中微子核子散射研究

— Studies on neutrino-nucleon scattering —

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第一届中微子散射：理论、实验和唯象研讨会
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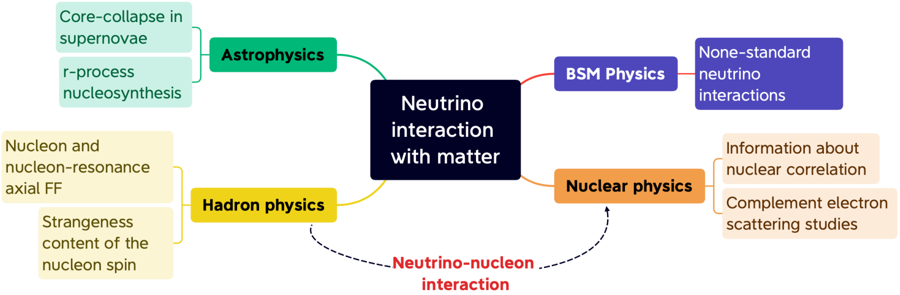
Contents

I. Introduction

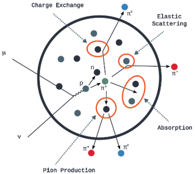
Neutrino interaction with matter

- At the heart of many interesting and relevant physical phenomena

[Neutrinos in particle physics , astronomy and cosmology, Z-Z. Xing and S. Zhou, 2010]



- Neutrino-nucleon scattering:



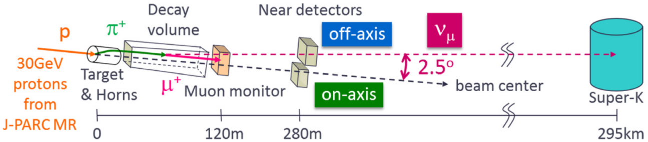
→ a bridge connecting hadron physics and nuclear physics

→Important contribution to the inclusive neutrino-nuclei (νA) cross section

Why neutrino-nucleon scattering?

❑ Oscillation experiments (e.g. T2K)

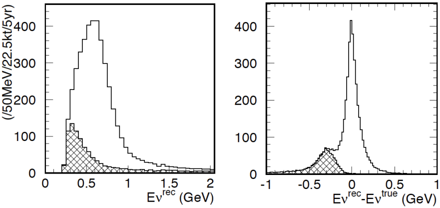
- survival probability of ν_μ: $P(\nu_\mu) = 1 - \sin^2 2\theta_{\mu\tau} \cdot \sin^2 \frac{\Delta m_{23} L}{E_\nu}$



❑ Source of experimental uncertainties

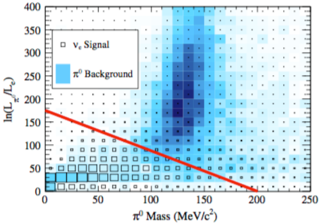
CC 1π:

- ☞ CCQE-like events: misiden. of pion
- ☞ to be subtracted for a good E_ν



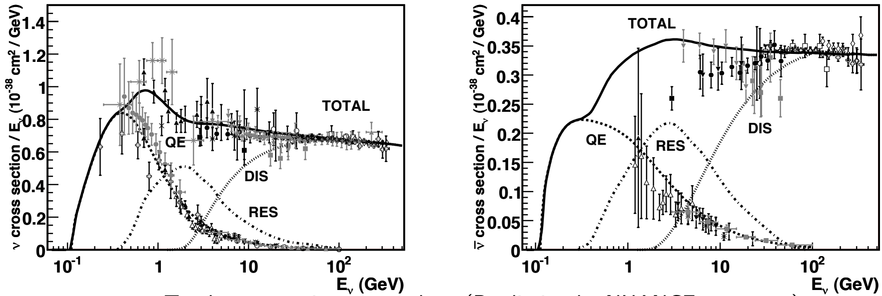
NC 1π:

- ☞ e-like background to $\nu_\mu \to \nu_e$ searches
- ☞ improved at T2K with a π^0 rejection cut



Cross section of neutrino-nucleon scattering

- Two types of processes:
  Charged-Current (CC) & Neutral-Current (NC) induced.
- Different processes in different energy regions
  - **CCQE** (or **NCE**)
  - **RES**: Predominantly $\Delta(1232)$ excitation $\Longrightarrow \Delta \to \pi N$ (99.4%)
  - **DIS**



Total cross section per nucleon (Prediction by NUANCE generator).

Left: ν-CC; Right: $\bar{\nu}$-CC.

II. Neutral current elastic scattering

J. M. Chen, Z. R. Liang and **DLY**, [arXiv:2403.17743 [hep-ph]]

Neutral current elastic scattering

- Low energies:

$$\mathrm{NCE}: \ \nu + N \to \nu + N \ , \quad \mathrm{CCQE}: \ \nu_\ell + N \to \ell + N$$

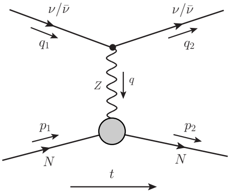
NCE processes are sensitive both to isovector and isoscalar weak current!

- Strangeness contribution to the nucleon spin $\Delta s = G_A^s(Q^2 = 0)$
  - 1980s, the E734 experiment at BNL: $0.45 \leq Q^2 \leq 1.05$ GeV^2
    [Ahrens et al., PRD1985]
  - 2010 & 2015, the MiniBooNE experiment at FNAL: $Q^2 \leq 2$ GeV^2
    [Aguilar-Arevalo et al., PRD2010 & PRD2015)]
  - The future MicroBooNE experiment in Argon: $0.1 \leq Q^2 \leq 1$ GeV^2
    [Ren, JPS Conf. Proc. 37, 020309 (2022).]
- Various parametrizations for form factors
  - Dipole parametrization
  - z expansion
  - ...

Chiral perturbation theory is a model-independent and systematical approach!

Kinematics & amplitude structure

- Kinematics: $\nu(q_1) + N(p_1) \to \nu(q_2) + N(p_2)$



$$\mathcal{M} = -\frac{G_F}{\sqrt{2}} L_\mu H^\mu \ ,$$

Leptonic part: $\quad L_\mu = \bar{\nu}(q_2)\gamma_\mu(1-\gamma_5)\nu(q_1)$,

Hadronic part: $\quad H^\mu = \langle N(p_2)|\mathcal{J}^\mu_{NC}(0)|N(p_1)\rangle$.

- Hadronic amplitude → 6 form factors (FFs)
  - Isospin structure: isovector (V) & isoscalar (S)

$$H^\mu = \chi_f^\dagger \left[\frac{\tau_a}{2} H_V^\mu + \frac{\tau_0}{2} H_S^\mu\right] \chi_i \ , \quad a=3,$$

$$H_V^\mu = (1-2\sin^2\theta_W)V_V^\mu - A_V^\mu \ , \quad H_S^\mu = -2\sin^2\theta_W V_S^\mu \ .$$

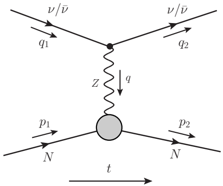
  - Lorentz decomposition:

$$V^\mu_{V,S} = \bar{\mathbf{u}}(p_2)\left[\gamma^\mu F_1^{V,S}(t) + \frac{i}{2m_N}\sigma^{\mu\nu}q_\nu F_2^{V,S}(t)\right]\mathbf{u}(p_1),$$

$$A^\mu_V = \bar{\mathbf{u}}(p_2)\left[\gamma^\mu\gamma_5 G_A(t) + \frac{q^\mu}{m}\gamma_5 G_P(t)\right]\mathbf{u}(p_1) \ .$$

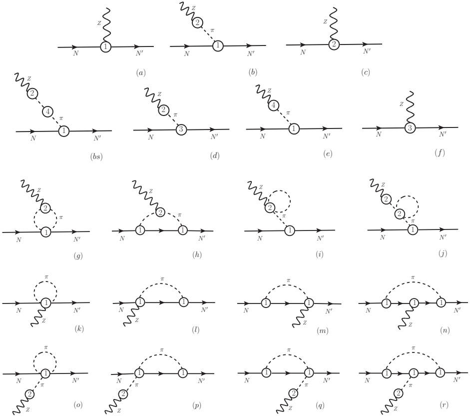
Form factors from BChPT

- Calculation up to $\mathcal{O}(p^3)$



black box

BChPT as a key



Feynman Diagrams

The Lagrangian

- **Covariant BChPT** in SU(2) case.
  - Nucleonic Lagrangian [Fettes et al Ann. Phys. (2000)]

$$\mathcal{L}_N = \bar{N}\left[i\not{D} - m + \frac{g}{2}\not{u}\gamma_5\right]N + \bar{N}\left[c_j\mathcal{O}_j^{(2)} + d_k\mathcal{O}_k^{(3)}\right]N + \cdots$$

  - Purely mesonic Lagrangian [Gasser and Leutwyler, Ann. Phys. (1984)] [Gasser et al., Nucl. Phys. B307 (1988)]

$$\mathcal{L}_\pi = \frac{F^2}{4}\mathrm{Tr}[\Delta_\mu U(\Delta^\mu U)^\dagger + \chi U^\dagger + U\chi^\dagger] + \sum_{j=3,4,6}\ell_j\mathcal{O}_j^{(4)}$$

- **Electro-weak interactions** enter through external fields

[c.f. Scherer and Schindler, 2011, Springer]

  - Charged weak bosons $W^\pm$:

$$r_\mu = 0, \quad l_\mu = -\frac{g}{\sqrt{2}}(V_{ud}W_\mu^+\tau_+ + h.c.)$$

  - Neural weak boson Z^0:

$$r_\mu = e\tan(\theta_W)Z_\mu^0\frac{\tau_3}{2}, \quad l_\mu = -\frac{g}{\cos(\theta_W)}Z_\mu^0\frac{\tau^3}{2} + e\tan(\theta_W)Z_\mu\frac{\tau_3}{2},$$

$$v_\mu^{(s)} = \frac{e\tan(\theta_W)}{2}Z_\mu^0$$

Form factors from BChPT

❑ Form factors in a chiral series

$$
\begin{aligned}
F_1^V(t) =& 1 - 2d_6 t + F_1^{V,\text{loops}} + F_1^{V,\text{wf}} \ , \\
F_2^V(t) =& c_6 + 2d_6 t + F_2^{V,\text{loops}} + F_2^{V,\text{wf}} \ , \\
F_1^S(t) =& 1 - 4d_7 t + F_1^{S,\text{loops}} + F_1^{S,\text{wf}} \ , \\
F_2^S(t) =& (c_6 + 2c_7) + 4d_7 t + F_2^{S,\text{loops}} + F_2^{S,\text{wf}} \ , \\
G_A(t) =& g + (4d_{16} M^2 + d_{22} t) + G_A^{\text{loops}} + G_A^{\text{wf}} \ , \\
G_P(t) =& \frac{2gm_N^2}{M^2 - t} + \frac{4m_N^2 M^2 (2d_{16} - d_{18})}{M^2 - t} + \frac{4gm_N^2 M^2 \ell_4}{F^2(M^2 - t)} - 2m_N^2 d_{22} \\
& - \frac{4gm_N^2 M^2 \left[M^2 \ell_3 + (M^2 - t)\ell_4\right]}{F^2 (M^2 - t)^2} + G_P^{\text{loops}} + G_P^{\text{wf}} \ ,
\end{aligned}
$$

Remarks:

- ☞ Wave function renormalization
- ☞ UV divergences: $\overline{\text{MS}} - 1$ subtraction with dimensional regularizaion
- ☞ Power counting breaking (PCB) issue: EOMS scheme
- ☞ No G_P for NCE

Observables for physical processes

- Differential cross sections

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2}=\frac{G_F^2 m_N^2}{8\pi E_\nu^2}\left[A(Q^2)\pm\frac{(s-u)}{m_N^2}B(Q^2)+\frac{(s-u)^2}{m_N^4}C(Q^2)\right]$$

  - Convenient scalar functions A, B and C: $(\eta=Q^2/4m_N^2)$

$$A(Q^2)\equiv 4\eta\left[\mathcal{G}_A^2(Q^2)\left(1+\eta\right)+4\eta\mathcal{F}_1(Q^2)\mathcal{F}_2(Q^2)-\left(\mathcal{F}_1^2(Q^2)-\eta\mathcal{F}_2^2(Q^2)\right)\left(1-\eta\right)\right]$$

$$B(Q^2)\equiv 4\eta\;\mathcal{G}_A(Q^2)\left(\mathcal{F}_1(Q^2)+\mathcal{F}_2(Q^2)\right)$$

$$C(Q^2)\equiv\frac{1}{4}\left[\mathcal{G}_A^2(Q^2)+\mathcal{F}_1^2(Q^2)+\eta\mathcal{F}_2^2(Q^2)\right]$$

  - Relationship between isospin and physical bases

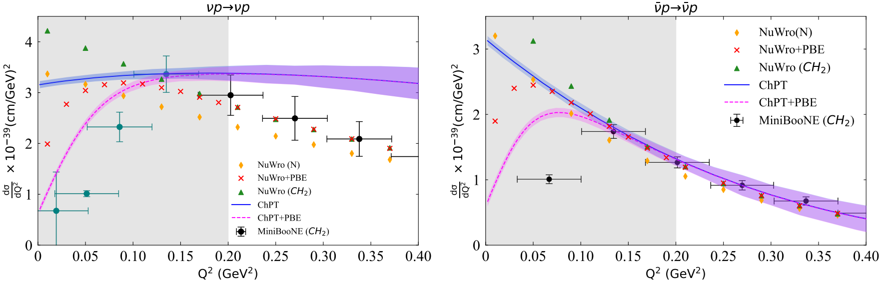
$$\mathcal{F}_i(t)=\cos 2\theta_W F_i^V(t)\frac{\mathcal{C}_3}{2}-2\sin^2\theta_W F_i^S(t)\frac{\mathcal{C}_0}{2}\;,\quad i=1,2,$$

$$\mathcal{G}_j(t)=G_j(t)\frac{\mathcal{C}_3}{2}\;,\quad j=A,P,$$

physical process	$\mathcal{C}_3$	$\mathcal{C}_0$	physical process	$\mathcal{C}_3$	$\mathcal{C}_0$
$\nu+p\to\nu+p$	1	1	$\nu+n\to\nu+n$	-1	1
$\bar\nu+p\to\bar\nu+p$	1	1	$\bar\nu+n\to\bar\nu+n$	-1	1

Differential cross section

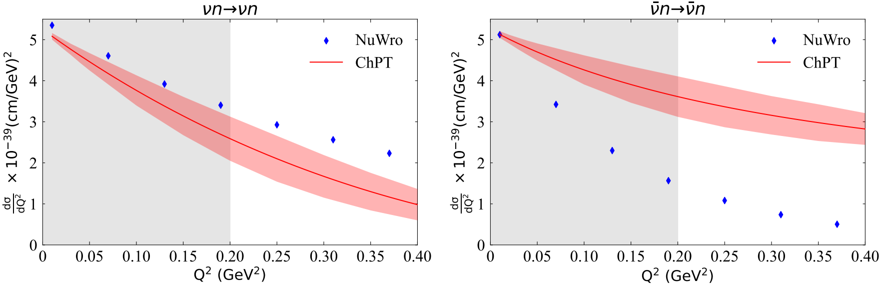
❑ Proton channels



- ☞ Pauli blocking effects
  → estimated by the nuclear models implemented in NuWro
- ☞ Contribution of strangeness axial form factor?

Differential cross section

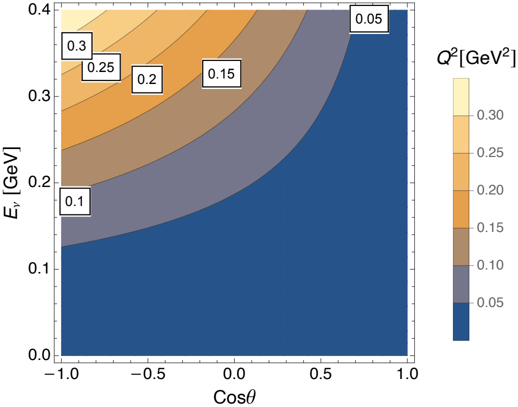
- Neutron channels



- No experimental data
- Large deviation from the NuWro results

Total cross section

- Valid energy region of BChPT

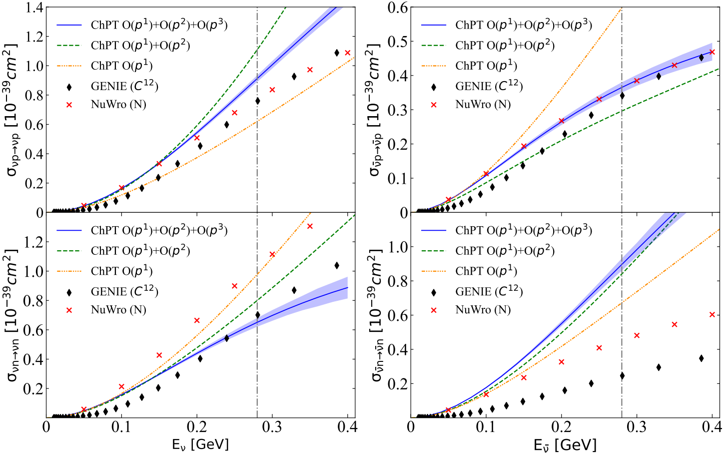


☞ Square of mom. transfer $Q^2 \leq 0.2$ GeV^2 $\longrightarrow$ neutrino energy $E_\nu \leq 0.28$ GeV

$$\boxed{\sigma = \int_{-1}^{+1} \frac{d\sigma}{dQ^2}\frac{dQ^2}{dx}dx} \;, \quad Q^2 = \frac{2m_N E_\nu^2}{2E_\nu + m_N}(1-x), \quad x = \cos\theta \;, \quad \theta \in [0, \pi]$$

Total cross section

❑ Order by order



- ☞ Large difference between NuWro and GENIE due to nuclear effects
- ☞ Our ChPT results deviate from the NuWro ones for neutron channels

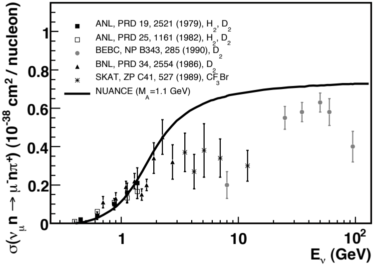
III. Weak pion production

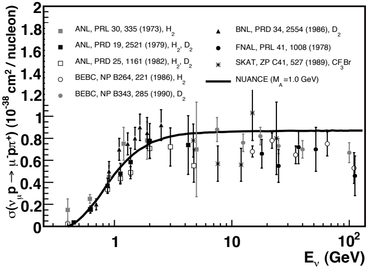
DLY, L. Alvarez-Ruso, A. N. Hiller Blin and M. J. Vicente Vacas, PRD2018

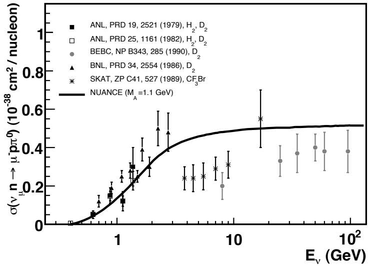
DLY L. Alvarez-Ruso and M. J. Vicente Vacas, PLB2019

Experimental data

- $CC1\pi$





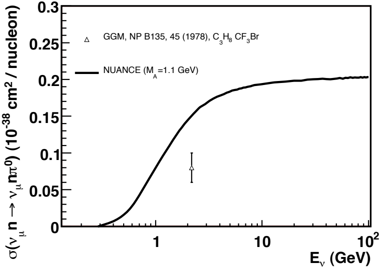


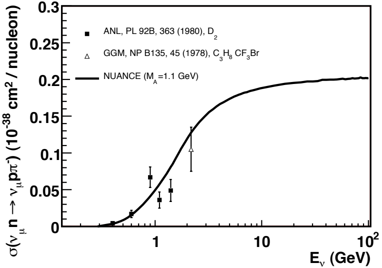
[Formaggio, Zeller, Rev. Mod. Phys. (2012)]

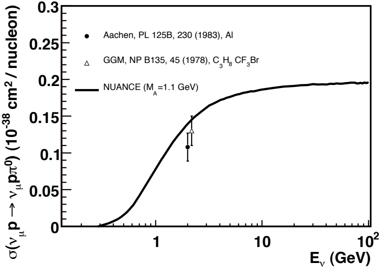
Experimental data

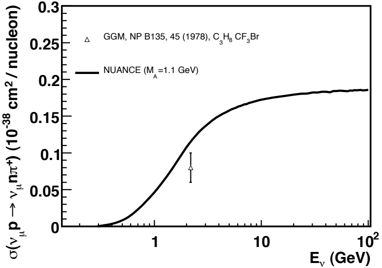
- NC1π: Very rare data below 1 GeV

[Formaggio, Zeller, Rev. Mod. Phys. (2012)]









Status of theoretical studies

❑ Isobar Models

- ☞ Δ and heavier resonances $\rightarrow$ nucleon-to-resonance form factors:

  [e.g., Llewellyn Smith, Phys. Rep. 3 (1972)] [Fogli and Nardulli, Nucl. Phys. B160 (1979)] [Rein and Sehgal, Ann. Phys. (1981)]

  - Real form factor from quark models
  - Conserved vector current $\rightarrow$ related to electromagnetic ones extracted from electron scattering data
  - PCAC $\rightarrow$ off-diagonal Goldberger-Treiman (GT) relation for the axial couplings

- ☞ Nonresonant mechanisms

  [Fogli and Nardulli, Nucl. Phys. B160 (1979)] [Bijtebier, Nucl. Phys. B21 (1970)] [Alevizos et al., J. Phys. G 3(1977)]

❑ Hernandez-Nieves-Valverde (HNV) Model

- ☞ Δ resonances & non-resonant terms $\rightarrow$ constrained by chiral symmetry at threshold [Hernandez, Nieves and Valverde, Phys. Rev. D (2007)]
- ☞ Final state interaction: imposing Watson's theorem [Alvarez-Ruso et al., Phys. Rev. D 93 (2016)]
- ☞ Unphysicsal spin-1/2 components: adding new contact terms

  [Hernandez and Nieves, Phys. Rev. D (2017)]

Status of theoretical studies

- Other Models:
  - ☞ Dynamical model: coupled-channel Lippmann Schwinger equation
    - Fulfilling Watson's theorem
    - PCAC $\rightarrow$ partially constrain the axial current in terms of πN scattering amplitude fitted to data [Nakamura, Kamano and Sato, Phys. Rev. D (2015)]
  - ☞ Chiral effective model with π, N, Δ together with σ, ρ, ω
    - Power counting only for tree diagrams [Serot and Zhang, Phys. Rev. C (2012)]
  - ☞ etc.

- Low energy regime:

  Chiral symmetry + Power counting + Perturbative Unitarity

- Baryon Chiral Perturbation Theory **(BChPT)**
  - ☞ Low-Energy theorems (axial only) at threshold using heavy baryon formalism

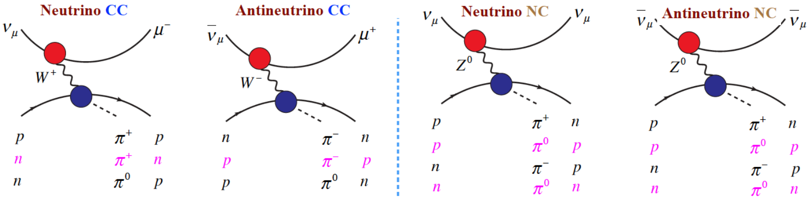
    [Bernard, Kaiser and Meißner, Phys. Lett. B (1994)]
  - ☞ Our work: One-loop analyses in relativistic BChPT with explicit Δs

    [DLY, Alvarez-Ruso, Hiller-Blin and Vicent-Vacas, Phys. Rev. D (2018)]

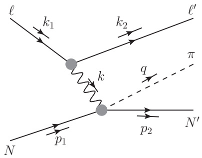
    [DLY, Alvarez-Ruso and Vicent-Vacas, Phys. Lett. B (2019)]

Leptonic and Hadronic parts

- Physical channels (3 for CC & 4 for NC)



- Amplitude structure:
  - One-boson approximation and $k^2 \ll M_B^2$
  - Leptonic part L_ν is well-known; Hadronic part H_μ needs to be investigated.



$$= i(2\pi)^4\delta^{(4)}(k_1+p_1-k_2-p_2-q)\frac{iN^2}{M_B^2}\underbrace{\langle\ell'|J_\nu(0)|\ell\rangle}_{L^\mu}\underbrace{\langle\pi N'|J_\mu(0)|N\rangle}_{H_\mu}$$

Convenient isospin decomposition

- Isospin even (+), isospin odd (-), isoscalar (0)

$$\langle \pi^b N' | J^a_\mu(0) | N \rangle = \chi_f^\dagger \left[\delta^{ba} H^+_\mu + i\epsilon^{bac}\tau^c H^- + \tau^b H^0_\mu \right] \chi_i$$

- The physical amplitudes constructed from the isospin amplitudes

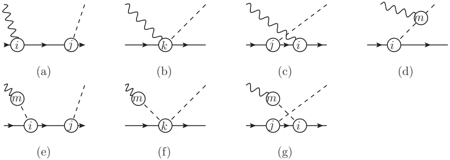
$$H_\mu(\text{physical process}) = a_+ H^+_\mu + a_- H^-_\mu + a_0 H^0_\mu$$

	Physical Process	a_+	a_-	a_0
NC	$Z^0 p \to p\pi^0$	1	0	1
	$Z^0 n \to n\pi^0$	1	0	-1
	$Z^0 n \to p\pi^-$	0	$-\sqrt{2}$	$\sqrt{2}$
	$Z^0 p \to n\pi^+$	0	$\sqrt{2}$	$\sqrt{2}$
CC	$W^+ p \to p\pi^+$ / $W^- n \to n\pi^-$	1	-1	0
	$W^+ n \to n\pi^+$ / $W^- p \to p\pi^-$	1	1	0
	$W^+ n \to p\pi^0$ / $W^- p \to n\pi^0$	0	$\sqrt{2}$	0

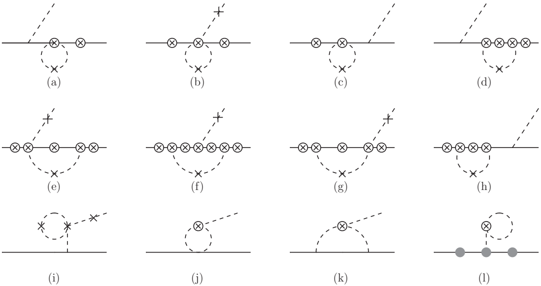
- The CC and NC amplitudes are related to each other
  - For CC, $H^\pm_\mu = \sqrt{2}\cos\theta_C (V^\pm_\mu - A^\pm_\mu)$, $H^0_\mu = 0$.
  - For NC, $H^\pm_\mu = (1 - 2\sin^2\theta_W) V^\pm_\mu - A^\pm_\mu$, $H^0_\mu = (-2\sin^2\theta_W) V^0_\mu$

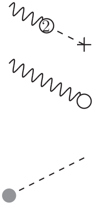
The hadronic amplitude

- Tree diagrams up through $O(p^3)$:



- All possible loop diagrams at $O(p^3)$:





89 diagrams & wave function renormalization & EOMS

Necessity of the Δ resonance

- Δ is strongly coupled to the final πN system
  - BR$(\Delta \to \pi N) \simeq 99.4\%$
  - Close to πN threshold: $\Delta = m_\Delta - m_N \sim 300$ MeV
- Strategy: the δ-counting [Pascalutsa and Phillips, Phys. Rev. C67 (2012)]
  - hierachy of scales: $M_\pi \sim p \ll \Delta \ll \Lambda \sim 4\pi F_\pi$
  - expanding parameter: $\delta = \frac{\Delta}{\Lambda} \sim \frac{M_\pi}{\Delta} \sim \frac{p}{\Delta} \longrightarrow \frac{1}{\not{p} - m_\Delta} = \frac{1}{\not{p} - m_N - \Delta} \sim p^{-\frac{1}{2}}$
- Counting rule:

$$\text{chiral order } D = 4L + \sum_k kV^{(k)} - 2I_\pi - I_N - \frac{1}{2} I_\Delta$$

  - only trees of $O(p^{3/2})$ and $O(p^{5/2})$
  - No loop diagrams with explicit Δ up through $O(p^3)$
- The width effect

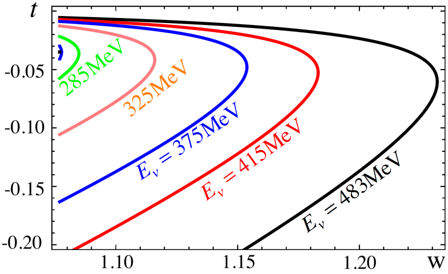
$$\frac{1}{m_\Delta^2 - s_\Delta} \to \frac{1}{m_\Delta^2 - i m_\Delta \Gamma_\Delta(s_\Delta) - s_\Delta}$$

Energy dependent width $\Gamma_\Delta(s_\Delta)$ calculated in the same scheme

[Gegelia et al, Phys. Lett. B(2016)]

Numerical settings

- ❑ Energies considered for $E_\nu \in [E_{\nu,th}, E_{\nu,\max} \equiv E_{\nu,th} + M_\pi]$
  - ☞ E.g., $E_{\nu,\max} = 415$ MeV for CC; $E_{\nu,\max} = 289$ MeV for NC
  - ☞ Well below the Δ peak $\to$ δ-counting is valid



W: CM energy of the final πN system (CC for example)

- ❑ Data for neutrino-induced single pion production off nucleons are very rare
- ❑ Values of the leading order constants

F_π	M_π	m_N	m_Δ	g_A	h_A
92.21	138.04	938.9	1232 MeV	1.27	1.43 ± 0.02

Low energy constants beyond LO

- Most of the LECs (16 out of 23) are previously determined from other processes or observables

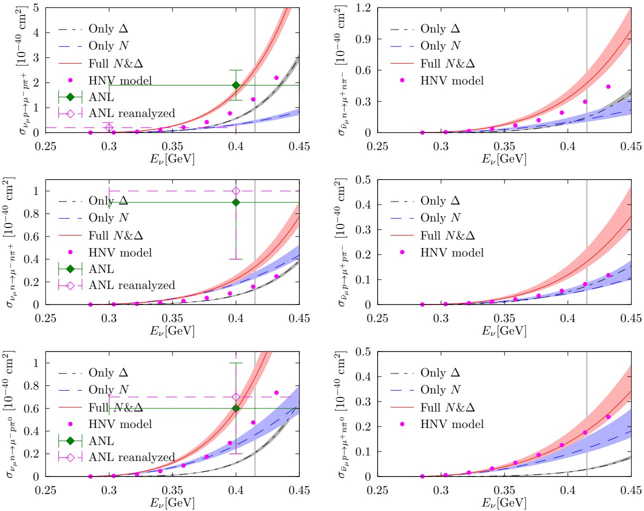
	LEC	Value	Source
$\mathcal{L}_{\pi\pi}^{(4)}$	$\bar{\ell}_6$	16.5 ± 1.1	$\langle r^2 \rangle_\pi$ [Gasser, Leutwyler 1984]
$\mathcal{L}_{\pi N}^{(2)}$	$\tilde{c}_1$	-1.00 ± 0.04	πN scattering [Alarcon et al. 2013 & Chen et al. 2013]
	$\tilde{c}_2$	1.01 ± 0.04	
	$\tilde{c}_3$	-3.04 ± 0.02	
	$\tilde{c}_4$	2.02 ± 0.01	
	$\tilde{c}_6$	1.35 ± 0.04	μ_p and μ_n [Bauer et al. 2012 & PDG2016]
	$\tilde{c}_7$	-2.68 ± 0.08	
$\mathcal{L}_{\pi N}^{(3)}$	d_{1+2}^r	0.15 ± 0.20	πN scattering [Alarcon et al. 2013 & Chen et al. 2013]
	d_3^r	-0.23 ± 0.27	
	d_5^r	0.47 ± 0.07	
	d_{14-15}^r	-0.50 ± 0.50	
	d_{18}^r	-0.20 ± 0.80	
	d_6^r	-0.70	$\langle r_E^2 \rangle_N$ [Fuchs et al. 2014]
	d_7^r	-0.47	
	d_{22}^r	0.96 ± 0.03	$\langle r_A^2 \rangle_N$ [Yao et al. 2017]
$\mathcal{L}_{\pi N\Delta}^{(2)}$	b_1	$(4.98 \pm 0.27)/m_N$	$\Gamma_\Delta^{\rm em}$ [Bernard et al 2012]

- The remaining unknown LECs $\to$ set to natural size

$$d_j^r = 0.0 \pm 1.0\ \text{GeV}^{-2}\ , \quad j \in \{1, 8, 9, 14, 20, 21, 23\}$$

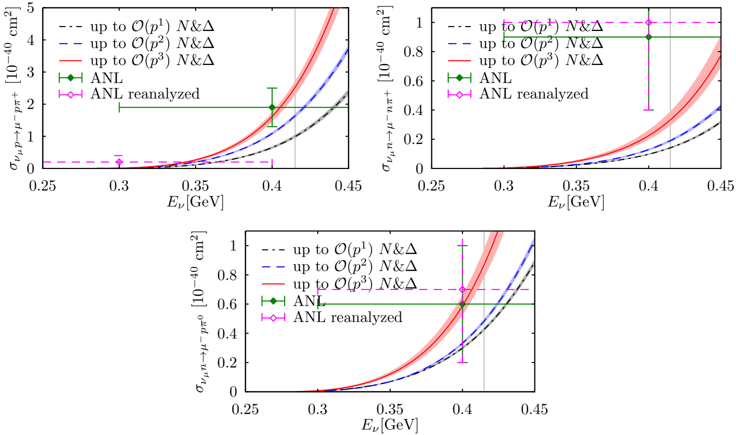
Cross sections for CC1π

- Fairly good agreement with the ANL data for most of the channels except for $\nu_\mu n \to \mu^- n\pi^+$



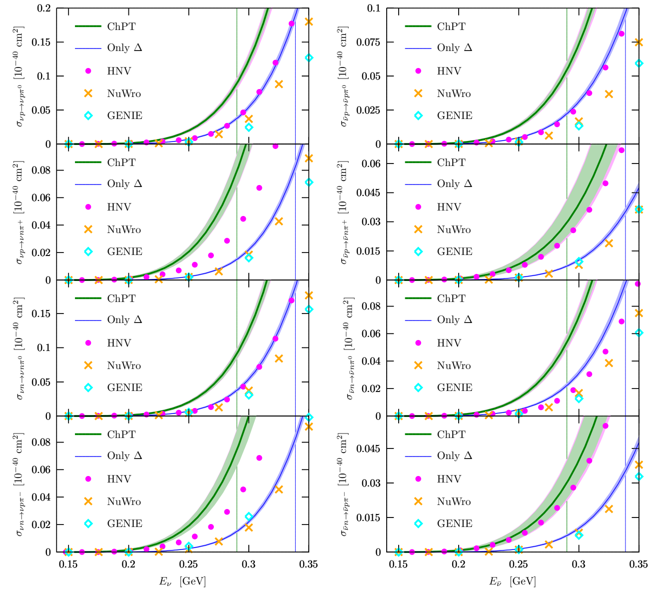
Cross sections for CC1π

- Order by order
  - Quite significant contribution when stepping from $O(p^2)$ and $O(p^3)$
  - Next-order effects could still be relevant (especially loops that πN can be put on-shell)



Cross sections for NC1π

- The $O(p^3)$ ChPT calculation produces considerably larger cross sections with respect to the HNV model in all reaction channels.
- Nuwro and GIENE results agree with the ChPT ones with Δ contribution.
- Non-resonant contribution is sizeable, not accounted by Nuwro and GIENE.



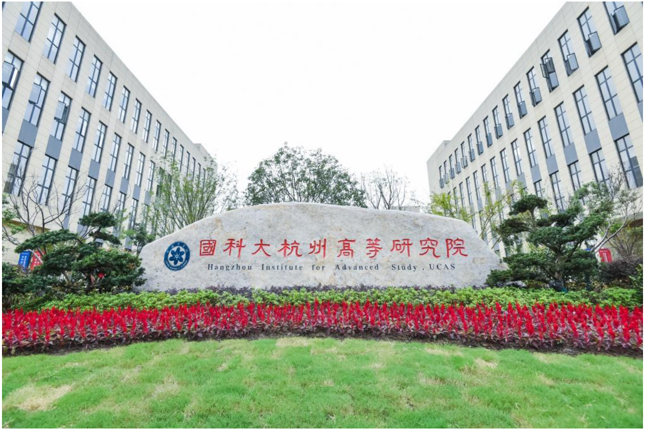
IV. Summary and outlook

Summary and outlook

- Systematically study the **NCE scattering** and **weak single pion production** off the nucleon for the first time within covariant BChPT up to $O(p^3)$.
  - The NCE cross sections are usefull for a precise determination of the strangeness axial vector form factor in future
  - The Δ-mechanism contributes significantly to all production channels
  - NC1π: Non-resonant contribution is sizeable which is not implemented in events generators like NuWro and GIENE

Provide a well-founded low energy benchmark for phenomenological models aimed at the description of weak pion production in the broad kinematic range of interest for current and future neutrino-oscillation experiments.

- Future application and improvement
  - Applied to study various low-energy theorems
  - Neutrion-nuclues scattering
  - Implement ChPT results in events generator?



Thank you very much for your attention!