Exploring hexaquark states from lattice QCD

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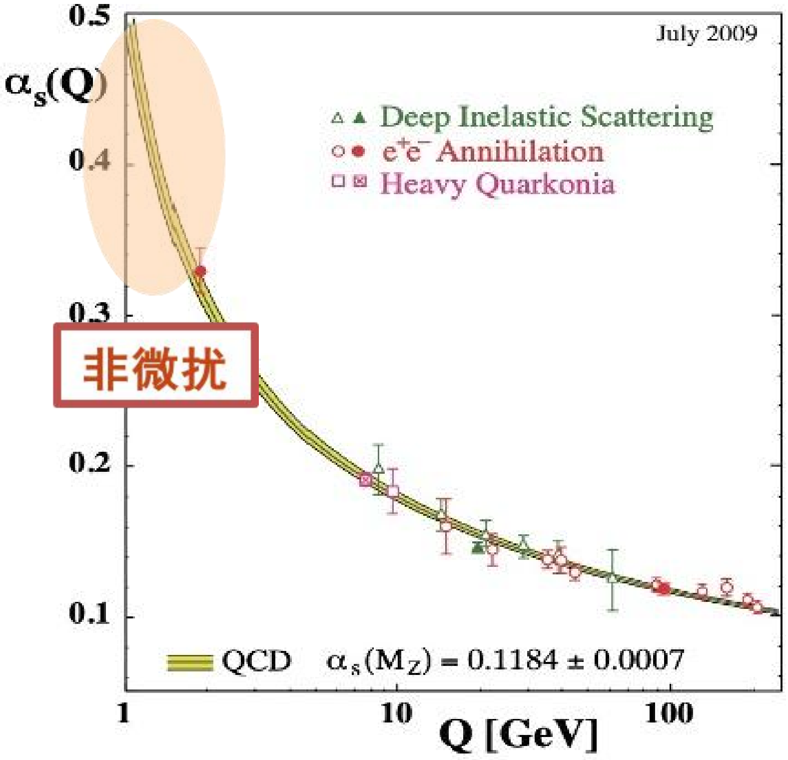
Sci. China-Phys. Mech. Astron. 67, 211011 (2024)

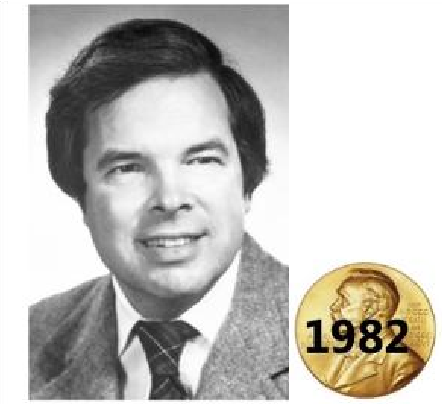
OUTLINE

- **Hidden-charm and hidden-strange hexaquark states from lattice QCD**
- **Preliminary test on $p-n$ and $p-\Lambda$ scattering**

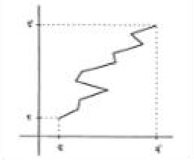
Background

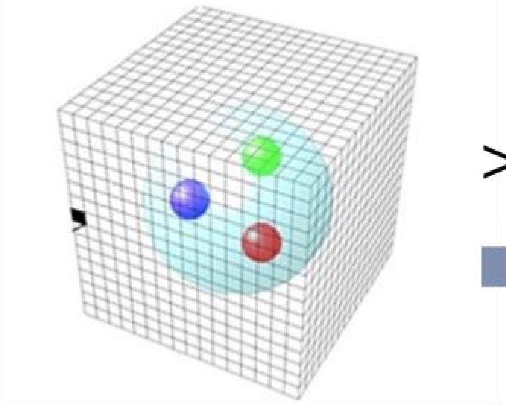
- 描述强相互作用的理论被称为量子色动力学QCD。
- 当能标降低时，相互作用增强，QCD进入到非微扰区域。
- 格点量子色动力学(Wilson, 1974)：从第一性原理出发的非微扰方法





K. G. Wilson





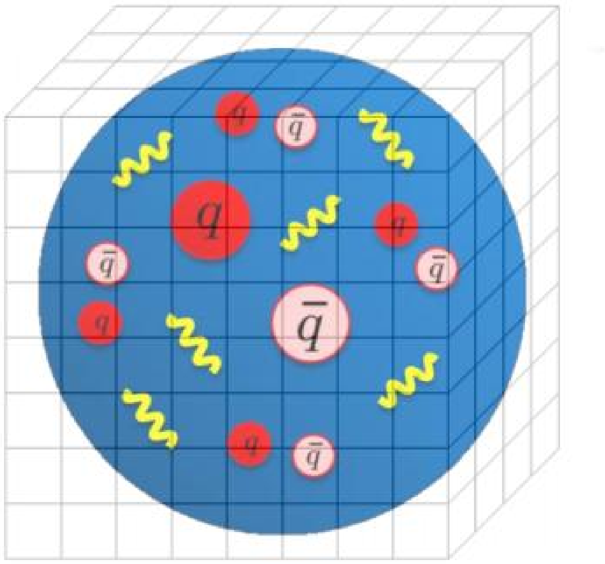
格点QCD

$> 2^{1000000}$

超级计算机

Background

$$S_E^{latt} = \underbrace{\sum \frac{6}{g^2} ReTr(1-U_P)}_{\text{wilson gauge action}} + \sum_q \underbrace{\bar{q}(D_\mu^{lat}\gamma_\mu + am_q)q}_{\text{lattice fermion action}}$$



- **Charm quark on discrete lattice:**

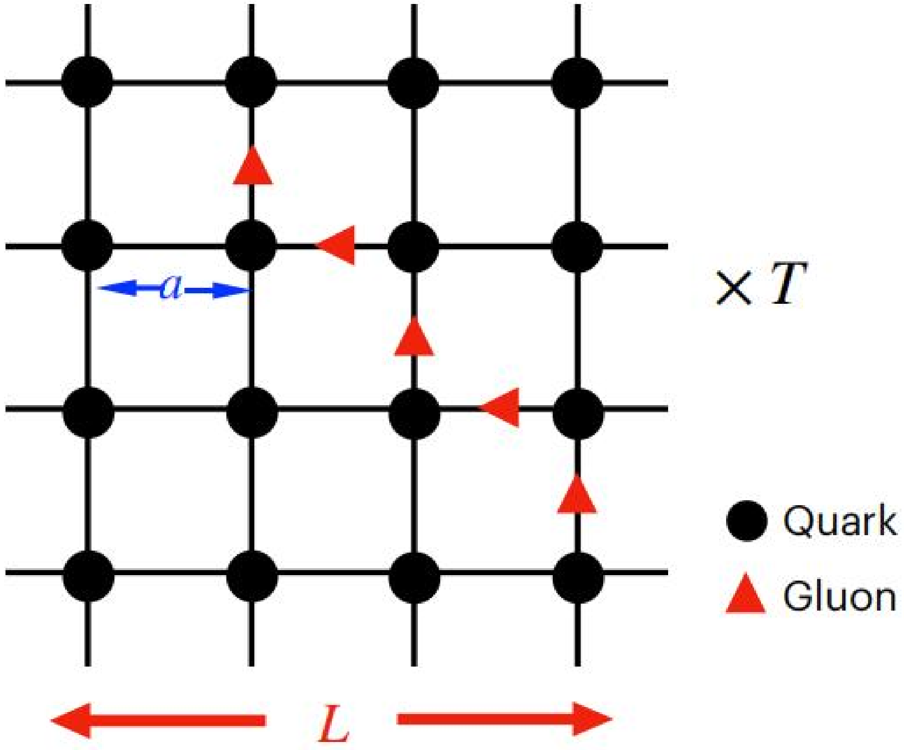
consider both IR and UV effects:

$$m_\pi L \gtrsim 4, \quad \text{and} \quad \mathrm{a}^{-1} \gg \text{mass scale}$$

for $m_\pi = m_\pi^{\text{phy}} \sim 140\text{MeV}$**, and** $m_c \simeq 1.3\text{GeV}$,

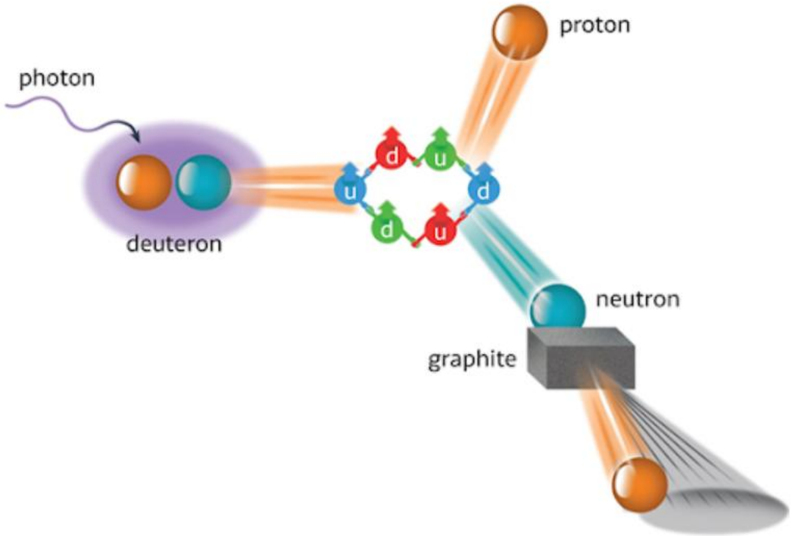
$$L \gtrsim 5.6\text{fm}$$

$$a^{-1} \gg 1.3\text{GeV} \simeq (0.15\text{fm})^{-1}$$



Hidden-charm and hidden-strange hexaquark

夸克模型定义的重子态、介子态及超出夸克模型的奇特态

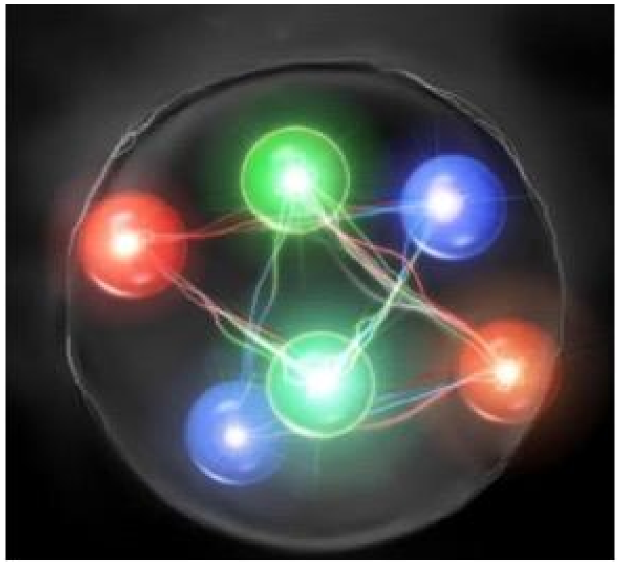


d*六夸克的形成过程

难点：1.如何有效地处理双重子关联函数中复杂的缩并。

2. source- type的选取

3.如何提高信噪比



六夸克态

Hidden-charm and hidden-strange hexaquark

格点上计算强子谱的步骤：

- 建立具有期望量子数的算符基 (basis of operators) $\{O_1, O_2, ...\}$,再构造关联函数矩阵：

$$C_{ij} = <0|\mathcal{O}_i\mathcal{O}_j^{\dagger}|0> = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

- 广义本征值求解问题(generalized eigenvalue problem)： $C_{ij}v_j^n(t) = \lambda_n(t) C_{ij}^0 v_j^n(t)$

- 特征值满足：$\lambda_n(t) \sim e^{-E_n t}(1 + e^{-\Delta E\, t})$

Hidden-charm and hidden-strange hexaquark

- 选取夸克组分$usc\bar{d}\overline{sc}$
- 构造算符

sink处：

0^{++} :

$$O_1^A(x) = \epsilon^{abc}\epsilon^{def}\left[u_a^T C\gamma_5 s_b\right]\left[\bar{d}_d C\gamma_5 \bar{s}_e^T\right]\times[\bar{c}_f c_c](x),$$

$$O_1^B(x) = [\bar{s}\gamma_5 u]\times\left[\bar{d}\gamma_5 s\right]\times[\bar{c}c](x),$$

source处（Coulomb wall source）：

$$\begin{aligned} O_{(s)}^A(t) = &\sum_{\mathbf{y}_i, i=1,6} \epsilon^{abc}\epsilon^{def}[u_a^T(\mathbf{y}_1)C\gamma_5 s_b(\mathbf{y}_2)] \\ &\times[\bar{d}_d(\mathbf{y}_3)C\gamma_5\bar{s}_e(\mathbf{y}_4)^T]\times[\bar{c}_f(\mathbf{y}_5)\gamma_x c_c(\mathbf{y}_6)], \\ O_{(s)}^B(t) = &\sum_{\mathbf{y}_i, i=1,6} \bar{s}(\mathbf{y}_1)\gamma_5 u(\mathbf{y}_2)\times\bar{d}(\mathbf{y}_3)\gamma_5 s(\mathbf{y}_4) \\ &\times\bar{c}(\mathbf{y}_5)\gamma_x c(\mathbf{y}_6), \end{aligned}$$

Hidden-charm and hidden-strange hexaquark

Table 2 Mass (GeV) for the ordinary hadrons[a)]

Hadron	Lattice	Exp.
K	0.4869(41)	0.4937/0.4976
D	1.8675(76)	1.864/1.870
D_s	1.9766(65)	1.968
Λ	1.074(48)	1.115
Ξ	1.354(22)	1.314
Ω	1.699(42)	1.672
Λ_c	2.348(59)	2.286
Ω_c	2.4380(68)	2.468
η_c	3.0041(20)	2.9839
J/ψ	3.0972(24)	3.0969

a) With the five ensembles of configurations, we have extracted the mass from the analysis of the two-point correlation functions and extrapolated it to the continuum and physical pion mass limit, and the errors are statistical. The experimental data are taken from Particle Data Group [56].

$$E^2 = m^2 + c_2 p^2 + c_3 p^4 a^2$$

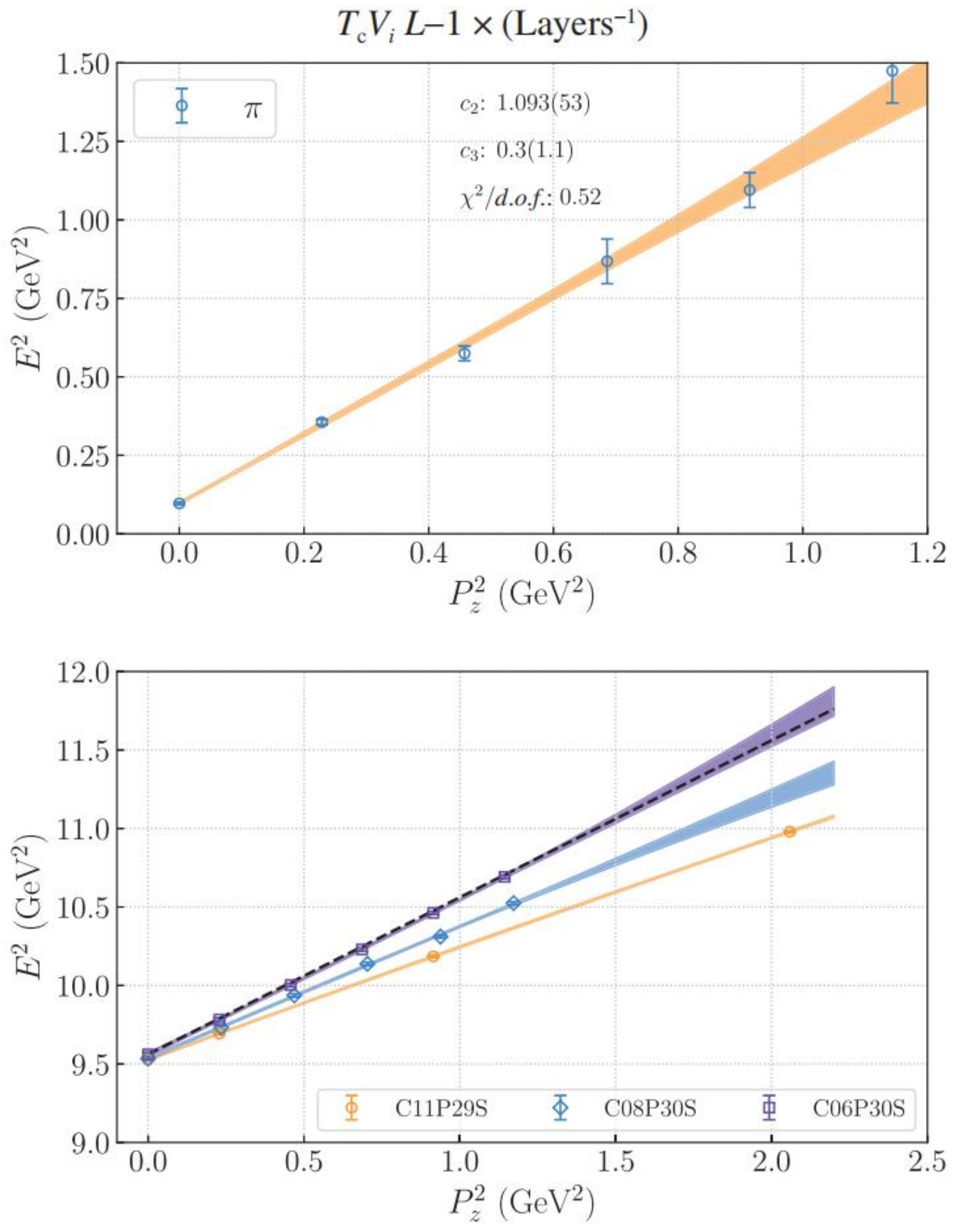


Figure 1 (Color online) Dispersion relation for π on the C06P30S ensemble (upper panel) and for J/ψ on the C11P29S, C08P30S, and C06P30S ensembles (lower panel).

Hidden-charm and hidden-strange hexaquark

四种量子数的六夸克态的有效质量(C06P30S)

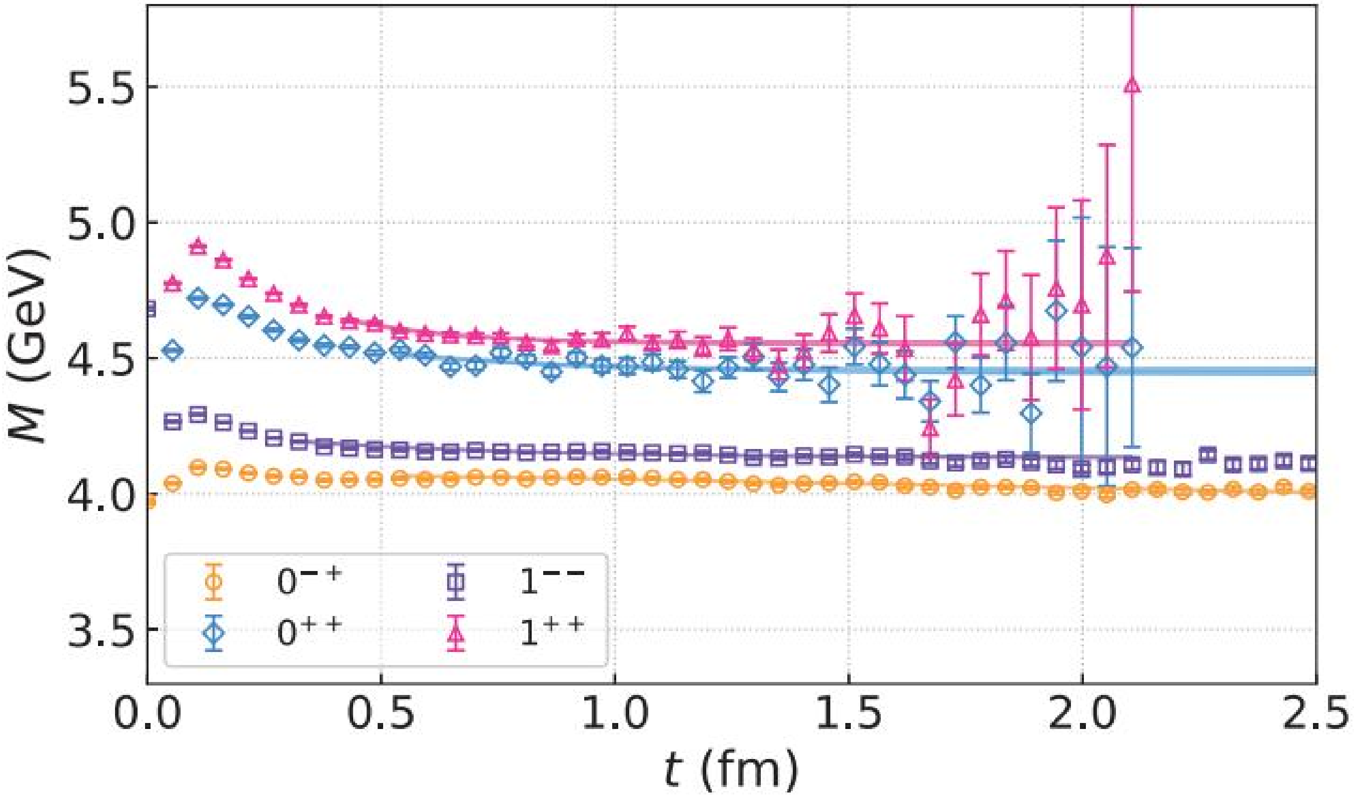


Figure 2 (Color online) Effective mass of the ground states for the hexaquarks on the four different quantum numbers on the C06P30S ensemble.

Hidden-charm and hidden-strange hexaquark

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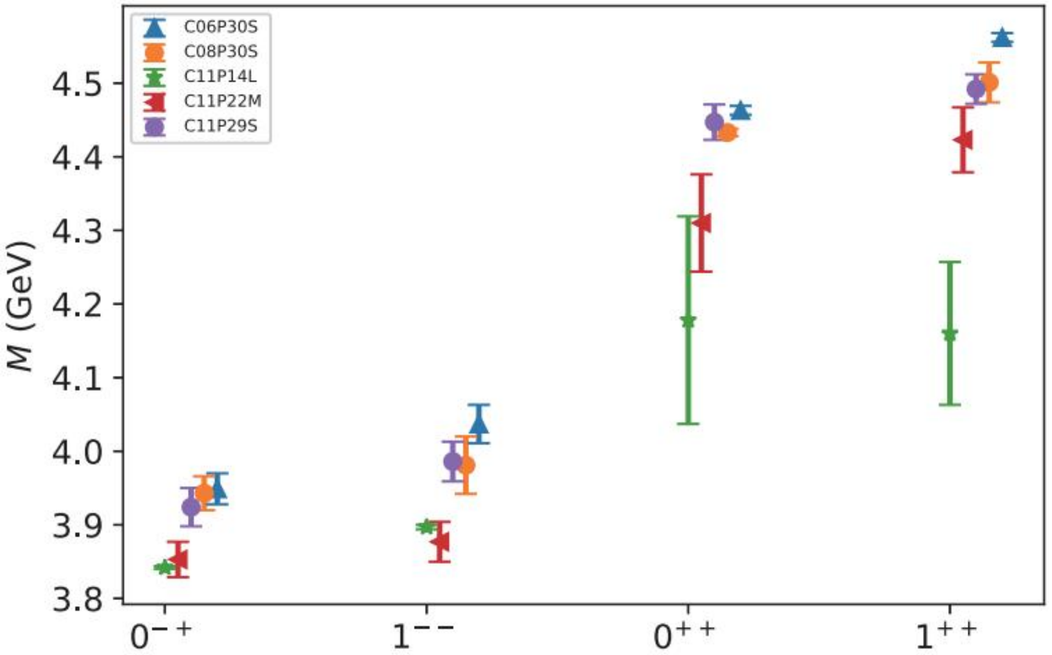


Figure 3 (Color online) Obtained masses of the different quantum numbers on the five ensembles. The horizontal axis 0^{-+}, 1^{--}, 0^{++}, and 1^{++} are correspond to the corresponding ground states.

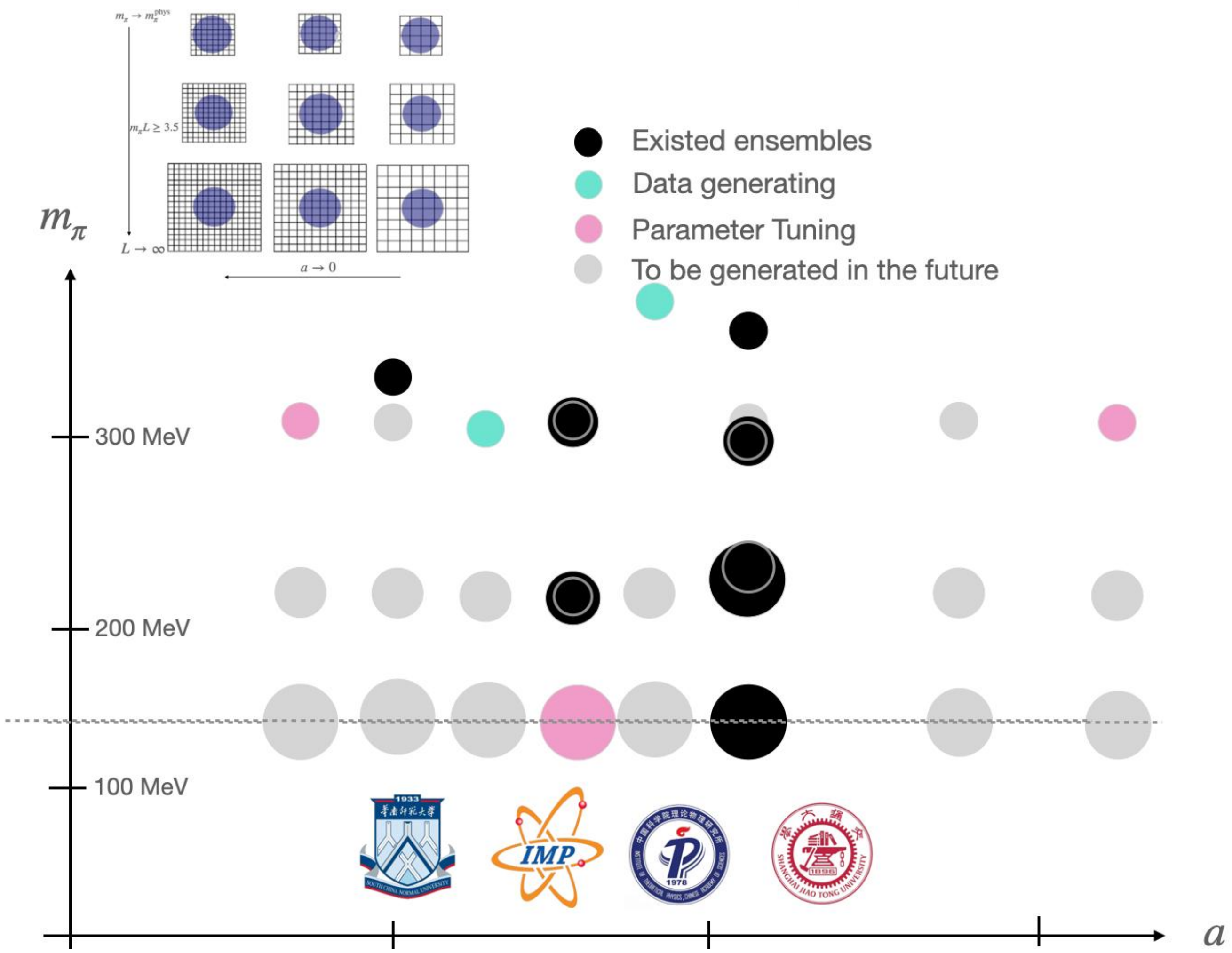
$$m_H(m_\pi, a) = m_{H,\text{phys}} + g_1^H(m_\pi^2 - m_{\pi,\text{phys}}^2) + g_2^H a^2$$

$I(J^{PC})$	Mass (GeV)
$1(0^{-+})$	3.865(44)
$1(1^{--})$	3.960(49)
$1(0^{++})$	4.12(13)
$1(1^{++})$	4.273(95)

Table 3 Obtained hexaquark masses (GeV) for the ground states at the physical pion mass after the chiral and continuum extrapolation

$\Xi_c\bar{\Xi}_c$阈值为4.938GeV，$K^+K^-\eta_c$阈值为3.971GeV

CLQCD ensembles



Ensemble	β	$L^3 \times T$	a (fm)	m_l^{b}	m_s^{b}	m_c^{b}	m_π	N_{meas}
C11P14L	6.20	$48^3 \times 96$	0.108	-0.2825	-0.2310	0.4800	135	203×48
C11P22M		$32^3 \times 64$		-0.2790	-0.2310	0.4800	222	451×20
C11P29S		$24^3 \times 72$		-0.2770	-0.2315	0.4780	284	432×26
C08P30S	6.41	$32^3 \times 96$	0.080	-0.2295	-0.2010	0.2326	297	653×26
C06P30S	6.72	$48^3 \times 144$	0.055	-0.1850	-0.1687	0.0770	312	136×80

$p-n$ and $p-\Lambda$ scattering

夸克组分：$p-n$为 $udu\ dud$
$p-\Lambda$为 $udu\ dus$

构造单重子算符

$$
\begin{aligned}
p_\sigma &= \epsilon^{abc}\frac{1}{\sqrt{2}}[u^a_\zeta(x)(C\gamma_5P_+)_{\zeta\xi}d^b_\xi(x) - d^a_\zeta(x)(C\gamma_5P_+)_{\zeta\xi}u^b_\xi(x)] \\
&\times [P_+(1-(-1)^\sigma i\gamma_1\gamma_2)]_{\sigma\rho}u^c_\rho(x) \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
n_\sigma &= \epsilon^{abc}\frac{1}{\sqrt{2}}[d^a_\zeta(x)(C\gamma_5P_+)_{\zeta\xi}u^b_\xi(x) - u^a_\zeta(x)(C\gamma_5P_+)_{\zeta\xi}d^b_\xi(x)] \\
&\times [P_+(1-(-1)^\sigma i\gamma_1\gamma_2)]_{\sigma\rho}d^c_\rho(x) \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_\sigma &= \epsilon^{abc}\frac{1}{\sqrt{2}}[d^a_\zeta(x)(C\gamma_5P_+)_{\zeta\xi}u^b_\xi(x) - u^a_\zeta(x)(C\gamma_5P_+)_{\zeta\xi}d^b_\xi(x)] \\
&\times [P_+(1-(-1)^\sigma i\gamma_1\gamma_2)]_{\sigma\rho}s^c_\rho(x) \qquad (3)
\end{aligned}
$$

$$p_\sigma(x) = \sum_\alpha w_\alpha^{[N]\sigma}u^{i(\alpha)}(x)d^{j(\alpha)}(x)u^{k(\alpha)}(x),$$

$$n_\sigma(x) = \sum_\alpha w_\alpha^{[N]\sigma}d^{i(\alpha)}(x)u^{j(\alpha)}(x)d^{k(\alpha)}(x).$$

α	$i(\alpha)$	$j(\alpha)$	$k(\alpha)$	$w_\alpha^{[N]0}$
1	0, 0	1, 1	0, 2	$-2\sqrt{2}$
2	0, 0	1, 2	0, 1	$2\sqrt{2}$
3	0, 1	1, 0	0, 2	$2\sqrt{2}$
4	0, 1	1, 2	0, 0	$-2\sqrt{2}$
5	0, 2	1, 0	0, 1	$-2\sqrt{2}$
6	0, 2	1, 1	0, 0	$2\sqrt{2}$
7	1, 0	0, 1	0, 2	$2\sqrt{2}$
8	1, 0	0, 2	0, 1	$-2\sqrt{2}$
9	1, 1	0, 0	0, 2	$-2\sqrt{2}$
10	1, 1	0, 2	0, 0	$2\sqrt{2}$
11	1, 2	0, 0	0, 1	$2\sqrt{2}$
12	1, 2	0, 1	0, 0	$-2\sqrt{2}$

PhysRevD.107.094508

$p-n$ and $p-\Lambda$ scattering

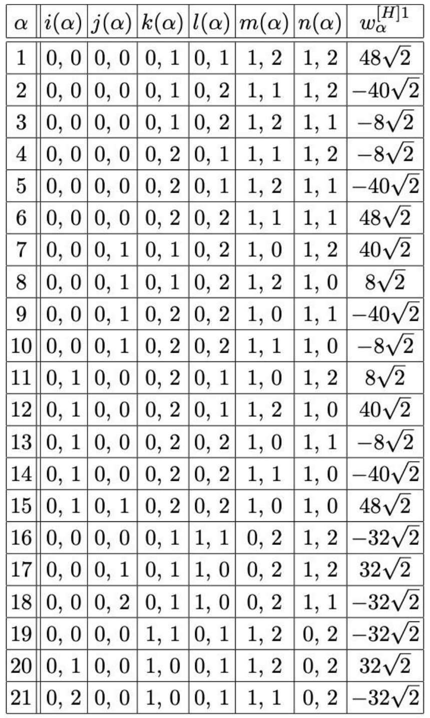
构造双重子(Dibaryon)算符和六夸克(Hexaquark)算符

$$D_{\rho\mathfrak{m}g}(t)=\sum_{\vec{x}_1,\vec{x}_2\in\Lambda_S}\psi_{\mathfrak{m}}^{[D]}(\vec{x}_1,\vec{x}_2)\sum_{\sigma,\sigma'}v^{\rho}_{\sigma\sigma'}\frac{1}{\sqrt{2}}\left[p_{\sigma g}(\vec{x}_1,t)n_{\sigma' g}(\vec{x}_2,t)\right.$$
$$\left.+(-1)^{1-\delta_{\rho 0}}n_{\sigma g}(\vec{x}_1,t)p_{\sigma' g}(\vec{x}_2,t)\right]$$

$$D_{\rho\mathfrak{m}g}(t)=\sum_{\vec{x}_1,\vec{x}_2\in\Lambda_S}\psi_{\mathfrak{m}}^{[D]}(\vec{x}_1,\vec{x}_2)\sum_{\alpha}w_{\alpha}^{[D]}u_g^{i(\alpha)}(\vec{x}_1,t)d_g^{j(\alpha)}(\vec{x}_1,t)u_g^{k(\alpha)}(\vec{x}_1,t)$$
$$\times d_g^{l(\alpha)}(\vec{x}_2,t)u_g^{m(\alpha)}(\vec{x}_2,t)d_g^{n(\alpha)}(\vec{x}_2,t),$$

$$H_{0\mathfrak{c}g}(t)=\sum_{\vec{x}\in\Lambda_S}\psi_{\mathfrak{c}}^{[H]}(\vec{x})\frac{1}{2}\left[p_{0g}(\vec{x},t)n_{1g}(\vec{x},t)-p_{1g}(\vec{x},t)n_{0g}(\vec{x},t)\right.$$
$$\left.+n_{0g}(\vec{x},t)p_{1g}(\vec{x},t)-n_{1g}(\vec{x},t)p_{0g}(\vec{x},t)\right],$$

$$H_{\rho\mathfrak{c}g}(t)=\sum_{\vec{x}\in\Lambda_S}\psi_{\mathfrak{c}}^{[H]}(\vec{x})\sum_{\alpha}w_{\alpha}^{[H]}u_g^{i(\alpha)}(\vec{x},t)d_g^{j(\alpha)}(\vec{x},t)u_g^{k(\alpha)}(\vec{x},t)$$
$$\times d_g^{l(\alpha)}(\vec{x},t)u_g^{m(\alpha)}(\vec{x},t)d_g^{n(\alpha)}(\vec{x},t).$$

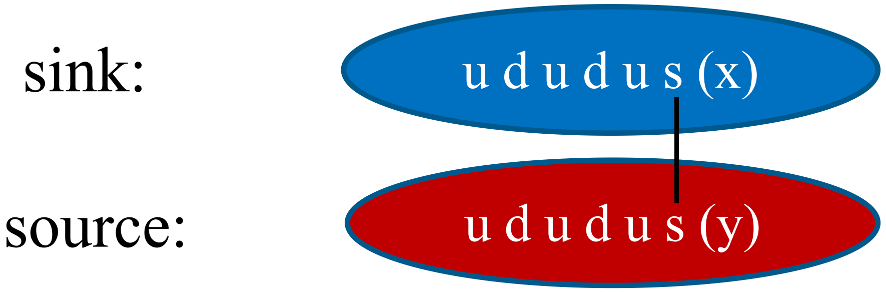


α	$i(\alpha)$	$j(\alpha)$	$k(\alpha)$	$l(\alpha)$	$m(\alpha)$	$n(\alpha)$	$w_\alpha^{[H]1}$
1	0, 0	0, 0	0, 1	0, 1	1, 2	1, 2	$48\sqrt{2}$
2	0, 0	0, 0	0, 1	0, 2	1, 1	1, 2	$-40\sqrt{2}$
3	0, 0	0, 0	0, 1	0, 2	1, 2	1, 1	$-8\sqrt{2}$
4	0, 0	0, 0	0, 2	0, 1	1, 1	1, 2	$-8\sqrt{2}$
5	0, 0	0, 0	0, 2	0, 1	1, 2	1, 1	$-40\sqrt{2}$
6	0, 0	0, 0	0, 2	0, 2	1, 1	1, 1	$48\sqrt{2}$
7	0, 0	0, 1	0, 1	0, 2	1, 0	1, 2	$40\sqrt{2}$
8	0, 0	0, 1	0, 1	0, 2	1, 2	1, 0	$8\sqrt{2}$
9	0, 0	0, 1	0, 2	0, 2	1, 0	1, 1	$-40\sqrt{2}$
10	0, 0	0, 1	0, 2	0, 2	1, 1	1, 0	$-8\sqrt{2}$
11	0, 1	0, 0	0, 2	0, 1	1, 0	1, 2	$8\sqrt{2}$
12	0, 1	0, 0	0, 2	0, 1	1, 2	1, 0	$40\sqrt{2}$
13	0, 1	0, 0	0, 2	0, 2	1, 0	1, 1	$-8\sqrt{2}$
14	0, 1	0, 0	0, 2	0, 2	1, 1	1, 0	$-40\sqrt{2}$
15	0, 1	0, 1	0, 2	0, 2	1, 0	1, 0	$48\sqrt{2}$
16	0, 0	0, 0	0, 1	1, 1	0, 2	1, 2	$-32\sqrt{2}$
17	0, 0	0, 1	0, 1	1, 0	0, 2	1, 2	$32\sqrt{2}$
18	0, 0	0, 2	0, 1	1, 0	0, 2	1, 1	$-32\sqrt{2}$
19	0, 0	0, 0	1, 1	0, 1	1, 2	0, 2	$-32\sqrt{2}$
20	0, 1	0, 0	1, 0	0, 1	1, 2	0, 2	$32\sqrt{2}$
21	0, 2	0, 0	1, 0	0, 1	1, 1	0, 2	$-32\sqrt{2}$

PhysRevD.107.094508

$p-n$ and $p-\Lambda$ scattering

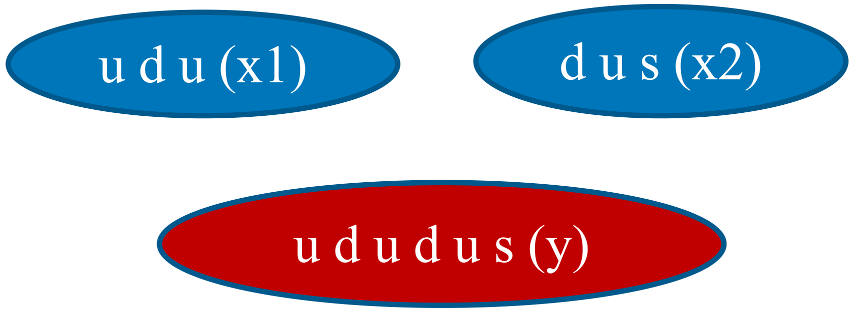
构造三种收缩方式的两点关联函数



sink: u d u d u s (x)

source: u d u d u s (y)

$$C^{[H,H]}_{\rho 0 g \rho' 0 g'}(t) = \sum_{\vec{x},\vec{y}\in\Lambda_{\mathcal{S}}} \psi^{[H]}_0(\vec{x}) \left\{\psi^{[H]}_0(\vec{y})\right\}^* \sum_{\alpha,\alpha'} w^{[H]\rho}_{\alpha} w^{[H]\rho'}_{\alpha'} \sum_{\mathcal{P}\in P^{[pn]}} \mathrm{sign}(\mathcal{P})$$
$$\times S^{\mathcal{P}[i(\alpha)]i'(\alpha')}_{gg'}(\vec{x},t;\vec{y},0)\, S^{\mathcal{P}[j(\alpha)]j'(\alpha')}_{gg'}(\vec{x},t;\vec{y},0)\, S^{\mathcal{P}[k(\alpha)]k'(\alpha')}_{gg'}(\vec{x},t;\vec{y},0)$$
$$\times S^{\mathcal{P}[l(\alpha)]l'(\alpha')}_{gg'}(\vec{x},t;\vec{y},0)\, S^{\mathcal{P}[m(\alpha)]m'(\alpha')}_{gg'}(\vec{x},t;\vec{y},0)\, S^{\mathcal{P}[n(\alpha)]n'(\alpha')}_{gg'}(\vec{x},t;\vec{y},0)$$

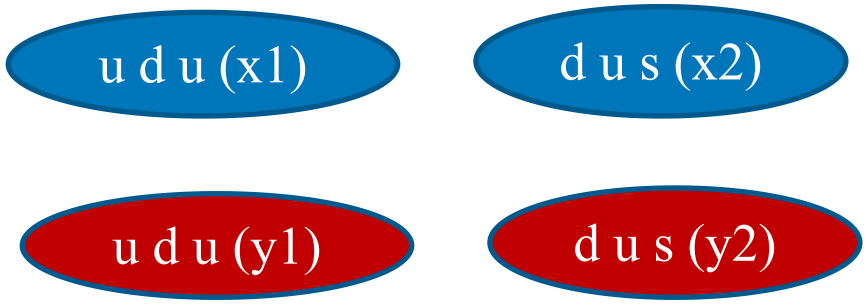


u d u (x1)

d u s (x2)

u d u d u s (y)

$$C^{[H,D]}_{\rho 0 g \rho' \mathfrak{m}' g'}(t) = \sum_{\vec{x},\vec{y}_1,\vec{y}_2\in\Lambda_{\mathcal{S}}} \psi^{[H]}_0(\vec{x}) \left\{\psi^{[D]}_{\mathfrak{m}'}(\vec{y}_1,\vec{y}_2)\right\}^* \sum_{\alpha,\alpha'} w^{[H]\rho}_{\alpha} w^{[D]\rho'}_{\alpha'} \sum_{\mathcal{P}\in P^{[pn]}} \mathrm{sign}(\mathcal{P})$$
$$\times S^{\mathcal{P}[i(\alpha)]i'(\alpha')}_{gg'}(\vec{x},t;\vec{y}_1,0)\, S^{\mathcal{P}[j(\alpha)]j'(\alpha')}_{gg'}(\vec{x},t;\vec{y}_1,0)\, S^{\mathcal{P}[k(\alpha)]k'(\alpha')}_{gg'}(\vec{x},t;\vec{y}_1,0)$$
$$\times S^{\mathcal{P}[l(\alpha)]l'(\alpha')}_{gg'}(\vec{x},t;\vec{y}_2,0)\, S^{\mathcal{P}[m(\alpha)]m'(\alpha')}_{gg'}(\vec{x},t;\vec{y}_2,0)\, S^{\mathcal{P}[n(\alpha)]n'(\alpha')}_{gg'}(\vec{x},t;\vec{y}_2,0)$$



u d u (x1)

d u s (x2)

u d u (y1)

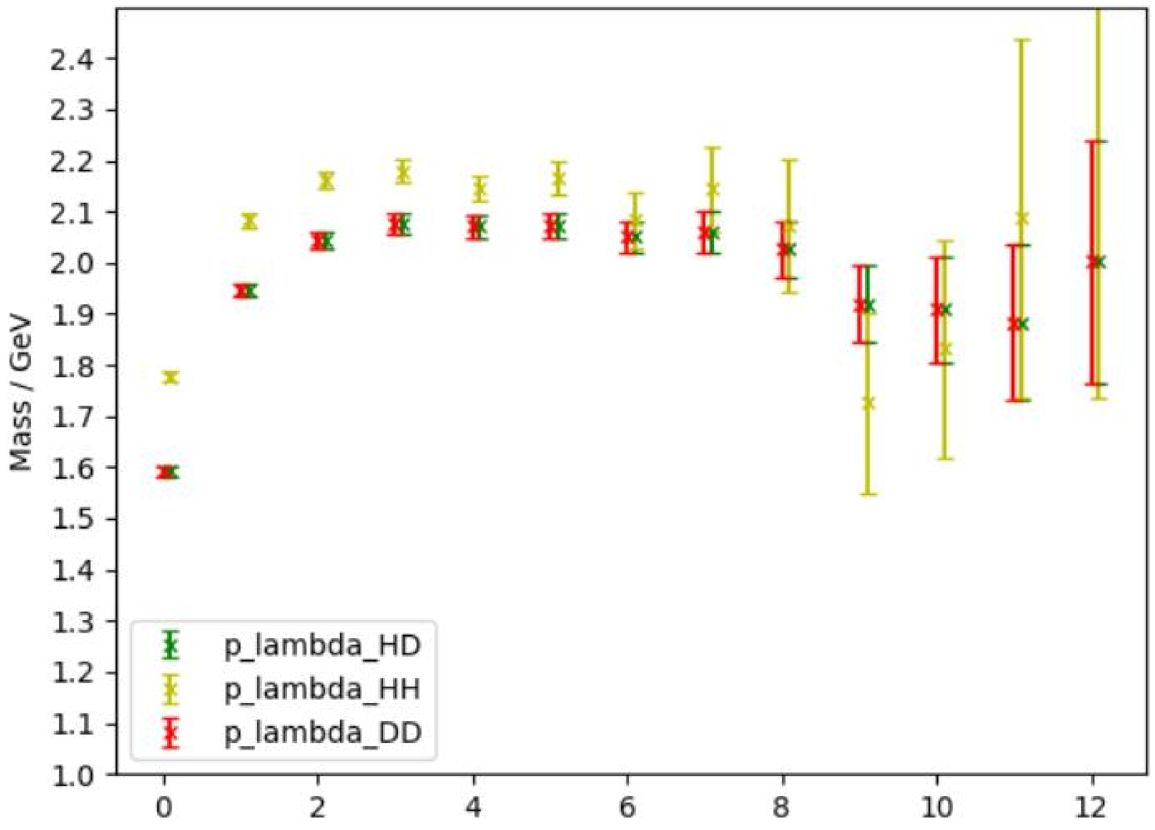
d u s (y2)

$$C^{[D,D]}_{\rho \mathfrak{m} g \rho' \mathfrak{m}' g'}(t) = \sum_{\vec{x}_1,\vec{x}_2,\vec{y}_1,\vec{y}_2\in\Lambda_{\mathcal{S}}} \psi^{[D]}_{\mathfrak{m}}(\vec{x}_1,\vec{x}_2) \left\{\psi^{[D]}_{\mathfrak{m}'}(\vec{y}_1,\vec{y}_2)\right\}^* \sum_{\alpha,\alpha'} w^{[D]\rho}_{\alpha} w^{[D]\rho'}_{\alpha'} \sum_{\mathcal{P}\in P^{[pn]}} \mathrm{sign}(\mathcal{P})$$
$$\times S^{\mathcal{P}[i(\alpha)]i'(\alpha')}_{gg'}\left(\vec{x}_{b_1(\mathcal{P})},t;\vec{y}_1,0\right) S^{\mathcal{P}[j(\alpha)]j'(\alpha')}_{gg'}\left(\vec{x}_{b_2(\mathcal{P})},t;\vec{y}_1,0\right)$$
$$\times S^{\mathcal{P}[k(\alpha)]k'(\alpha')}_{gg'}\left(\vec{x}_{b_3(\mathcal{P})},t;\vec{y}_1,0\right) S^{\mathcal{P}[l(\alpha)]l'(\alpha')}_{gg'}\left(\vec{x}_{b_4(\mathcal{P})},t;\vec{y}_2,0\right)$$
$$\times S^{\mathcal{P}[m(\alpha)]m'(\alpha')}_{gg'}\left(\vec{x}_{b_5(\mathcal{P})},t;\vec{y}_2,0\right) S^{\mathcal{P}[n(\alpha)]n'(\alpha')}_{gg'}\left(\vec{x}_{b_6(\mathcal{P})},t;\vec{y}_2,0\right),$$

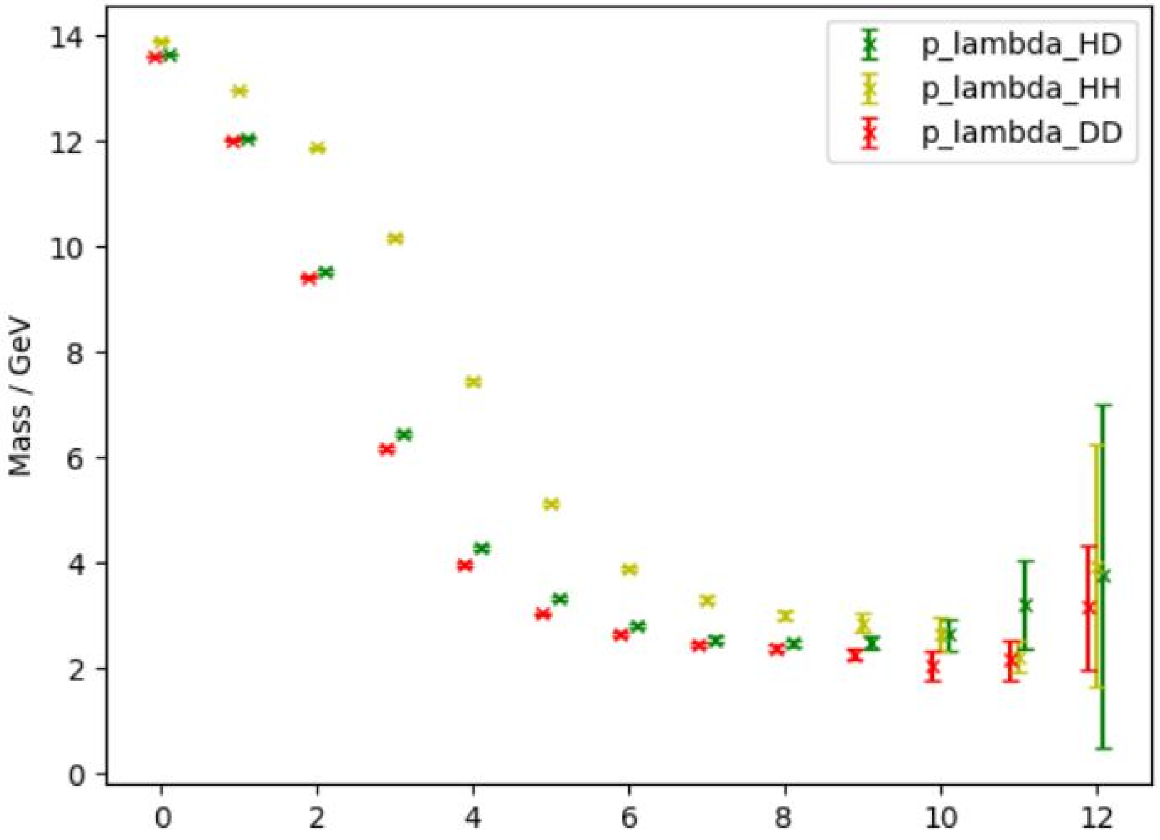
$p-\Lambda$ scattering (Preliminary)

$p-\Lambda$三种收缩方式的有效质量

Wall-source



point-source

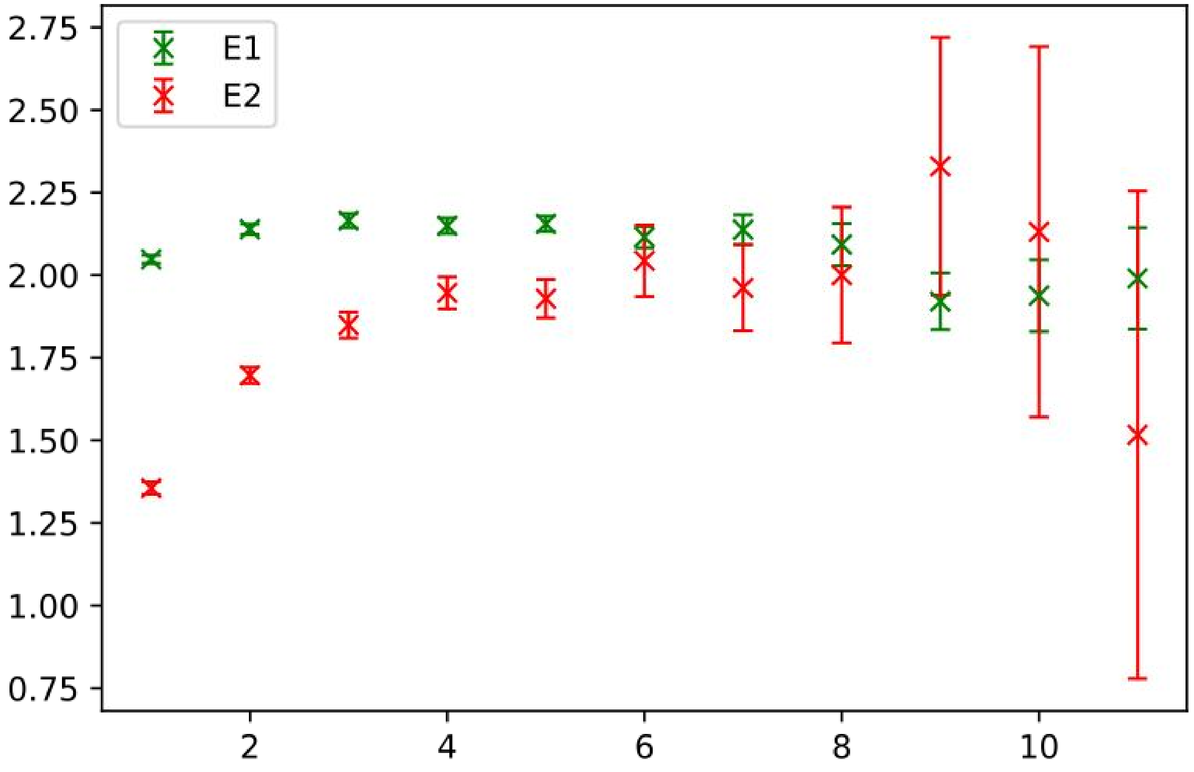


$p - \Lambda$ scattering (Preliminary)

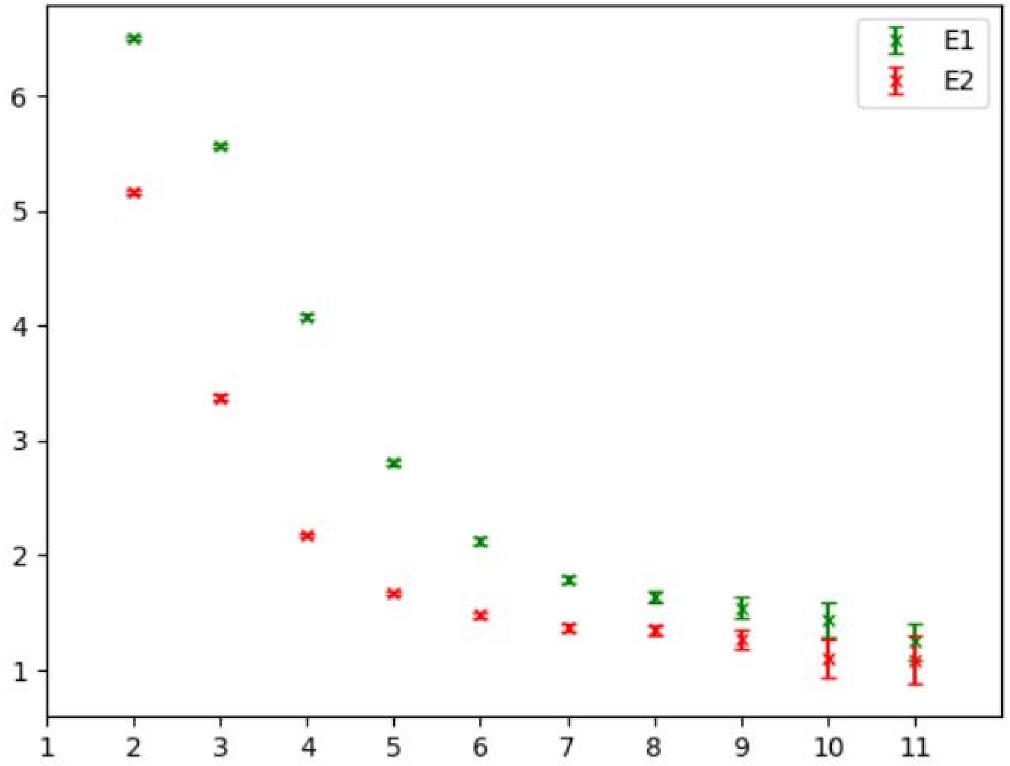
p-Λ 两点关联函数的变分分析

$$C = \begin{bmatrix} C\_HH & C\_HD \\ C\_HD & C\_DD \end{bmatrix}$$

Wall-source

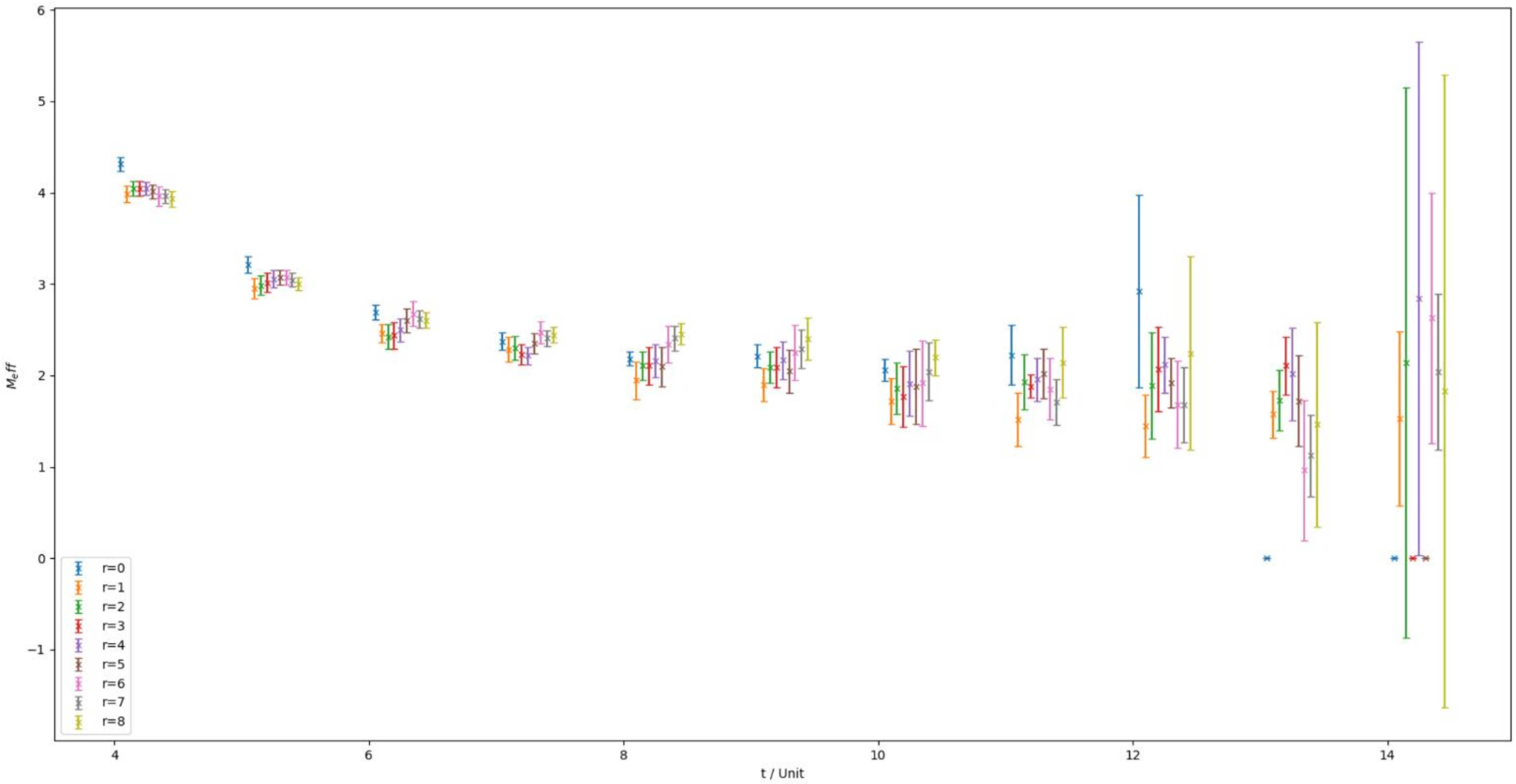


Point-source



$p-\Lambda$ scattering (Preliminary)

有效质量和 r 的关系



THANKS FOR ALL

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Sci. China-Phys. Mech. Astron. 67, 211011 (2024)