

第一届中微子散射：理论、实验、唯象研讨会 (ν STEP 2024)

2024年5月17日-20日，国科大杭州高等研究院

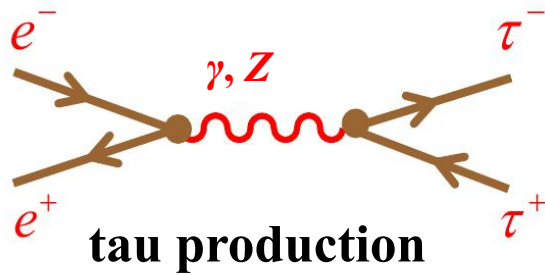
Charged current of neutrino in hadronic τ decays



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Mini-overview of hadronic tau decays and Resonance Chiral Theory



γ exchange dominates at tau-charm factory.

Z exchange dominates at CEPC.

Number of taus produced at e^+e^- colliders:

ALEPH: $\sim 3 \times 10^5$

BaBar /Belle: $\sim 1 \times 10^9$

Belle-II: $\sim 5 \times 10^{10}$

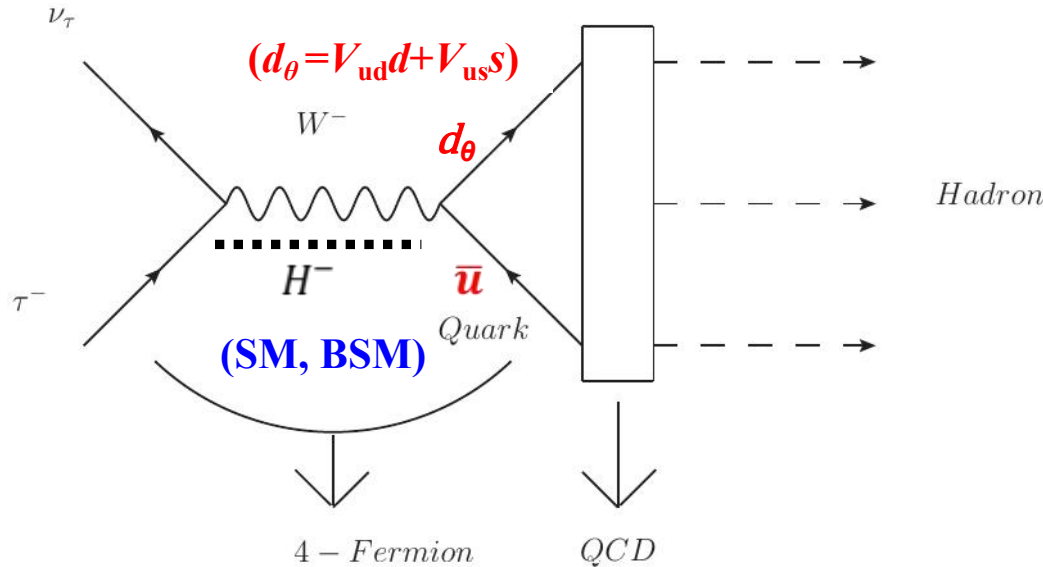
CEPC (Tera-Z factory): $\sim 3 \times 10^{10}$

STCF: $\sim 4 \times 10^{10}$ (**around 10% at threshold**)

Tau provides broad interests for particle physics:

- ✓ Precision tests for electroweak sector: V_{CKM} , lepton universality, g-2,
- ✓ Strong interactions: α_s , hadron resonances, chiral symmetry,
- ✓ Discoveries for new physics: cLFV, CPV,
- ✓ Possible way to study massive neutrinos: $\tau \rightarrow \pi\pi\pi\nu_4$,

Sketch for hadronic tau decays (similar for leptonic decays by dropping QCD part)



Theoretical tools: SM EFT + Chiral EFT

- **SM EFT $\rightarrow$ LEFT** [Cirigliano et al, '10] [Y.Liao et al., '21] [J.H.Yu et al., '21][F.Z.Chen et al, '22] ...

$$\mathcal{L}_{\text{eff}} = -\frac{G_\mu V_{uD}}{\sqrt{2}} \left[\left(1 + \epsilon_L^{D\ell}\right) \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell \cdot \bar{u} \gamma^\mu (1 - \gamma_5) D + \epsilon_R^{D\ell} \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell \cdot \bar{u} \gamma^\mu (1 + \gamma_5) D \right. \\ \left. + \bar{\ell} (1 - \gamma_5) \nu_\ell \cdot \bar{u} \left[\epsilon_S^{D\ell} - \epsilon_P^{D\ell} \gamma_5 \right] D + \frac{1}{4} \hat{\epsilon}_T^{D\ell} \bar{\ell} \sigma_{\mu\nu} (1 - \gamma_5) \nu_\ell \cdot \bar{u} \sigma^{\mu\nu} (1 - \gamma_5) D \right] + \text{h.c.},$$

ϵ_X parameterize various new physics at high energy scale

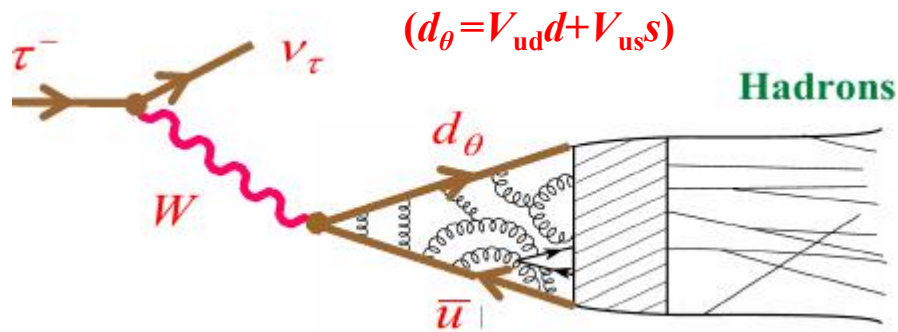
- **Chiral EFT**

[Gasser, Leutwyler, '83 '84]

$\mathcal{O}(p^4)$:

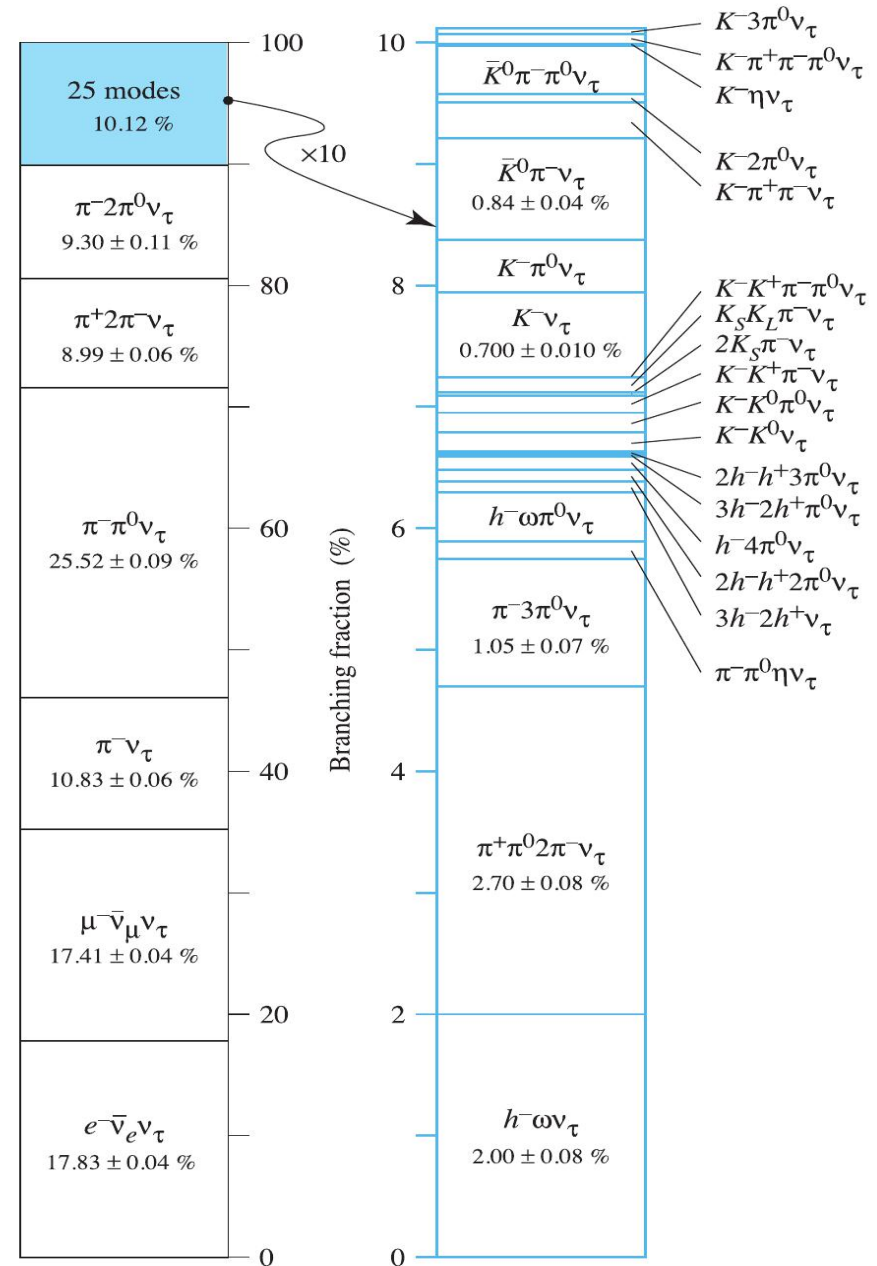
$$\mathcal{L}_2 = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle \quad \mathcal{L}_4^{\chi PT} = L_1 \langle u_\mu u^\mu \rangle^2 + L_2 \langle u_\mu u^\nu \rangle \langle u^\mu u_\nu \rangle + L_3 \langle u_\mu u^\mu u_\nu u^\nu \rangle + L_4 \langle u_\mu u^\mu \rangle \langle \chi_+ \rangle \\ + L_5 \langle u_\mu u^\mu \chi_+ \rangle + L_6 \langle \chi_+ \rangle^2 + L_7 \langle \chi_- \rangle^2 + \frac{L_8}{2} \langle \chi_+^2 + \chi_-^2 \rangle + \dots$$

Hadronic decays: a unique feature for tau lepton

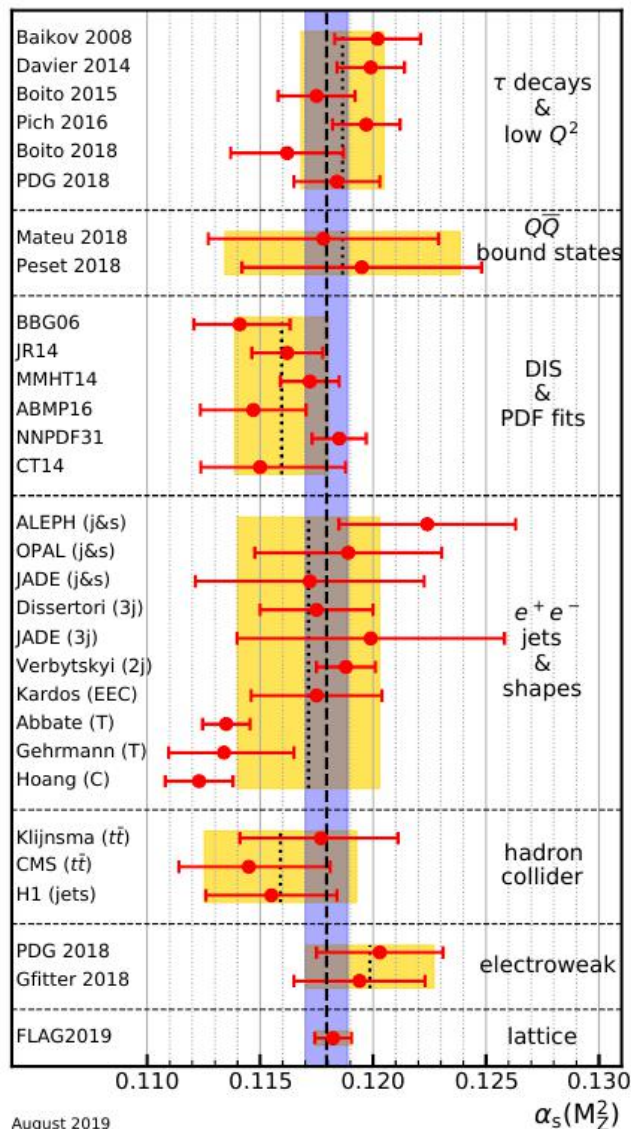


Valuable laboratory to study:
fundamental parameters &
rich hadron phenomenologies

But also challenging in theory:
broad energy range 0.14 ~ 1.8 GeV



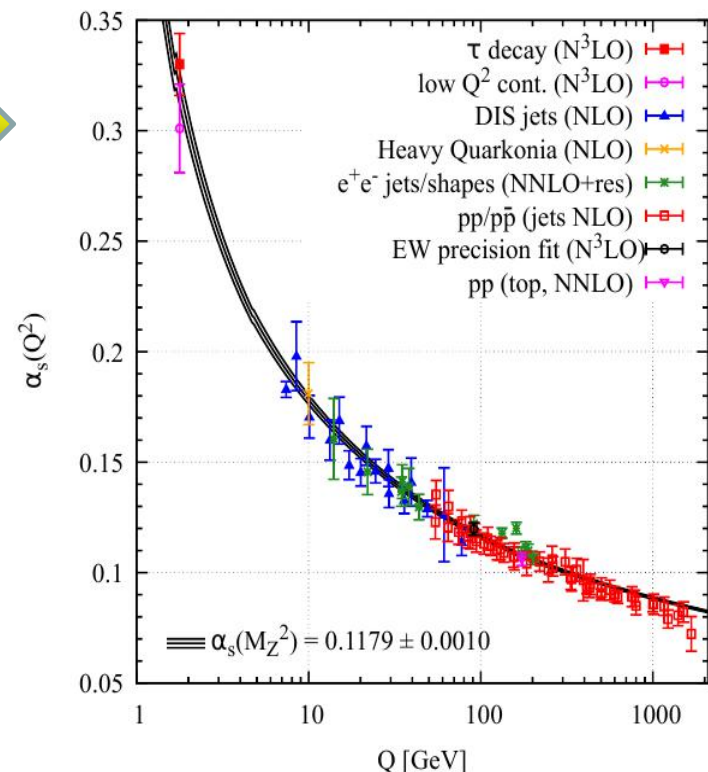
Strong coupling of QCD: α_s



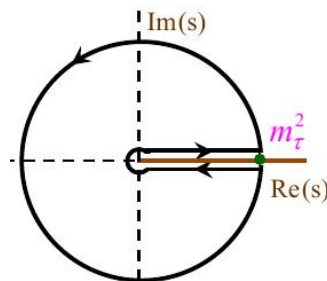
PDG



**Hadronic tau decay:
an invaluable source
to test the QCD
prediction of $\alpha_s(Q^2)$
below 2 GeV.**



$$R_\tau = \frac{\Gamma(\tau \rightarrow \nu_\tau \text{ hadrons})}{\Gamma(\tau \rightarrow \nu_\tau e \nu_e)} = \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \omega_J(s) \text{Im} \Pi^J(s)$$

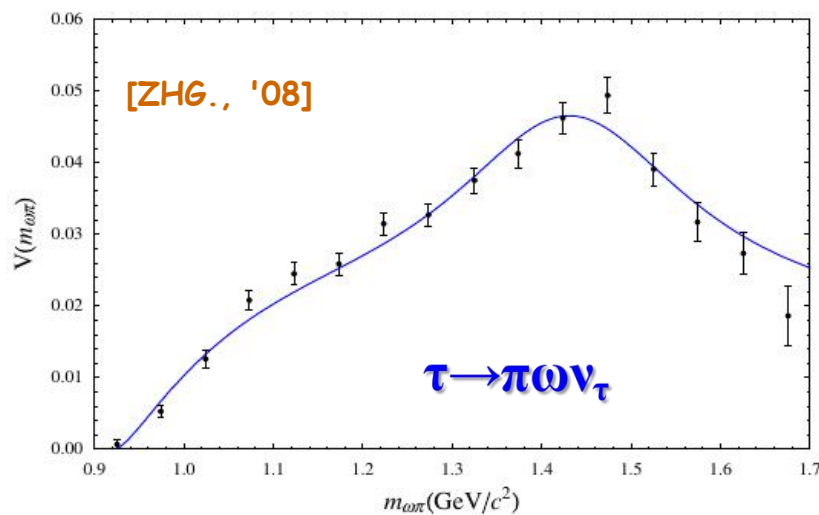
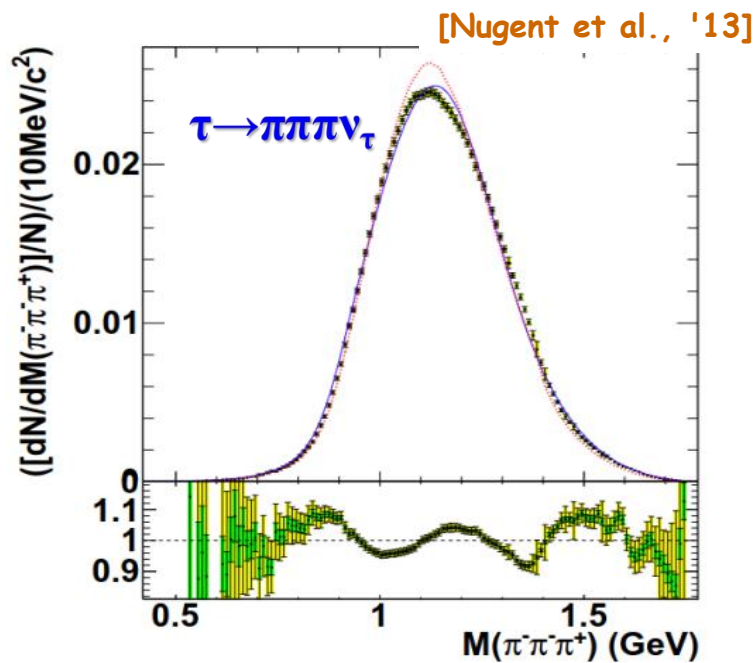
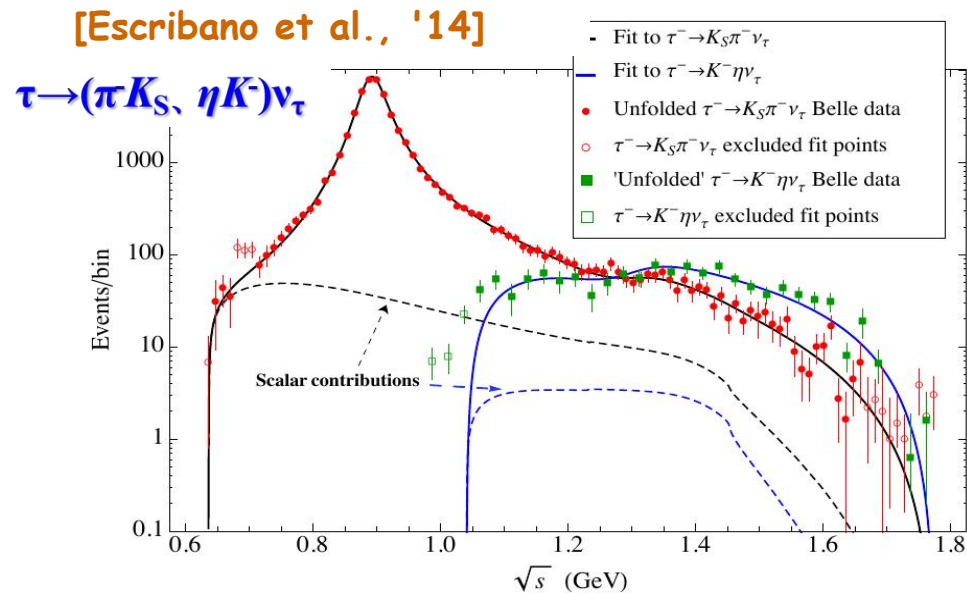
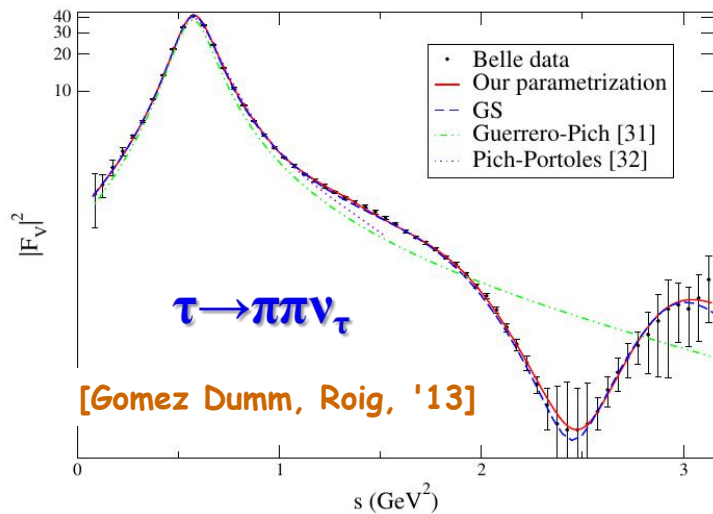


Exp

$$R_\tau = \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2} \omega_J(s) \Pi^J(s)$$

$$\Pi^J(s) \stackrel{\text{OPE}}{=} \sum_D \frac{C_D^J(s, \alpha_s(\mu), \mu) \langle O_D(\mu) \rangle}{(-s)^{D/2}}$$

Invariant-mass spectra for exclusive decays

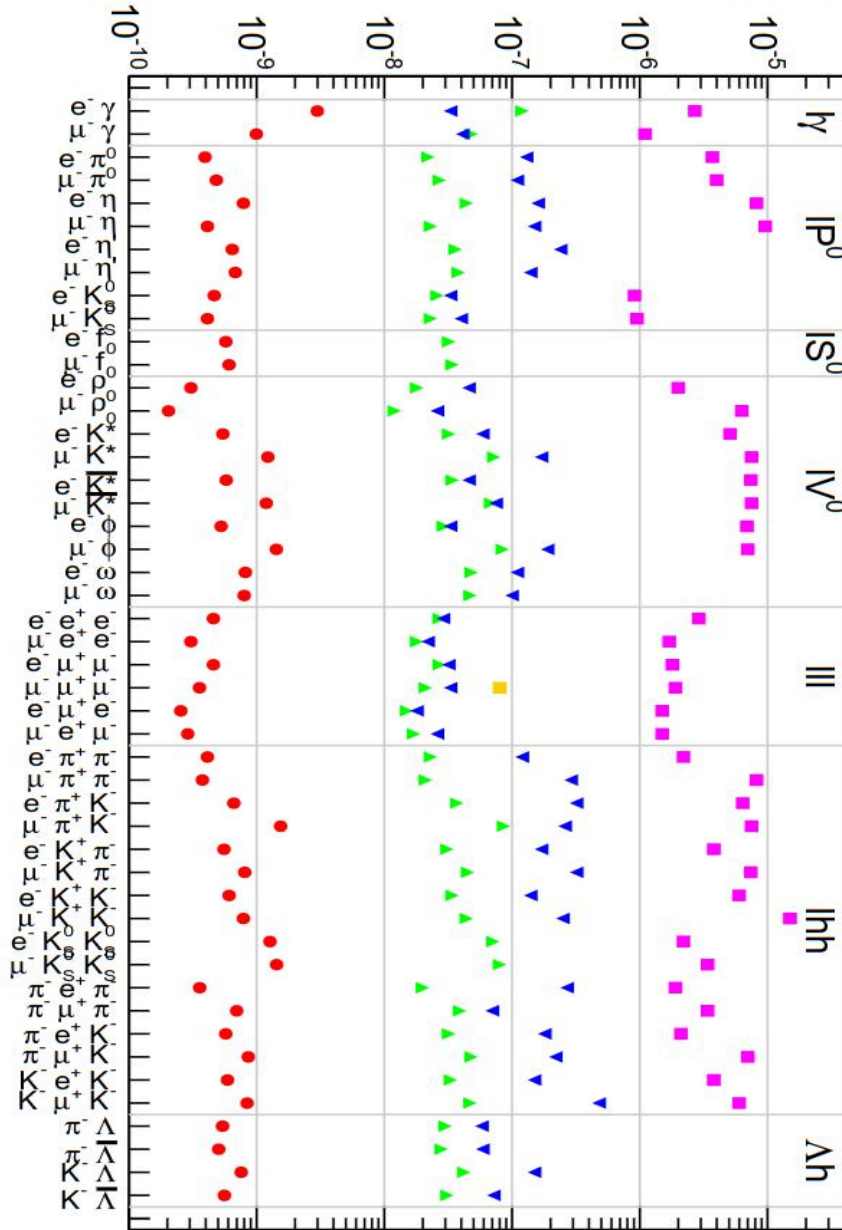


Hadron properties, Form factors,
Chiral dynamics,

➤ Charged lepton flavor violation in tau decays

90% C.L. upper limits for LFV τ decays

[Belle-II, '22]

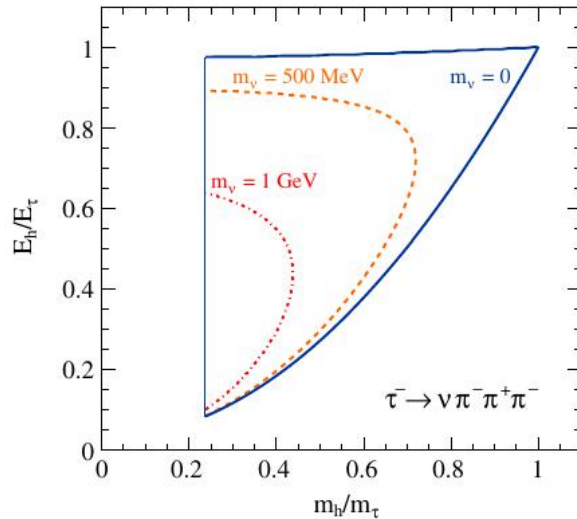


- **Not only statistic but also systematic uncertainties are important in $\tau \rightarrow l \gamma$**
- **Clean background makes $\tau \rightarrow l l' l''$ one of the best channels to search for LFV signals.**
- **$\tau \rightarrow l + \textit{hadrons}$ provides a different laboratory to probe different LFV origins, comparing with the pure leptonic processes.**

Proposal to search for massive neutrino in tau decay

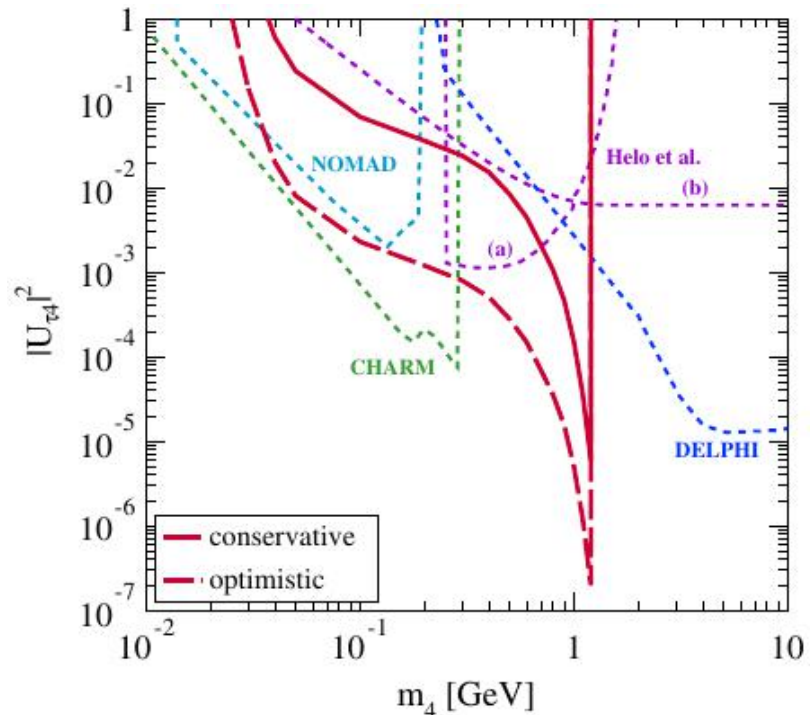
$\tau \rightarrow \pi\pi\pi\nu_4$

[Kobach, Dobbs, PRD'15]



$$\frac{d\Gamma_{\text{tot}}(\tau^- \rightarrow \nu h^-)}{dm_h dE_h} = (1 - |U_{\tau 4}|^2) \frac{d\Gamma(\tau^- \rightarrow \nu h^-)}{dm_h dE_h} \Big|_{m_\nu=0} + |U_{\tau 4}|^2 \frac{d\Gamma(\tau^- \rightarrow \nu h^-)}{dm_h dE_h} \Big|_{m_\nu=m_4}$$

Strong interaction of the $\pi\pi\pi$ [dominated by $a_1(1260)$] system will greatly affect the final results!



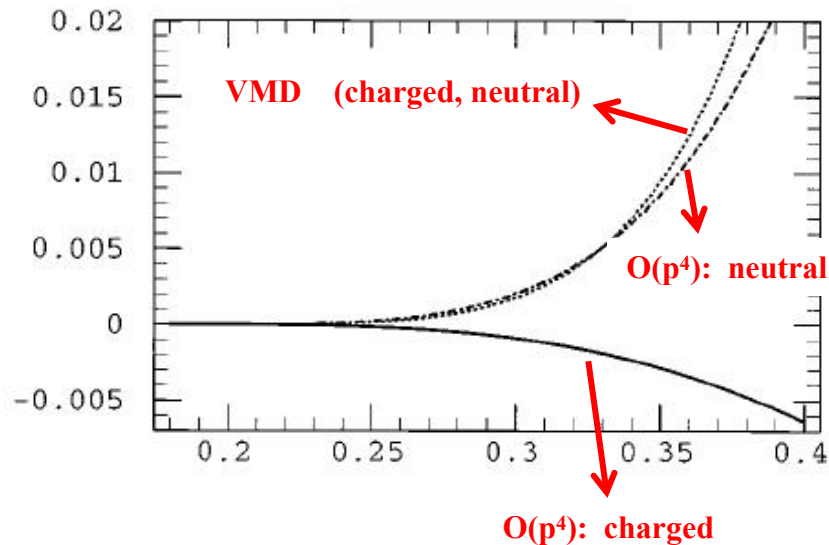
Chiral symmetry is RELEVANT to tau decays

Example: $\tau \rightarrow \nu_\tau \pi \pi \pi$ transition amplitudes in the low energy region
VMD models do not automatically respect chiral symmetry.

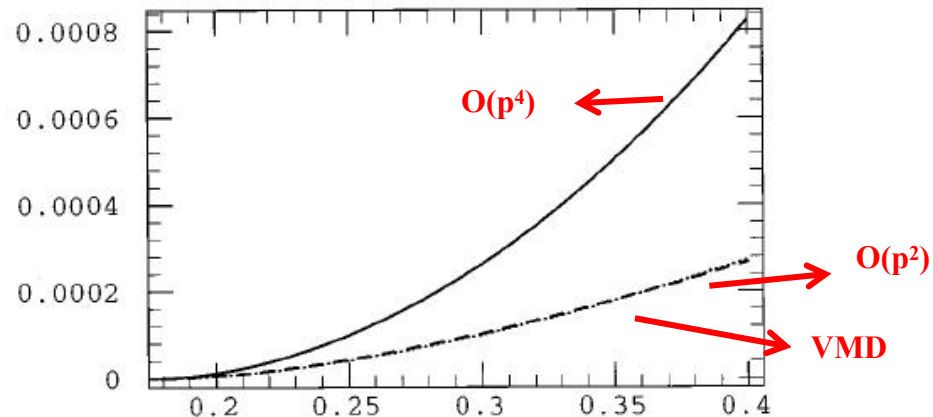
$$J_\alpha = -i \frac{2\sqrt{2}}{3f_\pi} \text{BW}_a(Q^2) (B_\rho(s_2) V_{1\alpha} + B_\rho(s_1) V_{2\alpha})$$

[Kuhn, Santamaria, ZPC'90]

W_D structure function



W_{SA} structure function (neutral channel)



[Colangelo, et al., PRD'96]

- Resonance chiral theory implements the constraint of chiral symmetry from the very beginning in the construction of the Lagrangians.

Resonance chiral theory (R χ T)

Chiral group: $G = SU(3)_L \times SU(3)_R$, $H = SU(3)_V$, $u(\phi) = G/H$

Resonances : $R \xrightarrow{G} h R h^\dagger, \quad h \in H$

pNGB and external sources : $X = u_\mu, \chi_\pm, f_\pm^{\mu\nu}, h_{\mu\nu}$

Operators	P	C	h.c.	chiral order
u	$u^\dagger$	u^T	$u^\dagger$	1
Γ_μ	Γ^μ	$-\Gamma_\mu^T$	$-\Gamma_\mu$	p
u_μ	$-u^\mu$	u_μ^T	u_μ	p
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^T$	$\pm\chi_\pm$	p^2
$f_{\mu\nu} \pm$	$\pm f_\pm^{\mu\nu}$	$\mp f_{\mu\nu}^T$	$f_{\mu\nu} \pm$	p^2
$h_{\mu\nu}$	$-h^{\mu\nu}$	$h_{\mu\nu}^T$	$h_{\mu\nu}$	p^2

Operators	P	C	h.c.
$V_{\mu\nu}$	$V^{\mu\nu}$	$-V_{\mu\nu}^T$	$V_{\mu\nu}$
$A_{\mu\nu}$	$-A^{\mu\nu}$	$A_{\mu\nu}^T$	$A_{\mu\nu}$
S	S	S^T	S
P	$-P$	P^T	P

Operators beyond minimal

[Cirigliano, *et al.*, '04]:

$$\mathcal{L}_{VAP} = \lambda_1^{VA} \langle [V^{\mu\nu}, A_{\mu\nu}] \chi_- \rangle + \dots,$$

[Ruiz-Femenia, Pich and Portolés, '03]

$$\mathcal{L}_{VVP} = d_1 \varepsilon_{\mu\nu\rho\sigma} \langle \{V^{\mu\nu}, V^{\rho\alpha}\} \nabla_\alpha u^\sigma \rangle + \dots,$$

$$\mathcal{L}_{VJP} = \frac{c_1}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{V^{\mu\nu}, f_+^{\rho\alpha}\} \nabla_\alpha u^\sigma \rangle + \dots,$$

$$d_4 \varepsilon_{\mu\nu\rho\sigma} \langle \{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\} u_\alpha \rangle$$

Minimal R χ T Lagrangian [Ecker, *et al.*, '89]

$$\mathcal{L}_{2V} = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

$$\mathcal{L}_{2A} = \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle,$$

$$\mathcal{L}_{2S} = c_d \langle S u_\mu u^\mu \rangle + c_m \langle S \chi_+ \rangle,$$

$$\mathcal{L}_{2P} = id_m \langle P \chi_- \rangle.$$

QCD dynamics in R χ T

- Low energy QCD: implemented from the construction of R χ T
- Intermediate energy: explicit resonance states
- **High energy information:** to match the same physical objects in R χ T and QCD, $\langle J(x_n) \cdots J(0) \rangle^{\text{R}\chi\text{T}} = \langle J(x_n) \cdots J(0) \rangle^{\text{QCD}}$.

For example: $\pi\pi$ vector form factor

$$\begin{aligned} [\mathcal{F}_{\pi\pi}^{\nu}(q^2)]^{\text{R}\chi\text{T}} &= 1 + \frac{F_V G_V}{F^2} \frac{q^2}{M_V^2 - q^2}, \\ [\mathcal{F}_{\pi\pi}^{\nu}(q^2)]^{\text{QCD}} &\rightarrow 0, \quad \text{for } q^2 \rightarrow \infty \end{aligned}$$

This leads to

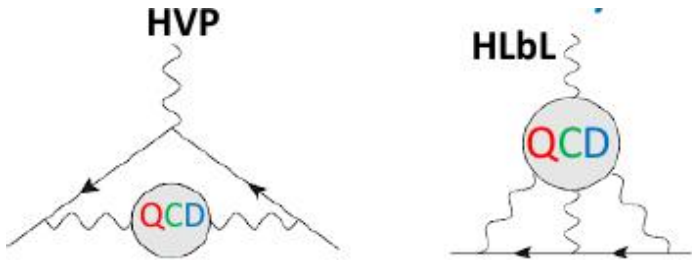
$$[\mathcal{F}_{\pi\pi}^{\nu}(q^2)]^{\text{R}\chi\text{T}} = [\mathcal{F}_{\pi\pi}^{\nu}(q^2)]^{\text{QCD}} \implies F_V G_V = F^2$$

Phenomenologies in $\tau \rightarrow \pi\pi\gamma\nu_\tau$

Why to focus on $\tau \rightarrow \pi\pi\gamma\nu_\tau$

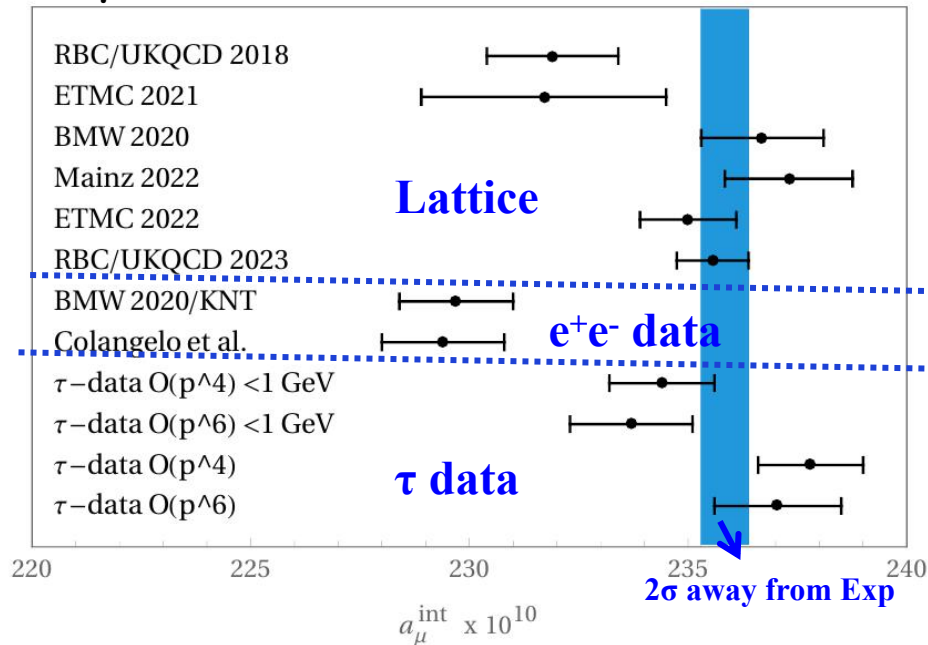
➤ Relevance to precise determination of a_μ

SM uncertainty dominated by

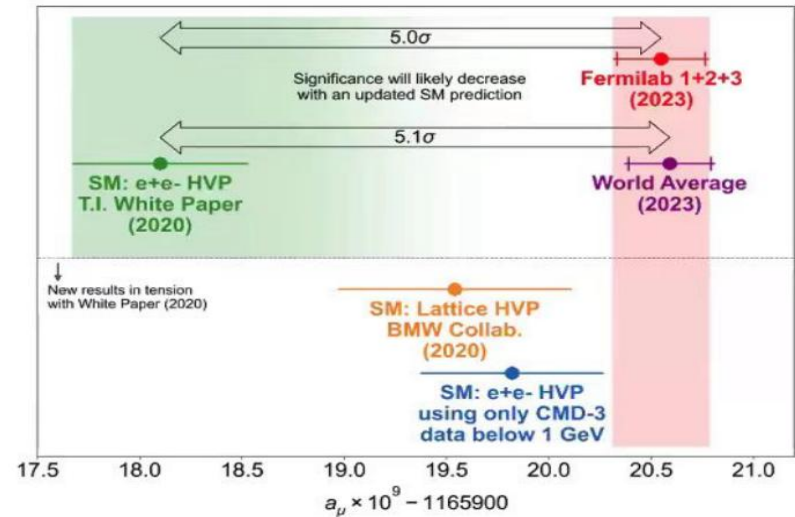


↓
Dominated by $\pi\pi$ ($> \sim 75\%$)

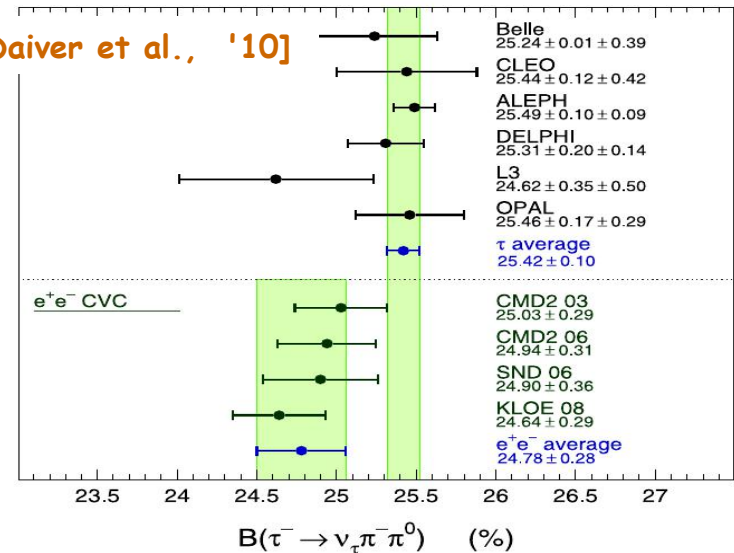
$a_\mu^{\text{HVP,LO}}$ [Masjuan et al., '23]

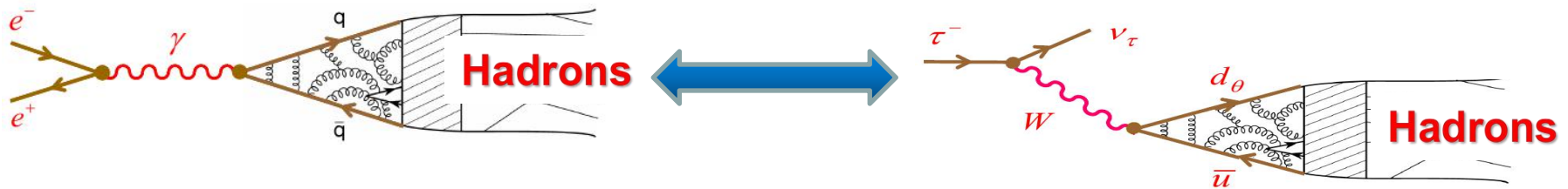


[Muon g-2, '23]



[Daiver et al., '10]





❖ Key problem in the matching: isospin breaking (IB) effects

IB corrections to a_μ

[Cirigliano et al., JHEP'02]

$$\Delta a_\mu^{\text{vacpol}} = \frac{1}{4\pi^3} \int_{4M_\pi^2}^{t_{\text{max}}} dt K(t) \left[\frac{K_\sigma(t)}{K_\Gamma(t)} \frac{d\Gamma_{\pi\pi[\gamma]}}{dt} \right] \times \left(\frac{R_{\text{IB}}(t)}{S_{\text{EW}}} - 1 \right)$$

$$R_{\text{IB}}(t) = \frac{1}{G_{\text{EM}}(t)} \frac{\beta_{\pi^+\pi^-}^3}{\beta_{\pi^+\pi^0}^3} \left| \frac{F_V(t)}{f_+(t)} \right|^2$$

EM corrections

Kinematics

IB effects in Form Factors

$G_{\text{EM}}(t) \sim$ virtual photon + real photon

Photon loops
in $\tau \rightarrow \pi\pi\nu_\tau$

Radiative decays:
 $\tau \rightarrow \pi\pi\gamma\nu_\tau$

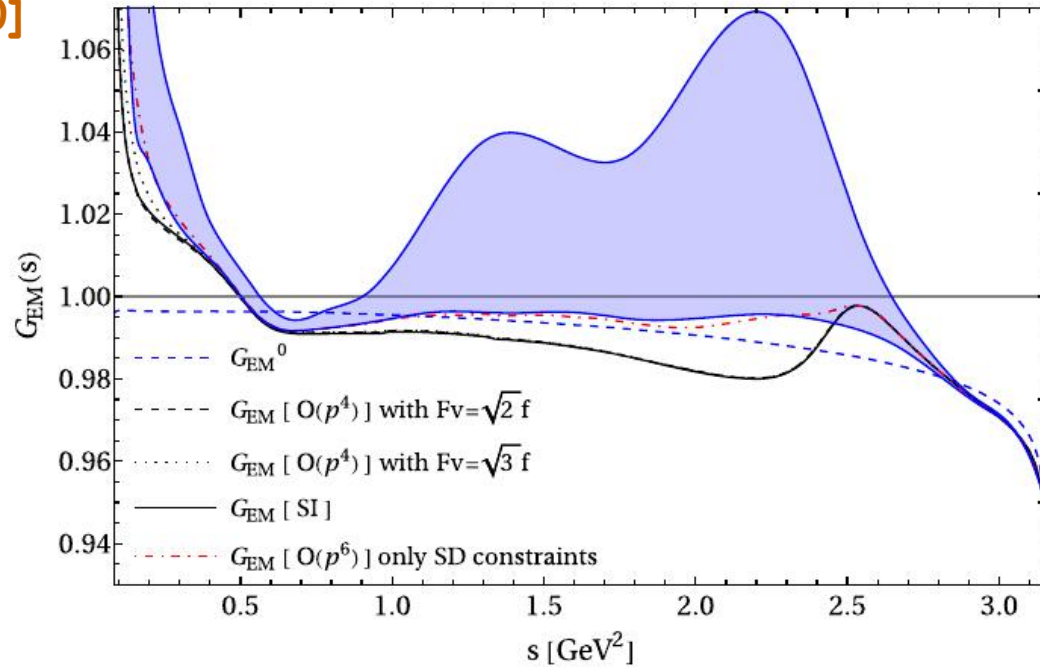


TABLE IV. Contributions to $\Delta a_\mu^{\text{HVP,LO}}$ in units of 10^{-11} using the dispersive representation of the form factor. From the two evaluations labeled $\mathcal{O}(p^4)$, the left (right) one corresponds to $F_V = \sqrt{2}F$ ($F_V = \sqrt{3}F$).

$[s_1, s_2]$	$\Delta a_{\mu, G_{\text{EM}}^{(0)}}^{\text{HVP,LO}}$	$\Delta a_{\mu, \text{SI}}^{\text{HVP,LO}}$	$\Delta a_{\mu, [\mathcal{O}(p^4)]}^{\text{HVP,LO}}$	$\Delta a_{\mu, [\mathcal{O}(p^4)]}^{\text{HVP,LO}}$	$\Delta a_{\mu, [\text{SD}]}^{\text{HVP,LO}}$	$\Delta a_{\mu, [\mathcal{O}(p^6)]}^{\text{HVP,LO}}$
$[4m_\pi^2, 1 \text{ GeV}^2]$	+17.8	-11.0	-11.3	-17.0	-32.4	-74.8 ± 44.0
$[4m_\pi^2, 2 \text{ GeV}^2]$	+18.3	-10.1	-10.3	-16.0	-31.9	-75.9 ± 45.5
$[4m_\pi^2, 3 \text{ GeV}^2]$	+18.4	-10.0	-10.2	-15.9	-31.9	-75.9 ± 45.6
$[4m_\pi^2, m_\tau^2]$	+18.4	-10.0	-10.2	-15.9	-31.9	-75.9 ± 45.6

Referenced value using the tau data to calculate a_μ

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (12.5 \pm 6.0) \times 10^{-10}$$

➤ CP violation in tau decays

$$A_{CP} = \frac{\Gamma(\tau^- \rightarrow \nu_\tau H) - \Gamma(\tau^+ \rightarrow \nu_\tau \bar{H})}{\Gamma(\tau^- \rightarrow \nu_\tau H) + \Gamma(\tau^+ \rightarrow \nu_\tau \bar{H})}$$

Intensive discussions on tau $\rightarrow K_s \pi \nu$

$$A_Q = \frac{\Gamma(\tau^+ \rightarrow \pi^+ K_S^0 \bar{\nu}_\tau) - \Gamma(\tau^- \rightarrow \pi^- K_S^0 \nu_\tau)}{\Gamma(\tau^+ \rightarrow \pi^+ K_S^0 \bar{\nu}_\tau) + \Gamma(\tau^- \rightarrow \pi^- K_S^0 \nu_\tau)}$$

$$\approx (0.36 \pm 0.01)\%$$

SM prediction

[Bigi et al., PLB'05]

[Grossman et al., JHEP'12]

[Lees et al., PRD'12]

[Cirigliano et al., PRL'18]

[Rendo et al., PRD'19]

[Chen et al., PRD'19 JHEP'20]

• • • • •

$$(-0.36 \pm 0.23_{\text{stat}} \pm 0.11_{\text{syst}})\%$$

BaBar

Other types of CPV observables: T-odd triple-product asymmetry

A typical T-odd kinematical variable:

$$\xi \equiv \varepsilon_{\mu\nu\rho\sigma} a^\mu b^\nu c^\rho d^\sigma \frac{\text{rest frame}}{\text{of particle } a} \vec{b} \cdot (\vec{c} \times \vec{d}) m_a / s_a$$

a, b, c, d: either momentum or spin

T transformation $(t \rightarrow -t, \vec{p} \rightarrow -\vec{p}, \dots): \bar{\xi} \rightarrow -\xi$

❖ When spin is involved, measurement of polarization is needed.

[Nelson, et al., PRD'94] [Tsai, PRD'95] [Datta, PRD'07] ...

❖ When focusing on the situation with four momenta, *i.e.*

$$\xi = \varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma \frac{\text{rest frame}}{\text{of particle 1}} \vec{p}_2 \cdot (\vec{p}_3 \times \vec{p}_4) m_1$$

In this case, there should be at least four particles in the final state!

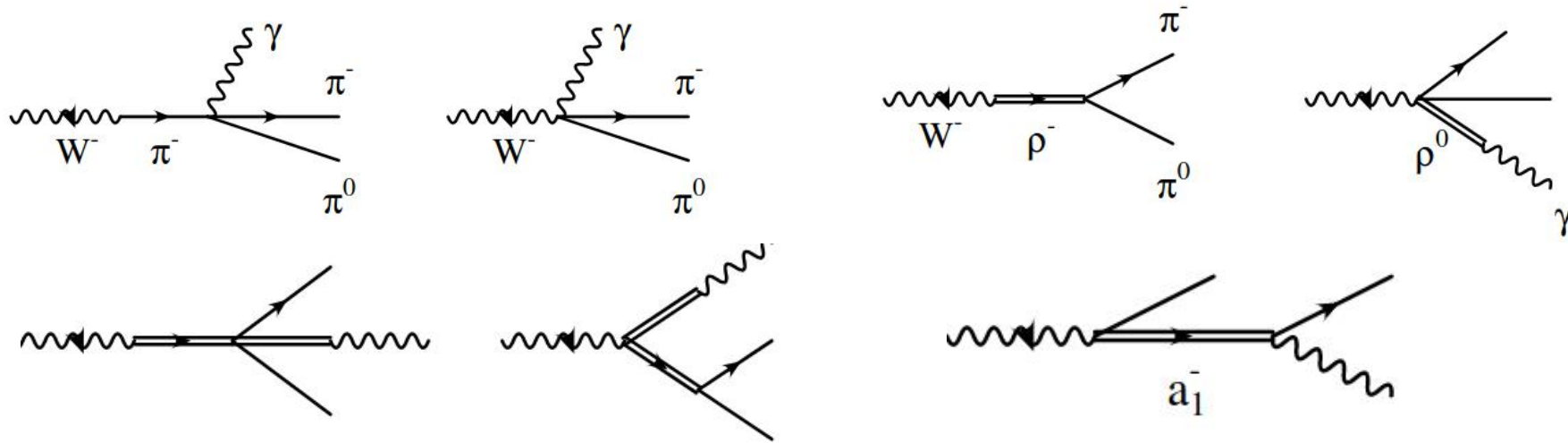
➤ Pro: Strong phase is not necessary for a CPV phenomenon using TPA.

Con: TPA could also be caused by the final-state interactions!

$\tau \rightarrow \pi\pi\gamma\nu_\tau$: good place to probe T-odd triple-product asymmetry

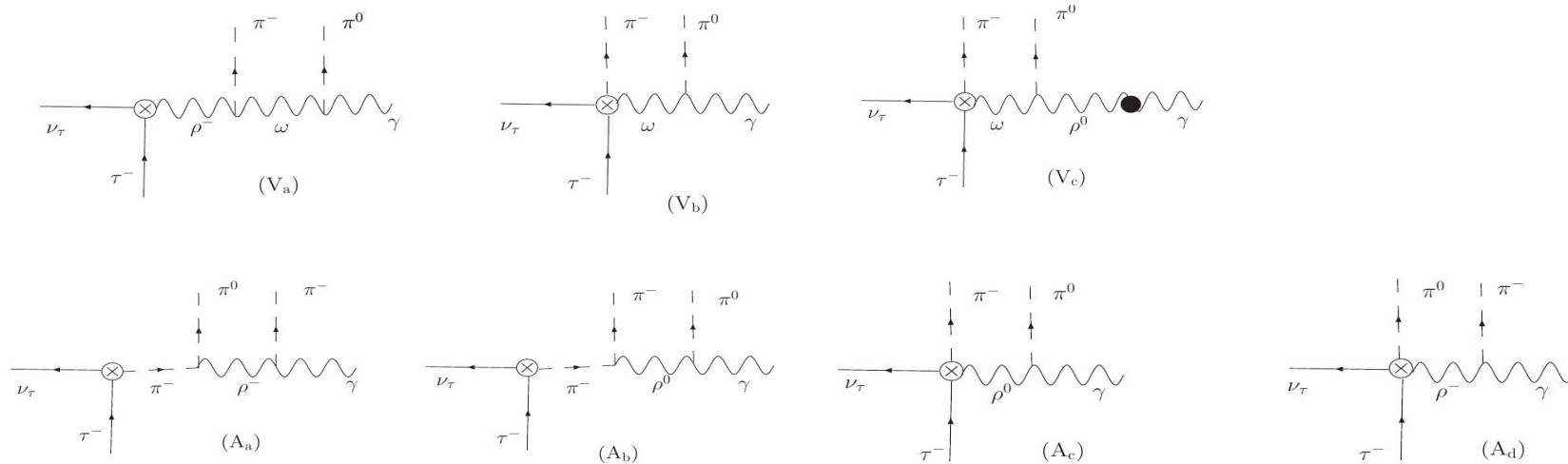
Minimal RChT contributions to $\tau \rightarrow \pi\pi\gamma\nu_\tau$

[Cirigliano et al., JHEP'02]



Contributions from VVP and VJP operators in RChT

[Chen, Duan, ZHG , JHEP'22]



High energy constraints to the resonance couplings

$$\int d^4x \int d^4y e^{i(p \cdot x + q \cdot y)} \langle 0 | T [V_\mu^a(x) V_\nu^b(y) P^c(0)] | 0 \rangle$$

$$= d^{abc} \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta \Pi_{\text{VVP}}(p^2, q^2, r^2),$$

$$\lim_{\lambda \rightarrow \infty} \Pi_{\text{VVP}}^{(8)}[(\lambda p)^2, (\lambda q)^2, (\lambda p + \lambda q)^2]$$

$$= \lim_{\lambda \rightarrow \infty} \Pi_{\text{VVP}}^{(0)}[(\lambda p)^2, (\lambda q)^2, (\lambda p + \lambda q)^2]$$

$$= -\frac{\langle \bar{\psi} \psi \rangle_0}{2\lambda^4} \frac{p^2 + q^2 + r^2}{p^2 q^2 r^2} [1 + \mathcal{O}(\alpha_s)] + \mathcal{O}\left(\frac{1}{\lambda^6}\right).$$

$$c_1 + 4c_3 = 0 \quad c_1 - c_2 + c_5 = 0 \quad c_5 - c_6 = \frac{N_C M_V}{64\sqrt{2}\pi^2 F_V}$$

$$d_1 + 8d_2 = -\frac{N_c M_V^2}{(8\pi F_V)^2} + \frac{F^2}{4F_V^2} \quad d_3 = -\frac{N_c M_V^2}{(8\pi F_V)^2} + \frac{F^2}{8F_V^2}$$

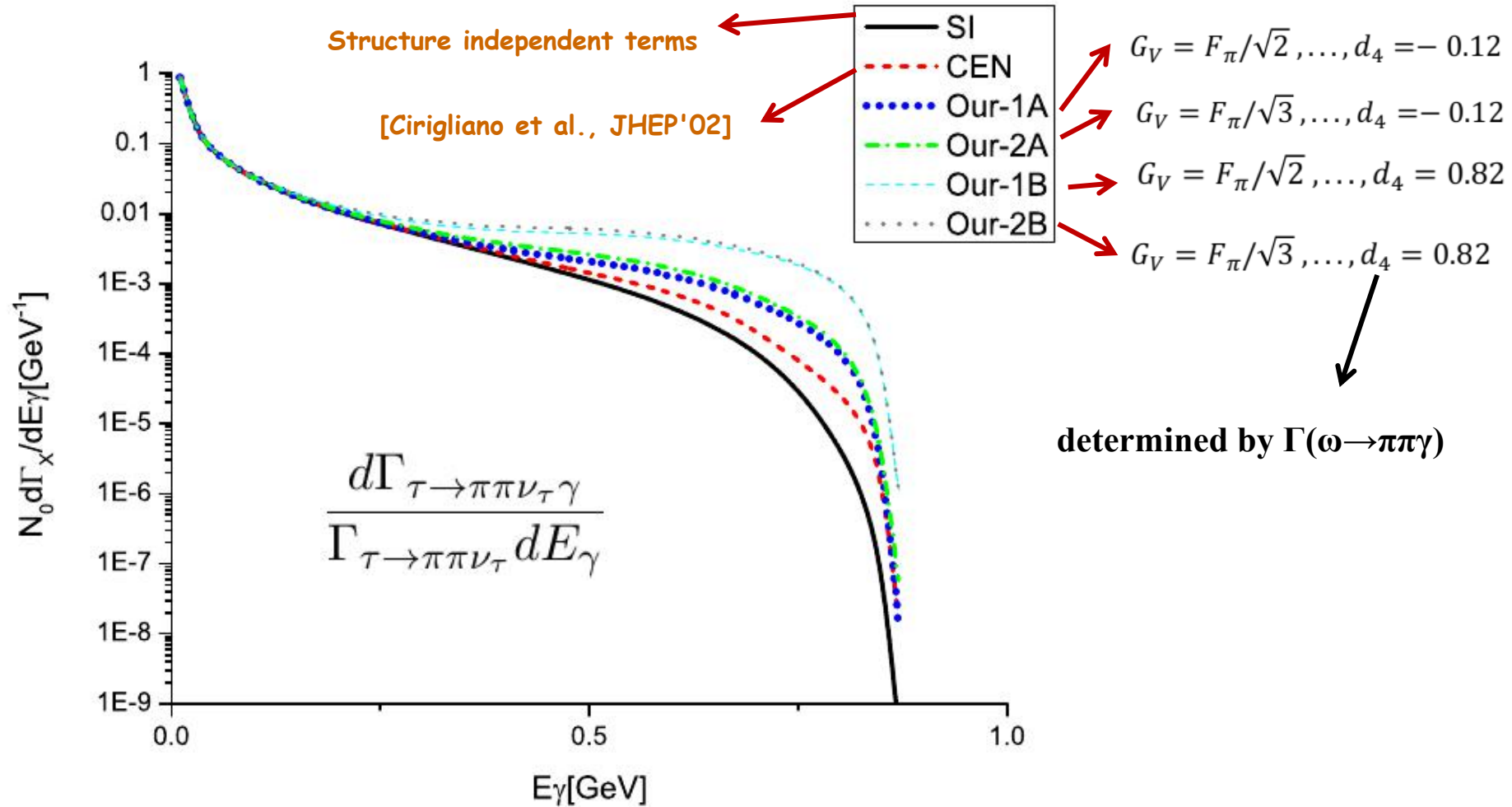
Other constraints from scattering and form factors

$$F_A = F_\pi, \quad F_V = \sqrt{2}F_\pi, \quad G_V = F_\pi/\sqrt{2}.$$

Or

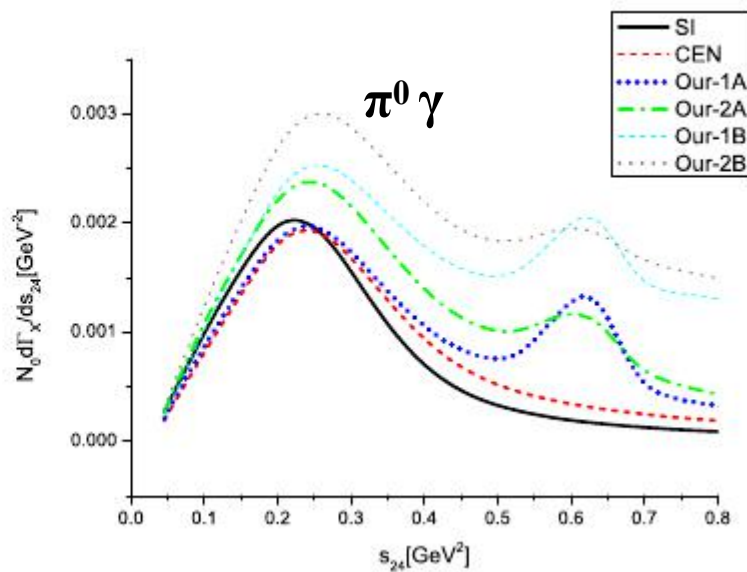
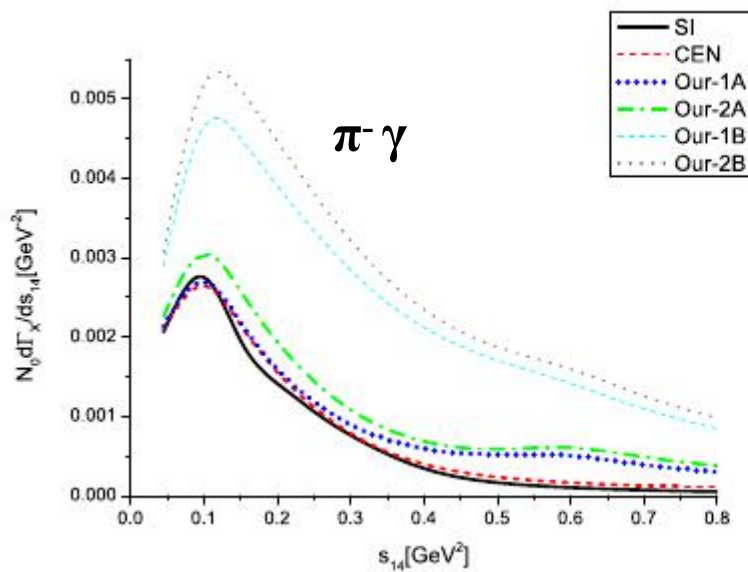
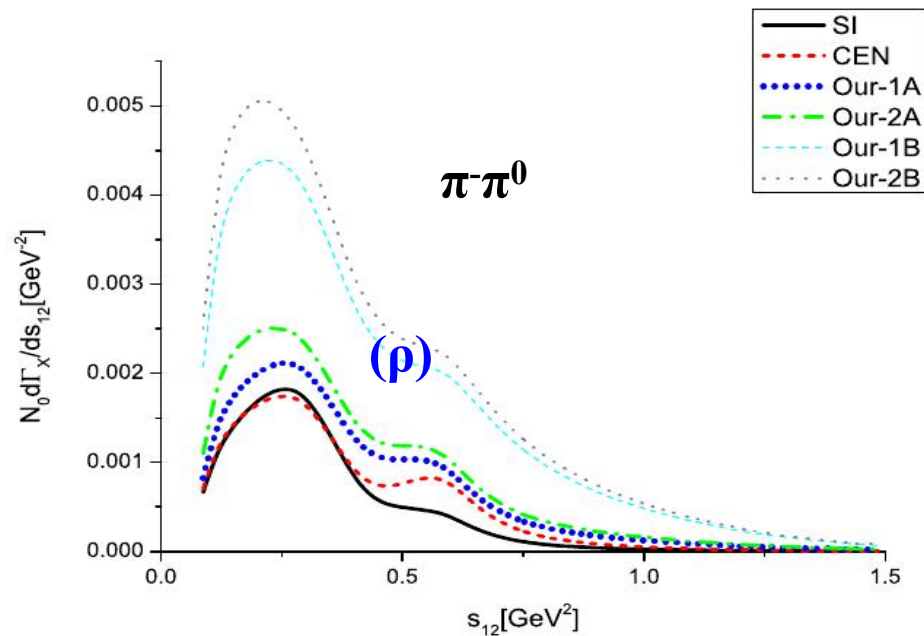
$$F_A = \sqrt{2}F_\pi, \quad F_V = \sqrt{3}F_\pi, \quad G_V = F_\pi/\sqrt{3}$$

Differential decay widths as a function of photon energies



- ❖ When the photon energy cutoff is around 300 MeV, the absolute branching ratio is predicted to be around 10^{-4} and it has a good chance to be well measured at Belle-II, STCF, CEPC,

Invariant-mass distributions of the $\pi\pi/\pi\gamma$ systems

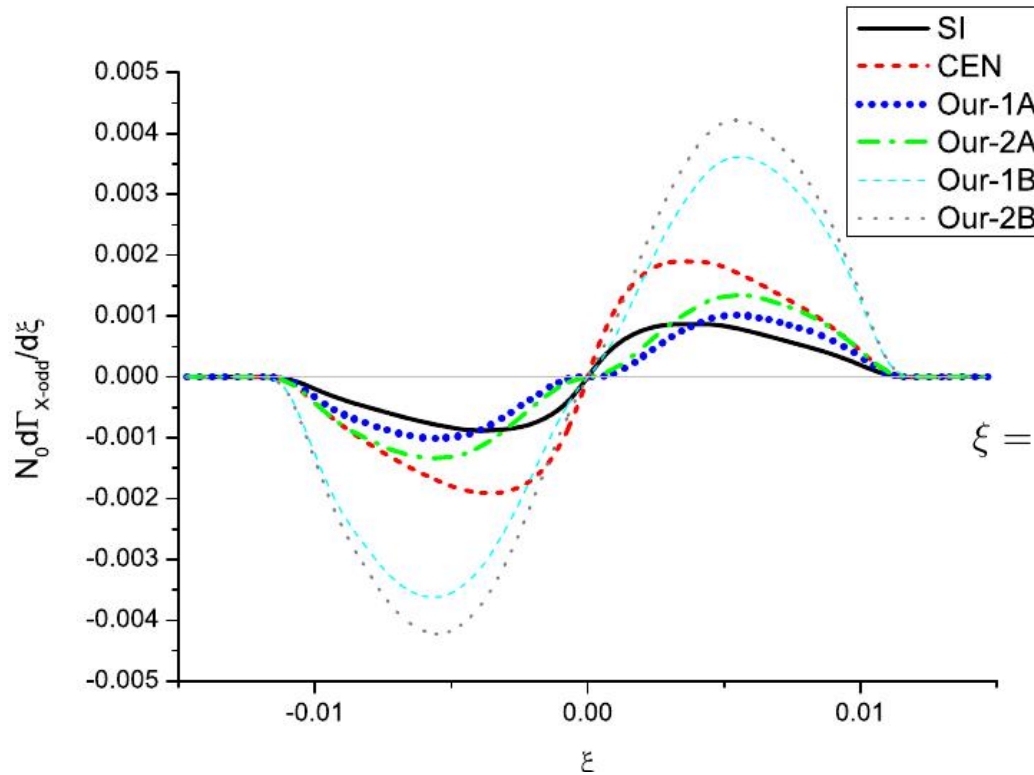


(ρ)

(ρ, ω)

Predictions of the T-odd asymmetry distribution in $\tau \rightarrow \pi\pi\gamma\nu_\tau$

[Chen, Duan, ZHG, JHEP'22]



$$\tau^-(P) \rightarrow \pi^-(p_1)\pi^0(p_2)\nu_\tau(q)\gamma(k)$$

$$\xi = \varepsilon_{\mu\nu\rho\sigma} P^\mu k^\nu p_1^\rho p_2^\sigma / m_\tau^4 \xrightarrow[\text{of } \tau]{\text{rest frame}} \vec{k} \cdot (\vec{p}_1 \times \vec{p}_2) / m_\tau^3$$

E_γ^{cut}	$A_\xi(\text{Our-1A})$	$A_\xi(\text{Our-2A})$	$A_\xi(\text{Our-1B})$	$A_\xi(\text{Our-2B})$
100 MeV	1.2/1.7/1.0/1.6	1.3/1.8/0.98/1.4	1.6/1.7/1.4/1.6	1.7/1.8/1.3/1.4
300 MeV	1.5/2.6/1.0/2.2	1.6/2.5/0.73/1.6	2.3/2.6/2.0/2.2	2.4/2.5/1.7/1.6
500 MeV	0.98/1.4/0.58/0.88	0.91/1.4/0.68/0.43	2.1/1.4/1.8/8.8	2.1/1.4/1.5/4.3

$$A_\xi = \frac{N_+ - N_-}{N_+ + N_-} \quad \text{with} \quad N_\pm = \int_{\xi \gtrless 0} d\Gamma$$

(numbers are multiplied by 10^{-2})

- The magnitudes of A_ξ for $\tau \rightarrow \pi\pi\gamma\nu_\tau$ are around **two orders larger** than those in $K_{l3\gamma}$. It has the good chance to be measured in Belle-II、STCF、CEPC

Prospects of revealing the genuine CPV signals

- CPV signals can be probed by taking the differences of A_ξ in $\tau \rightarrow \pi\pi^0\gamma\nu_\tau$ and $\tau^+ \rightarrow \pi^+\pi^0\gamma\nu_\tau$

$$A_\xi = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} \quad \overline{A}_{\bar{\xi}} = \frac{\overline{\Gamma}_+ - \overline{\Gamma}_-}{\overline{\Gamma}_+ + \overline{\Gamma}_-}$$

$$\overline{\Gamma}_+ = \frac{(2\pi)^4}{2m_\tau} \int_{\bar{\xi} > 0} d\Phi (\overline{\hat{M}}_0 + \bar{\xi} \overline{\hat{M}}_1), \quad \overline{\Gamma}_- = \frac{(2\pi)^4}{2m_\tau} \int_{\bar{\xi} < 0} d\Phi (\overline{\hat{M}}_0 + \bar{\xi} \overline{\hat{M}}_1)$$

$$\mathcal{M} = e G_F V_{ud}^* \epsilon^{*\mu}(k) \left\{ (1 + \mathbf{g}_V) F_\nu \bar{u}(q) \gamma^\nu (1 - \gamma_5) (m_\tau + \not{P} - \not{k}) \gamma_\mu u(P) \right. \\ \left. + [(1 + \mathbf{g}_V) V_{\mu\nu} - (1 - \mathbf{g}_A) A_{\mu\nu}] \bar{u}(q) \gamma^\nu (1 - \gamma_5) u(P) \right\}$$

$$\mathcal{A}_\xi = A_\xi - \overline{A}_{\bar{\xi}} \supset \text{Im}(\mathbf{g}_V^* \mathbf{g}_A) \text{Re}[F_V(t/u)^* A_i], \text{Im}(\mathbf{g}_V^* \mathbf{g}_A) \text{Re}(V_j^* A_i)$$

- Generally speaking, sizable hadronic contributions are also expected to enhance the CPV signals in $\tau \rightarrow \pi\pi\gamma\nu_\tau$.
- TPA in other types of τ decays could be also possible.

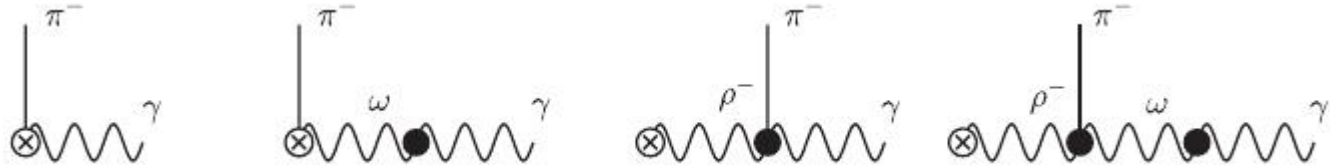
Other predictions:

- $\tau \rightarrow \pi/\mathbf{K} \gamma \nu_\tau$
- polarization effect in $\tau \rightarrow \omega \pi \nu_\tau$ (preliminary)

Predictions to $\tau \rightarrow \pi/K \gamma \nu_\tau$

[ZHG, Roig, PRD'10]

vector currents:

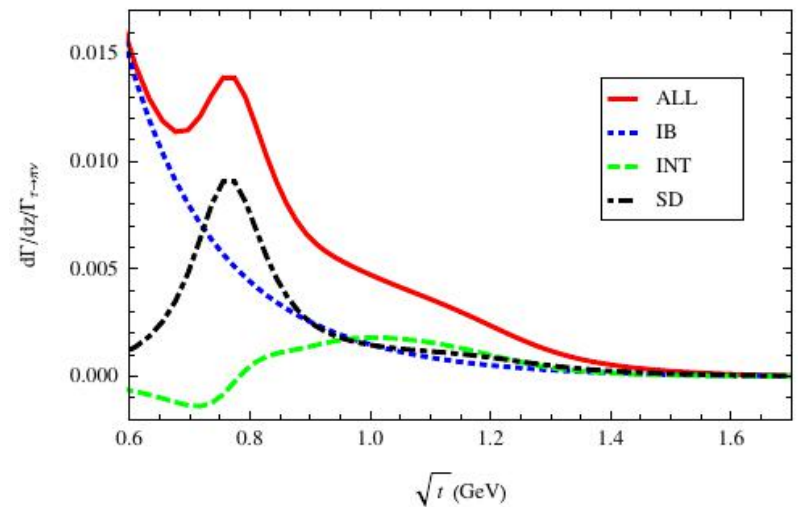


axial-vector currents:



	$E_\gamma=50 \text{ MeV}$	$E_\gamma=400 \text{ MeV}$
<i>IB</i>	13.09×10^{-3}	1.48×10^{-3}
<i>IB - V</i>	0.02×10^{-3}	0.04×10^{-3}
<i>IB - A</i>	0.34×10^{-3}	0.29×10^{-3}
<i>VV</i>	0.99×10^{-3}	0.73×10^{-3}
<i>VA</i>	~ 0	0.02×10^{-3}
<i>AA</i>	0.15×10^{-3}	0.14×10^{-3}
<i>ALL</i>	14.59×10^{-3}	2.70×10^{-3}

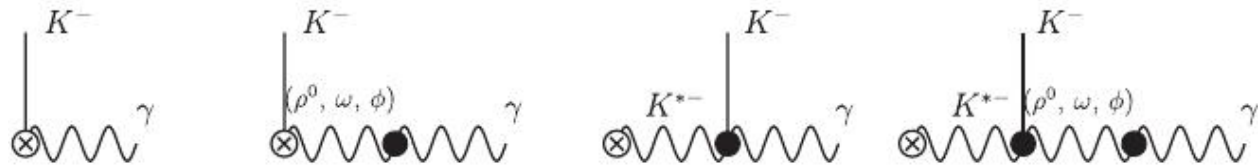
in units of $\text{Br}(\Gamma_{\tau \rightarrow \pi \nu_\tau}) \sim 11\%$



Exp measurement is still absent.

It has a good chance to be measured at BelleII, STCF, CEPC.

vector currents:

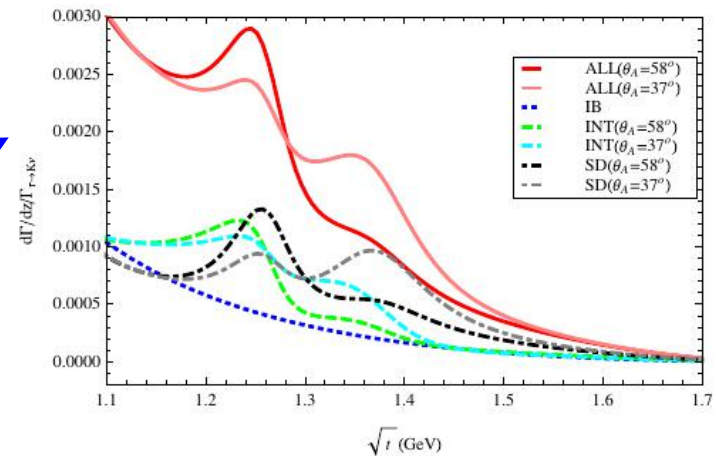
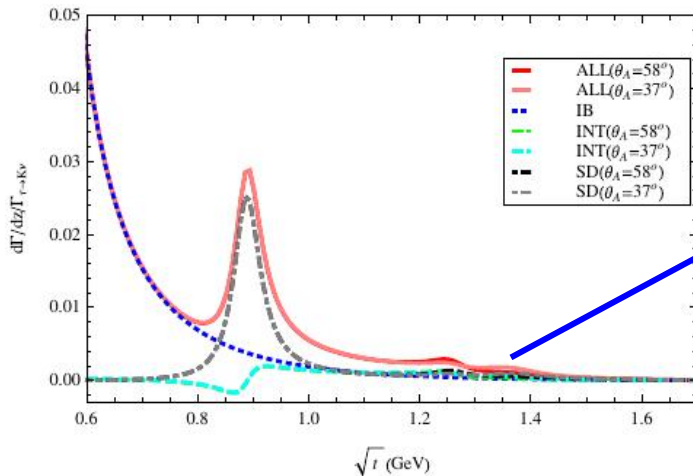


axial-vector currents:



	$E_\gamma=50 \text{ MeV}$ $c_4 = -0.07$ $ \theta_A = 58^\circ(37^\circ)$	$E_\gamma=50 \text{ MeV}$ $c_4 = 0$ $ \theta_A = 58^\circ(37^\circ)$	$E_\gamma=400 \text{ MeV}$ $c_4 = -0.07$ $ \theta_A = 58^\circ(37^\circ)$	$E_\gamma=400 \text{ MeV}$ $c_4 = 0$ $ \theta_A = 58^\circ(37^\circ)$
<i>IB</i>	3.64×10^{-3}	3.64×10^{-3}	0.31×10^{-3}	0.31×10^{-3}
<i>IB - V</i>	0.69×10^{-3}	0.10×10^{-3}	0.83×10^{-3}	0.12×10^{-3}
<i>IB - A</i>	$0.22(0.25) \times 10^{-3}$	$0.22(0.25) \times 10^{-3}$	$0.15(0.18) \times 10^{-3}$	$0.15(0.18) \times 10^{-3}$
<i>VV</i>	58.55×10^{-3}	1.30×10^{-3}	29.04×10^{-3}	0.66×10^{-3}
<i>VA</i>	$\sim 0(\sim 0)$	$\sim 0(\sim 0)$	$0.09(0.09) \times 10^{-3}$	$0.01(0.01) \times 10^{-3}$
<i>AA</i>	$0.13(0.16) \times 10^{-3}$	$0.13(0.16) \times 10^{-3}$	$0.12(0.15) \times 10^{-3}$	$0.12(0.15) \times 10^{-3}$
<i>ALL</i>	$63.23(63.29) \times 10^{-3}$	$5.39(5.45) \times 10^{-3}$	$30.54(30.60) \times 10^{-3}$	$1.37(1.43) \times 10^{-3}$

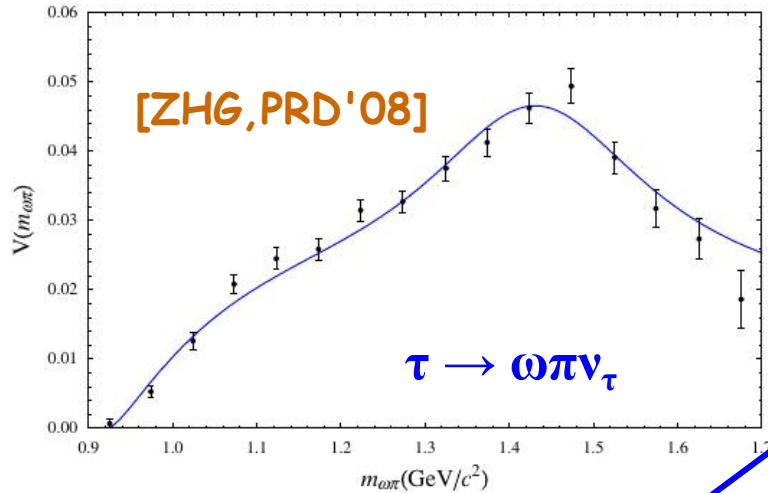
in units of $\text{Br}(\Gamma_{\tau \rightarrow K \nu_\tau}) \sim 0.7\%$



Polarized taus:

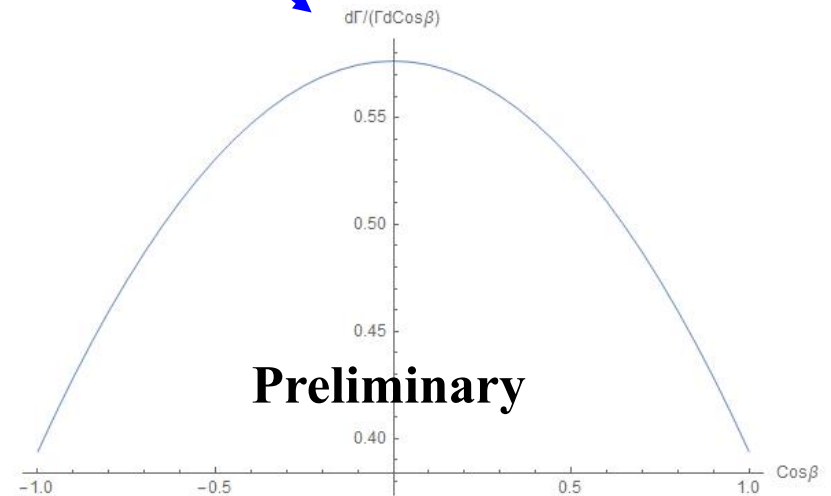
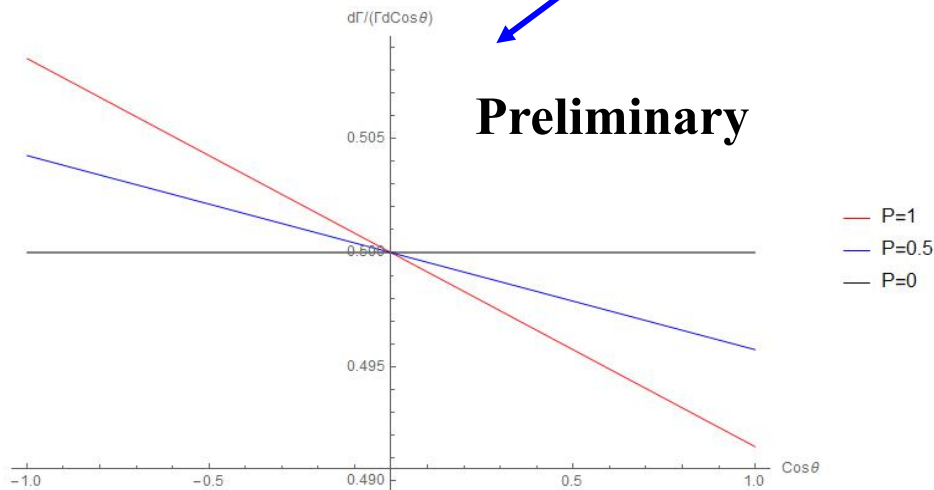
polarized beams are planned at future e^+e^- machines.

$$P_z = \frac{\text{number with spin up} - \text{number with spin down}}{\text{total number}}$$



(θ : the angle between the hadron system and tau spin direction at tau rest frame)

(β : angle between one hadron and the tau at two-hadron rest frame)



Summary

Tau offers a laboratory for a broad range of interesting topics:

- **Precision tests of SM: CKM, α_s , m_τ , lepton universality,**
- **Hadron interactions: light-flavor resonances, chiral symmetry, form factors, second-class currents,**
- **BSM tests:**
 - CPV (rate asym., triple-product asym.)**
 - LFV (lepton/radiative decays, hadron decays)**
 - Neutrino properties**
 -**
- **Resonance chiral theory offers a systematical tool to study the tau decays.**

Thanks for your patience !