

P -wave states T_{bb}^- from diquarks

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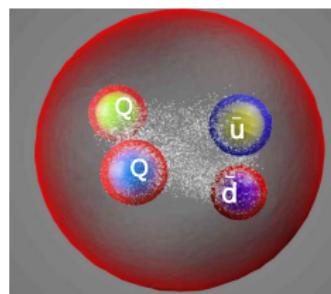
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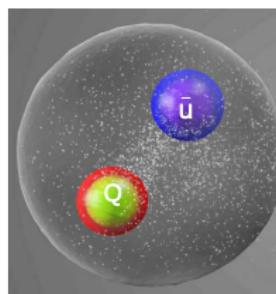
1. Background

- Theoretical explorations

- In the early 1980s, the states $QQ\bar{q}\bar{q}$ were pioneered
- Subsequently, various theoretical frameworks
 $T_{bb}^-(bb\bar{u}\bar{d})$ with 01^+ : the promising state.



Diquark configuration

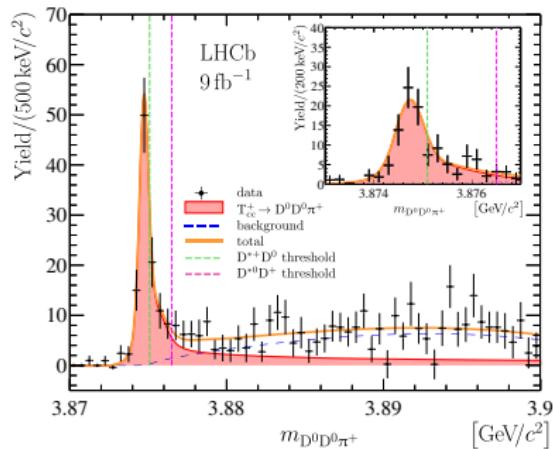


Meson-meson configuration

1. Background

- Discoveries in experiments

- In 2017, the doubly charmed baryon Ξ_{cc}^{++} [Phys. Rev. Lett. 119, 112001 \(2017\)](#)
- In 2021, the doubly charmed state T_{cc}^+ [Nature Commun. 13, 3351 \(2022\)](#)



Its binding energy and decay width are

$$E_b = -361 \pm 40 \text{ keV}, \Gamma = 47.8 \pm 1.9 \text{ keV}.$$

May be deuteronlike structure.

2. Quark model

- Model Hamiltonian

$$H_n = \sum_{i=1}^n \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - T_c + \sum_{i>j}^n (V_{ij}^{oge} + V_{ij}^{con} + V_{ij}^{obe} + V_{ij}^\sigma)$$

- One-gluon-exchange and quark confinement

$$V_{ij}^{oge} = \frac{\alpha_s}{4} \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \left(\frac{1}{r_{ij}} - \frac{2\pi\delta(\mathbf{r}_{ij})\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}{3m_i m_j} \right), \quad V_{ij}^{con} = -a_c \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c r_{ij}^2$$

- One Goldstone boson exchange

$$V_{ij}^{obe} = V_{ij}^\pi \sum_{k=1}^3 \mathbf{F}_i^k \mathbf{F}_j^k + V_{ij}^K \sum_{k=4}^7 \mathbf{F}_i^k \mathbf{F}_j^k + V_{ij}^\eta (\mathbf{F}_i^8 \mathbf{F}_j^8 \cos \theta_P - \sin \theta_P)$$

$$V_{ij}^\chi = \frac{g_{ch}^2}{4\pi} \frac{m_\chi^3}{12m_i m_j} \frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \left(Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} Y(\Lambda_\chi r_{ij}) \right), \quad \chi = \pi, K, \eta$$

- σ -meson exchange

$$V_{ij}^\sigma = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_\sigma^2 m_\sigma}{\Lambda_\sigma^2 - m_\sigma^2} \left(Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma} Y(\Lambda_\sigma r_{ij}) \right)$$

2. Quark model

- Meson spectrum and adjustable parameters [Phys. Rev. D 90, 054009 \(2014\)](#)

Mass unit in MeV and root-mean-square unit in fm.

State	D	D^*	D_s	D_s^*	\bar{B}	\bar{B}^*	\bar{B}_s	\bar{B}_s^*
Model prediction	1867	2002	1972	2140	5259	5301	5377	5430
PDG	1869	2007	1968	2112	5280	5325	5366	5416
$\langle r^2 \rangle^{\frac{1}{2}}$	0.68	0.82	0.52	0.69	0.73	0.77	0.57	0.62

Quark mass and Λ_0 unit in MeV, a_c unit in $\text{MeV}\cdot\text{fm}^{-2}$, r_0 unit in $\text{MeV}\cdot\text{fm}$ and α_0 is dimensionless.

Parameter	$m_{u,d}$	m_s	m_c	m_b	a_c	α_0	Λ_0	r_0
Value	280	512	1602	4936	40.78	4.55	9.17	35.06

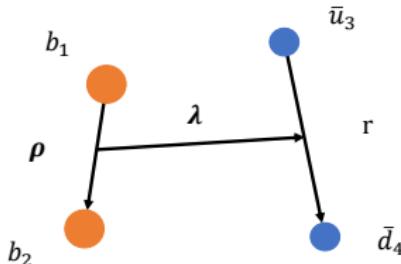
- Applied to the T_{cc}^+ , the model can match the experimental data well.

[C.R. Deng and S.L. Zhu, \$T_{cc}^+\$ and its partners, Phys. Rev. D 105, 054015 \(2022\);](#)

[C.R. Deng and S.L. Zhu, Decoding the double heavy tetraquark state \$T_{cc}^+\$, Science Bulletin 67, 1522](#)

3. Diquark configuration of T_{bb}^-

- Orbit of T_{bb}^-



$$\rho = \mathbf{r}_{b_1} - \mathbf{r}_{b_2}, \quad \mathbf{r} = \mathbf{r}_{\bar{u}_3} - \mathbf{r}_{\bar{d}_4}, \quad \lambda = \frac{\mathbf{r}_{b_1} + \mathbf{r}_{b_2}}{2} - \frac{\mathbf{r}_{\bar{u}_3} + \mathbf{r}_{\bar{d}_4}}{2}$$

- Gaussian expansion method

$$\phi_{l_x m_x}(\mathbf{x}) = \sum_{n_x=1}^{n_{x\max}} c_{n_x} N_{n_x l_x} x^{l_x} e^{-\nu_{n_x} x^2} Y_{l_x m_x}(\hat{\mathbf{x}}), \quad \mathbf{x} = \rho, \mathbf{r} \text{ and } \lambda$$

- Three P -wave excited modes

A ρ -mode, $l_\rho = 1$, $l_r = l_\lambda = 0$; B r -mode, $l_r = 1$, $l_\rho = l_\lambda = 0$;

C λ -mode, $l_\lambda = 1$, $l_r = l_\rho = 0$.

3. Diquark configuration of T_{bb}^-

- Color of T_{bb}^-

$$(\bar{\mathbf{3}}_{bb} \oplus \mathbf{6}_{bb}) \otimes (\mathbf{3}_{\bar{u}\bar{d}} \oplus \bar{\mathbf{6}}_{\bar{u}\bar{d}}) = \underbrace{(\bar{\mathbf{3}}_{bb} \otimes \mathbf{3}_{\bar{u}\bar{d}})}_{1 \oplus 8} \oplus \underbrace{(\bar{\mathbf{3}}_{bb} \otimes \bar{\mathbf{6}}_{\bar{u}\bar{d}})}_{8 \oplus \bar{10}} \oplus \underbrace{(\mathbf{6}_{bb} \otimes \mathbf{3}_{\bar{u}\bar{d}})}_{8 \oplus 10} \oplus \underbrace{(\mathbf{6}_{bb} \otimes \bar{\mathbf{6}}_{\bar{u}\bar{d}})}_{1 \oplus 8 \oplus 27}$$

- Only $\bar{\mathbf{3}} \otimes \mathbf{3} \rightarrow 1$ and $\mathbf{6} \otimes \bar{\mathbf{6}} \rightarrow 1$ are permitted. Color $\bar{\mathbf{3}}$ or $\mathbf{3}$: $\mathbf{c}_{bb} = \mathbf{c}_{\bar{u}\bar{d}} = 0$; Color $\mathbf{6}$ or $\bar{\mathbf{6}}$: $\mathbf{c}_{bb} = \mathbf{c}_{\bar{u}\bar{d}} = 1$.

- Spin of T_{bb}^-

- $s_{bb} = 0$ or 1 , $s_{\bar{u}\bar{d}} = 0$ or 1 ;
- Total spin $s = s_{bb} \oplus s_{\bar{u}\bar{d}}$;

$$s = \begin{cases} 0, & 1 \oplus 1 \text{ or } 0 \oplus 0 \\ 1, & 1 \oplus 1, 1 \oplus 0 \text{ or } 0 \oplus 1 \\ 2, & 1 \oplus 1 \end{cases}$$

- Isospin of T_{bb}^-

- $i_{bb} = 0$, $i_{\bar{u}\bar{d}} = 0$ or 1

3. Diquark configuration of T_{bb}^-

- Constraints due to the Pauli principle

$$[bb] : \mathbf{s}_{bb} + \mathbf{c}_{bb} + \mathbf{i}_{bb} + \mathbf{l}_\rho = \text{odd}$$

$$[\bar{u}\bar{d}] : \mathbf{s}_{\bar{u}\bar{d}} + \mathbf{c}_{\bar{u}\bar{d}} + \mathbf{i}_{\bar{u}\bar{d}} + \mathbf{l}_r = \text{even}$$

- Total wavefunction

$$\Phi_{IJ}^{T_{bb}^-} = \sum_{\alpha} c_{\alpha} \left[\Psi_{i_1 j_1 c_1 l_{\rho}}^{[bb]} \Psi_{i_2 j_2 c_2 l_r}^{[\bar{u}\bar{d}]} \phi_{l_{\lambda} m_{\lambda}}(\boldsymbol{\lambda}) \right]_{IJ}^{T_{bb}^-}$$

- Solving the four-body Schrödinger equation

$$(H_4 - E_4) \Phi_{IJ}^{T_{bb}^-} = 0$$

in the orbit-color-spin-isospin space composed of 1280 bases.

4. Natures of diquarks and correlations

- Average values of each interaction in the s-wave T_{bb}^-

$$\langle V_{ij} \rangle = \langle \Phi_{IJ}^{T_{bb}^-} | V_{ij} | \Phi_{IJ}^{T_{bb}^-} \rangle$$

$n^{2S+1}L_J$	Parts	T	V^{con}	V^{cm}	V^{coul}	V^η	V^π	V^σ	Parts	T	V^{con}	V^{cm}	V^{coul}	V^η	V^π	V^σ
1^3S_1	$[bb]_{\bar{3}}^1$	124	16	1	-199	0	0	0	$[bb]_{\bar{6}}^0$	51	-19	1	59	0	0	0
	$[\bar{u}\bar{d}]_{\bar{3}}^0$	789	55	-289	-257	57	-335	-40	$[\bar{u}\bar{d}]_{\bar{6}}^1$	249	-70	-10	68	-2	20	-14
	$[bb]_{\bar{3}}^1 - [\bar{u}\bar{d}]_{\bar{3}}^0$	210	124	-2	-340	0	0	0	$[bb]_{\bar{6}}^0 - [\bar{u}\bar{d}]_{\bar{6}}^1$	303	372	0	-764	0	0	0

- Natures of diquarks in s-wave

- Good diquark $[bb]$: $\mathbf{s}_{bb} = 1$, $\mathbf{c}_{bb} = 0$ ($\bar{3}_c$), $\mathbf{l}_\rho = 0$;
Coulomb interaction
- Good antidiquark $(\bar{u}\bar{d})$: $\mathbf{s}_{\bar{u}\bar{d}} = 0$, $\mathbf{c}_{\bar{u}\bar{d}} = 0$ (3_c), $\mathbf{l}_r = 0$, and $\mathbf{i}_{\bar{u}\bar{d}} = 0$;
Color magnetic, Coulomb, π -meson exchange
- Stronger correlation in the configuration $[bb]_{\bar{6}}^0 - [\bar{u}\bar{d}]_{\bar{6}}^1$.

4. Natures of diquarks and correlations

- Average values of each interaction in the P-wave T_{bb}^- with λ -mode

$n^{2S+1}L_J$	Parts	T	V^{con}	V^{cm}	V^{coul}	V^η	V^π	V^σ	Parts	T	V^{con}	V^{cm}	V^{coul}	V^η	V^π	V^σ
1^3S_1	$[bb]_3^1$	124	16	1	-199	0	0	0	$[bb]_6^0$	51	-19	1	59	0	0	0
	$[\bar{u}\bar{d}]_3^0$	789	55	-289	-257	57	-335	-40	$[\bar{u}\bar{d}]_6^1$	249	-70	-10	68	-2	20	-14
	$[bb]_3^1 - [\bar{u}\bar{d}]_3^0$	210	124	-2	-340	0	0	0	$[bb]_6^0 - [\bar{u}\bar{d}]_6^1$	303	372	0	-764	0	0	0
$1^3P_{0,1,2}^\lambda$	$[bb]_3^1$	115	18	1	-193	0	0	0	$[bb]_6^0$	41	-24	1	52	0	0	0
	$[\bar{u}\bar{d}]_3^0$	739	59	-273	-250	53	-316	-38	$[\bar{u}\bar{d}]_6^1$	203	-85	-7	60	-1	14	-11
	$[bb]_3^1 - [\bar{u}\bar{d}]_3^0$	282	216	-1	-232	0	0	0	$[bb]_6^0 - [\bar{u}\bar{d}]_6^1$	449	552	0	-604	0	0	0

- Natures of $[bb]$ and $[\bar{u}\bar{d}]$ are unchanged.
- Strong correlation becomes weak in the configuration $[bb]_6^0 - [\bar{u}\bar{d}]_6^1$.

4. Natures of diquarks and correlations

- Average values of each interaction in the P-wave T_{bb}^- with ρ -mode

$n^{2S+1}L_J$	Parts	T	v^{con}	v^{cm}	v^{coul}	v^η	v^π	v^σ	Parts	T	v^{con}	v^{cm}	v^{coul}	v^η	v^π	v^σ
1^3S_1	$[bb]_3^1$	124	16	1	-199	0	0	0	$[bb]_6^0$	51	-19	1	59	0	0	0
	$[\bar{u}\bar{d}]_3^0$	789	55	-289	-257	57	-335	-40	$[\bar{u}\bar{d}]_6^1$	249	-70	-10	68	-2	20	-14
	$[bb]_3^1 - [\bar{u}\bar{d}]_3^0$	210	124	-2	-340	0	0	0	$[bb]_6^0 - [\bar{u}\bar{d}]_6^1$	303	372	0	-764	0	0	0
$1^1P_1^\rho$	$[bb]_3^0$	138	39	0	-107	0	0	0	$[bb]_6^1$	97	-28	-7	44	0	0	0
	$[\bar{u}\bar{d}]_3^0$	775	55	-283	-253	56	-328	-40	$[\bar{u}\bar{d}]_6^1$	248	-70	-10	68	-2	20	-14
	$[bb]_3^0 - [\bar{u}\bar{d}]_3^0$	200	140	-4	-320	0	0	0	$[bb]_6^1 - [\bar{u}\bar{d}]_6^1$	302	396	-24	-740	0	0	0

- P-wave $[bb]_3^0$ is a good diquark, Coulomb interaction.
- $[\bar{u}\bar{d}]$ and the correlation between $[bb]$ and $[\bar{u}\bar{d}]$ just change a little bit.

4. Natures of diquarks and correlations

- Average values of each interaction in the P-wave T_{bb}^- with r -mode

$n^{2S+1}L_J$	Parts	T	V^{con}	V^{cm}	V^{coul}	V^η	V^π	V^σ	Parts	T	V^{con}	V^{cm}	V^{coul}	V^η	V^π	V^σ
1^3S_1	$[bb]\frac{1}{3}$	124	16	1	-199	0	0	0	$[bb]\frac{0}{6}$	51	-19	1	59	0	0	0
	$[\bar{u}\bar{d}]^0\frac{0}{3}$	789	55	-289	-257	57	-335	-40	$[\bar{u}\bar{d}]\frac{1}{6}$	249	-70	-10	68	-2	20	-14
	$[bb]\frac{1}{3}-[\bar{u}\bar{d}]^0\frac{0}{3}$	210	124	-2	-340	0	0	0	$[bb]\frac{0}{6}-[\bar{u}\bar{d}]\frac{1}{6}$	303	372	0	-764	0	0	0
$1^1P_1^r$	$[bb]\frac{1}{3}$	116	17	1	-195	0	0	0	$[bb]\frac{0}{6}$	41	-24	0	50	0	0	0
	$[\bar{u}\bar{d}]\frac{1}{3}$	435	218	4	-93	1	2	-5	$[\bar{u}\bar{d}]\frac{0}{6}$	375	-125	4	42	-1	-5	-4
	$[bb]\frac{1}{3}-[\bar{u}\bar{d}]\frac{1}{3}$	165	228	-6	-240	0	0	0	$[bb]\frac{0}{6}-[\bar{u}\bar{d}]\frac{0}{6}$	257	552	-2	-596	0	0	0

- P-wave $[\bar{u}\bar{d}]^0\frac{0}{6}$ is a good diquark, good \Rightarrow bad, bad \Rightarrow good.
- P-wave excitation in $[\bar{u}\bar{d}]$ dramatically changes the correlation between $[bb]$ and $[\bar{u}\bar{d}]$.

5. Bound states and binding mechanism

- Binding energy

$$\Delta E = E_4 - M_{\bar{B}^{(*)}} - M_{\bar{B}^{(*)}}$$

- Contribution from each interaction

$$\Delta \langle V_{ij} \rangle = \langle \Phi_{IJ}^{T_{bb}^-} | V_{ij} | \Phi_{IJ}^{T_{bb}^-} \rangle - \langle \Phi(\bar{B}^{(*)}\bar{B}^{(*)}) | V_{ij} | \Phi(\bar{B}^{(*)}\bar{B}^{(*)}) \rangle$$

- S-wave state T_{bb}^- , threshold $\bar{B}\bar{B}^*$

$n^{2S+1}L_J$	Color-spin, ratio	ΔE	ΔT	ΔV^{con}	ΔV^{cm}	ΔV^{coul}	ΔV^η	ΔV^π	ΔV^σ	$\langle \rho^2 \rangle^{\frac{1}{2}}$	$\langle r^2 \rangle^{\frac{1}{2}}$	$\langle \lambda^2 \rangle^{\frac{1}{2}}$
1^3S_1	$[bb]_{\frac{1}{3}}^1 [\bar{u}\bar{d}]_{\frac{0}{3}}^0, >99\%$	-215	479	-50	-259	-67	57	-335	-40	0.39	0.71	0.64
	$[bb]_{\frac{0}{6}}^0 [\bar{u}\bar{d}]_{\frac{1}{6}}^1, <1\%$	119	-38	40	94	19	-2	20	-14	0.60	1.13	0.53
	Mixing	-216	481	-51	-260	-68	57	-335	-40	0.39	0.71	0.64

- Compact bound state, $\Delta E = -216$ MeV.
- Binding mechanism
chromomagnetic and π -meson-exchange in the $[\bar{u}\bar{d}]_{\frac{0}{3}}^0$.
- Dominant configuration $[bb]_{\frac{1}{3}}^1 [\bar{u}\bar{d}]_{\frac{0}{3}}^0$.

5. Bound states and binding mechanism

- P-wave T_{bb}^- with $1^3P_{0,1,2}^\lambda$, no spin-orbit coupling, thresholds $\bar{B}\bar{B}^*$

$n^{2S+1}L_J$	Color-spin, ratio	ΔE	ΔT	ΔV^{con}	ΔV^{cm}	ΔV^{coul}	ΔV^η	ΔV^π	ΔV^σ	$\langle \rho^2 \rangle^{\frac{1}{2}}$	$\langle r^2 \rangle^{\frac{1}{2}}$	$\langle \lambda^2 \rangle^{\frac{1}{2}}$
$1^3P_{0,1,2}^\lambda$	$[bb]_3^1[\bar{u}\bar{d}]_3^0, > 99\%$	55	494	49	-243	56	53	-316	-38	0.40	0.74	0.91
	$[bb]_6^0[\bar{u}\bar{d}]_6^1, < 1\%$	511	51	199	22	237	-1	14	-11	0.67	1.25	0.72
	Mixing	55	494	49	-243	56	53	-316	-38	0.40	0.74	0.90

- No bound state, P-wave excitation between $[bb]$ and $[\bar{u}\bar{d}]$
- Dominant configuration $[bb]_3^1[\bar{u}\bar{d}]_3^0$
- Introduce spin-orbit interaction

5. Bound states and binding mechanism

- P-wave T_{bb}^- with $1^3P_1^\rho$, threshold $\bar{B}\bar{B}$

$n^{2S+1}L_J$	Color-spin, ratio	ΔE	ΔT	ΔV^{con}	ΔV^{cm}	ΔV^{coul}	ΔV^η	ΔV^π	ΔV^σ	$\langle \rho^2 \rangle^{\frac{1}{2}}$	$\langle r^2 \rangle^{\frac{1}{2}}$	$\langle \lambda^2 \rangle^{\frac{1}{2}}$
$1^1P_1^\rho$	$[bb]_3^0[\bar{u}\bar{d}]_3^0, >99\%$	-18	409	7	-210	91	56	-331	-40	0.60	0.72	0.65
	$[bb]_6^1[\bar{u}\bar{d}]_6^1, <1\%$	203	-53	69	39	144	-2	20	-14	0.72	1.13	0.53
	Mixing	-18	412	6	-213	89	56	-328	-40	0.60	0.72	0.65

- Compact bound state, $\Delta E = -18$ MeV.
- Binding mechanism
chromomagnetic and π -meson-exchange interaction in the $[\bar{u}\bar{d}]_3^0$.
- Dominant configuration $[bb]_3^0[\bar{u}\bar{d}]_3^0$.

5. Bound states and binding mechanism

- P-wave T_{bb}^- with $1^1P_1^r$, threshold $\bar{B}\bar{B}$

$n^{2S+1}L_J$	Color-spin, ratio	ΔE	ΔT	ΔV^{con}	ΔV^{cm}	ΔV^{coul}	ΔV^η	ΔV^π	ΔV^σ	$\langle \rho^2 \rangle^{\frac{1}{2}}$	$\langle r^2 \rangle^{\frac{1}{2}}$	$\langle \lambda^2 \rangle^{\frac{1}{2}}$
$1^1P_1^r$	$[bb]_{\frac{1}{2}}^1[\bar{u}\bar{d}]_{\frac{1}{2}}^1, < 1\%$	562	15	233	74	243	1	2	-5	0.40	1.42	0.71
	$[bb]_{\frac{6}{2}}^0[\bar{u}\bar{d}]_{\frac{6}{2}}^0, > 99\%$	480	-30	176	80	265	-1	-5	-4	0.67	1.53	0.57
	Mixing	480	-28	176	78	265	-1	-5	-4	0.67	1.53	0.57

- No bound state.
Strong chromomagnetic and π -meson-exchange interactions disappear in $[\bar{u}\bar{d}]$.
- Dominant configuration $[bb]_{\frac{6}{2}}^0[\bar{u}\bar{d}]_{\frac{6}{2}}^0$.

6. Summary

- Summary

- Good diquark $[bb]$ and good antiquark $[\bar{u}\bar{d}]$

S-wave: $[bb]_{\bar{3}}^1, [\bar{u}\bar{d}]_3^0$; P-wave: $[bb]_{\bar{3}}^0, [\bar{u}\bar{d}]_6^1$

- Diquark correlation in $[bb]_6[\bar{u}\bar{d}]_{\bar{6}}$ is stronger than $[bb]_{\bar{3}}[\bar{u}\bar{d}]_3$

Sometimes, the color configuration $[bb]_6[\bar{u}\bar{d}]_{\bar{6}}$ is dominant.

- Compact P-wave bound state T_{bb}^- with 00^-

Binding energy is about 18 MeV, orbit excitation occurs in the diquark $[bb]$.

Anti-diquark $[\bar{u}\bar{d}]_6^0$ is responsible for its binding mechanism.

Thank you for your attention!