

Global coupled-channel analysis of $e^+e^- \rightarrow c\bar{c}$ processes in $\sqrt{s} = 3.75 - 4.7$ GeV

arXiv: 2312.17658

Satoshi Nakamura (Shandong Univ.)

Collaborators: 彭海平, 周小蓉, 李旭紅 (USTC), 孙振田 (SDU)

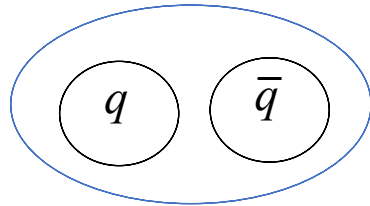
Introduction

Many exotic hadron candidates (often called XYZ) discovered by BESIII, Belle, LHCb, ... , experiments

Establishing the exotic hadrons (existence & structure) is highly controversial issue !

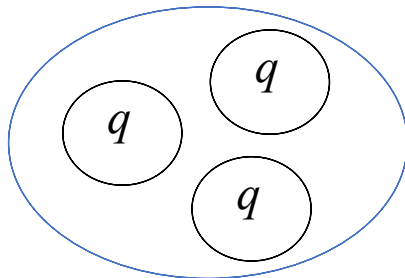
Conventional picture

Meson



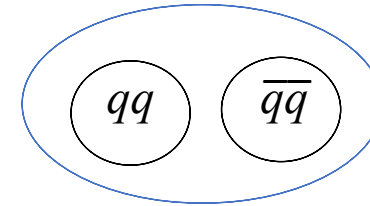
Quark-antiquark

Baryon

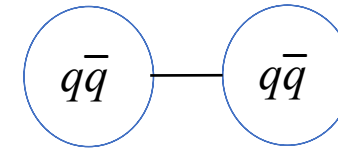


Three-quark

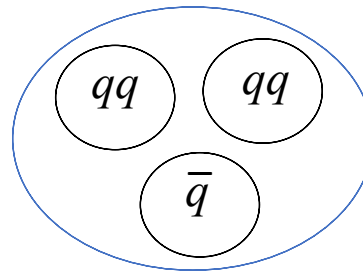
Exotic picture



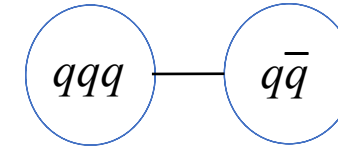
Diquark-antidiquark



meson-meson molecule



Diquark-diquark-antiquark

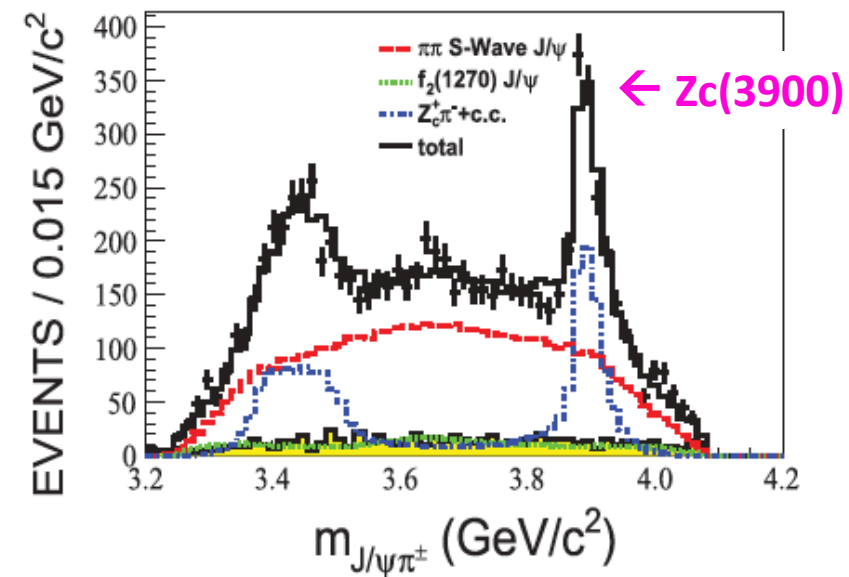
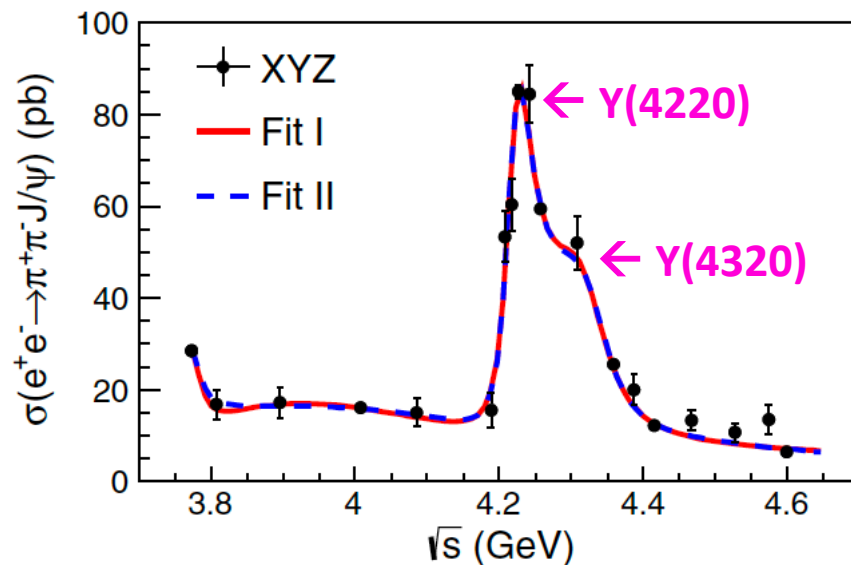


Meson-baryon molecule

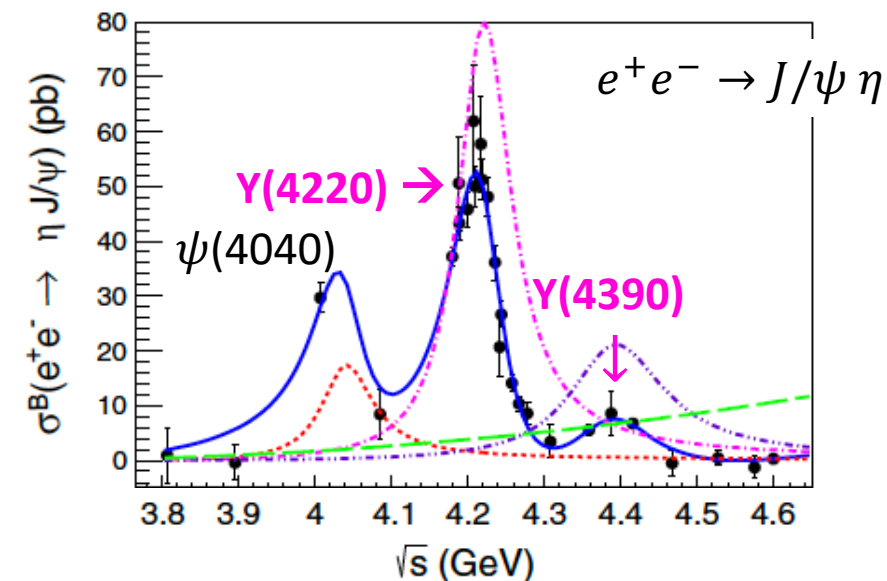
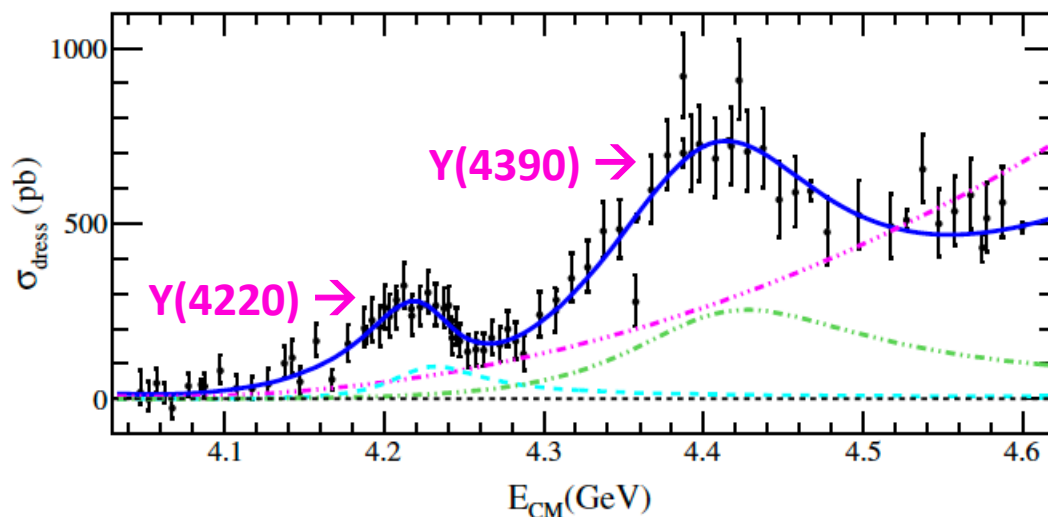
BESIII data for XYZ physics

(only selected ones)

$$e^+e^- \rightarrow J/\psi \pi^+\pi^- \rightarrow$$

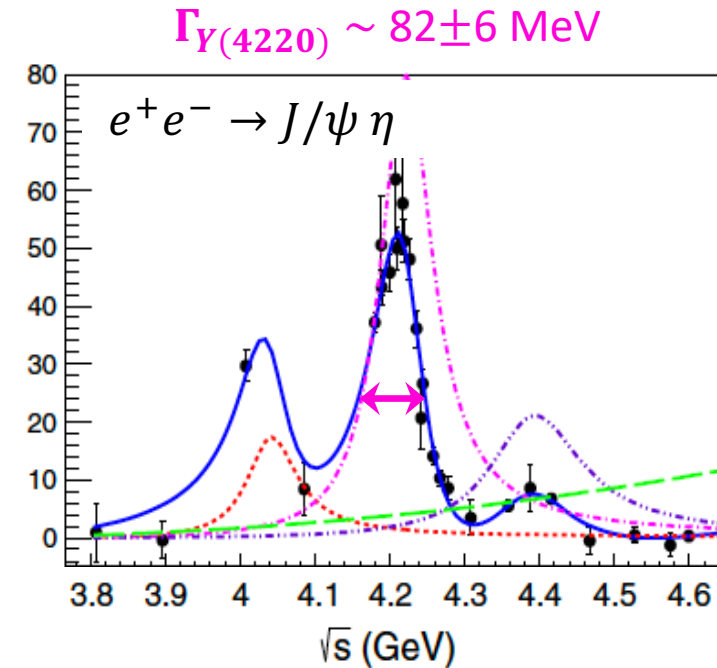
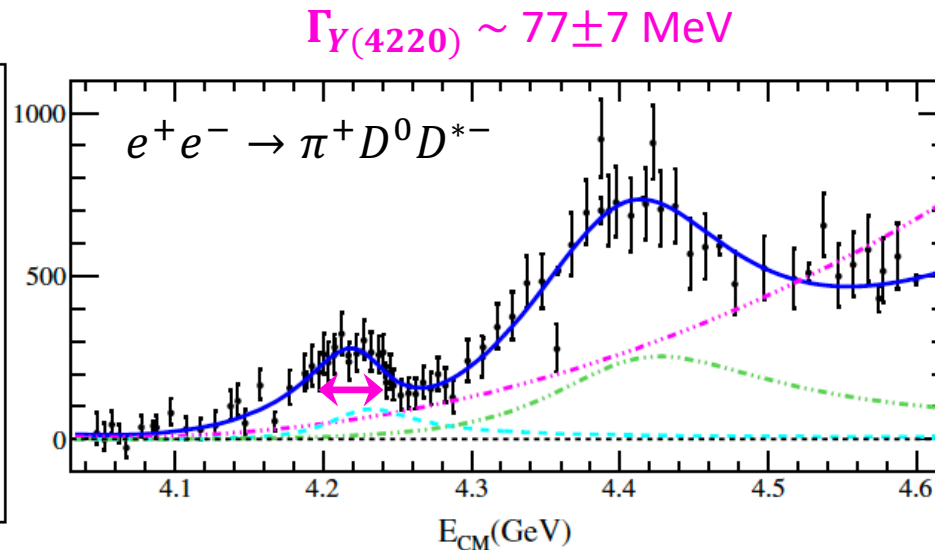
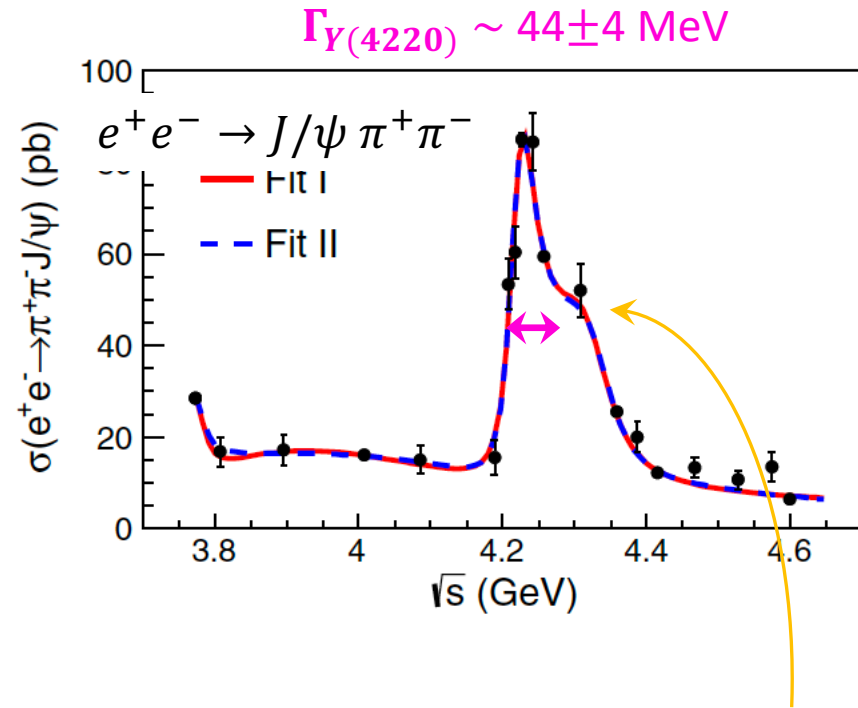


$$e^+e^- \rightarrow \pi^+ D^0 D^{*-}$$



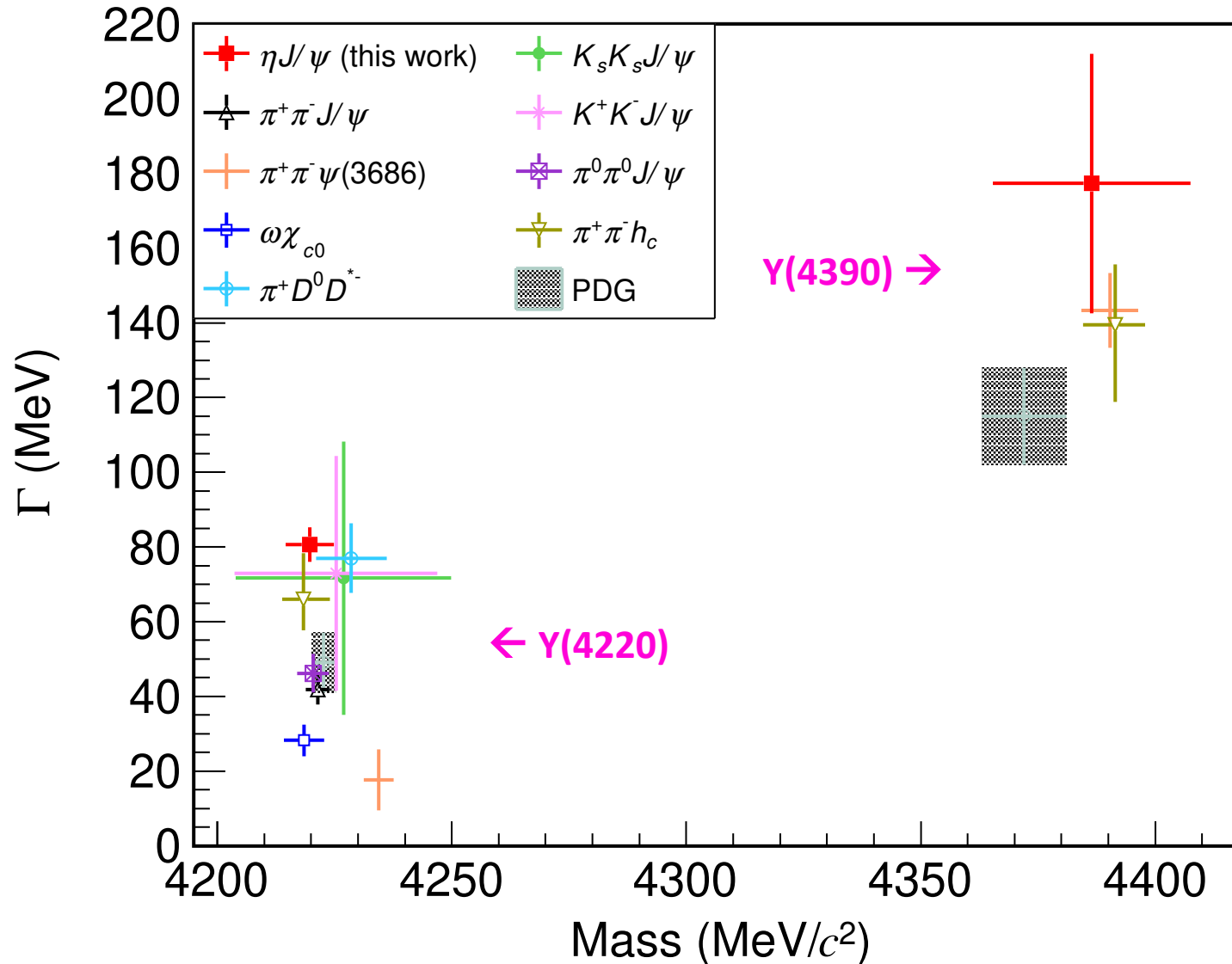
Outstanding question in XYZ physics : Υ width problem

- Why Υ states seem to have different widths for different final states ?



- Why $Y(4320)$ appears only in $e^+e^- \rightarrow J/\psi \pi\pi$?

Outstanding question in XYZ physics : Υ width problem



BESIII, arXiv:2310.03361

How to find solution to Υ width problem ?



Analyze different final states with different models (usual experimental method)

→ no simple relation between resonance parameters from different models → Υ width problem



Analyze different final states simultaneously with a unified model

- * how various charmonia interfere to create different lineshapes in different final states
- * kinematical effects (threshold opening, triangle singularity) change lineshapes in some processes

In other words,

To answer the Υ width problem,

we need to understand:

(i) vector charmonium pole structure (pole locations)

(ii) couplings of the poles with decay channels (residues)

To answer the Y width problem,
we need to understand:

- (i) **vector charmonium pole structure** (pole locations)
- (ii) **couplings of the poles with decay channels** (residues)

Now is the time to address vector charmonium poles !

BESIII accumulated high-quality data for various $e^+e^- \rightarrow c\bar{c}$ processes over wide energy region covering Y

$$e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi \eta^{(\prime)}, \chi_{c0}\omega \quad (\text{two-body final states})$$

$$e^+e^- \rightarrow \pi D^{(*)}\bar{D}^{(*)}, J/\psi\pi\pi, \psi'\pi\pi, h_c\pi\pi, J/\psi K\bar{K} \quad (\text{three-body final states})$$

$$e^+e^- \rightarrow \eta_c\rho\pi \quad (\rho \rightarrow \pi\pi) \quad (\text{four-body final states})$$

→ From analyzing these data simultaneously , possible to reliably extract poles

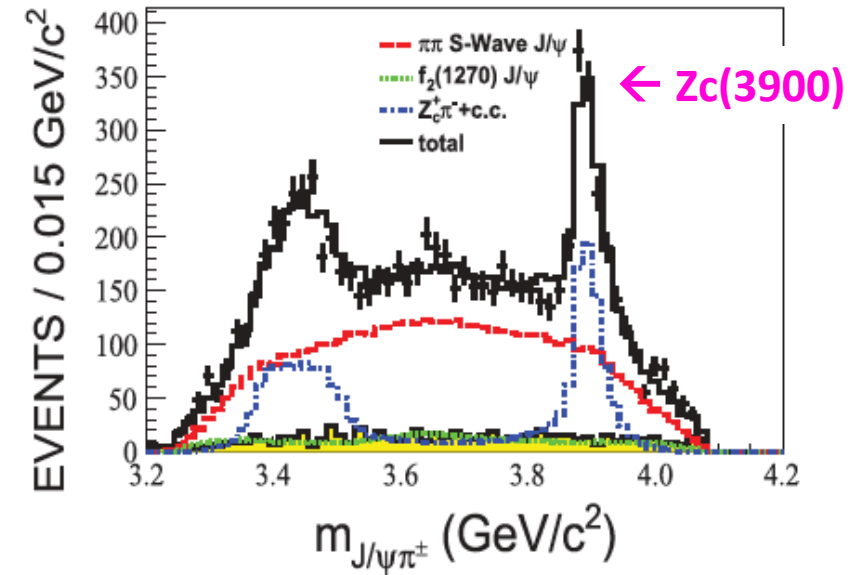
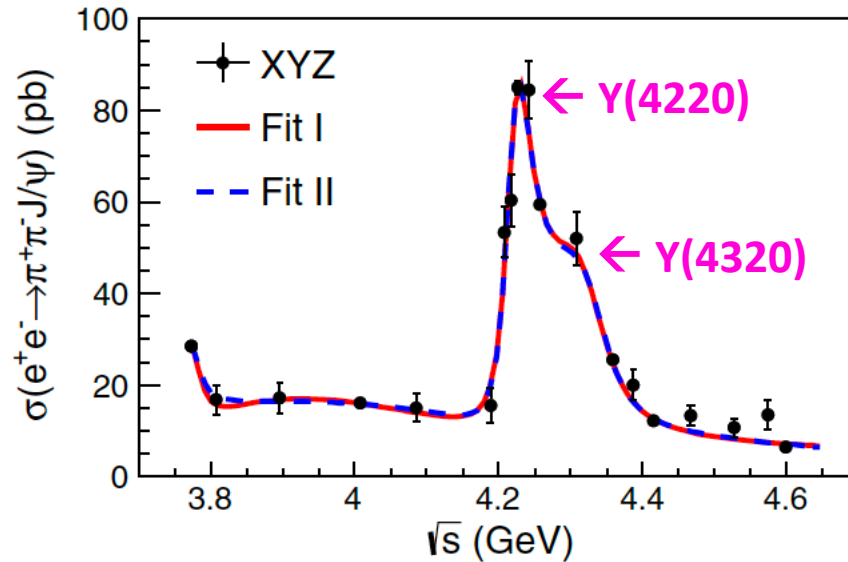
Important not only for Y states but also for well-established charmonium [$\psi(4040)$, $\psi(4160)$, $\psi(4415)$] because:

- Their properties were previously determined by simple Breit-Wigner fit to inclusive ($e^+e^- \rightarrow \text{hadrons}$) R values
- Analyzing precise exclusive data → More detailed and precise information

Understanding Y inevitably involves understanding Z_c

$Z_c(3900)$, $Z_c(4020)$: outstanding exotic candidates including $c\bar{c}u\bar{d}$

$$e^+e^- \rightarrow J/\psi \pi^+\pi^-$$



above process seems partly from $Y \rightarrow Z_c \pi \rightarrow (J/\psi \pi) \pi \rightarrow Y$ and Z_c properties should be highly correlated

Combined $e^+e^- \rightarrow c\bar{c}$ analysis inevitably include the above data with Z_c signals

$\rightarrow$ inevitably address Z_c properties as well as Y

This work

- **Global analysis** of BESIII and Belle data in $3.75 \leq \sqrt{s} \leq 4.7$ GeV with a unified coupled-channel model

$$e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi \eta^{(\prime)}, \chi_{c0}\omega \quad (9 \text{ two-body final states})$$

$$e^+e^- \rightarrow \pi D^{(*)}\bar{D}^{(*)}, J/\psi\pi\pi, \psi'\pi\pi, h_c\pi\pi, J/\psi K\bar{K} \quad (7 \text{ three-body final states})$$

$$e^+e^- \rightarrow \eta_c\rho\pi \quad (\rho \rightarrow \pi\pi) \quad (1 \text{ four-body final states})$$

- Fit both total cross sections and invariant mass distributions
- Extract vector charmonium (ψ , Y) and Z_c poles (mass, width)

Extraction of residues (branching fractions) and solution of Y width problem $\rightarrow$ near-future work

Related works previously done

Three-body model

* M. Cleven, Q. Wang, F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Zhao, PRD 90, 074039 (2014)

Analysis of $e^+e^- \rightarrow \pi D\bar{D}^*$, $J/\psi\pi\pi$, $h_c\pi\pi$ cross section and invariant mass in $4.1 \lesssim \sqrt{s} \lesssim 4.3$ GeV [Y(4230) region]

Pioneering works, but the data were very limited $\rightarrow$ limited conclusions on Y(4230) properties

* L. Detten, C. Hanhart, V. Baru, arXiv:2309.11970

Fitting data in Y(4230) region; more final states than the above

Our analysis includes significantly more
complete dataset
 $\rightarrow$ More reliable conclusion

Breit-Wigner fits to cross section data

* D.-Y. Chen, X. Liu, T. Matsuki, Eur. Phys. J. C 78, 136 (2018)

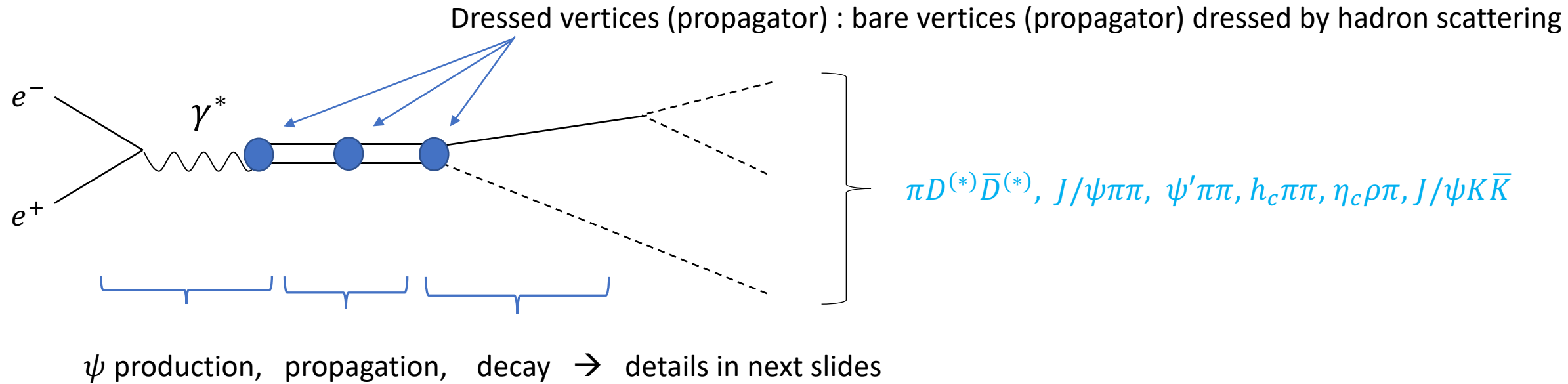
Fitting of $e^+e^- \rightarrow \pi D\bar{D}^*$, $J/\psi\pi\pi$, $h_c\pi\pi$ cross sections $\rightarrow$ Y(4320) and Y(4390) not necessary

* Z.-Y. Zhou, C.-Y. Li, Z. Xiao, arXiv:2304.07052

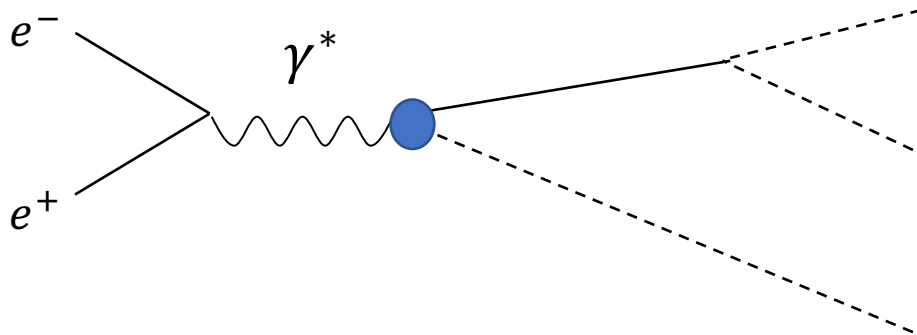
Fitting of $e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}$, $\pi D\bar{D}$ cross sections $\rightarrow$ $\psi(4160)$ is Y(4230)

MODEL

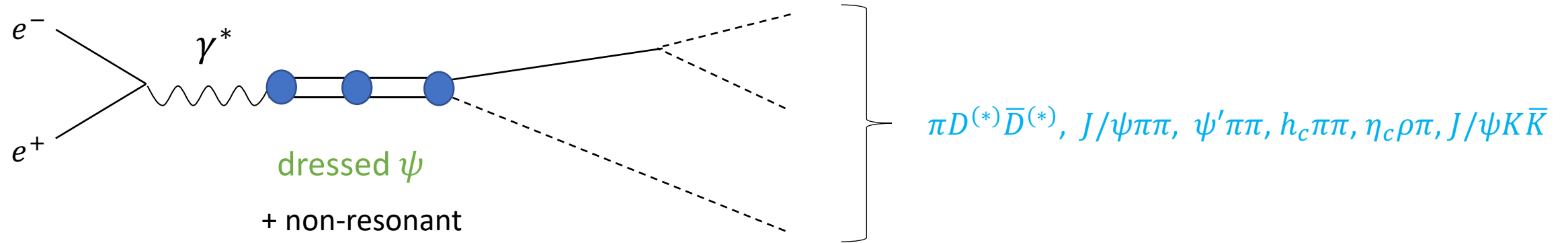
Full amplitude for $e^+e^- \rightarrow \pi D^{(*)} \bar{D}^{(*)}$, $J/\psi \pi \pi$, $\psi' \pi \pi$, $h_c \pi \pi$, $\eta_c \rho \pi$, $J/\psi K \bar{K}$ (three-body final states)



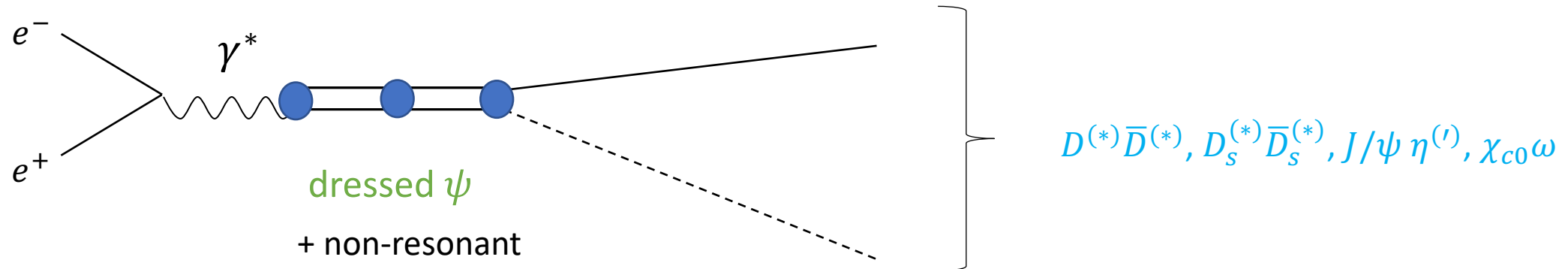
Non-resonant mechanisms are also included



Full amplitude for $e^+e^- \rightarrow \pi D^{(*)}\bar{D}^{(*)}, J/\psi\pi\pi, \psi'\pi\pi, h_c\pi\pi, \eta_c\rho\pi, J/\psi K\bar{K}$ (three-body final states)

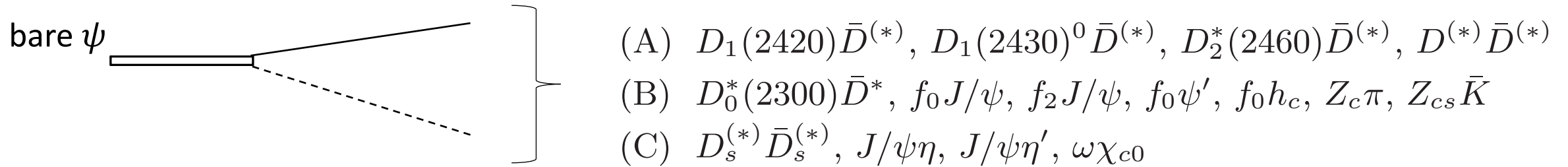


Full amplitude for $e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi\eta^{(\prime)}, \chi_{c0}\omega$ (two-body final states)



ψ and Y decays (bare vertices)

(quasi) two-body channels included; $J^{PC} = 1^{--}$



Group (A)

$D_1(2420), D_1(2430)^0, D_2^*(2460), D^* \rightarrow$ Breit-Wigner (BW) propagators; mass and width from PDG

$D_J^{(*)} \rightarrow D^{(*)}\pi$ coupling strength is determined, assuming the following decays saturate the width

$D_1(2420) \rightarrow D^*\pi$ (mainly d-wave decay); small s-wave coupling fixed by helicity angle distribution data

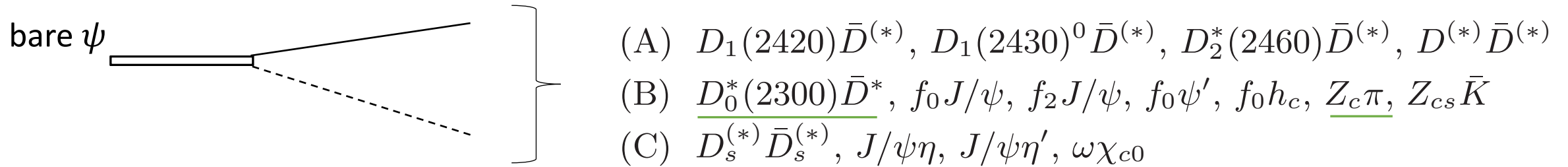
$D_1(2430)^0 \rightarrow D^*\pi$ (s-wave decay) Babar, PRD 82, 111101 (2010)

$D_2^*(2460) \rightarrow D^*\pi + D\pi; \Gamma(D\pi)/\Gamma(D^*\pi) \sim 1.5$

$D^* \rightarrow D\pi$

ψ and Y decays (bare vertices)

(quasi) two-body channels included; $J^{PC} = 1^{--}$

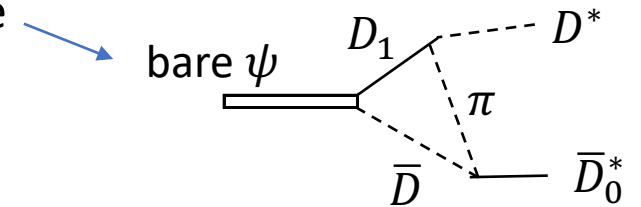


Group (B)

We do not include “bare $\psi \rightarrow D_0^*\bar{D}^*, Z_c\pi$ ”

bare ψ dominantly decays to two-body states; D_0^* and Z_c are probably not compact states

$D_0^*\bar{D}^*$ and $Z_c\pi$ channels are generated by coupled-channel effect like

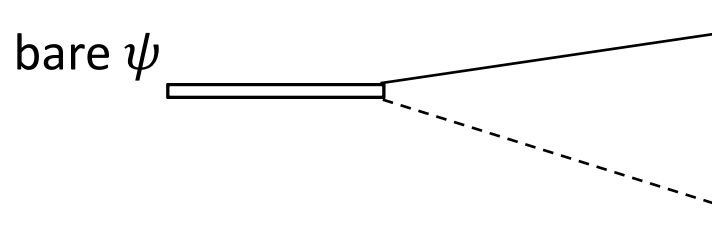


$D_0^*(2300) \rightarrow D\pi$ s-wave amplitude fitted to LQCD-based amplitude Albaladejo et al. PLB 767, 465 (2017)

D_0^* pole : $2104 - i 100$ MeV (ours) , $2105_{-8}^{+6} - i 102_{-12}^{+10}$ MeV (Albaladejo et al.)

ψ and Υ decays (bare vertices)

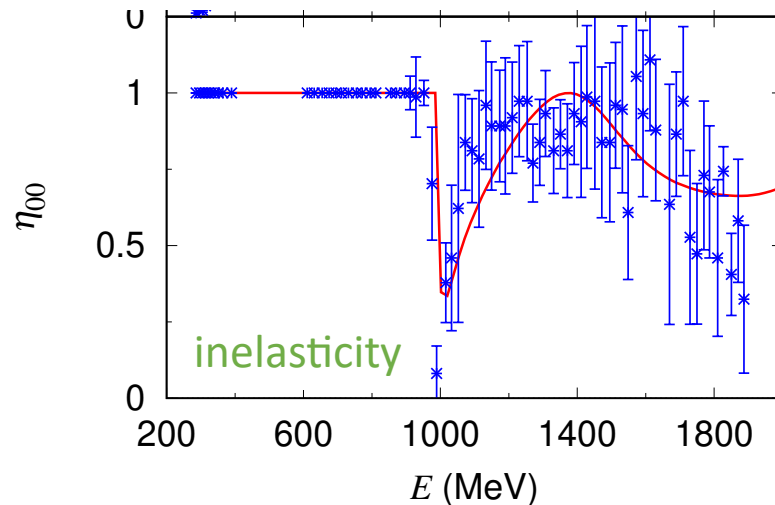
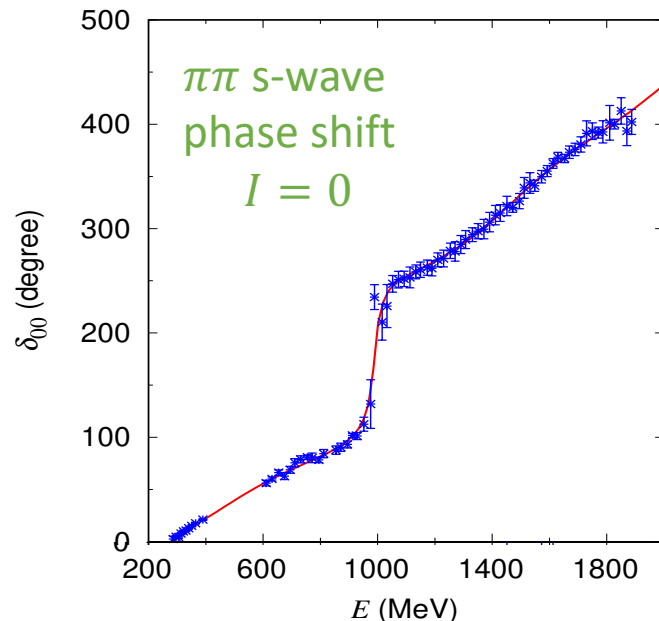
(quasi) two-body channels included; $J^{PC} = 1^{--}$



- (A) $D_1(2420)\bar{D}^{(*)}$, $D_1(2430)^0\bar{D}^{(*)}$, $D_2^*(2460)\bar{D}^{(*)}$, $D^{(*)}\bar{D}^{(*)}$
- (B) $D_0^*(2300)\bar{D}^*$, $f_0 J/\psi$, $f_2 J/\psi$, $f_0\psi'$, $f_0 h_c$, $Z_c\pi$, $Z_{cs}\bar{K}$
- (C) $D_s^{(*)}\bar{D}_s^{(*)}$, $J/\psi\eta$, $J/\psi\eta'$, $\omega\chi_{c0}$

Group (B)

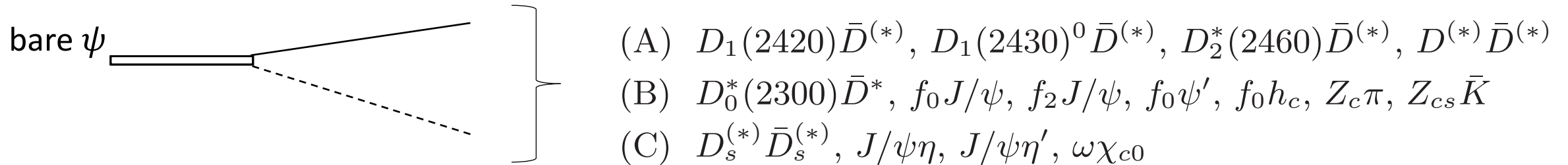
$f_0 [f_2] \rightarrow$ our $\pi\pi$ s[d]-wave amplitude fitted to empirical amplitude



$f_0(500), f_0(980), f_0(1370), f_2(1270)$
poles extracted $\rightarrow$ consistent with PDG

ψ and Υ decays (bare vertices)

(quasi) two-body channels included; $J^{PC} = 1^{--}$



Group (B) $Z_c : J^{PC} = 1^{+-}$ $D^*\bar{D} - D^*\bar{D}^* - J/\psi\pi - \psi'\pi - h_c\pi - \eta_c\rho$ couple—channel scattering amplitude
driven by contact interactions; s-wave interactions except $h_c\pi$ p-wave interaction

Z_c amplitude =

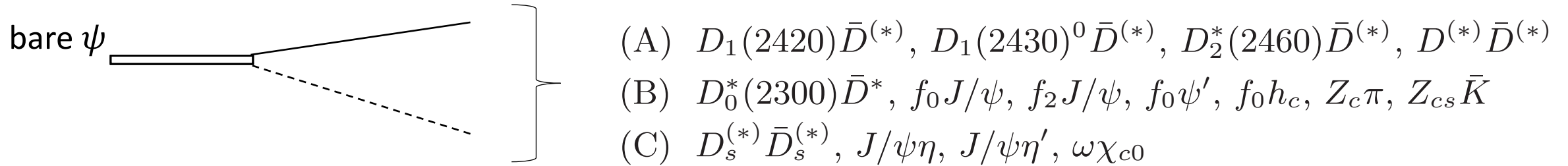
intermediate loops include all possible coupled-channels

$v_{D^*\bar{D}, D^*\bar{D}} = v_{D^*\bar{D}^*, D^*\bar{D}^*}$ (HQSS), no coupling between hidden-charm channels (e.g. $v_{J/\psi\pi, J/\psi\pi} = v_{J/\psi\pi, \psi'\pi} = 0$)

Nonzero couplings are determined by the global fit $\rightarrow$ poles may be generated if needed by data

ψ and Y decays (bare vertices)

(quasi) two-body channels included; $J^{PC} = 1^{--}$



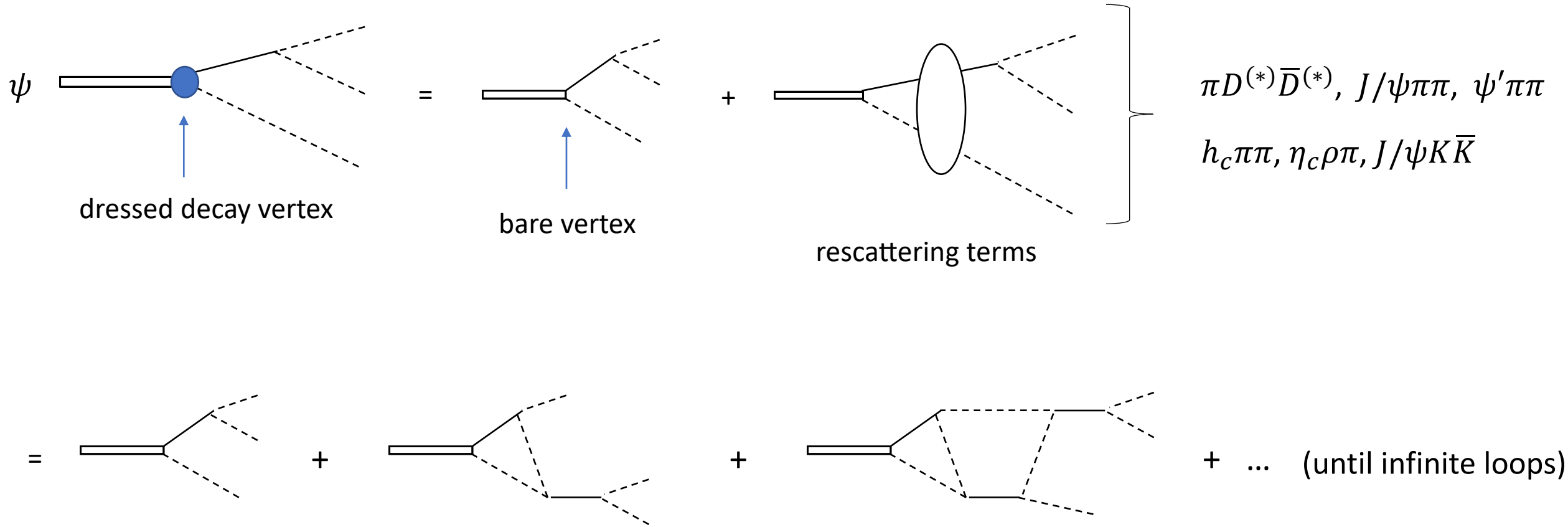
Group (B) $Z_{cs} : J^{PC} = 1^{+-} J/\psi K$ just for giving $\psi \rightarrow J/\psi K \bar{K}$ vertex, no pole

Group (C) treated as stable particles

Because of using BW for Group (A), three-body unitarity is not fully satisfied

But main mechanisms required by three-body unitarity are considered (next slide)

Three-body decay processes of ψ and Υ

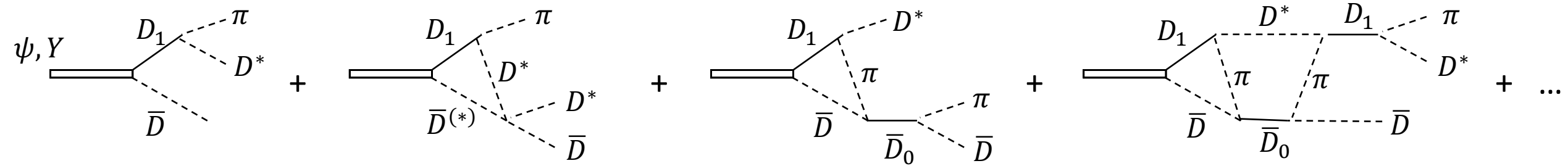


Final state interactions described by solution of Faddeev equation $\rightarrow$ Coupled-channels taken into account

Rescattering mechanisms (particle exchange) required by three-body unitarity are considered

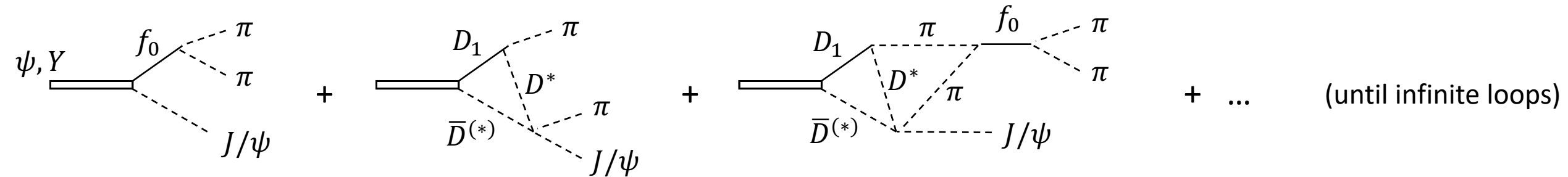
Three-body decay processes of ψ and Y

$$e^+e^- \rightarrow \pi D^* \bar{D}$$



(until infinite loops)

$$e^+e^- \rightarrow J/\psi \pi \pi$$

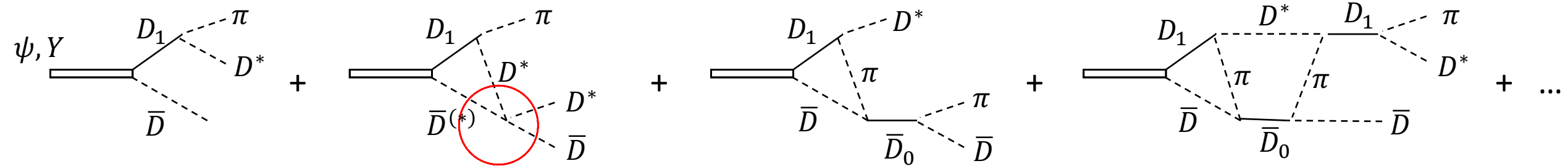


(until infinite loops)

Selected important diagrams; diagrams with more loops are usually more suppressed

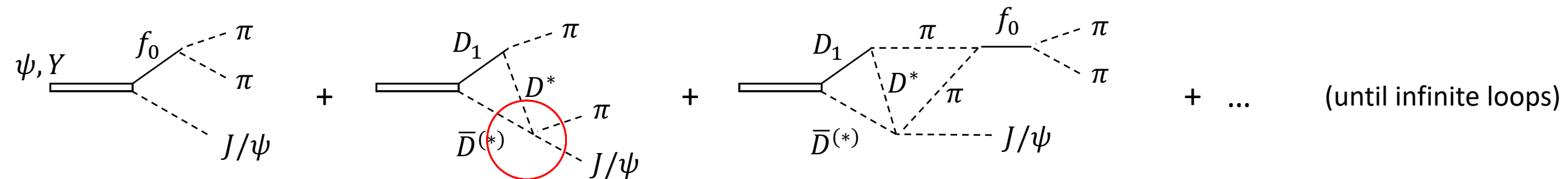
Different processes share the same interactions $\leftarrow$ unitarity requirement

$$e^+e^- \rightarrow \pi D^* \bar{D}$$



(until infinite loops)

$$e^+e^- \rightarrow J/\psi \pi \pi$$



(until infinite loops)

Z_c amplitude

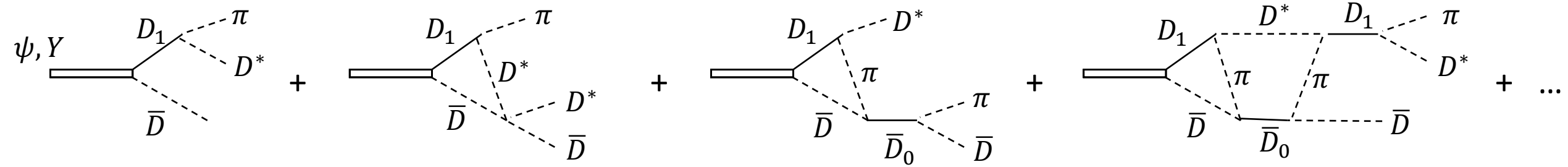
$D^* \bar{D} - D^* \bar{D}^* - J/\psi \pi - \psi' \pi - h_c \pi - \eta_c \rho$ coupled-channel scattering amplitude ($J^{PC} = 1^{+-}$)

$\rightarrow D^* \bar{D}$ and $D^* \bar{D}^*$ threshold cusps will be created in invariant mass distributions

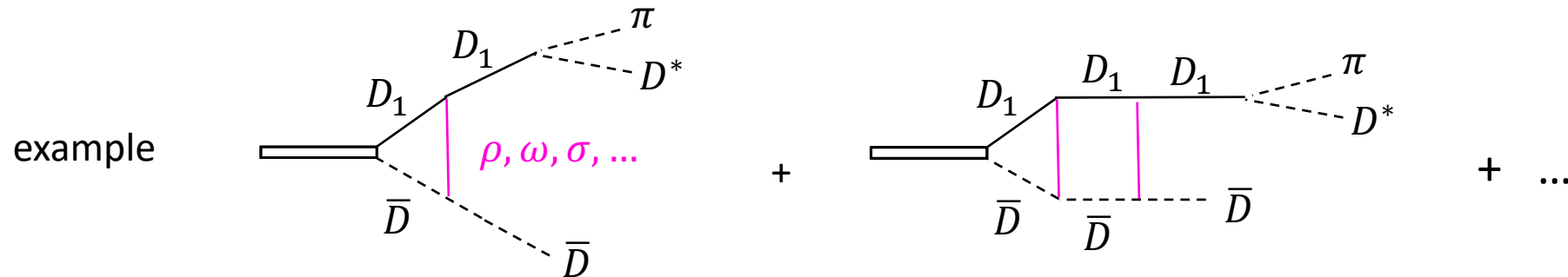
$Z_c(3900)$ and $Z_c(4020)$ poles may also be generated (if needed by data) to enhance the cusps

Three-body decay processes of ψ and Y

$$e^+e^- \rightarrow \pi D^* \bar{D}$$



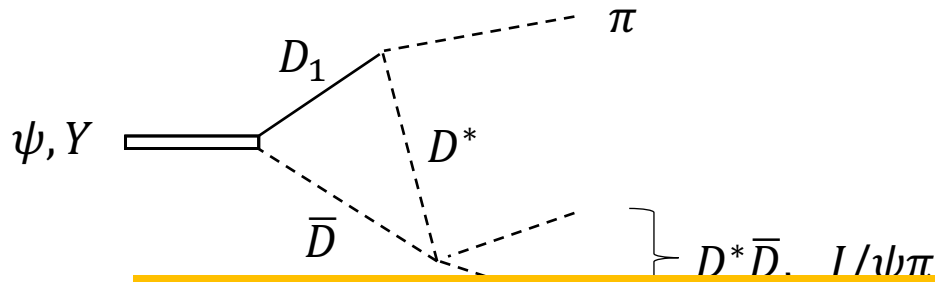
Addition, more mechanisms (off-shell, short-range) should exist $\rightarrow$ absorbed in bare couplings and masses of ψ, Y



Poorly understood mechanisms $\rightarrow$ fitting unknown coupling constants to data is computationally too expensive

$\rightarrow$ Some bare ψ states could be hadron-molecules

Triangle singularity (TS) from our model



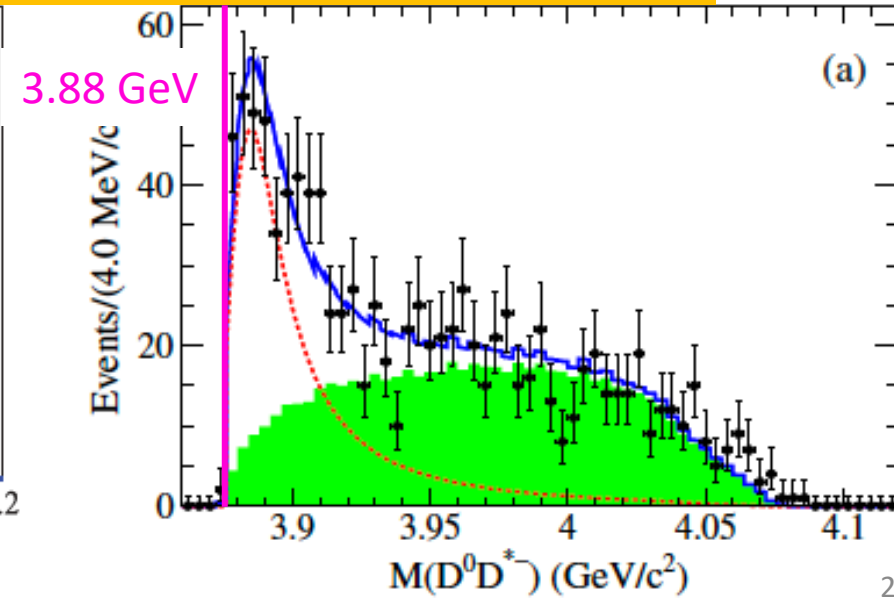
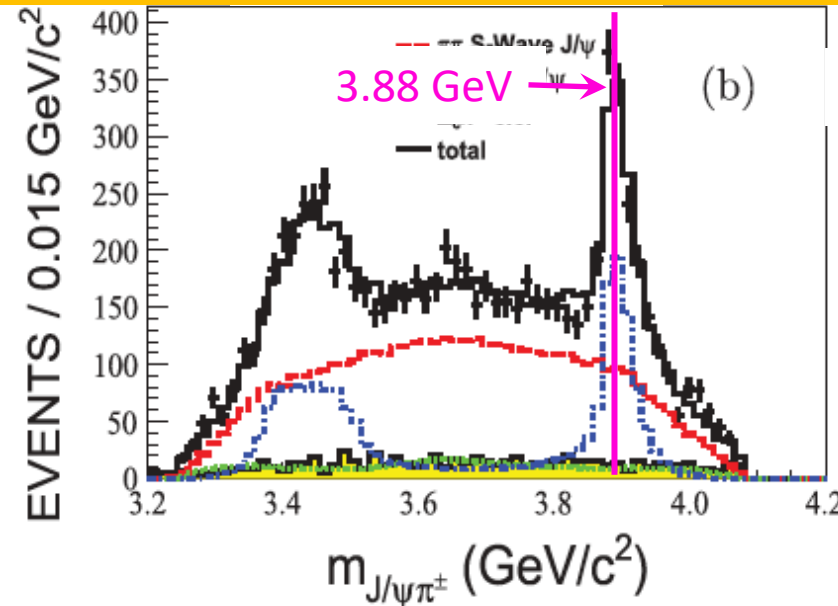
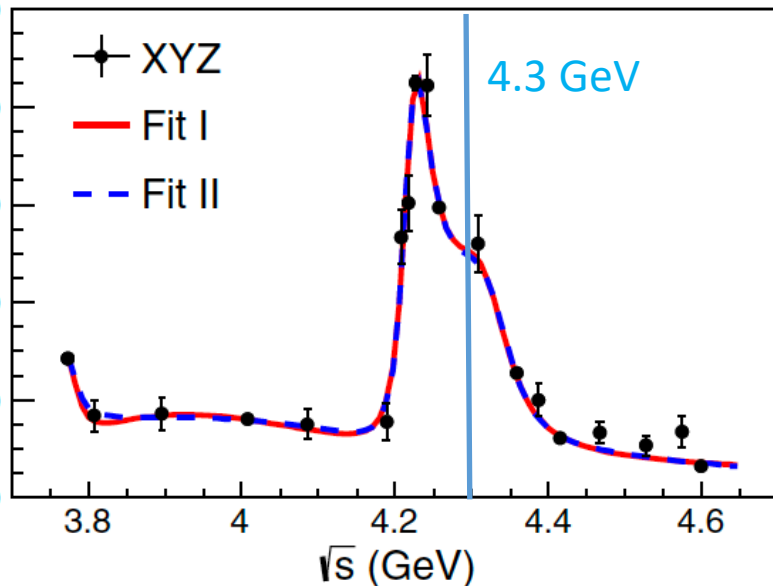
Kinematical condition for TS

Energy-momentum is conserved everywhere as classical process

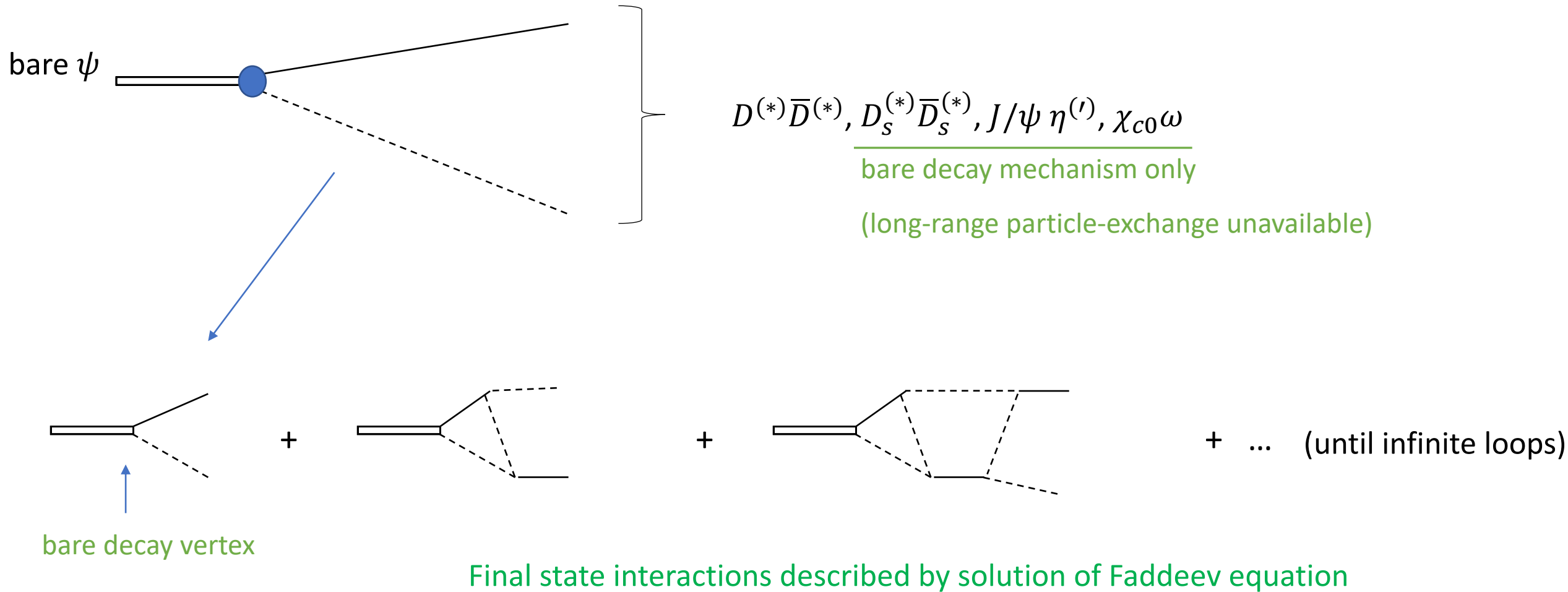
→ amplitude is significantly enhanced at

Data might indicate that $Y(4320)$ and Z_c peaks are TS effect (3.88 GeV)

→ to be examined in our analysis

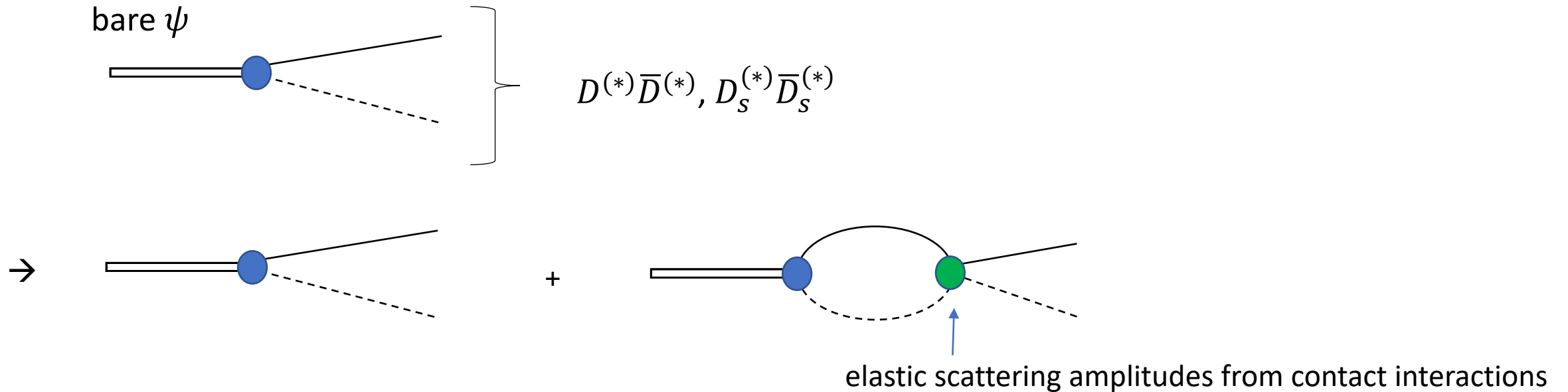


Two-body decay processes of ψ and Y

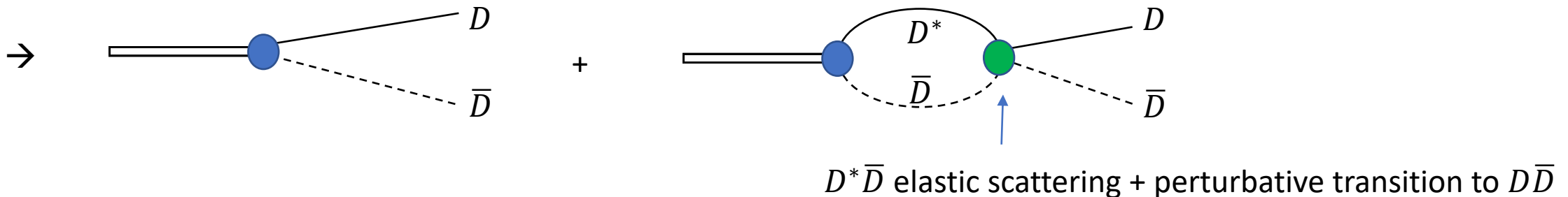


Two-body decay processes of ψ and Υ

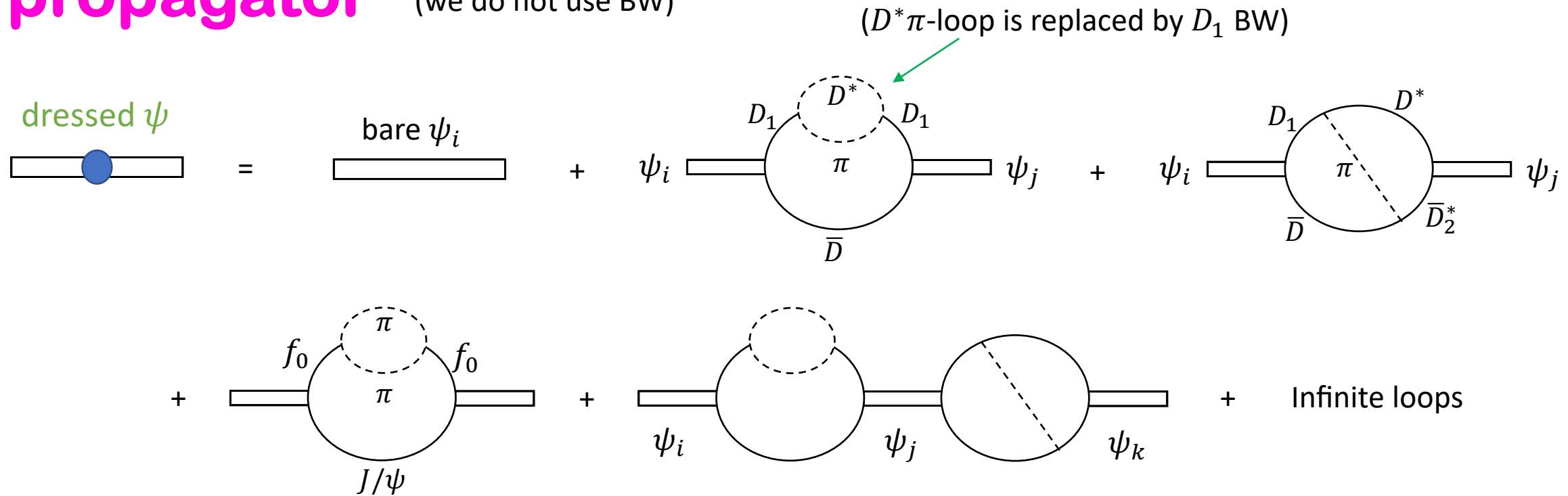
For $D^{(*)}\bar{D}^{(*)}$, $D_s^{(*)}\bar{D}_s^{(*)}$, moderately attractive interactions added $\rightarrow$ threshold enhancements $\rightarrow$ better fits



Exception: $D\bar{D}$ final state ($D^*\bar{D}$ threshold enhancement needed to fit data)



ψ, Y propagator (we do not use BW)



Charmonium poles are formed by non-perturbative couplings between bare ψ and $D_1 \bar{D}$, $f_0 J/\psi$, ...
 (= poles of dressed ψ propagator)

Unitary coupled-channel model : resonance pole (mass, width) and decay dynamics are explicitly related.

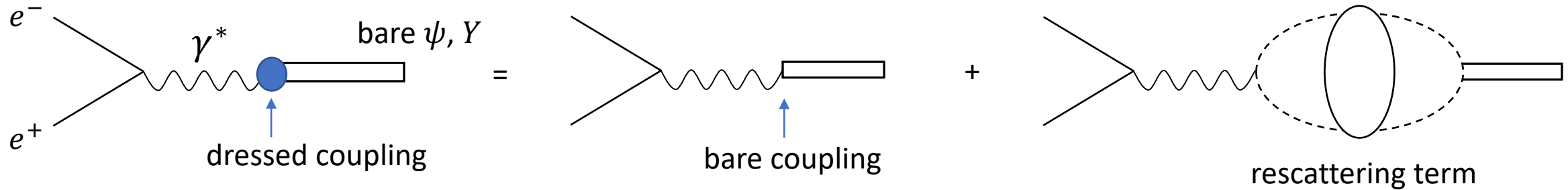
different (overlapping) resonances strongly couple (unitarity requirement)

Breit-Wigner model : decay dynamics are simulated by BW mass and width parameters

different (overlapping) resonances do not couple

ψ and Y production mechanisms

$e^+e^- \rightarrow c\bar{c}$ data in $3.75 \leq \sqrt{s} \leq 4.7$ GeV region $\rightarrow$ Charmonium excitations are important mechanism



Data determine how many bare states to be included (5 bare states) and which charmonium states exist

Expected states

$\psi(3770), \psi(4040), \psi(4160), \psi(4415), Y(4220), Y(4360)$

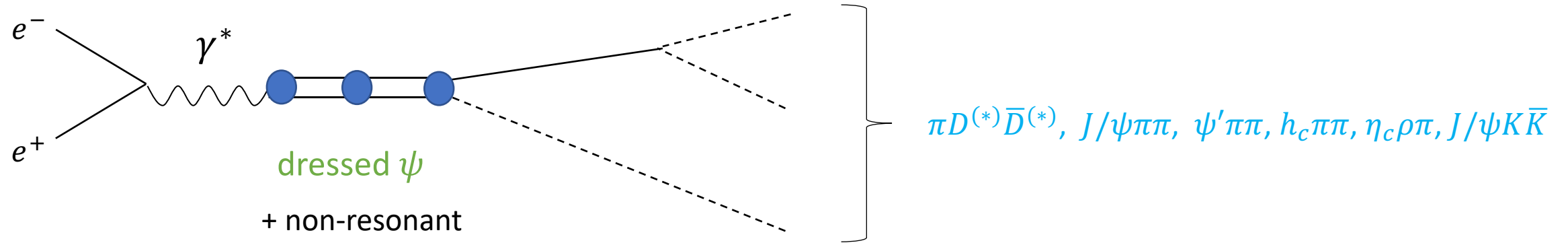
(Well-established) $c\bar{c}$

Exotic $D_1\bar{D}^{(*)}$ molecule, $c\bar{c}g$ hybrid ...

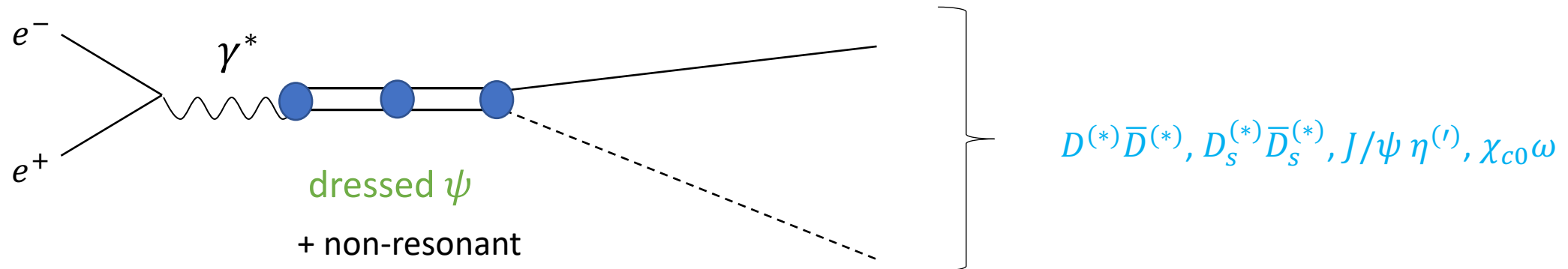
Data is not sufficient for coupled-channel analysis in $\sqrt{s} > 4.6$ GeV (final states including $s\bar{s}$ in particular)

$\rightarrow Y(4660)$ is included as a Breit-Wigner amplitude; not included in coupled-channel amplitude

Full amplitude for $e^+e^- \rightarrow \pi D^{(*)}\bar{D}^{(*)}, J/\psi\pi\pi, \psi'\pi\pi, h_c\pi\pi, \eta_c\rho\pi, J/\psi K\bar{K}$ (three-body final states)



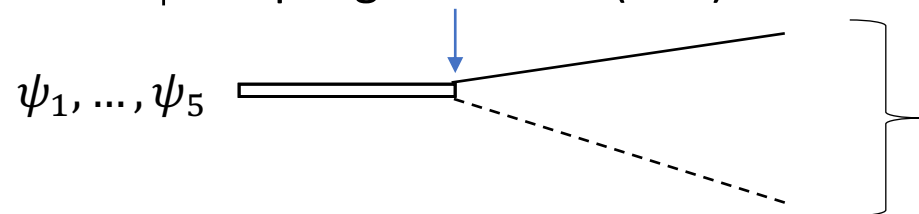
Full amplitude for $e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi \eta^{(\prime)}, \chi_{c0}\omega$ (two-body final states)



Fitting parameters in global analysis

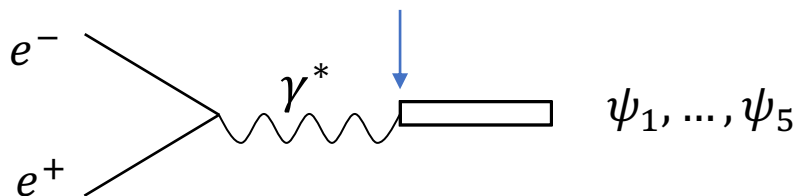
* bare ψ masses (5 bare states)

* bare ψ coupling constants (real)

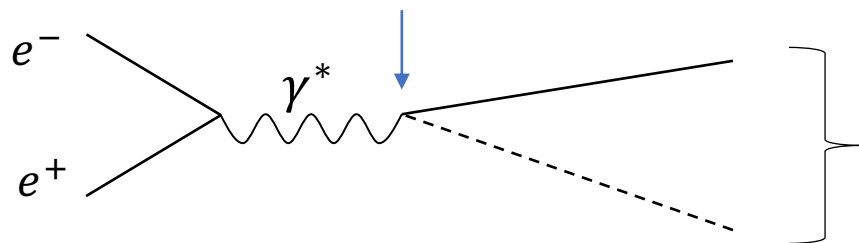


- (A) $D_1(2420)\bar{D}^{(*)}$, $D_1(2430)^0\bar{D}^{(*)}$, $D_2^*(2460)\bar{D}^{(*)}$, $D^{(*)}\bar{D}^{(*)}$
 (B) ~~$D_0^*(2300)\bar{D}^*$~~ , $f_0 J/\psi$, $f_2 J/\psi$, $f_0 \psi'$, $f_0 h_c$, ~~$Z_c \pi$~~ , $Z_{cs} \bar{K}$
 (C) $D_s^{(*)}\bar{D}_s^{(*)}$, $J/\psi \eta$, $J/\psi \eta'$, $\omega \chi_{c0}$

* bare photon- ψ coupling constants (real)



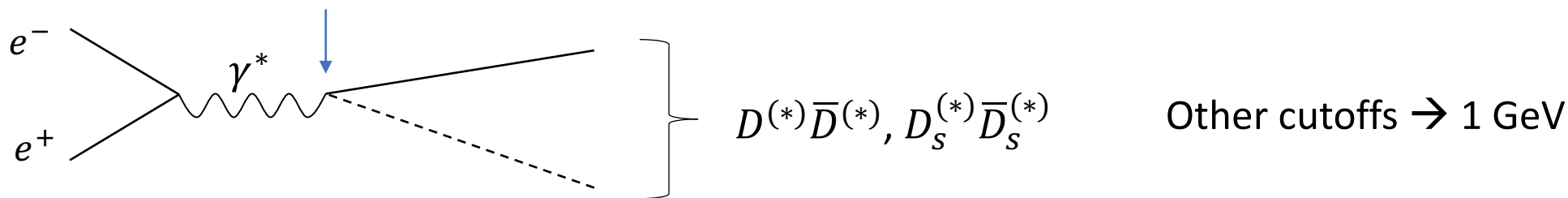
* non-resonant photon coupling constants (real)



- (A) $D_1(2420)\bar{D}^{(*)}$, $D_1(2430)^0\bar{D}^{(*)}$, $D_2^*(2460)\bar{D}^{(*)}$, $D^{(*)}\bar{D}^{(*)}$
 (B) ~~$D_0^*(2300)\bar{D}^*$~~ , $f_0 J/\psi$, $f_2 J/\psi$, $f_0 \psi'$, $f_0 h_c$, ~~$Z_c \pi$~~ , $Z_{cs} \bar{K}$
 (C) $D_s^{(*)}\bar{D}_s^{(*)}$, $J/\psi \eta$, $J/\psi \eta'$, $\omega \chi_{c0}$

Fitting parameters in global analysis

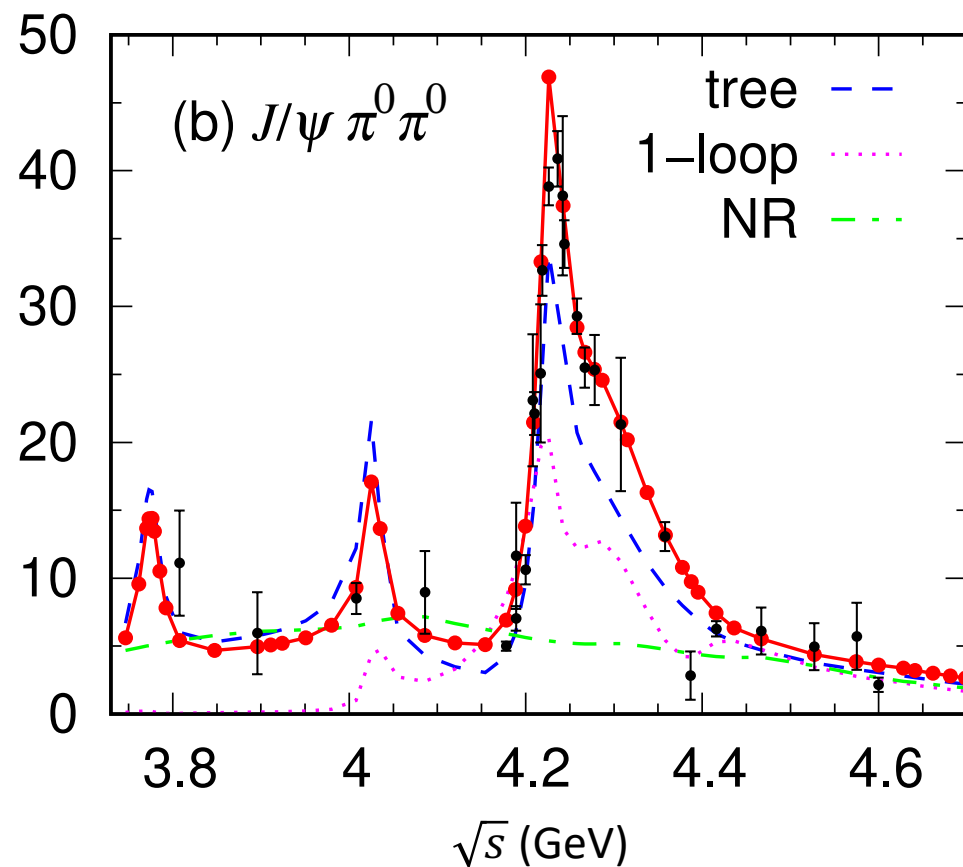
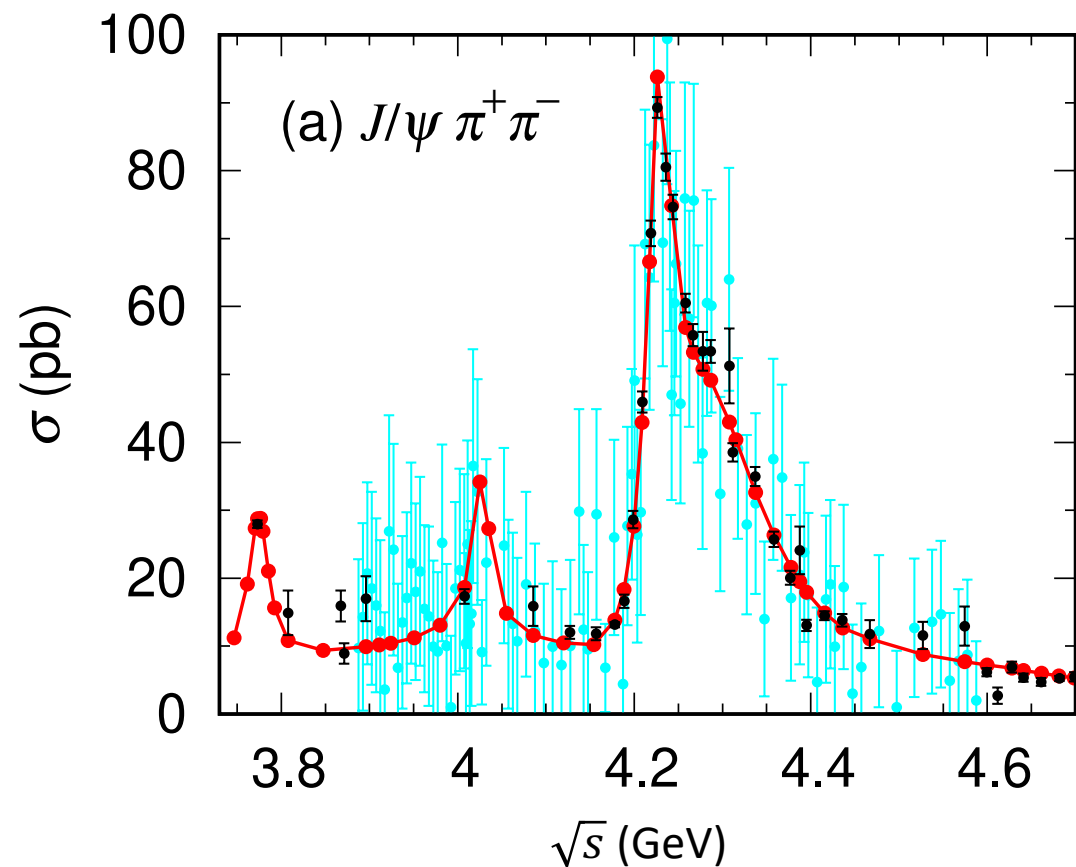
- * $\psi(4660)$ Breit-Wigner mass, width, $\psi(4660) \rightarrow f_0\psi'$ complex vertices
- * In $J^{PC} = 1^{+-}$ $D^*\bar{D} - D^*\bar{D}^* - J/\psi\pi - \psi'\pi - h_c\pi - \eta_c\rho$ couple—channel scattering amplitude (Z_c amplitude)
coupling constants: $v_{D^*\bar{D}, D^*\bar{D}}, v_{D^*\bar{D}, J/\psi\pi}, v_{D^*\bar{D}, \psi'\pi}$ etc.
- * Additional $D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}$ elastic contact interactions $\rightarrow$ coupling constants
- * Cutoffs (dipole form factors) to adjust energy dependence of nonresonant amplitudes



In total, 177 fitting parameters

Fit results

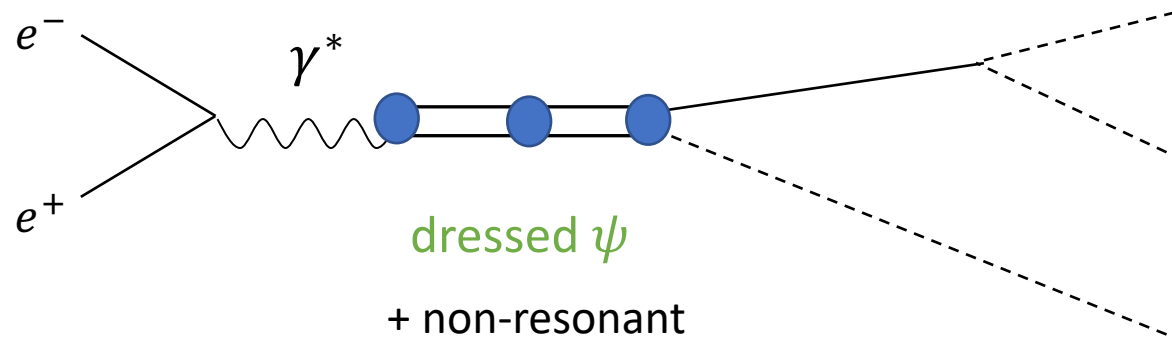
$$e^+e^- \rightarrow J/\psi \pi^+\pi^-, J/\psi \pi^0\pi^0$$



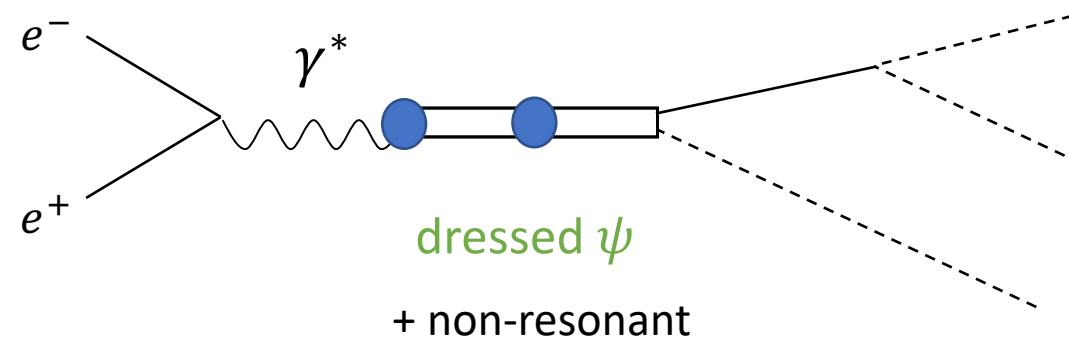
— Our fit
 — BESIII XYZ
 — BESIII R-scan

- Overall good agreement with data (our model is isospin symmetric, $\sigma(J/\psi\pi^+\pi^-) = 2 \times \sigma(J/\psi\pi^0\pi^0)$)
- Peaking structure at $\sqrt{s} \sim 4$ GeV is a consequence of the combined fit ($\psi(4040)$)
- Triangle singularity effect is seen in 1-loop contribution at $\sqrt{s} \sim 4.28$ GeV
 $\rightarrow$ Y(4320)-like enhancement in full calculation $\rightarrow$ Y(4320) is TS in our analysis

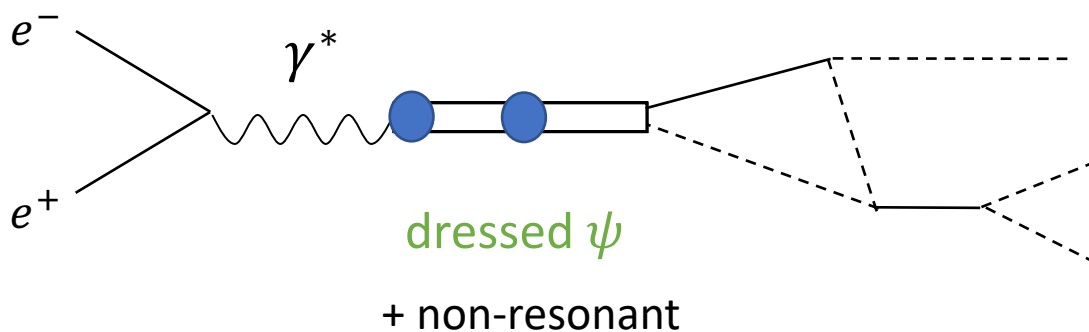
Full amplitude



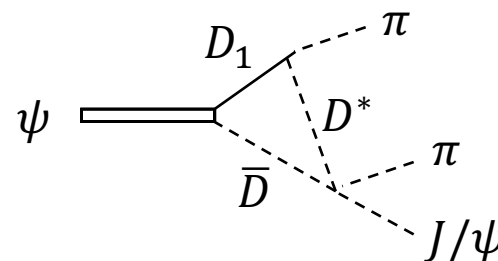
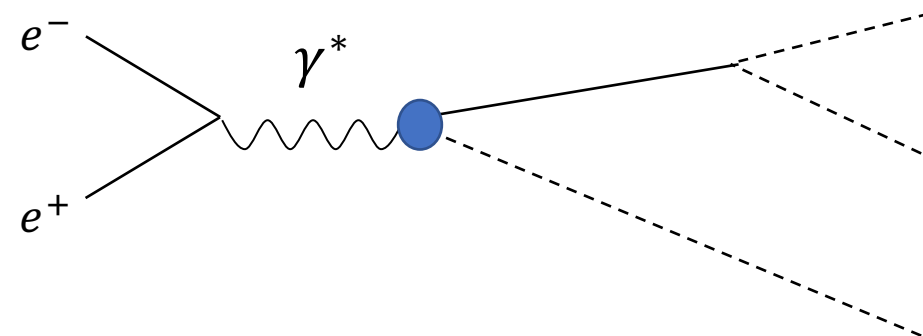
tree



1-loop



NR (non-resonant)



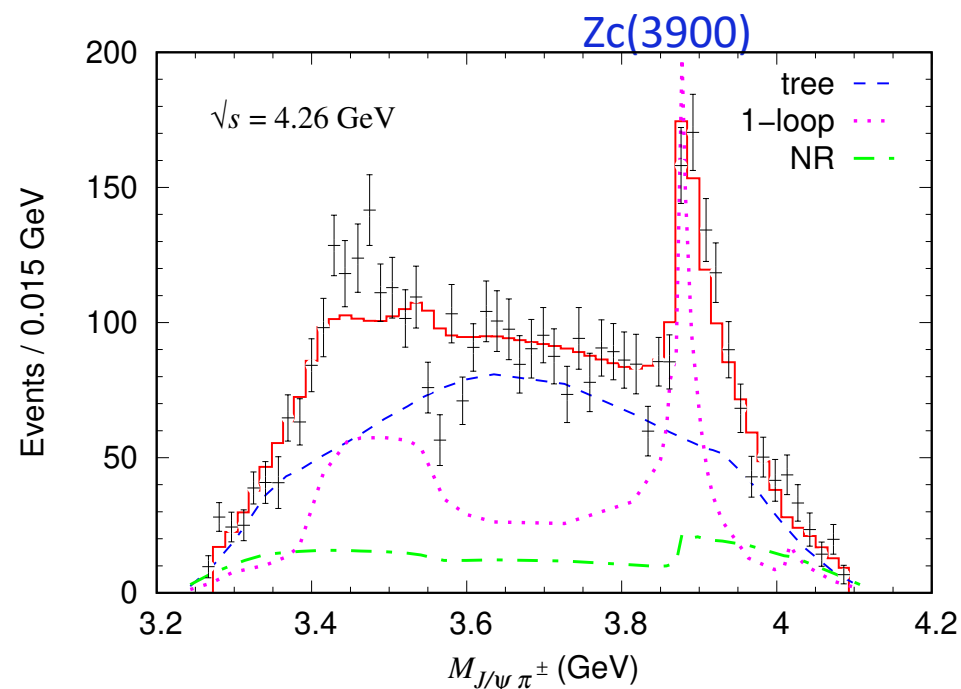
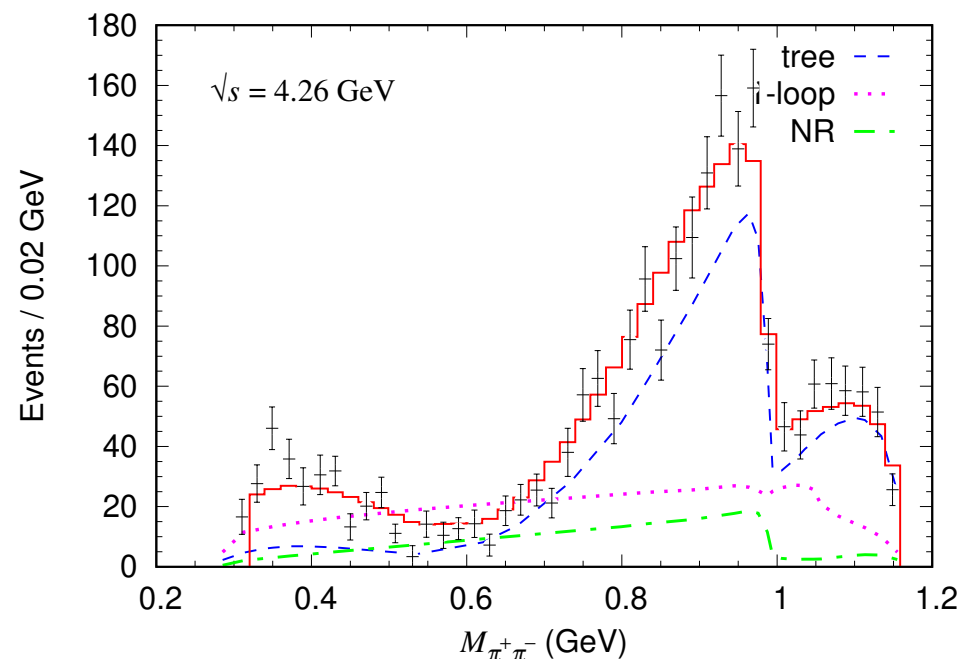
$D^* \bar{D}$ threshold cusp and/or
TS occurs from 1-loop

$$e^+e^- \rightarrow J/\psi \pi^+\pi^-$$

Fit to invariant masses

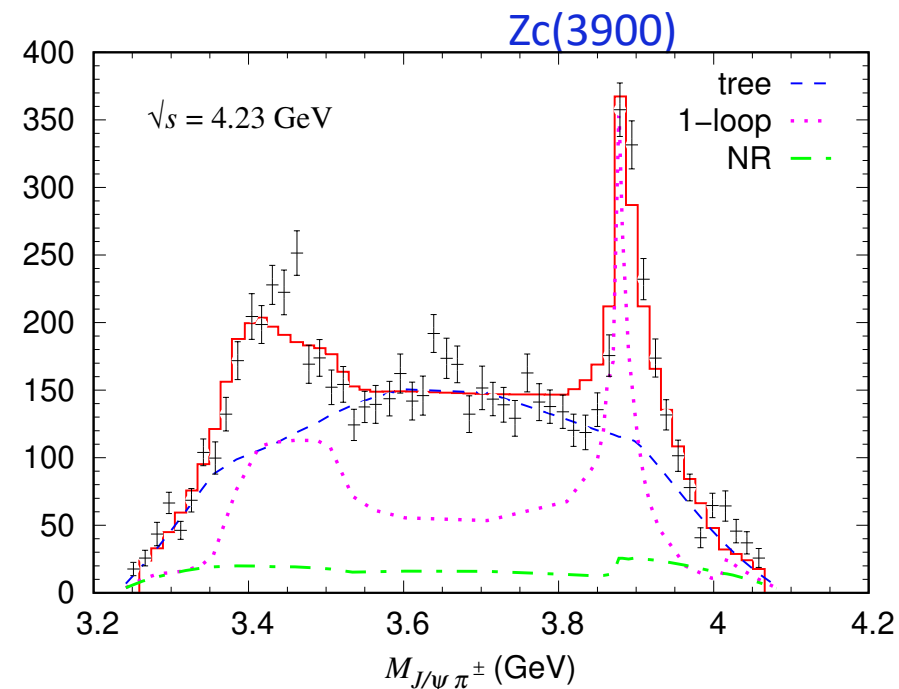
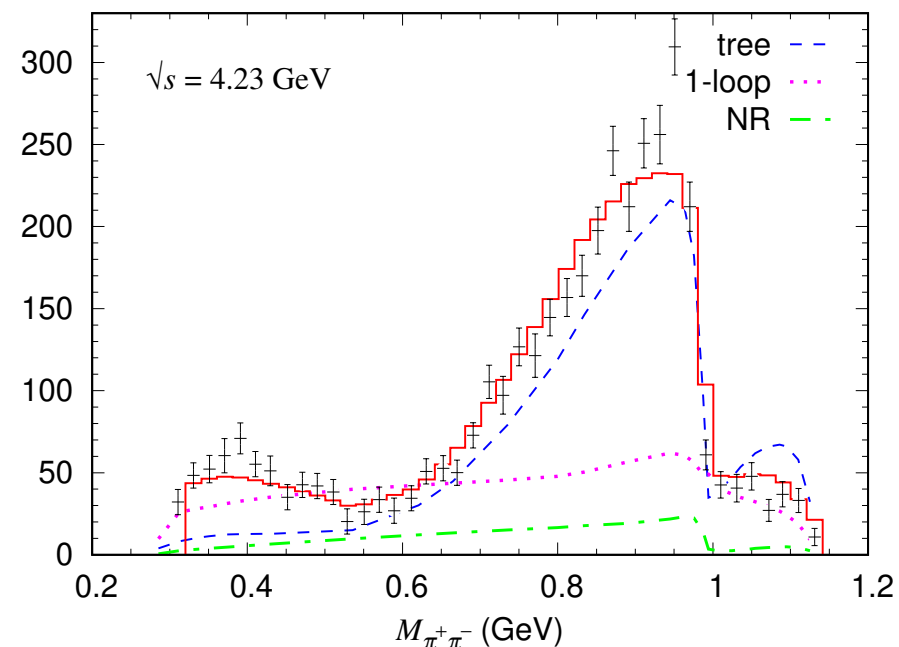
Zc(3900) peaks are well fitted

1-loop causes cusp
(a bit off TS condition)

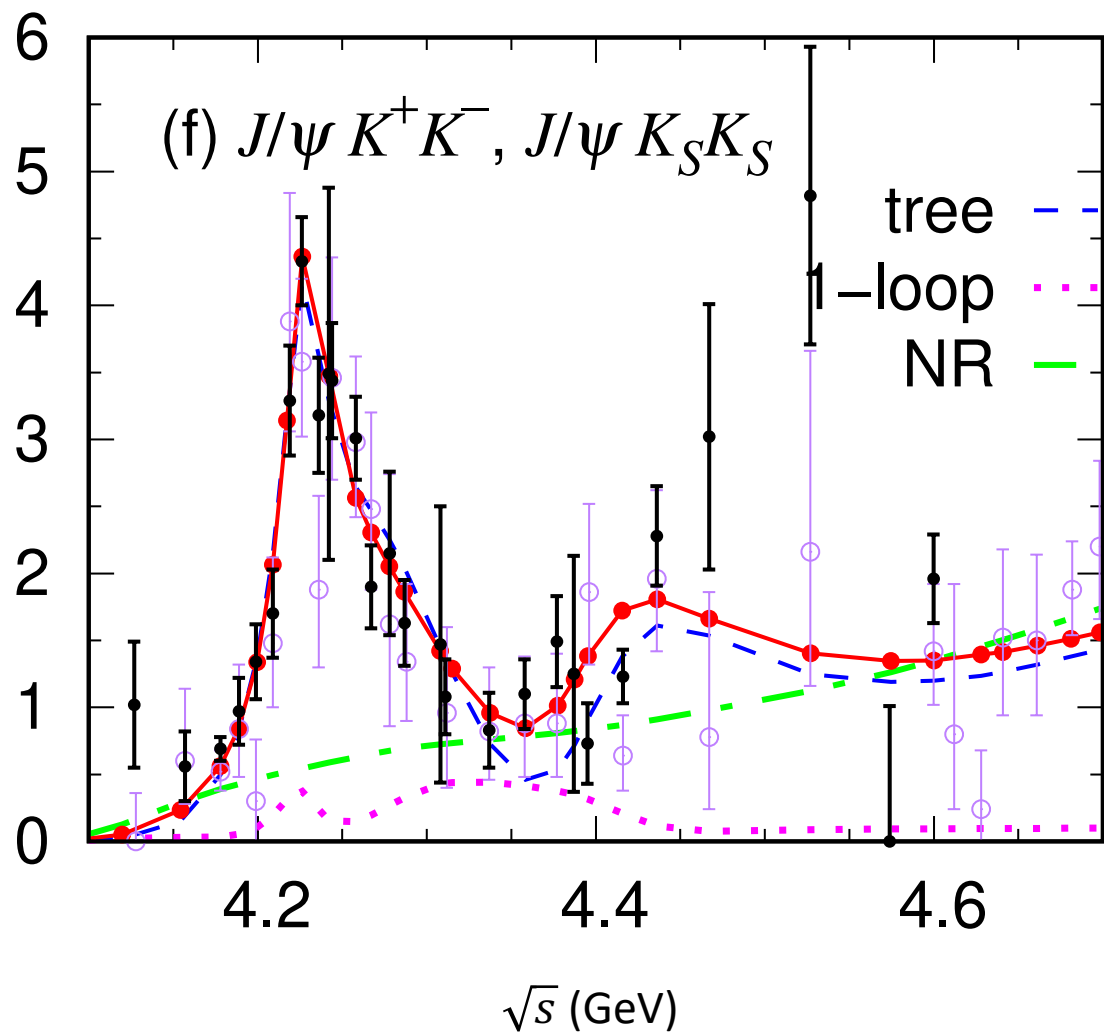


We will examine Zc(3900) pole

→ bound, resonance,
or virtual state



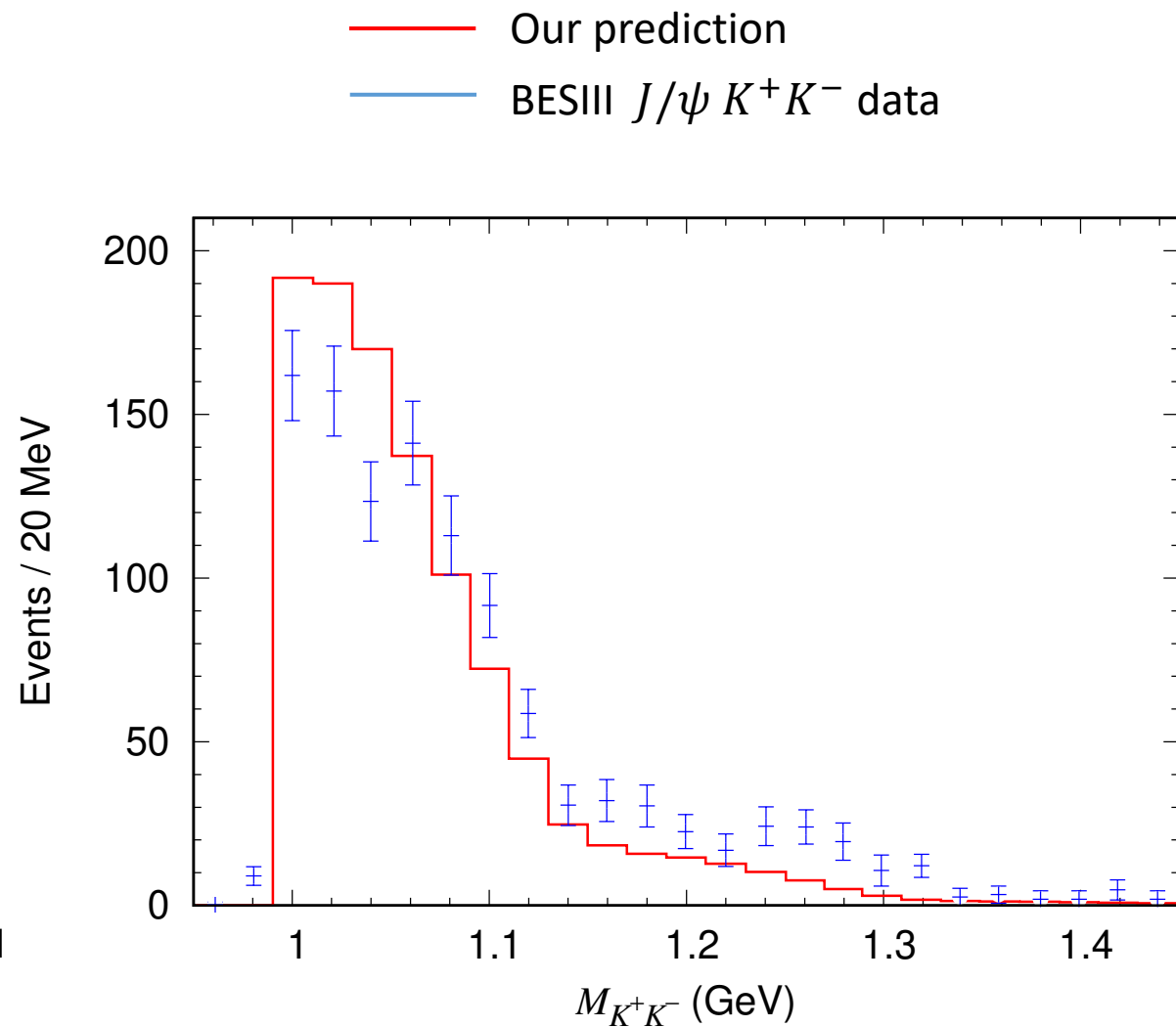
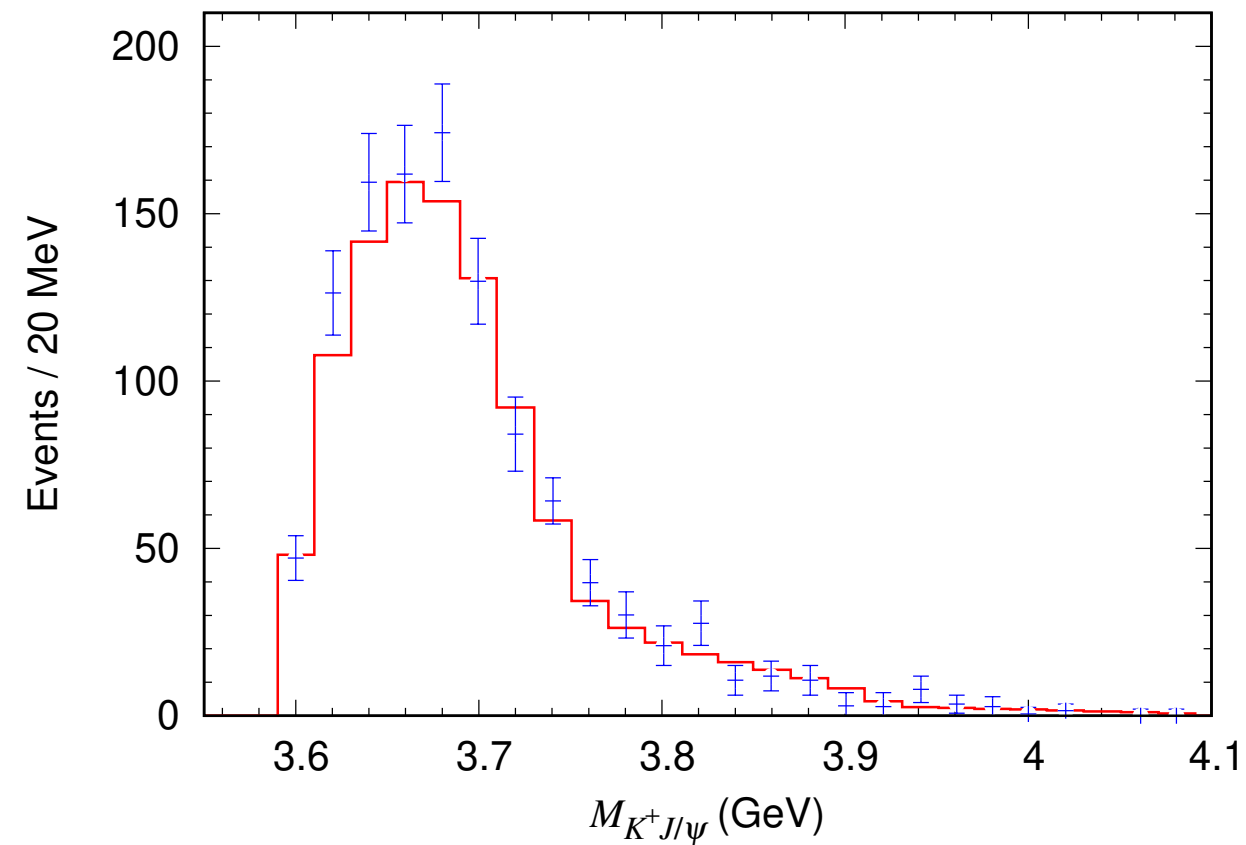
$$e^+e^- \rightarrow J/\psi K^+K^-, J/\psi K_S K_S$$



— Our fit
— BESIII $J/\psi K^+K^-$ data
- - - BESIII $J/\psi K_S K_S$ data

- Overall good agreement with data
(our model is isospin symmetric
 $\rightarrow \sigma(J/\psi K^+K^-) = 2 \times \sigma(J/\psi K_S K_S)$)
 - Model does not fit bump at ~ 4.5 GeV in $J/\psi K^+K^-$ data
 - * $J/\psi K_S K_S$ data do not show the same bump
 - * data largely fluctuate and error is large
- $\rightarrow$ our model does not have $Y(4500)$
 more precise data is important to pin-down
 the existence of $Y(4500)$

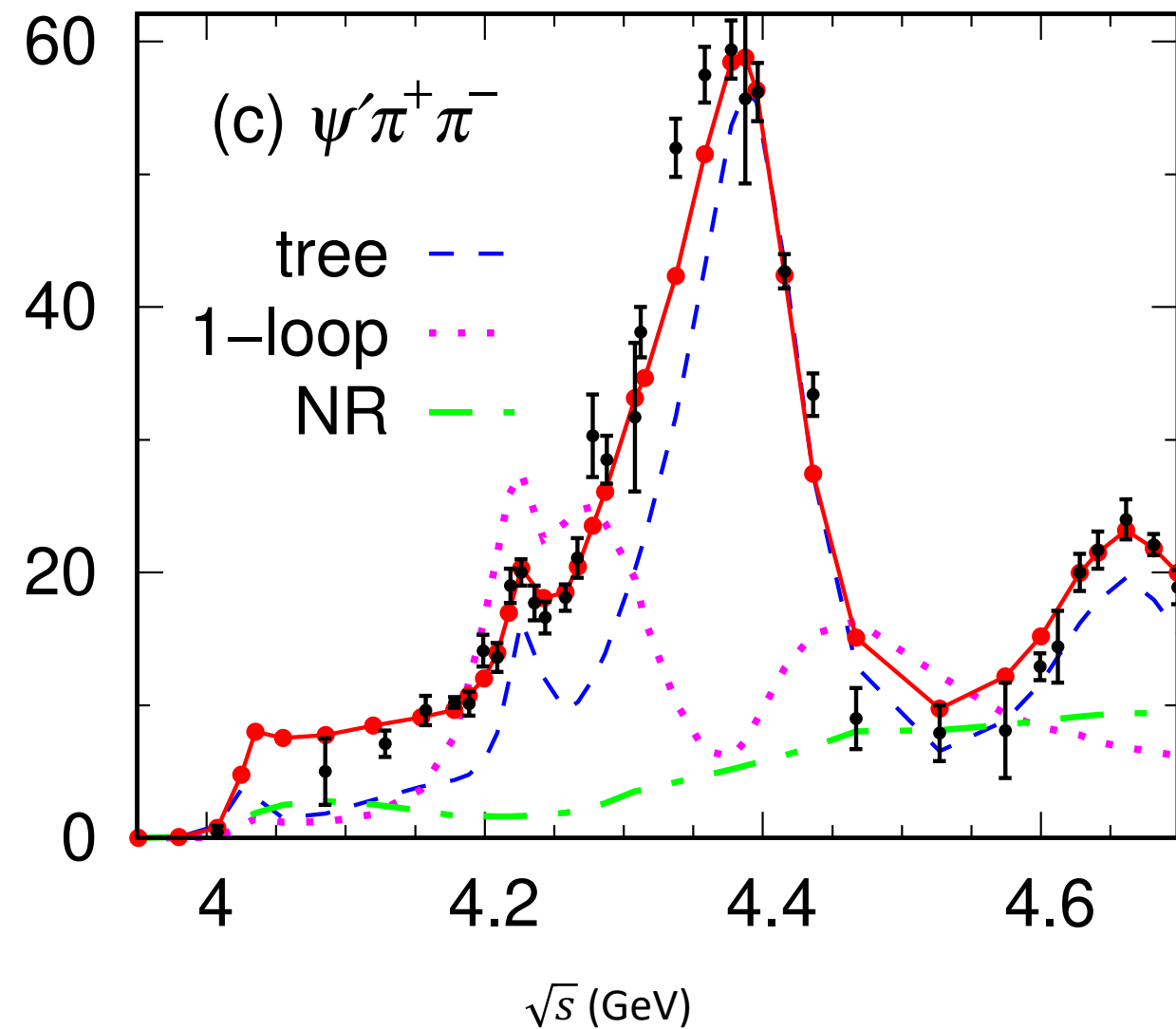
$$e^+e^- \rightarrow J/\psi K^+K^-$$



Data are sum of $\sqrt{s} = 4.1\text{--}4.6$ GeV data

Good agreement (this data is not included in our fit)

$$e^+e^- \rightarrow \psi' \pi^+ \pi^-$$



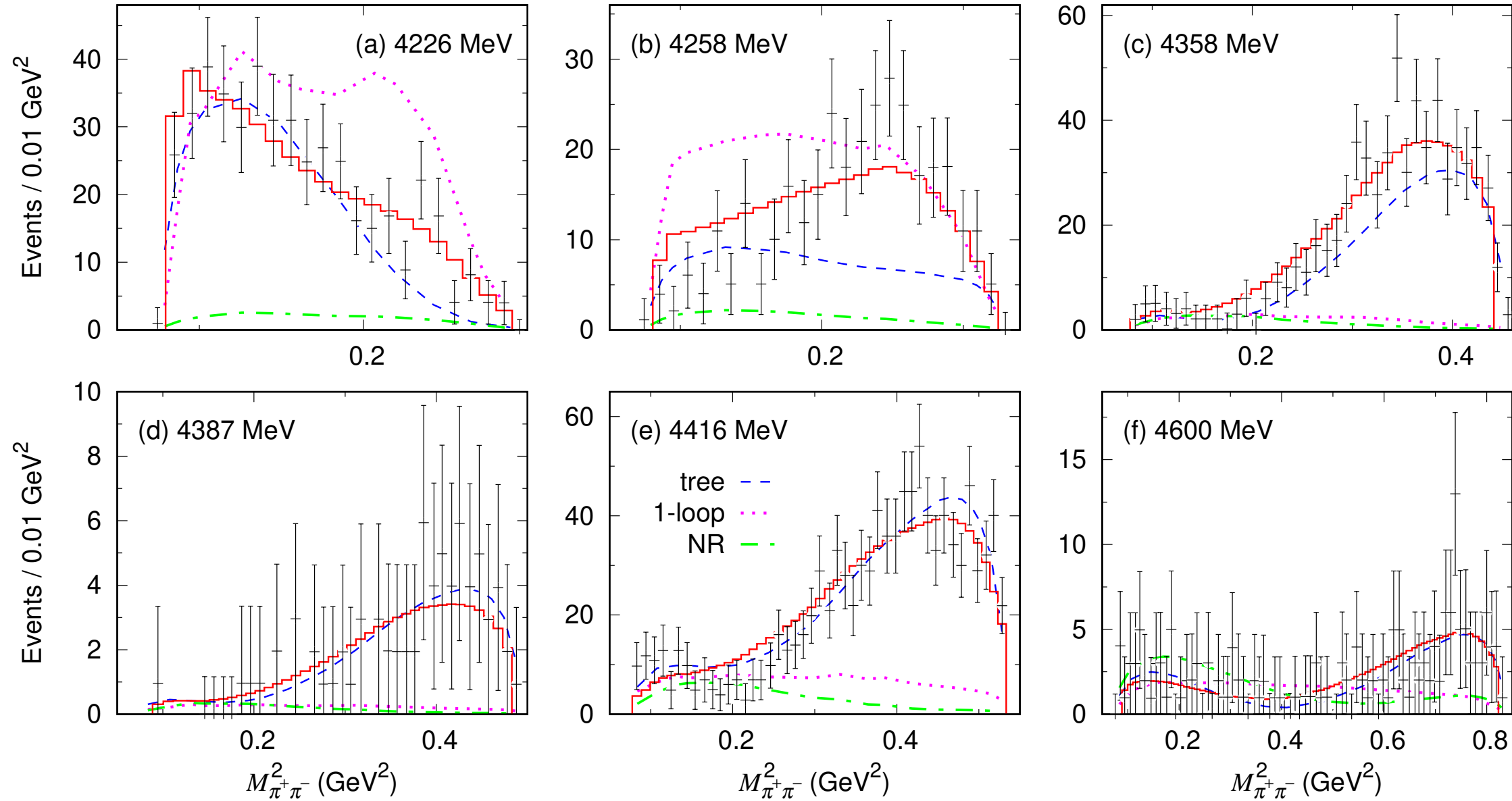
— Our fit
— BESIII data

- Overall good fit
- Enhancement at ~ 4.03 GeV is from $\psi(4040)$
 $\leftarrow$ consequence of coupled-channel fit
- 1-loop contribution is enhanced at
 ~ 4.28 GeV $\rightarrow D_1(2420)\bar{D}$ threshold
 ~ 4.45 GeV $\rightarrow D_1(2420)\bar{D}^*, D_2^*(2460)\bar{D}^*$ thresholds
 due to opening the thresholds, triangle singularity

$$e^+e^- \rightarrow \psi' \pi^+\pi^-$$

Fit to invariant mass distributions

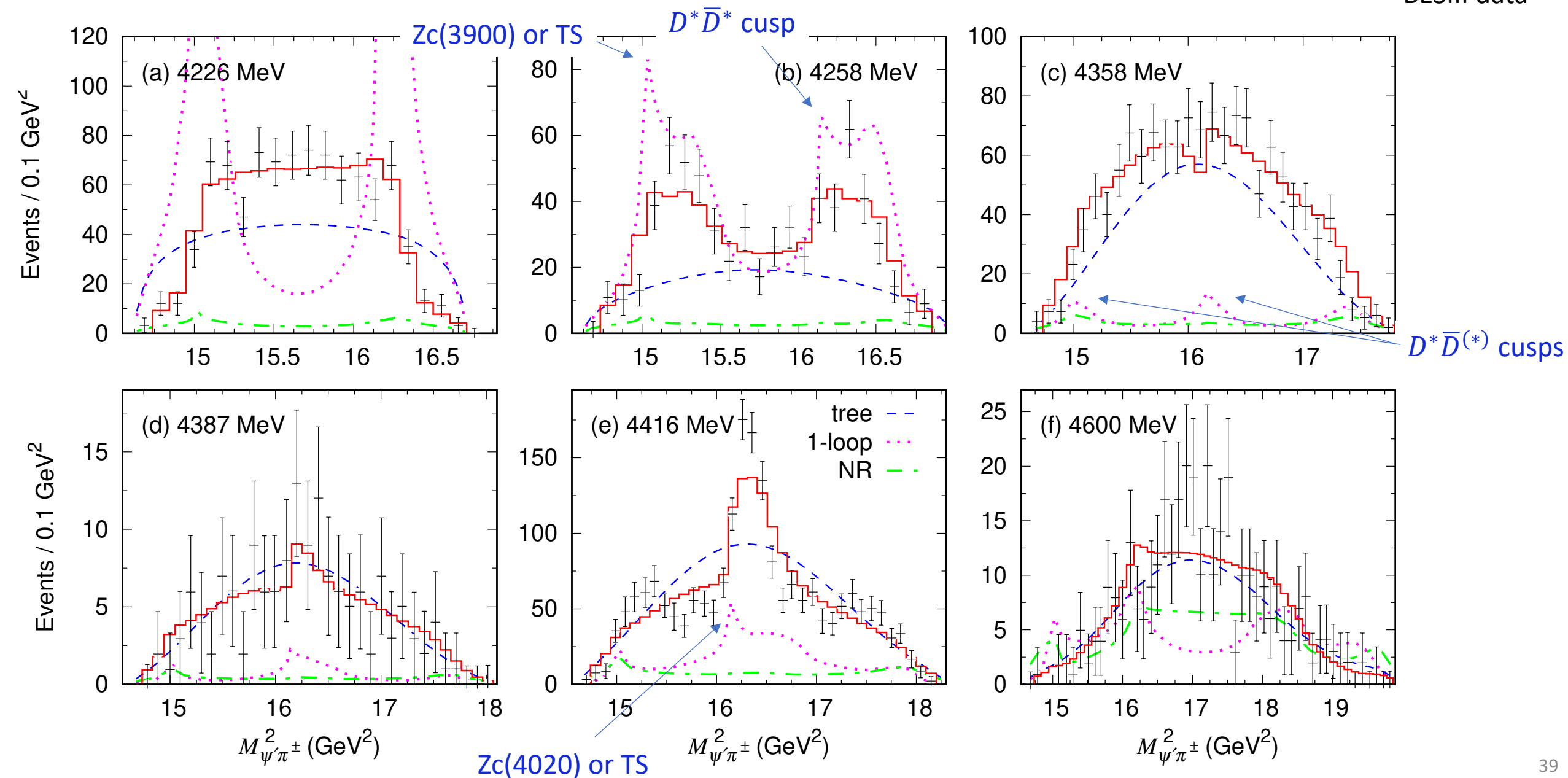
— Our fit
— BESIII data



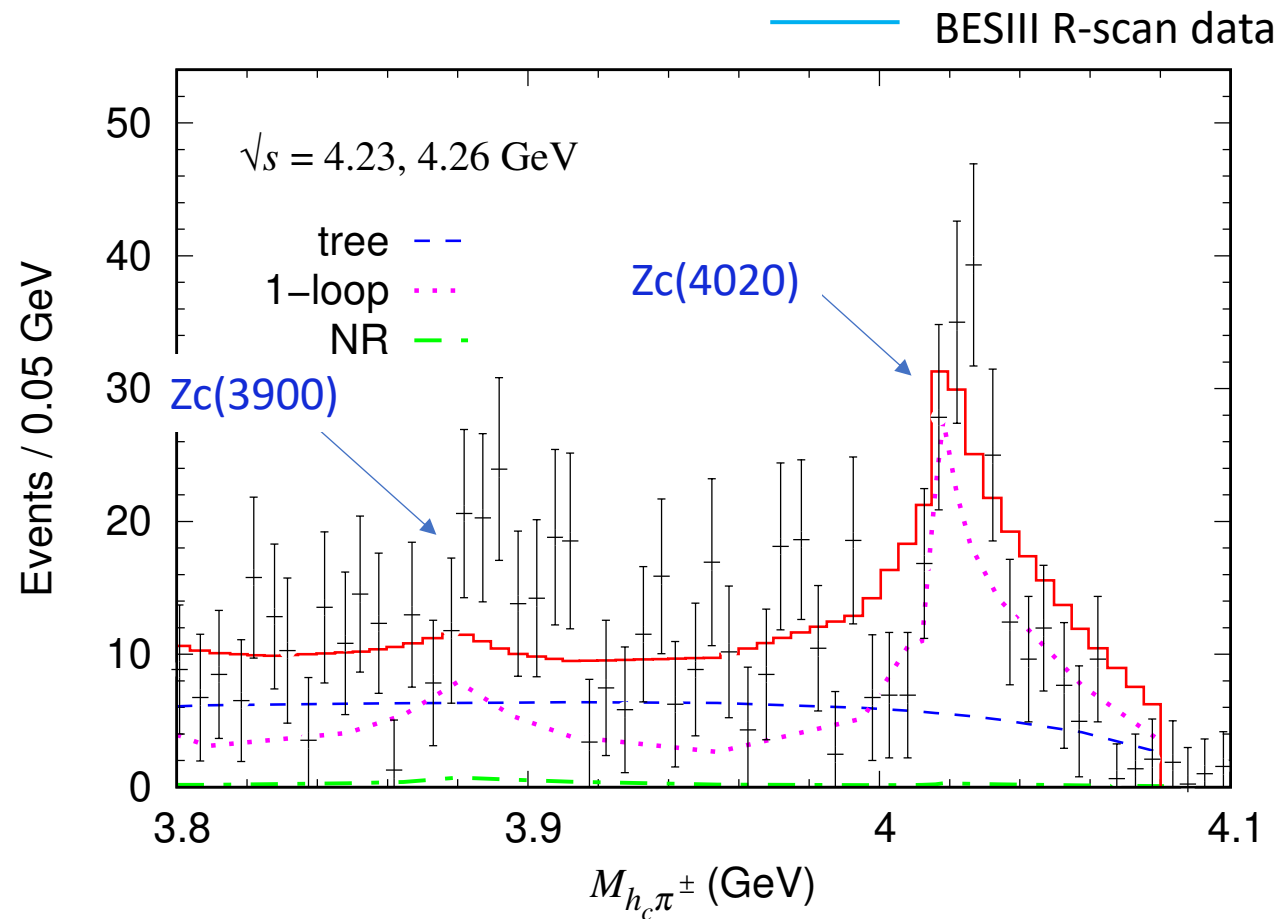
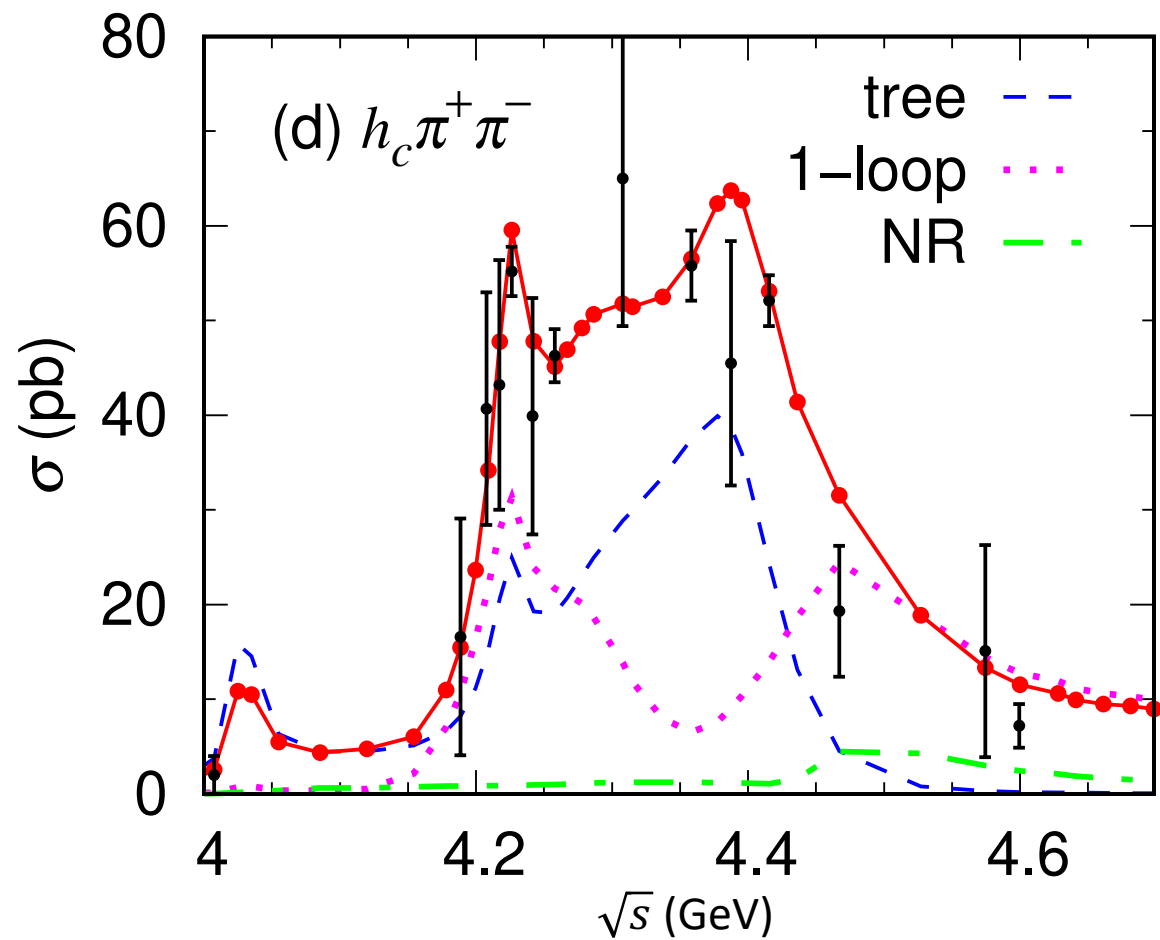
$$e^+e^- \rightarrow \psi' \pi^+ \pi^-$$

Fit to invariant mass distributions; many Zc or cusp or TS effects

— Our fit
— BESIII data



$$e^+e^- \rightarrow h_c \pi^+ \pi^-$$

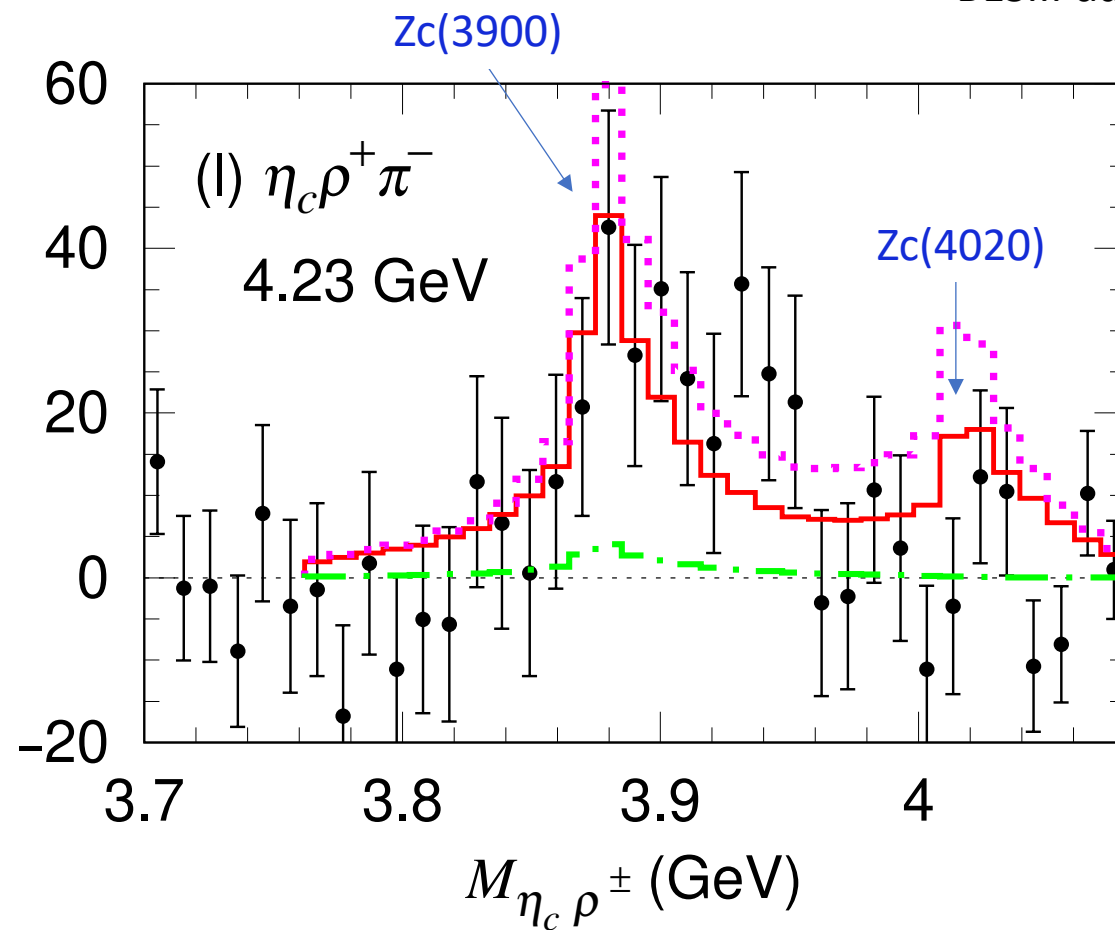
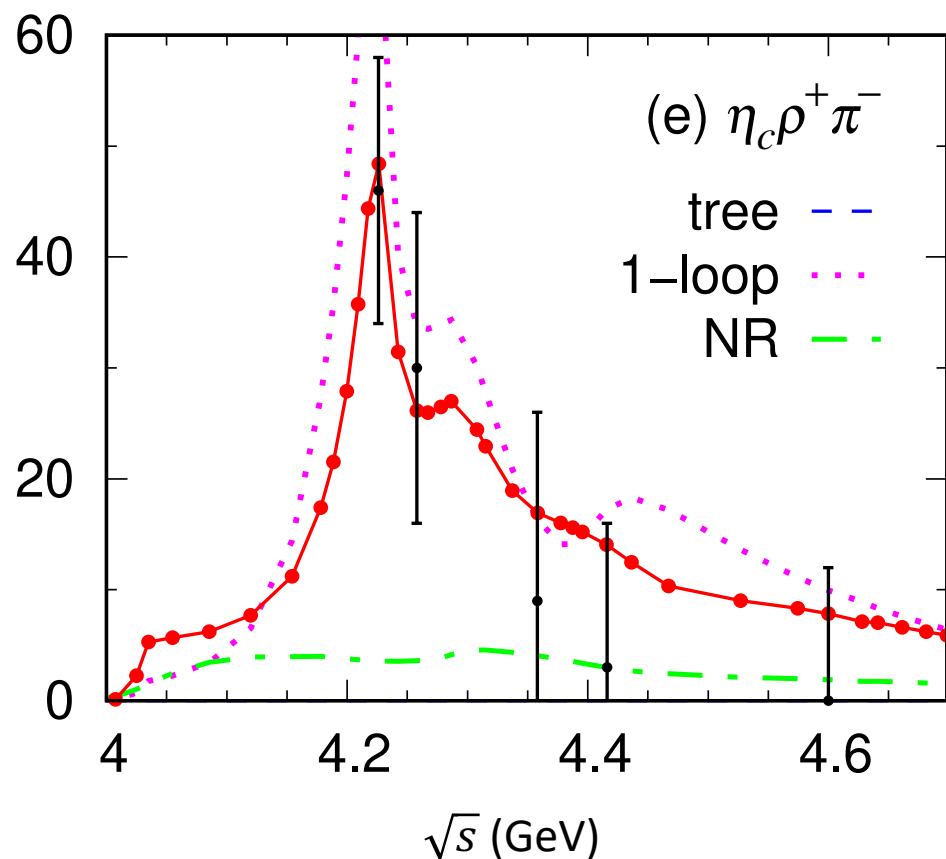


- Enhancement at ~ 4.03 GeV is from $\psi(4040) \leftarrow$ consequence of coupled-channel fit
- 1-loop contribution is enhanced due to opening the thresholds, triangle singularity
 ~ 4.28 GeV $\rightarrow D_1(2420)\bar{D}$ threshold ~ 4.45 GeV $\rightarrow D_1(2420)\bar{D}^*, D_2^*(2460)\bar{D}^*$ thresholds

$$e^+e^- \rightarrow \eta_c \rho^+ \pi^-$$

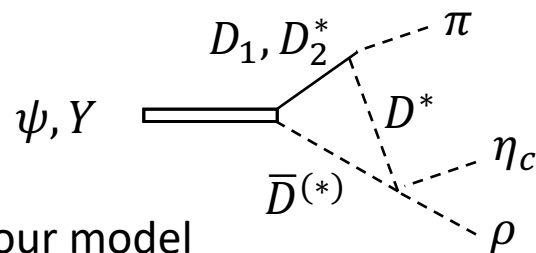
$\rho \rightarrow \pi\pi$ taken into account in calculation

— Our fit
— BESIII data



Mostly from 1-loop

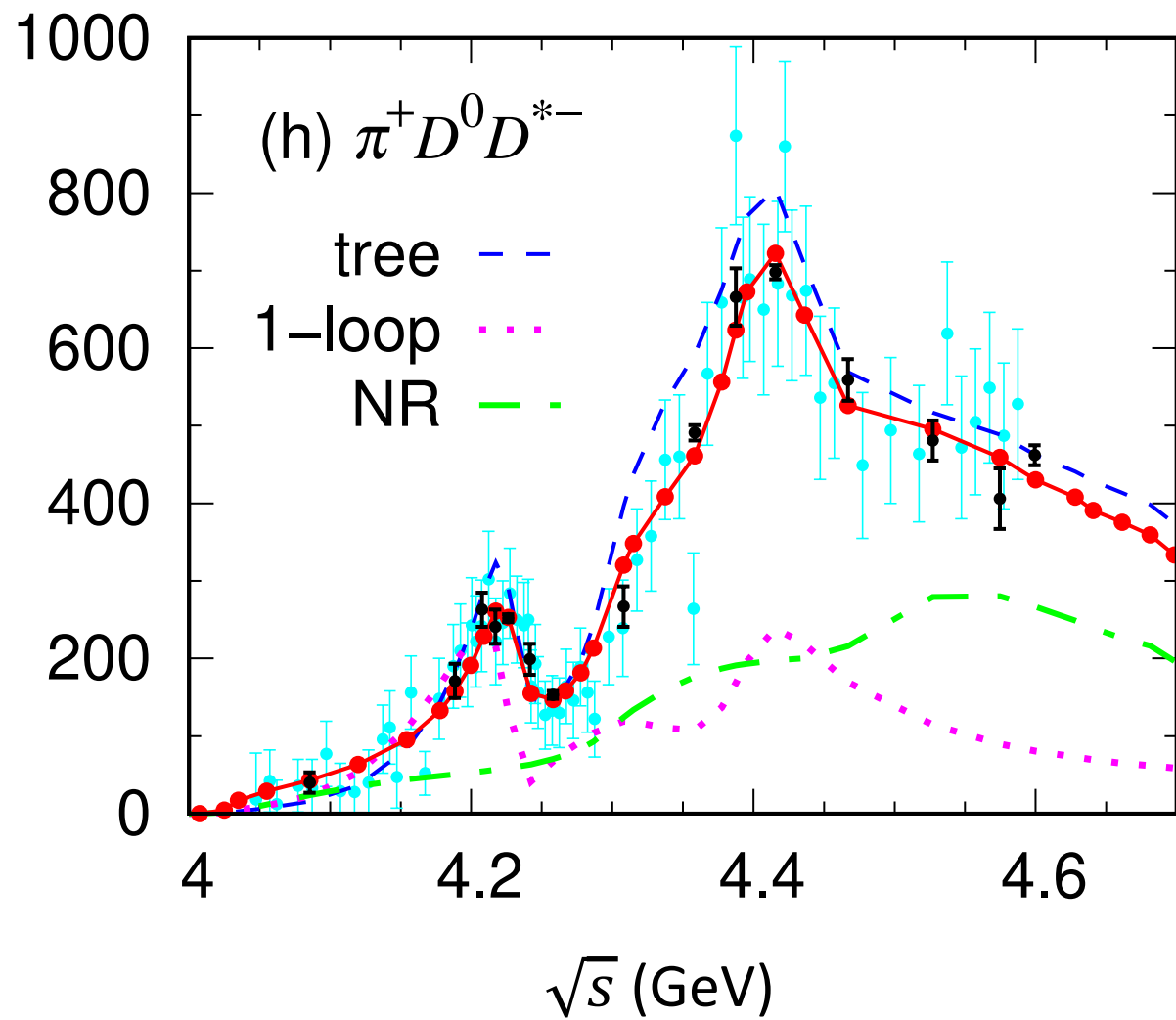
→



No tree mechanism for $\eta_c \rho \pi$ in our model

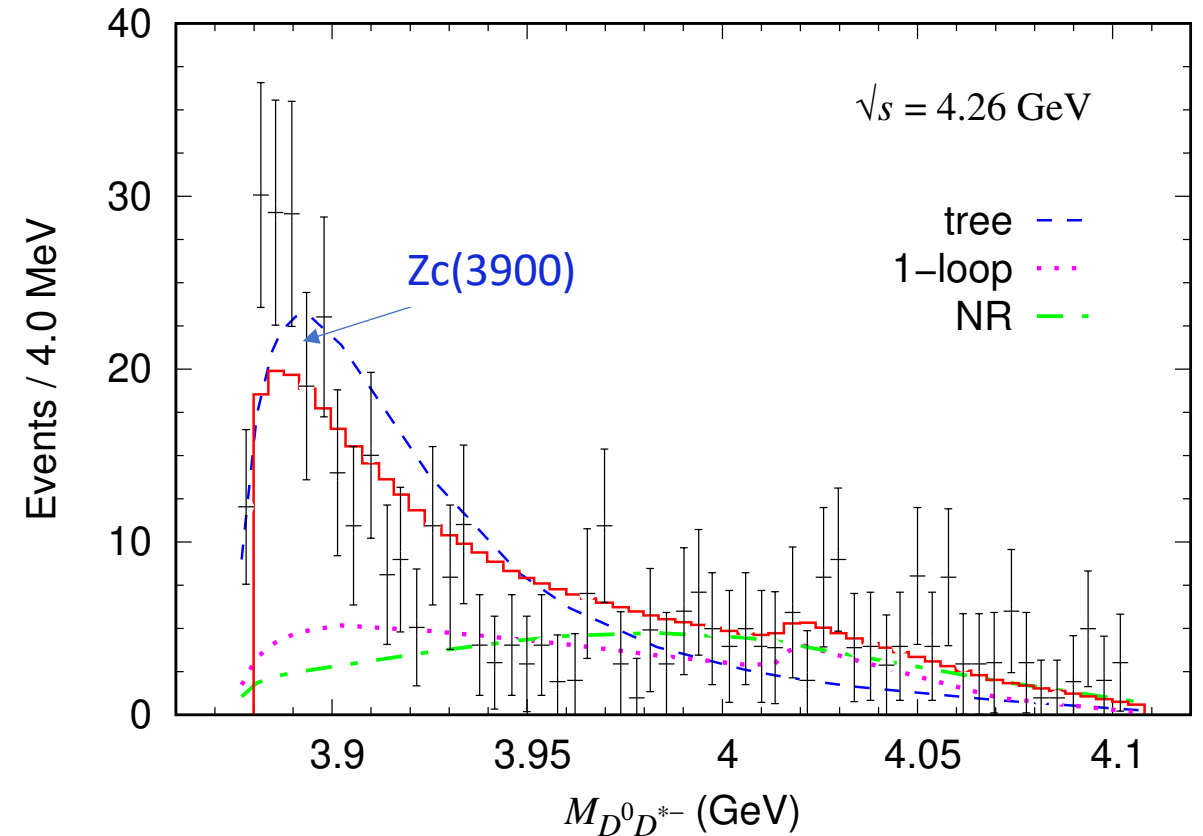
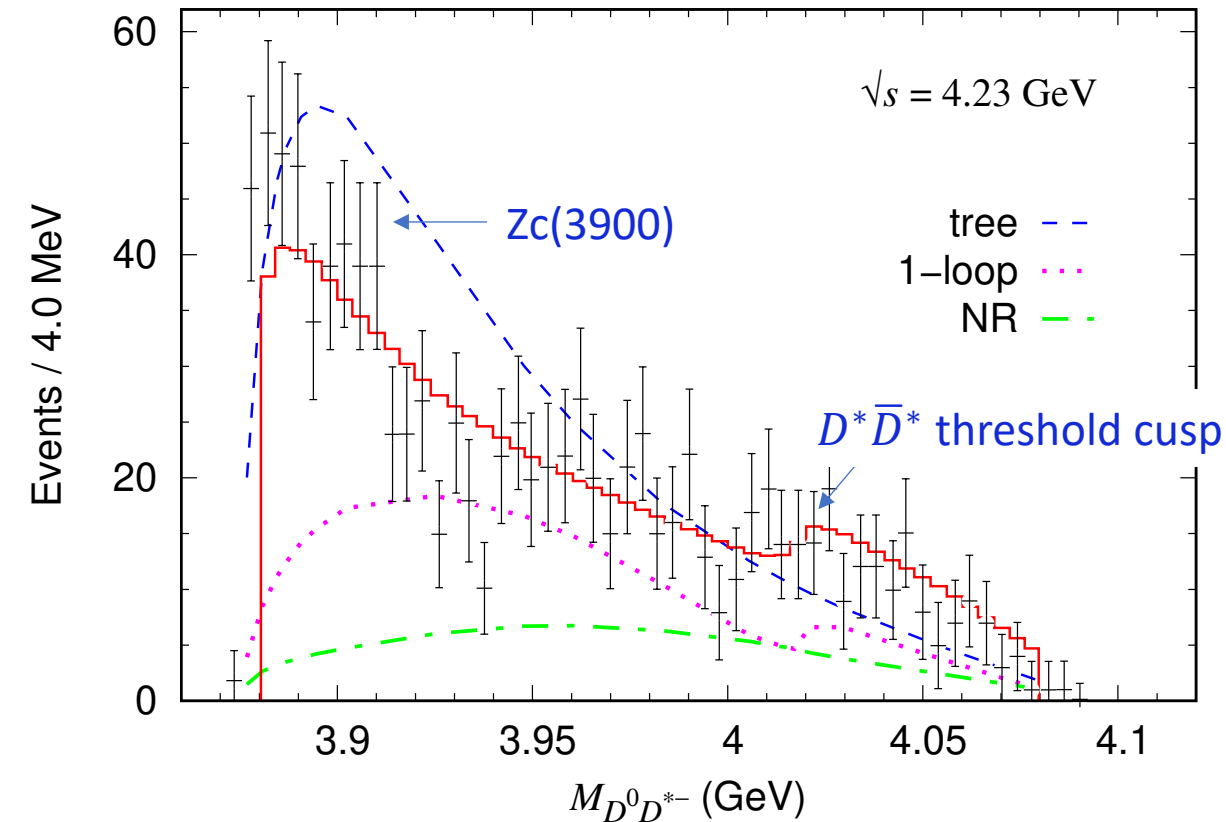
$Z_c(3900)$ peak is fitted

$$e^+e^- \rightarrow \pi^+ D^0 D^{*-}$$



$$e^+e^- \rightarrow \pi^+ D^0 D^{*-}$$

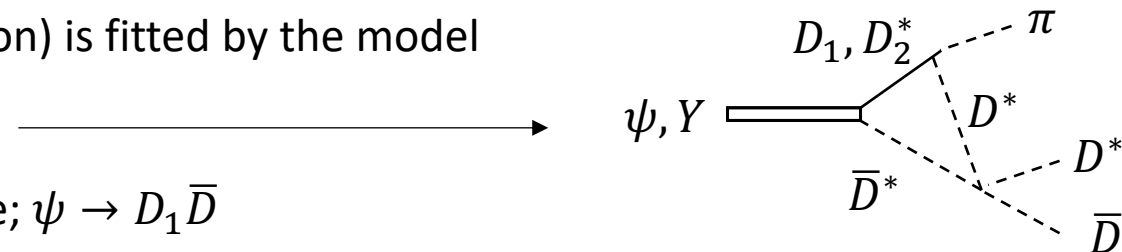
$D^0 D^{*-}$ invariant mass distributions



- Threshold enhancement (or $Z_c(3900)$ contribution) is fitted by the model

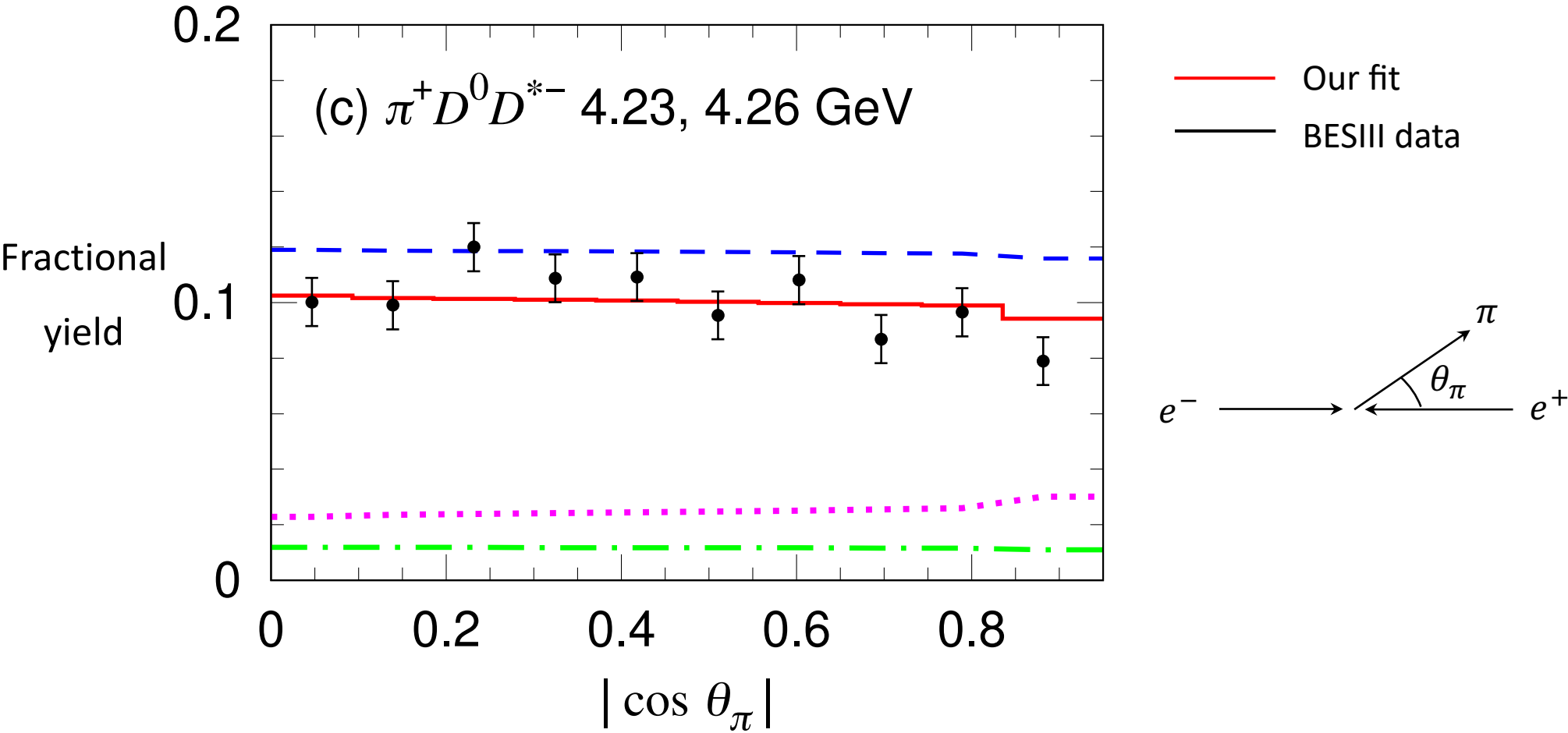
- $D^* \bar{D}^*$ threshold cusps are caused by

- $D^* \bar{D}$ threshold enhancement is mostly from tree; $\psi \rightarrow D_1 \bar{D}$



$$e^+e^- \rightarrow \pi^+ D^0 D^{*-}$$

Pion angle distributions from e^+e^- beam direction in total CM frame



Data are average of 4.23 GeV ($N = 418$) and 4.26 GeV ($N = 239$) data

$$e^+e^- \rightarrow \pi^+ D^0 D^{*-}$$

Conflict with BESIII analysis result

Conclusion from BESIII PRD 92, 092006 (2015)

we conclude that the $D\bar{D}_1(2420)$ contribution to our observed Born cross section is smaller than its relative systematic uncertainty.

Difficult to make our model consistent with this BESIII conclusion. Why ? Insufficient information !!

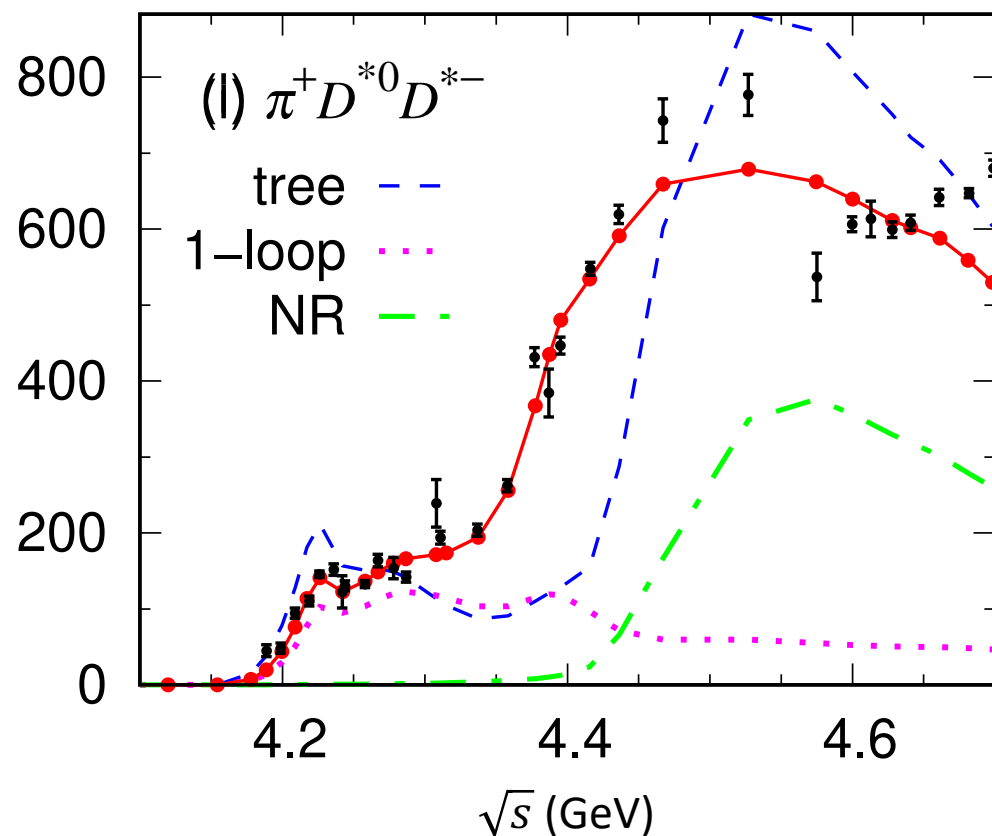
Hope BESIII to conduct amplitude analysis on this process, and present detailed results and/or Dalitz plots.

Without this information, $e^+e^- \rightarrow \pi^+ D^0 D^{*-}$ data cannot be well fitted, giving bad impact to the global fit overall

Most of previous theoretical models share the same problem

$$e^+e^- \rightarrow \pi^+(D^*\bar{D}^*)^-$$

— Our fit
— BESIII data



Good fit in < 4.45 GeV

Not good in higher energy region

BESIII reported a new charmonium at

$$M = 4675.3 \pm 29.5 \pm 3.5 \text{ MeV}, \Gamma = 218.3 \pm 72.9 \pm 9.3 \text{ MeV}$$

This state seems important to describe higher energy region

Inclusion of BW amplitude does not improve the fit

→ This state needs to be included in coupled-channel amplitude

The data (other final states) are not enough

for coupled-channel fit in > 4.6 GeV

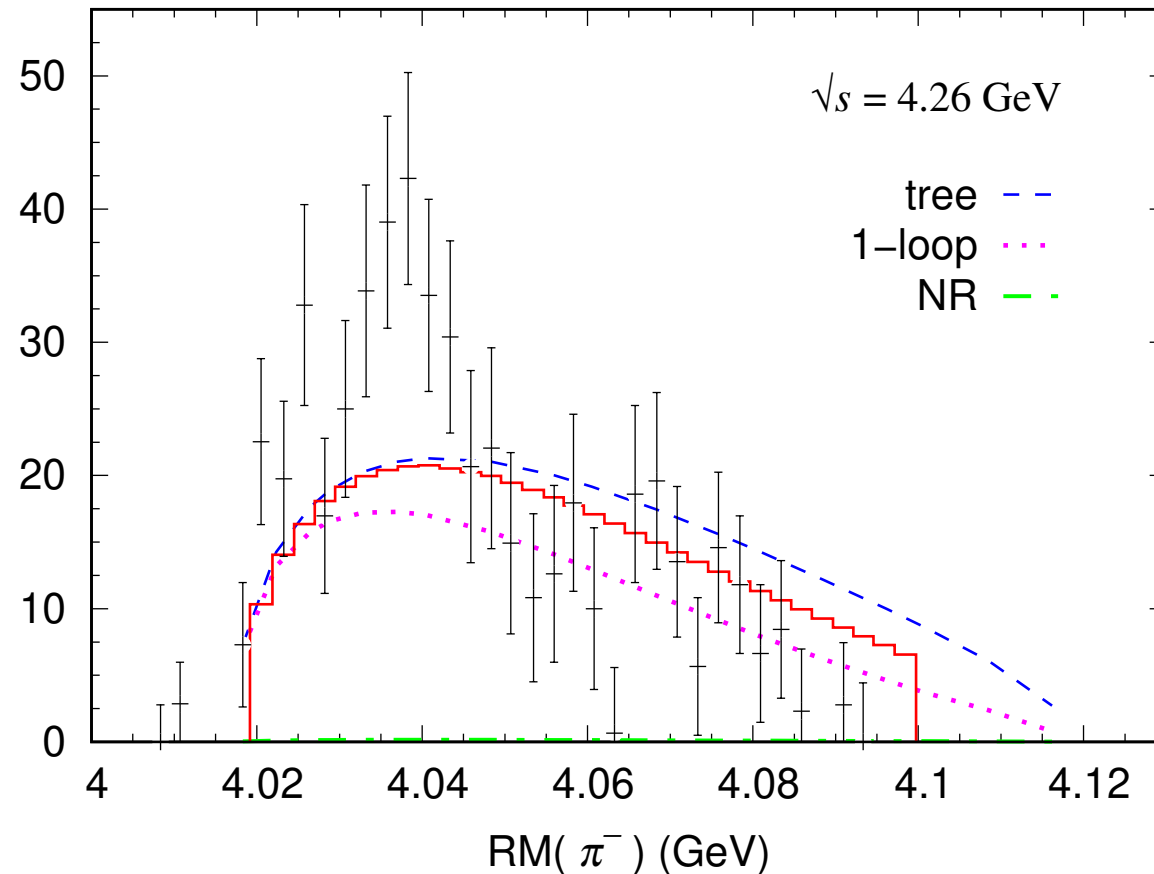
→ We wait for more data including $c\bar{c}s\bar{s}$ channels

$$\text{such as } D_{sJ}^{(*)}\bar{D}_s^{(*)}, KD^{(*)}\bar{D}_s^{(*)}$$

$$e^+e^- \rightarrow \pi^-(D^*\bar{D}^*)^+$$

$D^*\bar{D}^*$ invariant mass distributions (pion recoil mass)

— Our fit
— BESIII data



Fit does not seem good, however

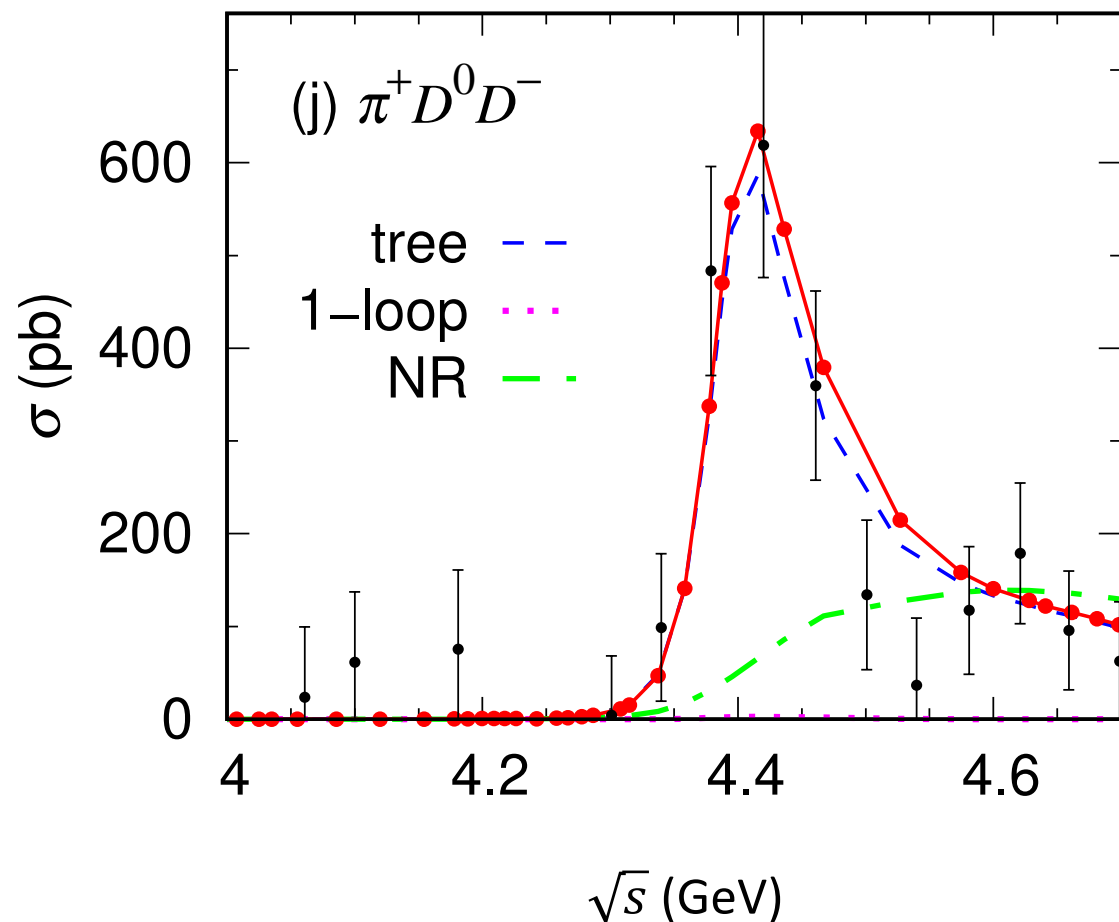
Kinematical end of the data ~ 4.09 GeV

Actual kinematical end ~ 4.12 GeV

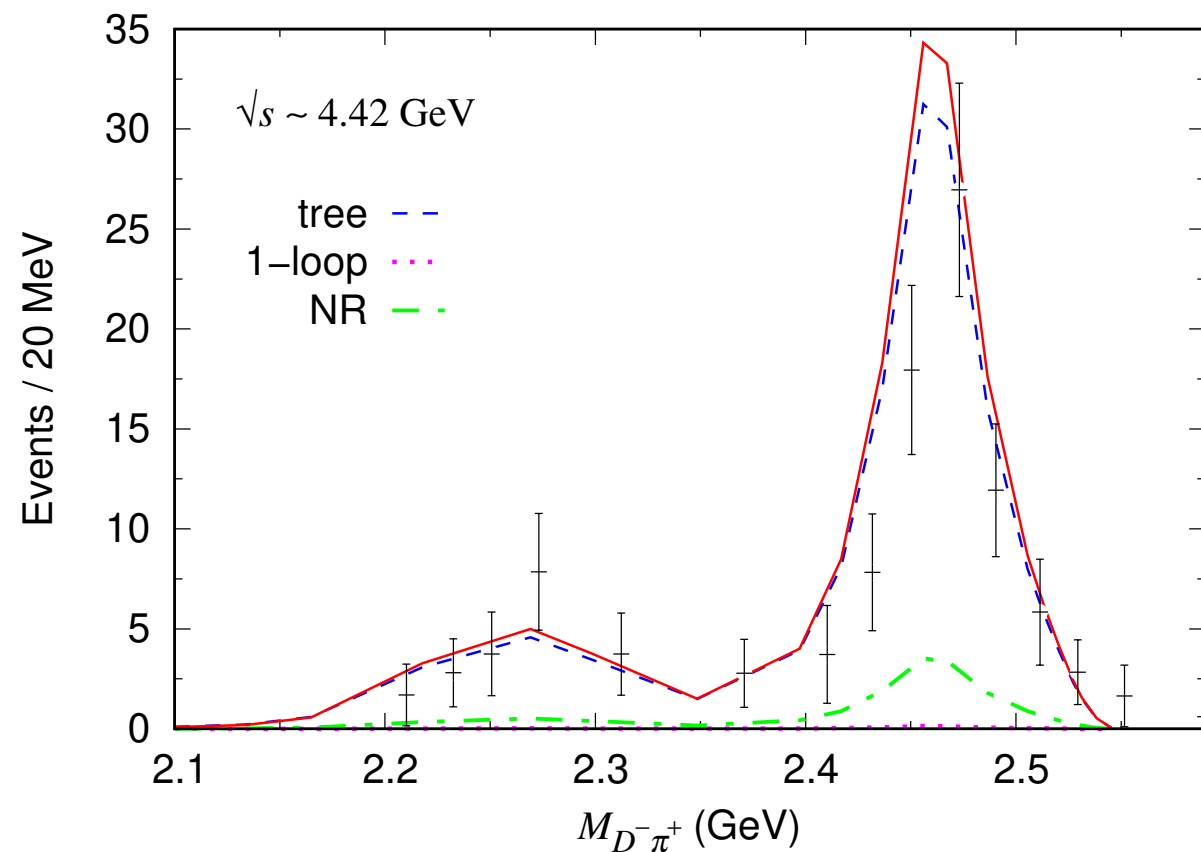
→ Efficiency correction seems significant for this lineshape data

Wait for efficiency corrected data (or MC output) for future improvement of coupled-channel model

$$e^+e^- \rightarrow \pi^+ D^0 D^-$$



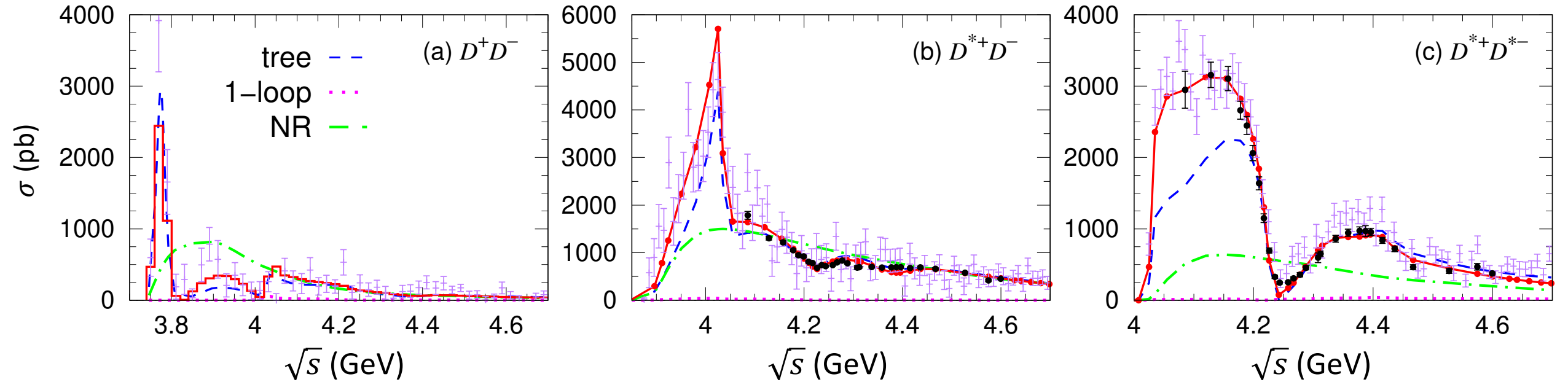
Clear $\psi(4420)$ peak is well fitted



Dominant $D_2^*(2460)$ contribution

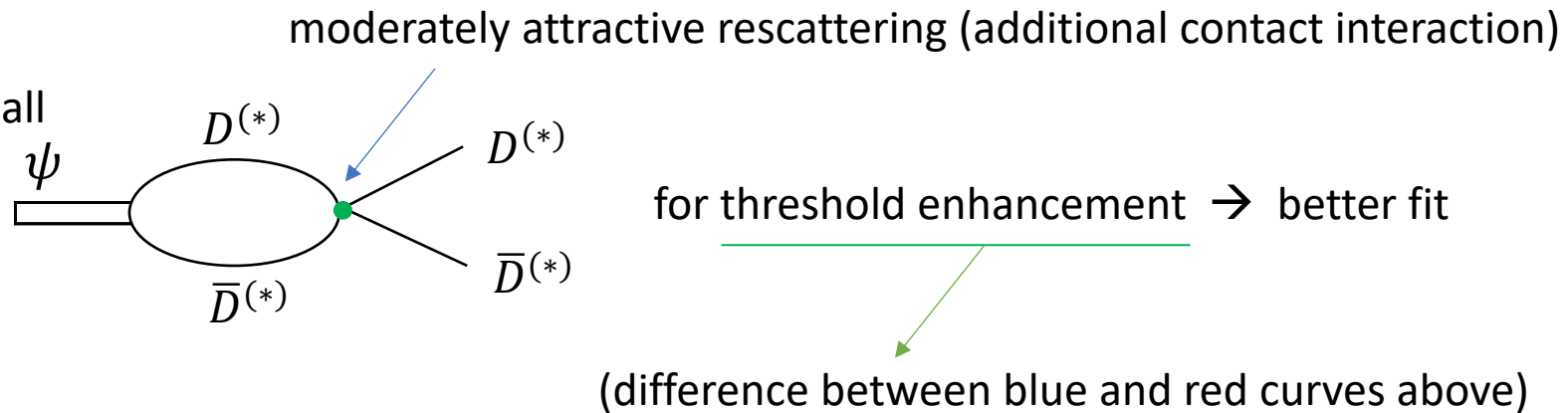
Hope to have a better quality data from BESIII ! $\rightarrow$ important for coupled-channel analysis

$$e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}$$

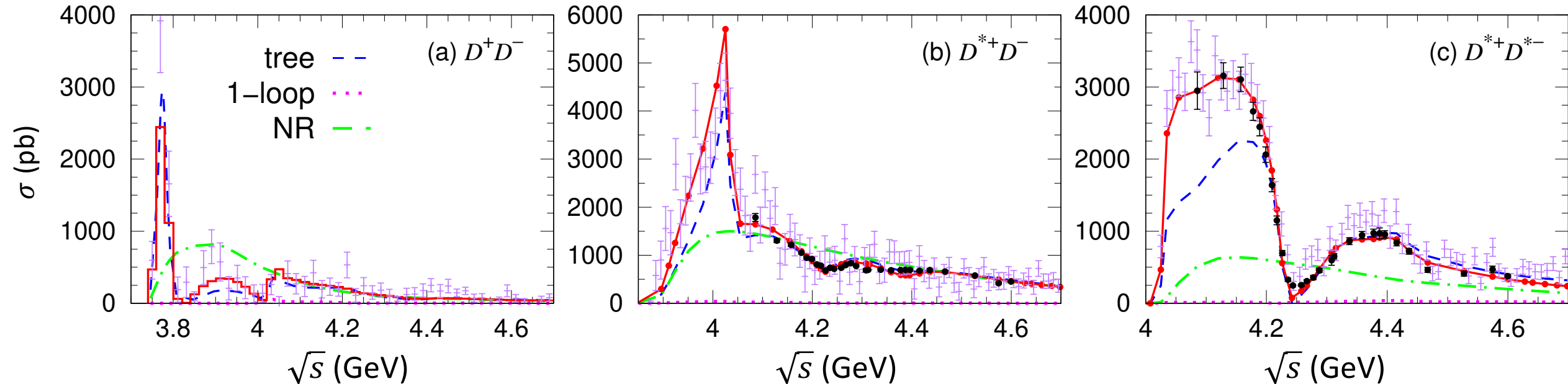


Tree dominates, 1-loop (particle exchange) is small

For $e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}$, $D_s^{(*)}\bar{D}_s^{(*)}$, we add

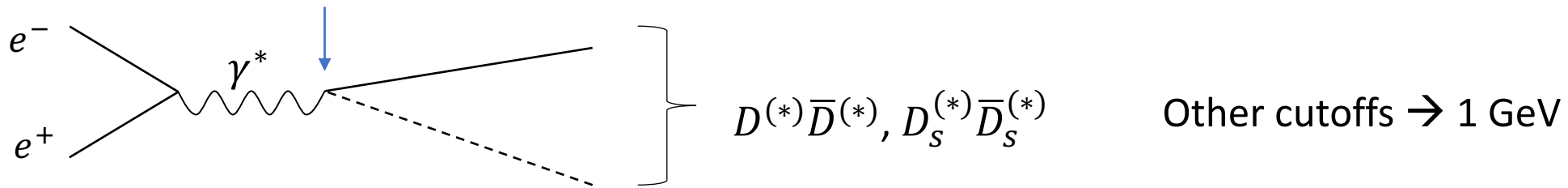


$$e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}$$

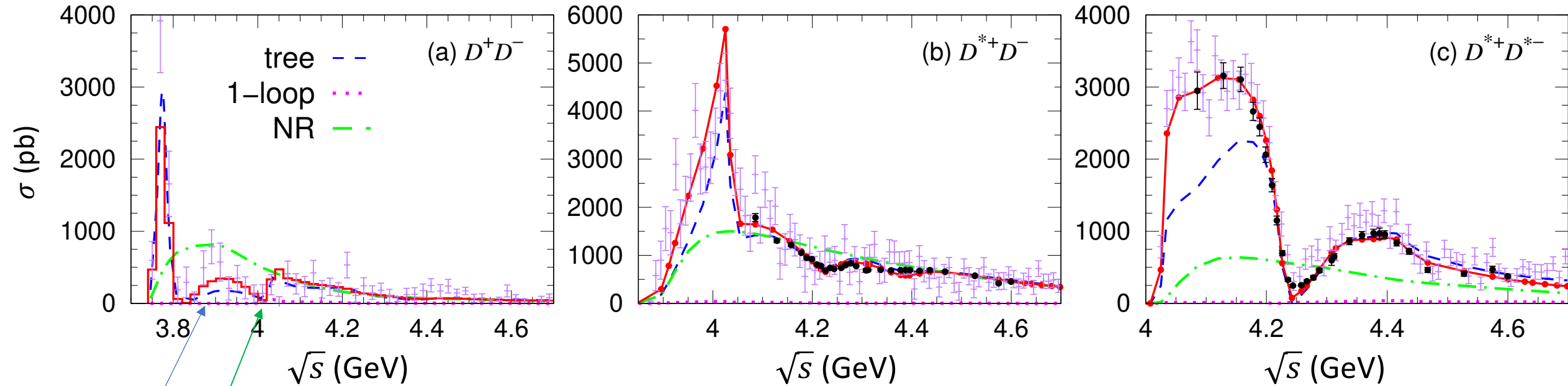


Energy dependence of NR contribution is important to fit data at higher energies

→ * Cutoffs (dipole form factors) to adjust energy dependence of nonresonant amplitudes

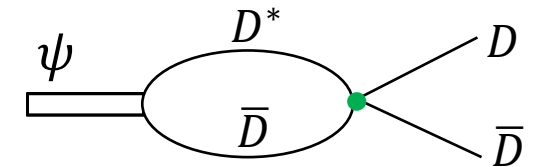


$$e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}$$

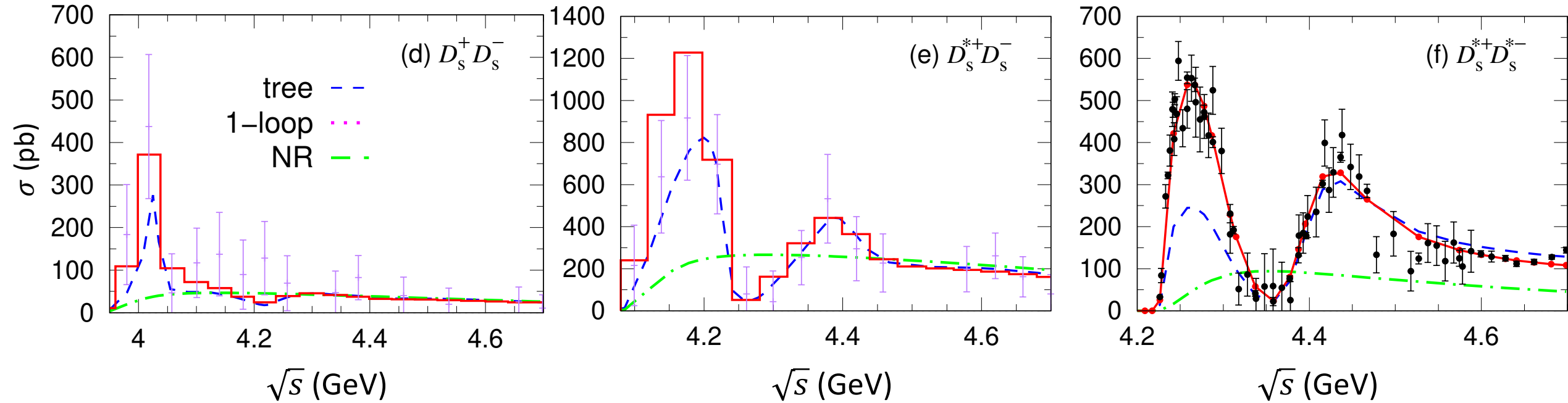


Interference between $\psi(3770)$ and non-resonant amplitudes + $D^*\bar{D}$ threshold cusp from
 $\rightarrow$ sharp dip and resonance-like peak at 3.9 GeV (not resonance)

Interference between $\psi(4040)$ and non-resonant amplitudes $\rightarrow$ sharp dip



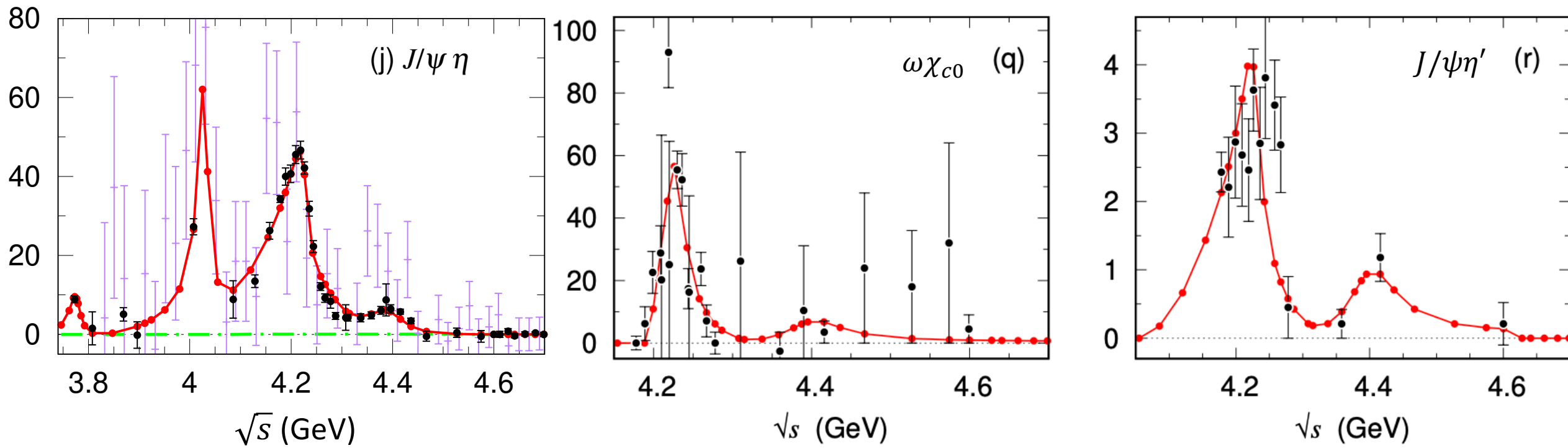
$$e^+e^- \rightarrow D_s^{(*)}\bar{D}_s^{(*)}$$



Similar observations as $e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}$

- Threshold enhancements needed for good fits (contact interactions added to bring blue to red curves)
- Energy dependence of NR contribution is important to fit data at higher energies (cutoff adjusted)

$$e^+e^- \rightarrow J/\psi \eta^{(\prime)}, \chi_{c0}\omega$$



For $J/\psi \eta$, a sharp peak appears at 4.02 GeV, as a consequence of coupled-channel fit

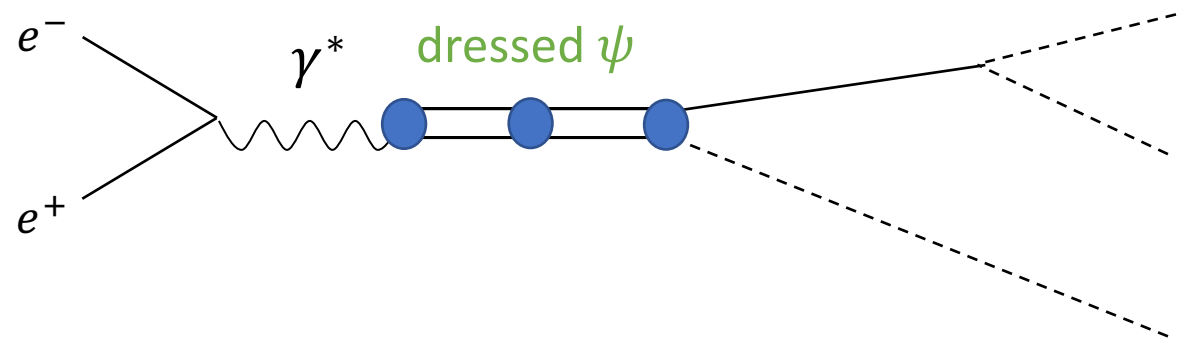
← BESIII does not have data point, but Belle data seems to favor this result

Poles and resonance properties

ψ, Y poles from their dressed propagator

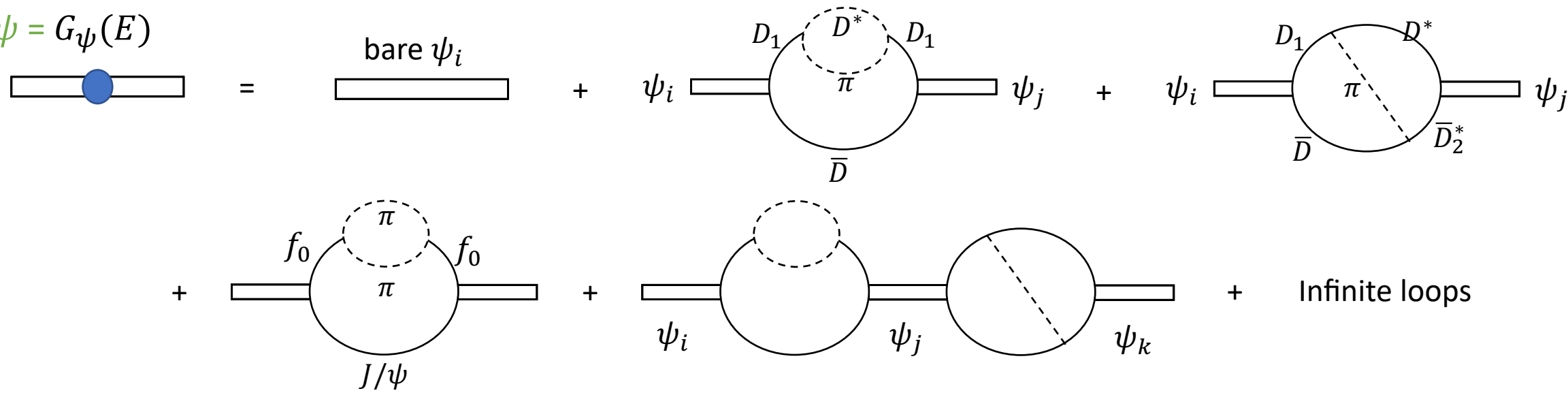
(we are not using BW)

Full amplitude



+ non-resonant

dressed $\psi = G_\psi(E)$



Search complex energy E_ψ where $G_\psi(E_\psi) = \infty$ (E_ψ : pole energy, pole position) by analytical continuation of $G_\psi(E)$

Resonance parameters

$$M = \text{Re}[E_\psi]$$
$$\Gamma = -2 \times \text{Im}[E_\psi]$$

BW fit →

This work		PDG		
M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	
3775	28	3778.1 ± 0.7	27.5 ± 0.9	$\psi(3770)$
4026	25	4039 ± 1	80 ± 10	$\psi(4040)$
4232	114	4191 ± 5	70 ± 10	$\psi(4160)$
4226	36	4222.5 ± 2.4	48 ± 8	$\psi(4230)$
4309	328	—	—	—
4369	183	4374 ± 7	118 ± 12	$\psi(4360)$
4394	93	4421 ± 4	62 ± 20	$\psi(4415)$
4690	106	4630 ± 6	72^{+14}_{-12}	$\psi(4660)$

When several poles are found nearby but on different Riemann sheets,
they correspond to the same state and only the one closest to the physical real energy is listed

Resonance parameters

$$M = \text{Re}[E_\psi]$$
$$\Gamma = -2 \times \text{Im}[E_\psi]$$

This work		PDG		
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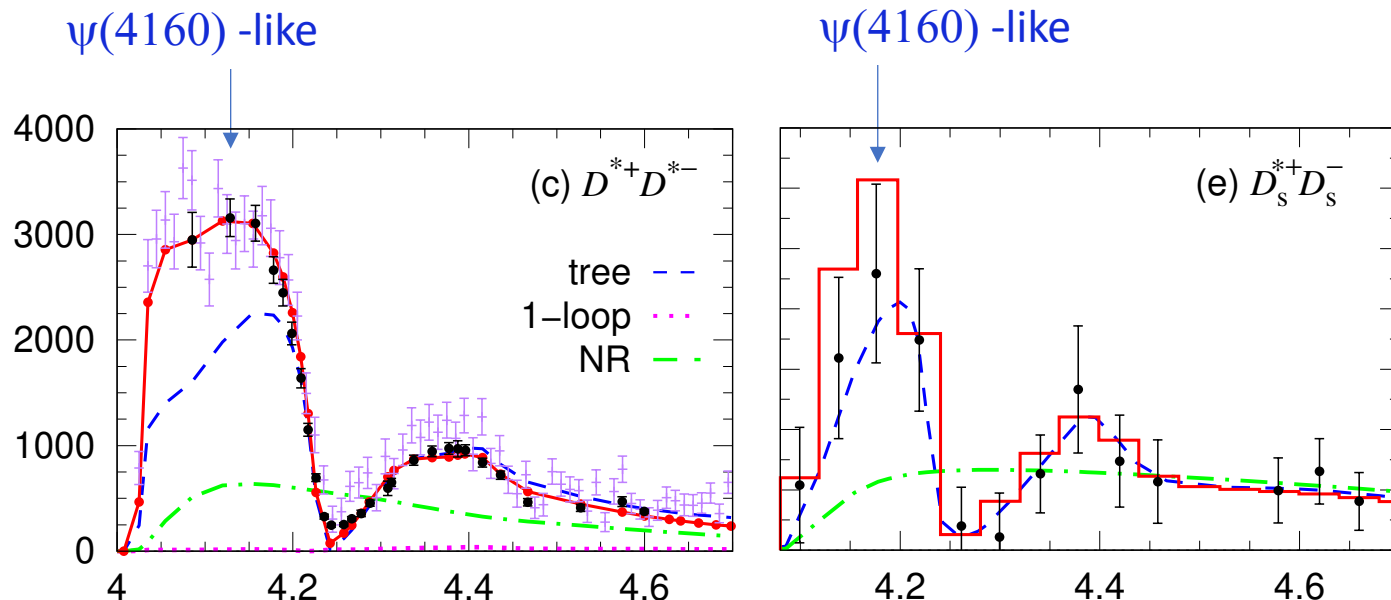
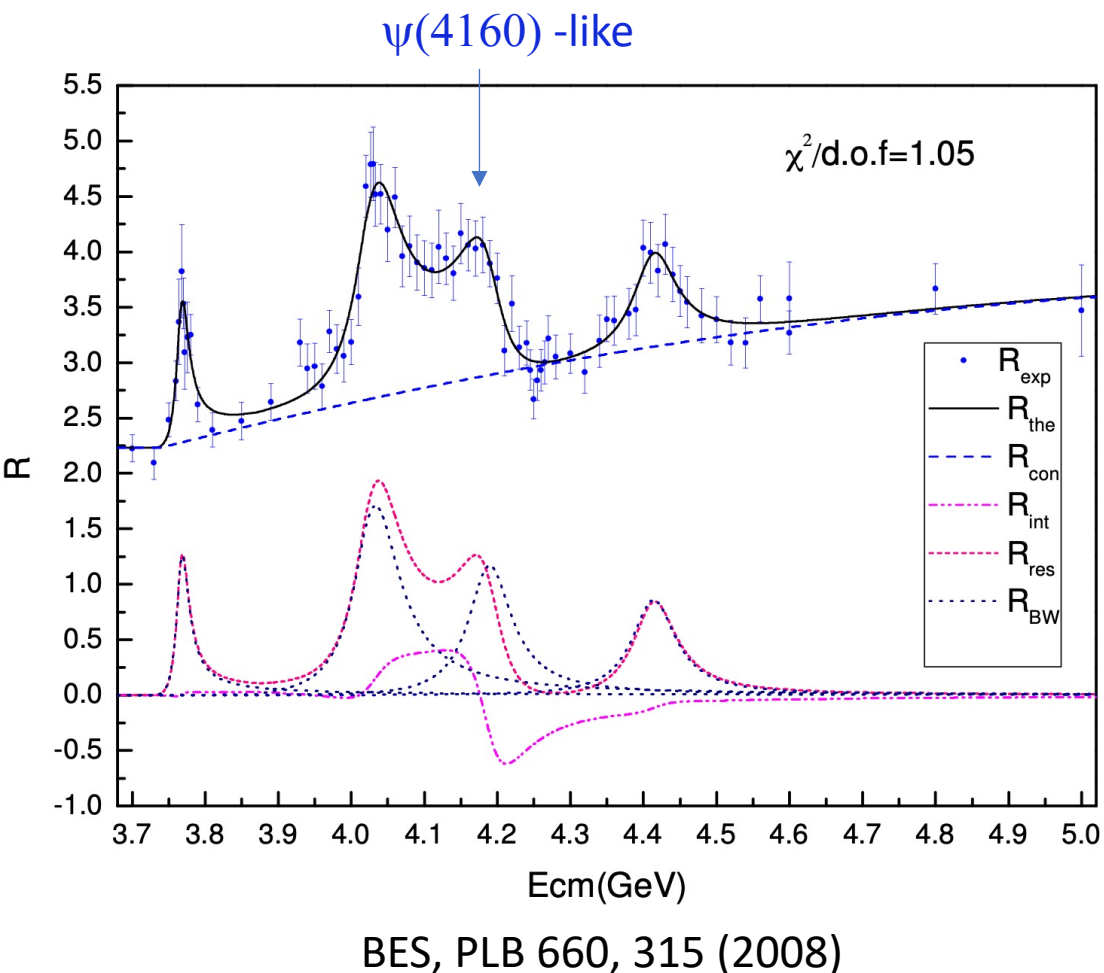
BW fit →

- $\psi(4040)$ width is significantly narrower
- (well-established) $\psi(4160)$ does not exist
- Two states at ~ 4230 MeV; $\psi(4230)$ and a broader one

Noticeable differences from PDG

No $\psi(4160)$ from our analysis

According to PDG, the clearest $\psi(4160)$ signal is seen in the R-scan data (BES2) and $B^+ \rightarrow K^+ \mu^+ \mu^-$ (LHCb)



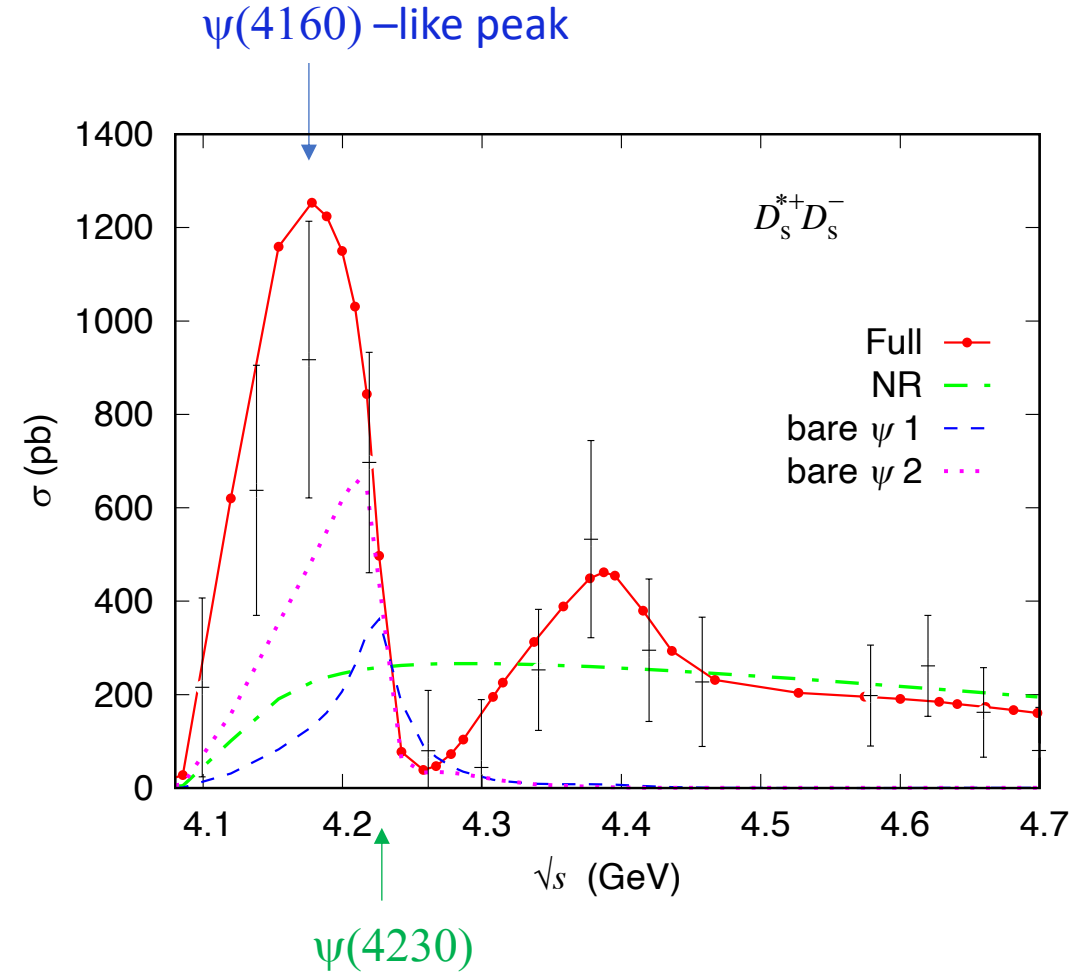
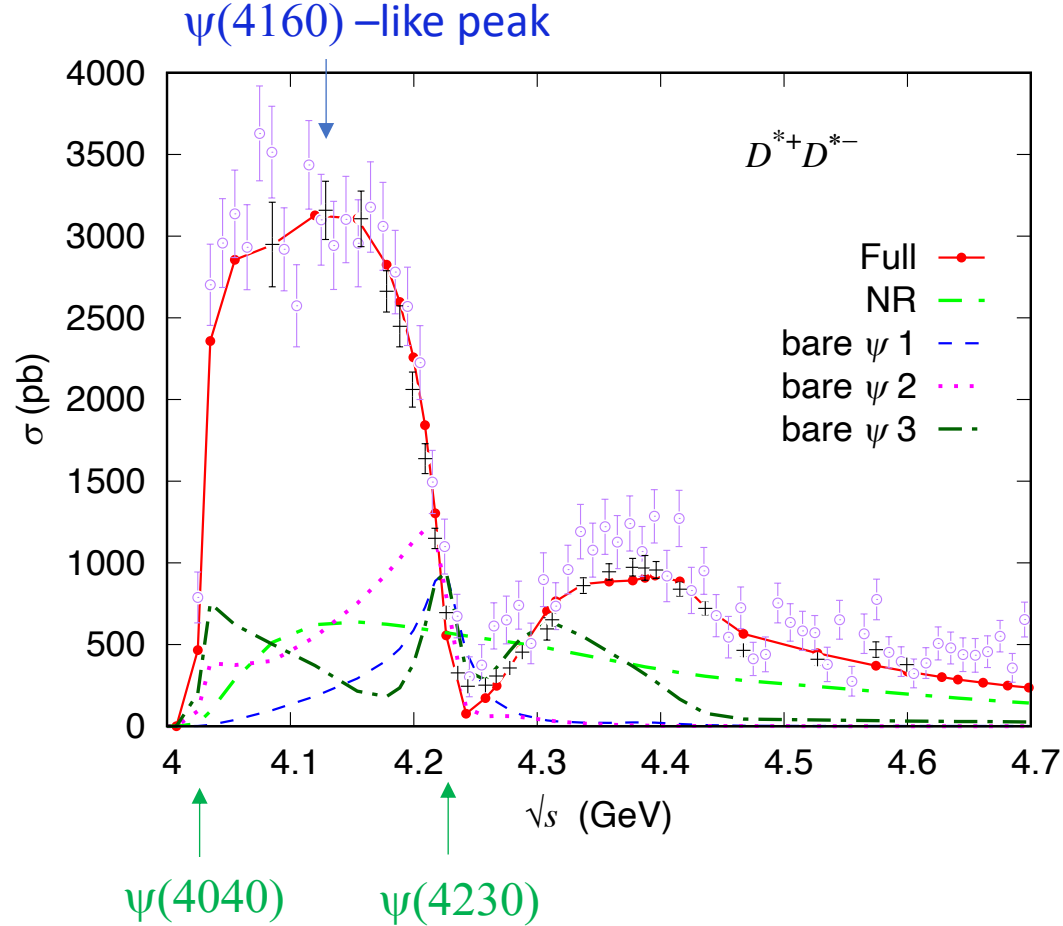
$\psi(4160)$ -like signals in exclusive processes

They can be described with interfering $\psi(4040)$ and $\psi(4230)$

→ $\psi(4160)$ signal in R may be caused by the same mechanism

$\psi(4040)$ and $\psi(4160)$ properties in PDG are from a simple BW fit to R value → artifacts may happen

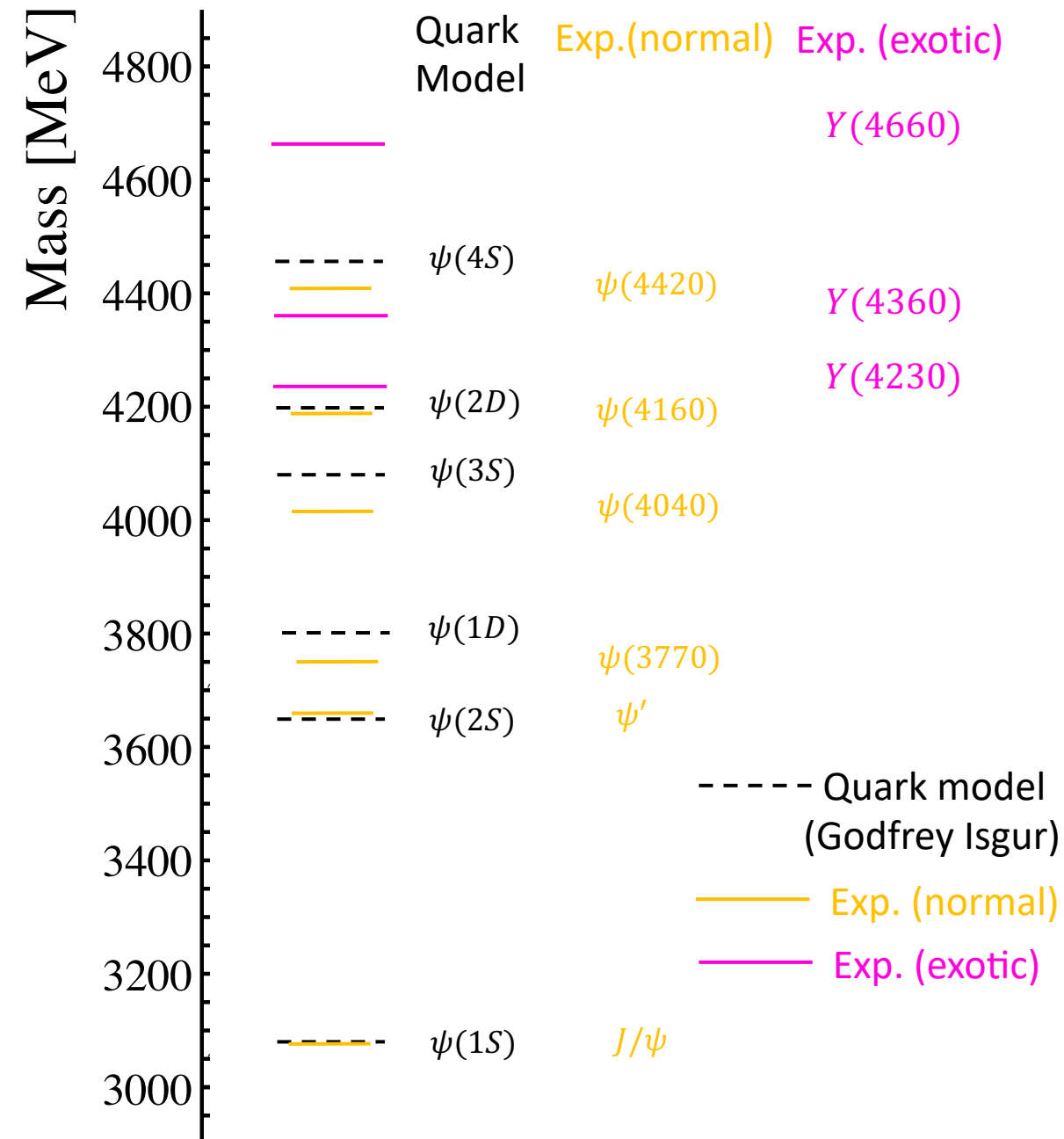
No $\psi(4160)$ from our analysis



In the above processes, $\psi(4160)$ -like peaks are from interfering $\psi(4040)$, $\psi(4230)$, and NR

In coupled-channel model, bare ψ states mix to form a resonance $\rightarrow$ each bare ψ decay has several resonance peaks

Charmonium spectrum ($J^{PC} = 1^{--}$)



No $\psi(4160) \rightarrow$ impact on Y

Y are considered exotic since not predicted by quark model

If $\psi(4160)$ does not exist, a natural assignment is

$$Y(4230) = \psi(2D) \quad (\text{conventional } c\bar{c})$$

Several theory papers proposed $Y(4230)$ as $D_1\bar{D}$ molecule

$\rightarrow Y(4230) \rightarrow D_1\bar{D}$ is main $Y(4230)$ decay mode

By examining the $Y(4230)$ pole residues,

we can support/disfavor this scenario (future work)

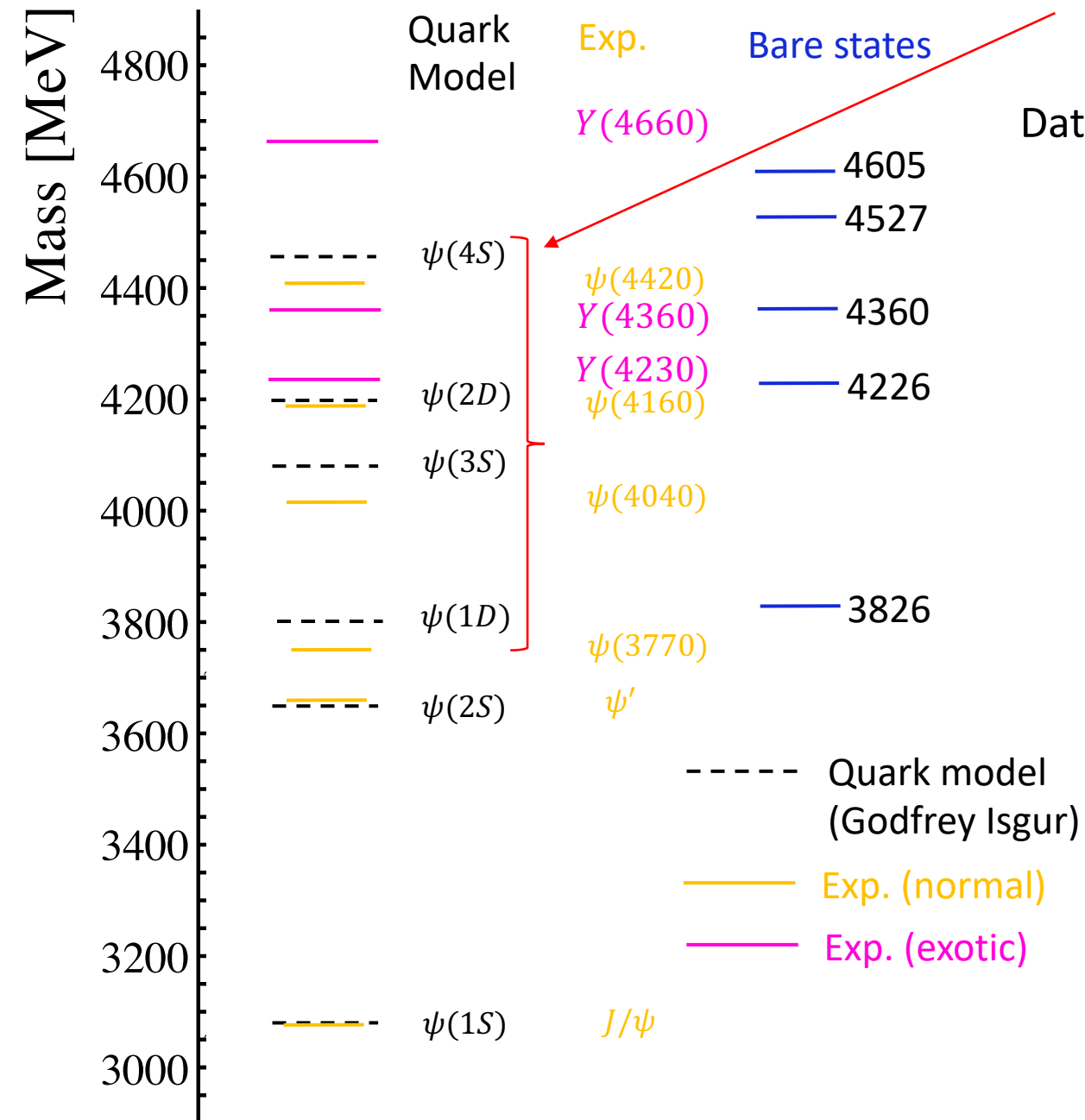
(From previous slide)

$Y(4230) \rightarrow D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}$ might occur more often

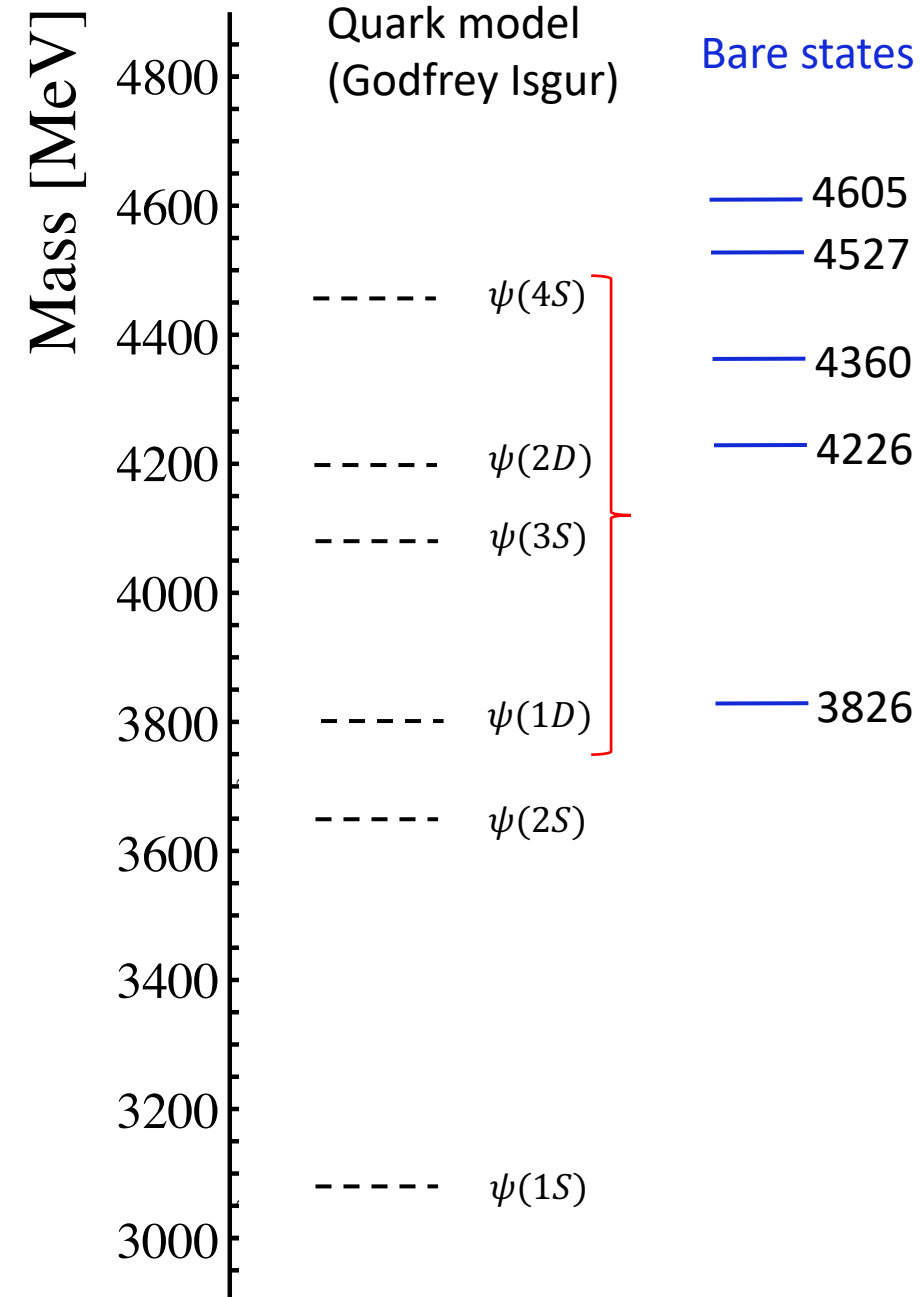
Charmonium spectrum ($J^{PC} = 1^{--}$)

Quark model predicts **four** states in the relevant energy region

Data require **five** bare states for achieving reasonable fit



Charmonium spectrum ($J^{PC} = 1^{--}$)



Quark model predicts **four** states in the relevant energy region

Data require **five** bare states for achieving reasonable fit

Conceptually, quark-model-state and our bare state is similar

→ Resonance without hadron-hadron continuum components

Very model-dependent argument/questions

One bare state is not accommodated in the quark model

→ Is it exotic bare state ?

Does it generate $Y(4230)$ and $Y(4360)$ after being dressed ?

Does it correspond to hybrid state predicted by LQCD ?

Liu et al., JHEP 07 (2012) 126

Our model alone cannot answer these interesting questions

Maybe possible by combining with structure model (quark model, etc.)

Relation between bare state and pole

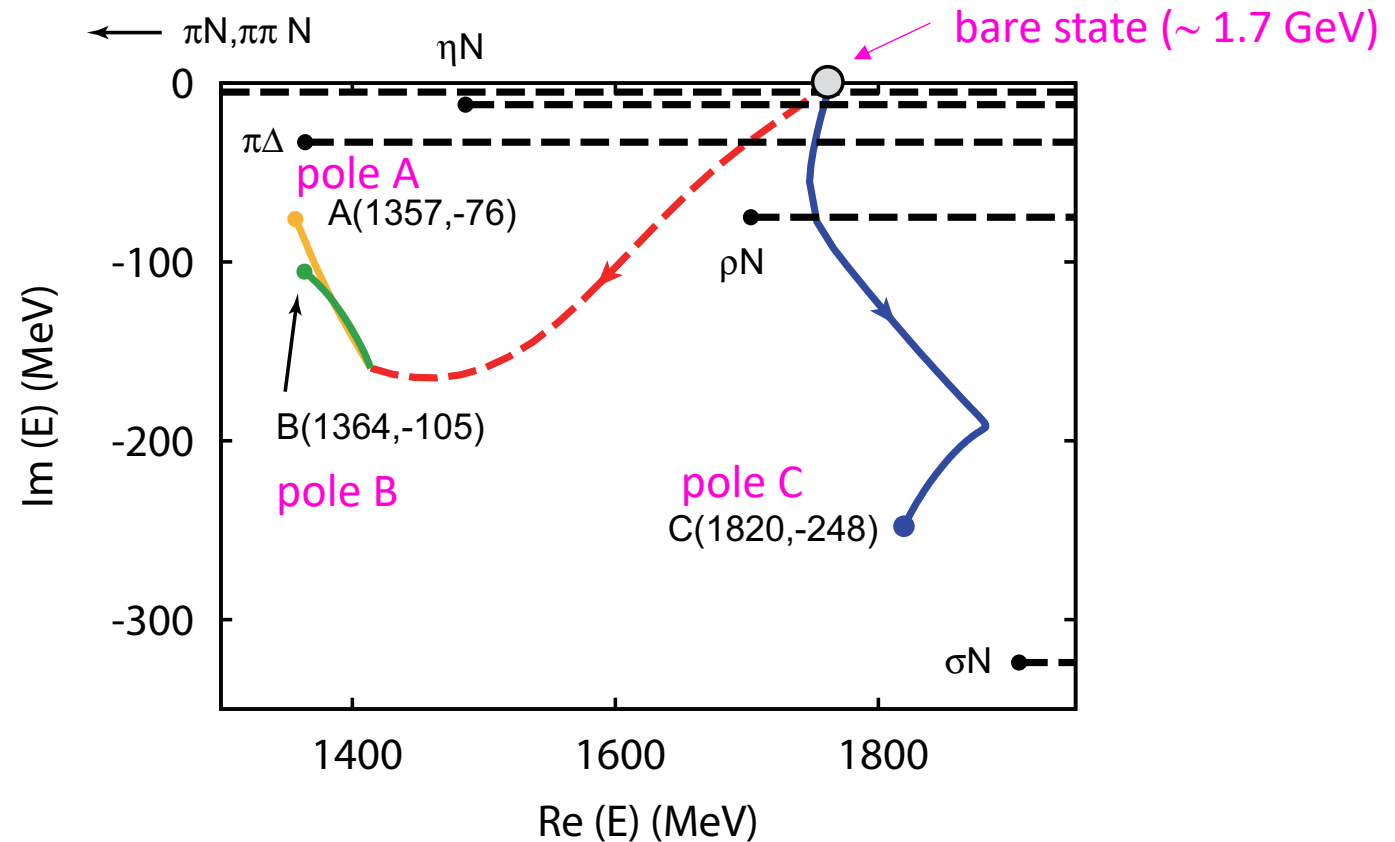
Data require **five** bare states

→ dressed by hadron continuum

→ **seven** poles

M (MeV)	Γ (MeV)
3775	28
4026	25
4232	114
4226	36
4309	328
4369	183
4394	93

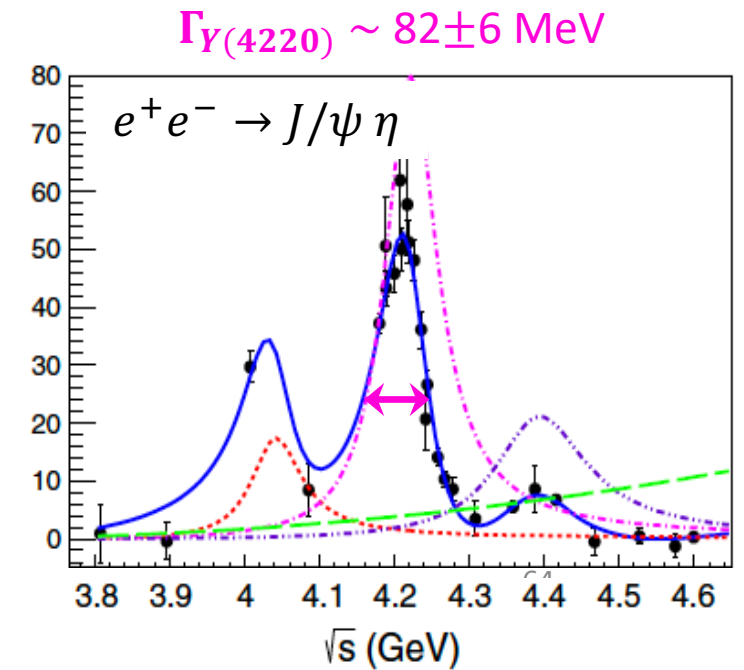
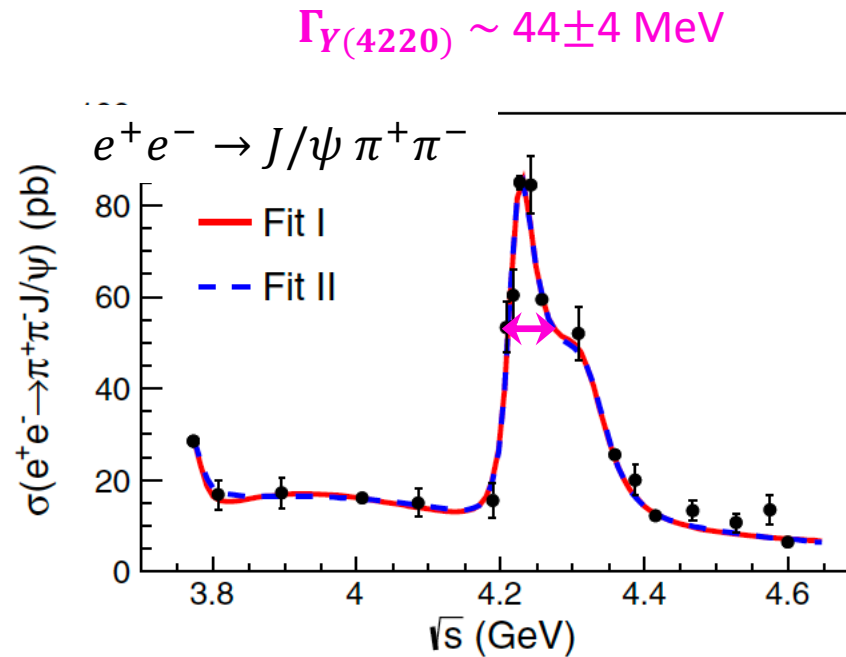
Similar finding in nucleon resonances Suzuki et al. (EBAC) PRL 104, 042302 (2010)



Future work : Which pair of poles come from the same bare state (mainly) ?

(speculation) Possible solution to Y width problem

	M (MeV)	Γ (MeV)
	3775	28
	4026	25
ψ_{wid}	4232	114
ψ_{nar}	4226	36
	4309	328
	4369	183
	4394	93



Two poles at $M \sim 4230$ (4380) MeV with narrow (ψ_{nar}) and wide (ψ_{wid}) widths. We can explain Y widths if:

$$\text{For } e^+e^- \rightarrow J/\psi \pi^+ \pi^- \quad |g_{\psi_{\text{nar}} \rightarrow J/\psi \pi \pi}| \gg |g_{\psi_{\text{wid}} \rightarrow J/\psi \pi \pi}|$$

$$\text{For } e^+e^- \rightarrow J/\psi \eta \quad |g_{\psi_{\text{nar}} \rightarrow J/\psi \eta}| \ll |g_{\psi_{\text{wid}} \rightarrow J/\psi \eta}|$$

$g_{\psi_{\text{nar}} \rightarrow J/\psi \pi \pi}$: pole residue

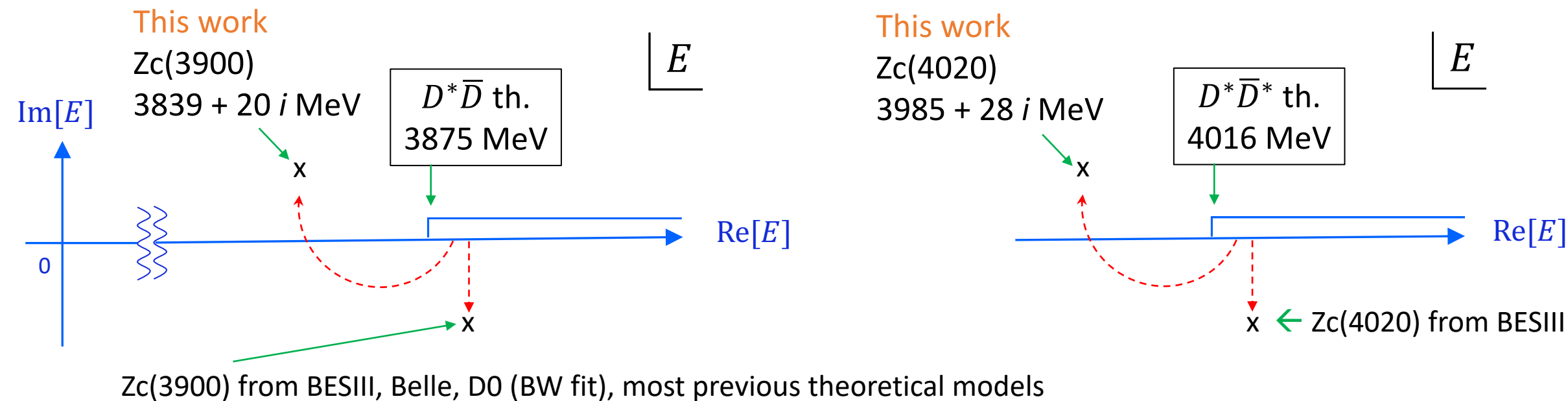
Residues will be extracted in near future, and address the Y width problem

Zc poles

from $J^{PC} = 1^{+-} D^* \bar{D} - D^* \bar{D}^* - J/\psi \pi - \psi' \pi - h_c \pi - \eta_c \rho$ couple—channel amplitude

This work		PDG		
E_{pole} (MeV)	RS	M (MeV)	Γ (MeV)	
$3839 + 20i$	(up)	3887.1 ± 2.6	28.4 ± 2.6	$Z_c(3900)$
$3985 + 28i$	(pu)	4024.1 ± 1.9	13 ± 5	$Z_c(4020)$

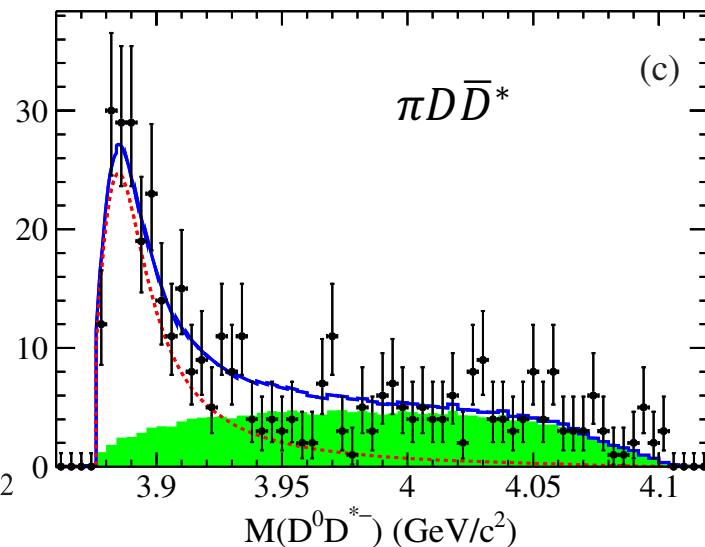
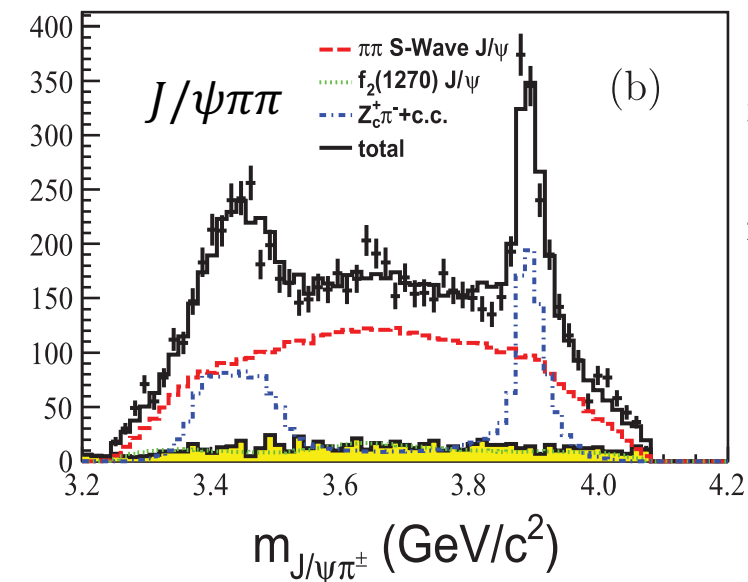
($S_{D^* \bar{D}}, S_{D^* \bar{D}^*}$)



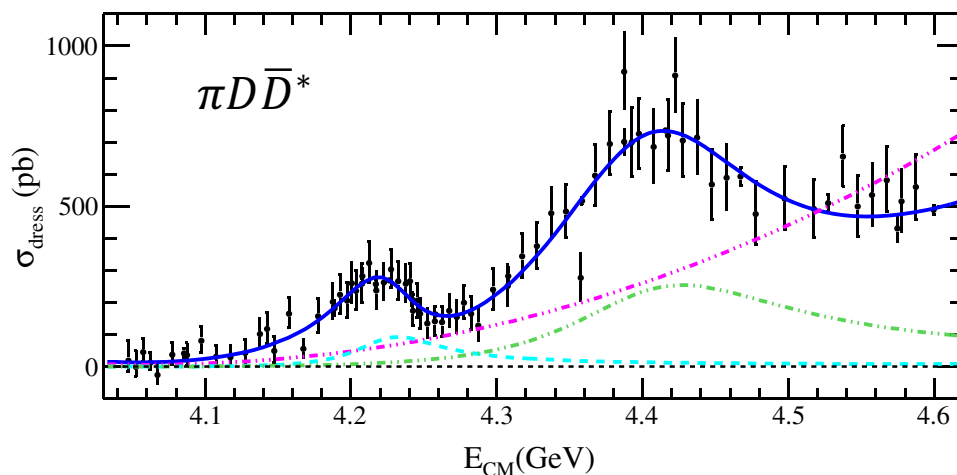
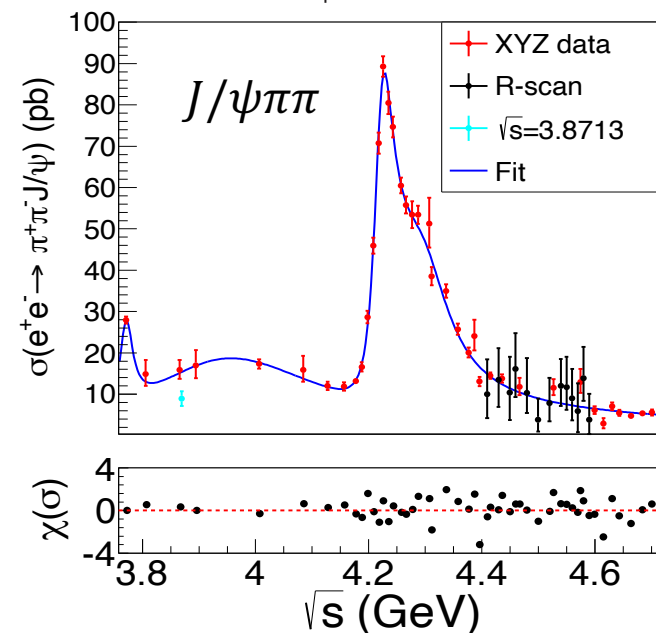
Zc from our analysis are virtual states, different from Breit-Wigner fit and most of previous theoretical analyses

In next slides, we make two points to support our result

Common problem in previous theoretical analyses on Zc(3900)



Invariant mass distributions (left, event numbers) are fitted to determine Zc(3900) pole
 $\rightarrow$ model's overall normalization is arbitrary but model has $\sigma(e^+e^- \rightarrow J/\psi\pi\pi)/\sigma(e^+e^- \rightarrow \pi D\bar{D}^*)$



In previous theoretical analyses,

cross sections (left) were not considered
 $\rightarrow \sigma(e^+e^- \rightarrow J/\psi\pi\pi)/\sigma(e^+e^- \rightarrow \pi D\bar{D}^*)$ from model is unchecked

Cross section data can test Zc production mechanism, Zc decay residues

$$\frac{\sigma(e^+e^- \rightarrow J/\psi\pi\pi)}{\sigma(e^+e^- \rightarrow \pi D\bar{D}^*)} \text{ should be checked to see if models are reasonable}$$

Our analysis cleared this problem

Present analysis result is consistent with lattice QCD

Previous LQCD analyses on $Z_c(3900)$ in:

Prelovsek et al. PLB 727, 172 (2013), PRD 91, 014504 (2015)
Chen et al. PRD 89, 094506 (2014)
Ikeda et al. (HAL QCD) PRL 117, 242001 (2016)
Cheung et al. (Hadron spectrum Collab.) JHEP 11, 033 (2017)

LQCD conclusion : $I = 1, J^{PC} = 1^{+-} D^* \bar{D}$ s-wave interaction is very weak,
disfavoring narrow $Z_c(3900)$ pole near $D^* \bar{D}$ threshold

Most of previous determinations of $Z_c(3900)$ pole are not consistent with LQCD

Summary and perspective

Summary

- Conducted global coupled-channel analysis of most of available $e^+e^- \rightarrow c\bar{c}$ data in $\sqrt{s} = 3.75 - 4.7$ GeV
Global coupled-channel analysis is common for N^* . The $e^+e^- \rightarrow c\bar{c}$ analysis now gets closer to the standard !
- Reasonable fits are obtained overall
- Vector charmonium and Z_c poles extracted
 - no $\psi(4160)$, but two poles at ~ 4230 MeV with different widths
 - Z_c poles are virtual poles at ~ 40 MeV below $D^*\bar{D}^{(*)}$ thresholds, consistent with LQCD results

Future

- Pole residues will be extracted $\rightarrow$ address Y width problem, structure of exotic candidates Y
- Fit efficiency-corrected, background-free Dalitz plots (not 1D fit) to fully consider experimental constraints on charmonium and Z_c properties
- Include $e^+e^- \rightarrow K\bar{D}_s^{(*)}D^{(*)}$ when cross sections become available $\rightarrow$ include higher charmonium states
 $\rightarrow$ address $Z_{cs}(3985)$ from global analysis