The study on three-body systems $\eta K^* \bar{K}^*$, $\pi K^* \bar{K}^*$ and $\mathrm{K K^* \bar{K}^*}$ by the Faddeev fixed-center approximation

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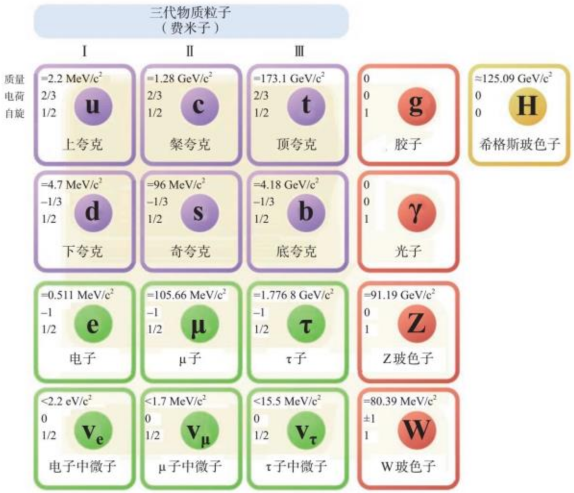
Part 1

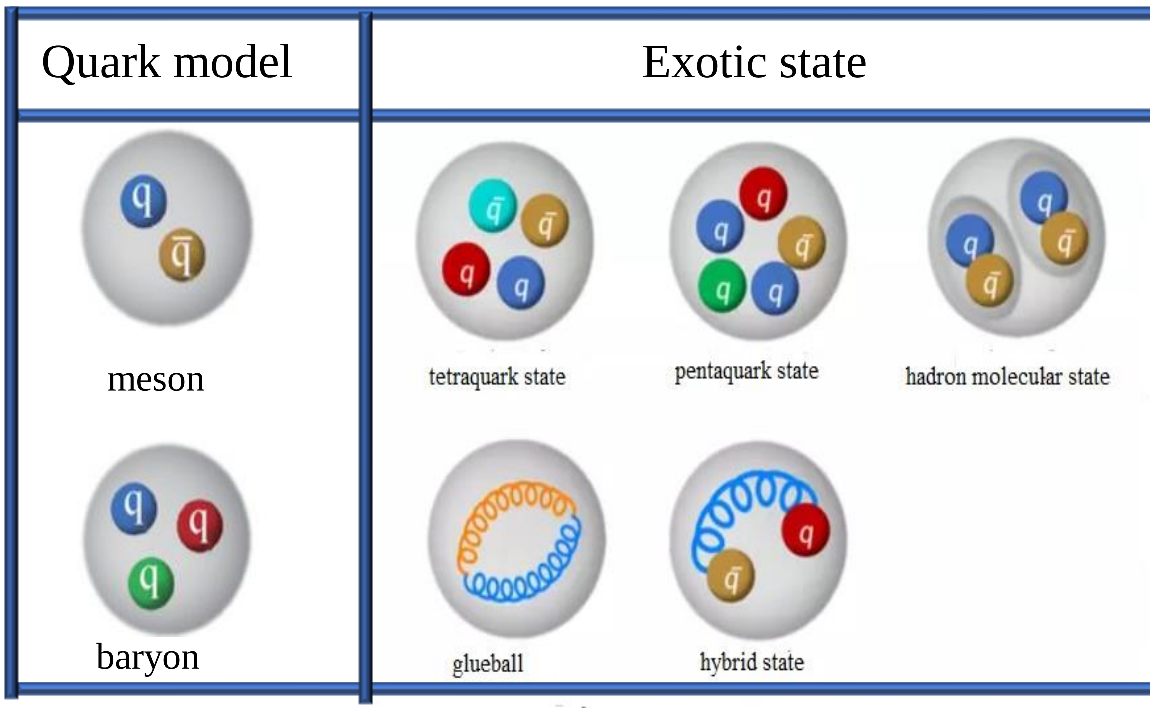
Introduction

- Background
- Three-body systems

Introduction

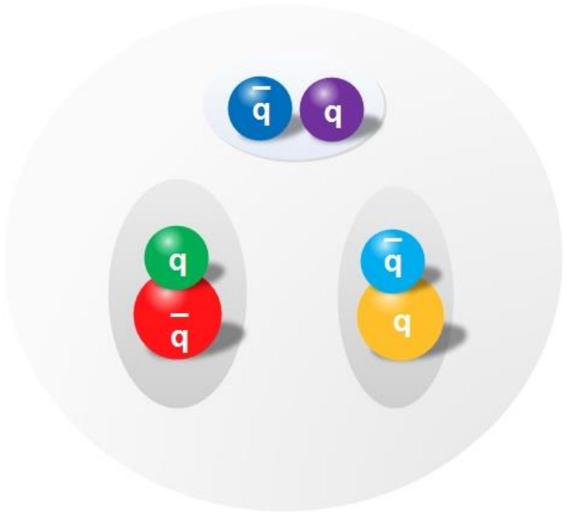
Background





Introduction

Three-body systems



$\pi K\bar{K} - \pi(1300)$ Phys. Rev. D 84 (2011) 7, 074027

$KK\bar{K} - K(1460)$ Phys. Rev. C 83 (2011) 6, 065205

$\eta K\bar{K} - \eta(1475)$ Phys. Rev. D 88 (2013) 11, 114024

$\rho K\bar{K} - \rho(1700)$ Eur. Phys. J. A 50 (2014), 67

$\phi K\bar{K} - \phi(2175)$ Phys. Rev. D 78 (2008) 7, 074031

$J/\Psi K\bar{K} - Y(4120)$ Phys. Rev. D 80 (2009) 9, 094012

$\pi\bar{K}K^* - \pi_1(1600)$ Phys. Rev. D 95 (2017) 5, 056014

$\eta\bar{K}K^*$ Phys. Rev. D 95 (2017) 5, 056014

……

We are interested in three-body system: $\eta K^*\bar{K}^*, \pi K^*\bar{K}^*, KK^*\bar{K}^*$

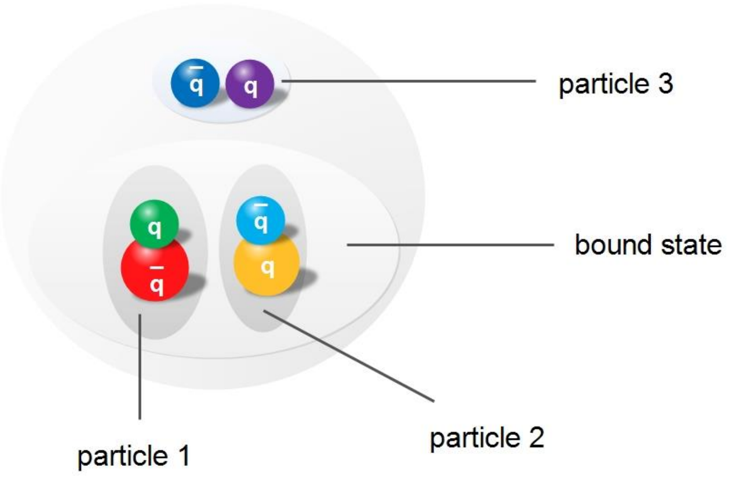
Part 2

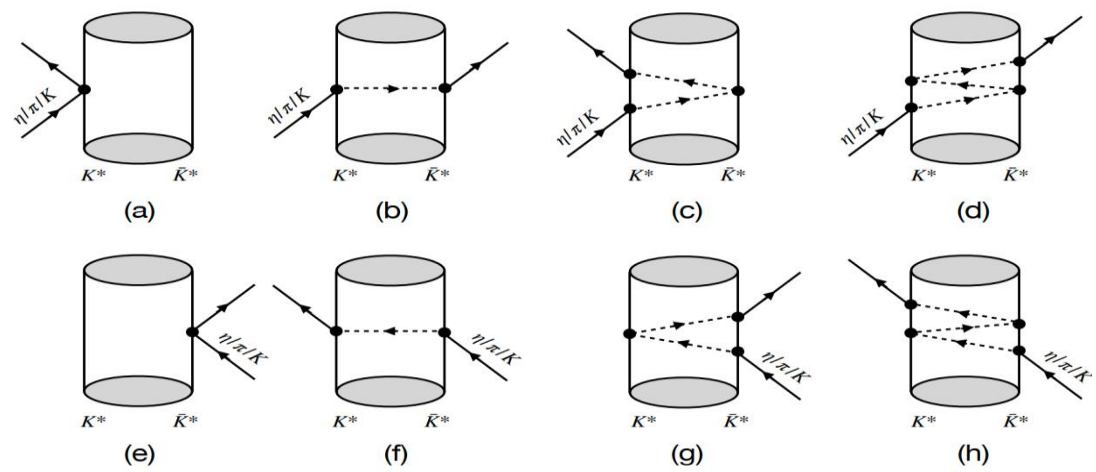
Formalism

- Fixed center approximation to Faddeev equations (FCA)
- Fixed center: vector-vector meson molecular state (V-V interaction)
- The interaction of vector mesons and pseudoscalar mesons

Formalism

Fixed center approximation to Faddeev equations (FCA)





$$T_1 = \tilde{t}_1 + \tilde{t}_1 G_0 T_2$$

$$T_2 = \tilde{t}_2 + \tilde{t}_2 G_0 T_1$$

$$T = T_1 + T_2$$

$$\Rightarrow \quad T = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1 \tilde{t}_2 G_0}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2}$$

$$T_{B_1} = FFS_1 \cdot \tilde{t}_1 + \tilde{t}_1 G_0 \tilde{t}_2 = (FFS_1 - 1)\tilde{t}_1 + T_1$$

$$T_{B_2} = (FFS_2 - 1)\tilde{t}_2 + T_2$$

$$T = T_{B_1} + T_{B_2}$$

$$\Rightarrow \quad T = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1 \tilde{t}_2 G_0}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2} + (FFS_1 - 1)\tilde{t}_1 + (FFS_2 - 1)\tilde{t}_2$$

Formalism

Fixed center approximation to Faddeev equations (FCA)

where

$$\tilde{t}_1 = \frac{M_B}{m_1} t_1$$

$$\tilde{t}_2 = \frac{M_B}{m_2} t_2$$

single-scattering form factor

$$FFS_1 = \frac{1}{2}\int_{-1}^{1} F_B(k_1) d(\cos\theta)$$

$$FFS_2 = \frac{1}{2}\int_{-1}^{1} F_B(k_2) d(\cos\theta)$$

$$k_1 = \frac{m_2}{m_1 + m_2} k\sqrt{2(1-\cos\theta)}$$

$$k_2 = \frac{m_1}{m_1 + m_2} k\sqrt{2(1-\cos\theta)}$$

$$k = \begin{cases} \dfrac{\sqrt{(s-(M_B+m_3)^2)(s-(M_B-m_3)^2)}}{2\sqrt{s}} & \sqrt{s} \geqslant M_B + m_3 \\ 0 & \sqrt{s} < M_B + m_3 \end{cases}$$

The loop function

$$G_0 = \frac{1}{2M_B}\int \frac{d^3\vec{q}}{(2\pi)^3} F_B(q) \frac{1}{q^{0^2} - \vec{q}^{\,2} - m^2 + i\varepsilon}$$

The form factor

$$F_B(q) = \frac{1}{N}\int_{\substack{|\vec{p}|<\Lambda \\ |\vec{p}-\vec{q}|<\Lambda}} d^3\vec{p}\, \frac{1}{M_B - w_1(\vec{p}) - w_2(\vec{p})} \frac{1}{m_{cls} - w_1(\vec{p}-\vec{q}) - w_2(\vec{p}-\vec{q})} \frac{1}{2w_1(\vec{p})} \frac{1}{2w_2(\vec{p})} \frac{1}{2w_1(\vec{p}-\vec{q})} \frac{1}{2w_2(\vec{p}-\vec{q})}$$

$$N = \int_{|\vec{p}|<\Lambda} d^3\vec{p} \left(\frac{1}{2w_1(\vec{p})} \frac{1}{2w_2(\vec{p})} \frac{1}{M_B - w_1(\vec{p}) - w_2(\vec{p})} \right)^2$$

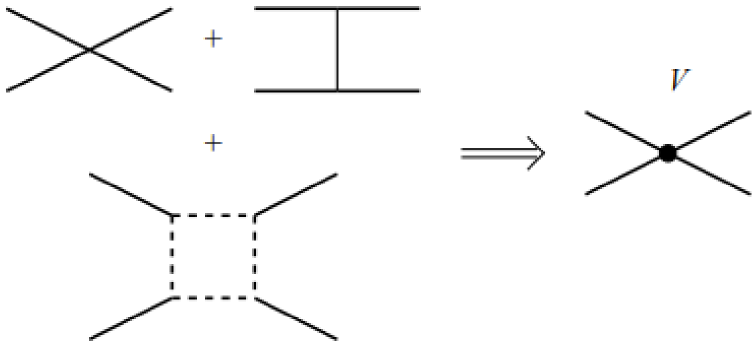
Formalism

Fixed center: vector-vector meson molecular state （V-V interaction）

L. S. Geng and E. Oset. Phys. Rev. D 79 (2009) 7, 074009

Hidden-gauge Lagrangian

$$\mathcal{L} = -\frac{1}{4}\left\langle \bar{V}_{uv}\bar{V}^{uv}\right\rangle + \frac{1}{2}M_v^2\left\langle\left[V_u - \frac{i}{g}\Gamma_u\right]^2\right\rangle$$



where

$$\bar{V}_{uv} = \partial_u V_v - \partial_v V_u - ig[V_u, V_v]$$

$$\Gamma_u = \frac{1}{2}\{u^+[\partial_u - i(v_u + a_u)]u + u[\partial_u - i(v_u - a_u)]u^+\}$$

$$u^2 = U = e^{\frac{i\sqrt{2}P}{f}} \qquad g = \frac{M_V}{2f}$$

$$\mathcal{L}_{VVVV} = \frac{1}{2}g^2\langle [V_u, V_v]V^u V^v\rangle$$

$$\mathcal{L}_{VVV} = ig\langle (V^u\partial_v V_u - \partial_v V_u V^u)V^v\rangle$$

$$\mathcal{L}_{VPP} = -ig\langle V_u[P, \partial^u P]\rangle$$

$$V_u = \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega-\pi^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \Phi \end{pmatrix}_u \quad P = \begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

Formalism

Fixed center: vector-vector meson molecular state （V-V interaction）

Three-body system: $\eta K^* \bar{K}^*, \pi K^* \bar{K}^*, K K^* \bar{K}^*$

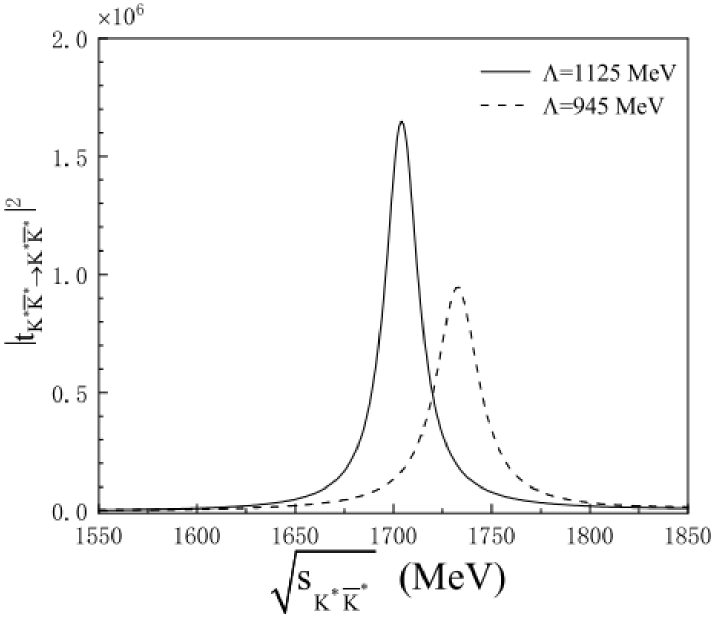
Fixed center: $K^* \bar{K}^*$

$I^G(J^{PC})$	Theory			PDG data		
	Pole position	Real axis $\Lambda_b = 1.4$ GeV	Real axis $\Lambda_b = 1.5$ GeV	Name	Mass	Width
$0^+(0^{++})$	(1512,51)	(1523,257)	(1517,396)	$f_0(1370)$	1200~1500	200~500
$0^+(0^{++})$	(1726,28)	(1721,133)	(1717,151)	$f_0(1710)$	1724 ± 7	137 ± 8
$0^-(1^{+-})$	(1802,78)	(1802,49)		h_1		
$0^+(2^{++})$	(1275,2)	(1276,97)	(1275,111)	$f_2(1270)$	1275.1 ± 1.2	$185.0^{+2.9}_{-2.4}$
$0^+(2^{++})$	(1525,6)	(1525,45)	(1525,51)	$f_2'(1525)$	1525 ± 5	73^{+6}_{-5}
$1^-(0^{++})$	(1780,133)	(1777,148)	(1777,172)	a_0		
$1^+(1^{+-})$	(1679,235)	(1703,188)		b_1		
$1^-(2^{++})$	(1569,32)	(1567,47)	(1566,51)	$a_2(1700)$??		
$1/2(0^+)$	(1643,47)	(1639,139)	(1637,162)	K_0^*		
$1/2(1^+)$	(1737,165)	(1743,126)		$K_1(1650)$?		
$1/2(2^+)$	(1431,1)	(1431,56)	(1431,63)	$K_2^*(1430)$	1429 ± 1.4	104 ± 4

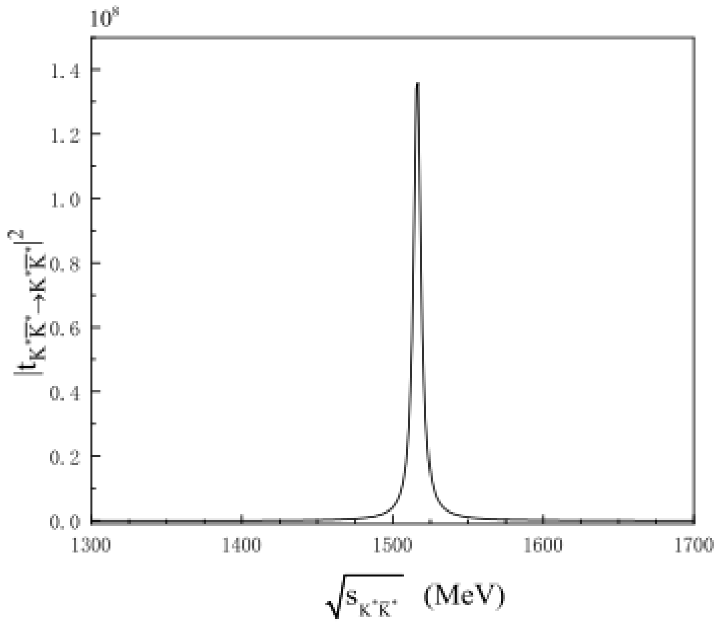
Formalism

Fixed center: vector-vector meson molecular state （V-V interaction）

$f_0(1710)$



$f_2'(1525)$



PDG

Mass (eva.)	1704 MeV ($\Lambda = 1125$ MeV)
Mass (ave.)	1733 MeV ($\Lambda = 945$ MeV)
Width	150 MeV

PDG

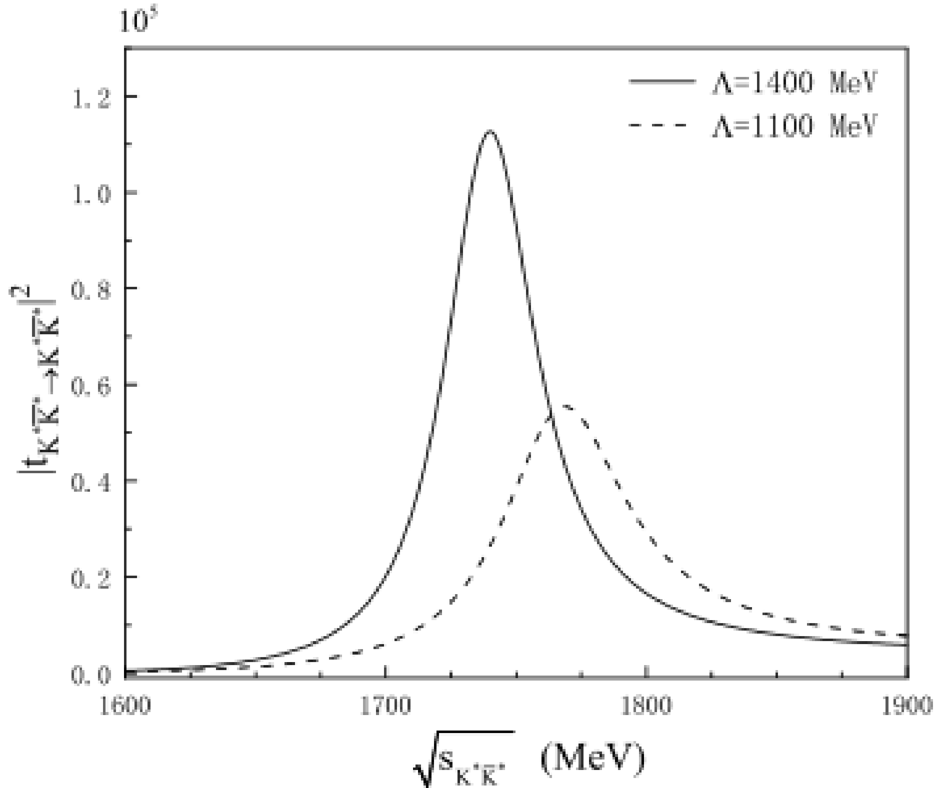
Mass	1517 MeV ($\Lambda = 1009$ MeV)
Width	86 MeV

Formalism

Fixed center: vector-vector meson molecular state （V-V interaction）

$a_0(1710)$

1769 MeV ($\Lambda = 1100$ MeV)	
1740 MeV ($\Lambda = 1400$ MeV)	



PDG

Mass	$1704 \pm 5 \pm 2$ MeV	BABR	2021
	$1817 \pm 8 \pm 20$ MeV	BES3	2022
Width	106 ± 15 MeV		

Formalism

The interaction between vector mesons and pseudoscalar mesons

L. Roca, E. Oset and J. Singh. Phys. Rev. D 72 (2005) 1, 014002

$$\mathcal{L} = -\frac{1}{4} Tr\{(\nabla_u V_v - \nabla_v V_u)(\nabla^u V^v - \nabla^v V^u)\}$$

where

$$\nabla_u V_v = \partial_u V_v + \left[\tilde{\Gamma}_u, \ V_v\right] \qquad \tilde{\Gamma}_u = \frac{1}{2}(u^+ \partial_u u + u \partial_u u^+) \qquad u^2 = U = e^{\frac{i\sqrt{2}P}{f}}$$

$$V_u = \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega-\pi^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \Phi \end{pmatrix}_u \quad P = \begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

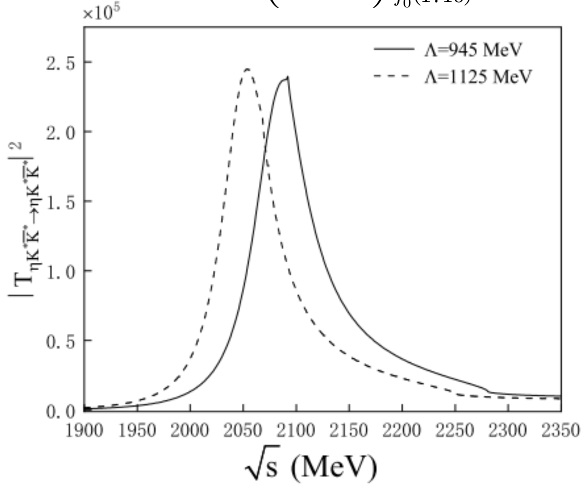
$$\mathcal{L}_{VVPP} = -\frac{1}{4f^2} Tr([V^u, \partial^v V_u][P, \partial_v P])$$

Part 3

Results

Results

$\eta - \left(K^*\bar{K}^*\right)_{f_0(1710)}$



Our work

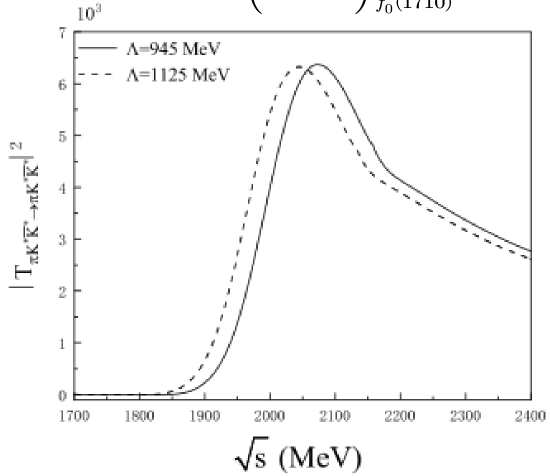
$\Lambda = 945$ MeV	2092 MeV
$\Lambda = 1125$ MeV	2054 MeV

PDG

further state: $\eta(2100)$

$2050^{\pm 36 \pm 181}_{-30 - 164}$ MeV	2016	BES3	[1]
2103 ± 50 MeV	1989	DM2	[2]

$\pi - \left(K^*\bar{K}^*\right)_{f_0(1710)}$



Our work

$\Lambda = 945$ MeV	2073 MeV
$\Lambda = 1125$ MeV	2031 MeV

PDG

further state: $\pi(2070)$

2075 ± 35 MeV	2001	Anisovich et al	[3]

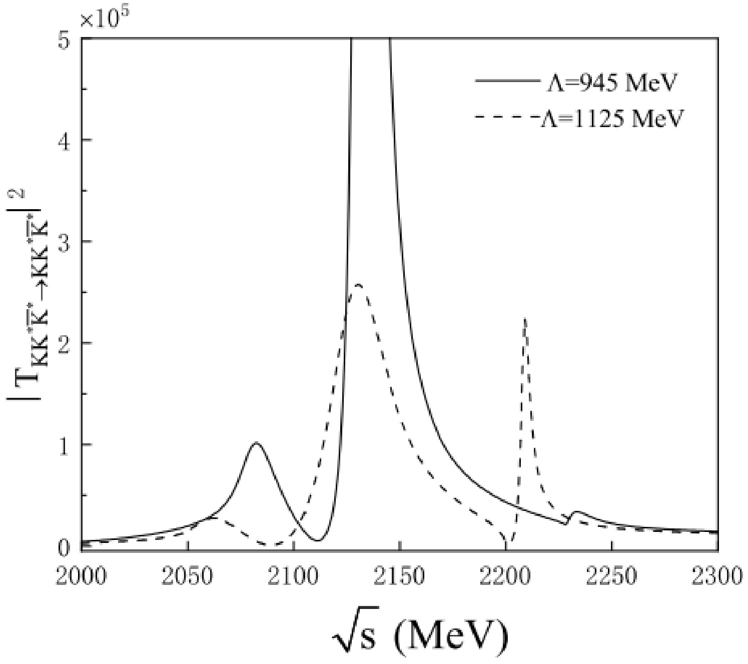
[1] *BESIII Collaboration*. Phys. Rev. D 93 (2016) 11, 112011.

[2] *DM2 Collaboration*. Phys. Rev. D 39 (1989), 701.

[3] Anisovich et al. Phys. Lett. B 517 (2001), 261-272.

Results

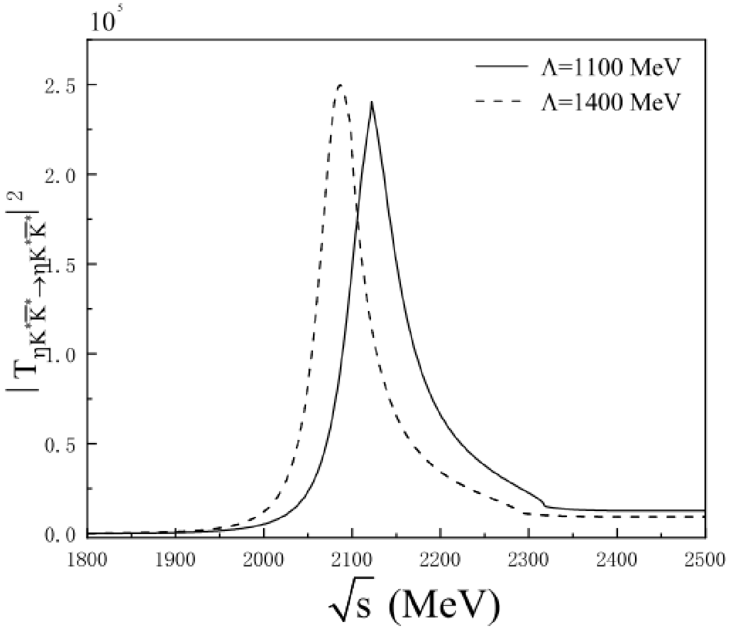
$K-\left(K^{*}\bar{K}^{*}\right)_{f_0(1710)}$



Our work

	State* 1	State 2	State*3
$\Lambda = 945$ MeV	2083 MeV	2134 MeV	2233 MeV
$\Lambda = 1125$ MeV	2062 MeV	2130 MeV	2209 MeV

$\eta-\left(K^{*}\bar{K}^{*}\right)_{a_0(1710)}$



Our work

$\Lambda = 1100$ MeV	2122 MeV
$\Lambda = 1400$ MeV	2086 MeV

PDG

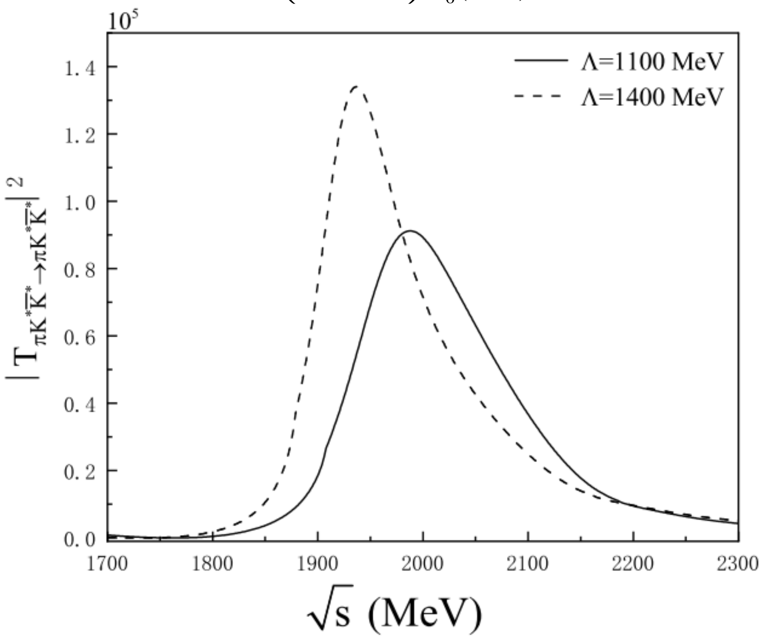
further state: $\pi(2070)$

2075 ± 35 MeV	2001	Anisovich et al	[1]

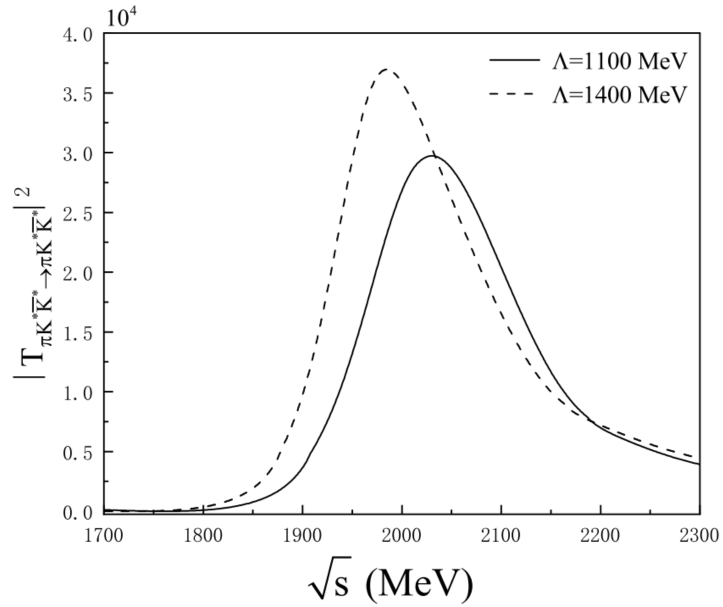
[1] Anisovich et al. Phys. Lett. B 517 (2001), 261-272.

Results

$\pi-\left(K^*\bar{K}^*\right)_{a_0(1710)}\quad I=0$



$\pi-\left(K^*\bar{K}^*\right)_{a_0(1710)}\quad I=1$



Our work

	$I=0$	$I=1$
$\Lambda=1100$ MeV	1988 MeV	2030 MeV
$\Lambda=1400$ MeV	1937 MeV	1997 MeV

PDG
further state: $\eta(2010)$

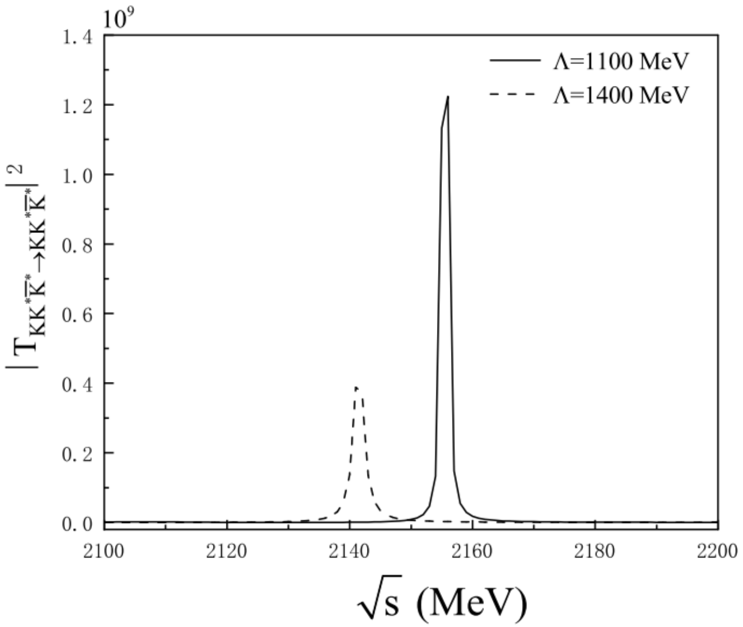
2010^{+35}_{-60} MeV	2000	Anisovich et al	[1]

PDG
further state: $\pi(2070)$

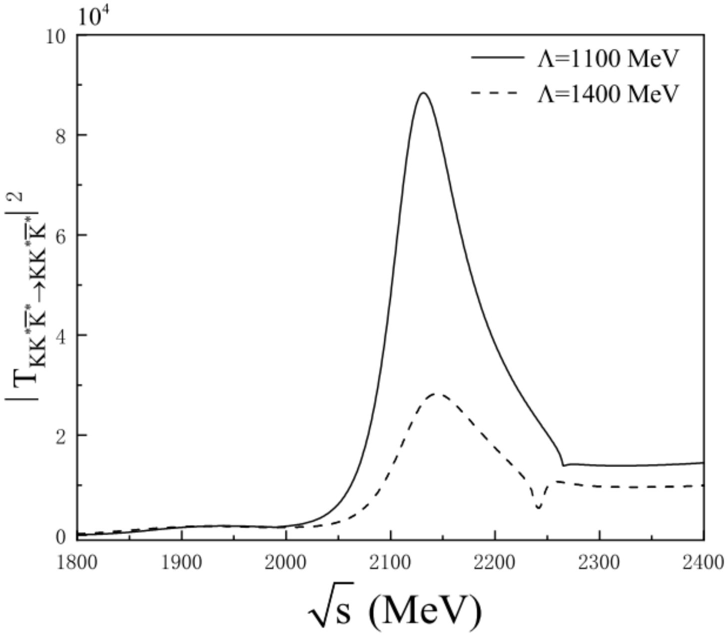
2075 ± 35 MeV	2001	Anisovich et al	[2]

[1] Anisovich et al. Phys. Lett. B 491 (2000), 47-58.
[2] Anisovich et al. Phys. Lett. B 517 (2001), 261-272

Results



$$K-\left(K^*\bar{K}^*\right)_{a_0(1710)} \quad I=\frac{1}{2}$$



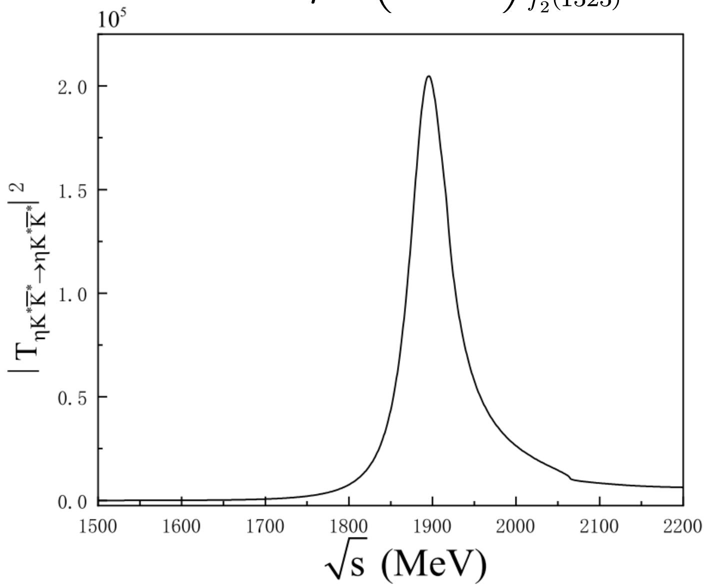
$$K-\left(K^*\bar{K}^*\right)_{a_0(1710)} \quad I=\frac{3}{2}$$

Our work

	$I=\frac{1}{2}$	$I=\frac{3}{2}$
$\Lambda=1100$ MeV	2156 MeV	2145 MeV
$\Lambda=1400$ MeV	2132 MeV	2138 MeV

Results

$\eta - \left(K^*\bar{K}^*\right)_{f_2'(1525)}$

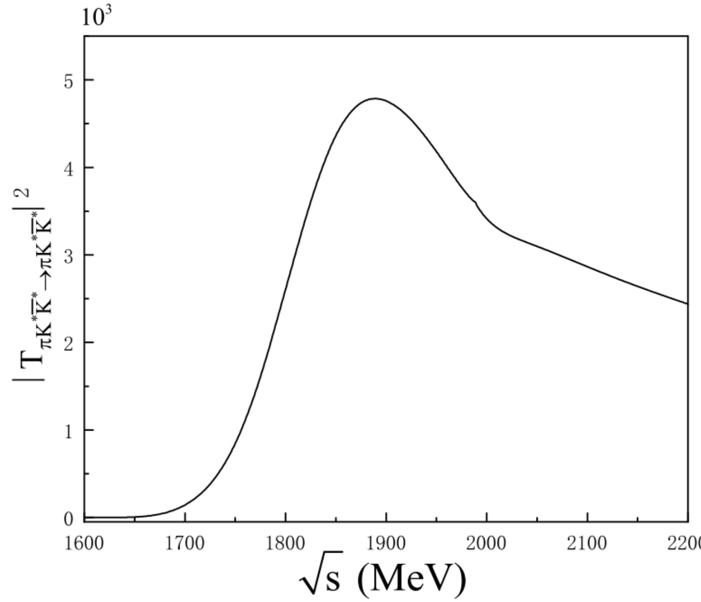


$\eta_2(1870)$

Unit in MeV.

Model	Our work	Hybrid State[1]	$s\bar{s}(^1D_2)$		$n\bar{n}(2^1D_2)$		PDG (ave.)
			GI[2]	VFV [3]	GI[2]	VFV [3]	
Mass	1896	1900	1890	1853	2130	1863	1842±8

$\pi - \left(K^*\bar{K}^*\right)_{f_2'(1525)}$



$\pi_2(1880)$

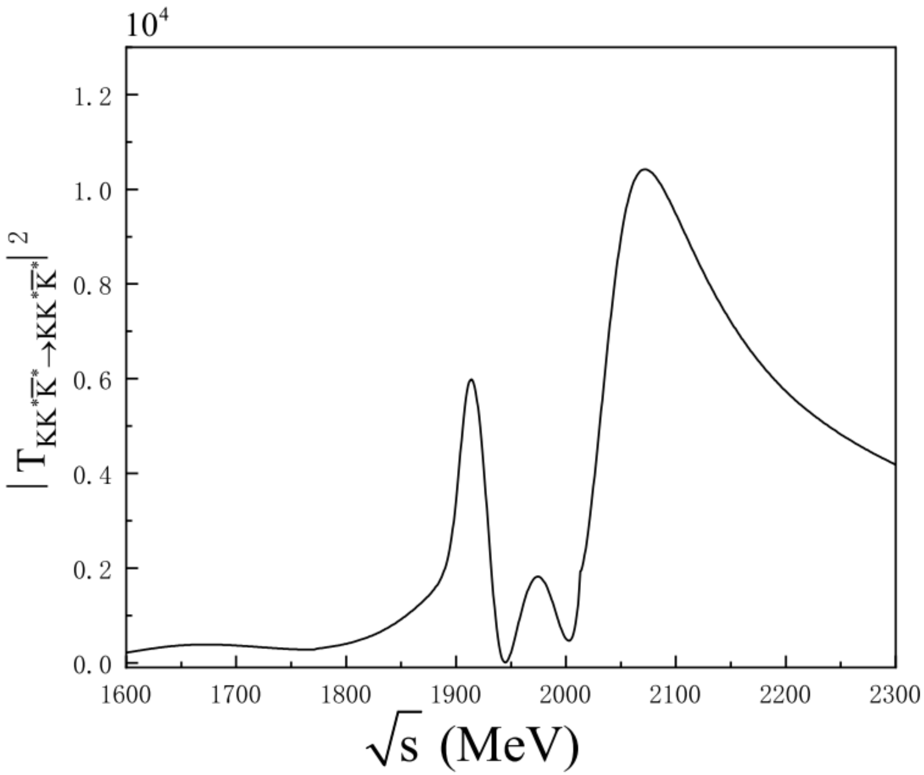
Unit in MeV.

Model	Our work	Hybrid state [4]	$q\bar{q}(2^1D_2)$ [2]	PDG
Mass	1889	1800-1900	2130	1879^{+26}_{-5}

[1] N. Isgur and J. E. Paton. Phys. Rev. D 31 (1985), 2910
[2] S. Godfrey and N. Isgur. Phys. Rev. D 32 (1985), 189.
[3] J. Vijande, F. Fernandez, and A. Valcarce. J. Phys. G 31 (2005), 481.
[4] T. Barnes, F. E. Close, and E. S. Swanson. Phys. Rev. D 52 (1995), 5242

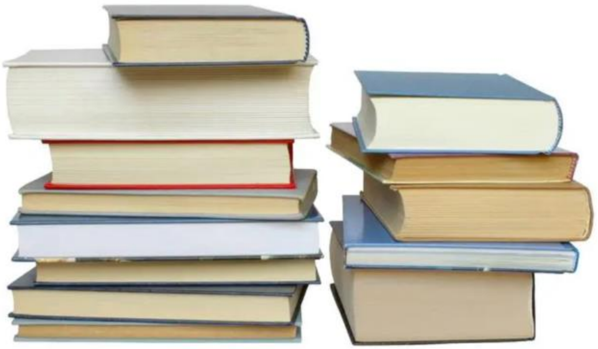
Results

$$K-\left(K^{*}\bar{K}^{*}\right)_{f_2^{'}(1525)}$$



Unit in MeV.

	State* 1	State* 2	State* 3
Mass	1914	1975	2072



Part 4

Summary

Summary

$\eta K^* \bar{K}^*$	$0^+(0^{-+})$	$\eta(2100)$
	$1^-(0^{-+})$	$\pi(2070)$
	$0^+(2^{-+})$	$\eta_2(1870)$

$\pi K^* \bar{K}^*$	$0^+(0^{-+})$	$\eta(2010)$
	$1^-(0^{-+})$	$\pi(2070)$
	$1^-(2^{-+})$	$\pi_2(1880)$

$K K^* \bar{K}^*$	$\frac{1}{2}(0^-)$	2080? 2150? 2230?
	$\frac{3}{2}(0^-)$	2140 ?
	$\frac{1}{2}(2^-)$	1910? 1980? 2070?

THANKS