Tetraquark Production in high-energy collisions and B decays



第七届强子谱和强子结构研讨会@成都, 2024.04.27

Fu-Sheng Yu Lanzhou University



- 1. Introduction
- 2. Production of Tcc at LHC
- 4. Summary

Outline

3. Topological diagrams of Tetraquark productions in B decays

研究不系统,但希望给不同的视角和看法

Introduction

- Exotic states: Compact tetraquarks? Molecules? Kinematic effects? Charmonium?
- Observables: Mass spectrum? Decays? Productions?
- Productions are very important to explore the nature of exotic states, complementary to their masses and decays.
- •Question again:
- What are the quarks in discussion? Constituent quarks?
- •QCD is related to current quarks.
- Proton has three constituent quarks at the low energy.
- •But it has a lot of sea quarks and gluons at high energy scale, whose structure is described by Parton Distribution Function.







- The structures of hadrons might be different at low and high scales.
- Degrees of freedom are different at low and high scales.
- •Example 1: PDF of proton.
- Example 2: EMC effect in nuclear physics.
- •Nucleus are not only the naive sum of nucleons.
- •Nucleons are not the good enough degrees of freedom in the nucleus at high energy scattering.

Introduction





- The structures of hadrons might be different at low and high scales.
- Degrees of freedom are different at low and high scales.
- •Example 1: PDF of proton.
- Example 2: EMC effect in nuclear physics.
- Example 3: excited N* states
- •Meson-baryon cloud ->3 quark ->pQCD



Introduction





Thanks to Ming-Zhu Liu's discussion



- 1. Introduction
- 2. Production of Tcc at LHC
- 4. Summary

Outline

3. Topological diagrams of Tetraquark productions in B decays

2. Double-charm tetraquark



- •LHCb observed $T_{cc}^+(3875)$ in 2021.
- Its mass is very close to the threshold of $D^0 D^{*+}$
- •So it is widely considered as a DD^* molecular state.
- Production of Tcc at LHC as a molecular state? [Shi-Yuan Li, Qian Wang, Jia-Jun Wu,...]

•After the discovery of double-charm baryon, it is natural to study double-charm tetraquarks



LHCb, Nature Phys. 18, 751 (2022)

Theoretical predictions

Chinese Physics C Vol. 45, No. 10 (2021) 103106

Discovery potentials of double-charm tetraquarks*

Qin Qin(秦溱)^{1†} Yin-Fa Shen(沈胤发)¹ Fu-Sheng Yu(于福升)^{2,3,4‡}

We find that their production cross sections at the LHCb with $\sqrt{s} = 13$ TeV reach $\mathcal{O}(10^4)$ pb, which indicate that the LHCb has collected $\mathcal{O}(10^8)$ such particles. Through the decay channels of $T^{\{cc\}}_{[\bar{u}\bar{d}]} \rightarrow D^+K^-\pi^+$ or $D^0D^+\gamma$ (if stable) or $T^{\{cc\}}_{[\bar{u}\bar{d}]} \rightarrow D^0D^{*+}$ (if unstable), it is highly hopeful that they get discovered at the LHCb in the near future. We also discuss the productions and decays of the double-charm tetraquarks at future Tera-Z factories.

branching fractions of $T_{[\bar{u}\bar{d}]}^{\{cc\}}$ decays is the same as the observed Ξ_{cc}^{++} . Comparing with the production rates between double-charm tetraquarks and baryons, and considering around 2×10^3 events of Ξ_{cc}^{++} with the current LHCb data, the signal yields of $T_{[\bar{u}\bar{d}]}^{\{cc\}}$ would be $\mathcal{O}(10^2)$ at LHCb, and will reach $\mathcal{O}(10^3)$ at LHCb Run III. Thus it is hopefully expected that the double-charm tetraquark will be observed in the near future. Although the production rates are smaller at the future Z factories, it is also expected to be observed at the Tera-Z factories due to the smaller backgrounds.

Qin, Shen, **FSY**, 2008.08026





2. Productio

Two steps to produce cc-tetraquarks at

1. cc jet by pQCD

- cc quarks are produced collinearly
- $M_{cc-jet} < 2m_c + \Delta M$
- ΔM is determined by B_c meson production
- $\sigma(p+p \to H_{cc} + X) = (2.2^{+2.0}_{-0.6}) \times$

2. cc jet → fragmentation
into hadrons (
$$T_{cc}$$
, Ξ_{cc})

high energy of
$$\sqrt{s} = 13$$
 TeV
 $\vec{c} \quad \vec{c} \quad \vec{c$

Qin, Shen, **FSY**, 2008.08026

Fragmentation

- Different from ordinary heavy mesons and baryons with excited states decaying into the ground states,
- The excited cc-tetraquarks would directly decay into DD mesons, but not the ground states
- Primarily production *v.s.* final production
- The ground-state fragmentation fraction is



$$\frac{f_{\Lambda_c}}{f_{\Lambda_c + \Sigma_c + \Lambda_c^*}} = 0.48$$

Belle, arXiv:1706.06791 LHCb, 1902.06794



Results of production

Convoluting the cc-diquark jet and the fragmentation

$$\mathcal{B}(Z \to T^{\{cc\}}_{[\bar{u}\bar{d}]} + X) = (1.1^{+1.0}_{-0.4}) \times 10^{-6} \longrightarrow 10^{6} T^{\{cc\}}_{[\bar{u}\bar{d}]} \text{ for Tera-Z factory}$$

$$\sigma(pp \to T^{\{cc\}}_{[\bar{u}\bar{d}]} + X) = (1.5^{+0.7}_{-0.5}) \times 10^{4} \text{ pb} \longrightarrow 10^{8} T^{\{cc\}}_{[\bar{u}\bar{d}]} \text{ for LHCb 9 fb}^{-1}$$

Cross checking the results by double-charm baryon productions at LHC 13TeV

$$\sigma(\Xi_{cc}^{++}) = \sigma(\Xi_{cc}^{+}) \approx 47 \text{ nb} \longleftarrow$$

Qin, Shen, FSY, '20

$$\sigma(\Xi_{cc}^{++}) = \sigma(\Xi_{cc}^{+}) \approx 62 \text{ nb}$$

Chang, Qiao, Wang, Wu,

图像合理,没有可调参数

'06

Compared with $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$

Production

Decay
$$Br(\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+ \pi^+)$$

 $\sim Br(T_{cc} \rightarrow D^0 D^{*+}) Br(D^{*+} - 1/2)$
 $\longrightarrow Br(T_c)$

Fully reconstructed: $T_{cc}^+ \rightarrow D^0 D^{*+}$



1500 events of $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+ \xrightarrow{\times 1/16}$, 100 events of $T_{cc}^+ \rightarrow D^0 D^0 \pi^+$

Qin, Shen, **FSY**, 2008.08026



Further study on the structure of Tcc

• Molecule: 1) charmed mesons firstly, 2) then form Tcc at the hadron level



Jin, Li, Liu, Qin, Si, **FSY**, 2109.05678

- Fragmentation: quark level.
- To be tested: Production of iso-vector states is one order smaller than iso-scalar states

Qin, Shen, **FSY**, 2008.08026



2.8

mass (GeV/c²)

- 1. Introduction
- 2. Production of Tcc at LHC
- 4. Summary

Outline

3. Topological diagrams of Tetraquark productions in B decays

3. Exotic states observed in B decays

- to their masses and decays.
- •B decays are a good place for the production of exotic states.

$\Lambda_b \to \boldsymbol{J}/\boldsymbol{\psi}\boldsymbol{p}K$	$P_c(4312)/P_c(4440)/P_c(4457)$	$B^+ \rightarrow D_s^{+} \pi^+ D^-$	$T_{c\bar{s}0}(2900)^{++}$	
$\Xi_b \rightarrow J/\psi \Lambda K$ $P_{cs}(4459)$		$B^0 \rightarrow D_s^{\ +} \pi^- \overline{D}{}^0$	$T_{c\bar{s}0}(2900)^0$	
$B_s \rightarrow J/\psi p \bar{p}$ $P_c(4337)$		$B^+ \rightarrow D^- K^+ D^+$	$X_0(2900)/X_1(2900)$	
$B \rightarrow J/\psi \Lambda \bar{p} \qquad P_{cs}(4338)$		$B^+ \rightarrow D^0 \overline{D}^{*0} K^+$	X(3872)	
$B^0 \rightarrow J/\psi \pi^{\mp} K^{\pm}$	<i>Z_c</i> (4200)	$B^+ \rightarrow \boldsymbol{D_s}^+ \boldsymbol{D_s}^- K^+$	X(3960)	
$B^+ \rightarrow J/\psi \omega K^+$	X(3915)	$B^+ \rightarrow D_s^{+} \pi^0 \overline{D}{}^0$	D _{s0} (2317)	
$B^+ \rightarrow J/\psi\phi K^+$	$X(4140)/Z_{cs}(4000),$ etc.	$B^+ \rightarrow D_s^{*+} \pi^0 \overline{D}{}^0$	<i>D</i> _{s1} (2460)	
$B^0 \rightarrow \psi' \pi^{\mp} K^{\pm}$	Z _c (4430)	$\Lambda_b o \boldsymbol{D} \boldsymbol{p} \pi$	Λ _c (2940)	
$B^0 \rightarrow \chi_{c1} \pi^{\mp} K^{\pm}$	$Z_c(4051)/Z_c(4248)$			

which can help us get more information about tetraquarks?

• Productions are very important to explore the nature of exotic states, complementary

See Ming-Zhu Liu's talk in this afternoon

•Question: Does exist general theoretical method to study decays of B into tetraquarks,



Theoretical methods for hadronic weak decays

- QCD-factorization methods are most popular: QCDF, PQCD, SCET.
- They work for energetic final states. First-principle and very predictive.
- Successful for predictions on CPV of B decays.



- However, tetraquarks are usually heavy. Final states in B decays are not energetic enough. No factorization ansatz works.
- Topological diagrammatic approach is helpful to study decays of B into tetraquark.





Topological diagrams

- Decaying amplitudes are classified according to the weak flavour flows
- All the strong interaction effects are included.



Successful prediction on Ξ_{cc} discovery channel

Modes	$Br(first)(\times 10^{-3})$	$Br(final)(\times 10^{-3})$
$p(D^+/D^0\pi^+)$	<u>e</u> 8.	0.2
$p(D_s^+/D^0K^+)$	0.4	0.01
$(pK^{-}\pi^{+}/\Sigma^{+})(D^{+}/D^{0}\pi^{+})$	<u>e</u> 80.	2.
$(pK^{-}\pi^{+}/\Sigma^{+})(D_{s}^{+}/D^{0}K^{+})$	3.	0.1
$(\Lambda_c^+\pi^+)(\pi^+\pi^-)$	3.	0.2
$(\Lambda_c^+\pi^+)(K^+\pi^-)$	0.2	0.008
$(\Lambda_c^+\pi^+)(K^-\pi^+)$	<u>.</u> 50.	3.
$(\Lambda_c^+\pi^+)(K^+K^-)$	2.	0.08
$\Lambda_c^+\pi^+$	30.	1.
$\Lambda_c^+ K^+$	1.	0.06
$(\Lambda_c^+\pi^+K^-/\Xi_c^+)\pi^+$	<u> </u>	20.
$(\Lambda_c^+\pi^+K^-/\Xi_c^+)K^+$	20.	0.9
 discovery channel 	ls: Br=O(10 ^{-3~-}	4) Ξ_{cc}^{++} -
		34

Talk at LHCb in 2016.12



[FSY, Jiang, Li, Lu, Wang, Zhao, '17]



Topological diagrams of decays of B into tetraquarks



- No explicit values of decay amplitudes are given. But provide relations between decay channels.

Processes with identical topological diagrams share analogous branching fractions.



B decays into open-charm tetraquarks

Modes	Topological amplitudes	Experimental processes	E
$B^- \to T_{\bar{c}\bar{u}ds} D^{(*)+}$	$C V_{cb} V_{cs}^*$	$B^- \to D_s^{(*)-} \pi^- D^{(*)+}$ [32,33]	B
$\overline{B}^0 \to T_{\bar{c}\bar{d}us} D^{(*)0}$	$C V_{cb} V_{cs}^*$	$\overline{B}^0 \to D_s^{(*)-} \pi^+ D^{(*)0} $ [32,33]	\overline{B}
$B^- \rightarrow T_{\bar{c}\bar{u}ss} D_s^{(*)+}$	$\sqrt{2}C V_{cb}V_{cs}^*$	$B^- \rightarrow D_s^{(*)-} K^- D_s^{(*)+}$	
$\overline{B}^0 \to T_{\bar{c}\bar{d}ss} D_s^{(*)+}$	$\sqrt{2}C V_{cb}V_{cs}^*$	$\overline{B}^0 \to D_s^{(*)-} \overline{K}^0 D_s^{(*)+}$	
$B^- \rightarrow T_{cs\bar{u}\bar{d}} D^{(*)-}$	$T_3 V_{cb} V_{cs}^*$	$B^- \to D^{(*)+} K^- D^{(*)-}$ [30]	В
$\overline{B}^0 \to T_{cs\bar{u}\bar{d}}\overline{D}^{(*)0}$	$T_3 V_{cb} V_{cs}^*$	$\overline{B}^0 \to D^{(*)+} K^- \overline{D}^{(*)0}$	B
$B^- \to T_{cs\bar{u}\bar{u}}\overline{D}^{(*)0}$	$\sqrt{2}T_3 V_{cb}V_{cs}^*$	$B^- \rightarrow D^{(*)0} K^- \overline{D}^{(*)0}$	
$\overline{B}^0 \to T_{cs\bar{d}\bar{d}} D^{(*)-}$	$\sqrt{2}T_3 V_{cb}V_{cs}^*$	$\overline{B}^0 \to D^{(*)+} \overline{K}^0 D^{(*)-}$	
$\overline{B}_{s}^{0} \rightarrow T_{cd\bar{u}\bar{s}}\pi^{0}$	$\frac{1}{\sqrt{2}}(T_3 - T_2) V_{cb} V_{ud}^*$	$\overline{B}_s^0 \to D_s^{(*)+} \pi^- \pi^0$	\overline{B}
$\overline{B}^0_s \to T_{cd\bar{s}\bar{s}}K^-$	$\sqrt{2}T_3 V_{cb}V_{ud}^*$	$\overline{B}_s^0 \to D_s^{(*)+} K^0 K^-$	
$B^- \rightarrow T_{cd\bar{u}\bar{s}}K^-$	$(T_1 + T_3) V_{cb} V_{ud}^*$	$B^- \rightarrow D_s^{(*)+} \pi^- K^-$	B
$B^- \to T_{cd\bar{u}\bar{u}}\pi^0$	$(T_1 + T_3 - T_2) V_{cb} V_{ud}^*$	$B^- \rightarrow D^{(*)0} \pi^- \pi^0$	
$\overline{B}_s^0 \to T_{cd\bar{u}\bar{u}}K^+$	$\sqrt{2}T_1 V_{cb}V_{ud}^*$	$\overline{B}_s^0 \to D^{(*)0} \pi^- K^+$	
$\overline{B}_{s}^{0} \rightarrow T_{cd\bar{u}\bar{s}}\phi$	$T_1 V_{cb} V_{ud}^*$	$\overline{B}_{s}^{0} ightarrow D_{s}^{(*)+} \pi^{-} \phi$	B
$\overline{B}^0 \to T_{cd\bar{u}\bar{s}}\overline{K}^{(*)0}$	$(E+T_1) V_{cb} V_{ud}^*$	$\overline{B}^0 \to D_s^{(*)+} \pi^- \overline{K}^{(*)0}$	\overline{B}
$\overline{B}^0 \to T_{cd\bar{u}\bar{u}}\pi^+$	$\sqrt{2}(E+T_1) V_{cb} V_{ud}^*$	$\overline{B}^0 \rightarrow D^{(*)0} \pi^- \pi^+$	
$B^- \rightarrow T_{cs\bar{u}\bar{u}}K^{(*)0}$	$\sqrt{2}T_2 V_{cb}V_{ud}^*$	$B^- \to D^{(*)0} K^- K^{(*)0}$	
$\overline{B}^0 \to T_{cs\bar{u}\bar{d}}K^{(*)0}$	$(E+T_2) V_{cb} V_{ud}^*$	$\overline{B}^0 \to D^{(*)+} K^- K^{(*)0}$	B
$\overline{B}^0 \to T_{cs\bar{u}\bar{u}}K^+$	$\sqrt{2} E V_{cb} V_{ud}^*$	$\overline{B}^0 \to D^{(*)0} K^- K^+$	



B decays into open-charm tetraquarks

Modes	Topological amplitudes	Experimental processes	E
$B^- \to T_{\bar{c}\bar{u}ds} D^{(*)+}$	$C V_{cb} V_{cs}^*$	$B^- \to D_s^{(*)-} \pi^- D^{(*)+}$ [32,33]	В
$\overline{B}^0 \to T_{\bar{c}\bar{d}us} D^{(*)0}$	$C V_{cb} V_{cs}^*$	$\overline{B}^0 \to D_s^{(*)-} \pi^+ D^{(*)0} $ [32,33]	\overline{B}
$B^- \rightarrow T_{\bar{c}\bar{u}ss} D_s^{(*)+}$	$\sqrt{2}C V_{cb}V_{cs}^*$	$B^- \to D_s^{(*)-} K^- D_s^{(*)+}$	
$\overline{B}^0 \to T_{\bar{c}\bar{d}ss} D_s^{(*)+}$	$\sqrt{2}C V_{cb}V_{cs}^*$	$\overline{B}^0 \to D_s^{(*)-} \overline{K}^0 D_s^{(*)+}$	
$B^- \to T_{cs\bar{u}\bar{d}} D^{(*)-}$	$T_3 V_{cb} V_{cs}^*$	$B^- \to D^{(*)+} K^- D^{(*)-}$ [30]	B
$\overline{B}^0 \to T_{cs\bar{u}\bar{d}}\overline{D}^{(*)0}$	$T_3 V_{cb} V_{cs}^*$	$\overline{B}^0 \to D^{(*)+} K^- \overline{D}^{(*)0}$	\overline{B}
$B^- \to T_{cs\bar{u}\bar{u}}\overline{D}^{(*)0}$	$\sqrt{2}T_3 V_{cb}V_{cs}^*$	$B^- \rightarrow D^{(*)0} K^- \overline{D}^{(*)0}$	
$\overline{B}^0 \to T_{cs\bar{d}\bar{d}} D^{(*)-}$	$\sqrt{2}T_3 V_{cb}V_{cs}^*$	$\overline{B}^0 \to D^{(*)+} \overline{K}^0 D^{(*)-}$	
$\overline{B}_{s}^{0} \rightarrow T_{cd\bar{u}\bar{s}}\pi^{0}$	$\frac{1}{\sqrt{2}}(T_3 - T_2) V_{cb} V_{ud}^*$	$\overline{B}_s^0 \to D_s^{(*)+} \pi^- \pi^0$	\overline{B}
$\overline{B}_s^0 \to T_{cd\bar{s}\bar{s}}K^-$	$\sqrt{2}T_3 V_{cb}V_{ud}^*$	$\overline{B}_s^0 \to D_s^{(*)+} K^0 K^-$	
$B^- \to T_{cd\bar{u}\bar{s}}K^-$	$(T_1 + T_3) V_{cb} V_{ud}^*$	$B^- \rightarrow D_s^{(*)+} \pi^- K^-$	В
$B^- \to T_{cd\bar{u}\bar{u}}\pi^0$	$(T_1 + T_3 - T_2) V_{cb} V_{ud}^*$	$B^- \rightarrow D^{(*)0} \pi^- \pi^0$	
$\overline{B}_{s}^{0} \to T_{cd\bar{u}\bar{u}}K^{+}$	$\sqrt{2}T_1 V_{cb}V_{ud}^*$	$\overline{B}_s^0 \to D^{(*)0} \pi^- K^+$	
$\overline{B}_{s}^{0} \rightarrow T_{cd\bar{u}\bar{s}}\phi$	$T_1 V_{cb} V_{ud}^*$	$\overline{B}_{s}^{0} \rightarrow D_{s}^{(*)+} \pi^{-} \phi$	\overline{B}
$\overline{B}^0 \to T_{cd\bar{u}\bar{s}}\overline{K}^{(*)0}$	$(E+T_1) V_{cb} V_{ud}^*$	$\overline{B}^0 \to D_s^{(*)+} \pi^- \overline{K}^{(*)0}$	\overline{B}
$\overline{B}^0 \to T_{cd\bar{u}\bar{u}}\pi^+$	$\sqrt{2}(E+T_1) V_{cb} V_{ud}^*$	$\overline{B}^0 \rightarrow D^{(*)0} \pi^- \pi^+$	
$B^- \rightarrow T_{cs\bar{u}\bar{u}}K^{(*)0}$	$\sqrt{2}T_2 V_{cb}V_{ud}^*$	$B^- \to D^{(*)0} K^- K^{(*)0}$	
$\overline{B}^0 \to T_{cs\bar{u}\bar{d}}K^{(*)0}$	$(E+T_2) V_{cb} V_{ud}^*$	$\overline{B}^0 \to D^{(*)+} K^- K^{(*)0}$	\overline{B}
$\overline{B}^0 \to T_{cs\bar{u}\bar{u}}K^+$	$\sqrt{2} E V_{cb} V_{ud}^*$	$\overline{B}^0 \to D^{(*)0} K^- K^+$	



 $\bar{B}^0_{\rm s} \to D^{(*)0} K^0 \phi$ $\overline{B}^0 \to D^{(*)0} K^0 \overline{K}^{(*)0}$

 $\overline{B}^0 \to D^{(*)0} \overline{K}^0 K^{(*)0}$

- Suggest to search for all the processes with C or T_3 diagrams.
- It is helpful to explore the nature of observed $T^a_{c\bar{s}0}(2900)$ and $X_{0.1}(2900)$.
- It might be used to distinguish whether the exotic states are resonances or non-resonant kinematic effects.

 $B^- \to T_{cd\bar{u}\bar{s}}K^- \to D_s^+\pi^-K^-$

less resonances

21



Summary

- Productions are very important to investigate the nature of tetraquarks.
- •Double-charm tetraquark was predicted correctly for its production and decay, which was confirmed by LHCb's observation.
- Topological diagrammatic approach was proposed to the tetraquark production in Bmeson decays.
- Pictures at the high and low energy scales are different.

Thank you!

MB Contribution to electro-excitation of N(1535)1/2⁻

Is it a 3-quark state or a hadronic molecule?



Meson-baryon cloud may account for discrepancies at low Q².

Jefferson Lab QNP, September 5-9 2022 - Annalisa D'Angelo – Hadron Spectroscopy with CLAS and CLAS12-RG-K

N(1535)1/2⁻ is consistent with the 1st orbital excitation of the nucleon.



Excited Nucleon Structure

- •
- $(Q^2 < 5 \ GeV^2)$
- -High Q²: confinement to pQCD regime $(Q^2 > 5 GeV^2)$
- underlying quark mass distribution



Non-perturbative strong interaction

• Hadron spectrum: exotic XYZ states...

Chen, Chen, Liu, Zhu, '16; Esposito, Pilloni, Polosa, '16; Lebed, Mitchell, Swanson, '16; Guo, Hanhart, Meissner, Wang, Zhao, Zou, '17; Ali, Lange, Stone, '17; Olsen, Skwarnicki, Zieminska, '18 ...



- for example, $X_{0,1}(2900)$
- Unclear about their nature
 - Bound states & resonances:
 - compact tetraquarks
 - loosely molecular

- Kinematic effects:
 - triangle singularity
 - low energy production











Topological diagrams of decays of B into tetraquarks

Meson	Mode	Amplitudes	$\mathcal{B}_{ ext{exp}}(imes 10^{-5})$	$\mathcal{B}_{\mathrm{FAT}}(imes 10^{-5})$		
\overline{B}^0		$V_{cb}V_{ud}^{*}$				Br _{exp}
	$D_s^+K^-$	E $E \rightarrow E$	$3.45{\pm}0.32$	$3.0^{+0.4}_{-0.2}\pm 0.0\pm 0.3$	$B^- \to D^- X_0 (2900)^0$	$(1.23 \pm 0.41) \times 10^{-5}$
	$D_s^{*+}K^-$	$E \propto \chi^{E} e^{\imath \phi^{L}}$	2.19 ± 0.30	$2.2^{+0.3}_{-0.1}\pm 0.0\pm 0.3$	$B^- \rightarrow D^- X_1 (2900)^0$	$(6.73 \pm 2.26) \times 10^{-5}$
	$D_{s}^{+}K^{*-}$	E	3.5 ± 1.0	$3.8^{+0.5}_{-0.2}\pm 0.0\pm 0.6$	<i>D F D M</i> ₁ (2)00)	(0.75 ± 2.20) × 10
\overline{B}^0		$V_{cb}V_{us}^*$				
	$D^0 \overline{K}^0$	C	5.2 ± 0.7	$4.0 \pm 0.0 \pm 1.0 \pm 0.0$		\overline{c}
	$D^{*0}\overline{K}^0$	$C \propto \chi^C e^{i\phi^C}$	3.6 ± 1.2	$4.5^{+0.2}_{-0.3}\pm 0.9\pm 0.5$		D^{-}
	$D^0 \overline{K}^{*0}$	C	4.2 ± 0.6	$3.7 \pm 0.2 \pm 0.7 \pm 0.2$	5	
Meson	Mode	Amplitudes	$\mathcal{B}_{ ext{exp}}(imes 10^{-3})$	$\mathcal{B}_{ m FAT}(imes 10^{-3})$	Å	s
\overline{B}_{s}^{0}		$V_{cb}V_{ud}^{*}$				$rac{}{}$ $X_{0,1}(2900)$
	$D_s^+\pi^-$	T factorization	3.04 ± 0.23	$3.02\pm 0.00\pm 0.6\pm 0.01$	$B^ \bar{u}$	\bar{u}
	$D_s^{*+}\pi^-$	T	2.0 ± 0.5	$2.71 \pm 0.00 \pm 0.54 \pm 0.01$		$\overline{T_2}$
	$D_s^+ ho^-$	T	7.0 ± 1.5	$7.86 \pm 0.00 \pm 1.57 \pm 0.79$		- 3

[Zhou, Wei, Qin, Li, FSY, Lu, '15]

 Processes with identical topological diagrams share analogous branching fractions, regardless of what spins and flavor SU(3) representations of the final states.

[Chen, Han, Lu, Wang, **FSY**, '20]



B decays into hidden-charm tetraquarks

Modes	Topological amplitudes	Experimental processes	Experimental processes	
$B^- \to T_{c\bar{c}s\bar{u}}\phi$	$T_2 V_{cb} V_{cs}^*$	$B^- \rightarrow D^{(*)0} D_s^{(*)-} \phi$	$B^- \rightarrow J/\psi K^- \phi$ [29]	$) \rightarrow B^{-} \bigcirc \qquad \stackrel{b}{\longrightarrow} \bigcirc \qquad \stackrel{c}{\longrightarrow} \bigcirc \qquad 7$
$\overline{B}^0 \to T_{c\bar{c}s\bar{d}}\phi$	$T_2 V_{cb} V_{cs}^*$	$\overline{B}^0 \to D^{(*)+} D_s^{(*)-} \phi$	$\overline{B}^0 \to J/\psi \overline{K}^{(*)0} \phi$	\overline{u} \overline{u} \overline{u}
$\overline{B}^0 \to T_{c\bar{c}u\bar{d}}K^-$	$T_2 V_{cb} V_{cs}^*$	$\overline{B}^0 \to D^{(*)+} \overline{D}^{(*)0} K^-$	$\overline{B}^0 \rightarrow J/\psi \pi^+ K^-$ [24]	7 (4000) = 1 HCb 2021
$B^- \to T_{c\bar{c}d\bar{u}}\overline{K}^{(*)0}$	$T_2 V_{cb} V_{cs}^*$	$B^- \rightarrow D^{(*)0} D^{(*)-} \overline{K}^{(*)0}$	$B^- \to J/\psi \pi^- \overline{K}^{(*)0}$	$Z_{cs}(4000)$, LIICD 2021
$\overline{B}_{s}^{0} \to T_{c\bar{c}d\bar{s}}\overline{K}^{(*)0}$	$(T_2 + E) V_{cb} V_{cs}^*$	$\overline{B}_s^0 \to D_s^{(*)+} D^{(*)-} \overline{K}^{(*)0}$	$\overline{B}_s^0 \to J/\psi K^0 \overline{K}^{(*)0}$	
$\overline{B}_s^0 \to T_{c\bar{c}u\bar{s}}K^-$	$(T_2 + E) V_{cb} V_{cs}^*$	$\overline{B}_s^0 \to D_s^{(*)+} \overline{D}^{(*)0} K^-$	$\overline{B}_s^0 \to J/\psi K^+ K^-$	S
$\overline{B}_{s}^{0} \rightarrow T_{c\bar{c}u\bar{d}}\pi^{-}$	$E V_{cb}V_{cs}^*$	$\overline{B}_s^0 \to D^{(*)+} \overline{D}^{(*)0} \pi^-$	$\overline{B}^0_s \to J/\psi \pi^+ \pi^-$	\bar{u}
$\overline{B}_{s}^{0} \rightarrow T_{c\bar{c}d\bar{u}}\pi^{+}$	$E V_{cb}V_{cs}^*$	$\overline{B}^0_s \to D^{(*)0} D^{(*)-} \pi^+$	$\overline{B}^0_s \to J/\psi \pi^- \pi^+$	
$\overline{B}^0 \to T_{c\bar{c}s\bar{u}}\pi^+$	$T_1 V_{cb} V_{cs}^*$	$\overline{B}^0 \rightarrow D^{(*)0} D_s^{(*)-} \pi^+$	$\overline{B}^0 \to J/\psi K^- \pi^+$	\xrightarrow{b} \xrightarrow{c} \overline{c} T_c
$B^- \rightarrow T_{c\bar{c}s\bar{d}}\pi^-$	$T_1 V_{cb} V_{cs}^*$	$B^- \to D^{(*)+} D_s^{(*)-} \pi^-$	$B^- \rightarrow J/\psi \overline{K}^0 \pi^-$	\overline{B}^0 () \overline{A} \overline{A}
$B^- \rightarrow T_{c\bar{c}s\bar{u}}\pi^0$	$\frac{1}{\sqrt{2}}T_1 V_{cb}V_{cs}^*$	$B^- \to D^{(*)0} D_s^{(*)-} \pi^0$	$B^- \to J/\psi K^- \pi^0$	$\begin{array}{c} & a \\ & & T_2 \end{array}$
$\overline{B}^0 \to T_{c\bar{c}s\bar{d}}\pi^0$	$\frac{1}{\sqrt{2}}T_1 V_{cb}V_{cs}^*$	$\overline{B}^0 \rightarrow D^{(*)+} D_s^{(*)-} \pi^0$	$\overline{B}^0 \to J/\psi \overline{K}^0 \pi^0$	Z _c (4200) ⁺ , Belle 2014
$\overline{B}_s^0 \to T_{c\bar{c}s\bar{d}}K^{(*)0}$	$(T_1 + E) V_{cb} V_{cs}^*$	$\overline{B}_{s}^{0} \to D^{(*)+} D_{s}^{(*)-} K^{(*)0}$	$\overline{B}_s^0 \to J/\psi \overline{K}^0 K^{(*)0}$	
$\overline{B}_s^0 \to T_{c\bar{c}s\bar{u}}K^+$	$(T_1 + E) V_{cb} V_{cs}^*$	$\overline{B}_s^0 \to D^{(*)0} D_s^{(*)-} K^+$	$\overline{B}_s^0 \to J/\psi K^- K^+$	

[Qin, Qiu, **FSY**, '23]



S







B decays into hidden-charm tetraquarks

Modes	Topological amplitudes	Experimental processes	Experimental processes
$B^- \to T_{c\bar{c}s\bar{u}}\phi$	$T_2 V_{cb} V_{cs}^*$	$B^- \rightarrow D^{(*)0} D_s^{(*)-} \phi$	$B^- \rightarrow J/\psi K^- \phi$ [29]
$\overline{B}^0 \to T_{c\bar{c}s\bar{d}}\phi$	$T_2 V_{cb} V_{cs}^*$	$\overline{B}^0 ightarrow D^{(*)+} D^{(*)-}_s \phi$	$\overline{B}^0 \to J/\psi \overline{K}^{(*)0} \phi$
$\overline{B}^0 \to T_{c\bar{c}u\bar{d}}K^-$	$T_2 V_{cb} V_{cs}^*$	$\overline{B}^0 \to D^{(*)+} \overline{D}^{(*)0} K^-$	$\overline{B}^0 \rightarrow J/\psi \pi^+ K^-$ [24]
$B^- \to T_{c\bar{c}d\bar{u}}\overline{K}^{(*)0}$	$T_2 V_{cb} V_{cs}^*$	$B^- \rightarrow D^{(*)0} D^{(*)-} \overline{K}^{(*)0}$	$B^- \to J/\psi \pi^- \overline{K}^{(*)0}$
$\overline{B}_{s}^{0} \to T_{c\bar{c}d\bar{s}}\overline{K}^{(*)0}$	$(T_2 + E) V_{cb} V_{cs}^*$	$\overline{B}_s^0 \to D_s^{(*)+} D^{(*)-} \overline{K}^{(*)0}$	$\overline{B}_s^0 \to J/\psi K^0 \overline{K}^{(*)0}$
$\overline{B}_s^0 \to T_{c\bar{c}u\bar{s}}K^-$	$(T_2 + E) V_{cb} V_{cs}^*$	$\overline{B}_s^0 \to D_s^{(*)+} \overline{D}^{(*)0} K^-$	$\overline{B}_s^0 \to J/\psi K^+ K^-$
$\overline{B}_{s}^{0} \rightarrow T_{c\bar{c}u\bar{d}}\pi^{-}$	$E V_{cb}V_{cs}^*$	$\overline{B}_{s}^{0} \rightarrow D^{(*)+}\overline{D}^{(*)0}\pi^{-}$	$\overline{B}_{s}^{0} \rightarrow J/\psi \pi^{+}\pi^{-}$
$\overline{B}_{s}^{0} \rightarrow T_{c\bar{c}d\bar{u}}\pi^{+}$	$E V_{cb}V_{cs}^*$	$\overline{B}_s^0 \rightarrow D^{(*)0} D^{(*)-} \pi^+$	$\overline{B}_s^0 \to J/\psi \pi^- \pi^+$
$\overline{B}^0 \to T_{c\bar{c}s\bar{u}}\pi^+$	$T_1 V_{cb} V_{cs}^*$	$\overline{B}^0 \rightarrow D^{(*)0} D_s^{(*)-} \pi^+$	$\overline{B}^0 \to J/\psi K^- \pi^+$
$B^- \rightarrow T_{c\bar{c}s\bar{d}}\pi^-$	$T_1 V_{cb} V_{cs}^*$	$B^- \to D^{(*)+} D_s^{(*)-} \pi^-$	$B^- \rightarrow J/\psi \overline{K}^0 \pi^-$
$B^- \to T_{c\bar{c}s\bar{u}}\pi^0$	$\frac{1}{\sqrt{2}}T_1 V_{cb}V_{cs}^*$	$B^- \to D^{(*)0} D_s^{(*)-} \pi^0$	$B^- \rightarrow J/\psi K^- \pi^0$
$\overline{B}^0 \to T_{c\bar{c}s\bar{d}}\pi^0$	$\frac{1}{\sqrt{2}}T_1 V_{cb}V_{cs}^*$	$\overline{B}^0 \rightarrow D^{(*)+} D_s^{(*)-} \pi^0$	$\overline{B}^0 \to J/\psi \overline{K}^0 \pi^0$
$\overline{B}_s^0 \to T_{c\bar{c}s\bar{d}}K^{(*)0}$	$(T_1 + E) V_{cb} V_{cs}^*$	$\overline{B}_s^0 \to D^{(*)+} D_s^{(*)-} K^{(*)0}$	$\overline{B}_s^0 \to J/\psi \overline{K}^0 K^{(*)0}$
$\overline{B}_s^0 \to T_{c\bar{c}s\bar{u}}K^+$	$(T_1 + E) V_{cb} V_{cs}^*$	$\overline{B}_s^0 \to D^{(*)0} D_s^{(*)-} K^+$	$\overline{B}_s^0 \to J/\psi K^- K^+$

[Qin, Qiu, **FSY**, '23]

- Suggest to search for all the processes with T_2 diagrams.
- It is helpful to explore the nature of observed $Z_{cs}(4000)$ and $Z_{c}(4200)$.
- It might be used to distinguish whether the exotic states are resonances or non-resonant kinematic effects.

 $\overline{B}{}^0 \to T_{c\bar{c}u\bar{d}}K^- \quad v.s. \quad B^- \to T_{c\bar{c}d\bar{u}}\overline{K}^{*0}$







2. Double-charm tetraquark

• heavy diquark, light anti-diquark: easily understood, analogous to Λ_c and Ξ_{cc}



- high-energy direct production: not possible kinematic effects
- $bb\bar{u}\bar{d}$ and $bc\bar{u}\bar{d}$ have been extensively studied during the past few years many references... but difficult to be observed due to their hard productions and small decay rates Ali, Parkhomenko, Qin, Wang, '18
- $cc\bar{u}d$ is the lightest double-heavy tetraquark state, possible to be observed





If observed by directly produced at pp collision, it must be a resonance, but **not kinematic effect**

• $bb\bar{u}d$ and $bc\bar{u}d$ have been extensively studied during the past few years Eichten, Quigg, PRL2017; Karliner, Rosner, PRL2017; S.Q.Luo, K.Chen, X.Liu, S.L.Zhu, EPJC2017; and many other references... but difficult to be observed due to their hard productions and small decay rates

• $cc\bar{u}d$ is the lightest double-heavy tetraquark state, possible to be observed



cc-Tetraquark

Ali, Parkhomenko, Qin, Wang, 2018

- cc quarks are produced collinearly
- cc diquark jet requires a jet definition, such as the invariant mass

$$M_{cc-jet} < 2m_c + \Delta M$$

• The jet-resolution parameter ΔM is determined by B_c meson production, $b\bar{c} \rightarrow B_c$

$$\Delta M = \begin{cases} (2.0^{+0.5}_{-0.4}) \text{ GeV, for LHCb,} \\ (2.7^{+1.3}_{-0.5}) \text{ GeV, for Z factori} \end{cases}$$

The results of production rate

$$\sigma(p+p \to H_{cc} + X) = (2.2^{+2.0}_{-0.6})$$

 $\mathcal{B}(Z \to H_{cc} + X) = (3.0^{+2.7}_{-0.9}) \times 10^{-5}$



$) \times 10^5 \text{ pb}$ for LHC

for Tera-Z factory



Qin, FSY, '20

Successful prediction on charm CPV

Meson	Mode	Representation	\mathcal{B}_{exp} (%)	$\mathcal{B}_{ ext{fit}}$ (%)
$\overline{D^0}$	$\overline{K^-\pi^+}_{ar{m{ u}}^0=0}$	$V_{cs}^* V_{ud}(T+E)$	3.91 ± 0.08 2.28 ± 0.00	3.91 ± 0.17
	$ar{K}^0 \eta \ ar{K}^0 n'$	$V_{cs}^* V_{ud} \left[\frac{1}{\sqrt{2}} (C + E) \cos \phi - E \sin \phi \right]$ $V_{cs}^* V_{ud} \left[\frac{1}{\sqrt{2}} (C + E) \sin \phi + E \cos \phi \right]$	2.38 ± 0.09 0.96 ± 0.06 1.90 ± 0.11	$\begin{array}{r} 2.30 \pm 0.08 \\ 0.98 \pm 0.05 \\ 1.91 \pm 0.09 \end{array}$
D^+	$ar{K}^0\pi^+$	$V_{cs}^* V_{ud} (T+C)$	3.07 ± 0.10	3.08 ± 0.36
D_s^+	$ar{K}^0K^+ \ \pi^+\pi^0$	$V_{cs}^* V_{ud}(C+A)$	2.98 ± 0.17 < 0.037	2.97 ± 0.32 0
	$\pi^+\eta\\pi^+\eta'$	$V_{cs}^* V_{ud}(\sqrt{2}A\cos\phi - T\sin\phi)$ $V_{cs}^* V_{ud}(\sqrt{2}A\sin\phi + T\cos\phi)$	1.84 ± 0.15 3.95 ± 0.34	$ 1.82 \pm 0.32 \\ 3.82 \pm 0.36 $



 $C = (2.61 \pm 0.08)e^{-i(152 \pm 1)^{\circ}}$, $T = 3.14 \pm 0.06$, $A = (0.39^{+0.13}_{-0.09})e^{i(31^{+20}_{-33})^{\circ}}$ $E = (1.53^{+0.07}_{-0.08})e^{i(122\pm2)^{\circ}},$ Cheng, Chiang,'10

Tree diagrams are determined by data of branching fractions Understand the dynamics at 1GeV

Relate the penguins to the trees, with the known dynamics at 1GeV

32



Successful prediction on charm CPV



Saur, FSY, Sci.Bull.2020

$$\Delta A_{CP} = A_{CP}(D^0 \to K^+ K^-) - A_{CP}(D^0 \to \pi^+ \pi^-)$$

Th: the only predictions of O(10⁻³)

CC: topological approach + QCDF H.Y.Cheng, C.W.Chiang, 2012 LLY: factorization-assisted topology (FAT) H.n.Li, C.D.Lu, F.S.Yu, 2012

Exp: LHCb, PRL122, 211803 (2019)

Topological diagrammatic approach successfully predicted the charm CPV !!!

Relations between topological diagrams

- Hierarchy of topological diagrams in heavy quark expansion
 - SCET: $|C/T| \sim |C'/T| \sim |E/T| \sim O(\Lambda_{QCD}/m_Q)$ Leibovich, Ligeti, Stewart, Wise, 2004
 - charm decay: $IC/TI \sim IC'/TI \sim IE/TI \sim O(\Lambda_{QCD}/m_c) \sim 1$
- BESIII measurements on Λ_c^+ decays are very helpful



BESIII, 2016

Successful prediction on Ξ_{cc} discovery channel

- What decaying channel to search for double-charm baryons is an important problem before 2017.
- Predictions are required for largest branching fraction.

- Topological diagrammatic approach may help.
- Less data can be used to estimate the order of magnitude of the branching fractions







