The heaviest QED atom and tau mass

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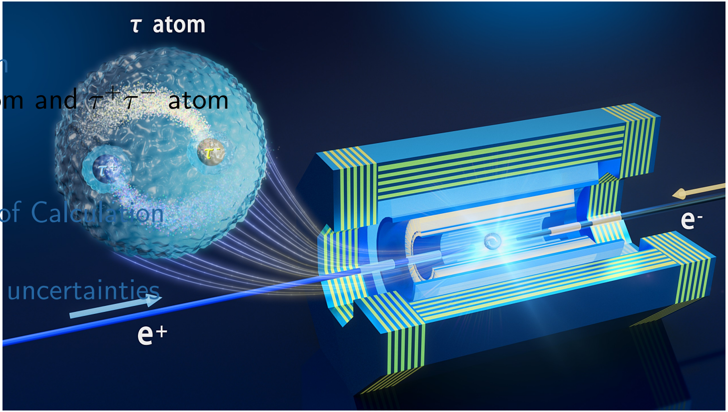
Outline



1. Introduction
   - QED atom and $\tau^+\tau^-$ atom
   - τ mass
2. The frame of Calculation
3. Reduce the uncertainties
4. Summary

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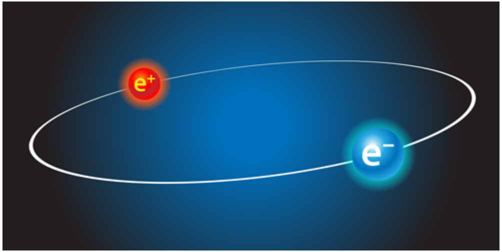
QED atom

1. QED atoms (e^+e^-, μ^+e^-, τ^+e^-, $\mu^+\mu^-$, $\tau^+\mu^-$, $\tau^+\tau^-$) are composed of unstructured and point-like lepton pairs, simple than the hydrogen formed of a proton and an electron.
2. The properties of QED atoms have been studied to test QED, fundamental symmetries, New Physics, gravity, and so on (hep-ex/0106103, 0912.0843, 1710.01833, 1802.01438, Phys.Rept. 975 (2022) 1-61).
3. Only positronium (e^+e^-) and muonium (μ^+e^-) had been discovered in 1951 and 1960 respectively.

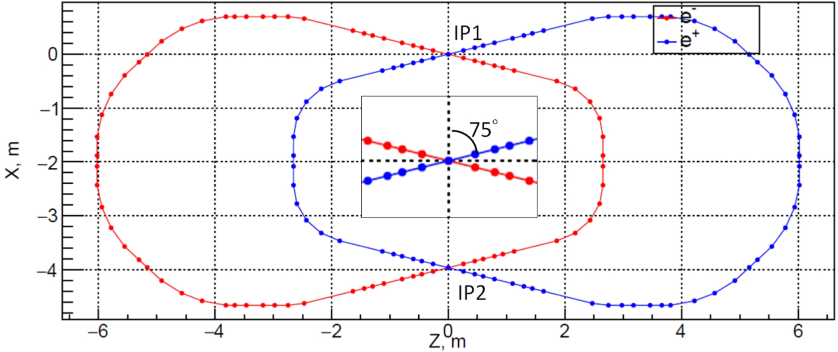
Positronium

1. Positronium was discovered by Martin Deutsch in 1951.
2. "I'm really glad that I did not get the Nobel Prize in 1956. It would have spoiled my life."
3. Positronium in medicine and biology: Nature Reviews Physics 1 (2019)527, Rev. Mod. Phys. 95 (2023) 021002.



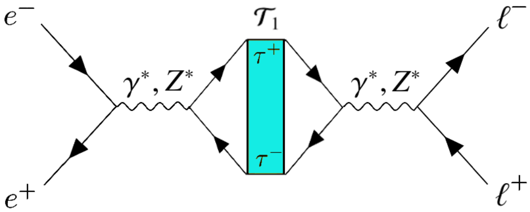
New colliders for true muonium

1. DIMUS: super-compact Dimuonium Spectroscopy collider at Fermilab, 2203.07144.
2. True muonium @ e^+e^- colliders with standard crossing angle, 2309.11683.

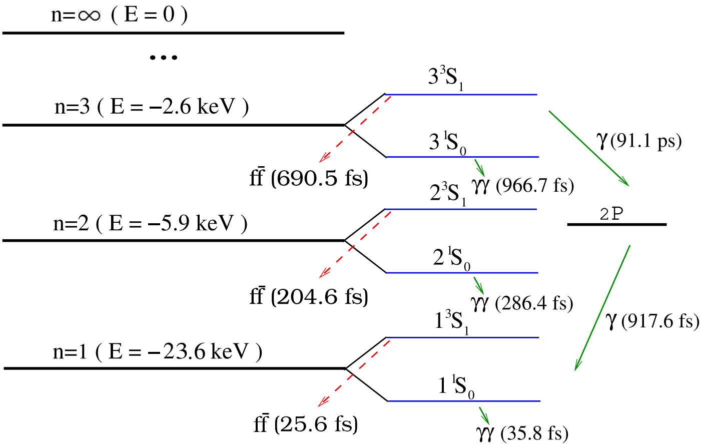


$\tau^+\tau^-$ atom

1. $\tau^+\tau^-$ atom is the smallest QED atom for Bohr radius is 30.4 fm (Moffat:1975uw)
2. $\tau^+\tau^-$ atom is is named tauonium (Avilez:1977ai,Avilez:1978sa), ditauonium (2204.07269, 2209.11439), and true tauonium (2202.02316).
3. We named them following charmonium just as $J_\tau(nS)$ for $n^{2S+1}L_J = n^3S_1$ and $J^{PC} = 1^{--}$, $\chi_{\tau J}(nP)$ for $n^{2S+1}L_J = n+1^3P_J$ and $J^{PC} = J^{++}$.
4. The production η_τ (2202.02316), and J_τ (2302.07365).



The spectroscopy of $\tau^+\tau^-$ atom, 2204.07269



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τ

Need more precise measurements m_τ, Γ_τ, $(g-2)_\tau$ in PDG 2022

τ

$$J = \frac{1}{2}$$

Mass $m = 1776.86 \pm 0.12$ MeV

$(m_{\tau^+} - m_{\tau^-})/m_{\text{average}} < 2.8 \times 10^{-4}$, CL = 90%

Mean life $\tau = (290.3 \pm 0.5) \times 10^{-15}$ s

$c\tau = 87.03$ μm

Magnetic moment anomaly > -0.052 and < 0.013, CL = 95%

$\text{Re}(d_\tau) = -0.220$ to 0.45×10^{-16} $e$cm, CL = 95%

$\text{Im}(d_\tau) = -0.250$ to 0.0080×10^{-16} $e$cm, CL = 95%

m_τ and lepton universality, 1405.1076

- The lepton universality can be tested as

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\tau_\mu}{\tau_\tau}\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{B(\tau \to e\nu\bar{\nu})}{B(\mu \to e\nu\bar{\nu})}(1+F_W)(1+F_\gamma), \tag{1}$$

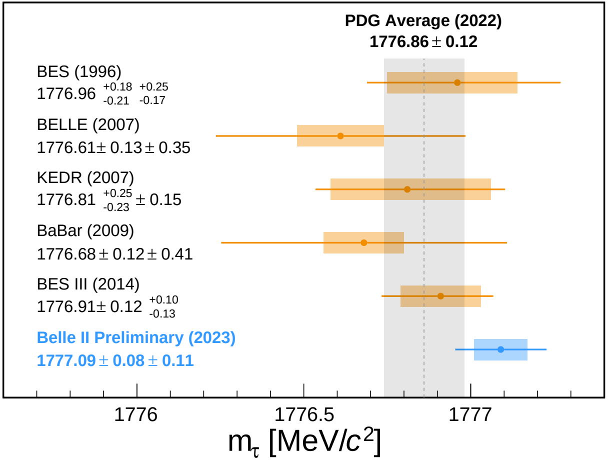
- BESIII measurement, 1405.1076

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = 1.0016 \pm 0.0042, \tag{2}$$

- Estimate uncertainty of τ mass

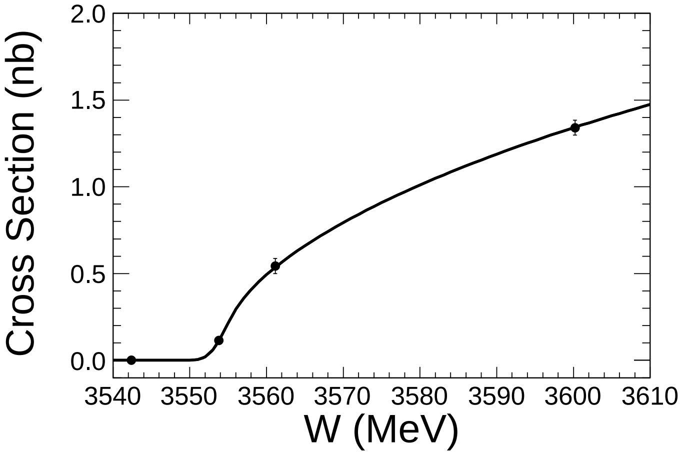
$$(\Delta m_\tau)^2 \sim \left(\frac{\partial m_\tau}{\partial N_\tau}\right)^2 (\Delta N_\tau)^2 \sim \left(\frac{\partial m_\tau}{\mathcal{L}\,\varepsilon\,\partial\sigma_\tau}\right)^2 (N_\tau) \sim \frac{\sigma_\tau}{\mathcal{L}\,\varepsilon\left(\frac{\partial\sigma_\tau}{\partial m_\tau}\right)^2}, \tag{3}$$

Measured m_τ, 175 M enents with 190 fb^{-1}, Belle II 2305.19116



m_τ measurement at BESIII, 1405.1076

Scan	$E_{\rm CM}$ (MeV)	$\mathcal{L}$(nb^{-1})
J/ψ	3088.7	78.5 ± 1.9
	3095.3	219.3 ± 3.1
	3096.7	243.1 ± 3.3
	3097.6	206.5 ± 3.1
	3098.3	223.5 ± 3.2
	3098.8	216.9 ± 3.1
	3103.9	317.3 ± 3.8
τ	3542.4	4252.1 ± 18.9
	3553.8	5566.7 ± 22.8
	3561.1	3889.2 ± 17.9
	3600.2	9553.0 ± 33.8
ψ'	3675.9	787.0 ± 7.2
	3683.7	823.1 ± 7.4
	3685.1	832.4 ± 7.5
	3686.3	1184.3 ± 9.1
	3687.6	1660.7 ± 11.0
	3688.8	767.7 ± 7.2
	3693.5	1470.8 ± 10.3



m_τ measurement at BESIII, 1405.1076

final state	1		2		3		4		total	
	Data	MC	Data	MC	Data	MC	Data	MC	Data	MC
ee	0	0	4	3.7	13	12.2	84	76.1	101	92.0
$e\mu$	0	0	8	9.1	35	31.4	168	192.6	211	233.1
$e\pi$	0	0	8	8.6	33	29.7	202	184.4	243	222.6
eK	0	0	0	0.5	2	1.8	16	16.9	18	19.3
$\mu\mu$	0	0	2	2.9	8	9.2	49	56.3	59	68.4
$\mu\pi$	0	0	4	3.9	11	14.1	89	86.7	104	104.7
μK	0	0	0	0.2	3	0.8	7	9.0	10	10.1
$\pi\pi$	0	0	1	2.0	5	7.7	57	54.0	63	63.8
πK	0	0	1	0.3	0	0.8	10	8.2	11	9.3
KK	0	0	0	0.0	1	0.1	1	0.3	2	0.4
$e\rho$	0	0	3	6.1	19	20.6	142	132.0	164	158.7
$\mu\rho$	0	0	8	3.3	8	11.8	52	63.3	68	78.5
$\pi\rho$	0	0	5	3.4	15	10.8	97	96.0	117	110.2
Total	0	0	44	44.2	153	151.2	974	975.7	1171	1171.0

New data taking scenario at BESIII, from Zhang Jianyong TAU2018

Data comparison

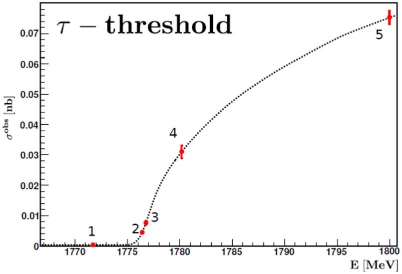
	J/ψ (pb⁻¹)	ψ' (pb⁻¹)	τ (pb⁻¹)				
			3540 MeV	3553 MeV	3554 MeV	3560 MeV	3600 MeV
2011	1.5	7.5	4.3	0	5.6	3.9	9.6
2018	32.6	67.2	25.5	42.6	27.1	8.3	13.9

New data taking scenario at BESIII, from Zhang Jianyong TAU2018

Three energy regions:

- **Low energy region**
  Point 1, 14 pb^{-1}, to determine background
- **Near threshold**
  Point 2, 39 pb^{-1} and point 3, 26 pb^{-1}, to determine tau mass
- **High energy region**
  Point 4, 7 pb^{-1} for X^2 check
  Point 5, 14 pb^{-1} to determine detection efficiency

Total lum. ~100pb^{-1},
uncertainty: 0.1MeV



We obtain more than 130 pb^{-1} tau scan data!

2018/09/24-28 Zhang Jianyong 16

$e^+e^- \to \tau^+\tau^- \to \nu X^- \bar{\nu} X^+$ around the $\tau^+\tau^-$ production threshold

1. Updated cross sections

$$\sigma_{ex}(W, m_\tau, \Gamma_\tau, \delta_w) = \int_{m(J_\tau)}^{\infty} dW' \frac{e^{-\frac{(W-W')^2}{2\delta_w^2}}}{\sqrt{2\pi}\delta_w} \int_0^{1-\frac{m(J_\tau)^2}{W'^2}} dx F(x, W') \frac{\bar{\sigma}(W'\sqrt{1-x}, m_\tau, \Gamma_\tau)}{|1-\Pi(W'\sqrt{1-x})|^2}.$$

2. Cross sections in BESIII, 1405.1076

$$\sigma(E_{\rm CM}, m_\tau, \delta_w^{\rm BEMS}) = \frac{1}{\sqrt{2\pi}\delta_w^{\rm BEMS}} \int_{2m_\tau}^{\infty} dE'_{\rm CM} e^{\frac{-(E_{\rm CM}-E'_{\rm CM})^2}{2(\delta_w^{\rm BEMS})^2}} \int_0^{1-\frac{4m^2}{E'^2_{\rm CM}}} dx F(x, E'_{\rm CM}) \frac{\sigma_1(E'_{\rm CM}\sqrt{1-x}, m_\tau)}{|1-\prod(E_{\rm CM})|^2}$$

3. Difference: shift $2m_\tau$ to $m(J_\tau)$ in the range of integration and add Γ_τ as a variable of the cross sections after including $J_\tau(nS)$ atom.

Cross sections from $J_\tau(nS)$

1. Then we get the $J_\tau(nS)$ contribution the cross section

$$\bar{\sigma}^{J_\tau}(W) = (3.11 \pm 0.02)\ \delta(\frac{W - 2m_\tau}{\mathrm{MeV}})\ \mathrm{pb} \tag{4}$$

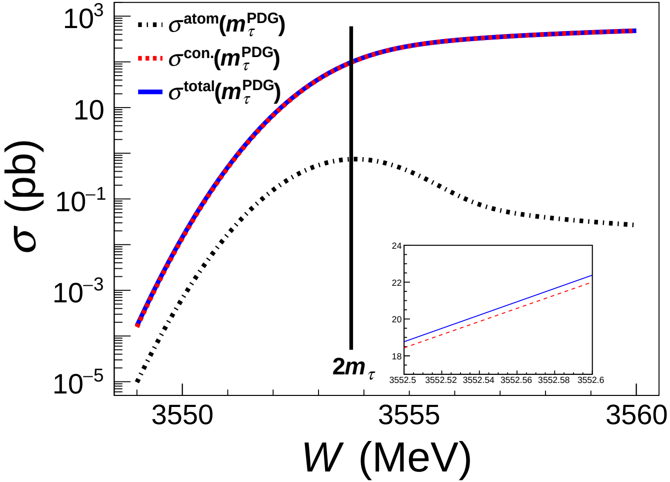
2. Updated $\bar{\sigma}(W, m_\tau, \Gamma_\tau)$

$$\bar{\sigma}(W) = (3.11 \pm 0.02)\delta(\frac{W - 2m_\tau}{\mathrm{MeV}})\ \mathrm{pb} + \theta(W - 2m_\tau)\bar{\sigma}_{Continue}(W) \tag{5}$$

3. Continue $\bar{\sigma}_{Continue}(2m_\tau)$

$$\bar{\sigma}_{Continue}(2m_\tau) = 236\ \mathrm{pb} \tag{6}$$

Cross sections from $J_\tau(nS)$



Uncertainties

1. The measured cross secitons

$$\sigma^{X^+Y^-\not{E}}(W) = \frac{N^{X^+Y^-\not{E}}(W)}{\mathcal{L}\,\varepsilon\, f_{ISR}\, f_{VP}} \tag{7}$$

2. Uncertainties are all larger than 0.1%.

Uncertainties	$\mathcal{L}$	ε	f_{ISR}	f_{VP}
BESIII	$> 0.5\%$	$> 1\%$	$\sim 0.5\%$	$> 0.14\%$
SCTF	$> 0.1\%$	-	$> 0.1\%$	-

3. We introduce $R_{X^+Y^-\not{E}}$, ratio of the cross sections, as

$$R_{X^+Y^-\not{E}}(W, \delta_W, m_\tau) = \frac{\sigma(W, m_\tau, \Gamma_\tau, \delta_W)}{\sigma^{\mu^+\mu^-}(W, \delta_W)}. \tag{8}$$

Numbers of the events

TABLE III: Numbers of $e^+e^- \to X^+Y^-\not{E}$ and $\mu^+\mu^-$ events and their statistical uncertainties in the pseudoexperiments with $m_\tau = m_\tau^{\text{PDG}}$.

i	$\mathcal{L}_i/\text{fb}^{-1}$	W_i/MeV	$N^{\text{data}}_{X^+Y^-\not{E},\, i}$	$N^{\text{data}}_{\mu^+\mu^-,\, i}$
1	5	3549.00	$0.1^{+1.2}_{-0.1}$	$(1.1764 \pm 0.0003) \times 10^7$
2	500	3552.56	$(8.772 \pm 0.009) \times 10^5$	$(1.17394 \pm 0.00003) \times 10^9$
3	1000	3555.83	$(2.4052 \pm 0.0005) \times 10^7$	$(2.34331 \pm 0.00005) \times 10^9$

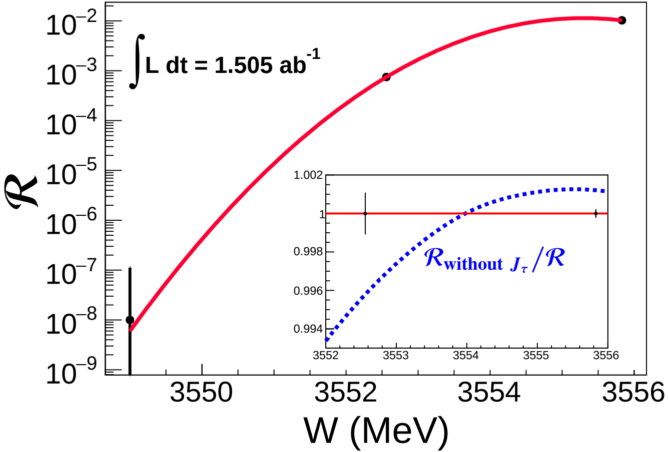
Determine χ^2 and m_τ

1. A least-square fit is applied

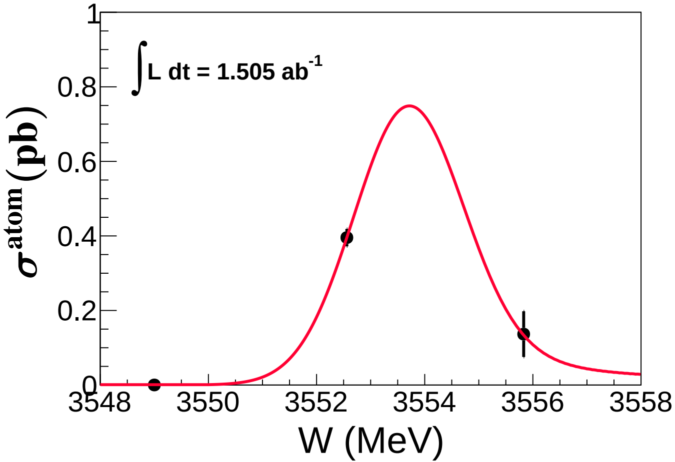
$$\chi^2 = \sum_{i=1}^{3} \left(\frac{\mathcal{R}_i^{\rm data} - \hat{\mathcal{R}}_i(m_\tau)}{\Delta \mathcal{R}_i^{\rm data}} \right)^2 , \tag{9}$$

2. Where $\mathcal{R}_i^{\rm data} = \frac{N^{\rm data}_{X^+Y^-\not{E},\, i}}{N^{\rm data}_{\mu^+\mu^-,\, i}}$ and $\Delta\mathcal{R}_i^{\rm data}$ is its statistical uncertainty.
3. And $\hat{\mathcal{R}}_i(m_\tau)$ is the expected ratio at the τ mass m_τ to be determined from the fit.

Ratio of the events



The cross section of J_τ



The systematic uncertainties σ_{m_τ}

TABLE IV: The systematic uncertainties of the m_τ (σ_{m_τ}) in keV.

Sources	σ_{m_τ}/keV
Energy scale of W_2	0.72
Energy scale of W_3	0.35
Energy spread δ_W	0.59
Efficiency	0.04
Theory	0.07
Systematic uncertainties	0.99

$$m_\tau = (1\ 776\ 860.00 \pm 0.25\ (\text{stat.}) \pm 0.99\ (\text{syst.}))\ \text{keV}$$

Summary

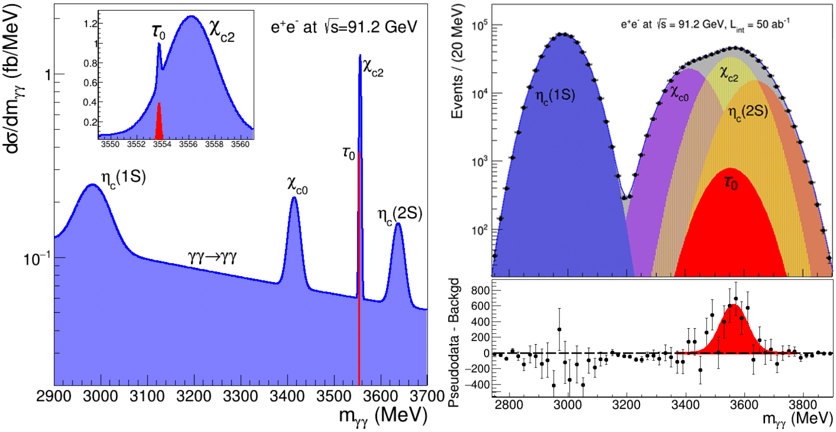
1. We show that the $\tau^+\tau^-$ atom with $J^{PC} = 1^{--}$, J_τ, can be observed with a significance larger than 5σ with a $1.5\,\mathrm{ab}^{-1}$ data sample at the proposed high luminosity experiments STCF and SCTF, by measuring the cross section ratio of the processes $e^+e^- \to X^+Y^-\not{E}$ and $e^+e^- \to \mu^+\mu^-$.
2. With the same data sample, the τ lepton mass can be measured with a precision of 1 keV, a factor of about 100 improvement over the existing world best measurements.

Thanks

$\gamma\gamma \to \eta_\tau \to \gamma\gamma$, 2202.02316

Colliding system, c.m. energy, $\mathcal{L}_{\text{int}}$, exp.	$\sigma \times \mathcal{B}_{\gamma\gamma}$						$N \times \mathcal{B}_{\gamma\gamma}$	
	$\eta_c(1S)$	$\eta_c(2S)$	$\chi_{c,0}(1P)$	$\chi_{c,2}(1P)$	LbL	$\mathcal{T}_0$	$\mathcal{T}_0$	$\chi_{c,2}(1P)$
e^+e^- at 3.78 GeV, 20 fb^{-1}, BES III	120 fb	3.6 ab	15 ab	13 ab	30 ab	0.25 ab	–	–
e^+e^- at 10.6 GeV, 50 ab^{-1}, Belle II	1.7 fb	0.35 fb	0.52 fb	0.77 fb	1.7 fb	0.015 fb	750	38 500
e^+e^- at 91.2 GeV, 50 ab^{-1}, FCC-ee	11 fb	2.8 fb	3.9 fb	6.0 fb	12 fb	0.11 fb	5 600	$3 \cdot 10^5$
p-p at 14 TeV, 300 fb^{-1}, LHC	7.9 fb	2.0 fb	2.8 fb	4.3 fb	6.3 fb	0.08 fb	24	1290
p-Pb at 8.8 TeV, 0.6 pb^{-1}, LHC	25 pb	6.3 pb	8.7 pb	13 pb	21 pb	0.25 pb	0.15	8
Pb-Pb at 5.5 TeV, 2 nb^{-1}, LHC	61 nb	15 nb	21 nb	31 nb	62 nb	0.59 nb	1.2	62

$\gamma\gamma \to \eta_\tau \to \gamma\gamma$ at Z pole, 2202.02316



$e^+e^- \to J_\tau \to \mu^+\mu^-$ at STCF, 2302.07365

TABLE IV: Cross sections and expected number of events for the s-channel production of ortho-ditauonium ($\mathcal{T}_1$), and for the $\tau^+\tau^-$ and (background) $\mu^+\mu^-$ continua, in e^+e^- at $\sqrt{s} \approx m_{\mathcal{T}}$ at various facilities. The last column lists the expected signal statistical significance.

Colliding system, $\sqrt{s}$ ($\delta_{\sqrt{s}}$ spread), $\mathcal{L}_{\rm int}$, experiment	σ			N			$S/\sqrt{B}$
	$\mathcal{T}_1$	$\tau^+\tau^-$	$\mu^+\mu^-$	$\mathcal{T}_1$	$\mathcal{T}_1 \to \mu^+\mu^-$	$\mu^+\mu^-$	
e^+e^- at 3.5538 GeV (1.47 MeV), 5.57 pb^{-1}, BES III	1.9 pb	117 pb	6.88 nb	10.4	2.1	38 300	0.01σ
e^+e^- at $\sqrt{s} \approx m_{\mathcal{T}}$ (1.24 MeV), 140 pb^{-1}, BES III	2.2 pb	103 pb	6.88 nb	310	63	$9.63 \cdot 10^5$	0.06σ
e^+e^- at $\sqrt{s} \approx m_{\mathcal{T}}$ (1 MeV), 1 ab^{-1}, STCF	2.6 pb	95 pb	6.88 nb	$2.6 \cdot 10^6$	$5.3 \cdot 10^5$	$6.88 \cdot 10^9$	6.4σ
e^+e^- at $\sqrt{s} \approx m_{\mathcal{T}}$ (100 keV), 0.1 ab^{-1}, STCF	22 pb	46 pb	6.88 nb	$2.2 \cdot 10^6$	$4.5 \cdot 10^5$	$6.88 \cdot 10^8$	17σ

1. $S/\sqrt{B}$ is 6.4 σ (17 σ) with 1 ab^{-1} data and $\delta_W = 1(0.1)$ MeV.
2. With monochromatized beams can also provide a very precise extraction of the tau lepton mass with at least $\mathcal{O}$(25 keV) uncertainty.

Monochromatization @ $e^+e^- \to H$ @ FCC-ee, EPJP 137 (2022) 1, 31

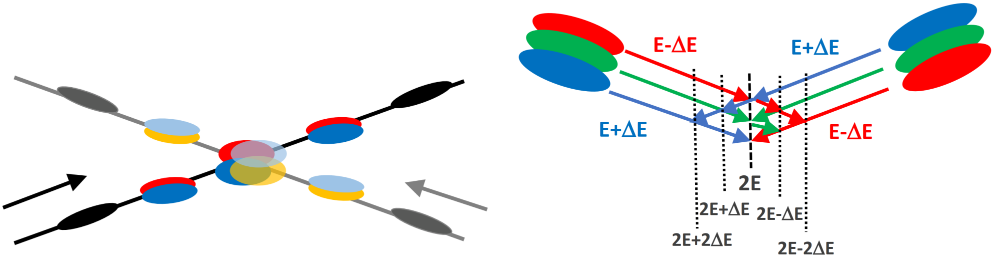


Fig. 1. FCC-ee monochromatization scheme featuring interaction-point dispersion of opposite sign for the two colliding beams, with (left) or without crab crossing and integrated resonance scan (right). Different colours schematically indicate bunch portions with slightly different energies.

$\bar{\sigma}(W, m_\tau, \Gamma_\tau)$, orthogonal perfect normalized basis, 1312.4791

1. $\bar{\sigma}(W, m_\tau, \Gamma_\tau)$

$$\bar{\sigma}(W, m_\tau, \Gamma_\tau) = \frac{4\pi\alpha^2}{3W^2}\frac{24\pi}{W^2}\mathrm{Im}\left[G_{\bar{\nu}X^+\nu X^-}(0,0,W-2m_\tau)\right], \qquad (10)$$

2. $G_{\bar{\nu}X^+\nu X^-}(\vec{r}, \vec{r}\,', E)$ represents a Green function of $\tau^+\tau^-$ currents in the non-relativistic effective theory, where $\tau^+\tau^-$ decay to $\bar{\nu}X^+\nu X^-$

$$G_{\bar{\nu}X^+\nu X^-}(\vec{r}, \vec{r}\,', E) = \sum_n \frac{\psi_n(\vec{r})\psi_n^*(\vec{r}\,')}{E_n - E - i\epsilon} Br[n \to \bar{\nu}X^+\nu X^-] + \int \frac{d^3\vec{k}}{2\pi^3}\frac{\psi_{\vec{k}}(\vec{r})\psi_{\vec{k}}^*(\vec{r}\,')}{E_{\vec{k}} - E - i\epsilon}, \qquad (11)$$

3. Then

$$\bar{\sigma}(W) = \bar{\sigma}^{J_\tau}(W) + \bar{\sigma}(W)_{continue} \qquad (12)$$

Breit-Wigner formula

1. Green function approach to bound states is consistent with Breit-Wigner formula for a narrow bound states

$$\bar{\sigma}^{J_\tau}(W) = \sum_n \frac{6\pi^2}{W^2}\delta(W - m(J_\tau(nS)))\Gamma(J_\tau(nS) \to e^+e^-)Br(J_\tau(nS) \to \bar{\nu}X^+\nu X^-) \quad (13)$$

2. Ignore the binding Energy of $J_\tau(nS)$ for it much less than δ_w

$$\bar{\sigma}^{J_\tau}(W) = \frac{6\pi^2}{W^2}\delta(W - 2m_\tau)\sum_n \Gamma(J_\tau(nS) \to e^+e^-)Br(J_\tau(nS) \to \bar{\nu}X^+\nu X^-) \quad (14)$$

Decay mode of $J_\tau(nS)$

$$\begin{aligned}\Gamma_{total}(J_\tau(nS)) &= \Gamma_{Ani}(J_\tau(nS)) + \Gamma_{Weak}(J_\tau(nS)) + \Gamma_{E1}(J_\tau(nS)) \\ \Gamma_{Ani}(J_\tau(nS)) &= (2+R)\Gamma(J_\tau(nS) \to e^+e^-) \\ \Gamma_{Weak}(J_\tau(nS)) &= 2\Gamma(\tau \to \nu X^-) \qquad (15)\end{aligned}$$

Parameters

1. Parameters

$$m_\tau = m_\tau^{\rm PDG} = 1776.86\,{\rm MeV}, \quad R = 2.342 \pm 0.0645,$$
$$\Gamma_\tau = 2.2674 \pm 0.0039\,{\rm meV}, \qquad \delta_W = 1\ {\rm MeV},$$
$$\alpha(0) = 1/137.036, \qquad \Delta\alpha_{had}(m_{J_\tau}) = (74 \pm 7) \times 10^{-4}. \tag{16}$$

2. The resulting NLO expression for $\bar{\sigma}^{J_\tau}(W)$ is given by

$$\bar{\sigma}^{J_\tau}(W) = (3.11 \pm 0.02)\, \delta\left(\frac{W - 2m_\tau + 13.8\ {\rm keV}}{1\ {\rm MeV}}\right)\ {\rm pb}, \tag{17}$$

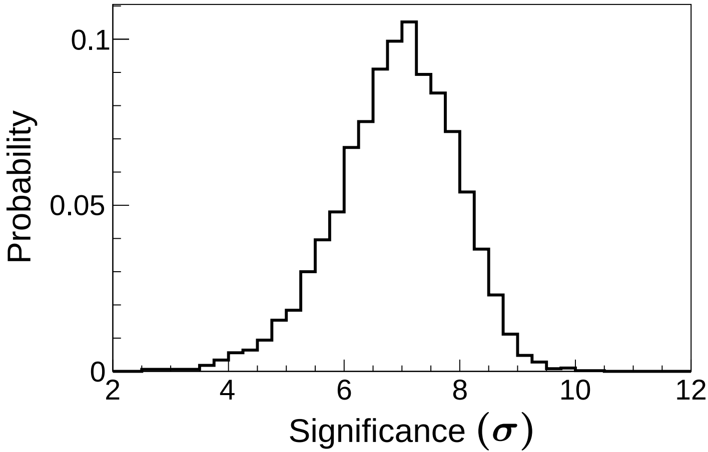
where $13.8\ {\rm keV} = \sum_n B_n Br^{J_\tau(nS)}_{X^+Y^-\not{E}} \Gamma^{J_\tau(nS)}_{e^+e^-} / \sum_n Br^{J_\tau(nS)}_{X^+Y^-\not{E}} \Gamma^{J_\tau(nS)}_{e^+e^-}$.

Decay mode of $J_\tau(nS)$

TABLE II: The decay data of $J_\tau(nS)$ in meV.

n	$\Gamma^{J_\tau(nS)}_{e^+e^-}$	$2\Gamma_\tau$	$\Gamma^{J_\tau(nS)}_{E1}$	$\Gamma^{J_\tau(nS)}_{\text{total}}$	$\Gamma^{J_\tau(nS)}_{e^+e^-} Br^{J_\tau(nS)}_{X^+Y^-\not{E}}$
1	6.484	4.535	0.0000	32.695	0.899
2	0.808	4.535	0.0000	8.044	0.455
3	0.239	4.535	0.0072	5.573	0.195
$\sum_{n=1}^{\infty}$					1.795 ± 0.012

The statistical significance distribution in 10^5 sets pseudoexperiments



The significance of $J_\tau(nS)$ as a function of $m_\tau^{\rm Natural} - m_\tau^{\rm PDG}$.