

Higgs properties and new physics beyond the SM

Bin Yan

Institute of High Energy Physics

2024年理论物理前沿与交叉科学研讨会

April 26-30, 2024

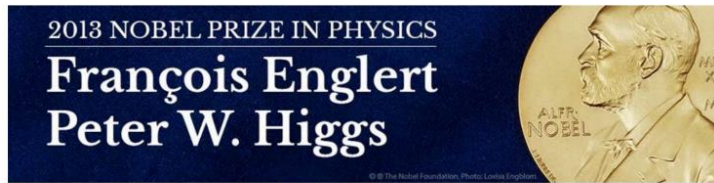
Bin Yan, C.-P. Yuan, PRL 127 (2021) 051801

Hai Tao Li, Bin Yan, C.-P. Yuan, PRL 131(2023) 041802

Xin-Kai Wen, Bin Yan, Zhite Yu, C.-P. Yuan, PRL 131 (2023) 241801

Bin Yan, C. Lee, JHEP 03 (2024) 123

The Era of the Higgs Physics

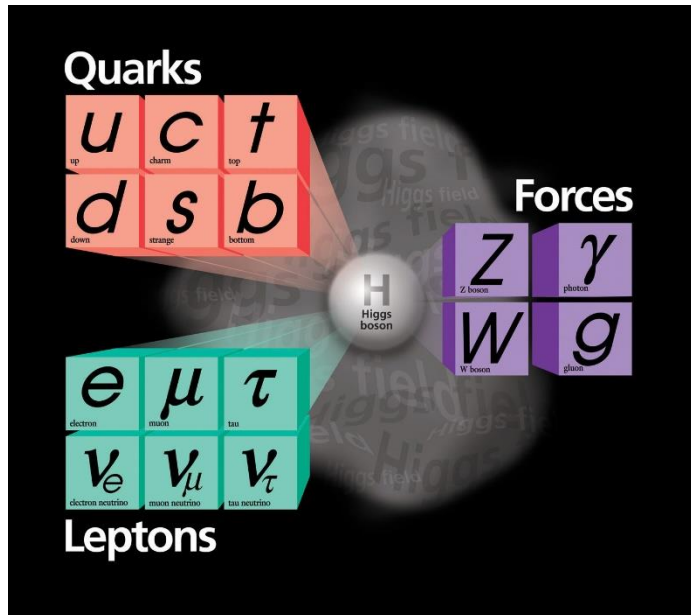


8 October 2013

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2013 to

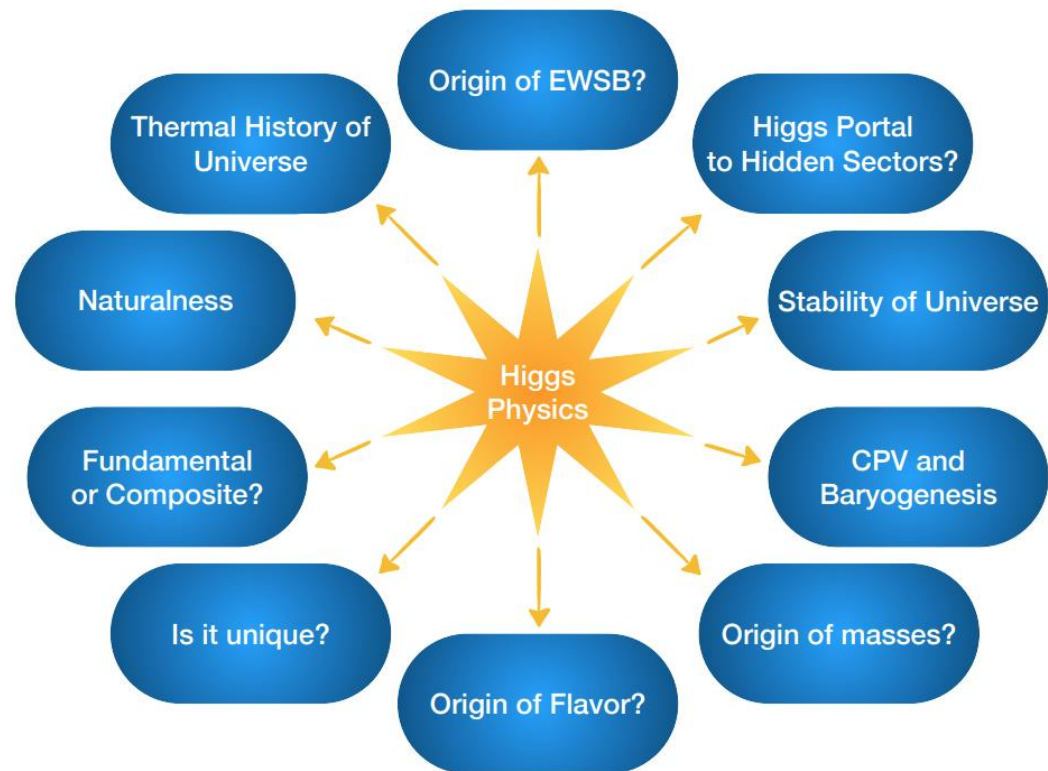
François Englert and Peter Higgs

"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"

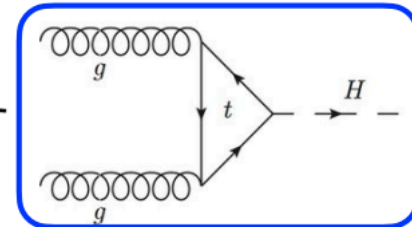
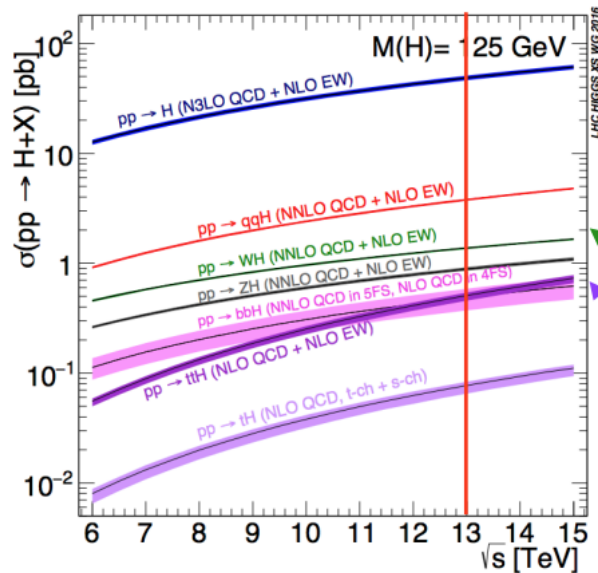


Understanding of origin of mass of subatomic particles

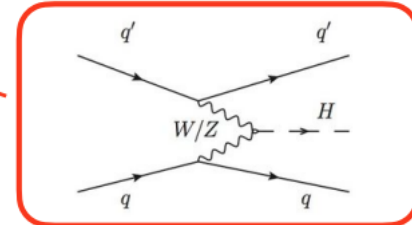
Snowmass 2021, 2209.07510



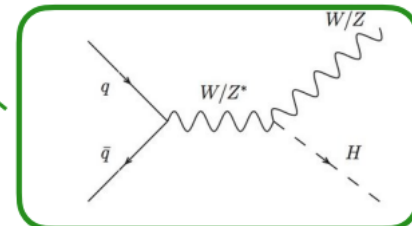
Higgs production @LHC



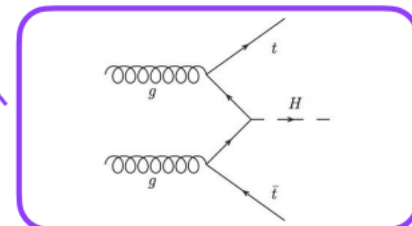
N³LO



N³LO



NNLO

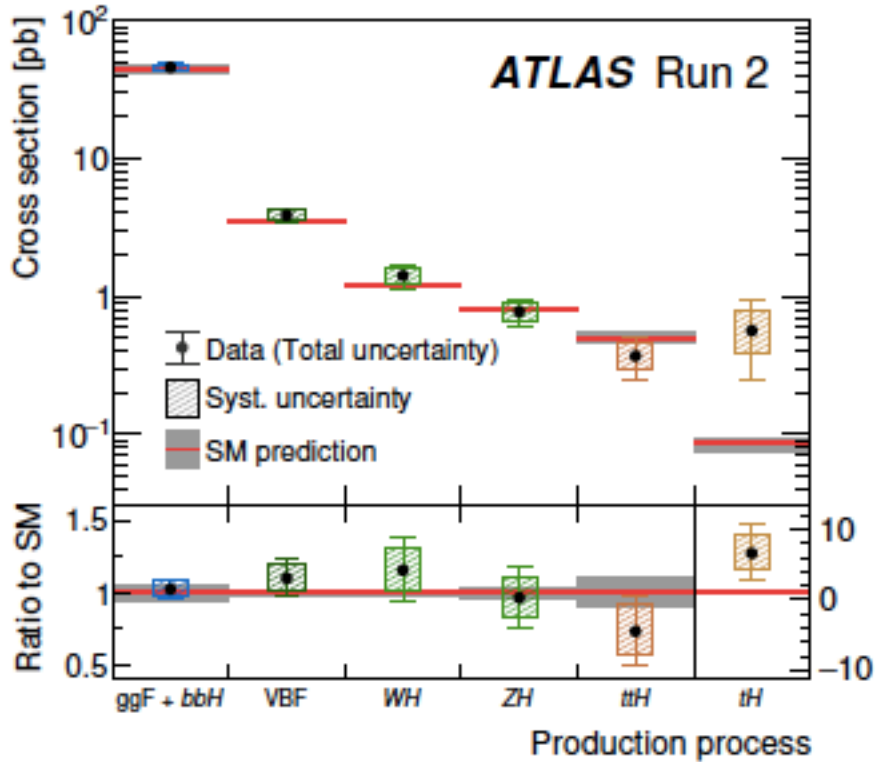


**NLO +
approx
NNLO**

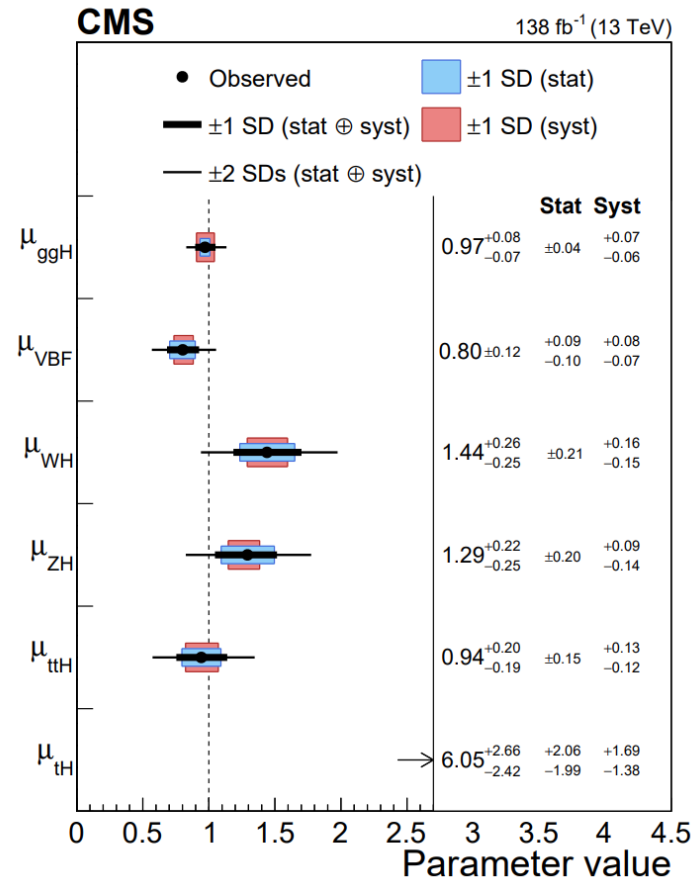
$\sqrt{s}$ (TeV)	Production cross section (in pb) for $m_H = 125$ GeV					
	ggF	VBF	WH	ZH	ttH	total
1.96	$0.95^{+17\%}_{-17\%}$	$0.065^{+8\%}_{-7\%}$	$0.13^{+8\%}_{-8\%}$	$0.079^{+8\%}_{-8\%}$	$0.004^{+10\%}_{-10\%}$	1.23
7	$16.9^{+5\%}_{-5\%}$	$1.24^{+2\%}_{-2\%}$	$0.58^{+3\%}_{-3\%}$	$0.34^{+4\%}_{-4\%}$	$0.09^{+8\%}_{-14\%}$	19.1
8	$21.4^{+5\%}_{-5\%}$	$1.60^{+2\%}_{-2\%}$	$0.70^{+3\%}_{-3\%}$	$0.42^{+5\%}_{-5\%}$	$0.13^{+8\%}_{-13\%}$	24.2
13	$48.6^{+5\%}_{-5\%}$	$3.78^{+2\%}_{-2\%}$	$1.37^{+2\%}_{-2\%}$	$0.88^{+5\%}_{-5\%}$	$0.50^{+9\%}_{-13\%}$	55.1
14	$54.7^{+5\%}_{-5\%}$	$4.28^{+2\%}_{-2\%}$	$1.51^{+2\%}_{-2\%}$	$0.99^{+5\%}_{-5\%}$	$0.60^{+9\%}_{-13\%}$	62.1

The measurements @ LHC

Nature 607 (2022)7917,52-59



Nature 607 (2022)7917,60-68



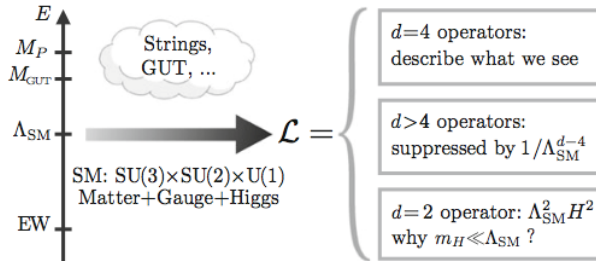
The data agrees with the SM prediction very well

The Framework for the Higgs physics

1. The κ framework for the couplings:

BSM physics is expected to affect the production modes and decay channels by a SM like interactions

2. The Standard Model Effective Field Theory



Linear realized EFT

Higgs is a **fundamental particle**
Weak interacting



W. Buchmuller, D. Wyler 1986

B. Grzadkowski et al, 2010

L. Lehman, A. Marin, 2015

B. Henning et al, 2015

H-L. Li et al, 2020

Murphy, 2020

$$\mathcal{L} = \frac{C_6}{\Lambda^2} \mathcal{O}_6 + \frac{C_8}{\Lambda^4} \mathcal{O}_8 + \dots$$

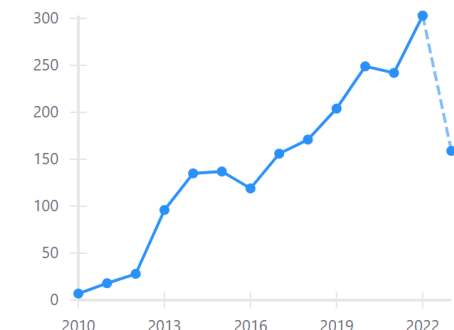
W. Buchmuller, D. Wyler 1986

Citations per year



B. Grzadkowski et al, 2010

Citations per year



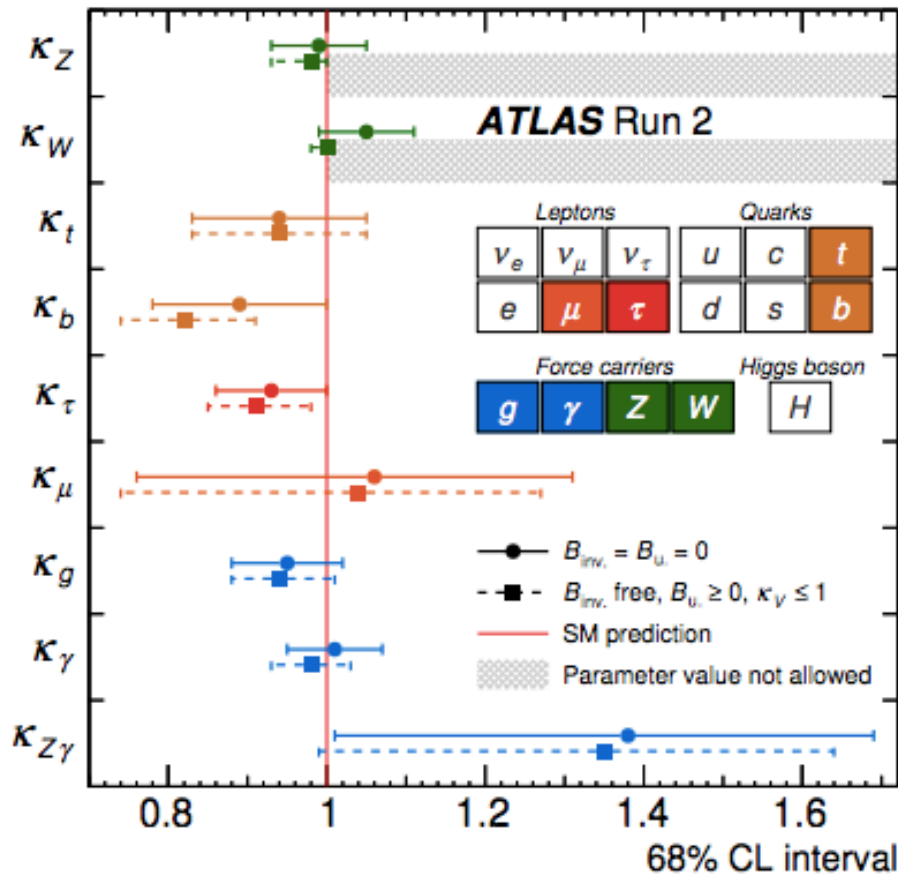
3. Higgs Effective Field Theory

Callan, Coleman, Wess, Zumino, 1969

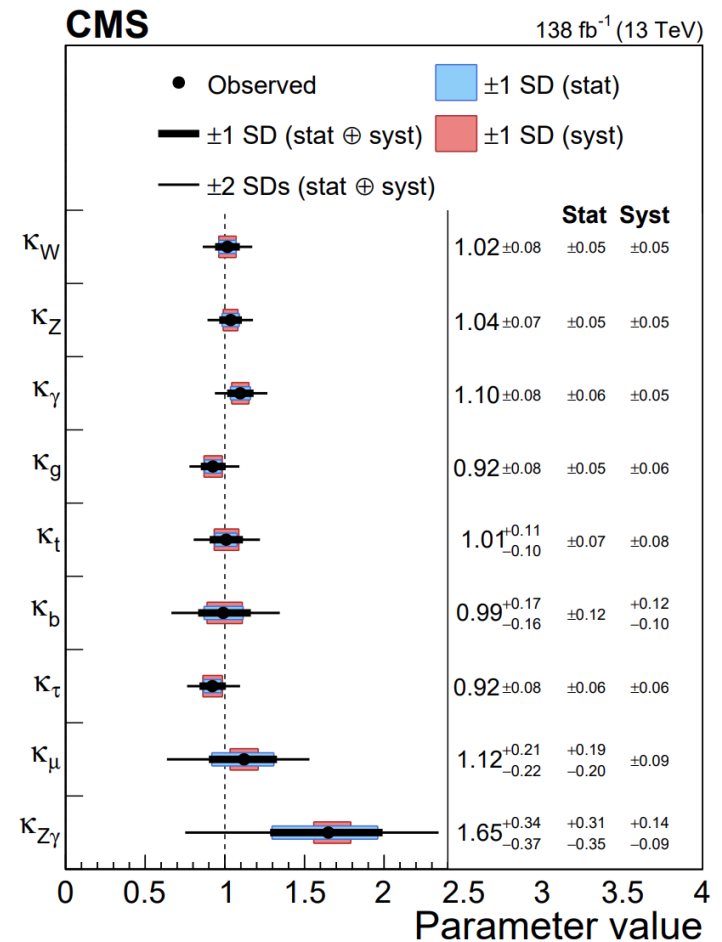
The electroweak chiral Lagrangian+light Higgs, A.C. Longhitano, 1980,....

Higgs couplings @LHC

Nature 607 (2022)7917,52-59



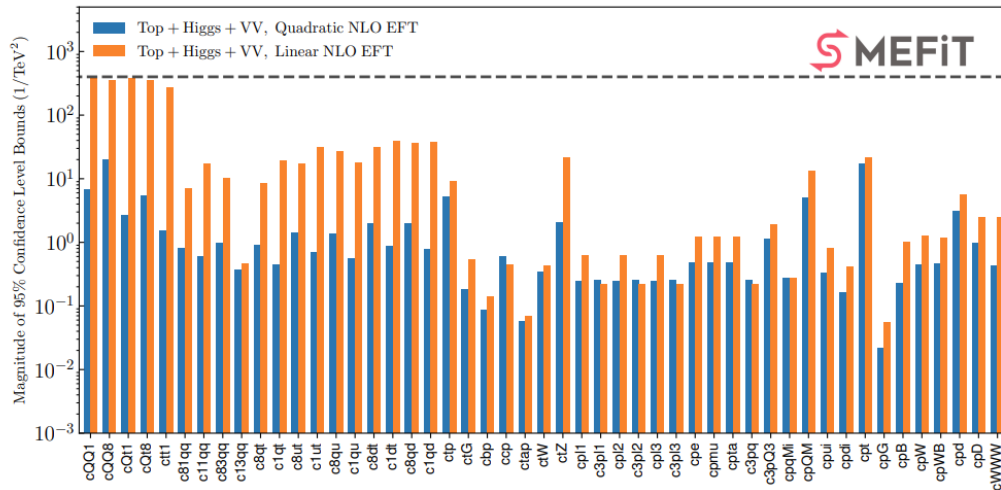
Nature 607 (2022)7917,60-68



The data agrees with the SM prediction very well

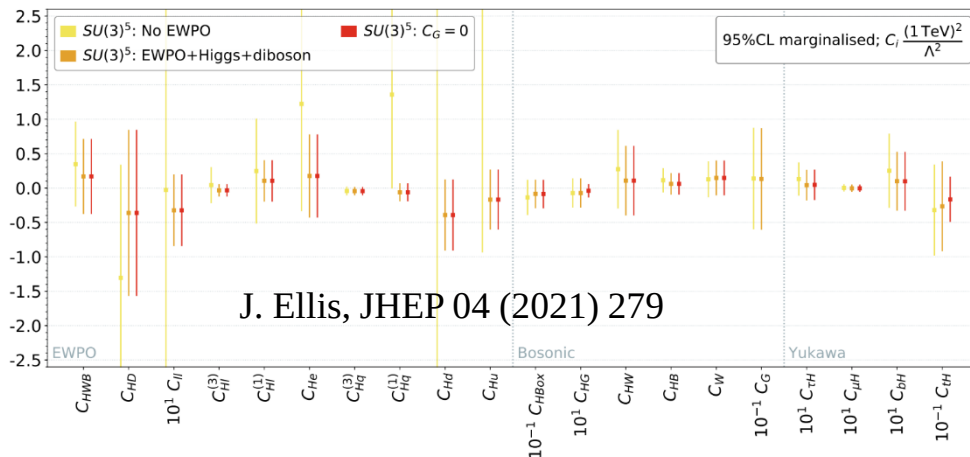
Global Analysis @ SMEFT

SMEFiT Collaboration, JHEP 11 (2021) 089



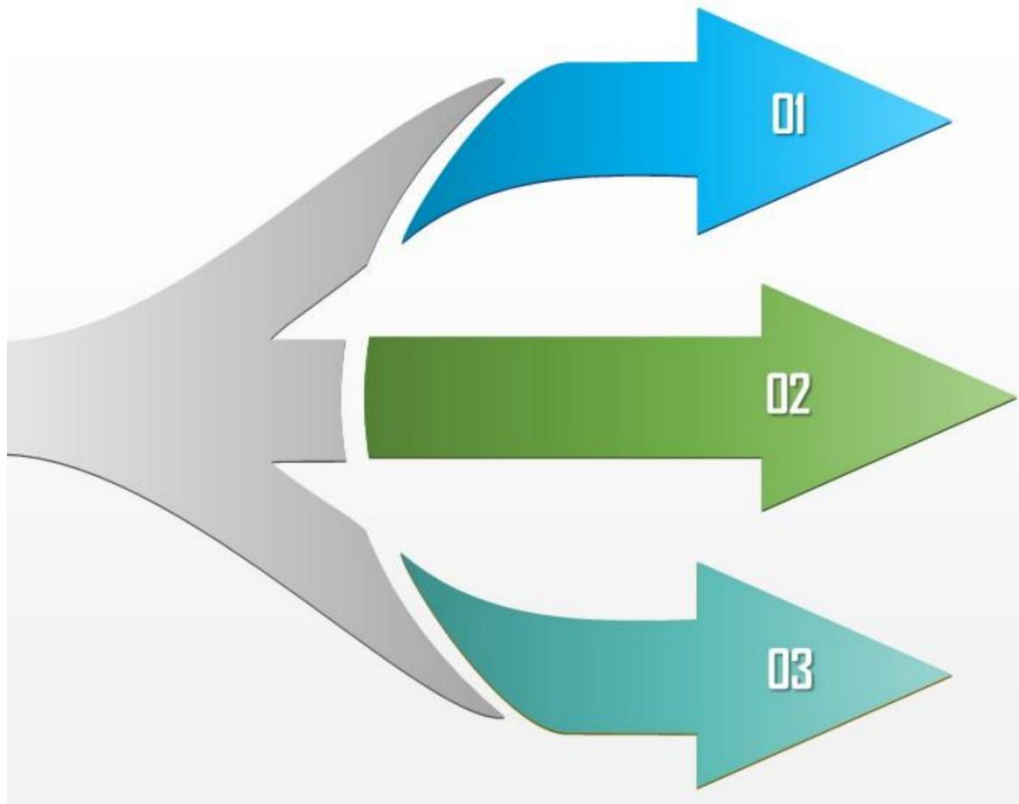
The SMEFT approach allows for the combination

- ◆ Higgs data
- ◆ Electroweak precision observables
- ◆ Diboson production
- ◆ Top quark Physics



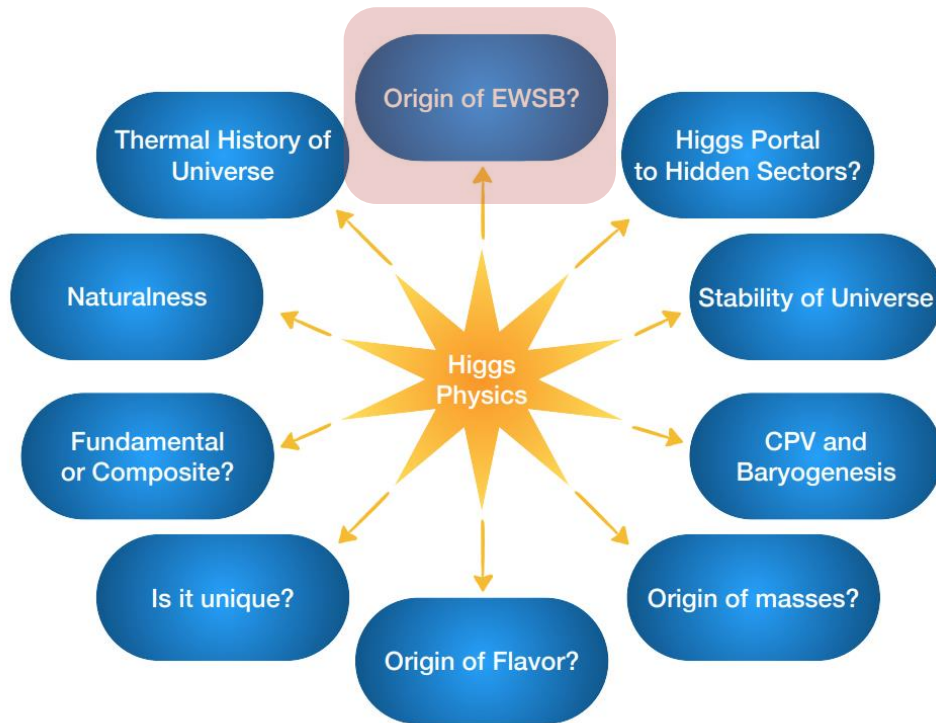
SMEFT is becoming one of the standard tool for the LHC experimental analysis

So, what's the next step for the Higgs physics (BSM) from the theoretical point of view?

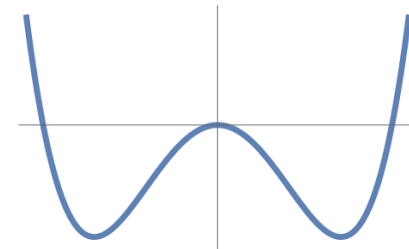


- Global analysis with more processes; the combination of low energy and high energy measurements
- QCD and EW correction to reduce the theoretical uncertainties
- New observables and new measurements

Testing the EWSB @ LHC



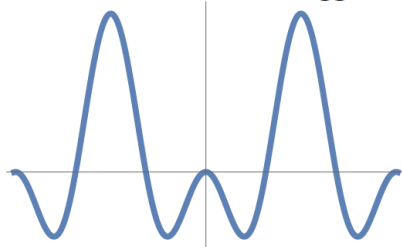
Landau-Ginzburg Higgs



$$V(\phi) = -m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

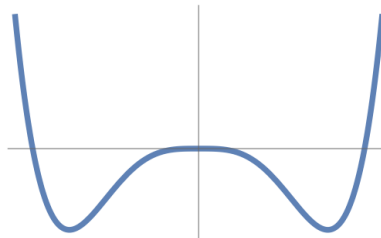
Agrawal, Saha, Xu, Yu, Yuan, 2019

Pseudo-Goldstone Higgs



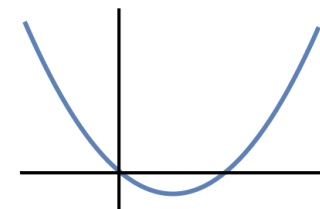
$$V(\phi) = a \sin^2(\phi/f) + b \sin^4(\phi/f)$$

Coleman Weinberg Higgs



$$V(\phi) = \lambda (\phi^\dagger \phi)^2 + \epsilon (\phi^\dagger \phi)^2 \log \frac{\phi^\dagger \phi}{\mu^2}$$

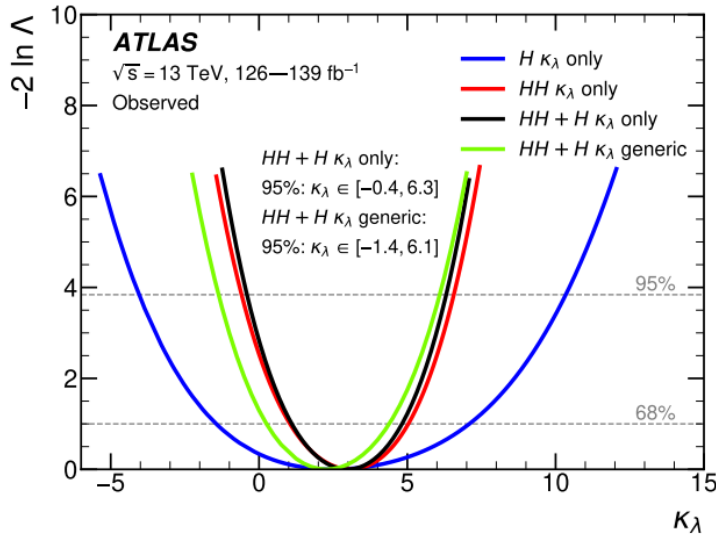
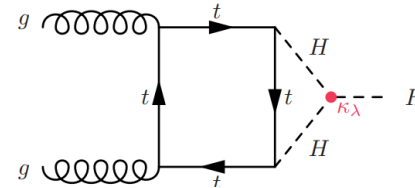
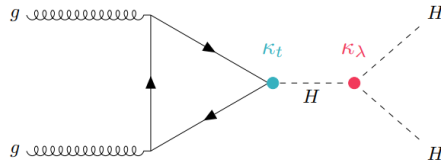
Tadpole-induced Higgs



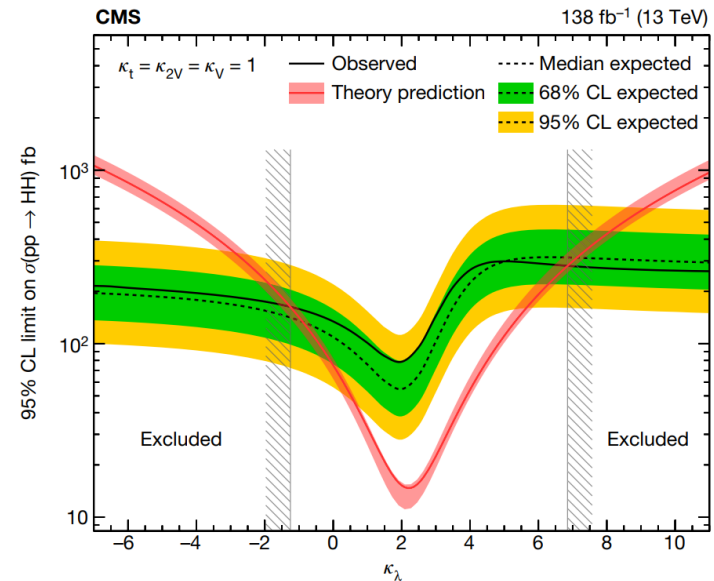
$$V(\phi) = -\mu^3 \sqrt{\phi^\dagger \phi} + m^2 \phi^\dagger \phi$$

Testing the EWSB @ LHC

To determine the Higgs potential shape is challenge!

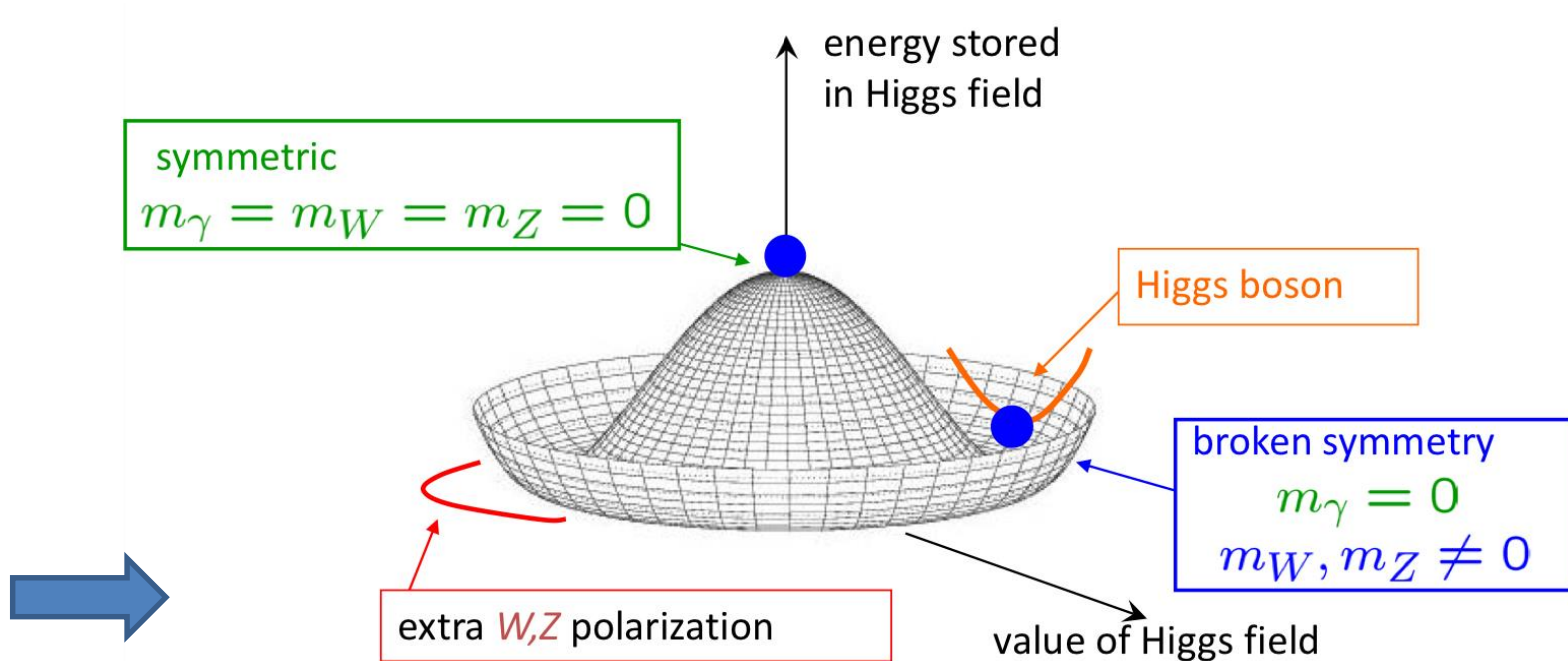


ATLAS, PRD108 (2023)5, 052003



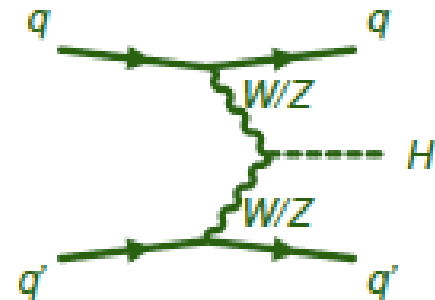
Nature 607 (2022) 60

Testing the EWSB @ LHC



Precisely determine the Higgs gauge couplings are also important for testing the EWSB

$$\mathcal{L}_{hVV} = \kappa_W g_{hWW}^{\text{SM}} h W_\mu^+ W^{-\mu} + \frac{\kappa_Z}{2} g_{hZZ}^{\text{SM}} h Z_\mu Z^\mu$$

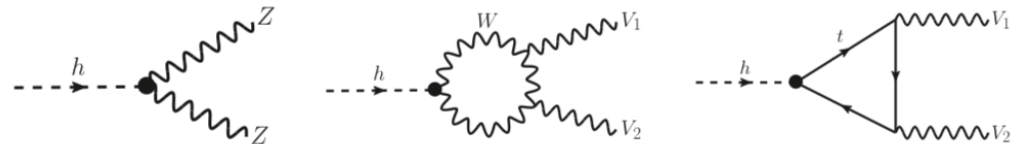


Higgs couplings and EWSB

- The **magnitude** of the Higgs gauge couplings
- The **relative sign** between hWW and hZZ couplings

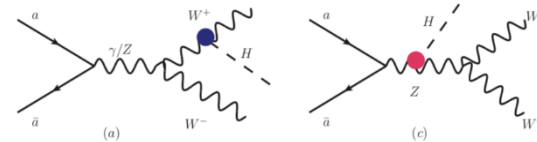
Y. Chen et al, PRL 2016

- ❑ Interference between tree and loop level in Higgs decay

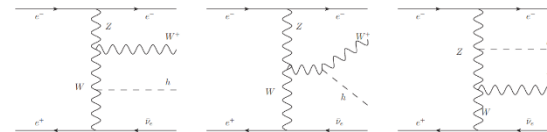


- ❑ Lepton Colliders

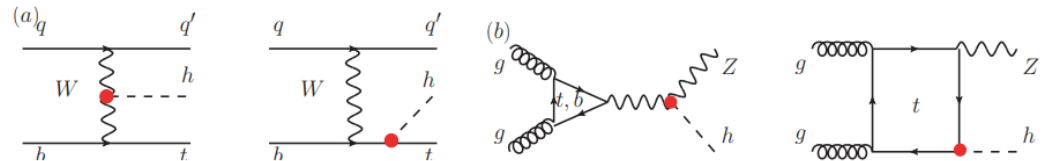
C.W Chiang, X. G. He and G. Li, JHEP08(2018) 126



D. Stolarski, Y. Wu, PRD 102 (2020)3, 033006



- ❑ th and Zh production

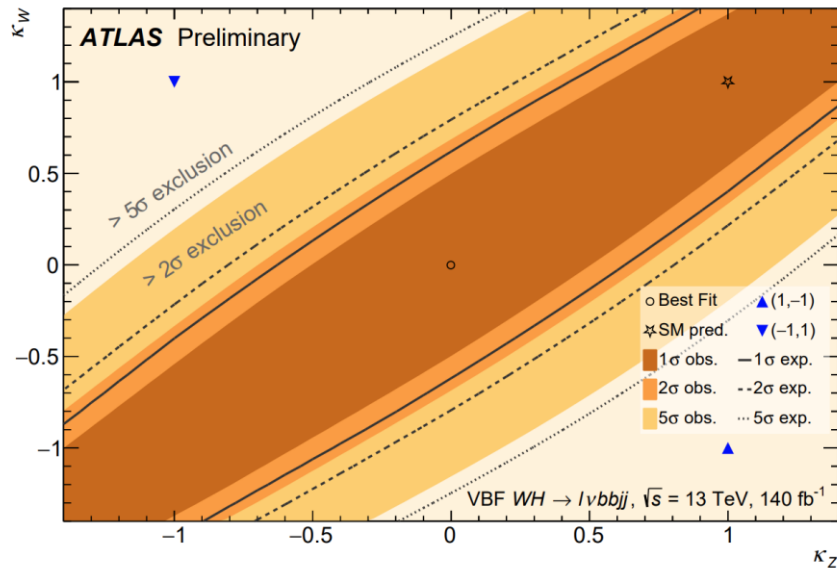


The data favors the same sign

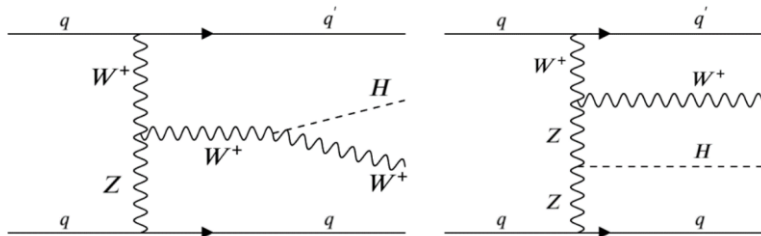
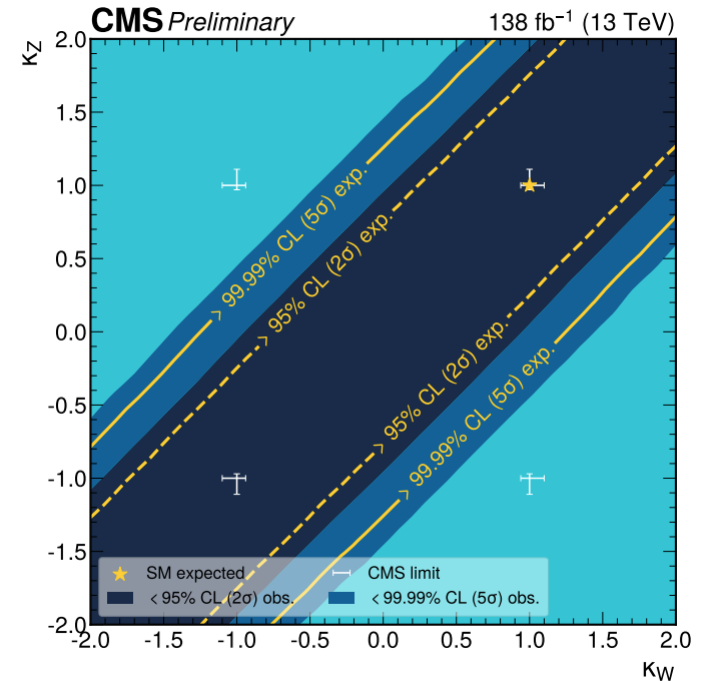
K. P. Xie and Bin Yan, PLB 820 (2021) 136515

Higgs couplings and EWSB

ATLAS-CONF-2023-057



CMS-PAS-HIG-23-007

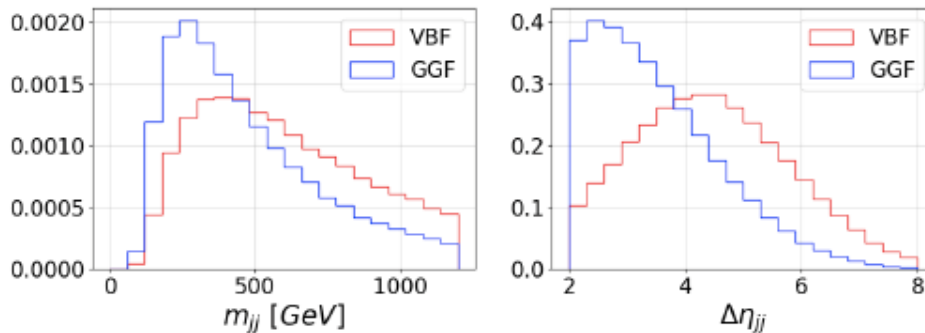
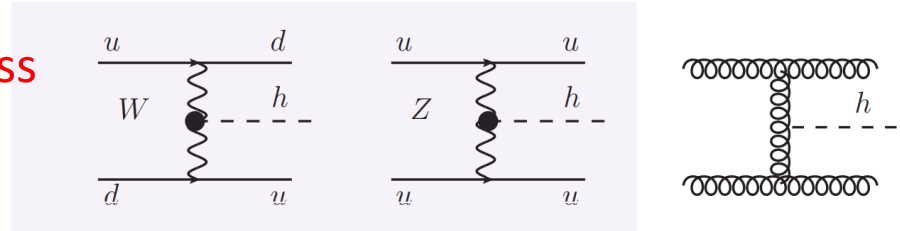


The **magnitude** of the Higgs gauge couplings would be the key task for testing EWSB

Higgs production mechanisms

VBF Higgs production is the main process to verify the Higgs gauge couplings

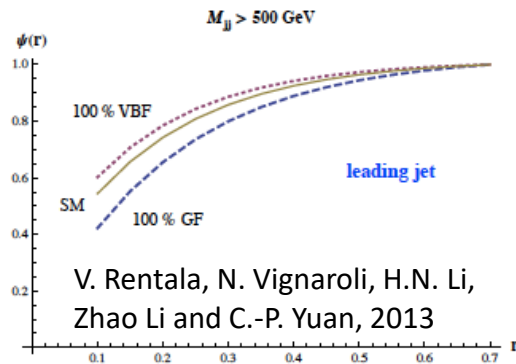
- The rapidity gap and the invariant mass of the two jets



V.D. Barger, K.m.Cheung, T. Han, J. Ohnemus and D. Zeppenfeld, 1991
N. Kauer, T. Plehn, D. L. Rainwater and D. Zeppenfeld, 2001

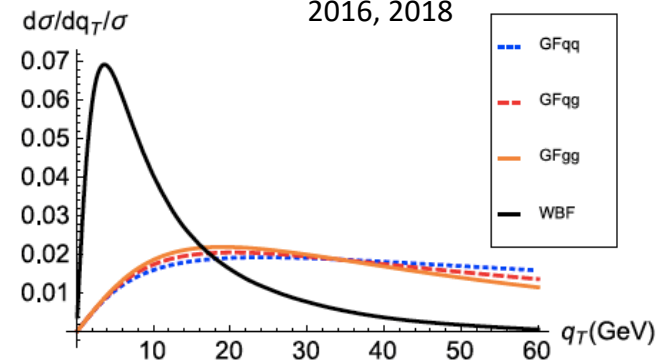
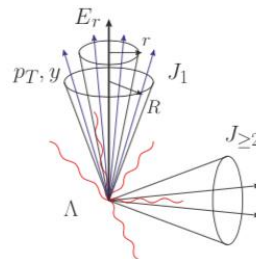
.....

- Soft gluon radiation effects: Jet energy profile, TMD effects



V. Rentala, N. Vignaroli, H.N. Li, Zhao Li and C.-P. Yuan, 2013

$$\Psi_J(r) = \frac{\sum_{i,d_i,\hat{n} < r} E_T^i}{\sum_{i,d_i,\hat{n} < R} E_T^i}$$



P. Sun, C.-P. Yuan and F. Yuan, 2016, 2018

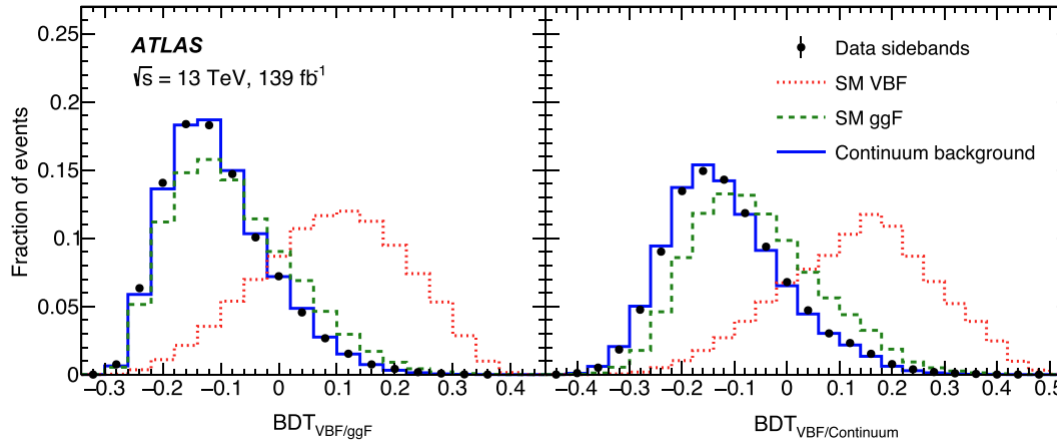
Higgs production mechanisms

Variable	Definition	VBF-ggF separation	VBF-yy separation
m_{jj}	Invariant mass of dijet	0.218	0.241
$\Delta\eta_{jj}$	Pseudo-rapidity separation of dijet	0.152	0.219
p_T^{Hjj}	Transverse momentum of Higgs+jj system	0.127	0.230
$\Delta\Phi_{\gamma\gamma,jj}$	Azimuthal angle between diphoton and dijet systems	0.120	0.186
$\Delta R_{\gamma,j}^{min}$	Minimum ΔR between one of the two leading photons and the corresponding leading jets	0.108	0.204
η^{Zepp}	$ \eta_{\gamma\gamma} - (\eta_{j1} + \eta_{j2})/2 $	0.060	0.078
$p_{Tt}^{\gamma\gamma}$	Diphoton p_T projected perpendicular to the diphoton thrust axis	0.011	0.040

Table 7: Variables used for VBF categorization and their separation power.

Soft gluon radiation effects: TMD effects

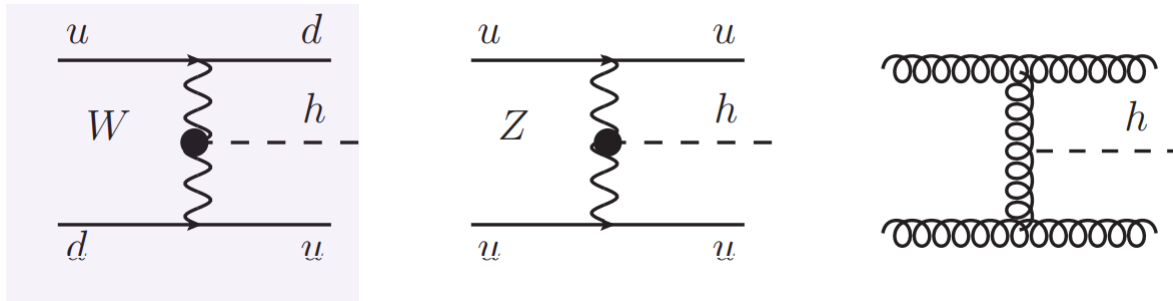
ATLAS, Phys.Rev.Lett. 131 (2023) 6, 061802



The VBF Higgs production can be well separated from the GGF process

Higgs production mechanisms

Discriminating W-boson fusion, Z-boson fusion and gluon fusion Higgs production



H. T. Li, Bin Yan, C.-P. Yuan, PRL 131 (2023) 4, 041802



Separating the W boson's contribution from the VBF Higgs production is an important task for determining the Higgs gauge coupling

The key observable: Jet Charge

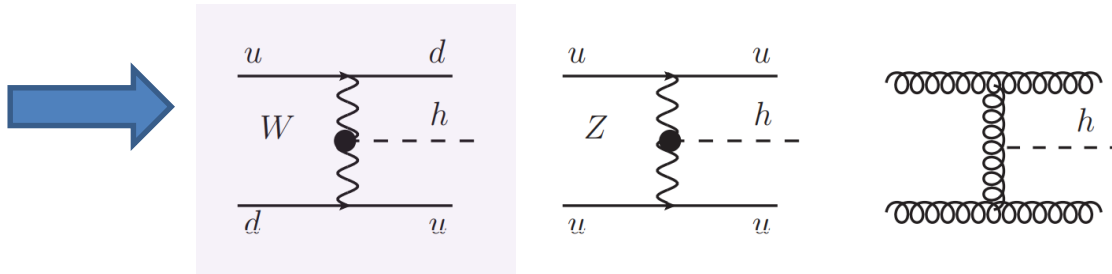
W: opposite sign for the two jet charges

Z: same or opposite sign for the two jet charges

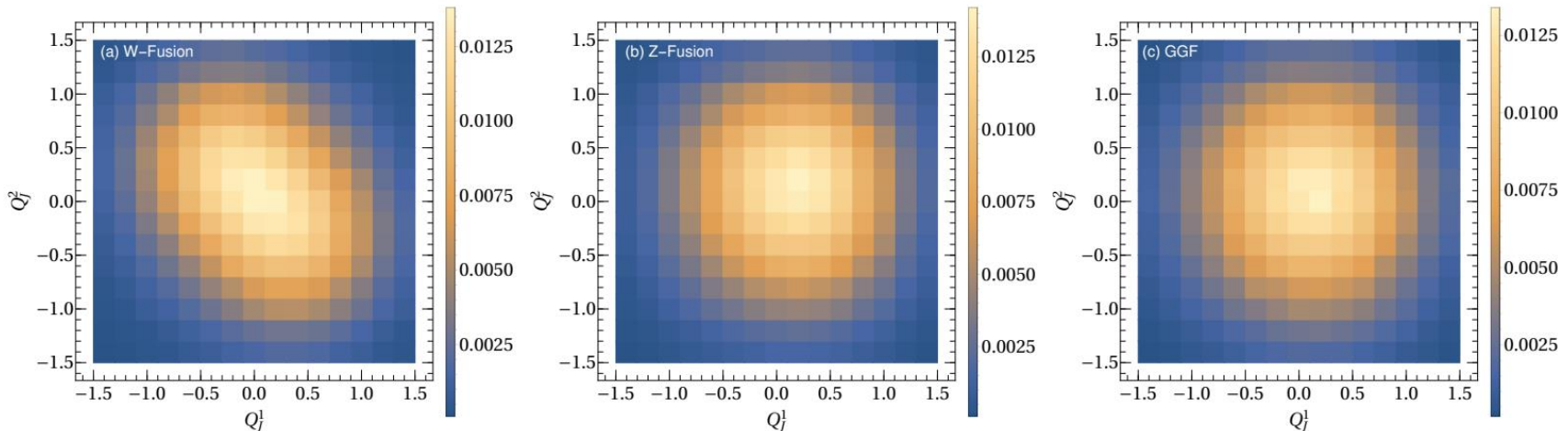
G: the sign of the jet charge is arbitrary

Higgs couplings @ VBF

The key observable: **Jet Charge**



H. T. Li, **Bin Yan**, C.-P. Yuan,
PRL 131 (2023) 4, 041802



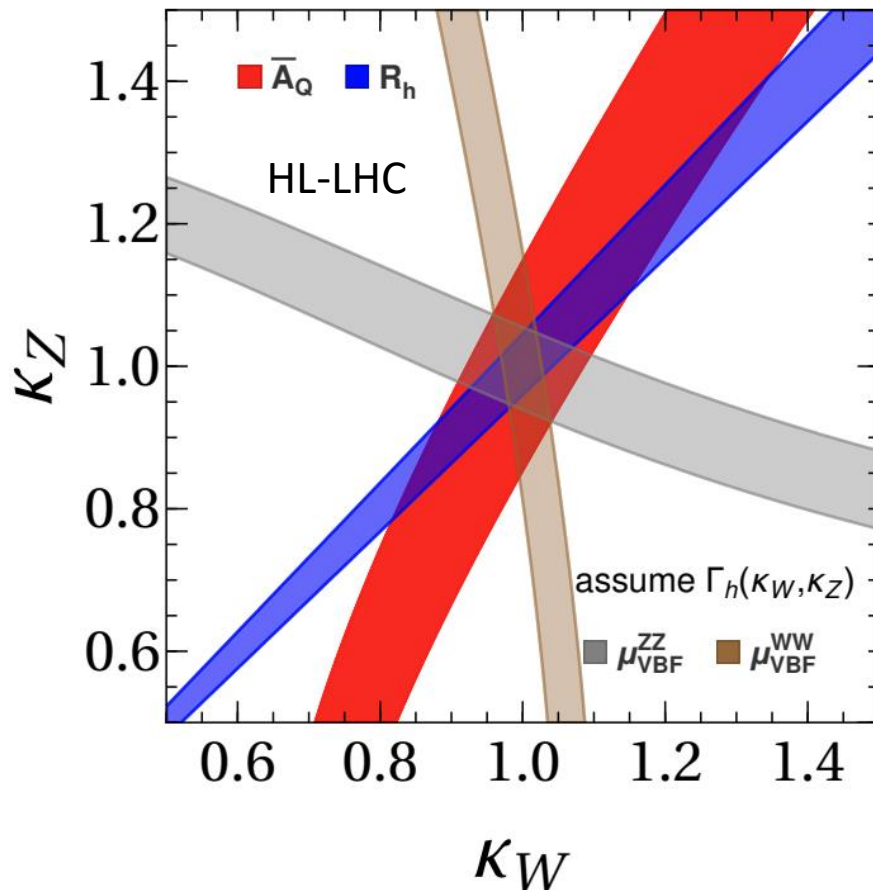
opposite sign for the
two jet charges

same or opposite sign

the sign of the jet
charge is arbitrary

Higgs couplings @ VBF

$$h \rightarrow 4\ell/2\ell 2\nu_\ell$$



$$Q^{(\pm)} = |Q_J^1 \pm Q_J^2|$$

$$\bar{A}_Q^{\text{tot}} = \frac{f_W \langle Q^{(-)} \rangle_W + f_Z \langle Q^{(-)} \rangle_Z + f_G \langle Q^{(-)} \rangle_G}{f_W \langle Q^{(+)} \rangle_W + f_Z \langle Q^{(+)} \rangle_Z + f_G \langle Q^{(+)} \rangle_G}$$

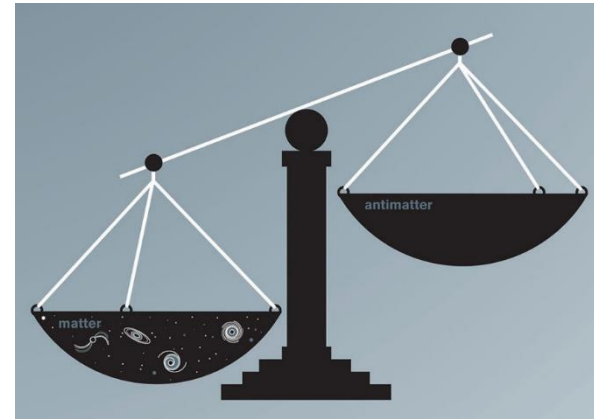
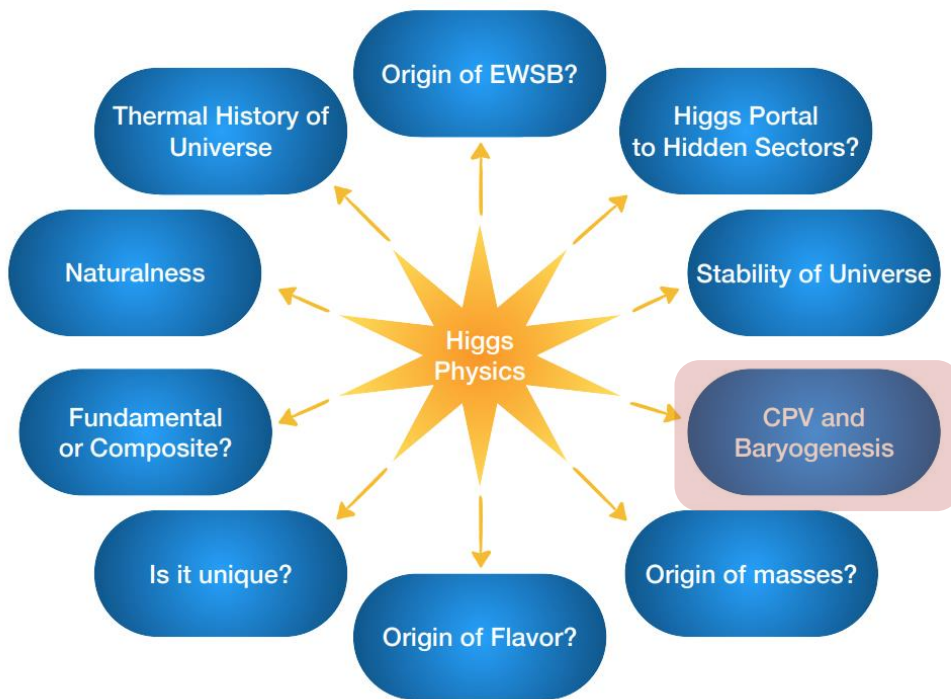
$$R_h = \frac{\mu(gg \rightarrow h \rightarrow WW^*)}{\mu(gg \rightarrow h \rightarrow ZZ^*)} = \frac{\kappa_W^2}{\kappa_Z^2}$$

$$\kappa_V = \frac{g_{hVV}}{g_{hVV}^{\text{SM}}}$$

H. T. Li, **Bin Yan**, C.-P. Yuan,
PRL 131 (2023) 4, 041802

The limits from R_h and jet charge asymmetry **are not depending** on the assumption of the **Higgs width**

Higgs CP violation



Sakharov Criteria (1967)

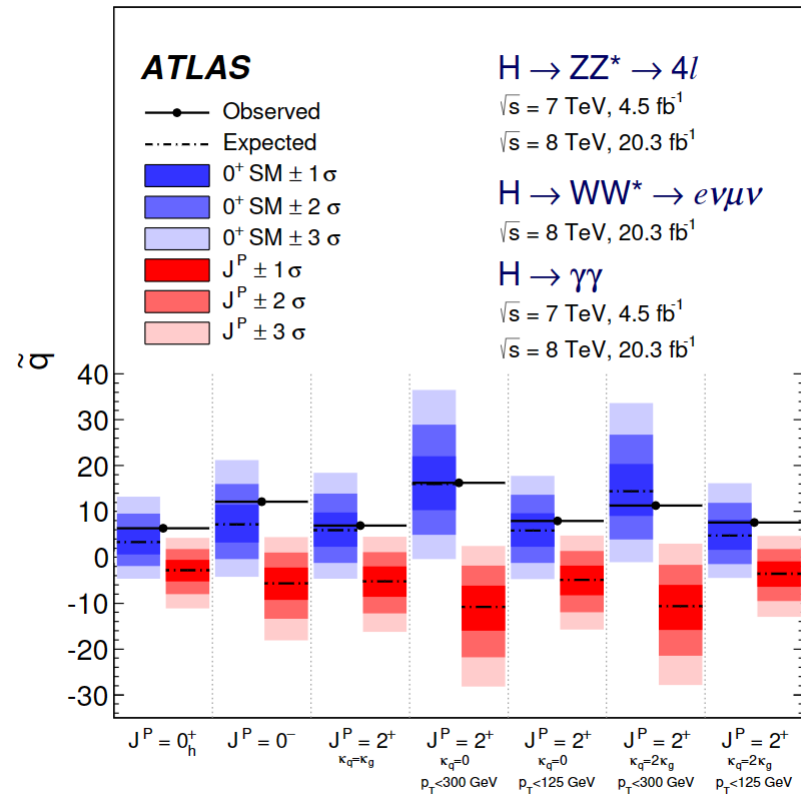
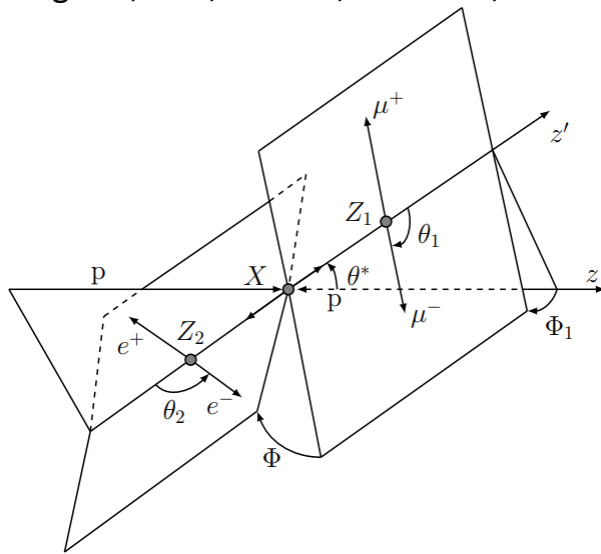
- B violation
- C & **CP violations**
- Departure from the equilibrium

Higgs CP violation

PDG 2022

Higgs boson CP property:

e.g. Bolognesi, Gao, Gritsan, Melnikov, Schulze, 2012

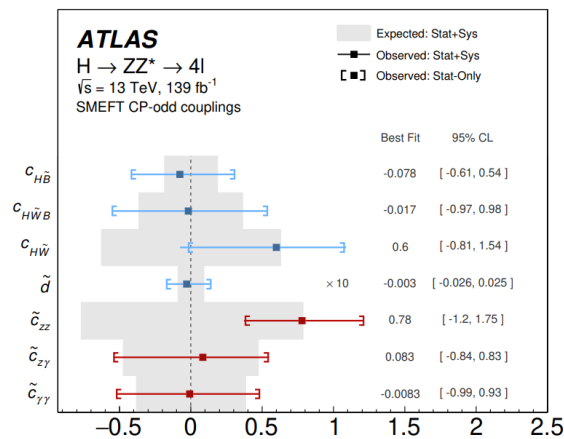


- A purely CP-odd Higgs has been excluded
- A CP-mixture Higgs boson is still possible

Higgs CP violation

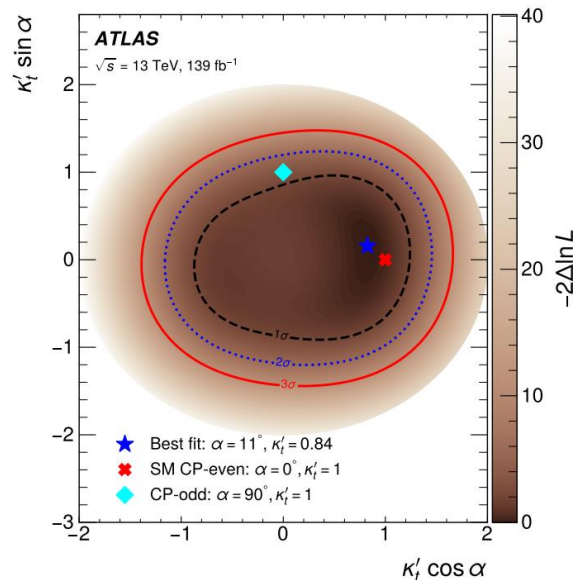
- CP-odd interactions with **gauge bosons** (loop induced operators) ATLAS, 2304.09612

Operator	Structure	Coupling
Warsaw Basis		
$O_{\Phi\tilde{W}}$	$\Phi^\dagger\Phi\tilde{W}_{\mu\nu}^I W^{\mu\nu I}$	$c_{H\tilde{W}}$
$O_{\Phi\tilde{W}B}$	$\Phi^\dagger\tau^I\Phi\tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$c_{H\tilde{W}B}$
$O_{\Phi\tilde{B}}$	$\Phi^\dagger\Phi\tilde{B}_{\mu\nu} B^{\mu\nu}$	$c_{H\tilde{B}}$
Higgs Basis		
$O_{hZ\tilde{Z}}$	$hZ_{\mu\nu}\tilde{Z}^{\mu\nu}$	$\tilde{c}_{ZZ}$
$O_{hZ\tilde{A}}$	$hZ_{\mu\nu}\tilde{A}^{\mu\nu}$	$\tilde{c}_{Z\gamma}$
$O_{hA\tilde{A}}$	$hA_{\mu\nu}\tilde{A}^{\mu\nu}$	$\tilde{c}_{\gamma\gamma}$



- CP-odd interactions with **fermions**

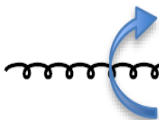
Gunion, He, PRL 76, 4468 (1996)
 Boudjema, Godbole, Guadagnolo, Mohan, PRD 92, 015019 (2015)
 Mileo, Kiers, Szynekman, Crane, Gegner, JHEP 07, 056 (2016)
 Gritsan, Rntsch, Schulze, Xiao, PRD 94, 055023 (2016)
 S. Amor Dos Santos et al, PRD 96, 013004 (2017)
 Kobakhidze, Liu, Wu, Yue, PRD 95 (2017) 1, 015016
 Gouveia et al, 1801.04954
 Goncalves, Kong, Kim, JHEP 06, 079 (2018)
 Ren, Wu, Yang, 1901.05627
 ATLAS, PRL 125 (2020) 6, 061802
 CMS, PRL 125 (2020) 6, 061801
 Q.-H. Cao, K.-P. Xie, H. Zhang, R. Zhang, CPC45 (2021) 2, 023117
 Zhite Yu and C.-P. Yuan, 2211.00845

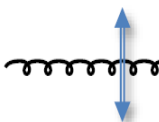


ATLAS: 2303.05974
 CMS: 2208.02686

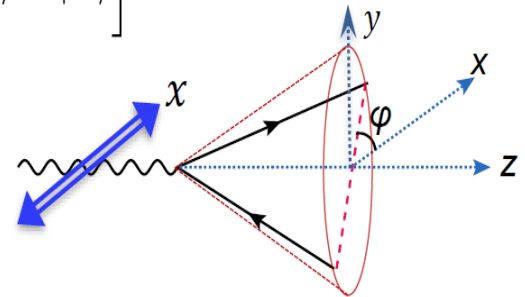
New polarization observables

Linear polarization vs. helicity/circular polarization

helicity pol.  $|\pm 1\rangle$

linear pol.  $|x\rangle = -\frac{1}{\sqrt{2}}[|+\rangle - |-\rangle]$, $|y\rangle = \frac{i}{\sqrt{2}}[|+\rangle + |-\rangle]$

 $|e^{+i\phi} \pm e^{-i\phi}|^2 \rightarrow 2(1 \pm \cos 2\phi)$



Interference of helicity λ_1 and λ_2 causes azimuthal distributions

$$\cos(\lambda_1 - \lambda_2)\phi, \quad \sin(\lambda_1 - \lambda_2)\phi$$


CP even


CP odd



Useful probes of new physics

New polarization observables

Linear polarization/transversely polarized effects for NP searches

- ❑ Boosted top quark (hadronic mode), linear polarization of W boson

Zhite Yu, C.-P. Yuan, PRL 129 (2022) 11,11

- ❑ Linear polarization of photon @ ultraperipheral heavy ion collisions (UPCs)

Ding Yu Shao, Bin Yan, Shu-Run Yuan, Cheng Zhang, 2310.14153, accepted by Science China Physics, Mechanics & Astronomy

- ❑ Transversely polarized effects @ lepton collider

Xin-Kai Wen, Bin Yan, Zhite Yu, C.-P. Yuan, PRL 131 (2023) 241801

- ❑ Transversely polarized effects @ EIC

R. Boughezal et al, PRD 107 (2023) 7,075028

Hao-Lin Wang, Xin-Kai Wen, Hongxi Xing, Bin Yan, 2401.08419, accepted by PRD

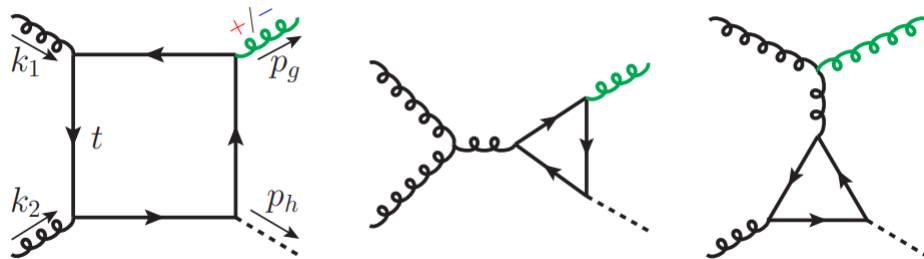
- ❑ Lam-Tung relation of Drell-Yan process

Xu Li, Bin Yan, C.-P. Yuan, 2404.xxxxx

This new polarization observable would be important for probing the CP violation, and becoming popular

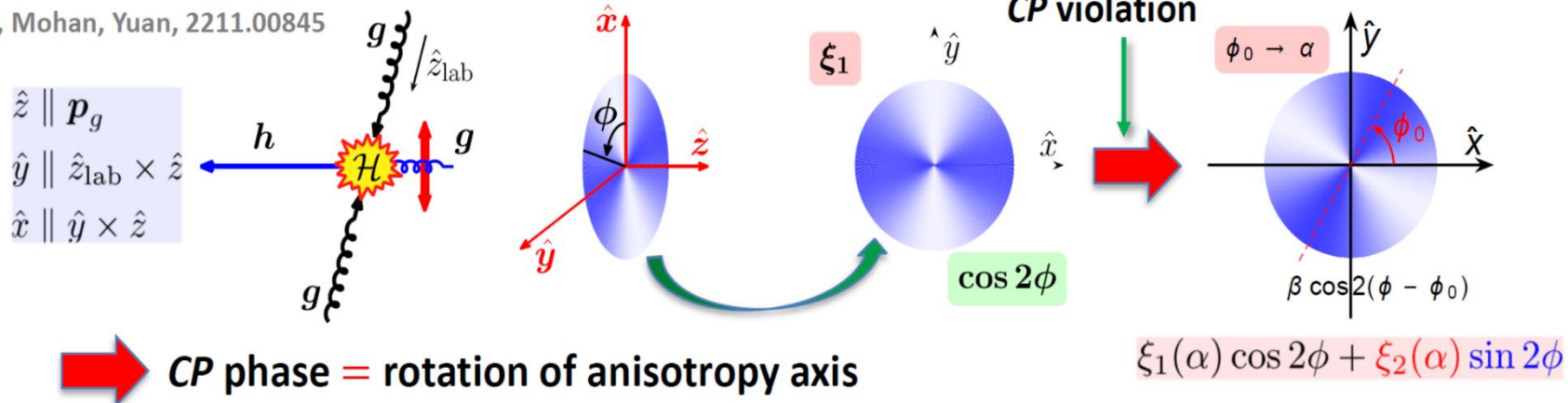
New polarization observables

Linear polarization of gluon



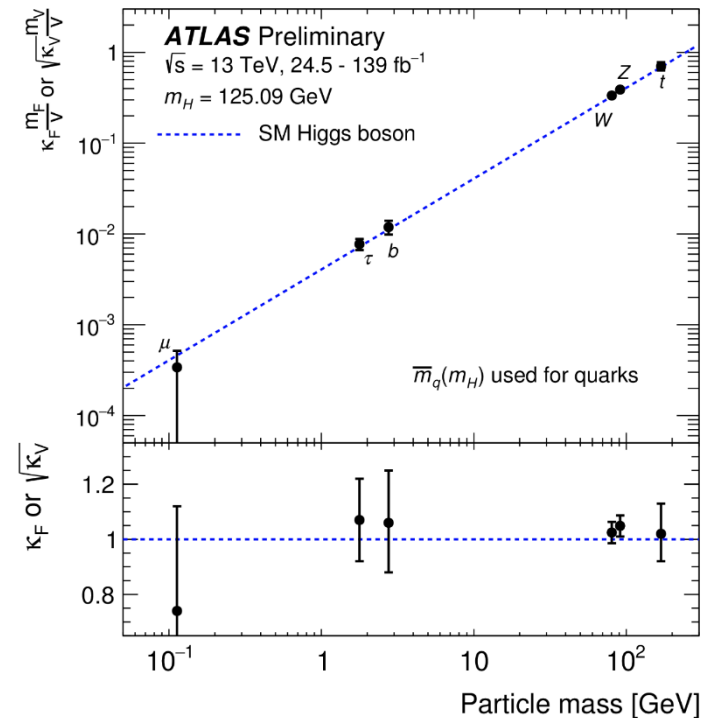
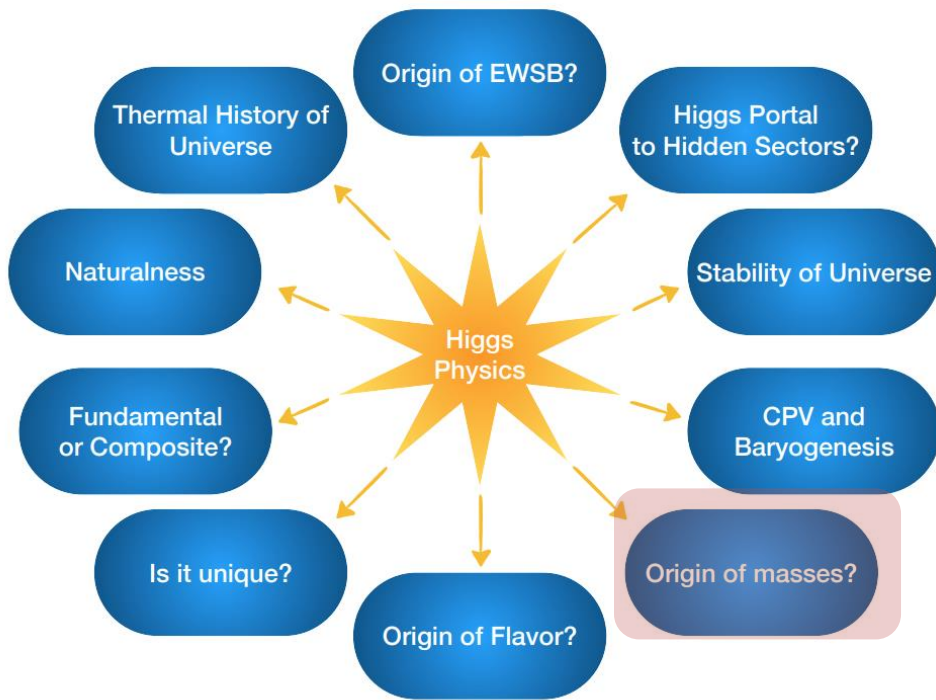
$$\rho_{\lambda\lambda'} = \frac{1}{2} (1 + \boldsymbol{\xi} \cdot \boldsymbol{\sigma})_{\lambda\lambda'} = \frac{1}{2} \begin{pmatrix} 1 + \xi_3 & \xi_1 - i \xi_2 \\ \xi_1 + i \xi_2 & 1 - \xi_3 \end{pmatrix}$$

Yu, Mohan, Yuan, 2211.00845



C.-P. Yuan's talk @ MBI 2023

Higgs Yukawa couplings



All fundamental particles get their mass from Higgs boson vev



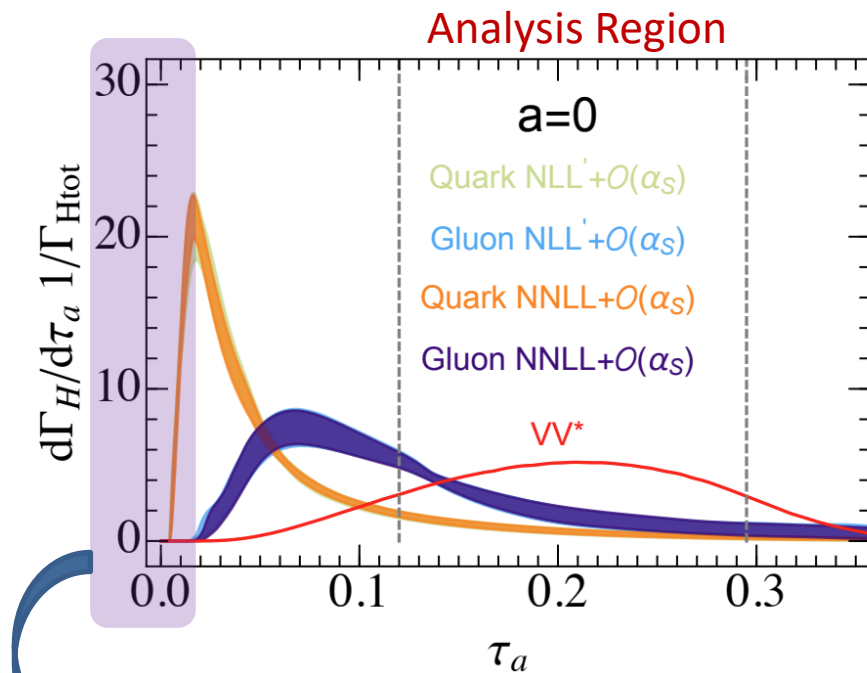
How about light quarks?
Does Higgs mechanism still work?

Higgs Yukawa couplings

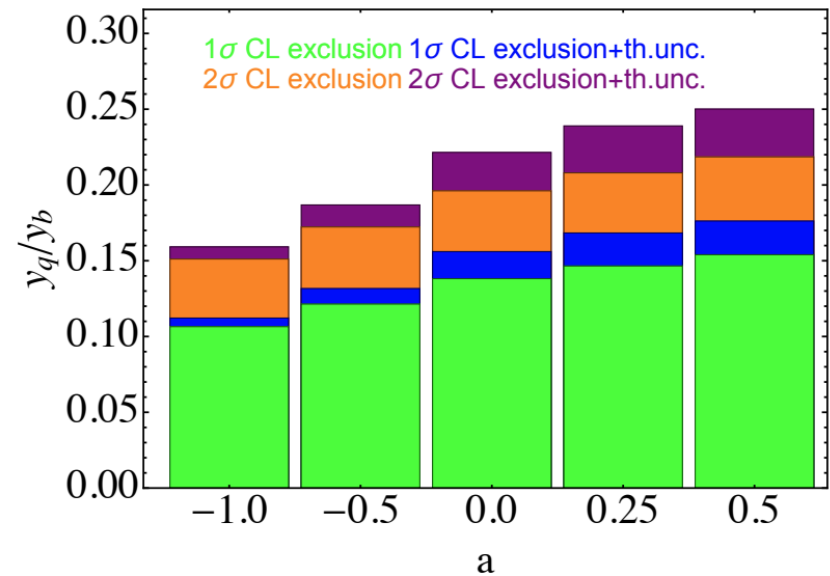
$$e(X) = \frac{1}{Q} \sum_{i \in X} |p_{\perp}^i| f_e(\eta_i)$$

$$f_{\tau_a}(\eta) = e^{-|\eta|(1-a)}$$

Angularity distributions are very different for quark and gluon final state

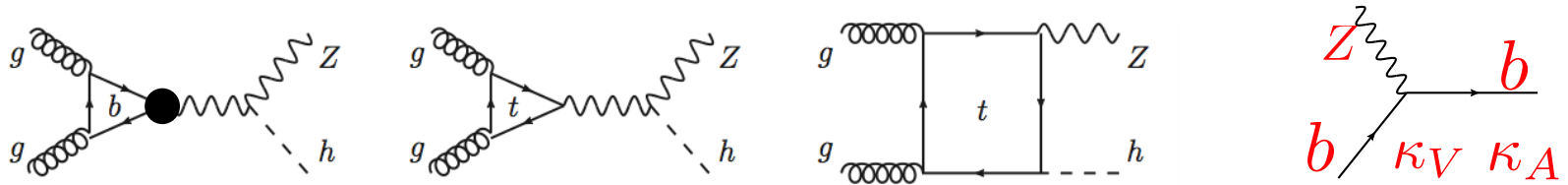


Sensitive to non-perturbative assumptions



Electroweak properties for the SM

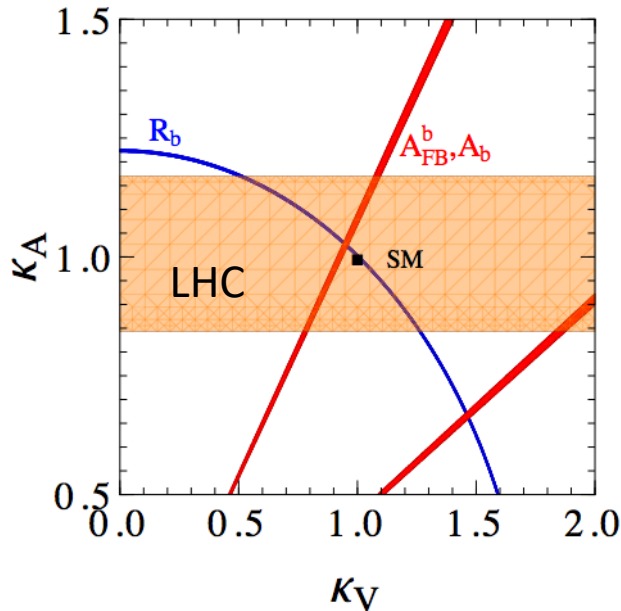
The precise measurements for the SM Higgs production can also test the **electroweak properties of the SM**



Bin Yan, C.-P. Yuan, PRL 127 (2021) 5, 051801

$$\mathcal{L} = \bar{b} \gamma_\mu (\kappa_V g_V - \kappa_A g_A \gamma_5) b Z_\mu$$

The degeneracy of the anomalous Zbb could be resolved by the LHC data



The other possible methods:

Bin Yan, C.-P. Yuan, Shu-Run Yuan, PRD 108 (2023) 5, 053001

F. Bishara, Zhuoni Qian, JHEP 10 (2023) 088

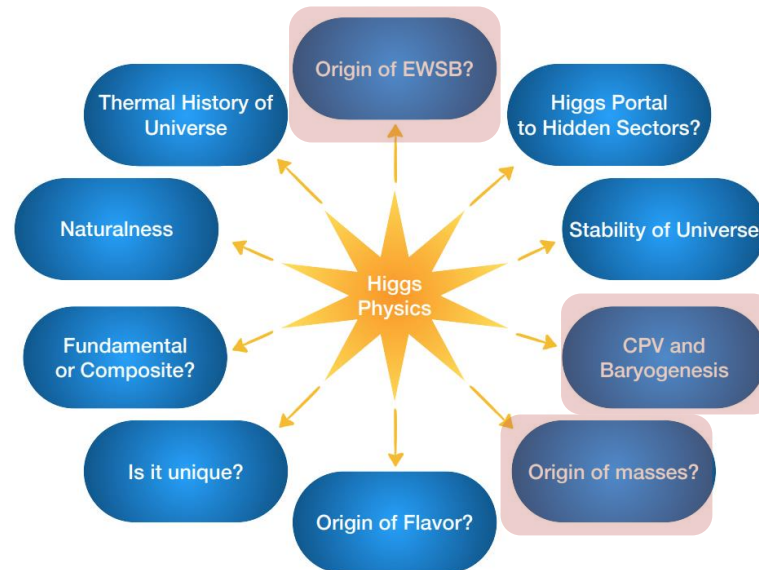
Hongxin Dong, Peng Sun, Bin Yan, C.-P. Yuan, PLB 829 (2022) 137076

Hai Tao Li, Bin Yan, C.-P. Yuan, PLB 833 (2022) 137300

Bin Yan, Zhite Yu, C.-P. Yuan, PLB 822 (2021) 136697

Summary

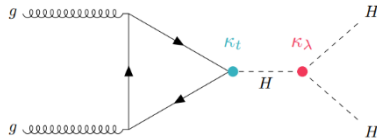
- The properties of the Higgs boson are closely related to many fundamental questions of particle physics



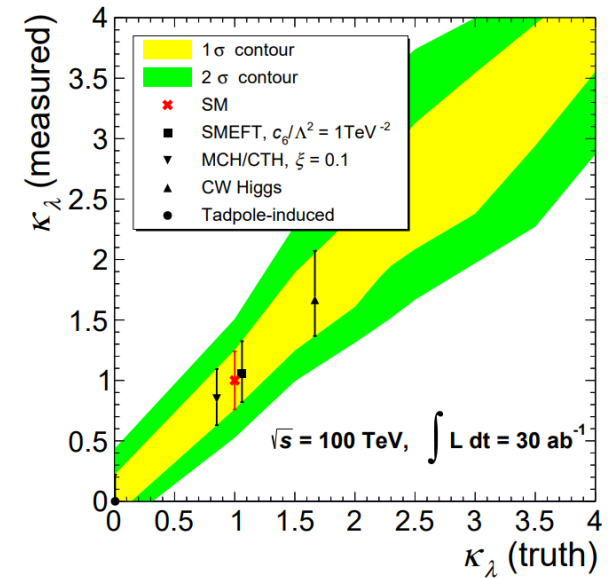
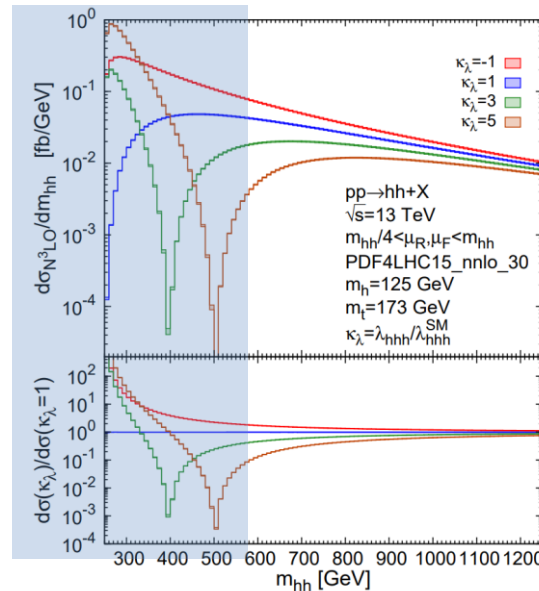
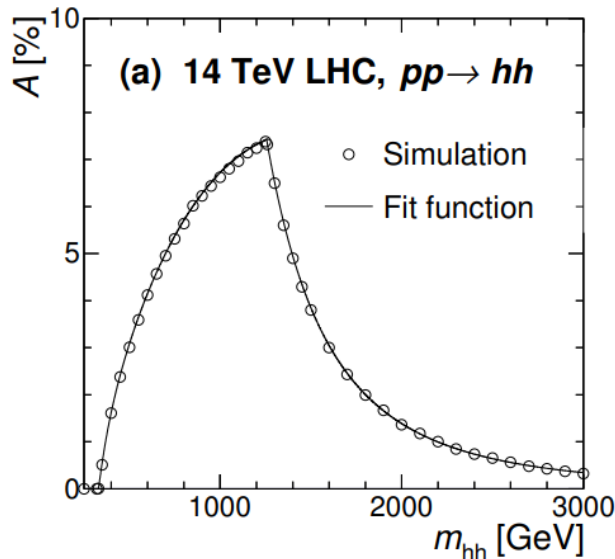
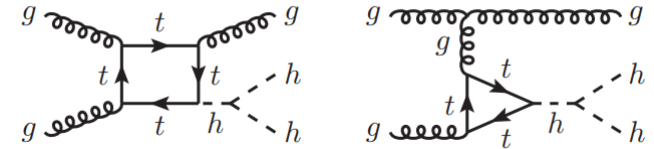
Thank you!

Testing the EWSB @ LHC

Current experimental searches mainly focus on the **high di-Higgs invariant mass region**



The low di-Higgs invariant mass region is more sensitive to the Higgs shape

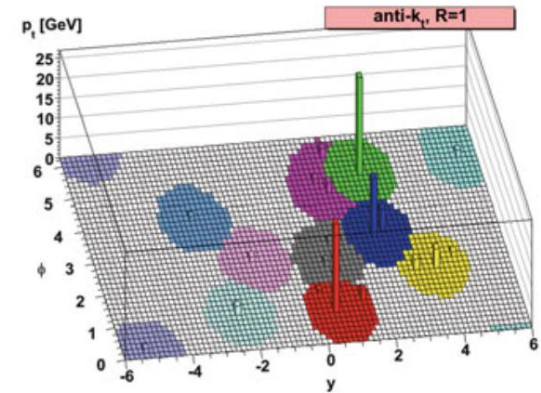
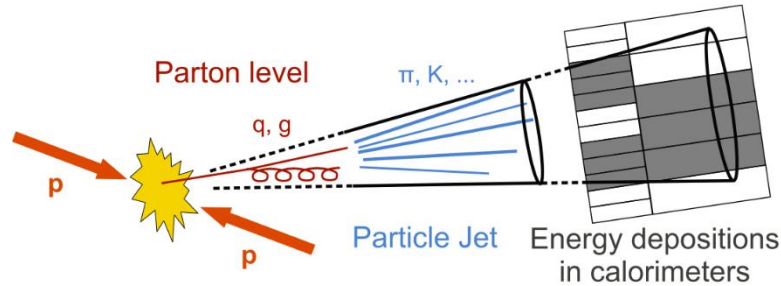


Q.-H. Cao, Bin Yan, D.-M. Zhang, H. Zhang, PLB 752 (2016) 285-290

L. B. Chen, H. T. Li, H. S. Shao, J. Wang, PLB 803 (2020) 135292, JHEP 03 (2020) 072

K. Chai, J.-H. Yu, H. Zhang, PRD 107(2023) 5,055031

Jet charge definition



Transverse-momentum-weighting scheme:

$$Q_J = \frac{1}{(p_T^j)^\kappa} \sum_{i \in jet} Q_i (p_T^i)^\kappa, \quad \kappa > 0$$

κ : To regulate the sensitivity of the soft gluon radiation

R.D. Field and R.P. Feynman, NPB136,1(1978)

- SCET calculation
- Quark/gluon jet discrimination
- Nuclear medium effects
- Quark flavor structure
- Non-perturbative model
- Electroweak and Higgs physics

D. Krohn et al, PRL, 2013, W.J.Waalewijn, PRD, 2012

K.Fraser and M.D. Schwartz, JHEP, 2018, Zhong-Bo Kang, Xiaohui Liu, et al, PRD, 2021

H. T. Li and I. Vitev, PRD, 2020, PRL, 2021

Zhong-Bo Kang, Xiaohui Liu, et al, PRD, 2021, + Ding Yu Shao, PRL, 2020

Zhong-Bo Kang et al, PRL, 2023

H. T. Li, [Bin Yan](#) and C.-P. Yuan, PLB 2022, PRL 2023

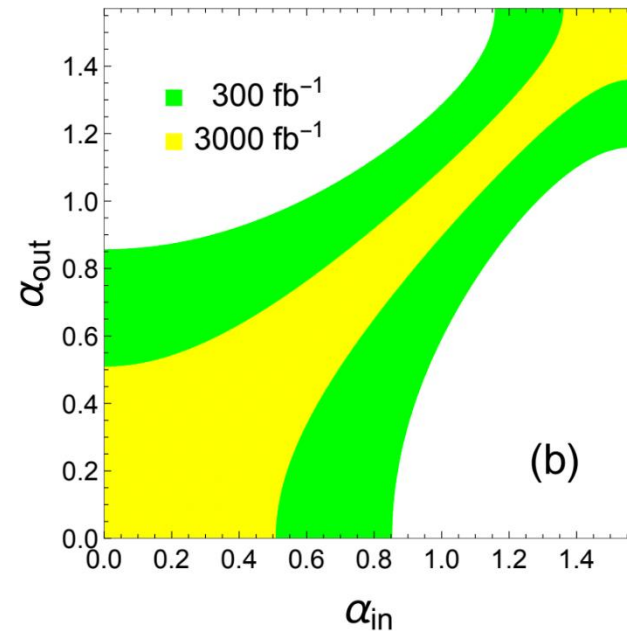
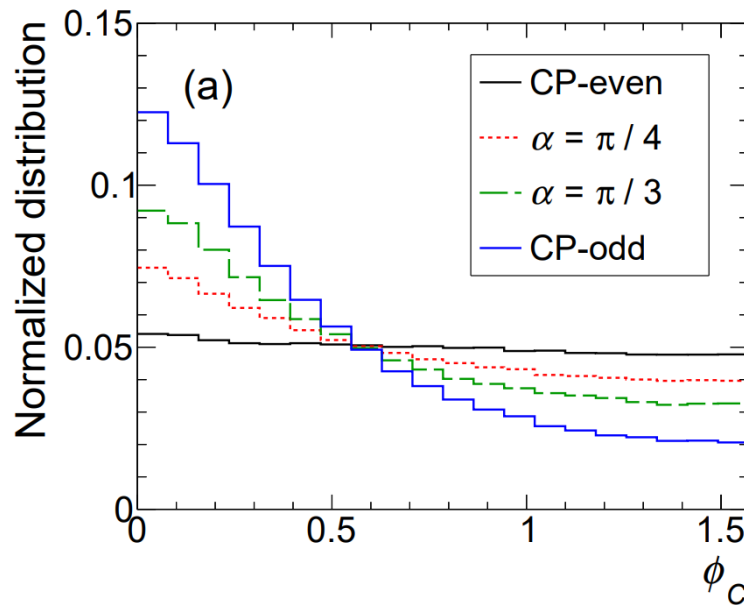
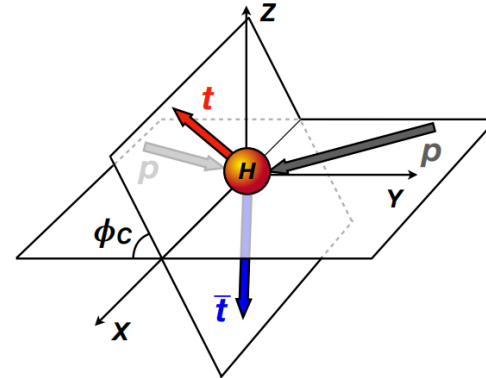
Xiao-Rui Wang, [Bin Yan](#), PRD 2023

H. Cui, M. Zhao, Y. Wang, H. Liang, Manqi Ruan, 2023

Higgs CP violation

$$\mathcal{L} = y_f h \bar{f} (\cos \alpha_f + i \gamma_5 \sin \alpha_f) f$$

$$\phi_C = \arccos \left| (\mathbf{n}_{p_1} \times \mathbf{n}_{p_2}) \cdot (\mathbf{n}_t \times \mathbf{n}_{\bar{t}}) \right|$$



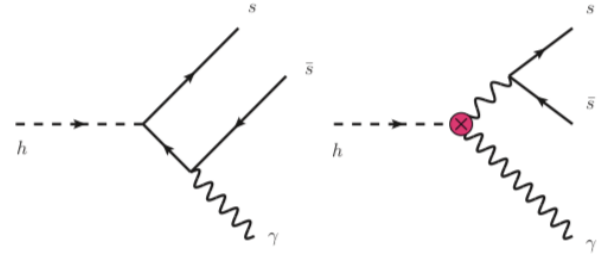
Light quark Yukawa couplings@LHC

A. Rare decay: $h \rightarrow J/\Psi \gamma$ ($\phi \gamma, \rho \gamma, \omega \gamma$)

G. T. Bodwin, F. Petriello, S. Stoynev, M. Velasco, PRD88 (2013) 5, 053003

A. L. Kagan, G. Perez, F. Petriello, Y. Soreq, S. Stoynev, PRL114 (2015) 10, 101802

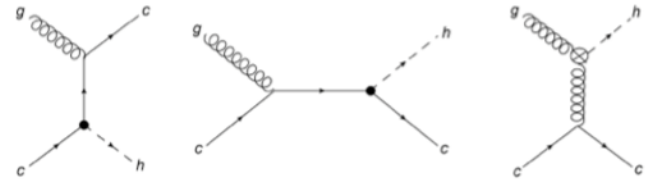
e.g. 14 TeV HL-LHC $y_s/y_b < 0.39$ $y_c/y_c^{\text{SM}} < 220$



B. Higgs+charm production

I. Brivio, F. Goertz, G. Isidori, PRL115 (2015) 21, 211801

e.g. 14 TeV HL-LHC $y_c/y_c^{\text{SM}} < 2.5$

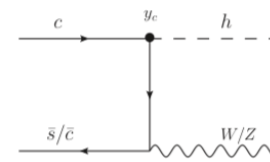


C. Higgs data global analysis:

G. Perez, Y. Soreq, E. Stamou, K. Tobioka, PRD92(2015)3, 033016, PRD93(2016)1, 013001

Y. Zhou, PRD93(2016) 1, 013019

e.g. 14 TeV HL-LHC $y_c/y_c^{\text{SM}} < 6.2$



D. Higgs p_T analysis:

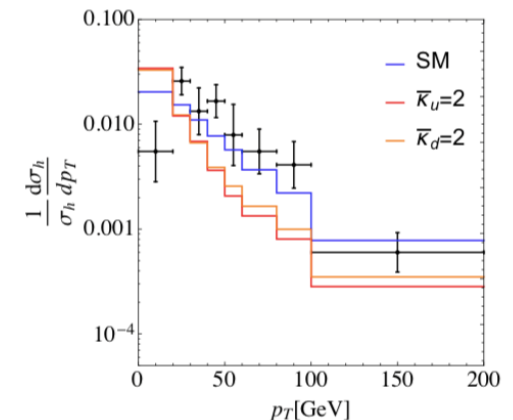
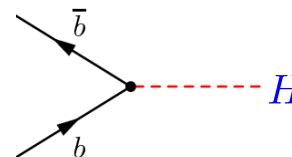
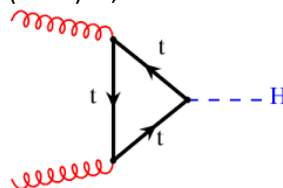
Y. Soreq, H.X. Zhu, J. Zupan, JHEP 12(2016)045

F. Bishara, U. Haisch, P. F. Monni, E. Re, PRL 118(2017)12, 121801

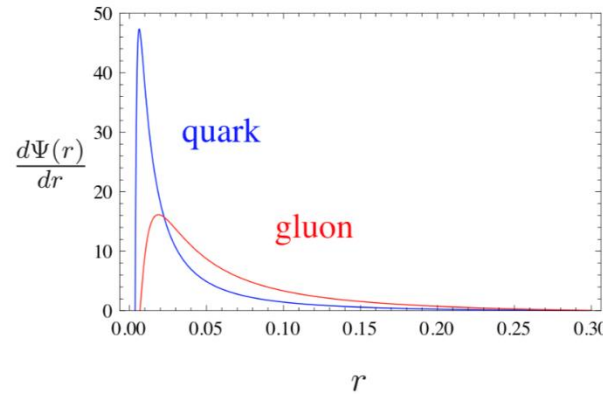
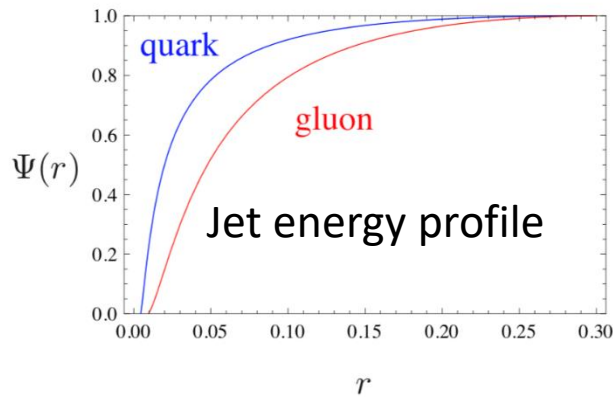
G. Bonner, H. E. Logan, 1608.04376

$y_{u,d}/y_b < 0.4 \sim 0.5$

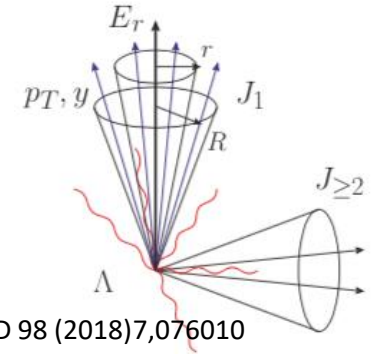
Soft gluon radiation



Light quark Yukawa couplings@ e^+e^-

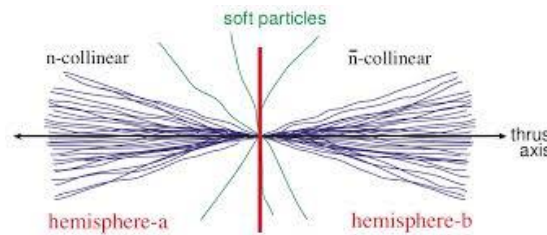
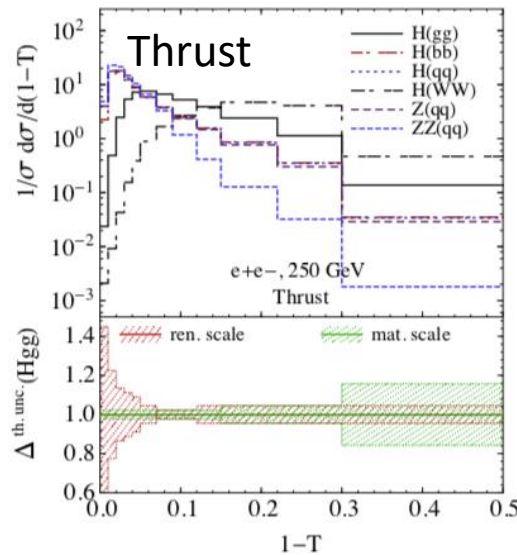


$$\Psi_J(r) = \frac{\sum_{i,d_i,\hat{n} < r} E_T^i}{\sum_{i,d_i,\hat{n} < R} E_T^i}$$



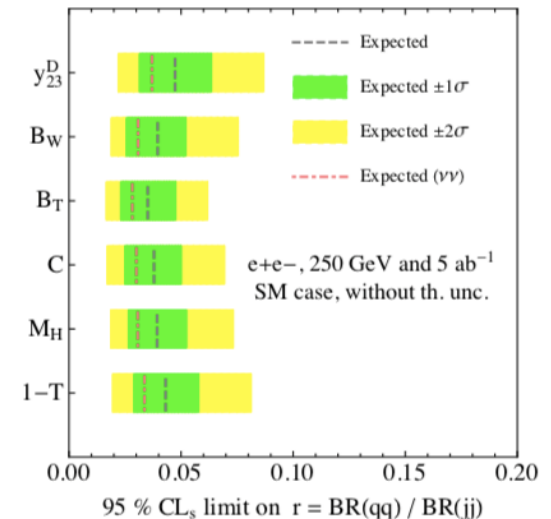
H. N. Li, Z. Li and C.-P. Yuan, PRL 107 (2011)152001; Y. T. Chien, I. Vitev, JHEP 12(2014)061

J. Isaacson, H.N. Li, Z. Li and C.-P. Yuan, PLB 771 (2017)619-623; G. X. Li, Z. Li, Y.D. Liu, Y. Wang, X. R. Zhao, PRD 98 (2018)7,076010



$$T = \max_{\vec{n}} \left(\frac{\sum_i |p_i \cdot \vec{n}|}{\sum_i |p_i|} \right)$$

$$y_{u,d,s}/y_b < 0.091$$



Event shapes

One class of event shapes:

$$e(X) = \frac{1}{Q} \sum_{i \in X} |p_{\perp}^i| f_e(\eta_i)$$

Examples:

Thrust $f_{1-T}(\eta) = e^{-|\eta|}$ Brandt, Peyrou, Sosnowski, Wroblewski, 64; Farhi, 77

Jet broadening $f_B(\eta) = 1$ Catani, Turnock, Webber, 92

C-Parameter $f_C(\eta) = \frac{3}{\cosh(\eta)}$ Ellis, Ross, Terrano, 81

Angularities $f_{\tau_a}(\eta) = e^{-|\eta|(1-a)}$ Berger, Kucs, Sterman, 03 (relatively new)

G. Bell, A. Hornig, C. Lee, J. Talbert, JHEP01(2019)147 The proportions of two jet-like and three-or-more jet like events

