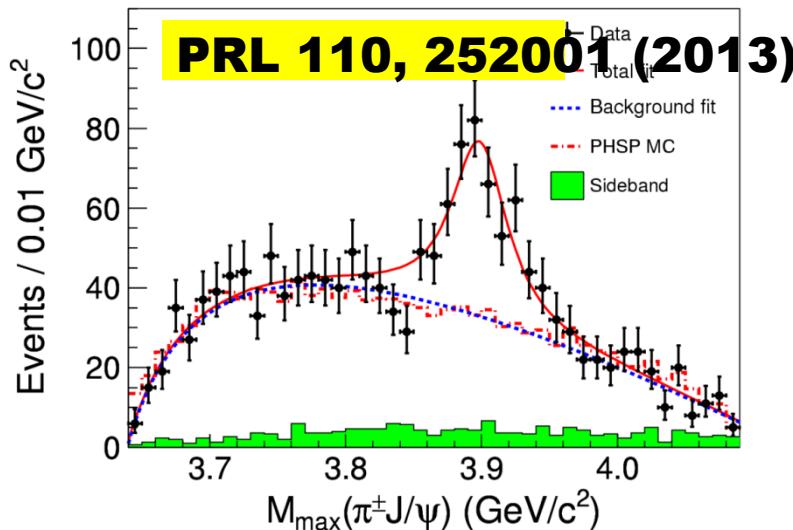
# $Z_c(3900)$ confirmed by other experiments

**BESIII : Mar. 24, 2013**

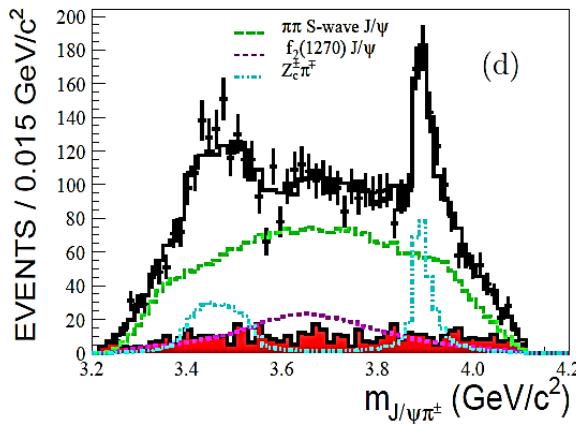
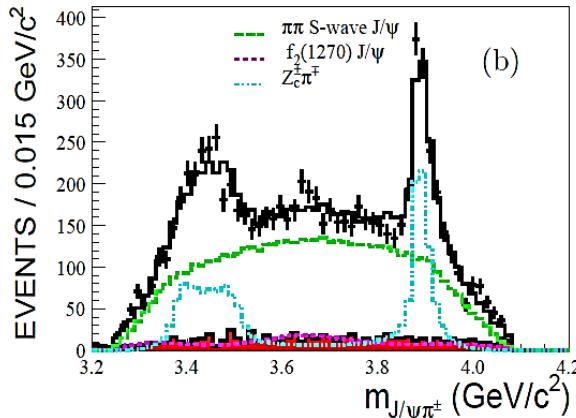
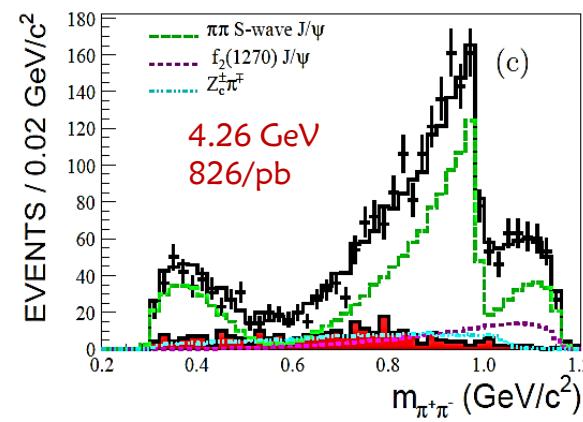
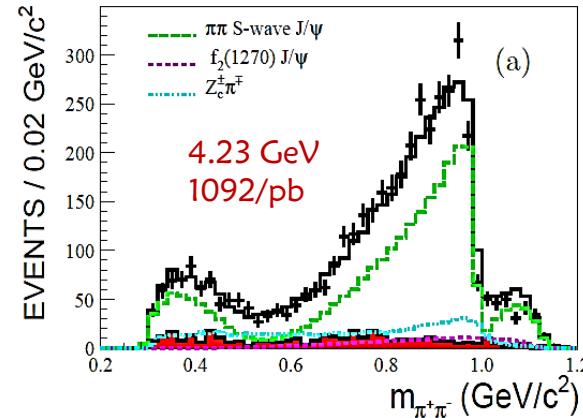
**Belle : Mar. 30, 2013**

**CLEOc : Apr. 10, 2013**

**$Z_c$  established !**



# Spin and parity measurement



- ✓ simultaneous fit of two data sets
- ✓ helicity amplitude for  $e^+e^- \rightarrow \gamma^* \rightarrow R(\pi\pi)\psi$  &  $Z_c\psi, \psi \rightarrow l^+l^-$
- ✓ Isobar model:  
 $\sigma f_0, f_0(1370), (1270), Z_c^\pm$
- ✓  $Z_c^\pm$  as  $1^+$  state

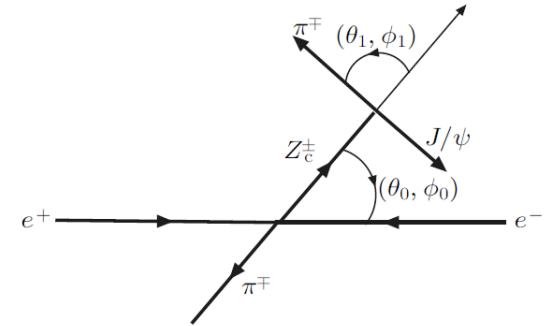
# ✓ Z<sub>c</sub>(3900) 自旋宇称的角分布分析

1.  $e^+ e^- \rightarrow \pi^\pm Z_c^\mp$

$$\frac{dN}{d \cos \theta_0} \propto \begin{cases} \sin^2 \theta_0 & (J^P = 0^-) \\ 1 + \alpha_0 \cos^2 \theta_0 & (J^P = 1^+) \\ 1 + \cos^2 \theta_0 & (J^P = 1^-) \\ 1 + \alpha_0 \cos^2 \theta_0 & (J^P = 2^-) \\ 1 + \cos^2 \theta_0 & (J^P = 2^+) \end{cases}$$

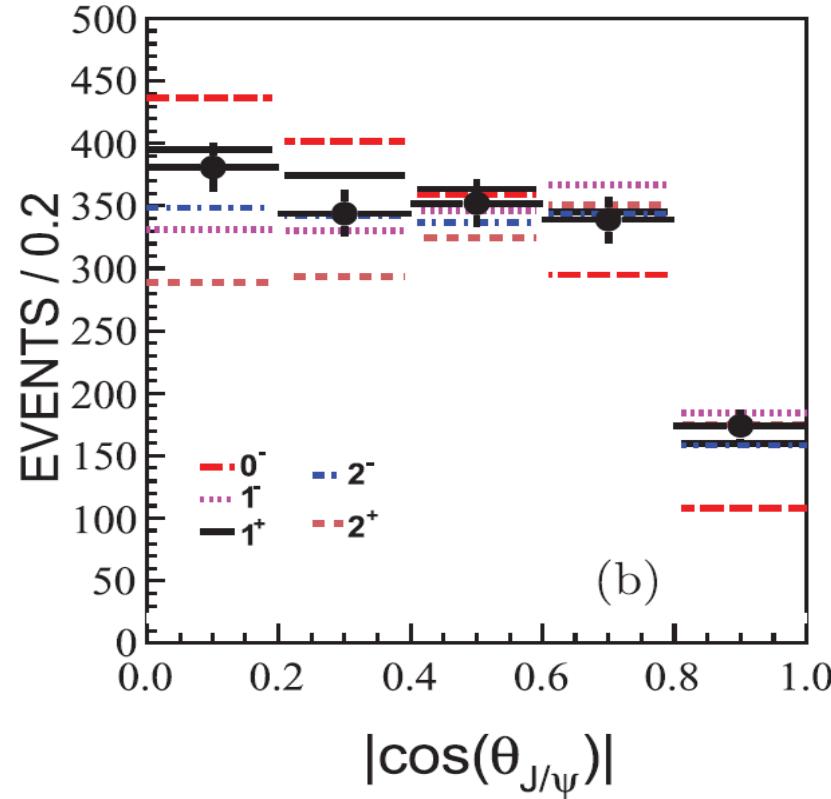
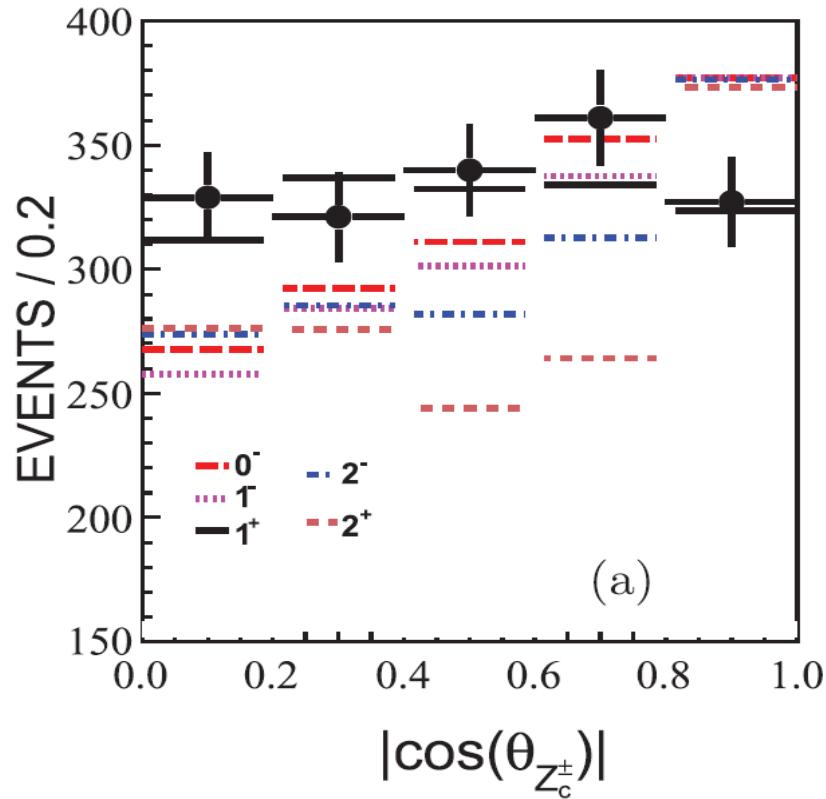
2.  $Z_c^\mp \rightarrow \pi^\mp J/\psi$

$$\frac{dN}{d \cos \theta_1} \propto \begin{cases} 1 & (J^P = 0^-) \\ 1 + \alpha_1 \cos^2 \theta_1 & (J^P = 1^+) \\ 1 + \cos^2 \theta_1 & (J^P = 1^-) \\ 1 + \alpha_1 \cos^2 \theta_1 + \alpha_2 \cos^4 \theta_1 & (J^P = 2^-) \\ 1 - 3 \cos^2 \theta_1 + 4 \cos^4 \theta_1 & (J^P = 2^+) \end{cases}$$



Ref. Chin. Phys. Lett. Vol. 33,  
061401 (2016)

✓  $Z_c(3900)$  自旋宇称的角分布检验和统计显著性



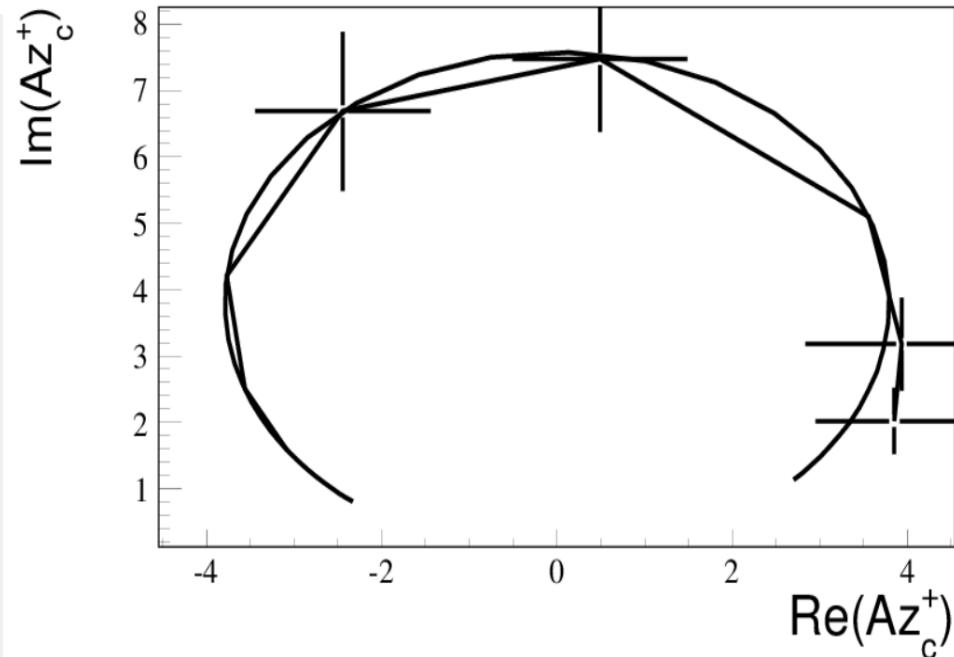
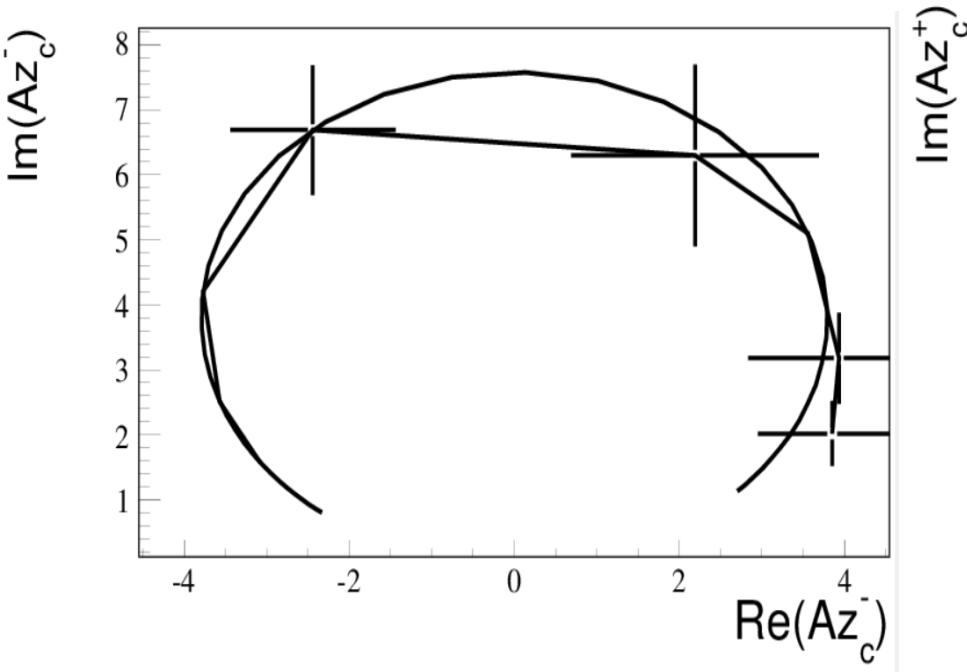
## ✓ Z<sub>c</sub>(3900) 统计显著性

- $H_0$  hypothesis:  $J^P = 1^+$ ;  $H_1$  hypothesis:  $J^P = 0^-$  or  $(1^-, 2^\pm)$ ;
- Statistics:  $t \equiv -2 \ln \lambda = 2[\ln L_{\max}(H_1) - \ln L_{\max}(H_0)]$ ,
- $p$ -value :  $p(t_{\text{obs}}) = \int_{t_{\text{obs}}}^{\infty} \chi^2(t; r) dt.$
- Significance

$$\int_{-S}^S \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 1 - p(t_{\text{obs}}) = \int_0^{t_{\text{obs}}} \chi^2(t; r) dt.$$

Hypothesis	$\Delta(-2 \ln L)$	$\Delta(\text{ndf})$	Significance
$1^+$ over $0^-$	94.0	13	$7.6\sigma$
$1^+$ over $1^-$	158.3	13	$10.8\sigma$
$1^+$ over $2^-$	151.9	13	$10.5\sigma$
$1^+$ over $2^+$	96.0	13	$7.7\sigma$

# ✓ $Z_c^{\pm}$ Argand plot



# $X(2085)$ 自旋和宇称的测量

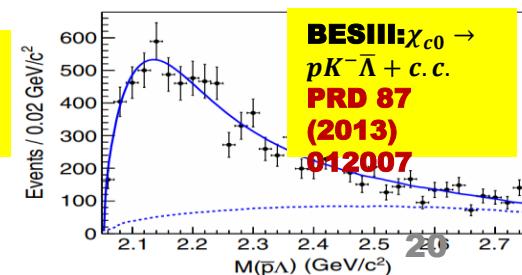
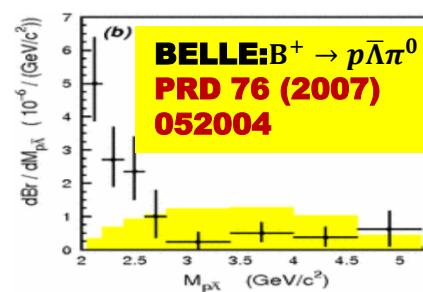
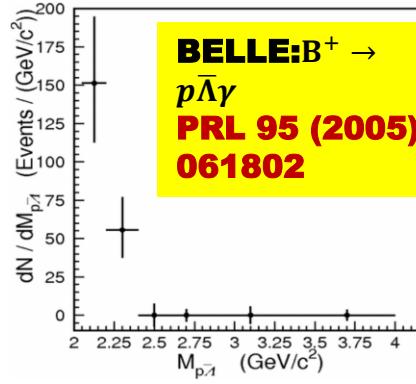
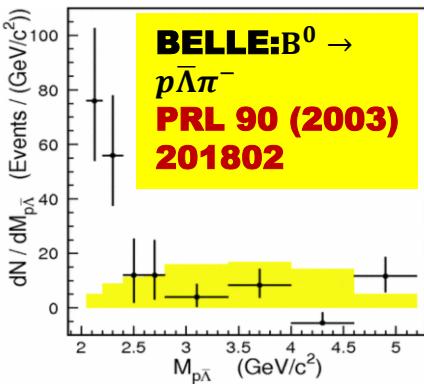
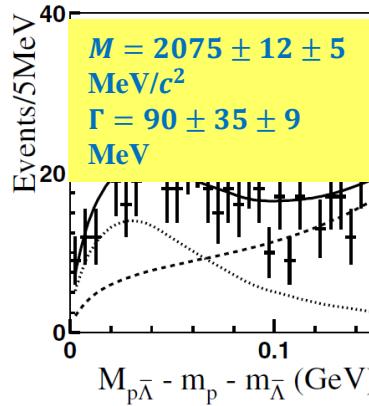
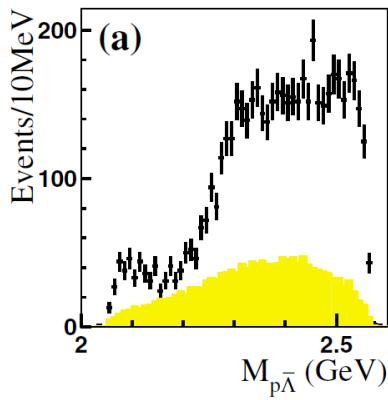
$$e^+ e^- \rightarrow K^- X(2085)^+, X(2085)^+ \rightarrow p \bar{\Lambda}$$

$$M_{\text{pole}} = 2084^{+4}_{-2} \pm 9 \text{ MeV}, \Gamma = 58^{+4}_{-3} \pm 25 \text{ MeV}$$

Ablikim, M., et.al, (BESIII) Phys.Rev.Lett., 131, 151901(2023)

# $p\bar{\Lambda}$ threshold enhancement $X(2085)$

**BESII:**  $J/\psi \rightarrow pK^-\bar{\Lambda} + c.c.$   
**PRL 93 (2004) 112002**

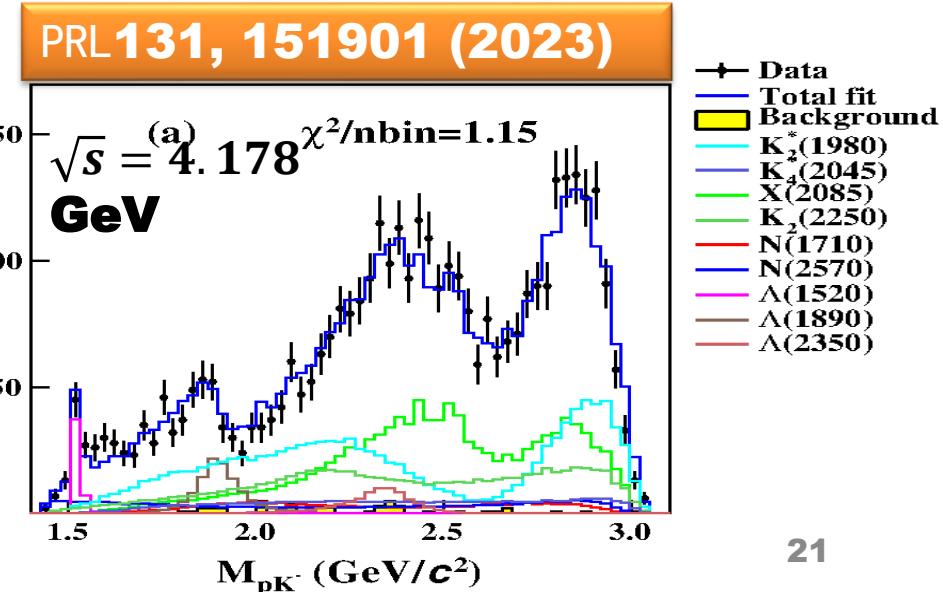
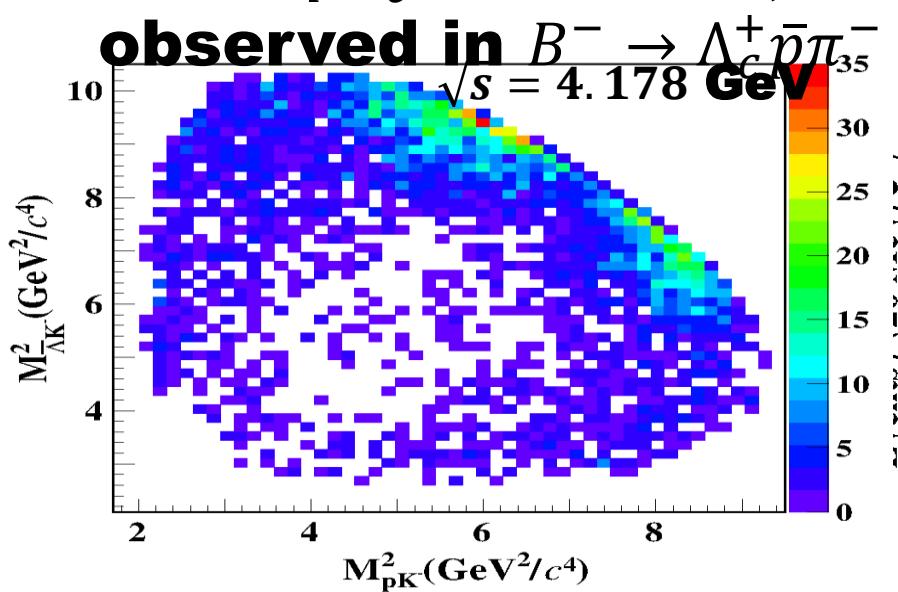


## Motivation

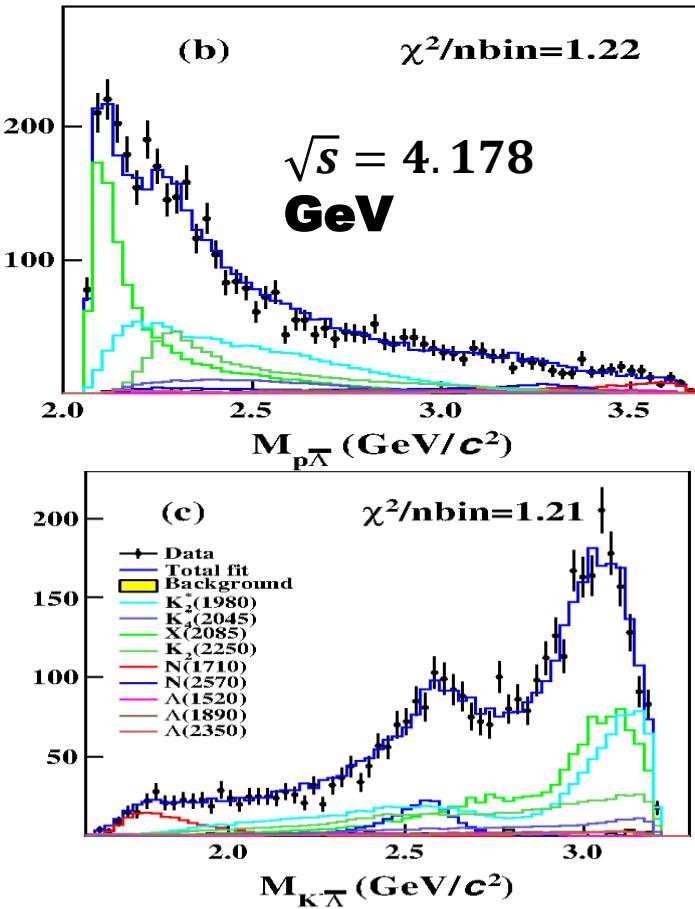
- Observed at BESII in 2004
- Similar structure was seen in several B meson and charmonium decays
- Investigated theoretically under scenario of quark model, FSI and chiral effective theory

# $X(2085)$ in $e^+e^- \rightarrow \gamma^* \rightarrow pK^-\bar{\Lambda} + c.c.$

- $X(2085)$ [**K(2085)**] first observed in  $J/\psi \rightarrow pK^-\Lambda + c.c.$ , but spin-parity not measured.
- Similar evidence found in  $B \rightarrow p\Lambda\pi$ , and  $\psi', \chi_{cJ} \rightarrow pK^-\Lambda$ .
- Near  $p\Lambda_c$  threshold, an enhancement also observed in  $B^- \rightarrow \Lambda_c^+ p \pi^-_{35}$



# $X(2085)$ in $e^+e^- \rightarrow \gamma^* \rightarrow pK^-\bar{\Lambda} + c.c.$

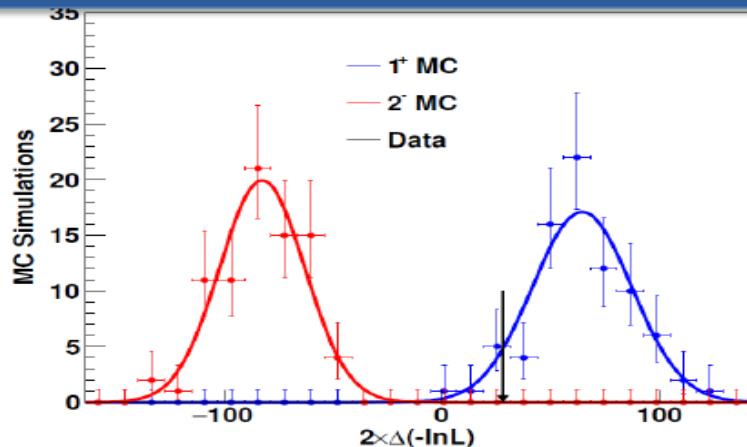


- $X(2085), J^P = 1^+$
- $M_{pole} = (2084^{+4}_{-2} \pm 9) \text{ MeV}$
- $\Gamma_{pole} = (58^{+4}_{-3} \pm 25) \text{ MeV}$

	$\Delta \ln \mathcal{L}$	$\Delta ndf$	Significance
$1^+$ over $0^-$	40.6	4	8.3
$1^+$ over $1^-$	30.2	2	7.5
$1^+$ over $2^+$	44.8	2	9.2
$1^+$ over $2^-$	13.8	0	5.3

$\sqrt{s}$ (GeV)	$M_{pole}$ (MeV/ $c^2$ )	$\Gamma_{pole}$ (MeV)
4.008	$2085 \pm 14$	$50 \pm 16$
4.178	$2085 \pm 6$	$62 \pm 10$
4.226	$2088 \pm 10$	$68 \pm 12$
4.258	$2083 \pm 11$	$48 \pm 10$
4.416	$2088 \pm 13$	$56 \pm 12$
4.682	$2092 \pm 10$	$54 \pm 10$
Average	$2086 \pm 4$	$56 \pm 5$

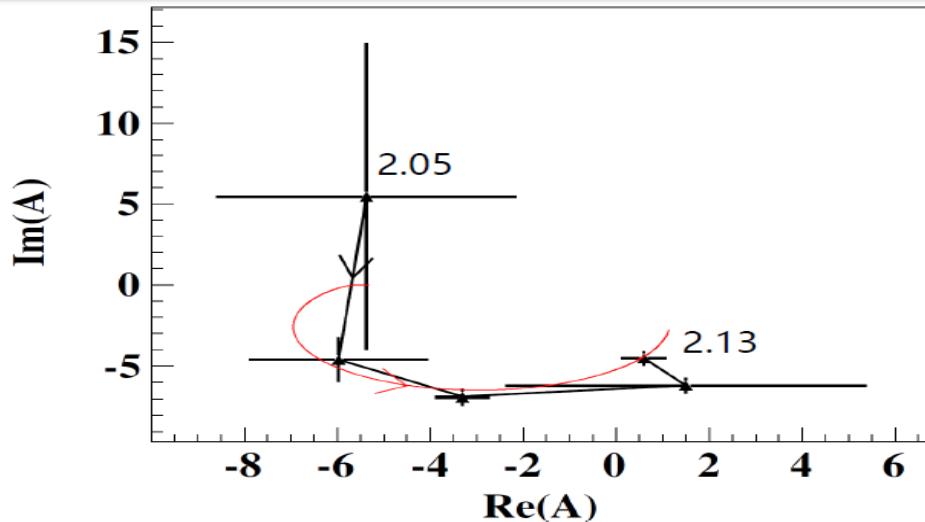
# Check significance with toy MC method



	$\Delta \ln \mathcal{L}$	$\Delta ndf$	Significance
1 <sup>+</sup> over 0 <sup>-</sup>	40.6	4	8.3
1 <sup>+</sup> over 1 <sup>-</sup>	30.2	2	7.5
1 <sup>+</sup> over 2 <sup>+</sup>	44.8	2	9.2
1 <sup>+</sup> over 2 <sup>-</sup>	13.8	0	5.3

- ✓ The statistical significances of  $1^+$  over  $0^-$ ,  $1^-$ , and  $2^+$  are obtained with  $\Delta \ln L = \ln L^{1^+} - \ln L^{J^P}$  and  $\Delta ndf$
- ✓ For  $1^+$  over  $2^-$ , the  $\Delta ndf$  is assumed to be 1, following PRL 115 (2015), 072001.
- ✓ The approach based on MC simulation is checked. The  $t = -2 \ln(L^{2^-}/L^{1^+})$  distribution suggests a statistical significance  $5.6 \sigma$

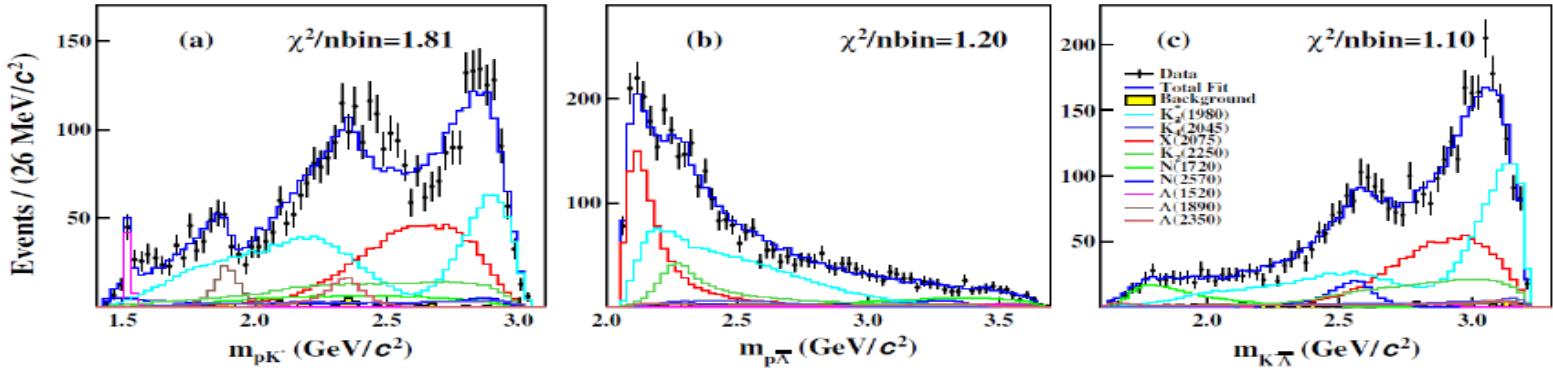
# X(2085) Argand plot



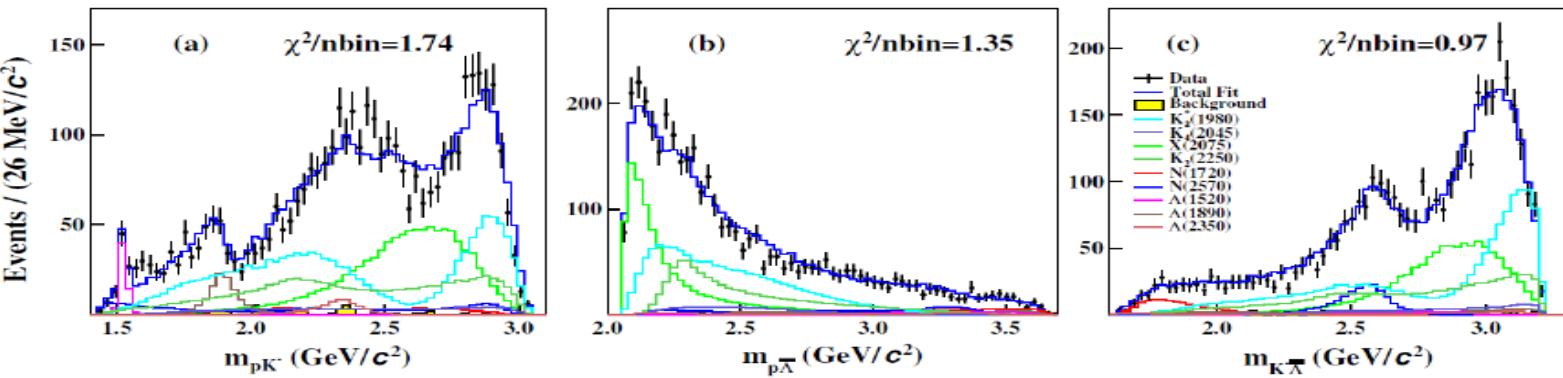
- From 2.05 – 2.13 GeV with step size 20 MeV
- BW function replaced with a complex number  $A_i$  in point  $i$
- Cubic interpolation between two points
- **No conclusion** whether  $X(2075)$  exhibits the characteristics of a resonance or not.

# check alternative $J^P$

$J^P = 0^-$

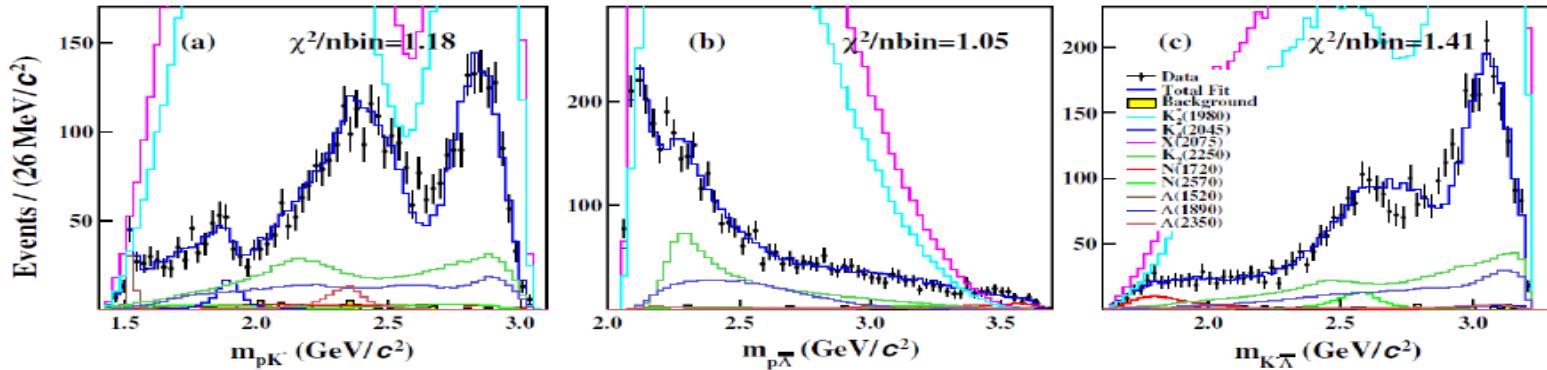


$J^P = 1^-$

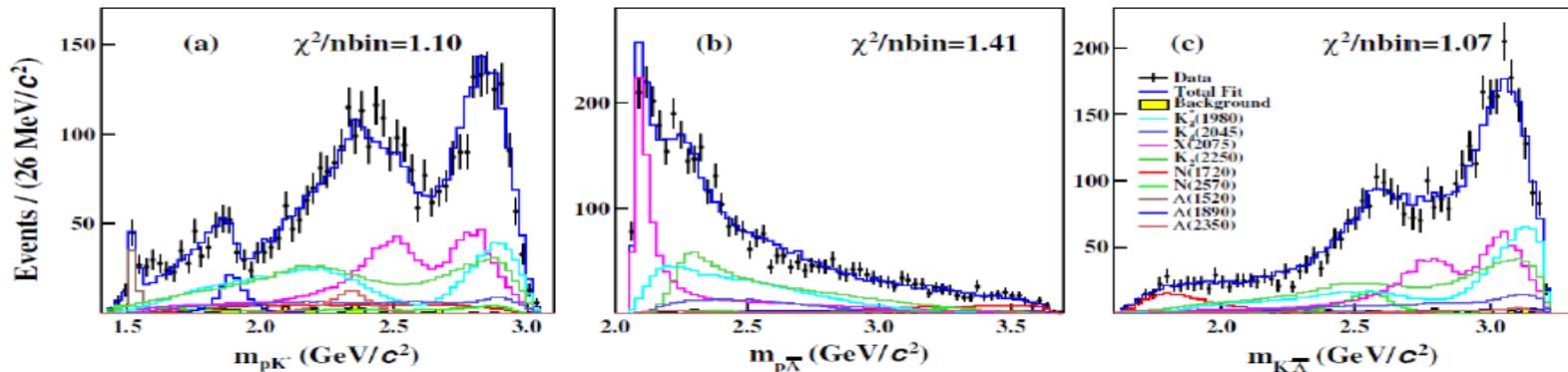


# check alternative $J^P$

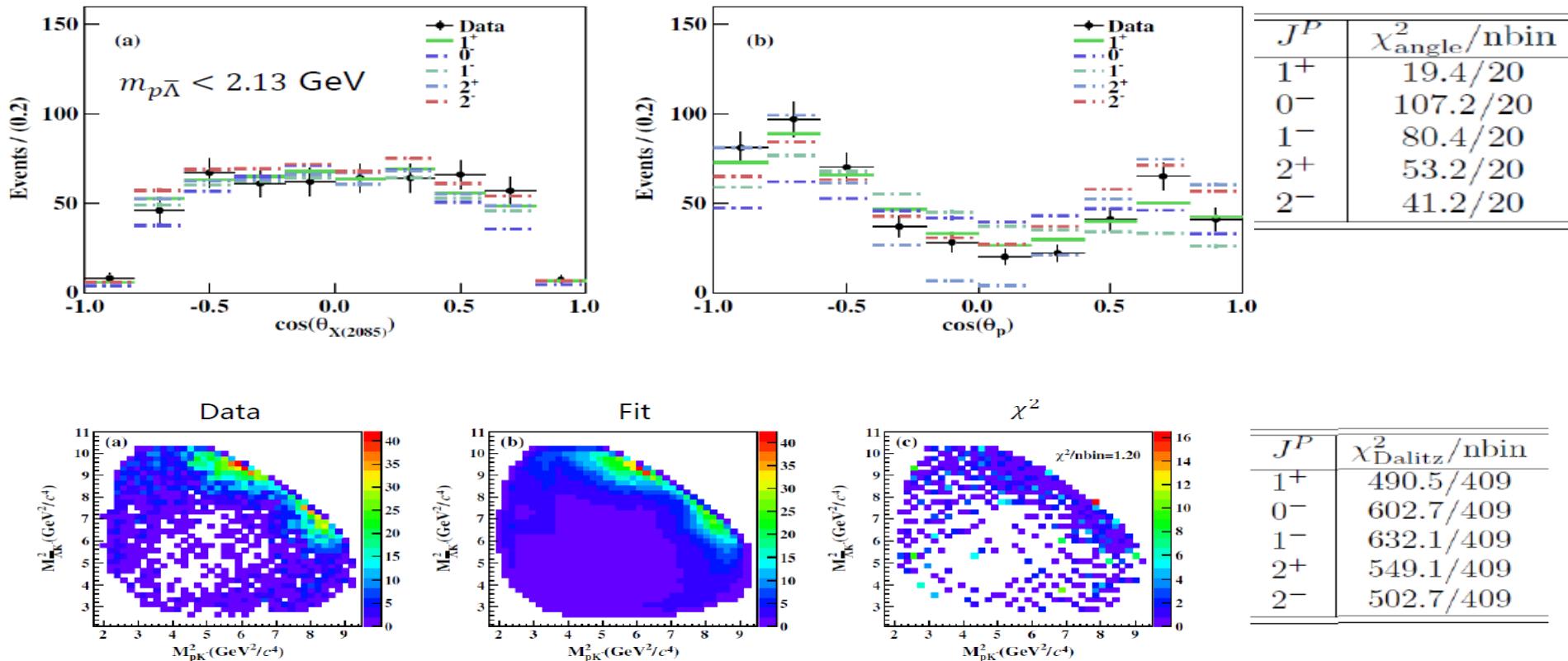
$J^P = 2^+$



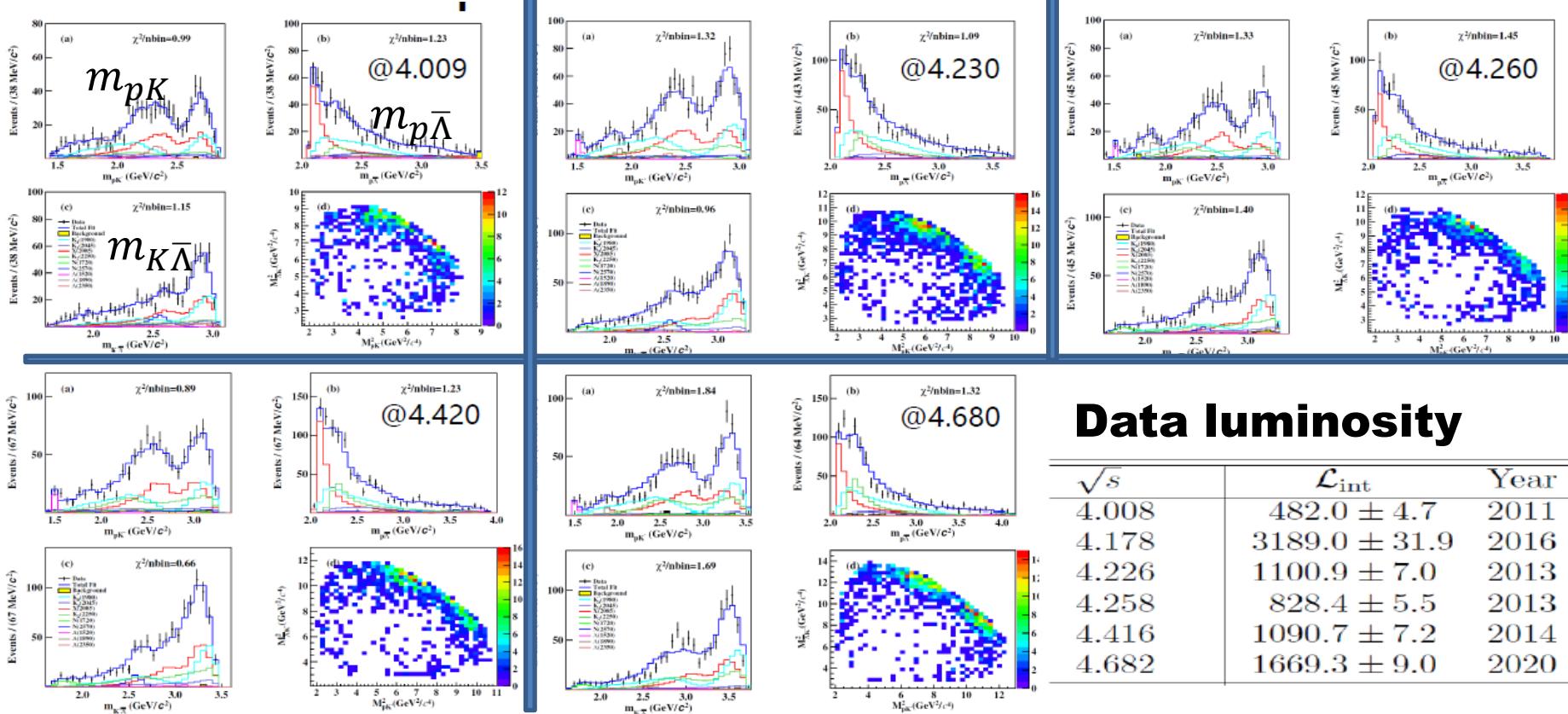
$J^P = 2^-$



# Data favor $1^+$ hypothesis



# Data at other five points



## Data luminosity

$\sqrt{s}$	$\mathcal{L}_{\text{int}}$	Year
4.008	$482.0 \pm 4.7$	2011
4.178	$3189.0 \pm 31.9$	2016
4.226	$1100.9 \pm 7.0$	2013
4.258	$828.4 \pm 5.5$	2013
4.416	$1090.7 \pm 7.2$	2014
4.682	$1669.3 \pm 9.0$	2020

# 其他自旋-宇称量子数测量的范例

1. M. Ablikim et al. (BESIII Collaboration), Observation of an Isoscalar Resonance with **Exotic  $J^{PC} = 1^{-+}$**  Quantum Numbers in  $J/\psi \rightarrow \gamma\eta\eta'$ , Phys.Rev.Lett. 129, 192002 (2022);
2. M. Ablikim et al. (BESIII Collaboration), companion paper, Phys. Rev. D 106, 072012 (2022).
3. M. Ablikim, et.al., (BESIII) , Model-Independent Determination of the Spin of the  $\Omega^-$  and Its Polarization Alignment in  $\psi(2S) \rightarrow \Omega^-\bar{\Omega}^+$ , Phys. Rev. Lett. 126 (2021) 9, 092002.
4. M. Ablikim , et.al., (BESIII), Determination of spin and parity of  $D^{*(s)}$  mesons, Phys. Lett., B846, 138245(2023)

# Summary

- **Spin-parity measurements using hypothesis testing**
- **Distinguishing spin parity through angular distribution, angular moment analysis, and fit quality**
- **Exotic spin-parity measurements as a tool for identifying exotic states.**

**Thanks for your attention!**

# backup