Vortex ion beams

Dmitry Karlovets

ITMO University, St. Petersburg

27.02.2024

d.karlovets@gmail.com

PBT2024

Outline

1. Introduction:
    - Twisted photons,
    - Twisted matter waves: vortex electrons, neutrons, atoms, their scattering, etc.
    - Angular-momentum-dominated beams
2. Vortex ions: possible generation strategies
3. Generalized measurements and entanglement in collisions
4. Conclusion and outlook

Introduction: twisted photons

Classical light:

- Plane waves
- Spherical waves
- Cylindrical waves

Each wave can be represented as a superposition of the waves from the other set

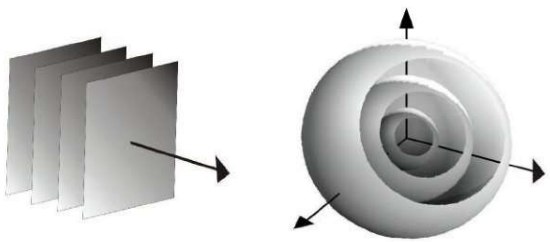
Quantum light:

- Plane-wave photons
- Spherical (waves of) photons
- Cylindrical (waves of) photons

Each function in a set can be represented as a superposition of the functions from the other set

or the twisted photons

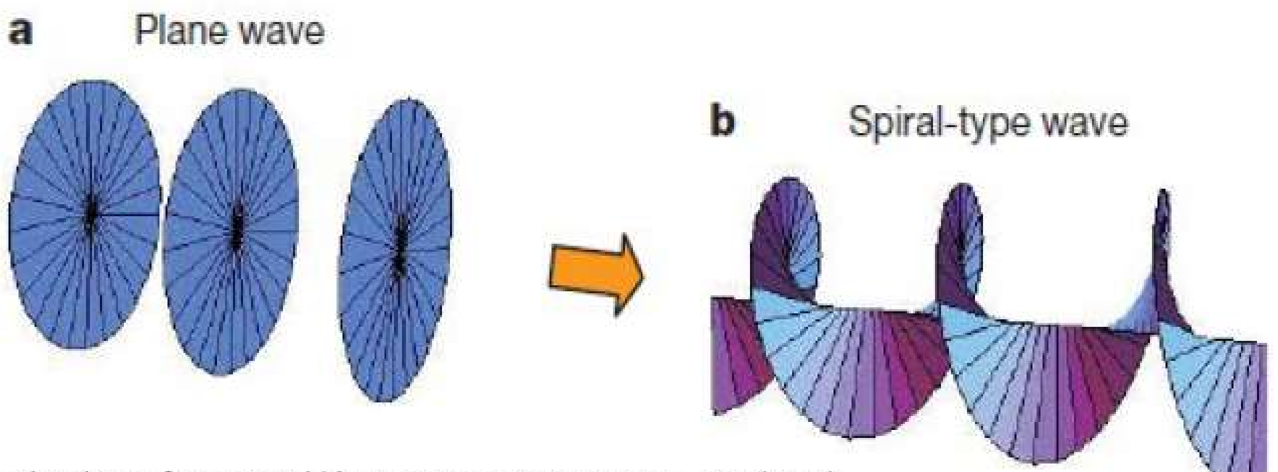
Plane vs. spherical



https://www.researchgate.net/profile/Miikka-Tikander-2/publication/242641740/figure/fig1/AS:298423984115712@1448161230440/Propagation-of-a-plane-waves-and-b-spherical-waves_W640.jpg

Introduction: twisted photons

- Plane-wave photons: k_x, k_y, k_z + polarization
- Spherical (waves of) photons: $|\boldsymbol{k}|$, l, m (= L_z) + polarization
- The cylindrical (twisted) photons: $k_\perp$, k_z, m (= L_z) + polarization



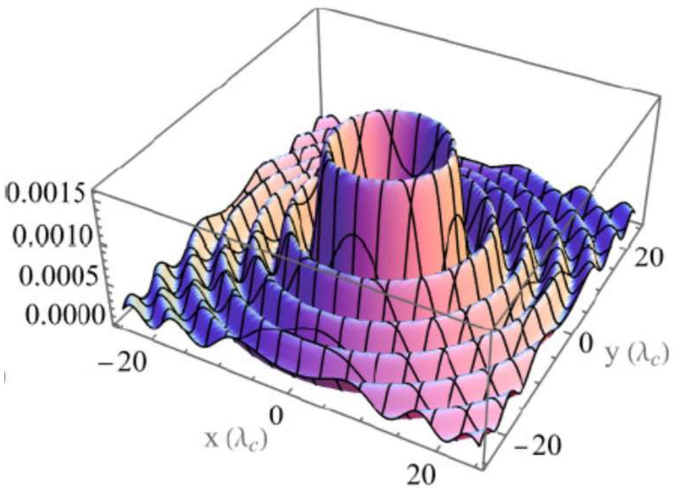
The picture from M. Uchida, A. Tonomura, Nature **464**, 737 (2010)

Introduction: twisted photons

The uncertainty relation:
the definite L_z implies undefined azimuthal angle!

$$\langle(\Delta L_z)^2\rangle\langle(\Delta \sin\phi_r)^2\rangle \geq \frac{1}{4}\langle\cos\phi_r\rangle^2$$

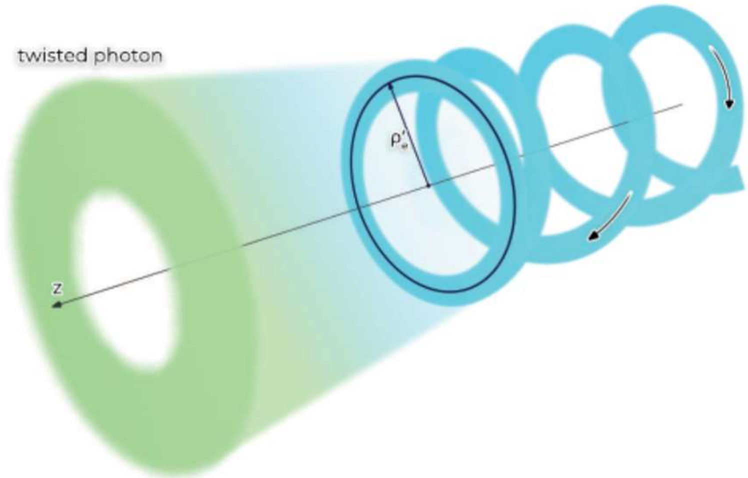
Doughnut- (or bagel-, Brezel-) shaped spatial distribution



P. Carruthers, M.M. Nieto,
Rev. Mod. Phys. **40**, 411 (1968)

Twisted EM waves (photons) are naturally generated

- In cylindrical waveguides
- By charged particles via non-linear Thomson or Compton scattering
- By synchrotron radiation in strong magnetic field
- Nearby rotating black holes
- Via Cherenkov, transition radiation, Smith-Purcell radiation, during channeling, etc.
- In helical undulators (say, at FELs)
- Nearly always when the particle trajectory is helical



Twisted photons generated in neutron stars

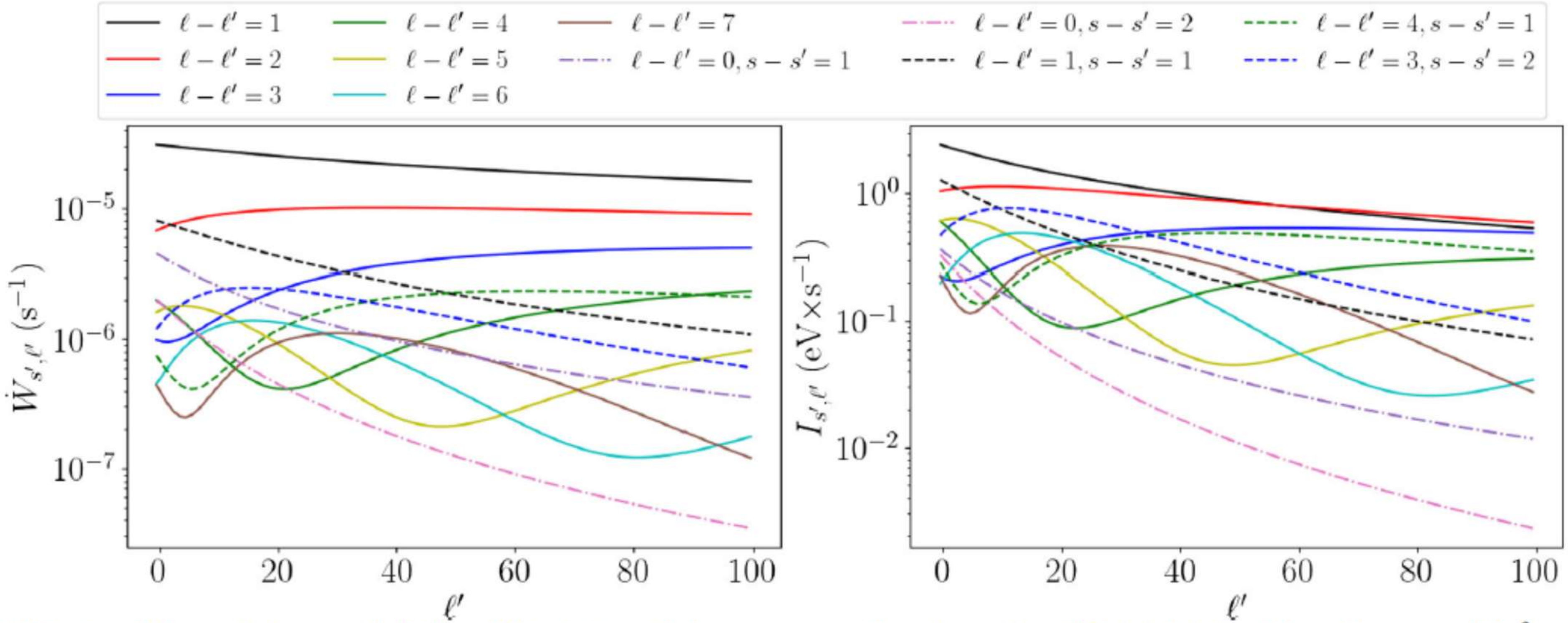


FIG. 1. The emission probability (31) (left) and the corresponding intensity (32) (right) for $H = H_c$, $p_z = 10^{-3}mc$, and no spin flip. For the solid lines, $s = s' = 20$; the dashed lines correspond to the twisted photons with a simultaneous change of the radial quantum number, $s \to s' \neq s$; and the dash-dotted lines correspond to the untwisted photons with the TAM $j_z = \ell - \ell' = 0$.

DK, Di Piazza, PRD 108, 063007 (2023); Pavlov, DK, PRD 109, 036017 (2024)

Introduction: twisted photons, vortex electrons, atoms, etc.

- Vortex ions: generation strategies

- Vortex beams via generalized measurements

- Conclusion

Introduction: twisted matter waves

$$\psi_{\mathbf{k}}(\mathbf{r}) = \exp(\mathrm{i}\mathbf{k}\mathbf{r}),$$ – plane waves

$$\psi_{klm}(\mathbf{r}) = \sqrt{\frac{2\pi}{kr}} J_{l+1/2}(kr)\, Y_{lm}(\theta_r, \varphi_r),$$ – spherical waves

$$\psi_{\varkappa m k_z}(\mathbf{r}) = J_m(\varkappa\rho) \exp\left[\mathrm{i}(m\varphi_r + k_z z)\right]$$ – cylindrical waves (Bessel beams)

$$\hat{l}_z = [\hat{\boldsymbol{r}} \times \hat{\boldsymbol{p}}]_z = -i\frac{\partial}{\partial\phi_r}$$

K.Y. Bliokh, et al., PRL 99 (2007) 190404

B.A. Knyazev, V.G. Serbo, Phys.-Usp. 61 449 (2018)

Twisted matter waves: Exact solutions to free Schrödinger equation

The standard Laguerre-Gaussian packet (*orthogonal* set):

$$\psi_{\ell,n}(\rho,t) = N\frac{\rho^{|\ell|}}{(\sigma_\perp(t))^{|\ell|+1}}\, L_n^{|\ell|}\left(\frac{\rho^2}{(\sigma_\perp(t))^2}\right) \exp\left\{i\ell\phi_r - i(2n+|\ell|+1)\arctan(t/t_d) - \frac{\rho^2}{2(\sigma_\perp(t))^2}(1 - it/t_d)\right\}, \qquad \int d^2\rho\, |\psi_{\ell,n}(\rho,t)|^2 = 1,$$

$$\sigma_\perp(t) = \sigma_\perp(0)\sqrt{1+\frac{t^2}{t_d^2}}, \qquad \sigma_\perp(0) = 1/\sigma_p,$$

The elegant Laguerre-Gaussian packet (*bi-orthogonal* set):

$$\psi_{\ell,n}(\rho,t) = N\frac{\rho^{|\ell|}}{(\sigma_\perp(t))^{n+|\ell|+1}}\, L_n^{|\ell|}\left(\frac{\rho^2}{2(\sigma_\perp(t))^2}(1 - it/t_d)\right)$$

$$\times \exp\left\{i\ell\phi_r - i(n+|\ell|+1)\arctan(t/t_d) - \frac{\rho^2}{2(\sigma_\perp(t))^2}(1 - it/t_d)\right\}$$

D.K., New J. Phys. **23** (2021) 033048

Introduction: twisted photons, vortex electrons, atoms, etc.

- Vortex ions: generation strategies
- Vortex beams via generalized measurements
- Conclusion

Introduction: twisted matter waves

The first generation of vortex electrons

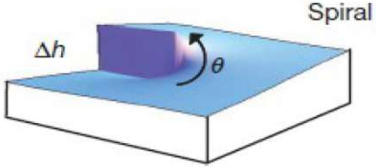
- The highest electron energy is 300 keV (TEM)

- The highest angular momentum is ~ 1000!

(the magnetic moment is much larger than the Bohr magneton!)

- The smallest spot size is 0.1 nm!

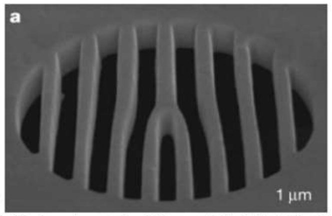
Atomic scale!



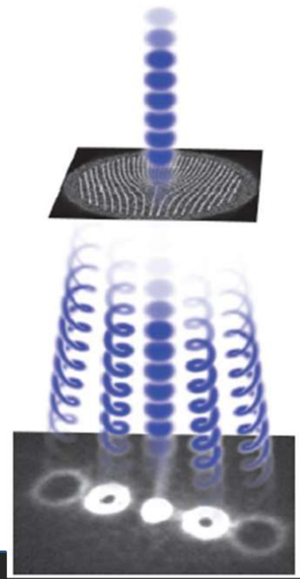
M. Uchida, A. Tonomura, Nature **464**, 737 (2010)

Diffraction grating with an edge dislocation made of Pt foil

Spiral phase plate



J. Verbeeck, et al., Nature **467**, 301 (2010)



M. Uchida and A. Tonomura, Nature 464, 737 (2010),
J. Verbeeck, et al., Nature 467 (2010) 301–304,
B. J. McMorran, et al., Science 331, 192 (2011)

Introduction: twisted matter waves

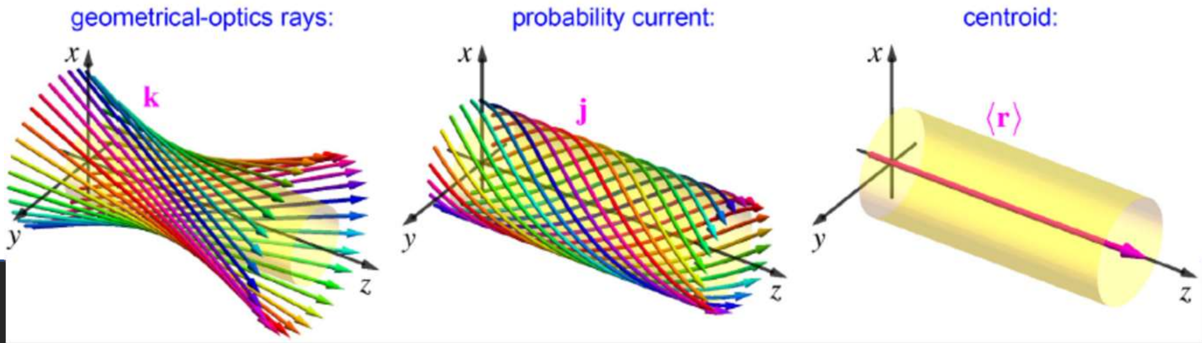
The vortex electron/muon/proton/ion magnetic moment can be huge!

$$\mathbf{M} = \frac{1}{2c}\frac{\int \mathbf{r}\times\mathbf{j}_e\, d^3\mathbf{r}}{\int \rho\, d^3\mathbf{r}} = \frac{e}{2m_e c}\langle\mathbf{L}\rangle$$

$$\mu \propto \mu_B(\mathbf{l} + 2\mathbf{s})$$

If the OAM is large, $l_z \equiv m \gg \hbar$, then $\mu \gg \mu_B$!

Vortex particles instead of spin-polarized ones!



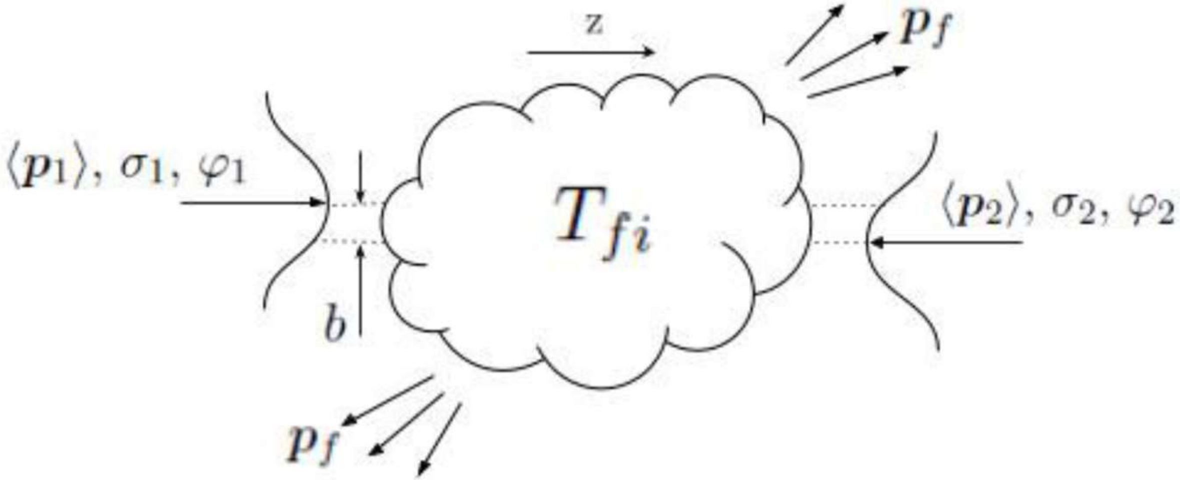
K.Y. Bliokh et al. / Physics Reports 690 (2017) 1–70

Introduction: twisted matter waves

Spin → Orbital Angular Momentum (OAM),
Total AM = OAM+spin

Relativistic symmetry conserves total momentum,
not spin or OAM separately!

However, a wave has OAM only if it is spatially localized



Introduction: twisted photons, vortex electrons, atoms, etc.

- Vortex ions: generation strategies
- Vortex beams via generalized measurements
- Conclusion

Spin-orbital coupling of a relativistic fermion

$$\boldsymbol{\mu}_f = \boldsymbol{\mu}_s + \boldsymbol{\mu}_b,$$

$$\boldsymbol{\mu}_b = \frac{1}{2}\left\langle \boldsymbol{u} \times \frac{\partial \varphi_\ell(p)}{\partial \boldsymbol{p}} \right\rangle = \hat{\boldsymbol{z}}\, \ell \left\langle \frac{1}{2\varepsilon} \right\rangle \simeq \hat{\boldsymbol{z}}\, \ell\, \frac{1}{2\bar{\varepsilon}} \left[1 - \frac{\sigma^2}{2m^2} \left(|\ell| + \frac{1}{2} + \frac{m^2}{\bar{\varepsilon}^2} \right) \right]$$

OAM contribution

Non-paraxial corrections

Bohr magneton

$$\boldsymbol{\mu}_s = \left\langle \frac{1}{(2\varepsilon)^2} \left(\boldsymbol{\zeta}(\varepsilon + m) + \frac{\mathbf{p}(\mathbf{p}\boldsymbol{\zeta})}{\varepsilon + m} \right) \right\rangle \simeq \boldsymbol{\zeta}\, \frac{1}{2\bar{\varepsilon}} \left\{ 1 - \frac{\sigma^2}{2m^2} \left[\frac{1}{2} + \frac{3}{2}\frac{m}{\bar{\varepsilon}} + \frac{1}{2}\frac{m^2}{\bar{\varepsilon}^2} - \frac{3}{2}\frac{m^3}{\bar{\varepsilon}^3} - \frac{m}{\bar{\varepsilon} + m} \left(\frac{3}{2} - 2\frac{m^2}{\bar{\varepsilon}^2} - \frac{3}{2}\frac{m^3}{\bar{\varepsilon}^3} \right) + |\ell| \left(1 + \frac{m}{\bar{\varepsilon}} - \frac{m}{\bar{\varepsilon} + m} \right) \right] \right\}$$

DK, PRA **98**, 012137 (2018)

Introduction: twisted matter waves

$$d\sigma_{\text{gen}} = \frac{dW}{L}.$$

The generalized cross section

$$dW = |S_{fi}|^2 \prod_f V \frac{d^3 p_f}{(2\pi)^3} = \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3} \mathcal{L}(\boldsymbol{p}_i, \boldsymbol{k}) d\sigma(\boldsymbol{p}_i, \boldsymbol{k}),$$

The probability

$$\mathcal{L}(\boldsymbol{p}_i, \boldsymbol{k}) = \upsilon(\boldsymbol{p}_i) \int d^4x d^3R e^{i\boldsymbol{k}\boldsymbol{R}} n_1(\boldsymbol{r}, \boldsymbol{p}_1, t) n_2(\boldsymbol{r} + \boldsymbol{R}, \boldsymbol{p}_2, t)$$

The correlator

$$L = \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} d^4x \upsilon(\boldsymbol{p}_i) n_1(\boldsymbol{r}, \boldsymbol{p}_1, t) n_2(\boldsymbol{r}, \boldsymbol{p}_2, t).$$

The luminosity
via the Wigner functions

$$\upsilon(\boldsymbol{p}_i) = \frac{\sqrt{(p_{1\mu} p_2^{\mu})^2 - m_1^2 m_2^2}}{\varepsilon_1(\boldsymbol{p}_1)\varepsilon_2(\boldsymbol{p}_2)} = \sqrt{(\boldsymbol{u}_1 - \boldsymbol{u}_2)^2 - [\boldsymbol{u}_1 \times \boldsymbol{u}_2]^2},$$

Introduction: twisted matter waves

When the wave packets are wide:

$$d\sigma_{\rm gen} = d\sigma^{\rm incoh} + d\sigma^{\rm int} + \mathcal{O}((\delta p)^2),$$

$$d\sigma^{\rm incoh} = \frac{dW^{\rm incoh}}{L},$$

$$dW^{\rm incoh} = \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} d^4 x v(\boldsymbol{p}_i) n_1(\boldsymbol{r}, \boldsymbol{p}_1, t)\, n_2(\boldsymbol{r}, \boldsymbol{p}_2, t) d\sigma^{(\rm pw)}(\boldsymbol{p}_i).$$

Even the first term is NOT yet the plane-wave cross section!

DK, Serbo, PRD 101, 076009 (2020)

Introduction: twisted matter waves

The correction due to quantum interference:

$$d\sigma^{\rm int} = -\frac{1}{L}\int \frac{d^3 p_1}{(2\pi)^3}\frac{d^3 p_2}{(2\pi)^3} d^4 x v(\boldsymbol{p}_i) n_1(\boldsymbol{r},\boldsymbol{p}_1,t)\frac{\partial n_2(\boldsymbol{r},\boldsymbol{p}_2,t)}{\partial \boldsymbol{r}} d\sigma^{(\rm pw)}(\boldsymbol{p}_i)\partial_{\Delta\boldsymbol{p}}\zeta^{(\rm pw)}_{fi}(\boldsymbol{p}_i)$$

$$M^{(\rm pw)}_{fi} = |M^{(\rm pw)}_{fi}|\exp\{i\zeta^{(\rm pw)}_{fi}\},$$

$$\partial_{\Delta\boldsymbol{p}} = \frac{\partial}{\partial \boldsymbol{p}_1} - \frac{\partial}{\partial \boldsymbol{p}_2},$$

$$\zeta^{(\rm pw)}_{fi} = \arctan\frac{{\rm Im}M^{(\rm pw)}_{fi}}{{\rm Re}M^{(\rm pw)}_{fi}} = {\rm inv}$$

DK, Serbo, PRD 101, 076009 (2020)

Introduction: twisted photons, vortex electrons, atoms, etc.

- Vortex ions: generation strategies
- Vortex beams via generalized measurements
- Conclusion

Introduction: twisted matter waves

ИТМО

Eur. Phys. J. C (2016) 76:661
DOI 10.1140/epjc/s10052-016-4399-8

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Experimental Physics

Measurement of elastic pp scattering at $\sqrt{s} = 8$ TeV in the Coulomb–nuclear interference region: determination of the ρ-parameter and the total cross-section

TOTEM Collaboration

$$d\sigma^{\mathrm{PW}} \propto |M_{fi}^{\mathrm{em}} + M_{fi}^{\mathrm{strong}}|^2$$

With the vortex particles, the cross section does depend on the amplitude's phase already at the tree level!

Introduction: twisted photons, vortex electrons, atoms, etc.

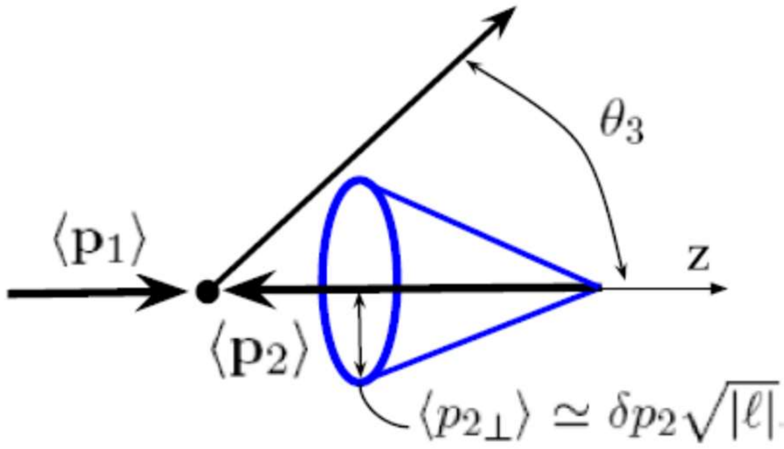
- Vortex ions: generation strategies
- Vortex beams via generalized measurements
- Conclusion

Collision of a Gaussian packet with a vortex packet

S-channel at $\sqrt{s} \gg m_\mu$ $\quad e^+(p_1)e^-_{\rm tw}(p_2(\phi_2)) \to \mu^+(p_3)\mu^-(p_4)$

$$\frac{d\sigma^{\rm incoh}_{\rm CM}}{d\Omega_3} = \frac{d\sigma^{\rm (pw)}_{\rm CM}}{d\Omega_3}\left(1 + \frac{p_\perp^2}{s_0}g(\theta_3) + \mathcal{O}\left(\frac{p_\perp^4}{s_0^2}\right)\right),$$

$$g(\theta_3) = -\frac{\cos\theta_3}{1+\cos^2\theta_3}(2 + \cos\theta_3 - 4\cos^2\theta_3 + 5\cos^3\theta_3),$$



Leptons: $\delta p_2 \lesssim 1$ keV, $\quad \sqrt{s_0} > 1$ GeV $\rightarrow$ $\frac{p_\perp^2}{s_0} \sim \frac{(\delta p_2)^2}{s_0}|\ell| \lesssim 10^{-12}|\ell|$

Hadrons: $p_{\rm (tw)}p \to X, \quad p_{\rm (tw)}\bar{p} \to X, \quad e\,p_{\rm (tw)} \to ep,$ etc.

$$\frac{p_\perp^2}{s_0} \sim \frac{(\delta p_p)^2}{s_0}|\ell| \lesssim 10^{-8}|\ell|$$

Introduction: twisted photons, vortex electrons, atoms, etc.

- Vortex ions: generation strategies
- Vortex beams via generalized measurements
- Conclusion

Quantum interference

$$d\sigma^{\text{int}} = -2\sigma_{12}^2 \frac{1}{L} \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} v(p_i) I_\ell^{\text{corr}}(\boldsymbol{p}_i; \boldsymbol{b}) d\sigma^{(\text{pw})}(\boldsymbol{p}_i)$$
$$\times \left(\boldsymbol{b}_{\text{eff}} - \frac{\sigma_{12}^2}{\sigma_{12}^2 (\Delta \boldsymbol{u}_\perp)^2 + \sigma_{12,z}^2 (\Delta u_z)^2} \Delta \boldsymbol{u} (\Delta \boldsymbol{u} \boldsymbol{b}_{\text{eff}}) \right) \cdot \partial_{\Delta \boldsymbol{p}} \zeta_{fi}^{(\text{pw})}(\boldsymbol{p}_i),$$

An effective impact-parameter
(akin to Goos-Hänchen shift in optics):

$$\boldsymbol{b}_{\text{eff}} = \boldsymbol{b} + \ell \frac{\boldsymbol{p}_2 \times \hat{\boldsymbol{z}}}{p_{2\perp}^2}.$$

Scattering assymetry:

$$\mathcal{A} = \frac{d\sigma_{\text{gen}}(\boldsymbol{b}_{\text{eff}}) - d\sigma_{\text{gen}}(-\boldsymbol{b}_{\text{eff}})}{d\sigma_{\text{gen}}(\boldsymbol{b}_{\text{eff}}) + d\sigma_{\text{gen}}(-\boldsymbol{b}_{\text{eff}})} = \frac{d\sigma^{\text{int}}(\boldsymbol{b}_{\text{eff}})}{d\sigma^{\text{incoh}}(\boldsymbol{b}_{\text{eff}})}$$

DK, Serbo, PRD 101, 076009 (2020)

Introduction: twisted matter waves

LETTER

doi:10.1038/nature15265

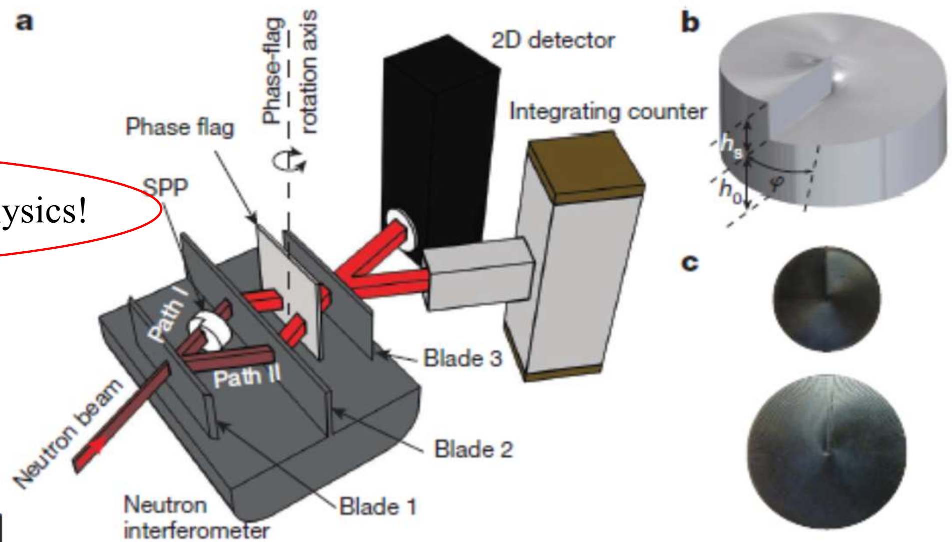
Controlling neutron orbital angular momentum

Charles W. Clark[1], Roman Barankov[2], Michael G. Huber[3], Muhammad Arif[3], David G. Cory[4,5,6,7] & Dmitry A. Pushin[6,8]

At the National Institute of Standards and Technology (NIST)

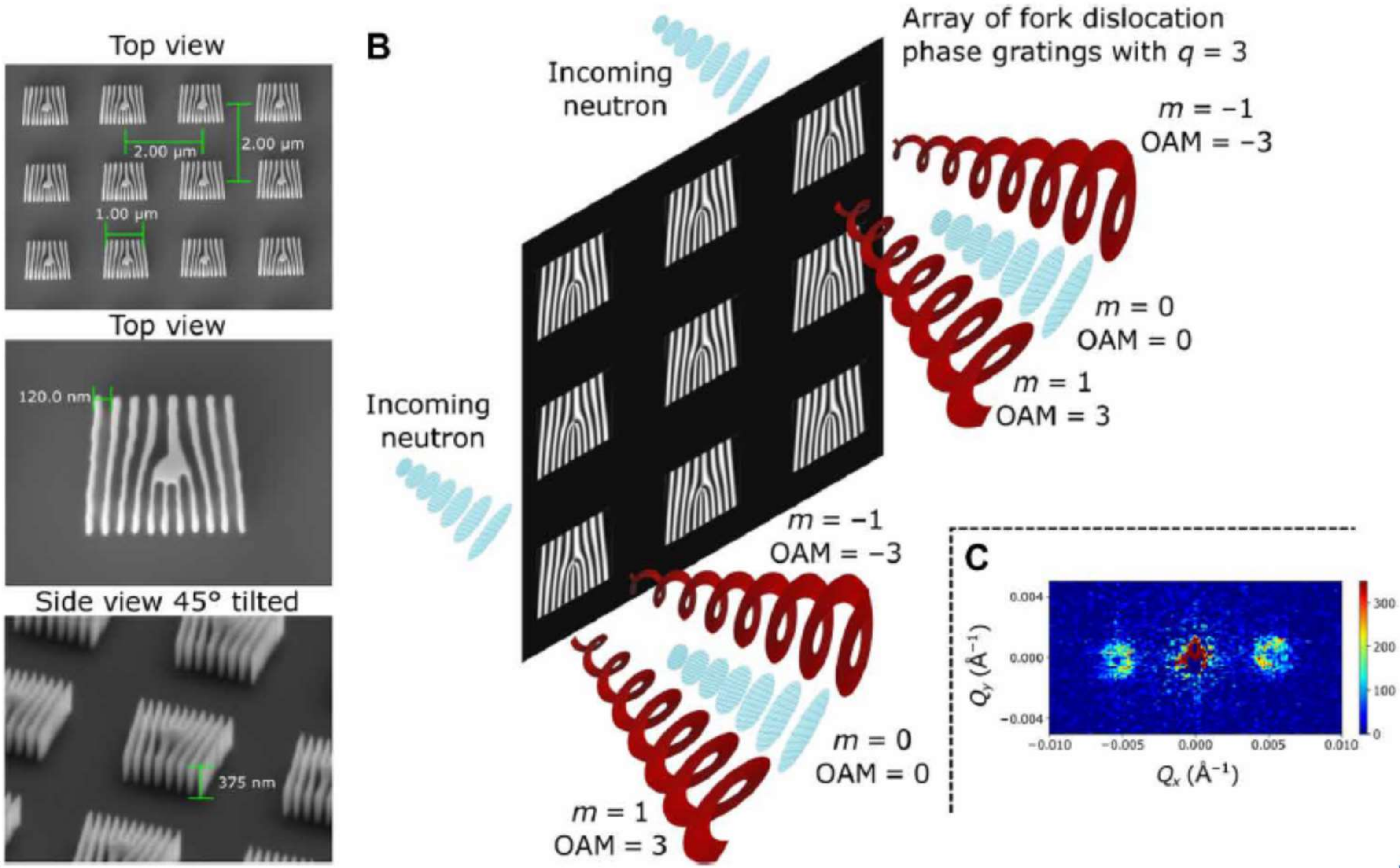
New means for neutron optics and low-energy nuclear physics!

E = 11 meV, wavelength = 0.27 nm,
sub-micrometer transverse coherence length



Ch. W. Clark, et al., Nature **525**, 504 (2015)

The more recent technique: arrays with $N = 2500$ on silicon substrates



Sarenac *et al.*, *Sci. Adv.* **8**, eadd2002 (2022)

Introduction: twisted photons, vortex electrons, atoms, etc.

- Vortex ions: generation strategies

- Vortex beams via generalized measurements

- Conclusion

What's the use of twisted neutrons?

Plane-wave scattering amplitude

Strong amplitude → a ; EM amplitude → $i\mathbf{B}\boldsymbol{\sigma}$

$$f_{\lambda\lambda'}(\mathbf{n},\,\mathbf{n}') = w_{\lambda'}^{\prime\dagger}(a + i\mathbf{B}\boldsymbol{\sigma})w_{\lambda}, \quad \mathbf{B} = \beta\,\frac{\mathbf{n}\times\mathbf{n}'}{(\mathbf{n}-\mathbf{n}')^2},$$

$$\beta = \frac{\mu_n Z e^2}{m_p c^2} = -Z \times 2.94 \times 10^{-16}\ \text{cm},$$

$$\mathbf{n} = \mathbf{p}/p \text{ and } \mathbf{n}' = \mathbf{p}'/p \qquad \mu_n = -1.91$$

For thermal neutrons with the energies ~ 25 meV and ${}^{197}_{79}\mathrm{Au}$

$$\varepsilon \equiv |\beta/a| \approx 0.03, \quad |(\mathrm{Im}\,a)/a| \approx 2\times 10^{-4}$$

The standard cross section summed over spin states of final neutrons:

$$\frac{d\sigma^{(\mathrm{st})}(\mathbf{e}_z,\,\mathbf{n}',\,\boldsymbol{\zeta})}{d\Omega'} = |a|^2 + \frac{1}{4}[\beta\cot(\theta'/2)]^2 - \beta\,\zeta_\perp\,(\mathrm{Im}\,a)\,\cot(\theta'/2)\sin(\varphi'-\varphi_\zeta)$$

Transverse polarization of the incoming neutron (↑ $\zeta_\perp$)

No sensitivity to:

1. The neutron's helicity
2. Re a

J. Schwinger, Phys. Rev. **73**, 407 (1948)

Introduction: twisted photons, vortex electrons, atoms, etc.

- Vortex ions: generation strategies
- Vortex beams via generalized measurements
- Conclusion

For a twisted neutron

$$W_{\lambda}^{(m)}(\theta, \theta', \varphi', \mathbf{b}) = \sum_{\lambda'} \left| F_{\lambda\lambda'}^{(m)}(\theta, \theta', \varphi', \mathbf{b}) \right|^2 = \frac{1}{2}\, \Sigma^{(m)} + \lambda\, \Delta^{(m)}$$

Depends on Re a!

Neutron helicity!

Helicity asymmetry:

$$A_{\lambda} = \frac{W_{\lambda=1/2}^{(m)} - W_{\lambda=-1/2}^{(m)}}{W_{\lambda=1/2}^{(m)} + W_{\lambda=-1/2}^{(m)}} = \frac{\Delta^{(\mathrm{m})}}{\Sigma^{(\mathrm{m})}}$$

For a target of a finite width,
we average the probability with

$$n(\mathbf{b} - \mathbf{b}_t) = \frac{1}{2\pi \sigma_t^2}\, e^{-(\mathbf{b}-\mathbf{b}_t)^2/(2\sigma_t^2)}$$

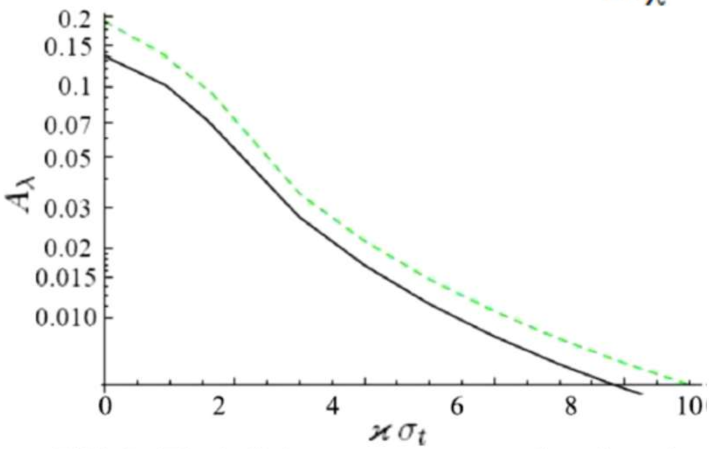


FIG. 9. The helicity asymmetry as a function of $\varkappa\sigma_t$ where σ_t is a width of the $^{197}_{79}$Au mesoscopic target for $m = 5/2$, $\theta' = 0.04$ rad, $\theta = 0.07$ rad, and $\varepsilon = 0.03$, $b_t = \varphi_t = 0$. The case Im $a > 0$ is shown by the black solid line, while the case Im $a < 0$ is shown by the green dashed line.

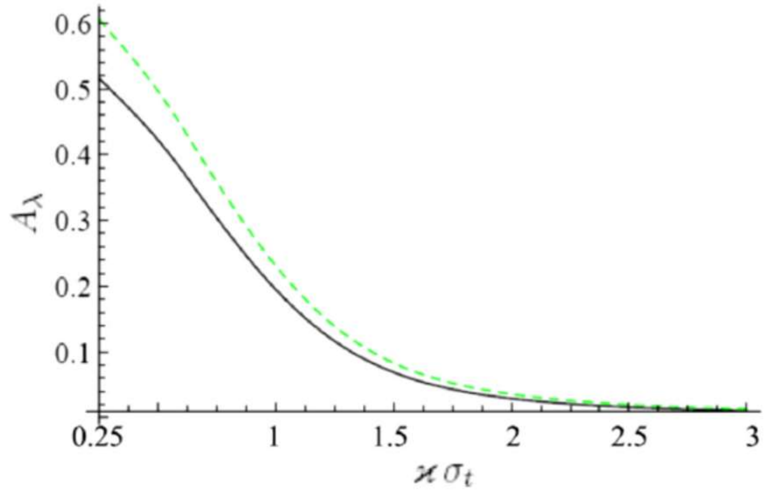


FIG. 8. The helicity asymmetry as a function of $\varkappa\sigma_t$ where σ_t is a width of the $^{197}_{79}$Au mesoscopic target for $m = 1/2$, $\theta' = 0.03$ rad, $\theta = 0.06$ rad, and $\varepsilon = 0.03$, $b_t = \varphi_t = 0$. The case Im $a > 0$ is shown by the black solid line, while the case Im $a < 0$ is shown by the green dashed line.

Afanasev, DK, Serbo,
PRC **103**, 054612 (2021)

Introduction: twisted matter waves

RESEARCH

RESEARCH ARTICLE

QUANTUM PHYSICS

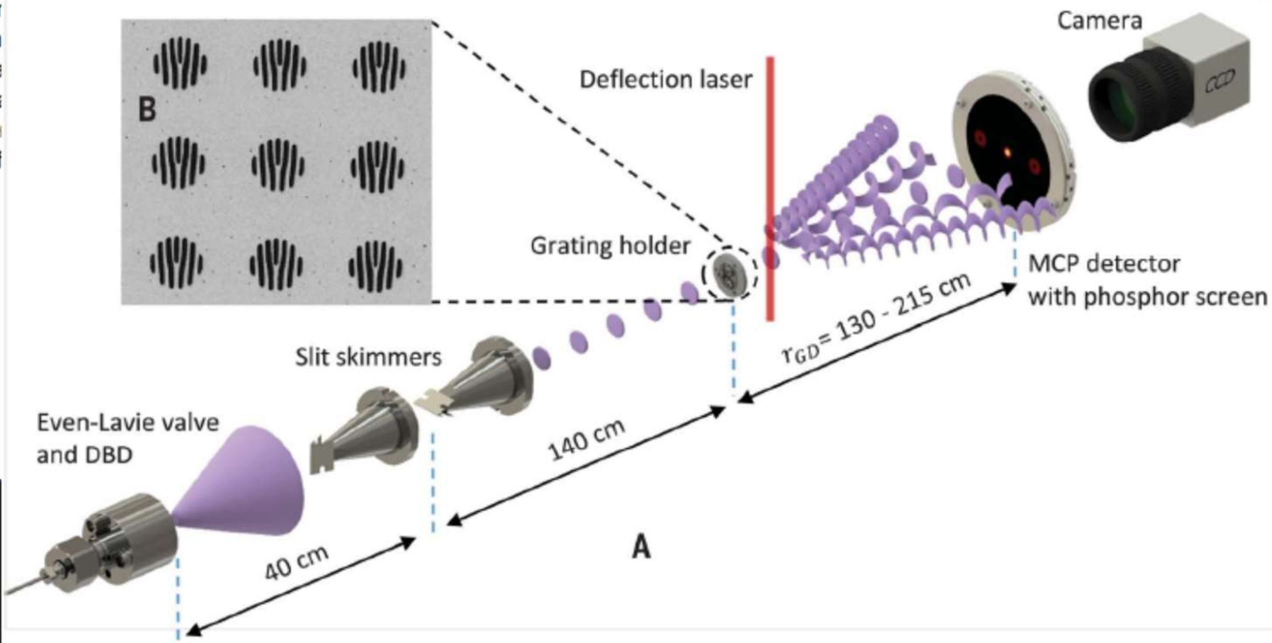
Vortex beams of atoms and molecules

Alon Luski[1]†, Yair Segev[1]†‡, Rea David[1], Ora Bitton[1], Hila Nadler[1], A. Ronny Barnea[2], Alexey Gorlach[3], Ori Cheshnovsky[2], Ido Kaminer[3], Edvardas Narevicius[1]*

Angular momentum plays a central role in quantum mechanics, recurring in every length scale from the microscopic interactions of light and matter to the macroscopic behavior of superfluids. Vortex beams, carrying intrinsic orbital angular momentum (OAM), are now regularly gener photons and electrons. Thus far, the creation of a vortex beam of a demonstrated experimentally. We present vortex beams of atoms a supersonic beams of helium atoms and dimers off transmission gra be applied to most atomic and molecular gases. Our results may ope additional degree of freedom of OAM to probe collisions and alter f

Wavelength = 90 pm,
Transverse coherence length ~ 840 nm

August 2021!



Luski et al., Science 373, 1105 (2021)

Introduction: twisted photons, vortex electrons, atoms, etc.

- Vortex ions: generation strategies

- Vortex beams via generalized measurements

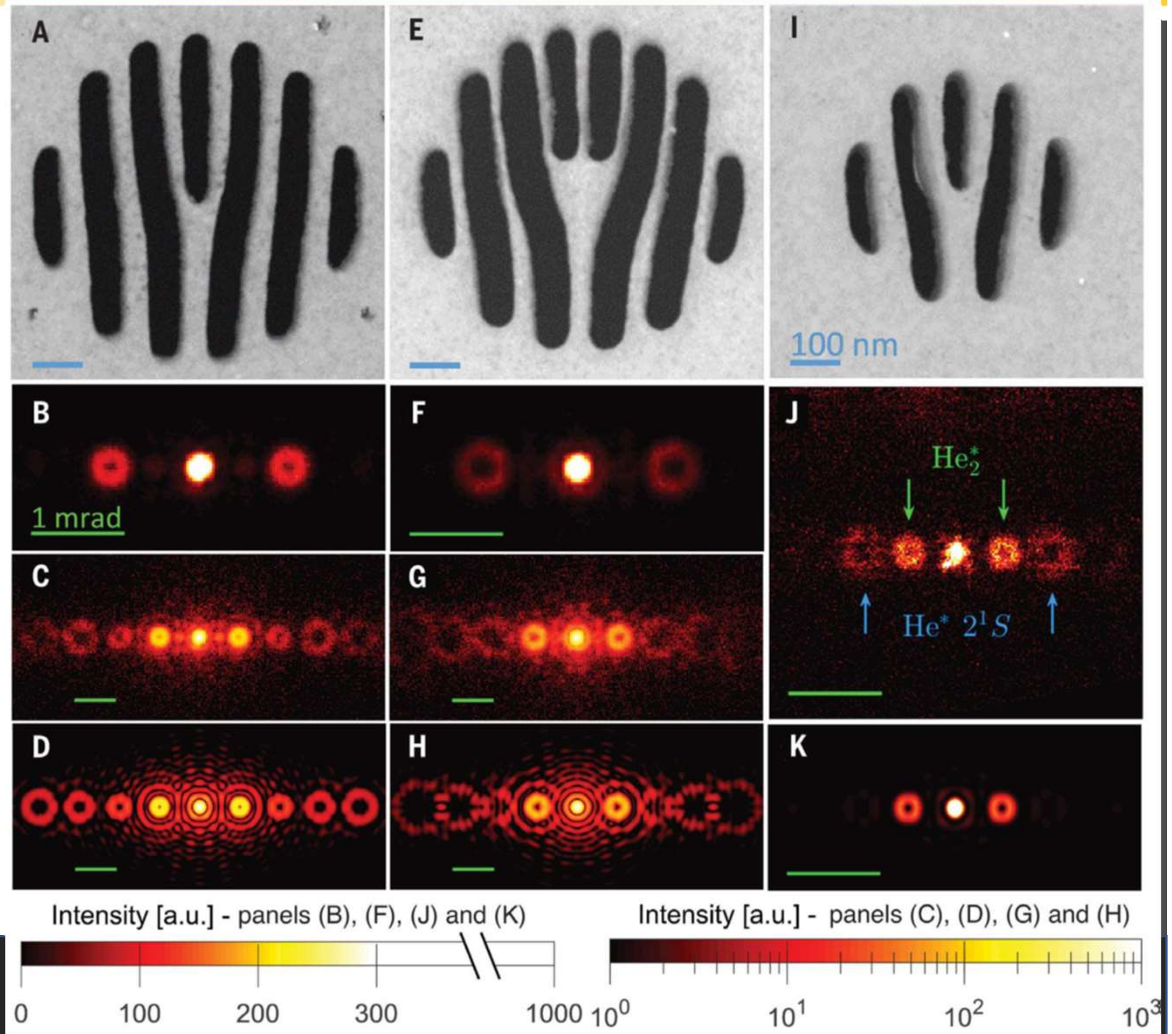
- Conclusion

Introduction: twisted matter waves

Helium and neon atoms and dimers

Wavelength = 90 pm,

Transverse coherence length ~ 840 nm



Luski et al., Science 373, 1105 (2021)

Introduction: twisted photons, vortex electrons, atoms, etc.

- Vortex ions: generation strategies
- Vortex beams via generalized measurements
- Conclusion

Angular-momentum-dominated beams

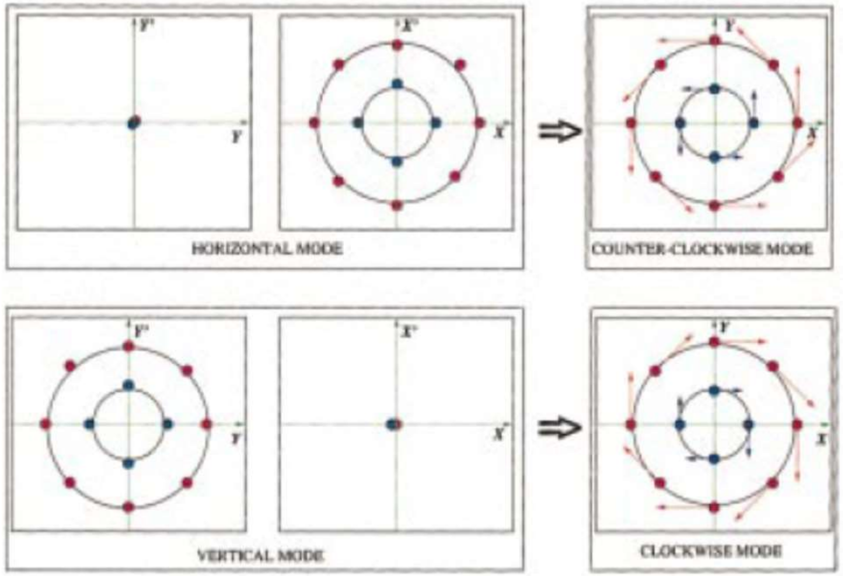
Planar-to-circular beam adapters:
analogous to Hermite-Gaussian → Laguerre-Gaussian conversion of light

- Round beams for circular colliders:

elimination of betatron resonances, increase of the beam lifetime.

- Flat beams for linear colliders:

to increase the luminosity and to suppress the beamstrahlung, and to enhance the efficiency of generation of em radiation from X-rays to THz (say, for Smith-Purcell radiation).



Burov A, Nagaitsev S and Derbenev Y, *Phys. Rev.* E **66** 016503, 2002

The Busch theorem: a charged particle in magnetic field gets vorticity

$$\hat{H} = \frac{(\hat{\boldsymbol{p}}^{\rm kin})^2}{2m} = \frac{(\hat{\boldsymbol{p}}^{\rm can})^2}{2m} - \omega_{\rm L}\hat{L}_z^{\rm can} + \frac{m}{2}\omega_{\rm L}^2\rho^2$$

$$\hat{\boldsymbol{p}}^{\rm can} = \hat{\boldsymbol{p}}^{\rm kin} + e\boldsymbol{A} = -{\rm i}\nabla$$

$$\hat{\boldsymbol{L}}^{\rm can} = \boldsymbol{r} \times \hat{\boldsymbol{p}}^{\rm can} \quad \text{and} \quad \hat{\boldsymbol{L}}^{\rm kin} = \boldsymbol{r} \times \hat{\boldsymbol{p}}^{\rm kin}$$

$$\langle \hat{L}_z^{\rm kin}\rangle = \ell - m\omega_{\rm L}\langle\rho^2\rangle = \ell - 2\,{\rm sgn}(e)\frac{\langle\rho^2\rangle}{\rho_{\rm H}^2} \qquad \rho_{\rm H} = \sqrt{\frac{4}{|e|H}} = 2\lambda_{\rm c}\sqrt{\frac{H_{\rm c}}{H}}$$

In quantum realm, the canonic OAM is an integer:

$$\langle \hat{L}_z^{\rm can}\rangle = \ell, \quad \ell = 0, \pm1, \pm2, \dots \qquad (\hbar=1)$$

The Busch theorem:
a charged particle in magnetic field gets vorticity

$$\langle \hat{L}_z^{\mathrm{kin}} \rangle = 0$$

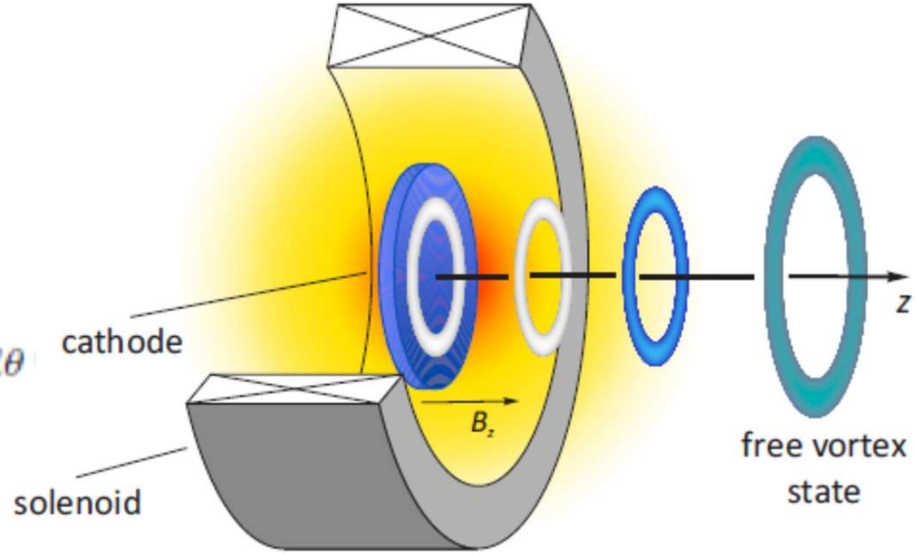
$$\ell = \frac{eH}{2}\langle \rho^2 \rangle = 2\,\mathrm{sgn}(e)\,\frac{\langle \rho^2 \rangle}{\rho_{\mathrm{H}}^2} = \frac{1}{2}\,\mathrm{sgn}(e)\,\frac{\langle \rho^2 \rangle}{\lambda_{\mathrm{c}}^2}\,\frac{H}{H_{\mathrm{c}}},$$

The flux of the field through the area of the beam (classical) or of the wave packet (quantum):

$$\langle \Phi \rangle = H\pi \langle \rho^2 \rangle$$

Akin to the Aharonov-Bohm effect:

$$\Psi \to \Psi \exp\left\{ i\theta \frac{q}{2\pi\hbar}\left\langle \oint A dl \right\rangle \right\} = \Psi\, e^{i\ell\theta}$$



Introduction: twisted photons, vortex electrons, atoms, etc.

- Vortex ions: generation strategies

- Vortex beams via generalized measurements

- Conclusion

6. *Berechnung der Bahn von Kathodenstrahlen im axialsymmetrischen elektromagnetischen Felde;* *von H. Busch*

Vor einiger Zeit habe ich eine Methode der e/m-Bestimmung angegeben[1]), die — ursprünglich nur zu Unterrichtszwecken ausgearbeitet — sich im Laufe der Versuche als sehr geeignet zu Präzisionsmessungen erwies.[2]) Im folgenden sollen die theoretischen Grundlagen der Methode mitgeteilt werden.

Das Meßverfahren beruht auf der bekannten Erscheinung, daß ein von einem Punkte P ausgehendes divergentes Kathodenstrahlbündel durch ein longitudinales, d. h. parallel zur Bündelachse gerichtetes Magnetfeld wieder in einem Punkte P' vereinigt, „fokussiert" wird. Aus der Entfernung l zwischen Brennpunkt P' und Ausgangspunkt P in Verbindung mit der Stärke $\mathfrak{H}$ des Magnetfeldes erhält man eine Beziehung zwischen der Elektronengeschwindigkeit v und ihrer spezifischen Ladung $\eta = \frac{e}{m}$, die, in üblicher Weise mit einer zweiten, etwa aus dem von den Elektronen durchfallenen Entladungspotential V zu gewinnenden Gleichung kombiniert, η und v einzeln zu berechnen gestattet.

Im Falle eines homogenen Magnetfeldes ist jene Beziehung sehr einfach; hier bilden die Elektronenbahnen die bekannten regelmäßigen Schraubenlinien und die besagte Beziehung lautet:

$$l = \frac{2\pi v}{\eta \mathfrak{H}} \cos \alpha, \tag{1}$$

worin α den Winkel bedeutet, den die Anfangsrichtung der Elektronenbahn mit $\mathfrak{H}$ bildet.[3])

1) H. Busch, Physik. Ztschr. 23. S. 438. 1922.

2) Eine solche Präzisionsbestimmung ist im hiesigen Physikalischen Institut im Gange und steht kurz vor dem Abschluß.

3) Zu beachten ist, daß wegen des Faktors $\cos \alpha$ die Abbildung des Punktes P in P' — in der Sprache der geometrischen Optik — nicht

Busch H 1926 Berechnung der Bahn von Kathodenstrahlen im axialsymmetrischen elektromagnetischen Felde *Ann. Phys.* **386** 974

Angular-momentum-dominated beams

PHYSICAL REVIEW ACCELERATORS AND BEAMS **21,** 014201 (2018)

Extension of Busch's theorem to particle beams

L. Groening* and C. Xiao
GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt D-64291, Germany

M. Chung†
Ulsan National Institute of Science and Technology, Ulsan 44919, Republic of Korea

(Received 24 July 2017; published 10 January 2018)

In 1926, H. Busch formulated a theorem for one single charged particle moving along a region with a longitudinal magnetic field [H. Busch, Berechnung der Bahn von Kathodenstrahlen in axial symmetrischen electromagnetischen Felde, Z. Phys. **81**, 974 (1926)]. The theorem relates particle angular momentum to the amount of field lines being enclosed by the particle cyclotron motion. This paper extends the theorem to many particles forming a beam without cylindrical symmetry. A quantity being preserved is derived, which represents the sum of difference of eigenemittances, magnetic flux through the beam area, and beam rms-vorticity multiplied by the magnetic flux. Tracking simulations and analytical calculations using the generalized Courant–Snyder formalism confirm the validity of the extended theorem. The new theorem has been applied for fast modeling of experiments with electron and ion beams on transverse emittance repartitioning conducted at FERMILAB and at GSI. Thus far, developments of beam emittance manipulations with electron or ion beams have been conducted quite decoupled from each other. The extended theorem represents a common node providing a short connection between both.

DOI: 10.1103/PhysRevAccelBeams.21.014201

III. BEAM VORTICITY

We choose the ansatz assigning $\mathcal{W}_A$ to the rotation $(\vec{\nabla}\times)$ of the mean, i.e., averaged over (x', y') space, beam angle $\vec{\bar{r}}'(x, y, s)$ being integrated over the beam rms-area, and finally multiplied by the twofold beam rms-area:

$$\mathcal{W}_A = 2A \int_A \left[\vec{\nabla} \times \vec{\bar{r}}'(x, y, s)\right] \cdot d\vec{A} \qquad (15)$$

being equivalent to

$$\mathcal{W}_A = 2A \oint_C \vec{\bar{r}}'(x, y, s) \cdot d\vec{C}, \qquad (16)$$

Angular-momentum-dominated beams

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 7, 123501 (2004)

Generation of angular-momentum-dominated electron beams from a photoinjector

Y.-E Sun,[1,*] P. Piot,[2,†] K.-J. Kim,[1,3] N. Barov,[4,‡] S. Lidia,[5] J. Santucci,[2] R. Tikhoplav,[6] and J. Wennerberg[2,§]
[1]*University of Chicago, Chicago, Illinois 60637, USA*
[2]*Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA*
[3]*Argonne National Laboratory, Argonne, Illinois 60439, USA*
[4]*Northern Illinois University, DeKalb, Illinois 60115, USA*
[5]*Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[6]*University of Rochester, Rochester, New York 14627, USA*
(Received 2 November 2004; published 22 December 2004)

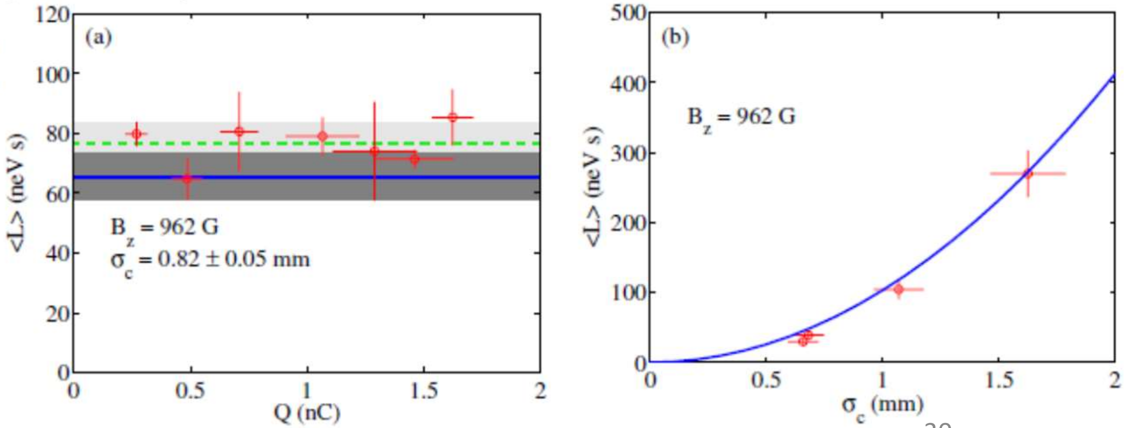
Various projects under study require an angular-momentum-dominated electron beam generated by a photoinjector. Some of the proposals directly use the angular-momentum-dominated beams (e.g., electron cooling of heavy ions), while others require the beam to be transformed into a flat beam (e.g., possible electron injectors for light sources and linear colliders). In this paper we report our experimental study of an angular-momentum-dominated beam produced in a photoinjector, addressing angular momentum on initial conditions. We also briefly discuss the removal of angu results of the experiment, carried out at the Fermilab/NICADD Photoinjector Labora in good agreement with theoretical and numerical models.

DOI: 10.1103/PhysRevSTAB.7.123501 PACS numbers: 29.27.

Up to <L> ~ 10^8*ħ!

- UV laser, cesium telluride photocathode
- e: **4 MeV/c → 16 MeV/c** after the booster cavity

<L> is conserved during the acceleration!



Canonical angular momentum versus charge (a) and photocathode drive-laser beam spot size (b).

Magnetized stripping foil technique for ions

PRL 113, 264802 (2014) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2014

Experimental Proof of Adjustable Single-Knob Ion Beam Emittance Partitioning

L. Groening,* M. Maier, C. Xiao, L. Dahl, P. Gerhard, O. K. Kester, S. Mickat, H. Vormann, and M. Vossberg
GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt D-64291, Germany

M. Chung
Ulsan National Institute of Science and Technology, Ulsan 698-798, Republic of Korea
(Received 26 September 2014; published 30 December 2014)

The performance of accelerators profits from phase-space tailoring by coupling of degrees of freedom. Previously applied techniques swap the emittances among the three degrees but the set of available emittances is fixed. In contrast to these emittance exchange scenarios, the emittance transfer scenario presented here allows for arbitrarily changing the set of emittances as long as the product of the emittances is preserved. This Letter is the first experimental demonstration of transverse emittance transfer along an ion beam line. The amount of transfer is chosen by setting just one single magnetic field value. The envelope functions (beta) and slopes (alpha) of the finally uncorrelated and repartitioned beam at the exit of the transfer line do not depend on the amount of transfer.

Nitrogen: Z from +3 to +7

The foil (carbon, 200 $\mu g/cm^2$, 30 mm in diameter)

The energies: from 10s to 100s MeV/u

Nuclear Instruments and Methods in Physics Research A 767 (2014) 153–158

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

ELSEVIER

Minimization of the emittance growth of multi-charge particle beams in the charge stripping section of RAON

CrossMark

Ji-Gwang Hwang [a], Eun-San Kim [a,*], Hye-Jin Kim [b,*], Dong-O Jeon [b]

[a] Department of Physics, Kyungpook National University, Daegu 702-701, Korea
[b] Rare Isotope Science Project, Institute for Basic Science, Jeonmin-dong, Yuseong-gu, Daejeon, Korea

ARTICLE INFO

Article history:
Received 20 April 2014
Received in revised form
26 July 2014
Accepted 13 August 2014
Available online 21 August 2014

Keywords:
Rare Isotope Science Project
Beam dynamics for multicharge particles
Emittance growth in dispersive section
Correction of high-order aberration

ABSTRACT

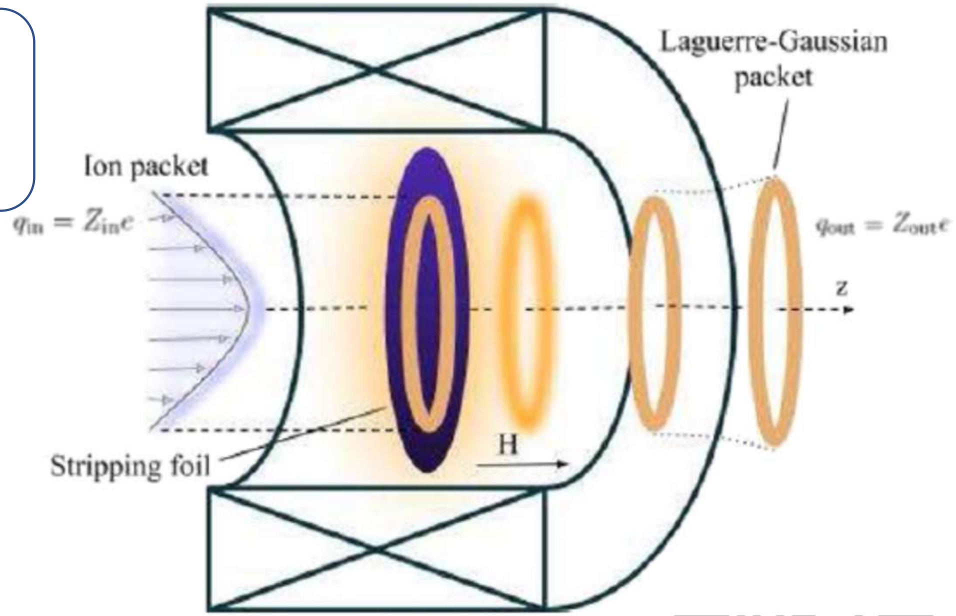
The charge stripping section of the Rare isotope Accelerator Of Newness (RAON), which is one of the critical components to achieve a high power of 400 kW with a short linac, is a source of transverse emittance growth. The dominant effects are the angular straggling in the charge stripper required to increase the charge state of the beam and chromatic aberrations in the dispersive section required to separate the selected ion beam from the various ion beams produced in the stripper. Since the main source of transverse emittance growth in the stripper is the angular straggling, it can be compensated for by changing the angle of the phase ellipse. Therefore the emittance growth is minimized by optimizing the Twiss parameters at the stripper. The emittance growth in the charge selection section is also minimized by the correction of high-order aberrations using six sextupole magnets. In this paper, we present a method to minimize the transverse emittance growth in the stripper by changing the Twiss parameters and in the charge selection section by using sextupole magnets.

Uranium: Z from +33-34 to +77-81

Magnetized stripping foil technique for vortex ions

The quantum Busch theorem for ions:

$$\ell = \frac{q_{\text{out}} - q_{\text{in}}}{2} H \langle \rho^2 \rangle = (Z_{\text{out}} - Z_{\text{in}}) \frac{eH}{2} \langle \rho^2 \rangle$$



Requirements:

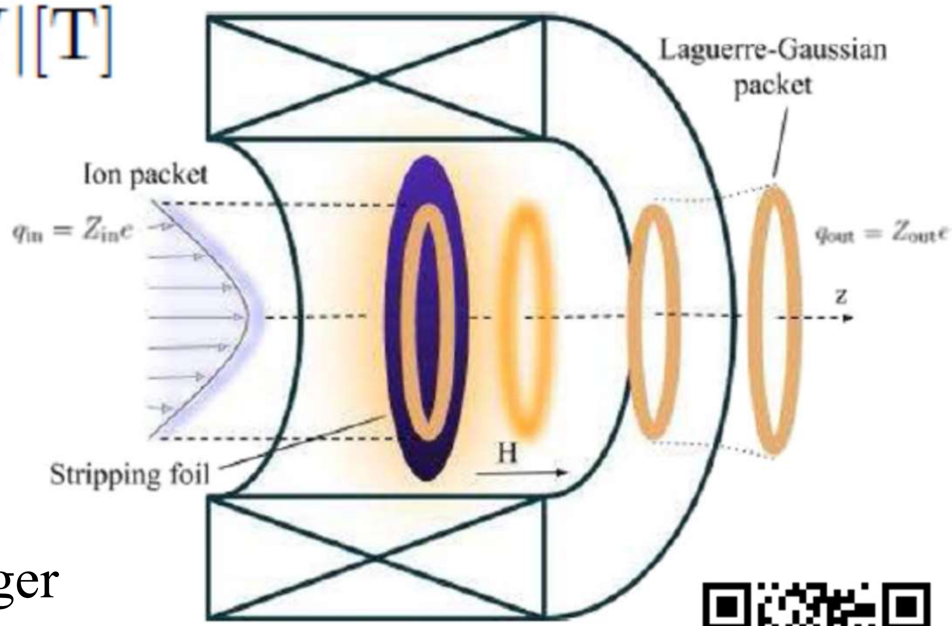
- Negligible space charge
- Large transverse coherence (much easier than for cathode emission!)
- No emittance degradation: small scattering in the target

K. Floettmann, D.K., PRA **102**, 043517 (2020); D.K., New J. Phys. **23** (2021) 033048

Magnetized stripping foil technique for vortex ions

The numerical estimate:

$$|\ell| \approx 1.5 \times 10^{-3}\, |Z_{\rm out} - Z_{\rm in}|\, \langle \rho^2 \rangle [{\rm nm}^2]\, |H| [{\rm T}]$$



Requirements:

- Fields of H ~ 1 T or larger
- The transverse coherence of the ion ~ 100 nm or larger

What are the distances to achieve this transverse coherence?

Electron wave packets: reference numbers

The rms-radii in SEMs, TEMs, electron accelerators, photo-electrons, etc.:

$$\sqrt{\langle \rho^2 \rangle} \sim \mathbf{1\text{-}100\ nm}$$

Example 1: a radius of the ground Landau state in the field H ~ 0.1-10 T is

$$\rho_{\mathrm{H}} = \sqrt{\frac{4}{|e|H}} \sim \mathbf{10\text{-}100\ nm}$$

Example 2: the transverse coherence length of an electron from a Tungsten photo-cathode or a field-emitter (at room temperature) is*

$$\sqrt{\langle \rho^2 \rangle} \sim \mathbf{0.5\ \text{-}\ 1\ nm}$$

*Ehberger D, et al., Phys. Rev. Lett. **114,** 227601 (2015)

Proton/light ion wave packets

Naively, the proton wave packets are m_p/m_e ~ 1800 times narrower:

$$\sqrt{\langle\rho^2\rangle} \sim \textbf{1-100 pm}$$

The general spreading law is

$$\langle\rho^2\rangle(t) = \langle\rho^2\rangle(0) + \frac{\partial\langle\rho^2\rangle(0)}{\partial t}\, t + \langle u_\perp^2\rangle\, t^2,$$

For the Laguerre-Gaussian beam this leads in the far-field (<z> >> z_R) to

$$\langle z\rangle = \frac{\langle p\rangle}{2n+|\ell|+1}\sqrt{\langle\rho^2\rangle(\langle z\rangle)}\sqrt{\langle\rho^2\rangle(0)} \equiv \frac{\rho\rho_0}{\lambda}\frac{1}{2n+|\ell|+1},$$

For the Gaussian mode, n= ell=0, this is simply the van Cittert–Zernike theorem!

D.K., New J. Phys. **23** (2021) 033048

Transverse coherence length of single ions

The conservative estimate

(say, for the beams available at GSI)

Consider $^{14}N^{3+}$ ions with the kinetic energy of 11.45 MeV/u and the wavelength of $\lambda \sim 0.1$ fm

If the initial transverse coherence length of the ion is as large as $\rho_0 \sim 1$ nm

then the micrometer-sized transverse coherence of $\rho \sim 1\,\mu$m

is achieved at the distance

z ~ 10 m!

(or less for pm-scale initial coherence)

One needs a several-meter long straight beam line
of a linac or a storage ring!

Accelerating vortex electrons, protons, ions, etc.

The real **in**homogeneous fields can be approximated as:

$$H_z(\rho, z) = H_z(0, z) - \frac{\rho^2}{4} H_z''(0, z) + \mathcal{O}(\rho^4),$$

$$H_\rho(\rho, z) = -\frac{\rho}{2} H'(z) + \mathcal{O}(\rho^3),$$

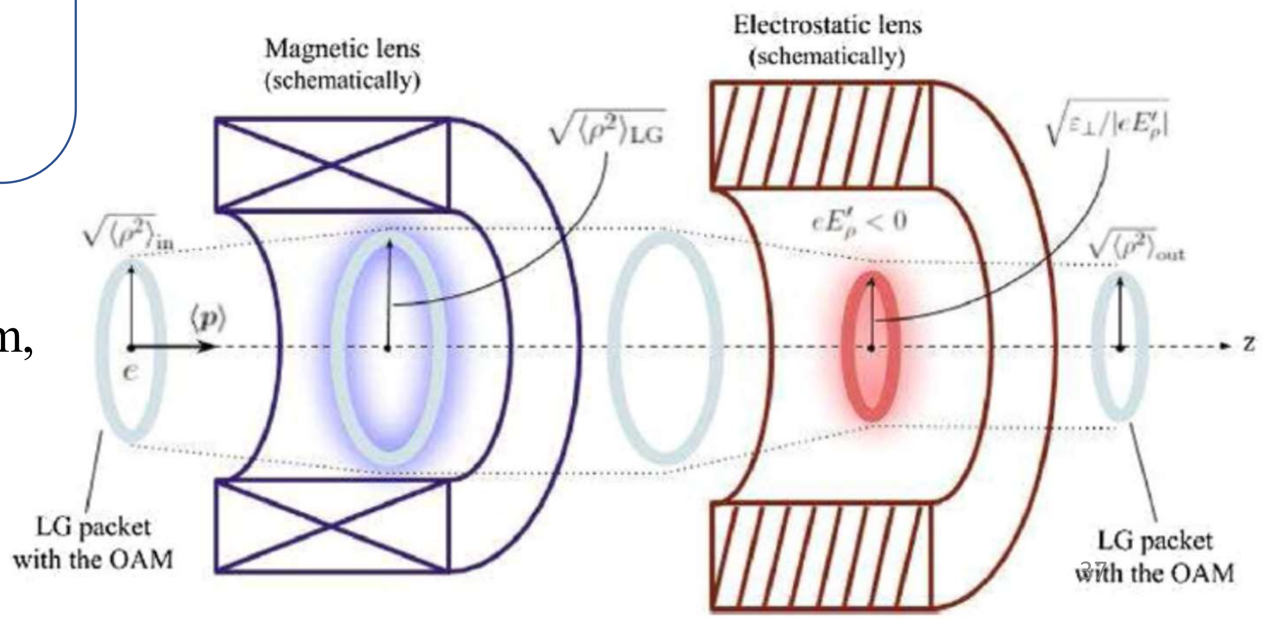
$$\boldsymbol{E} = E_\rho(\rho)\, \boldsymbol{e}_\rho + E_z\, \boldsymbol{e}_z,$$

In linear fields, the OAM, the beam quality, and the emittance are conserved!

The fields may be **in**homogeneous for a beam,

but still **homogeneous**

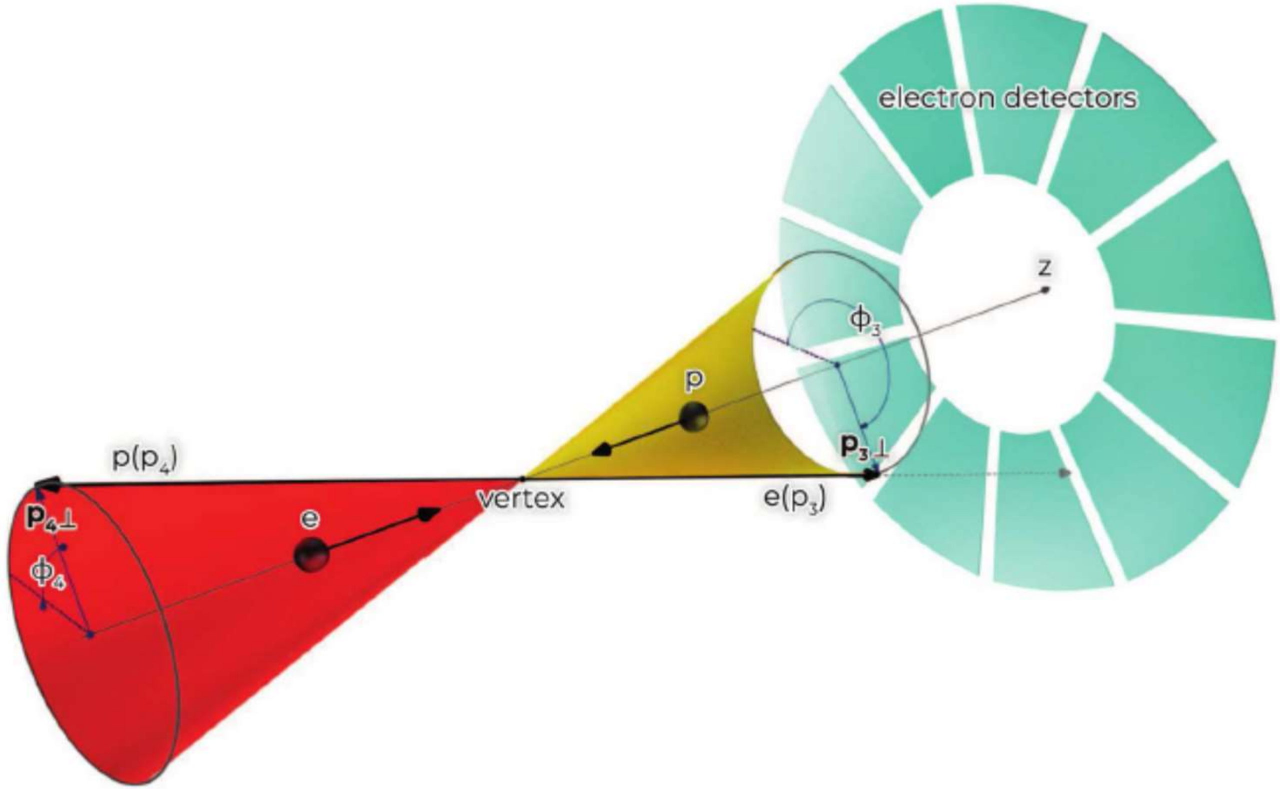
for the ion/proton/electron packet!



Generation of vortex particles via generalized measurements

Consider 2 → 2 scattering/annihilation:

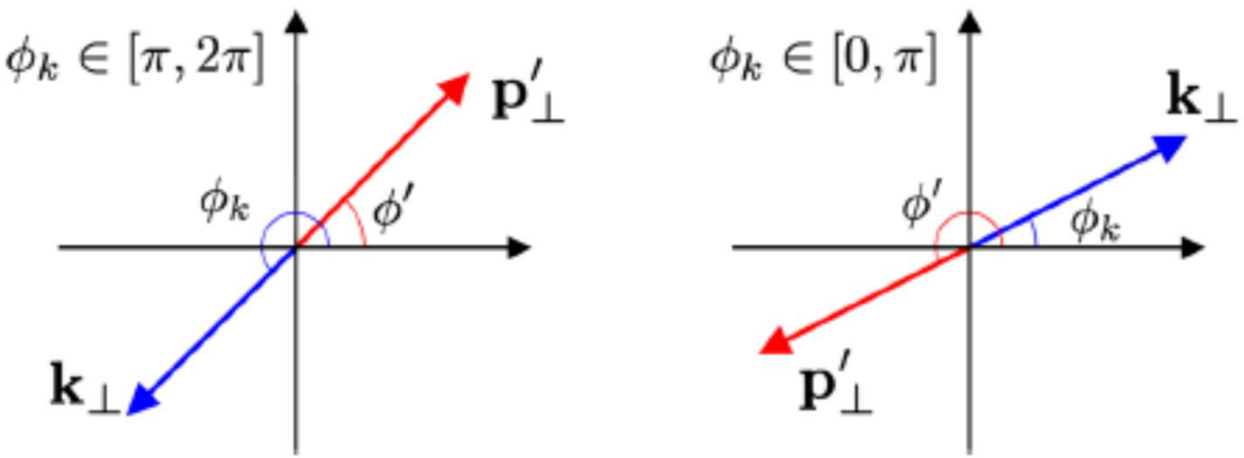
ep → *ep*, *e+mu* → *e+mu*, *gamma+p* → *gamma+p*, etc.



Generation of vortex particles via generalized measurements

The momentum conservation law implies

$$\delta(\mathbf{p}'_\perp + \mathbf{k}_\perp) = \delta(p'_x + k_x)\delta(p'_y + k_y) = \frac{1}{p'_\perp}\,\delta(p'_\perp - k_\perp)\left(\delta(\phi' - (\phi_k - \pi))\Big|_{\phi_k \in [\pi, 2\pi]} + \delta(\phi' - (\phi_k + \pi))\Big|_{\phi_k \in [0, \pi]}\right)$$



If one final particle is detected as a plane wave, the second one automatically turns out to be the plane wave with no vorticity!

Generation of vortex particles via generalized measurements

1. The projective (von Neumann) measurements: the errors are vanishing
2. The generalized (realistic) measurements: some errors can be finite:
   a. Without the loss of information
   b. With the loss of information

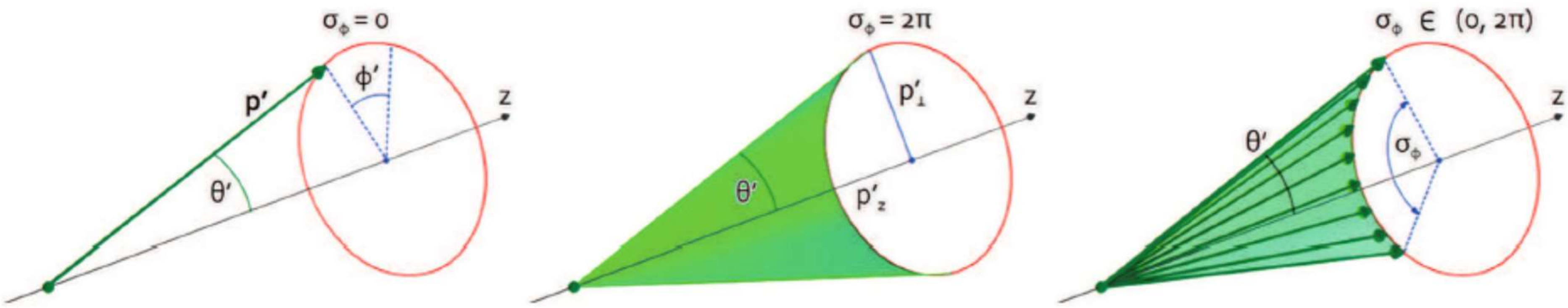
The electron is detected in a state

$$|e'\rangle = \int \frac{d^3 p'}{(2\pi)^3} f_p(\mathbf{p}')|\mathbf{p}', \lambda'\rangle$$

The detector function can be of the Gaussian form:

$$f_p(\mathbf{p}') \propto \prod_i \exp\left\{-(p_i' - \langle p_i \rangle)^2/(2\sigma_i)^2\right\}$$

Generation of vortex particles via generalized measurements



Projective: a plane-wave state

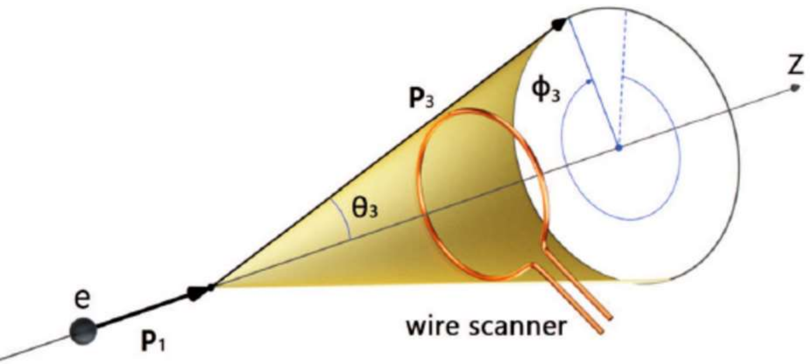
Projective: a Bessel beam

Generalized: a Bessel-like wave packet

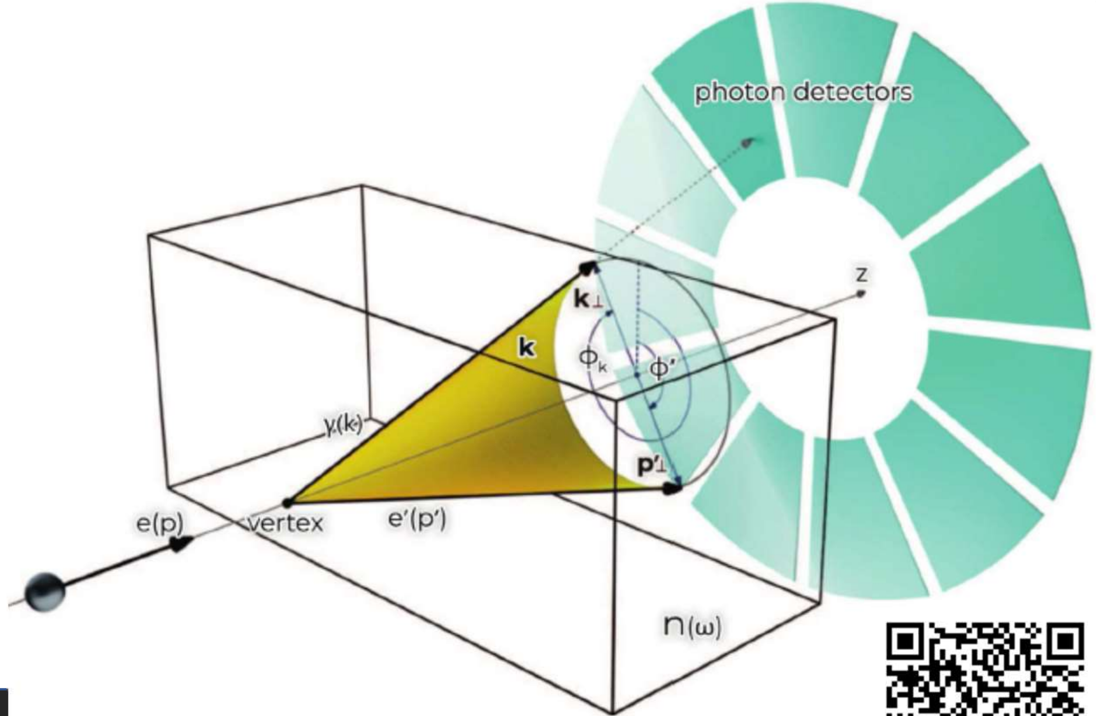
D.K., et al., Eur. Phys. J. C 83, 372 (2023)

Generation of vortex particles via generalized measurements

When the uncertainty approaches 2 pi:



Cherenkov radiation:
the electron scattering angle
is ~ 10^-5 – 10^-6!



D.K., et al., Eur. Phys. J. C 83, 372 (2023)

New effects with vortex atoms and ions

1. Non-dipolar coupling between bound electrons and the center-of-mass
2. Twisted light can also interact with the twisted center-of-mass:

   new transitions allowed, new absorption/emission lines

3. Quantum optical phenomena with twisted atoms/ions: entanglement, interferometry, Schrödinger cat states, etc. – *see also Igor Ivanov, Ann. Phys. (Berlin)* ***2021****, 2100128*
4. Relativistic twisted ions can resonantly scatter twisted photons (Gamma-factory)

   – *see also DK, Serbo, Surzhykov, PRA* ***104****, 023101 (2021)*

5. How to diagnose vorticity of ions?

a.) Interaction of twisted light with a twisted ion

b.) Listen to Anna Maiorova‘s very next talk!

Angular momentum as a degree of freedom in atomic collision

A. Maiorova[1,2], D. Karlovets, S. Fritzsche[1,2,3], A. Surzhykov[4,5,6], Th. Stöhlker[1,2,7]

[1] Helmholtz Institute Jena, Jena, Germany
[2] GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany
[3] Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität Jena, Jena, Germany
[4] Physikalisch–Technische Bundesanstalt, Braunschweig, Germany
[5] Institut für Mathematische Physik, Technische Universität Braunschweig, Braunschweig, Germany
[6] Laboratory for Emerging Nanometrology Braunschweig, Braunschweig, Germany
[7] Institut für Optik und Quantenelektronik, Friedrich-Schiller-Universität Jena, Jena, Germany

Summary

1. The established techniques for angular-momentum dominated beams of ions and electrons can be extended to quantum realm

2. Vortex beam of ions/protons can be generated:
    - By diffraction on fork gratings,
    - Via magnetized stripping foil technique,
    - Via generalized measurements technique

3. Vortex ions/protons/electrons:
    - Can be stored in Penning traps,
    - Can be accelerated in a linac,

4. In a storage ring, the longitudinal vorticity is not conserved.

department of PHYSICS

Thank you for your attention!

Special thanks to Andrei Surzhykov, Dima Glazov, Andrei Volotka, Valery Serbo, Igor Ivanov, Antonino Di Piazza as well to my group: George Sizykh, Alisa Chaikovskaia, Dima Grosman, Ilia Pavlov, and others

d.karlovets@gmail.com
https://physics.itmo.ru/ru/research-group/5430

The studies
are supported by the Russian Science Foundation,
Project No. 23-62-10026
https://rscf.ru/en/project/23-62-10026/