Generalized Parton Distributions with Polarized Beam and Target

Jianhui Zhang

The Chinese University of Hong Kong, Shenzhen

香港中文大學（深圳）
The Chinese University of Hong Kong, Shenzhen

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Outline

- Introduction and overview
- GPDs from phenomenology
- GPDs from theory
- Summary and outlook

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- GPDs from theory

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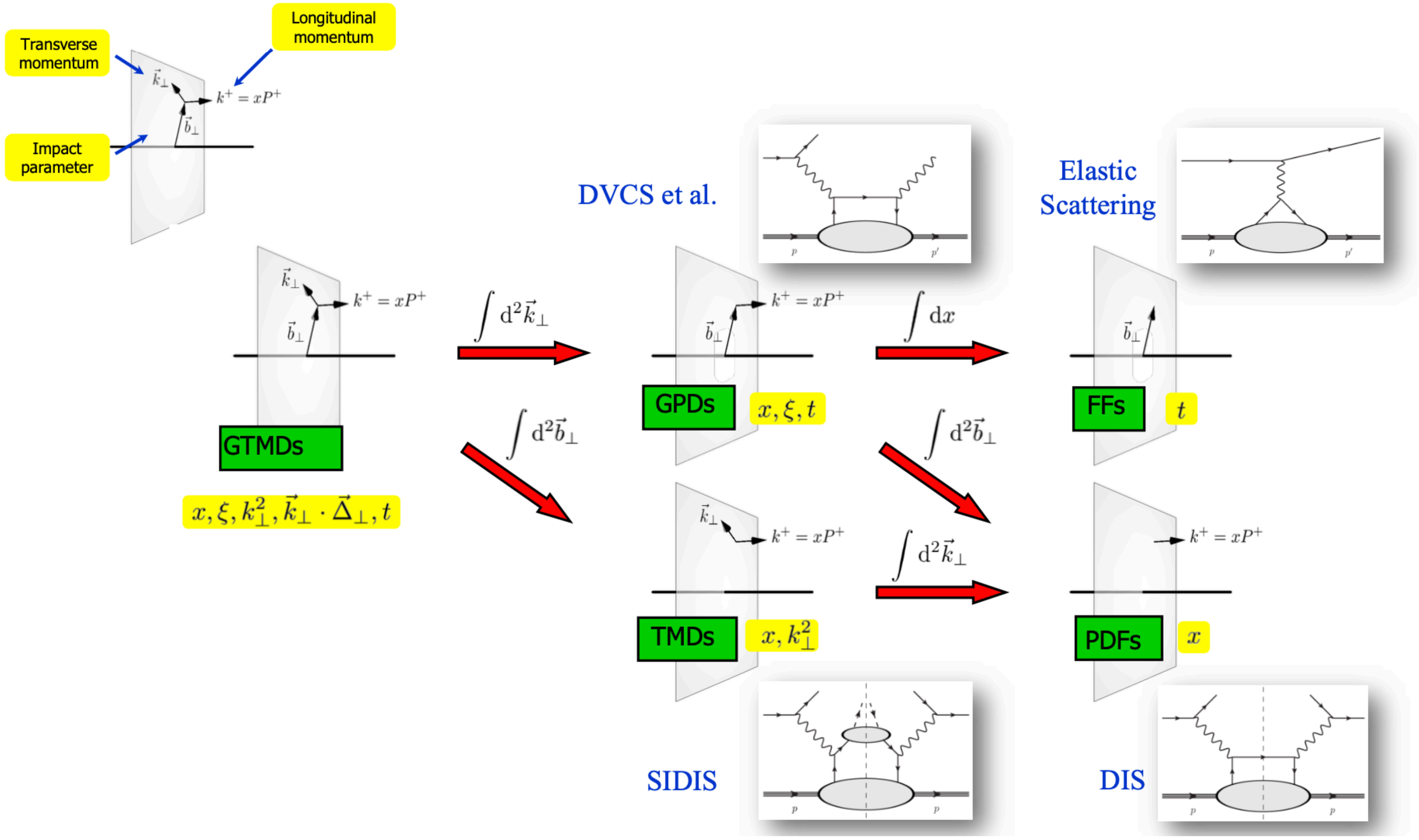
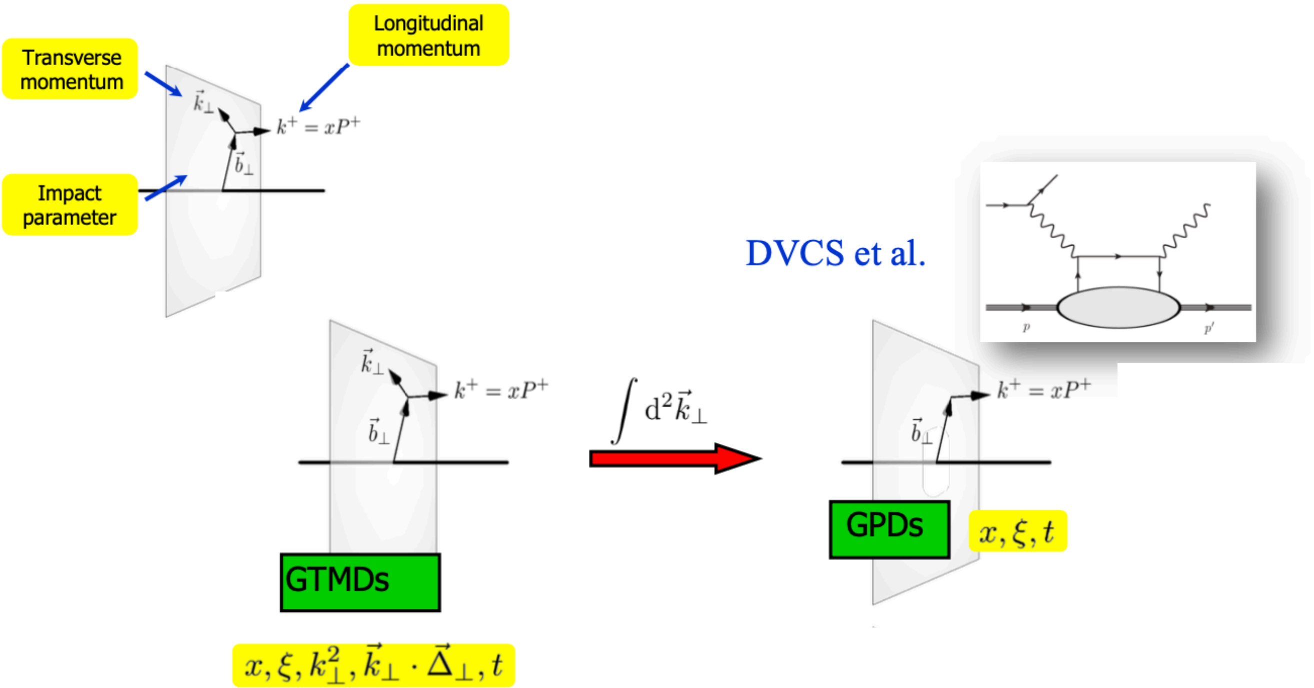
Some materials of this talk are blatantly taken from talks at the RBRC Workshop@BNL, and by P.-J. Lin, S. Niccolai, J. Wagner and others

Toward nucleon tomography



Generalized parton distributions



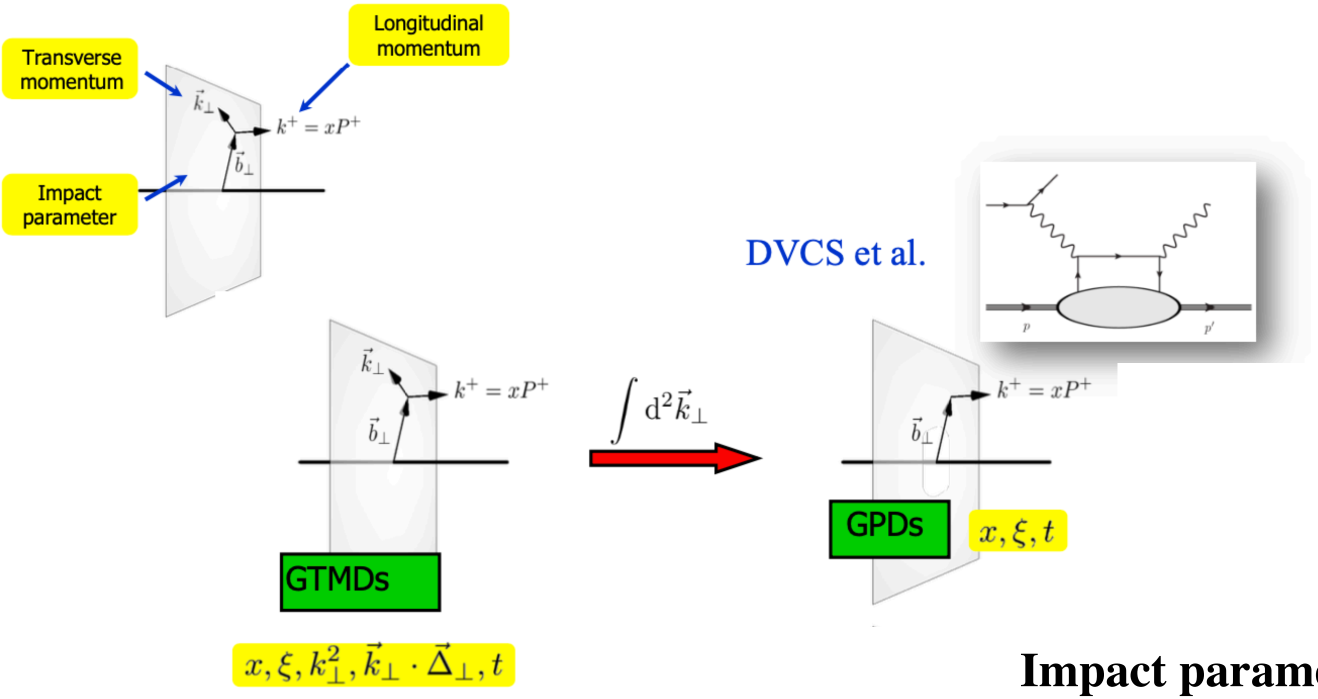
Generalized parton distributions
(Example: Unpol. quark)

$$F(x,\xi,t) = \frac{1}{2\bar{P}^+}\int \frac{d\lambda}{2\pi} e^{-ix\lambda}\langle P' | O_{\gamma^+}(\lambda n) | P\rangle = \frac{1}{2\bar{P}^+}\bar{u}(P')\left[H(x,\xi,t)\gamma^+ + E(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}\right]u(P)$$

$$O_{\gamma^+}(\lambda n) = \bar{\psi}(\frac{\lambda n}{2})\gamma^+ W(\frac{\lambda n}{2}, -\frac{\lambda n}{2})\psi(-\frac{\lambda n}{2}), \quad \bar{P} = \frac{P'+P}{2}, \quad \Delta = P' - P, \quad t = \Delta^2, \quad \xi = -\frac{\Delta^+}{2\bar{P}^+}$$

Mueller et al, FP 94', Ji, PRL 97', Radyushkin, PRD 99'

Generalized parton distributions



Impact parameter distribution

$$\mathsf{q}(x,b) = \int \frac{d\mathbf{q}}{(2\pi)^2} H(x, \xi = 0, t = -\mathbf{q}^2) e^{i\mathbf{q}\cdot\mathbf{b}}$$

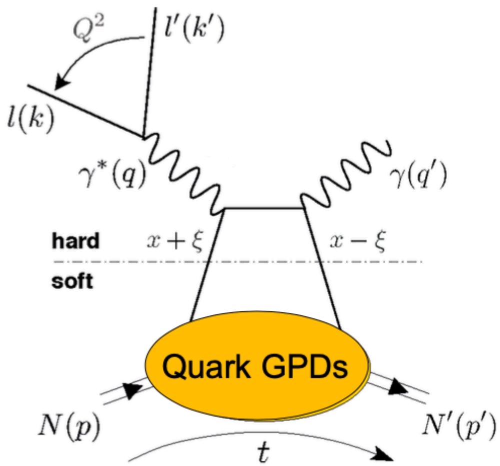
Generalized parton distributions

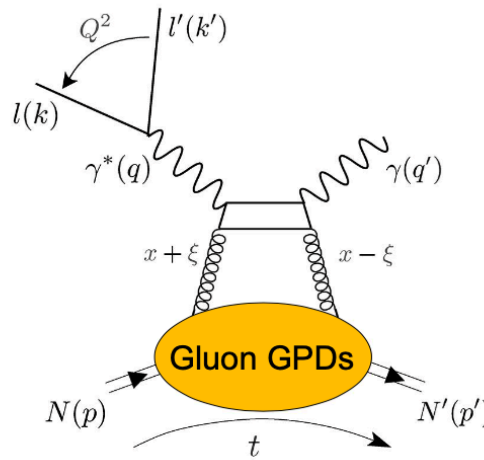
- Link PDFs and FFs
- Correlate transverse coordinate and longitudinal momentum of partons
- Shed light on angular momentum of partons

Angular momentum (Ji sum rule)

$$\mathbf{J_q} = \frac{1}{2}\int_{-1}^{1} dx\, x[H(x,\xi,0) + E(x,\xi,0)]$$

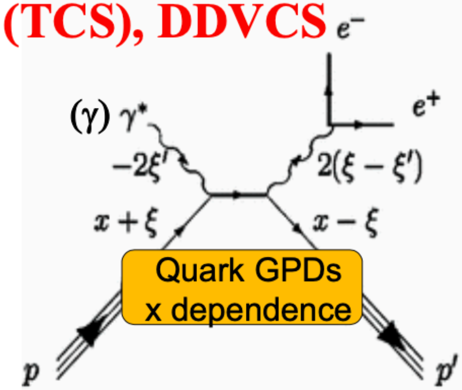
Exclusive reactions giving access to GPDs

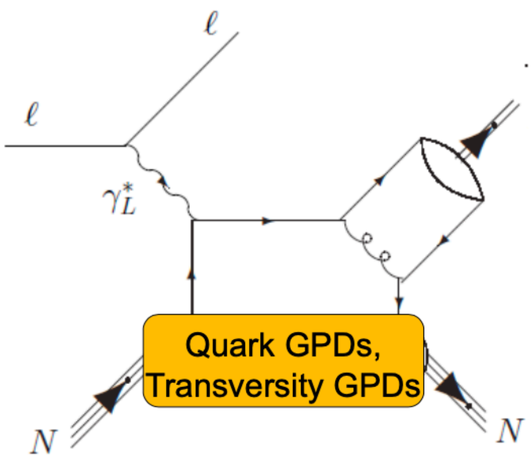




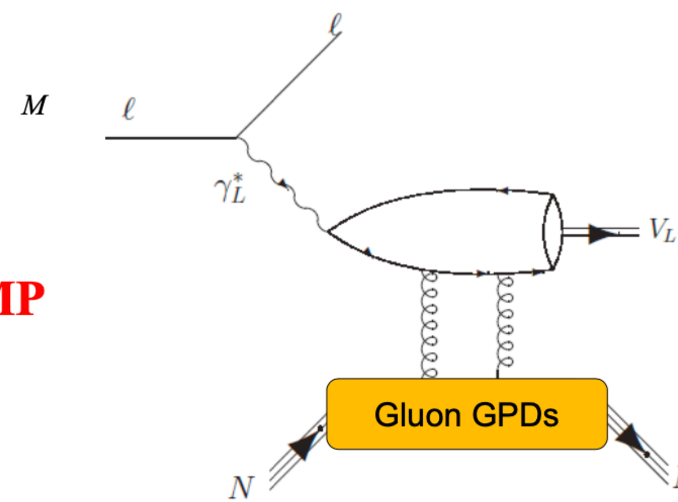
DVCS

(TCS), DDVCS



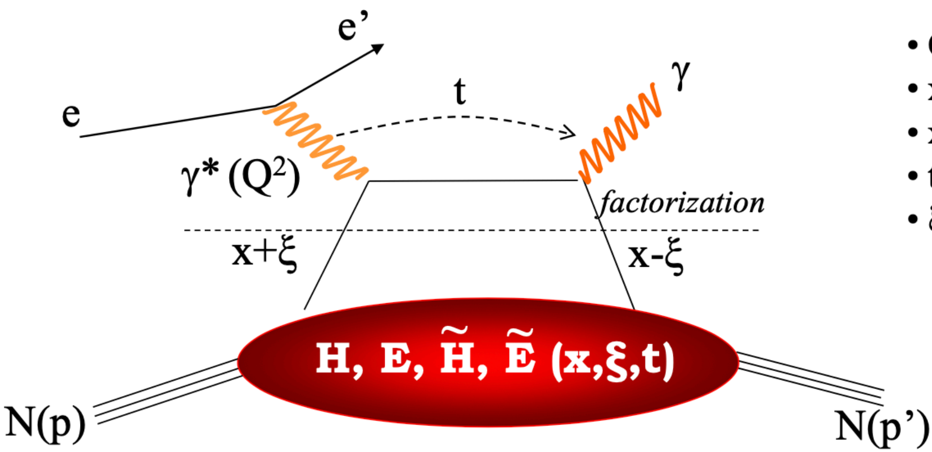


DVMP



Deeply Virtual Compton Scattering (DVCS)
Deeply Virtual Meson Production (DVMP)
Timelike Compton scattering (TCS)
Double DVCS (DDVCS)

GPDs from DVCS



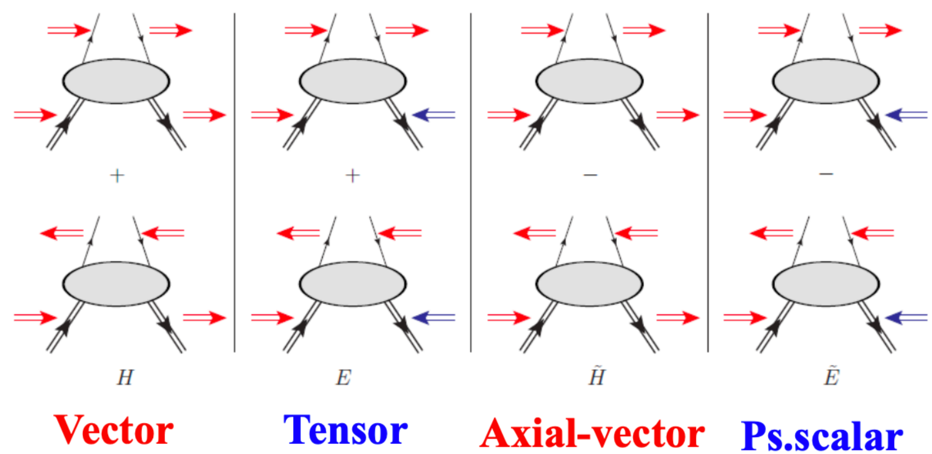
- $Q^2 = -(e-e')^2$
- $x_B = Q^2/2M\nu \quad \nu = E_e - E_{e'}$
- $x+\xi$, $x-\xi$ longitudinal momentum fractions
- $t = \Delta^2 = (p-p')^2$
- $\xi \cong x_B/(2-x_B)$

« **Handbag** » factorization, valid in the **Bjorken regime** (**high Q^2** and ν, fixed x_B), **$t \ll Q^2$**

4 GPDs for each quark flavor
(leading-order, leading twist, quark-helicity conservation)

conserve nucleon spin

flip nucleon spin



Golden channel giving access to 4 chiral-even GPDs

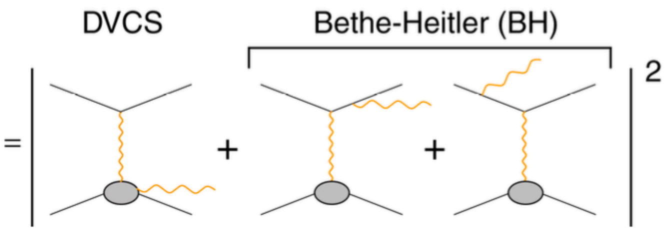
GPDs from DVCS

$$T^{DVCS} \sim P\int_{-1}^{+1} \frac{GPDs(x,\xi,t)}{x \pm \xi} dx \pm i\pi GPDs(\pm\xi,\xi,t) + \ldots$$

$$Re\mathcal{H}_q = e_q^2 P\int_0^{+1} \left(H^q(x,\xi,t) - H^q(-x,\xi,t)\right)\left[\frac{1}{\xi - x} + \frac{1}{\xi + x}\right] dx$$

$$Im\mathcal{H}_q = \pi e_q^2 \left[H^q(\xi,\xi,t) - H^q(-\xi,\xi,t)\right]$$

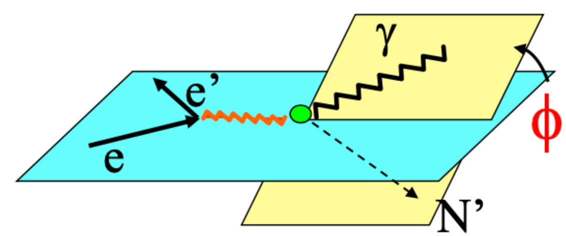
$\sigma(eN \longrightarrow eN\gamma) =$



$$\sigma \sim \left|T^{DVCS} + T^{BH}\right|^2$$

$$\Delta\sigma = \sigma^+ - \sigma^- \propto I(DVCS \cdot BH)$$

	Proton Neutron
Polarized beam, unpolarized target: $\Delta\sigma_{LU} \sim \sin\phi\ Im\{F_1\mathcal{H} + \xi(F_1+F_2)\widetilde{\mathcal{H}} - kF_2\mathcal{E} + \ldots\}$	$Im\{\mathcal{H}_p, \widetilde{\mathcal{H}}_p, \mathcal{E}_p\}$ $Im\{\mathcal{H}_n, \widetilde{\mathcal{H}}_n, \mathcal{E}_n\}$
Unpolarized beam, longitudinal target: $\Delta\sigma_{UL} \sim \sin\phi\ Im\{F_1\widetilde{\mathcal{H}} + \xi(F_1+F_2)(\mathcal{H} + x_B/2\,\mathcal{E}) - \xi kF_2\widetilde{\mathcal{E}}\}$	$Im\{\mathcal{H}_p, \widetilde{\mathcal{H}}_p\}$ $Im\{\mathcal{H}_n, \mathcal{E}_n\}$
Polarized beam, longitudinal target: $\Delta\sigma_{LL} \sim (A + B\cos\phi) Re\{F_1\widetilde{\mathcal{H}} + \xi(F_1+F_2)(\mathcal{H} + x_B/2\,\mathcal{E}) + \ldots\}$	$Re\{\mathcal{H}_p, \widetilde{\mathcal{H}}_p\}$ $Re\{\mathcal{H}_n, \mathcal{E}_n\}$
Unpolarized beam, transverse target: $\Delta\sigma_{UT} \sim \cos\phi\sin(\phi_s - \phi) Im\{k(F_2\mathcal{H} - F_1\mathcal{E}) + \ldots\}$	$Im\{\mathcal{H}_p, \mathcal{E}_p\}$ $Im\{\mathcal{H}_n\}$
Unpolarized beam and target, different lepton charges: $\Delta\sigma_C \sim \cos\phi\ Re\{F_1\mathcal{H} + \xi(F_1+F_2)\widetilde{\mathcal{H}} - kF_2\mathcal{E} + \ldots\}$	$Re\{\mathcal{H}_p, \widetilde{\mathcal{H}}_p, \mathcal{E}_p\}$ $Re\{\mathcal{H}_n, \widetilde{\mathcal{H}}_n, \mathcal{E}_n\}$



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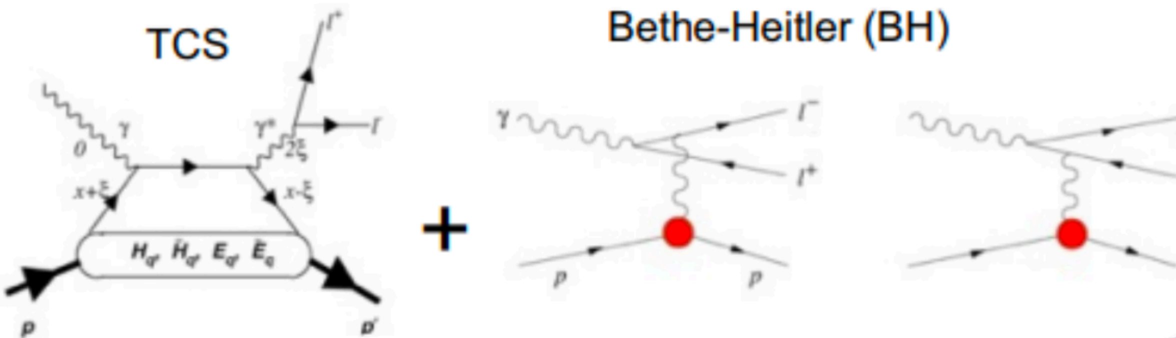
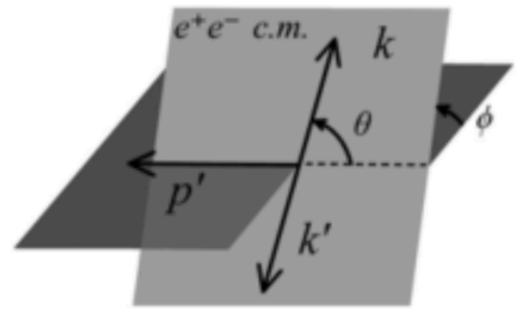
GPDs from TCS

TCS is the time-reversal symmetric process to DVCS:
The incoming photon is real, the outgoing photon is highly virtual and decays in a pair of leptons





$$\frac{d\sigma_{INT}}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = -\frac{\alpha_{em}^3}{4\pi s^2}\frac{1}{-t}\frac{M}{Q'}\frac{1}{\tau\sqrt{1-\tau}}\frac{L_0}{L}\Big[\cos\varphi\frac{1+\cos^2\theta}{\sin\theta}\mathrm{Re}\tilde{M}^{--} - \cos 2\varphi\sqrt{2}\cos\theta\,\mathrm{Re}\tilde{M}^{0-} + \cos 3\varphi\,\sin\theta\,\mathrm{Re}\tilde{M}^{+-} + O\Big(\frac{1}{Q'}\Big)\Big]$$

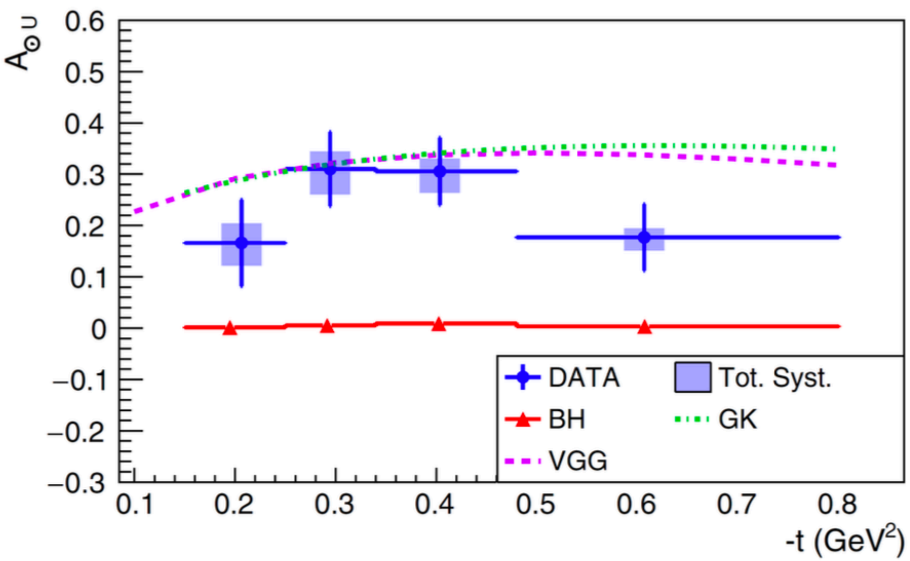
Incoming photon polarization

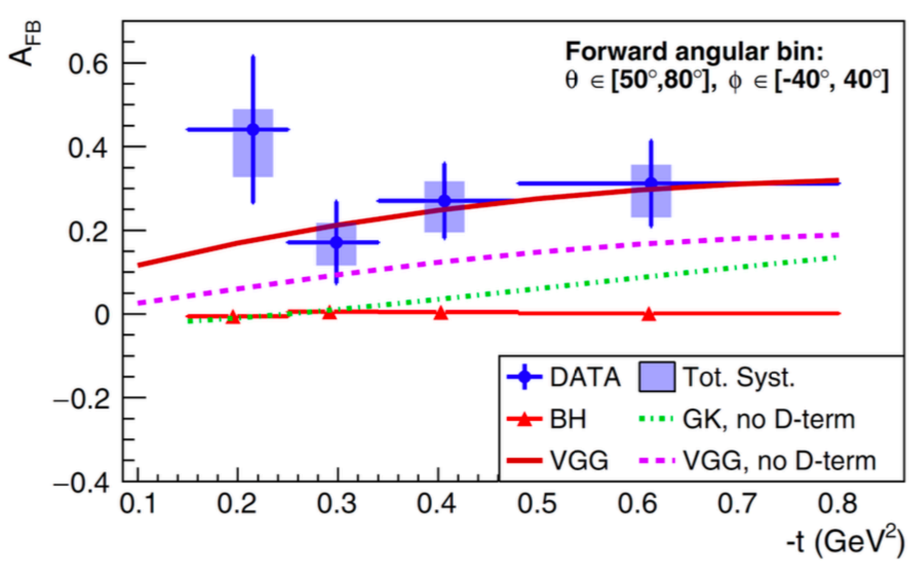
$$-\lambda\frac{\alpha_{em}^3}{4\pi s^2}\frac{1}{-t}\frac{M}{Q'}\frac{1}{\tau\sqrt{1-\tau}}\frac{L_0}{L}\Big[\sin\varphi\frac{1+\cos^2\theta}{\sin\theta}\mathrm{Im}\tilde{M}^{--} - \sin 2\varphi\sqrt{2}\cos\theta\,\mathrm{Im}\tilde{M}^{0-} + \sin 3\varphi\,\sin\theta\,\mathrm{Im}\tilde{M}^{+-} + O\Big(\frac{1}{Q'}\Big)\Big].$$

GPDs from TCS

- First ever Timelike Compton Scattering Measurement at CLAS
  Phys. Rev. Lett. 127, 262501 (2021)
- Photon polarization asymmetry $A_{\odot U} \sim sin\phi \cdot \mathrm{Im}\widetilde{M}^{--}$ → GPD universality
- Forward backward asymmetry $A_{FB} \sim cos\phi \cdot \mathrm{Re}\widetilde{M}^{--}$ → Access D-term

$$\tilde{M}^{--} = \left[F_1 \mathcal{H} - \xi (F_1 + F_2) \tilde{\mathcal{H}} - \frac{t}{4m_p^2} F_2 \mathcal{E} \right]$$

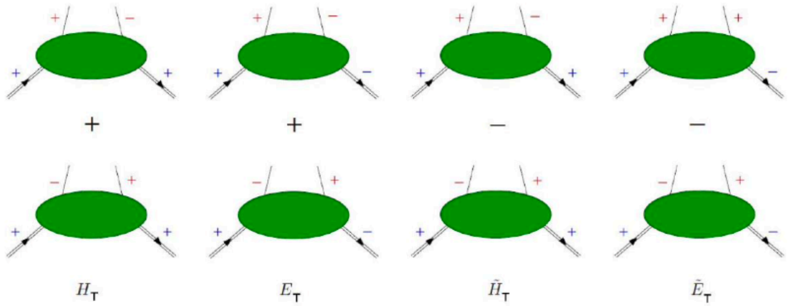




GPDs from beyond DVCS

Chiral-odd GPDs

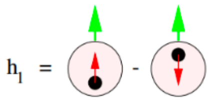
$H_T, \tilde{H}_T, E_T, \tilde{E}_T$



- 4 chiral-odd GPDs (parton helicity flip) at leading twist
- Difficult to access (helicity flip processes are **suppressed**)
- Chiral-odd GPDs are very **little constrained**
- Anomalous tensor magnetic moment:

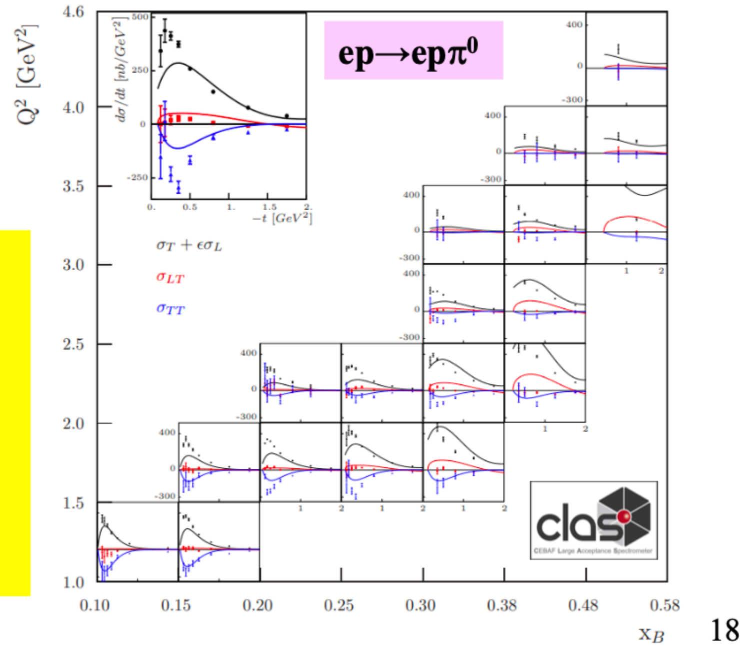
$$\kappa_T = \int_{-1}^{+1} dx\, \bar{E}_T(x,\xi,t=0) \qquad \bar{E}_T = 2\tilde{H}_T + E_T$$

- Link to the **transversity** PDF: $H_T^q(x,0,0) = h_1^q(x)$



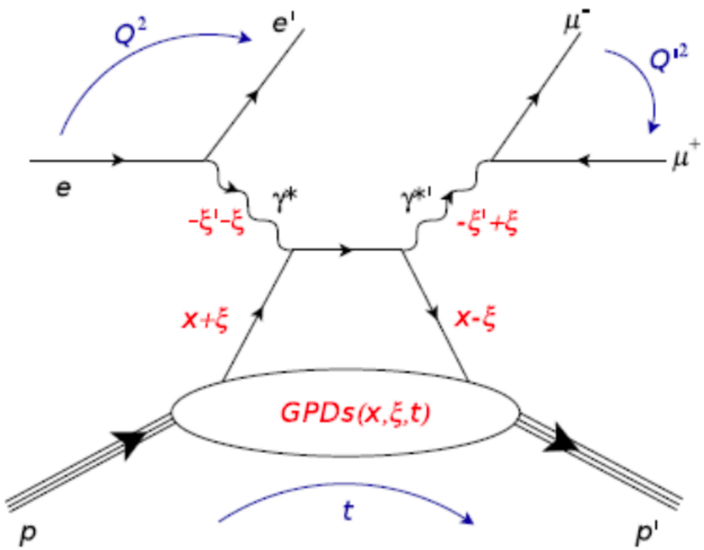
		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	H		$2\tilde{H}_T + E_T$
	L		$\tilde{H}$	$\tilde{E}_T$
	T	E	$\tilde{E}$	$H_T, \tilde{H}_T$

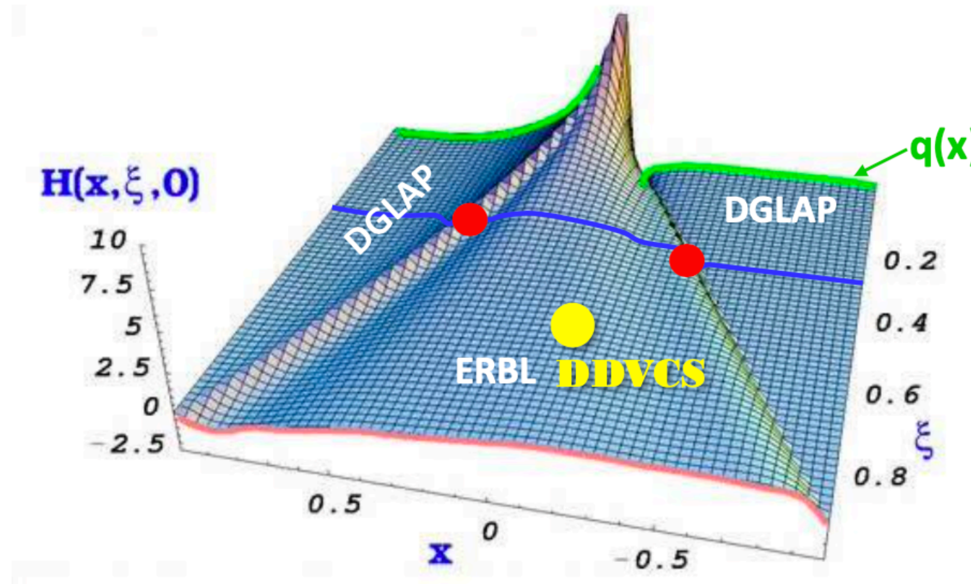
JLab data at 6 GeV (CLAS, Hall A) showed the first evidence of the sensitivity of *exclusive electroproduction of* ***pseudoscalar mesons*** to chiral-odd GPDs



GPDs from beyond DVCS

- DDVCS





Allows to access GPDs at $x \neq \pm\xi$, important for their modeling

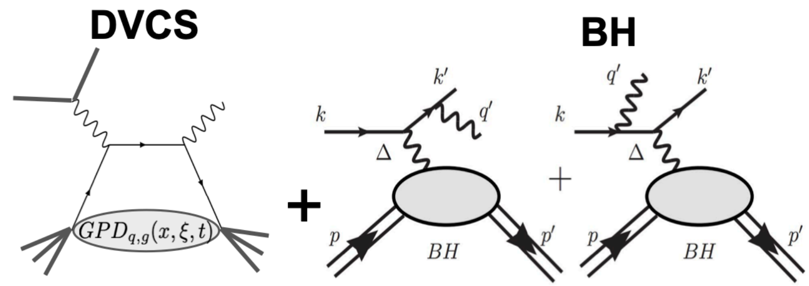
Experimental challenges

- Small cross section (~300 times smaller than DVCS)
- Need to detect muons

GPDs from phenomenology

- Extraction of GPDs from DVCS

$$\sigma = \sigma_{BH} + \sigma_{DVCS} + \sigma_I$$



$$\sigma_{BH}(x_{Bj},t,Q^2,E_b,\phi) = \frac{\Gamma}{t}\left[A_{UU}^{BH}\left(F_1^2+\tau F_2^2\right) + B_{UU}^{BH}\tau G_M^2(t)\right]$$

No CFFs

$$\sigma_{\mathcal{I}}(x_{Bj},t,Q^2,E_b,\phi) = \frac{\Gamma}{Q^2 t}\Big[A_{\mathcal{I}}(x_{Bj},t,Q^2,E_b,\phi)\Big(F_1(t)\Re e\mathcal{H}(x_{Bj},t,Q^2) + \tau F_2(t)\Re e\mathcal{E}(x_{Bj},t,Q^2)\Big)$$
$$+ B_{\mathcal{I}}(x_{Bj},t,Q^2,E_b,\phi)G_M(t)\Big(\Re e\mathcal{H}(x_{Bj},t,Q^2) + \Re e\mathcal{E}(x_{Bj},t,Q^2)\Big)$$
$$+ C_{\mathcal{I}}(x_{Bj},t,Q^2,E_b,\phi)G_M(t)\Re e\widetilde{\mathcal{H}}(x_{Bj},t,Q^2)\Big]$$

Linear CFFs: 3

$$\sigma_{DVCS}(x_{Bj},t,Q^2,E_b,\phi) = \frac{\Gamma}{Q^2}\frac{2}{1-\epsilon}\Big[(1-\xi^2)\Big[(\Re e\mathcal{H})^2 + (\Im m\mathcal{H})^2 + (\Re e\widetilde{\mathcal{H}})^2 + (\Im m\widetilde{\mathcal{H}})^2\Big]$$
$$+ \frac{t_o - t}{4M^2}\Big[(\Re e\mathcal{E})^2 + (\Im m\mathcal{E})^2 + \xi^2(\Re e\widetilde{\mathcal{E}})^2 + \xi^2(\Im m\widetilde{\mathcal{E}})^2\Big]$$
$$- 2\xi^2\Big(\Re e\mathcal{H}\,\Re e\mathcal{E} + \Im m\mathcal{H}\Im m\mathcal{E} + \Re e\widetilde{\mathcal{H}}\,\Re e\widetilde{\mathcal{E}} + \Im m\widetilde{\mathcal{H}}\Im m\widetilde{\mathcal{E}}\Big)\Big]$$

Quadratic CFFs: 8

Very complicated to disentangle all these pieces

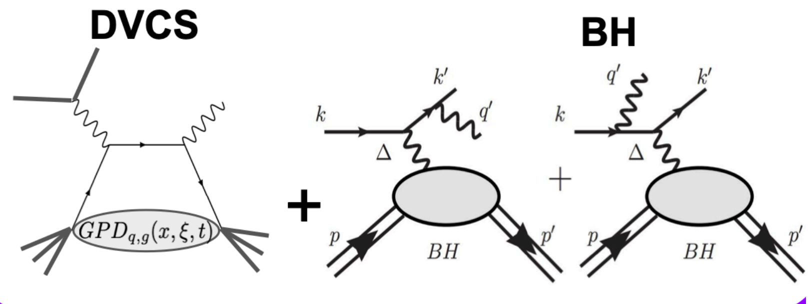
- Moreover,

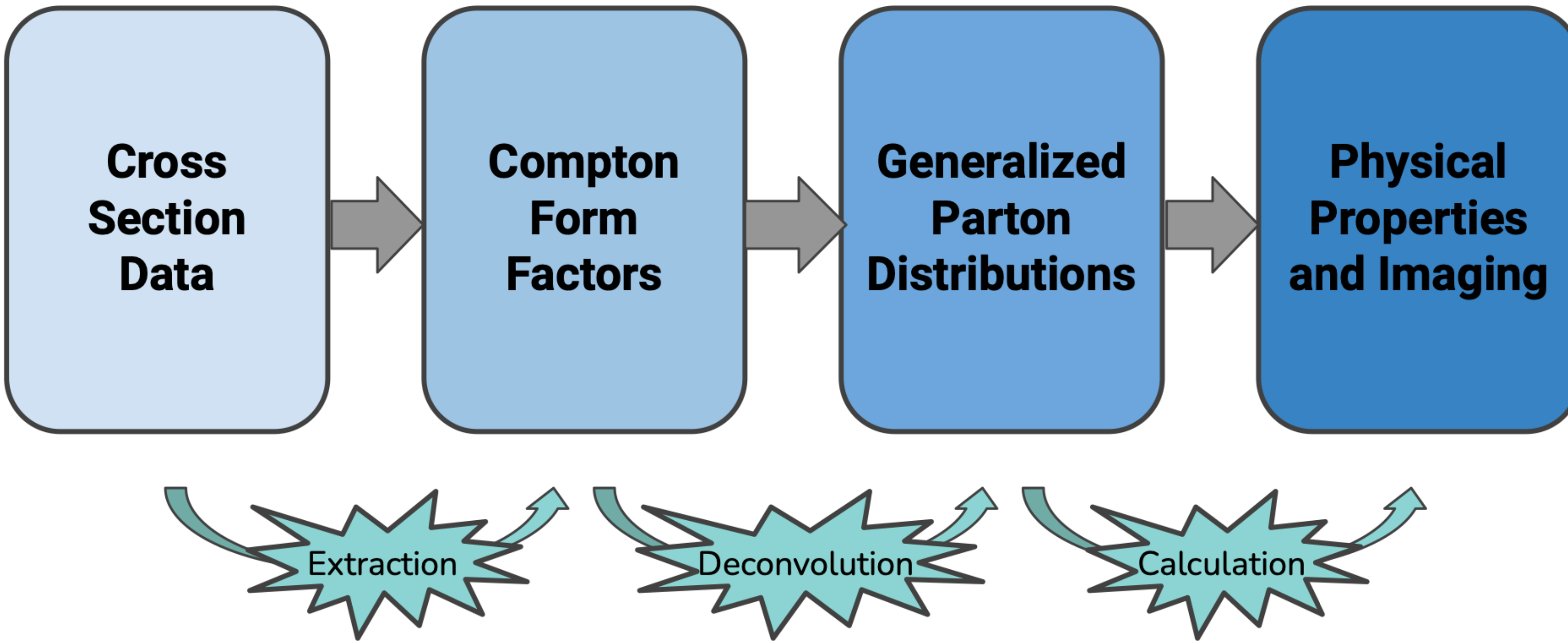
$$\mathcal{H}\left(\xi,t,Q^2\right) = \int_{-1}^{1}\frac{\mathrm{d}x}{\xi}\sum_{a=g,u,d,\ldots} C^a\left(\frac{x}{\xi},\frac{Q^2}{\mu_F^2},\alpha_S\left(\mu_F^2\right)\right)H^a\left(x,\xi,t,\mu_F^2\right)$$

GPDs from phenomenology

- Extraction of GPDs from DVCS

$$\sigma = \sigma_{BH} + \sigma_{DVCS} + \sigma_I$$





GPDs need to be parametrized

GPDs from phenomenology

- GPD parametrizations are subject to many physical constraints
  - Polynomiality property
  - Positivity constraint
  - Forward limit of certain GPDs reduces to PDFs
  - Dispersion relations between CFFs
  - Evolution at the perturbative scale

- Despite that, various models for GPD parametrization have been proposed for their extraction from experimental data **Kumericki et al, EPJA 16'**
  - Double distributions
  - Light-front wave functions
  - Conformal moments
  - …

GPDs from phenomenology

- Example of parametric fit **Moutarde et al, EPJC 18'**
  - Border function:
    For the GPDs H^q and $\widetilde{H}^q$ at $\xi = 0$ we use an Ansatz that is commonly used in phenomenological analyses of GPDs:

$$G^q(x,0,t) = \mathrm{pdf}_G^q(x)\ \exp(f_G^q(x)t)\ .$$

The profile function, $f_G^q(x)$, fixes the interplay between the x and t variables, and it is given by:

$$f_G^q(x) = A_G^q \log(1/x) + B_G^q(1-x)^2 + C_G^q(1-x)x\ ,$$

  - Skewness function:

$$g_G^q(x,\xi,t) = \frac{G^q(x,\xi,t)}{G^q(x,0,t)}\ ,$$

In our case:

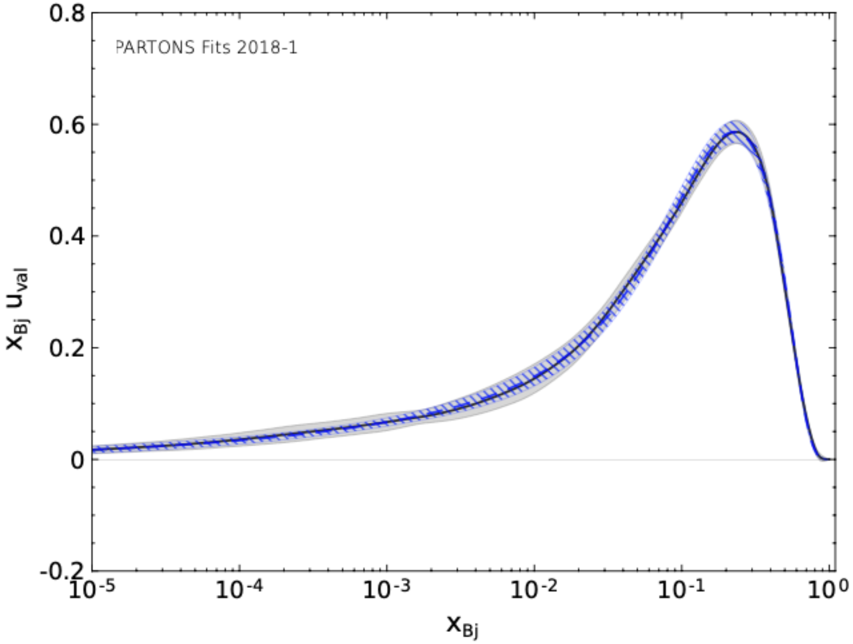
$$G^q(x,x,t) = G^q(x,0,t)\ g_G^q(x,x,t)\ ,$$

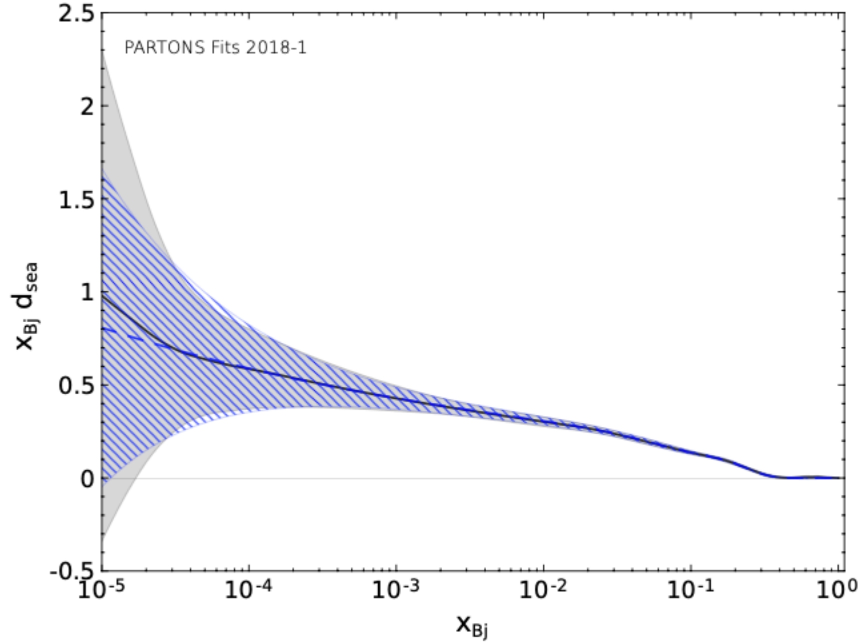
We assume the following form (suggested by F. Yuan, Phys. Rev. D69)

$$g_G^q(x,x,t) \equiv g_G^q(x,t) = \frac{a_G^q}{(1-x^2)^2}\,(1 + t(1-x)(b_G^q + c_G^q \log(1+x)))\ ,$$

GPDs from phenomenology

- Example of parametric fit **Moutarde et al, EPJC 18'**

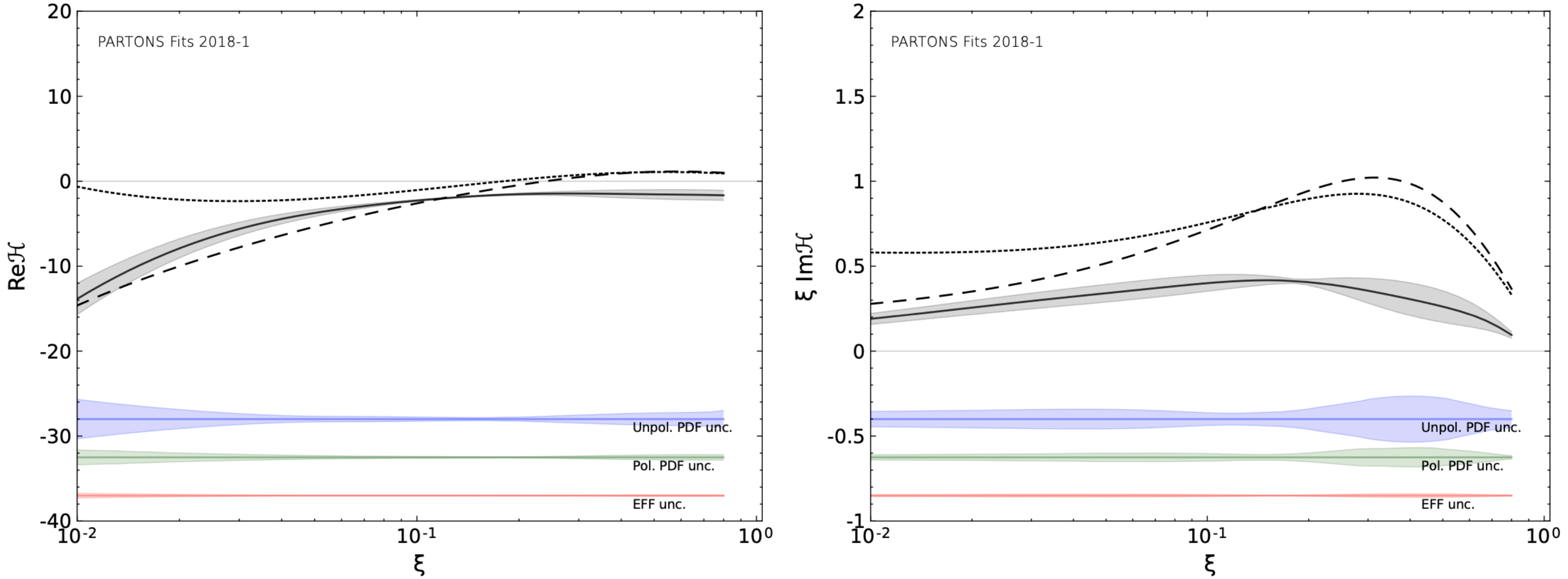




Comparison with PDFs from NNPDF

GPDs from phenomenology

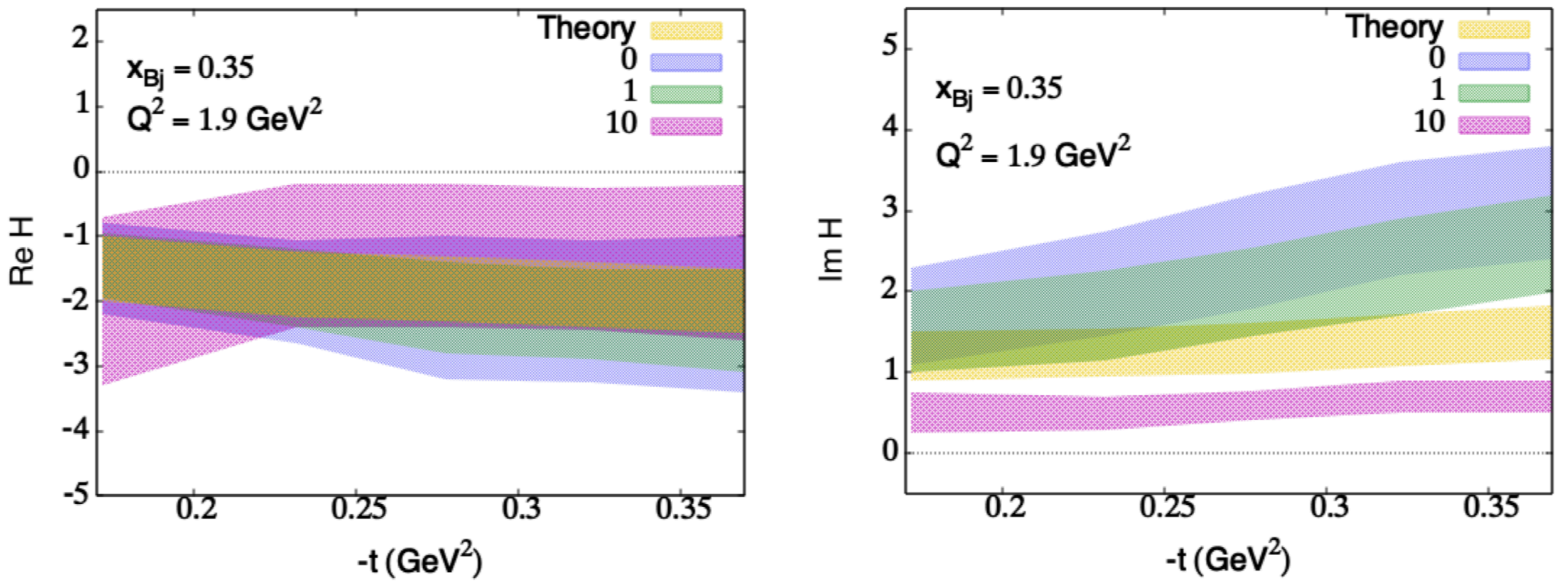
- Example of parametric fit **Moutarde et al, EPJC 18'**



CFF as a function of skewness

GPDs from phenomenology

- Also machine learning techniques have been applied to analyze exclusive scattering data **Almaeen et al, 22'**



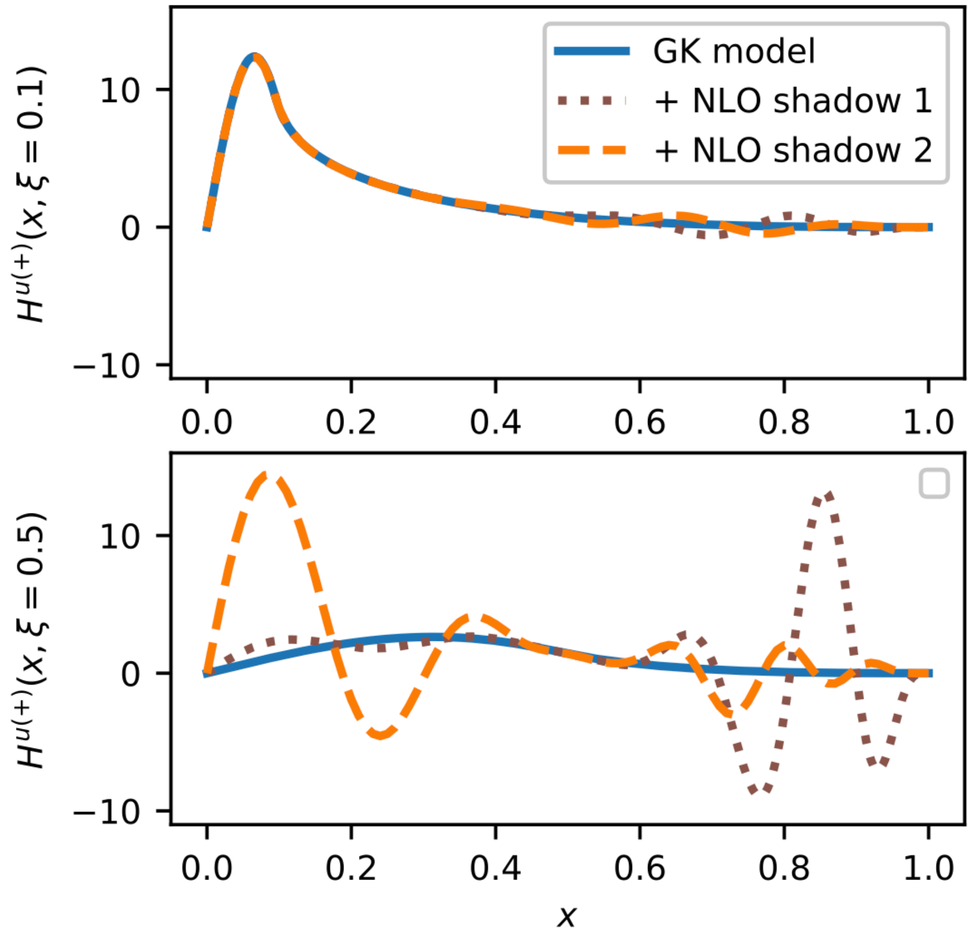
- ReH seems to converge regardless of the choice of the other CFFs
- ImH seems to converge at smaller momentum transfer and diverge at large t

GPDs from phenomenology

- Shadow GPDs and deconvolution of CFFs **Bertone et al, PRD 21'**

Shadow GPDs have considerable size, but

- At an initial scale do not contribute to CFFs and PDFs
- At other scales contribute negligibly

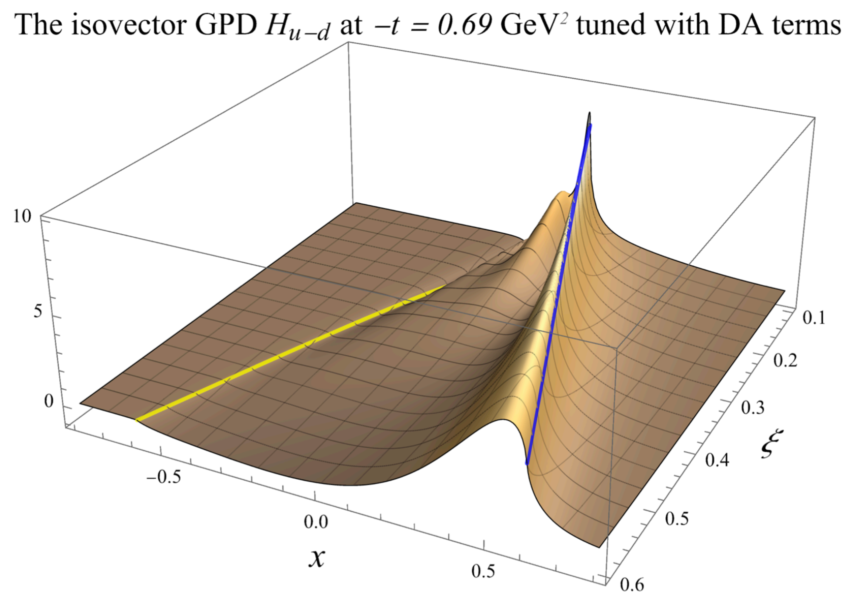


Such GPDs for DVCS have been found both at LO and NLO, making the deconvolution of CFFs an ill-posed problem

DDVCS and lattice calculations may help resolve this issue

GPDs from phenomenology

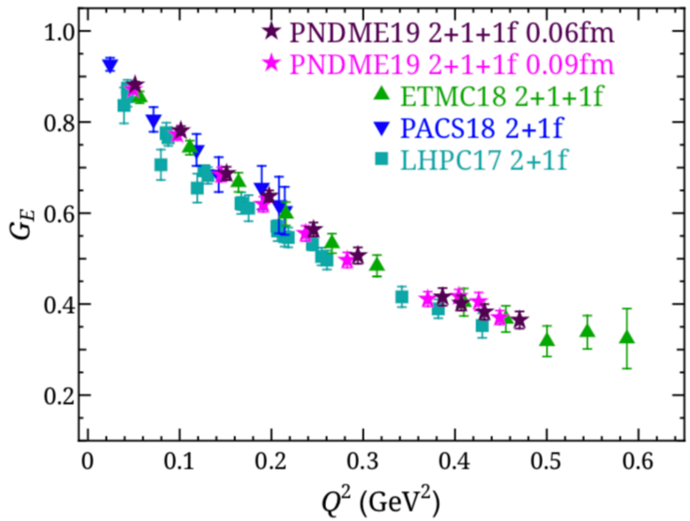
- Universal moment parametrization **Guo et al, JHEP 23'**

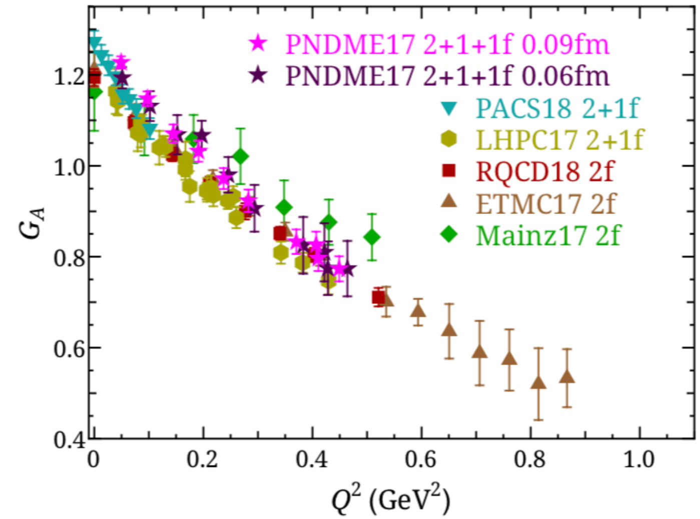


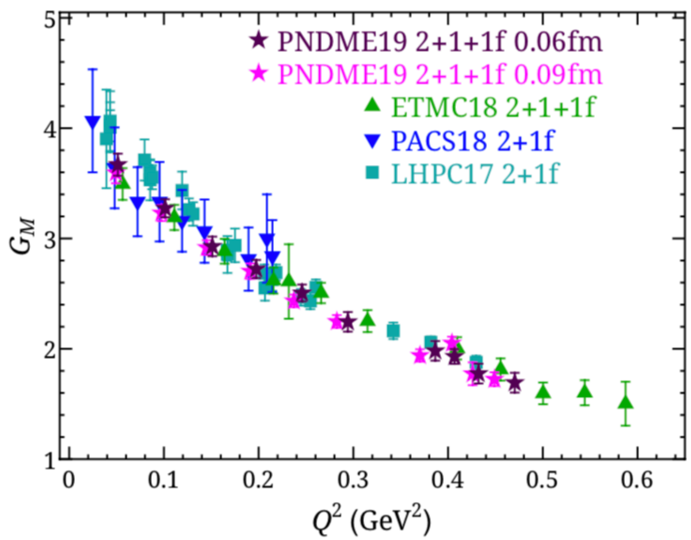
- Parametrize each GPD moment with certain number of parameters
- Take each moment to be a power series in skewness
- Fitting to PDFs, FFs, certain lattice calculations of GPDs and FFs, DVCS measurements

GPDs from theory

- Apart from phenomenological analysis, lattice QCD can provide important complementary inputs — **moments**







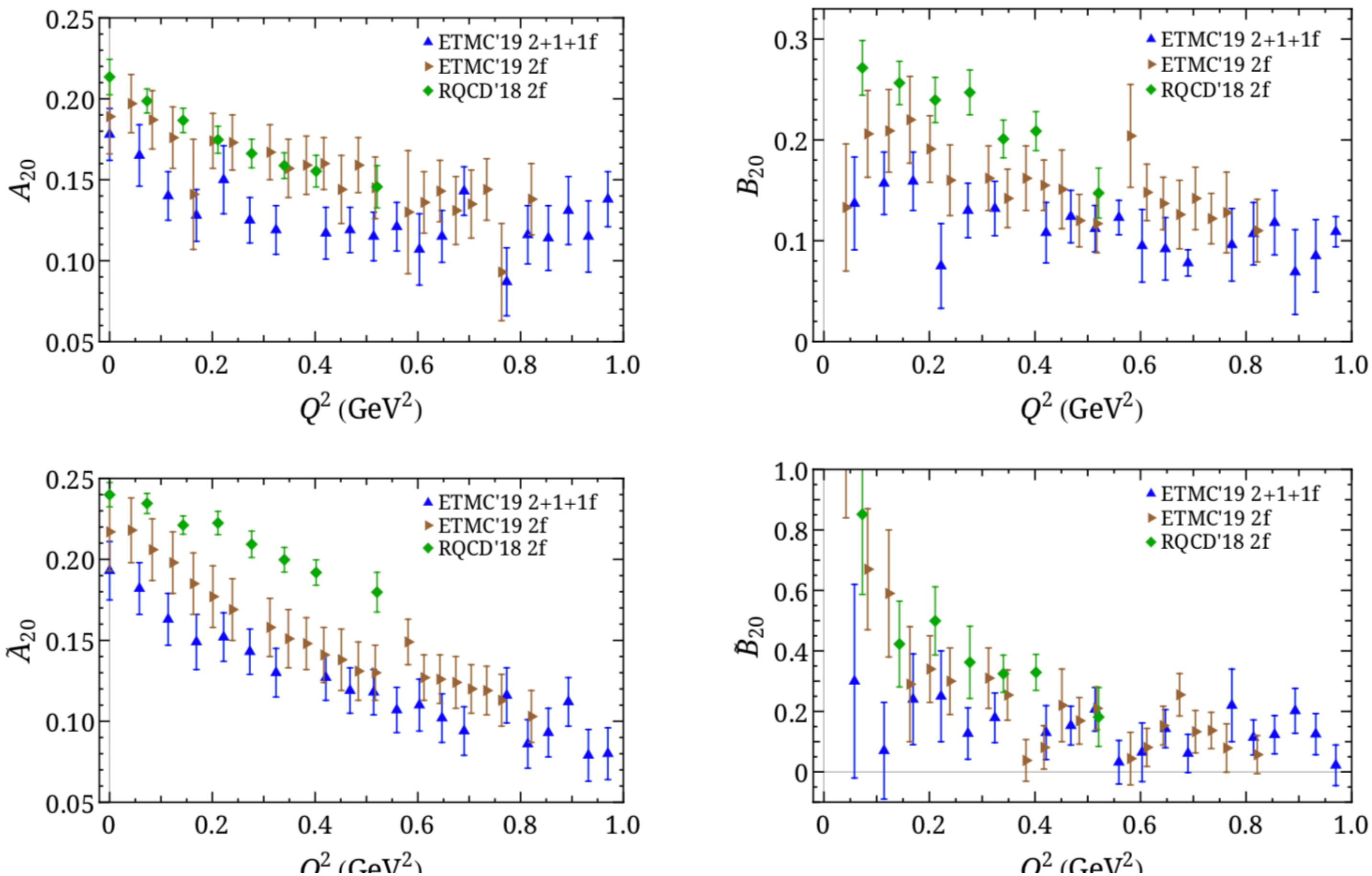
$$\langle N(p_f)|V_\mu^+(x)|N(p_i)\rangle =$$
$$\bar{u}^N\left[\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{2M_N}F_2(q^2)\right]u_N e^{iq\cdot x}$$

$$\langle N(p_f)|A_\mu^+(x)|N(p_i)\rangle =$$
$$\bar{u}_N\left[\gamma_\mu\gamma_5 G_A(q^2) + iq_\mu\gamma_5 G_P(q^2)\right]u_N e^{iq\cdot x}$$

Constantinou, JHZ et al, PPNP 21'

GPDs from theory

- Apart from phenomenological analysis, lattice QCD can provide important complementary inputs — **moments**

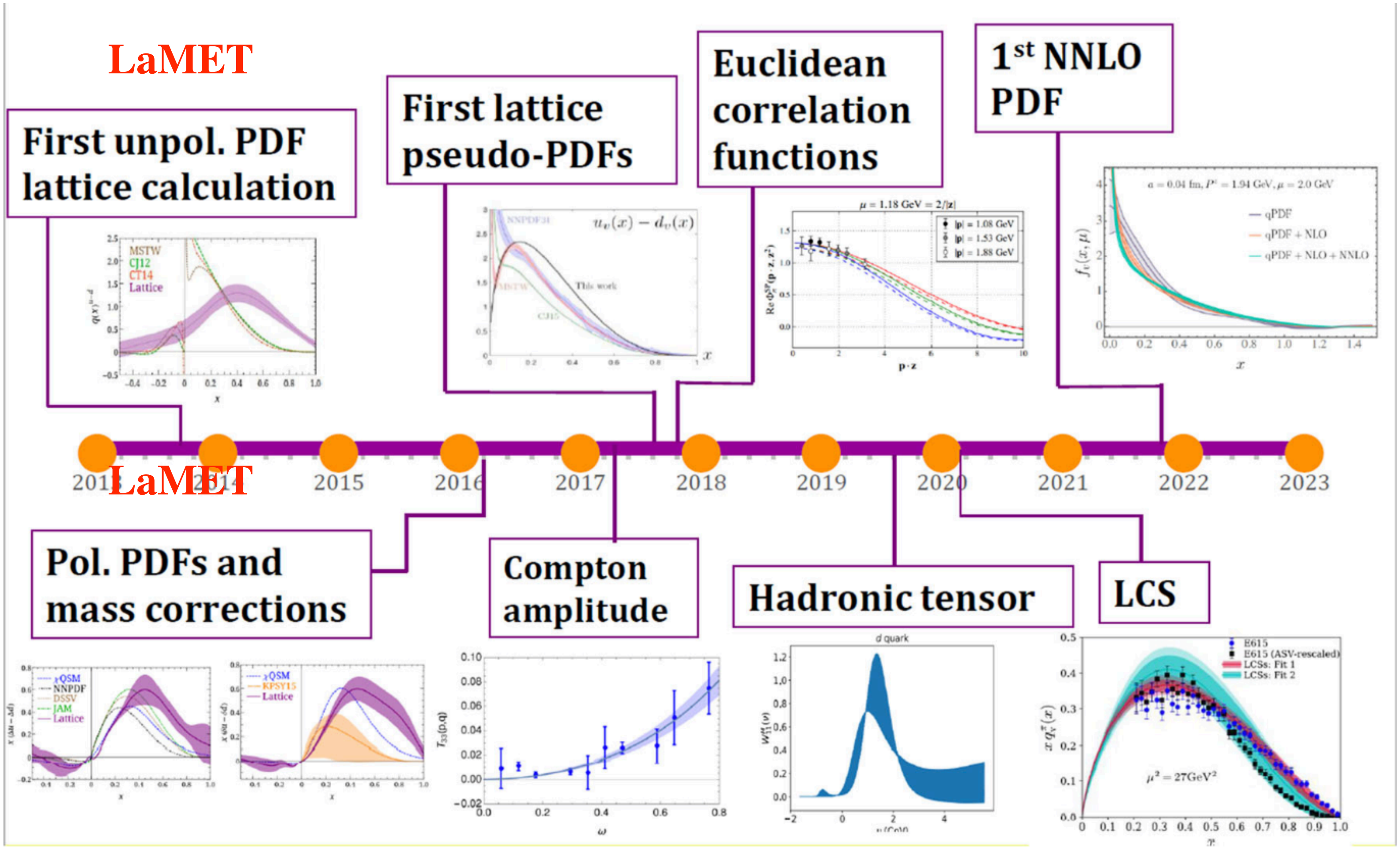


$$\langle N(p',s')|\mathcal{O}_V^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{1}{2}\left[A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m_N} + C_{20}(q^2)\,\frac{1}{m_N}q^{\{\mu}q^{\nu\}}\right]u_N(p,s),$$

$$\langle N(p',s')|\mathcal{O}_A^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{i}{2}\left[\tilde{A}_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}}\gamma^5 + \tilde{B}_{20}(q^2)\,\frac{q^{\{\mu}P^{\nu\}}}{2m_N}\gamma^5\right]u_N(p,s),$$

GPDs from theory

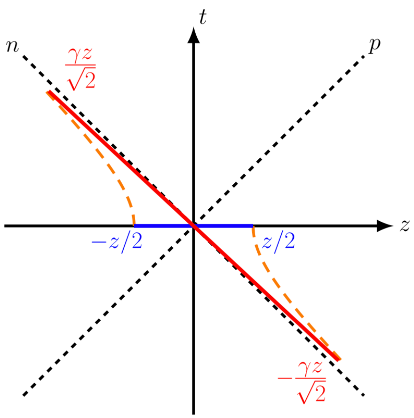
- Lattice QCD can provide important complementary inputs — **x-dependent distributions**
- Significant progress has been achieved along this line



H-W Lin, FBS 23'

GPDs from theory

- Lattice QCD can provide important complementary inputs — **x-dependent distributions**
- **A popular approach: Large-momentum effective theory (LaMET)**
  Ji, PRL 13' & SCPMA 14', Ji, JHZ et al, RMP 21'



$$q(x,\mu) = \int \frac{d\lambda}{4\pi} e^{ix\lambda} \langle P | \bar{\psi}(0) n \cdot \gamma L(0, \lambda n) \psi(\lambda n) | P \rangle, \quad n^2 = 0$$

$$\tilde{q}(y, P^z) = N \int \frac{dz}{4\pi} e^{-iyzP^z} \langle P | \bar{\psi}(0) \gamma^0 L(0, z) \psi(z) | P \rangle$$

$$\tilde{q}(y, P^z) = C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \otimes q(x,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{(yP^z)^2}, \frac{\Lambda^2_{QCD}}{((1-y)P^z)^2}\right)$$

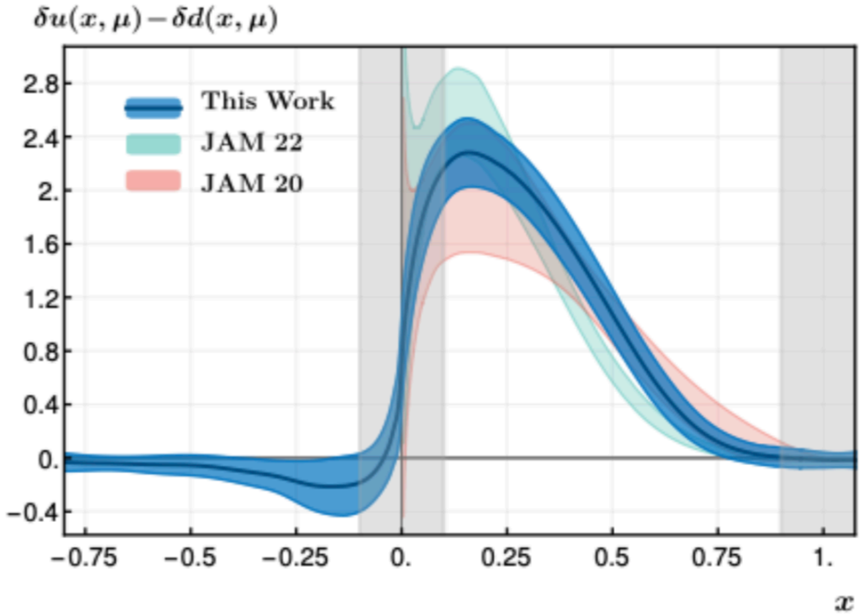
- Theory studies and/or lattice calculations available for
  - Collinear PDFs, distribution amplitudes
  - GPDs, TMDPDFs/wave functions
  - Higher-twist distributions, double parton distributions

A huge number of references…

Lattice results on x-dependent distributions

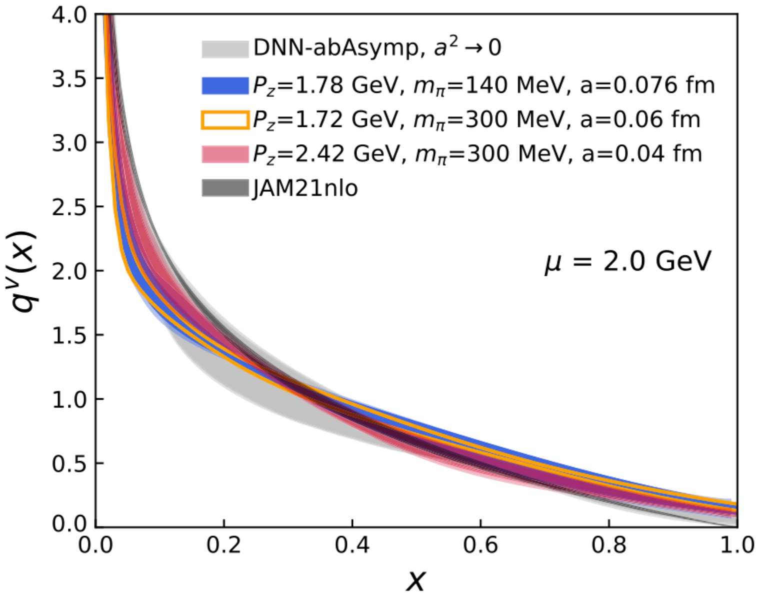
- Lattice QCD can provide important complementary inputs — **x-dependent distributions**
- **A popular approach: Large-momentum effective theory (LaMET)**
  Ji, PRL 13' & SCPMA 14', Ji, JHZ et al, RMP 21'
- Examples of the state-of-the-art:

Nucleon quark transversity PDF



Yao, JHZ et al (LPC) PRL 23'

Pion valence quark PDF



Gao et al, PRD 22'

Lattice results on GPDs

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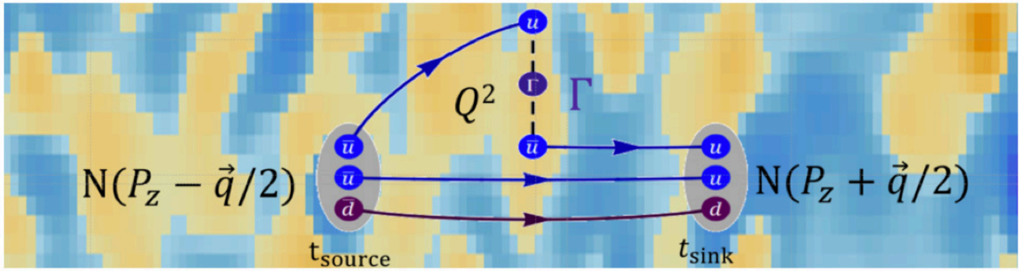
- Examples of the state-of-the-art:

Factorization relating lattice matrix elements to GPDs

$$\tilde{H}^{\pi}_{u-d}(x,\xi,t,P^z,\tilde{\mu}) = \int_{-1}^{1}\frac{dy}{|y|}C\left(\frac{x}{y},\frac{\xi}{y},\frac{\tilde{\mu}}{\mu},\frac{yP^z}{\mu}\right)H^{\pi}_{u-d}(y,\xi,t,\mu) + h.t.$$

$$\tilde{H}^{\pi}_{u-d}(x,\xi,t,P^z,\tilde{\mu}) = \int\frac{dz}{4\pi}e^{ixzP^z}\tilde{h}^R_{\rm lat}(z,P^z,t)$$

$$\tilde{h}_{\rm lat}(z,P^z,t,a) = \frac{P^z}{P_0}\langle N(\vec{P}+\frac{\vec{\Delta}}{2})\,|\,\bar{q}(z)\Gamma(\prod_n U_z(n\hat{z}))\tau_3 q(0)\,|\,N(\vec{P}-\frac{\vec{\Delta}}{2})\rangle$$



Lattice results on GPDs

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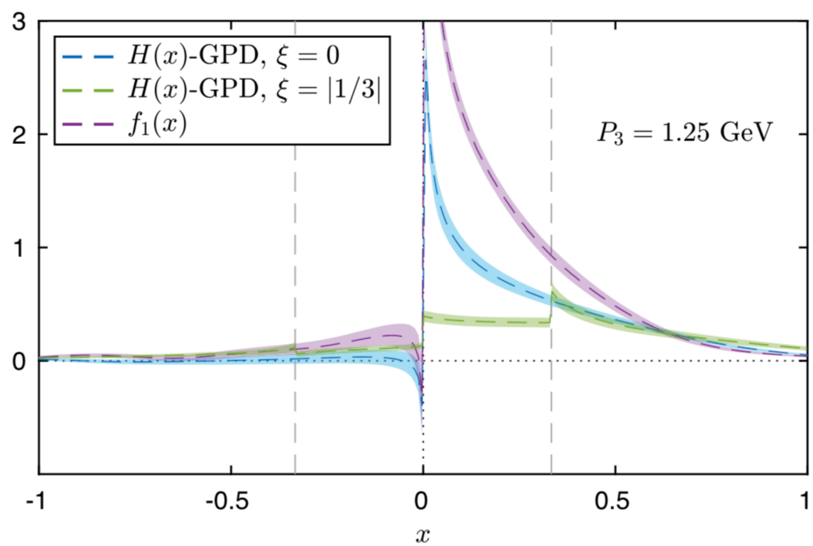
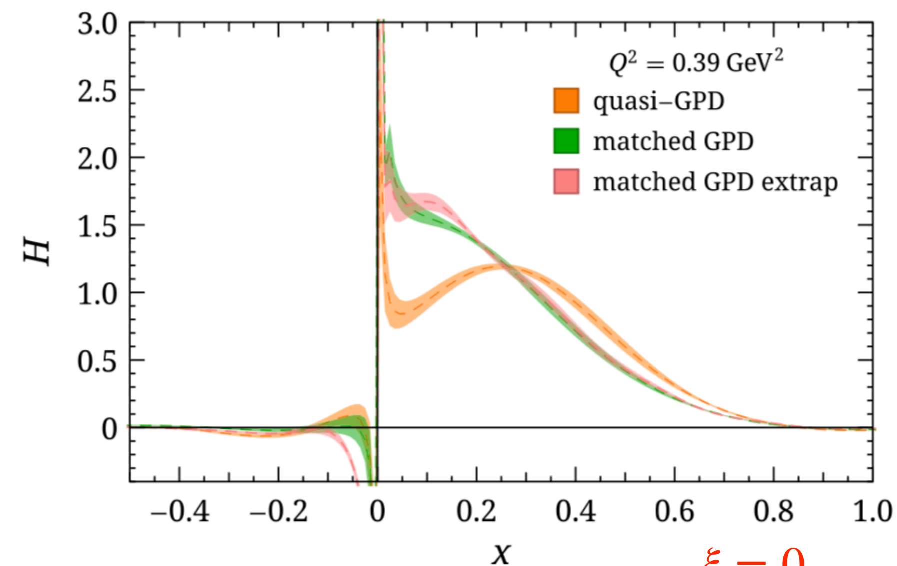
Nucleon quark unpolarized GPD



Alexandrou et al, PRL 20'



$\xi = 0$

Lin, PRL 21'

Lattice results on GPDs

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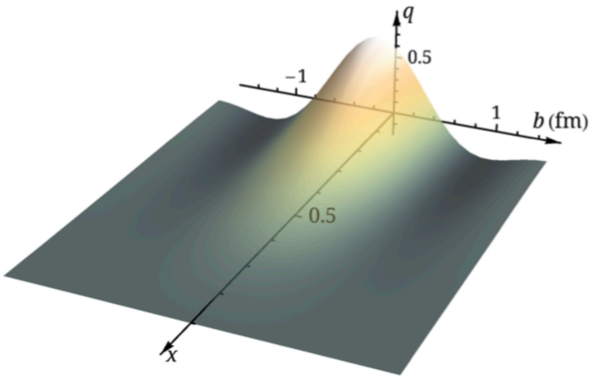
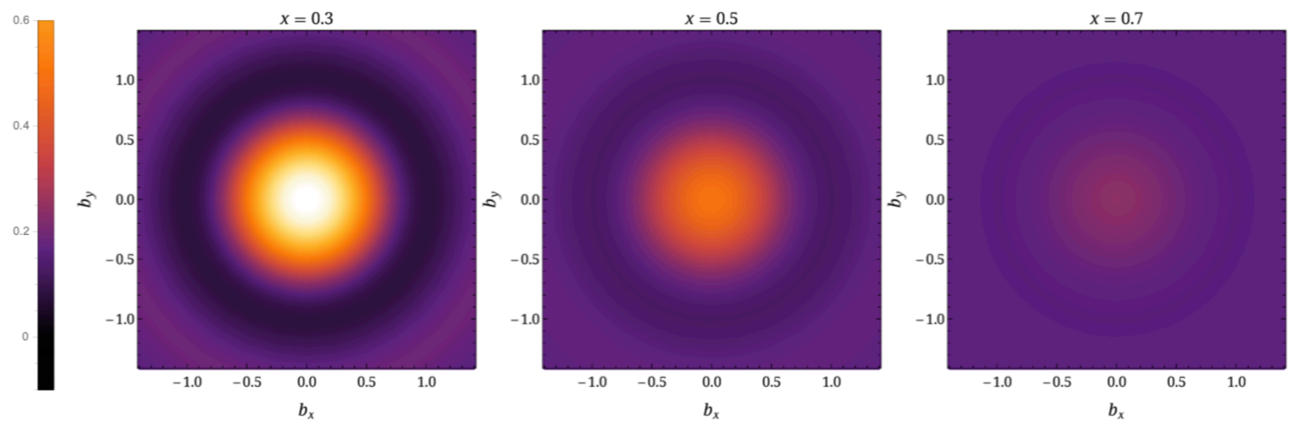
- Examples of the state-of-the-art:

Impact parameter distribution

$$\mathsf{q}(x,b) = \int \frac{d\mathbf{q}}{(2\pi)^2} H(x,\xi=0,t=-\mathbf{q}^2) e^{i\mathbf{q}\cdot\mathbf{b}}$$





Lin, PRL 21'

Lattice results on GPDs

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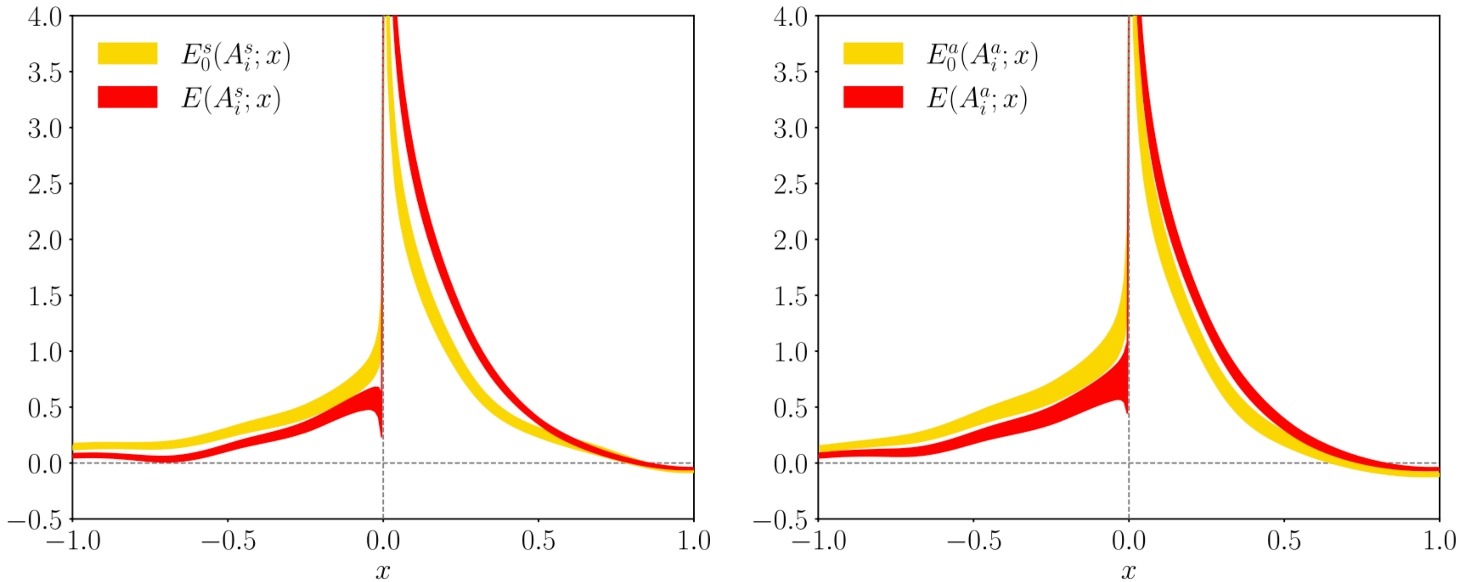
- Examples of the state-of-the-art:

Nucleon quark GPD in an asymmetric frame

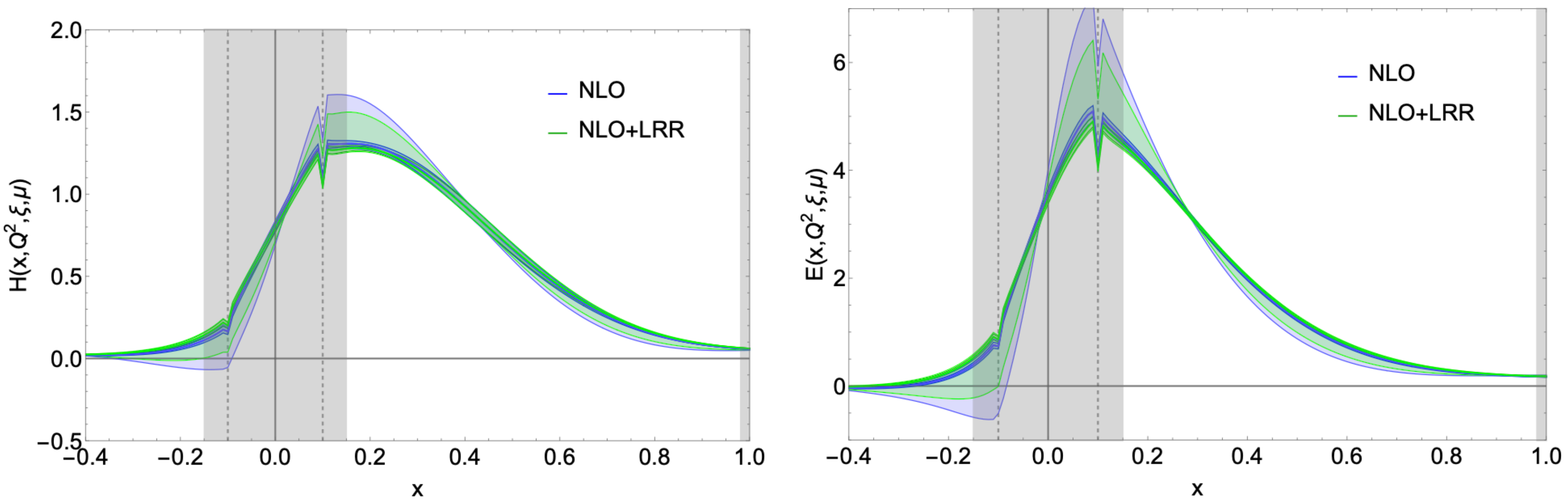
Bhattacharya et al, PRD 22'



- Leading-twist violates translational invariance, which can be restored by including kinematic higher-twist contributions **Braun, 23'**

Lattice results on GPDs

- Update by implementing recent theory developments in lattice calculations of GPDs



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$\xi = 0.1$

Holligan et al, 23'

Summary and outlook

- Determination of GPDs from experimental data is important goal of EIC and EicC, polarized beam and targets can shed more light on various CFFs

- Phenomenological fitting is very challenging due to the multi-dimensionality of GPDs

- Lattice QCD can provide complementary information to experimental data on the 3D structure of nucleons

- For simple quantities such as collinear PDFs, lattice calculations of x-dependent distributions have reached a stage where precision control becomes important

- Similar analyses shall be extended to the lattice calculations of GPDs, more to be expected