



**The study on three-body systems $\eta K^* \bar{K}^*$, $\pi K^* \bar{K}^*$ and $KK^* \bar{K}^*$
by the Faddeev fixed-center approximation**

Phys. Rev. D 107 (2023) 3, 034019

Phys. Rev. D 109 (2024) 1, 014012

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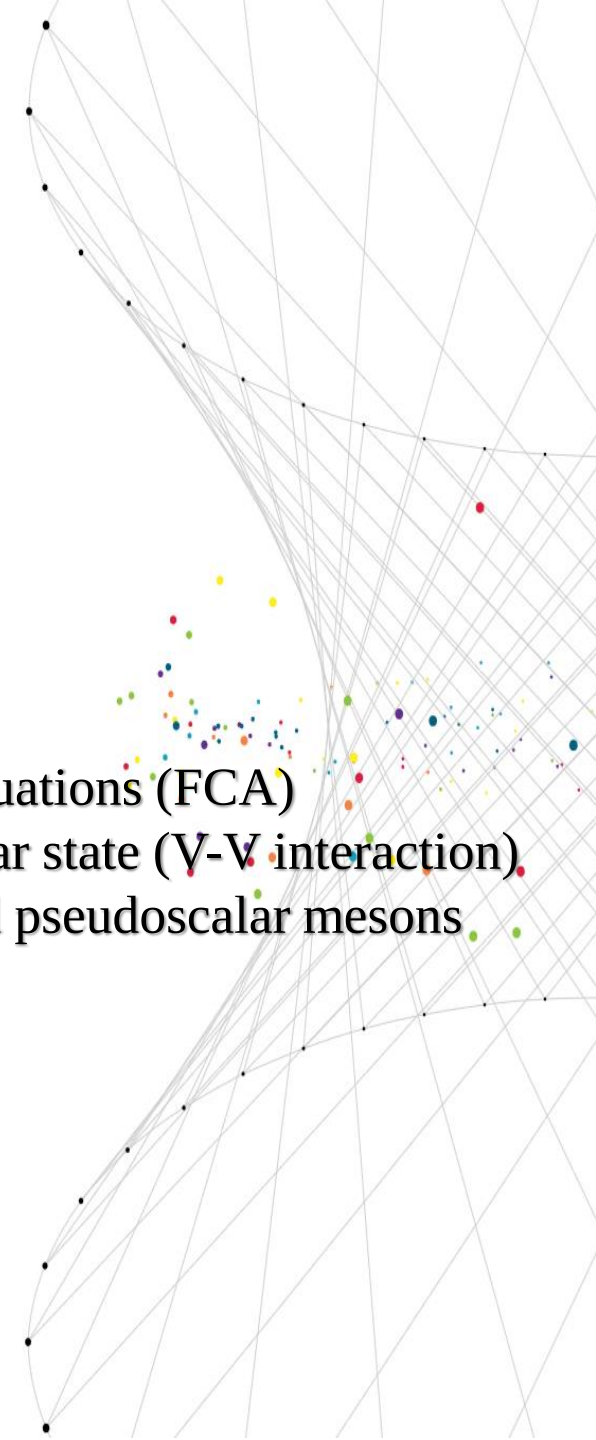
- Background
- Three-body systems

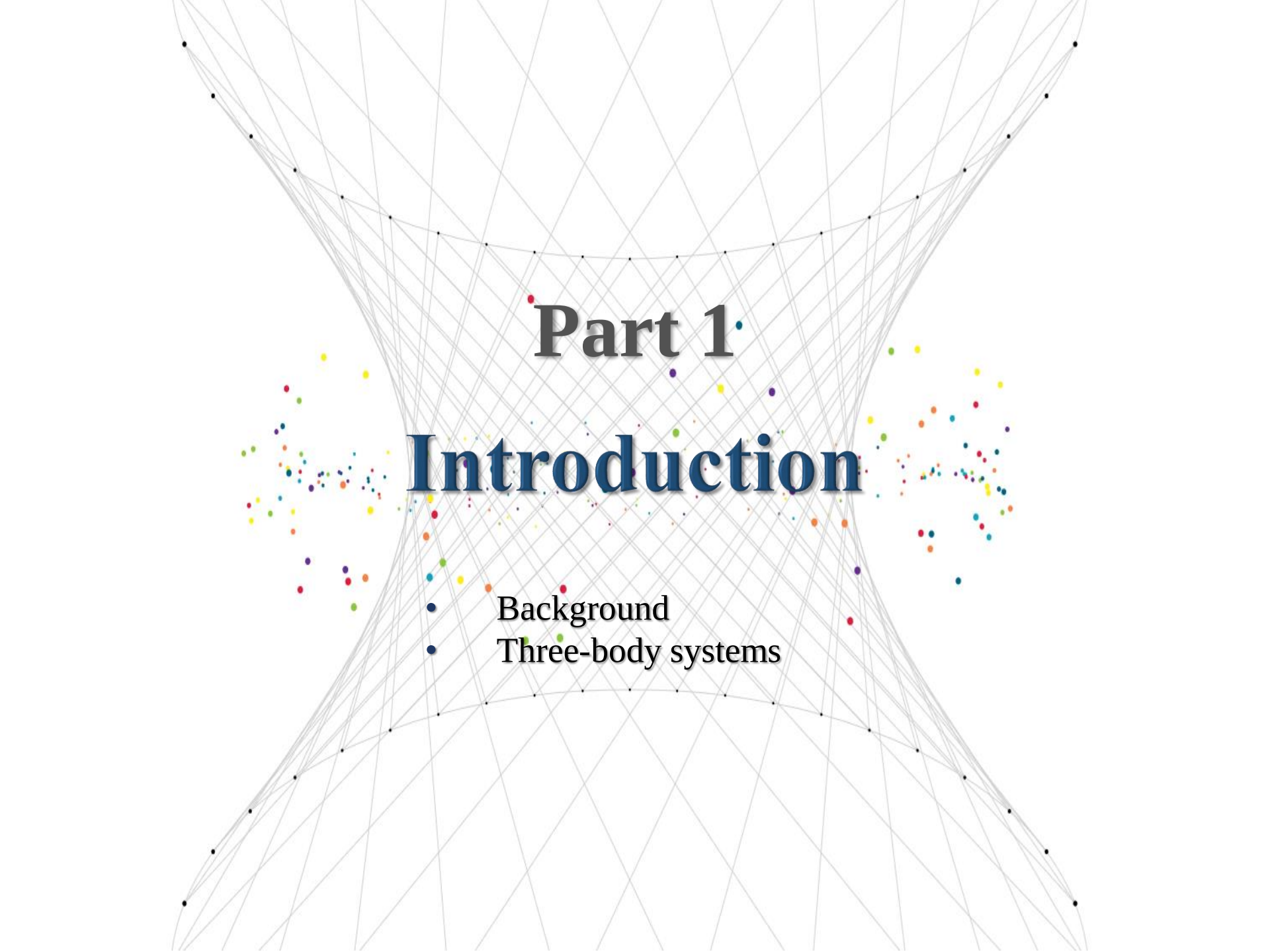
- **Formalism**

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- Fixed Center: vector-vector meson molecular state (V-V interaction)
- The interaction between vector mesons and pseudoscalar mesons

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Part 1

Introduction

- Background
- Three-body systems



Introduction

Background

三代物质粒子 (费米子)					
	I	II	III		
质量	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	0	$\approx 125.09 \text{ GeV}/c^2$
电荷	$2/3$	$2/3$	$2/3$	0	0
自旋	$1/2$	$1/2$	$1/2$	1	0
	u 上夸克	c 粲夸克	t 顶夸克	g 胶子	H 希格斯玻色子
	$\approx 4.7 \text{ MeV}/c^2$	$\approx 96 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	
	$-1/3$	$-1/3$	$-1/3$	0	
	$1/2$	$1/2$	$1/2$	1	
	d 下夸克	s 奇夸克	b 底夸克	γ 光子	
	$\approx 0.511 \text{ MeV}/c^2$	$\approx 105.66 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	0	
	-1	-1	-1	0	
	$1/2$	$1/2$	$1/2$	1	
	e 电子	μ μ 子	τ τ 子	Z Z玻色子	
	$< 2.2 \text{ eV}/c^2$	$< 1.7 \text{ MeV}/c^2$	$< 15.5 \text{ MeV}/c^2$	$\approx 80.39 \text{ MeV}/c^2$	
	0	0	0	± 1	
	$1/2$	$1/2$	$1/2$	1	
	ν_e 电子中微子	ν_μ μ 子中微子	ν_τ τ 子中微子	W W玻色子	

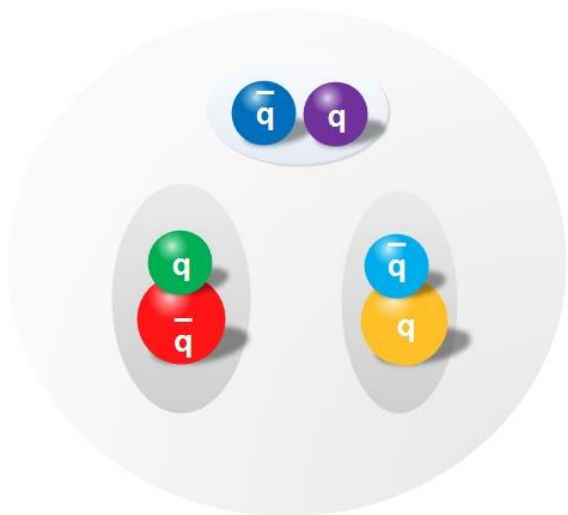


Quark model	Exotic state		
 meson	 tetraquark state	 pentaquark state	 hadron molecular state
 baryon	 glueball	 hybrid state	



Introduction

Three-body systems



$$\pi K \bar{K} - \pi(1300) \text{ Phys. Rev. D 84 (2011) 7, 074027}$$

$$K K \bar{K} - K(1460) \text{ Phys. Rev. C 83 (2011) 6, 065205}$$

$$\eta K \bar{K} - \eta(1475) \text{ Phys. Rev. D 88 (2013) 11, 114024}$$

$$\rho K \bar{K} - \rho(1700) \text{ Eur. Phys. J. A 50 (2014), 67}$$

$$\phi K \bar{K} - \phi(2175) \text{ Phys. Rev. D 78 (2008) 7, 074031}$$

$$J/\psi K \bar{K} - Y(4120) \text{ Phys. Rev. D 80 (2009) 9, 094012}$$

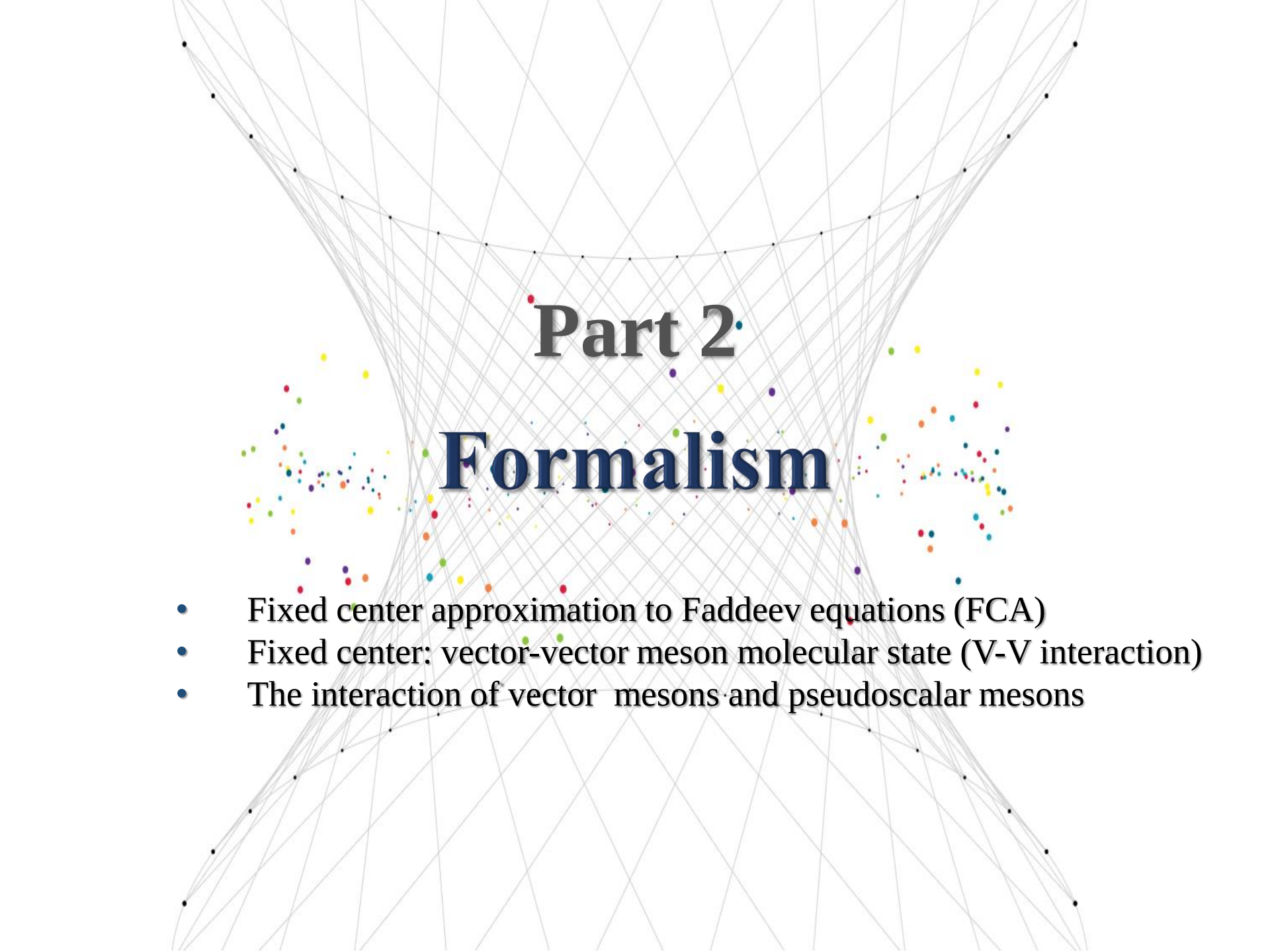
$$\pi \bar{K} K^* - \pi_1(1600) \text{ Phys. Rev. D 95 (2017) 5, 056014}$$

$$\eta \bar{K} K^* \text{ Phys. Rev. D 95 (2017) 5, 056014}$$

.....

We are interested in three-body system:

$$\eta K^* \bar{K}^*, \pi K^* \bar{K}^*, K K^* \bar{K}^*$$



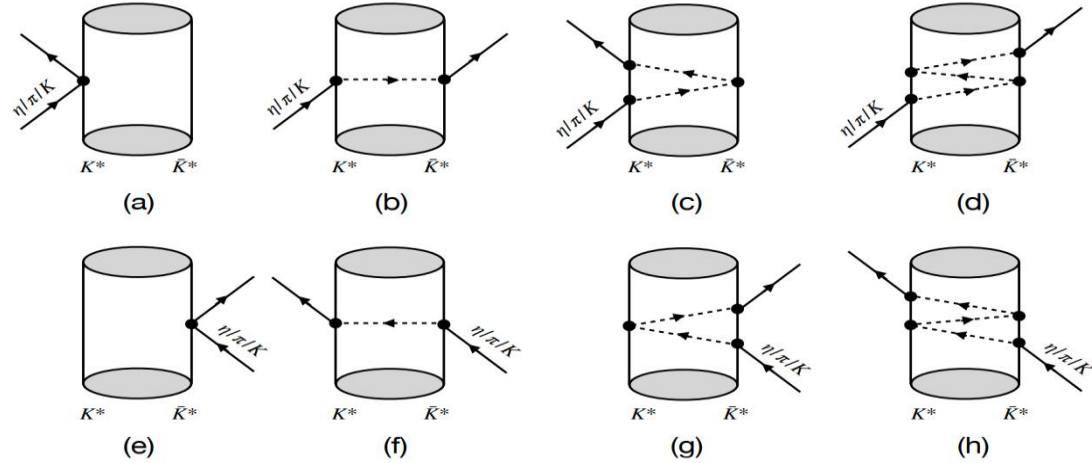
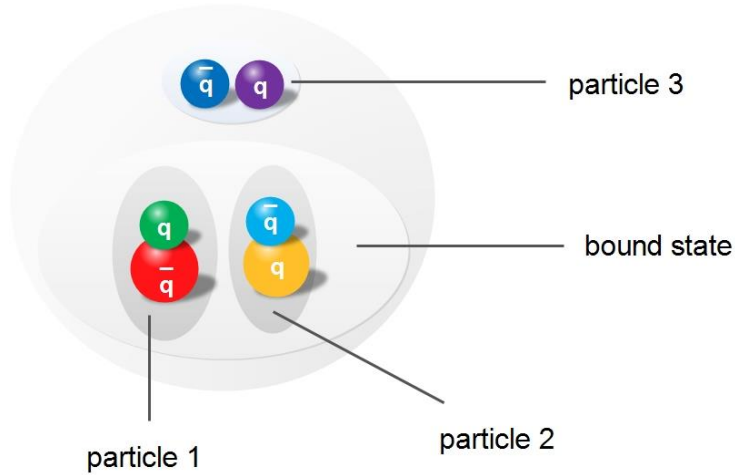
Part 2

Formalism

- Fixed center approximation to Faddeev equations (FCA)
- Fixed center: vector-vector meson molecular state (V-V interaction)
- The interaction of vector mesons and pseudoscalar mesons

Formalism

Fixed center approximation to Faddeev equations (FCA)



$$T_1 = \tilde{t}_1 + \tilde{t}_1 G_0 T_2$$

$$T_2 = \tilde{t}_2 + \tilde{t}_2 G_0 T_1$$

$$T = T_1 + T_2$$



$$T = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1 \tilde{t}_2 G_0}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2}$$

$$T_{B_1} = FFS_1 \cdot \tilde{t}_1 + \tilde{t}_1 G_0 \tilde{t}_2 = (FFS_1 - 1) \tilde{t}_1 + T_1$$

$$T_{B_2} = (FFS_2 - 1) \tilde{t}_2 + T_2$$

$$T = T_{B_1} + T_{B_2}$$



$$T = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1 \tilde{t}_2 G_0}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2} + (FFS_1 - 1) \tilde{t}_1 + (FFS_2 - 1) \tilde{t}_2$$



Formalism

Fixed center approximation to Faddeev equations (FCA)

where

$$\tilde{t}_1 = \frac{M_B}{m_1} t_1$$

$$\tilde{t}_2 = \frac{M_B}{m_2} t_2$$

single-scattering form factor

$$FFS_1 = \frac{1}{2} \int_{-1}^1 F_B(k_1) d(\cos\theta)$$

$$FFS_2 = \frac{1}{2} \int_{-1}^1 F_B(k_2) d(\cos\theta)$$

$$k_1 = \frac{m_2}{m_1 + m_2} k \sqrt{2(1 - \cos\theta)}$$

$$k_2 = \frac{m_1}{m_1 + m_2} k \sqrt{2(1 - \cos\theta)}$$

$$k = \begin{cases} \frac{\sqrt{(s - (M_B + m_3)^2)(s - (M_B - m_3)^2)}}{2\sqrt{s}} & \sqrt{s} \geq M_B + m_3 \\ 0 & \sqrt{s} < M_B + m_3 \end{cases}$$

The loop function

$$G_0 = \frac{1}{2M_B} \int \frac{d^3 \vec{q}}{(2\pi)^3} F_B(q) \frac{1}{q^0{}^2 - \vec{q}^2 - m^2 + i\epsilon}$$

The form factor

$$F_B(q) = \frac{1}{N} \int_{\substack{|\vec{p}| < \Lambda \\ |\vec{p} - \vec{q}| < \Lambda}} d^3 \vec{p} \frac{1}{M_B - w_1(\vec{p}) - w_2(\vec{p})} \frac{1}{m_{cls} - w_1(\vec{p} - \vec{q}) - w_2(\vec{p} - \vec{q})} \\ \frac{1}{2w_1(\vec{p})} \frac{1}{2w_2(\vec{p})} \frac{1}{2w_1(\vec{p} - \vec{q})} \frac{1}{2w_2(\vec{p} - \vec{q})}$$

$$N = \int_{|\vec{p}| < \Lambda} d^3 \vec{p} \left(\frac{1}{2w_1(\vec{p})} \frac{1}{2w_2(\vec{p})} \frac{1}{M_B - w_1(\vec{p}) - w_2(\vec{p})} \right)^2$$



Formalism

Fixed center: vector-vector meson molecular state (V-V interaction)

L. S. Geng and E. Oset. *Phys. Rev. D* 79 (2009) 7, 074009

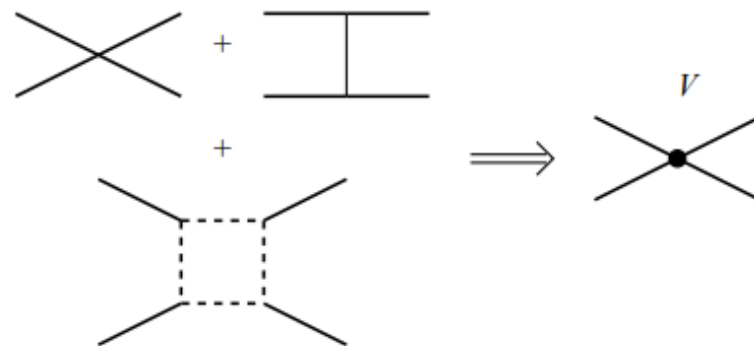
Hidden-gauge Lagrangian

$$\mathcal{L} = -\frac{1}{4} \langle \bar{V}_{uv} \bar{V}^{uv} \rangle + \frac{1}{2} M_v^2 \left\langle \left[V_u - \frac{i}{g} \Gamma_u \right]^2 \right\rangle$$

where

$$\bar{V}_{uv} = \partial_u V_v - \partial_v V_u - ig [V_u, V_v]$$

$$\Gamma_u = \frac{1}{2} \{ u^+ [\partial_u - i(v_u + a_u)] u + u [\partial_u - i(v_u - a_u)] u^+ \}$$



$$u^2 = U = e^{\frac{i\sqrt{2}P}{f}}$$

$$g = \frac{M_V}{2f}$$



$$\mathcal{L}_{VVV} = \frac{1}{2} g^2 \langle [V_u, V_v] V^u V^v \rangle$$

$$V_u = \begin{pmatrix} \frac{\omega + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega - \pi^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \Phi \end{pmatrix}_u \quad P = \begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

$$\mathcal{L}_{VV} = ig \langle (V^u \partial_v V_u - \partial_v V_u V^u) V^v \rangle$$

$$\mathcal{L}_{VPP} = -ig \langle V_u [P, \partial^u P] \rangle$$



Formalism

Fixed center: vector-vector meson molecular state (V-V interaction)

Three-body system: $\eta K^* \bar{K}^*, \pi K^* \bar{K}^*, K K^* \bar{K}^*$

Fixed center: $K^* \bar{K}^*$

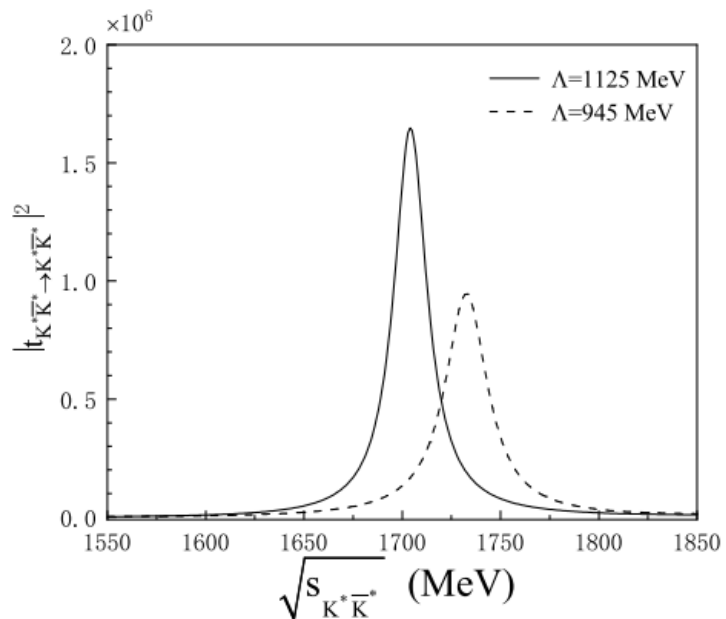
$I^G(J^{PC})$	Theory			PDG data		
	Pole position	Real axis		Name	Mass	Width
		$\Lambda_b = 1.4 \text{ GeV}$	$\Lambda_b = 1.5 \text{ GeV}$			
$0^+(0^{++})$	(1512,51)	(1523,257)	(1517,396)	$f_0(1370)$	1200~1500	200~500
$0^+(0^{++})$	(1726,28)	(1721,133)	(1717,151)	$f_0(1710)$	1724 ± 7	137 ± 8
$0^-(1^{+-})$	(1802,78)	(1802,49)		h_1		
$0^+(2^{++})$	(1275,2)	(1276,97)	(1275,111)	$f_2(1270)$	1275.1 ± 1.2	$185.0^{+2.9}_{-2.4}$
$0^+(2^{++})$	(1525,6)	(1525,45)	(1525,51)	$f'_2(1525)$	1525 ± 5	73^{+6}_{-5}
$1^-(0^{++})$	(1780,133)	(1777,148)	(1777,172)	a_0		
$1^+(1^{+-})$	(1679,235)	(1703,188)		b_1		
$1^-(2^{++})$	(1569,32)	(1567,47)	(1566,51)	$a_2(1700)??$		
$1/2(0^+)$	(1643,47)	(1639,139)	(1637,162)	K_0^*		
$1/2(1^+)$	(1737,165)	(1743,126)		$K_1(1650)?$		
$1/2(2^+)$	(1431,1)	(1431,56)	(1431,63)	$K_2^*(1430)$	1429 ± 1.4	104 ± 4



Formalism

Fixed center: vector-vector meson molecular state (V-V interaction)

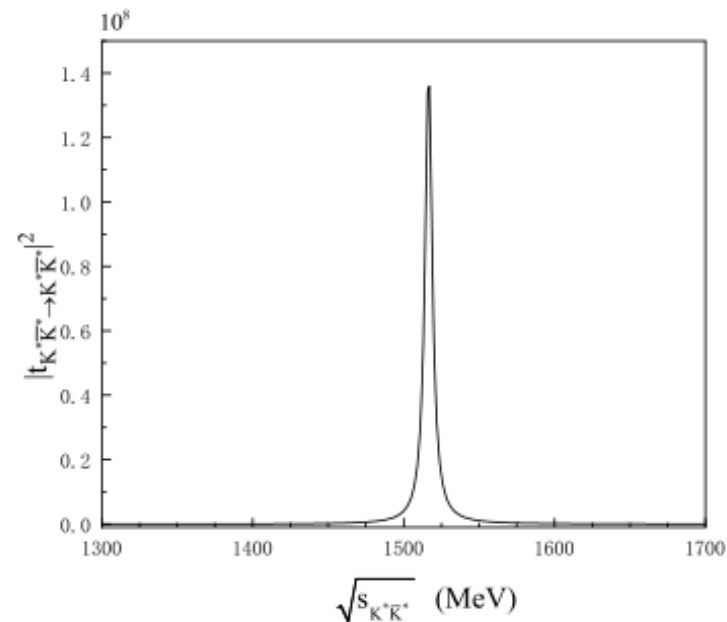
$f_0(1710)$



PDG

Mass (eva.)	1704 MeV ($\Lambda = 1125$ MeV)
Mass (ave.)	1733 MeV ($\Lambda = 945$ MeV)
Width	150 MeV

$f_2'(1525)$



PDG

Mass	1517 MeV ($\Lambda = 1009$ MeV)
Width	86 MeV



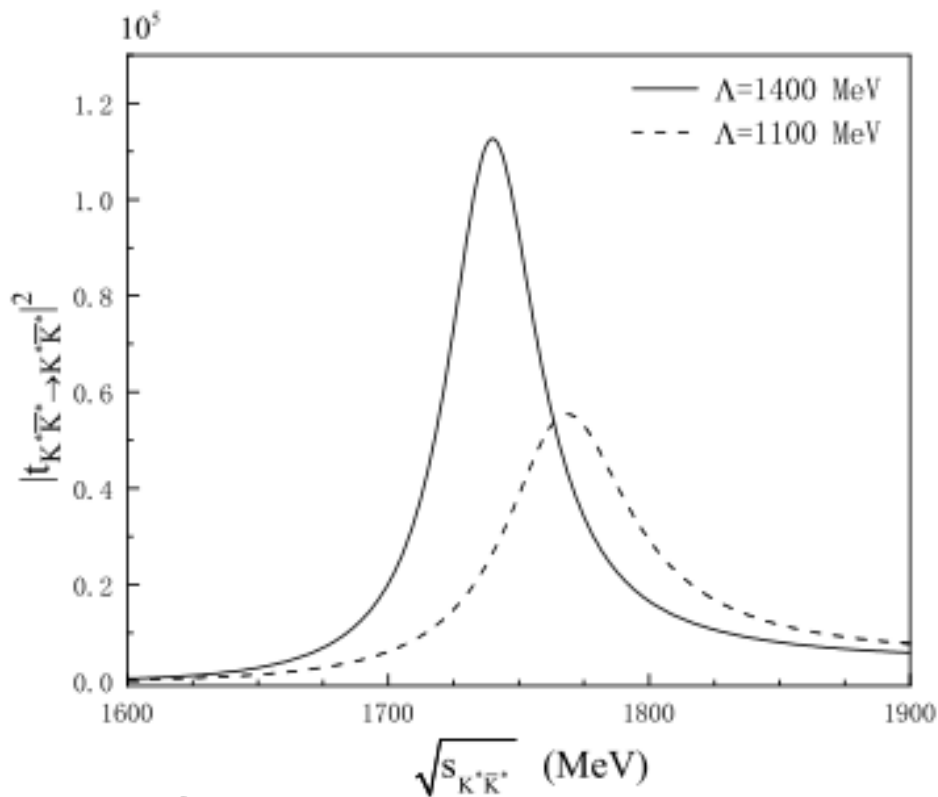
Formalism

Fixed center: vector-vector meson molecular state (V-V interaction)

$a_0(1710)$

1769 MeV ($\Lambda = 1100$ MeV)

1740 MeV ($\Lambda = 1400$ MeV)



PDG

Mass	$1704 \pm 5 \pm 2$ MeV	BABR	2021
	$1817 \pm 8 \pm 20$ MeV	BES3	2022
Width	106 ± 15 MeV		



Formalism

The interaction between vector mesons and pseudoscalar mesons

L. Roca, E. Oset and J. Singh. [Phys. Rev. D 72 \(2005\) 1, 014002](#)

$$\mathcal{L} = -\frac{1}{4} \text{Tr} \{ (\nabla_u V_v - \nabla_v V_u) (\nabla^u V^v - \nabla^v V^u) \}$$

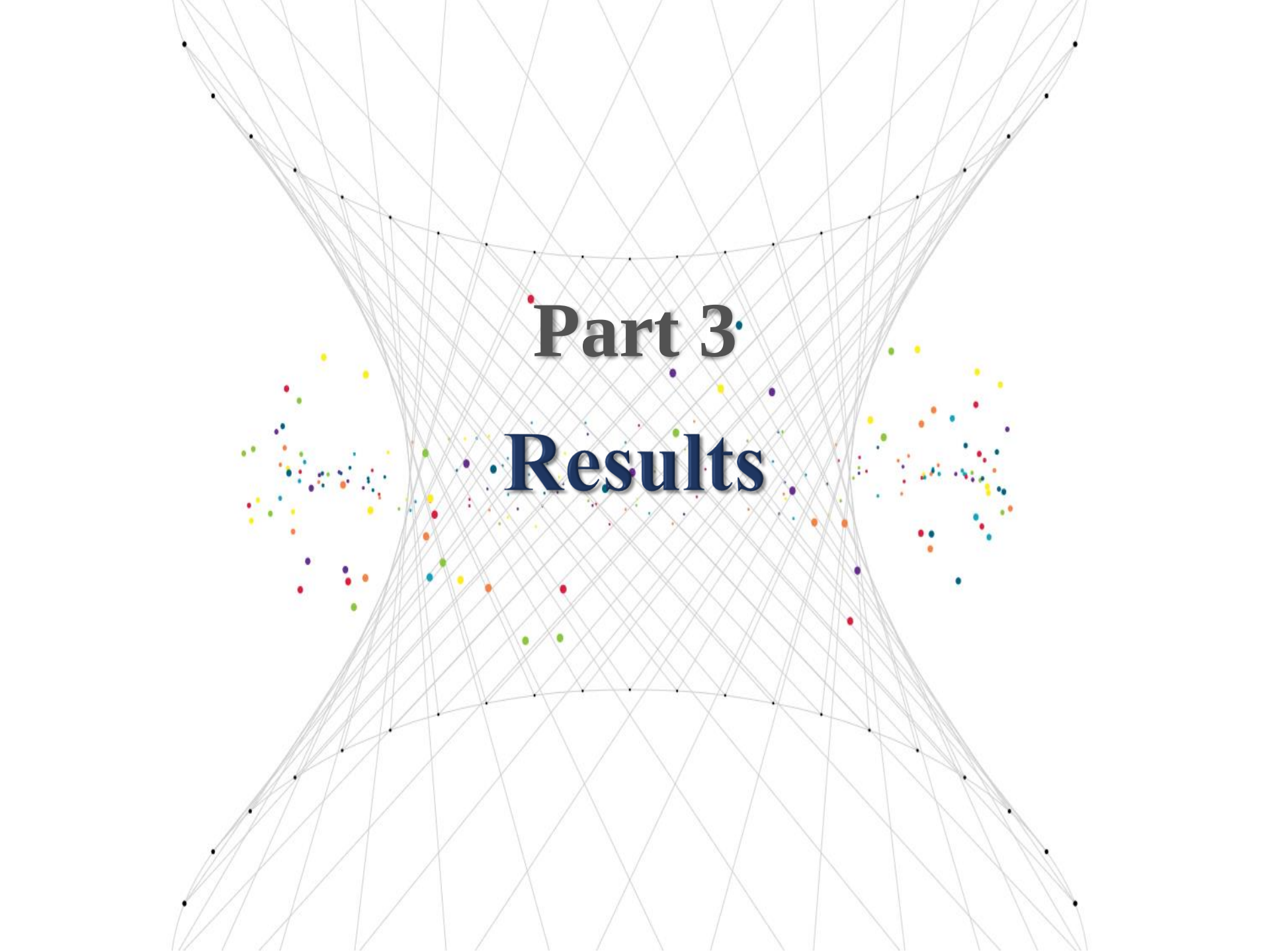
where

$$\nabla_u V_v = \partial_u V_v + [\tilde{\Gamma}_u, V_v] \quad \tilde{\Gamma}_u = \frac{1}{2} (u^+ \partial_u u + u \partial_u u^+) \quad u^2 = U = e^{\frac{i\sqrt{2}P}{f}}$$

$$V_u = \begin{pmatrix} \frac{\omega + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega - \pi^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \Phi \end{pmatrix}_u \quad P = \begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$



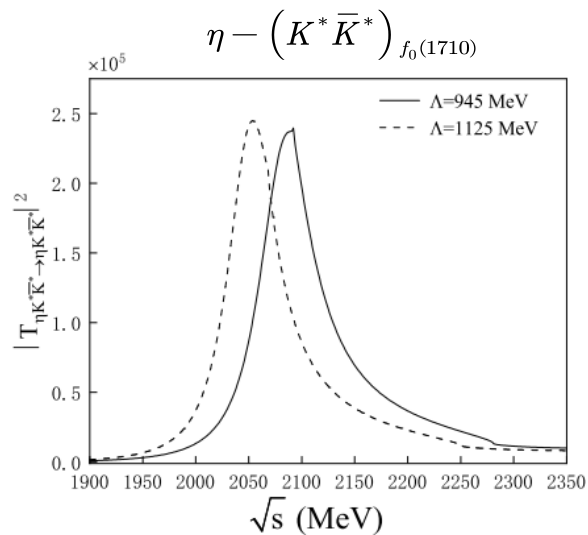
$$\mathcal{L}_{VPP} = -\frac{1}{4f^2} \text{Tr} ([V^u, \partial^v V_u] [P, \partial_v P])$$



The background features a complex geometric pattern of thin, light gray lines that form a hyperboloid of two sheets, creating a central void. Scattered throughout this structure are numerous small, colored dots in various colors including red, yellow, green, blue, orange, and purple. The dots are more densely clustered in some areas, particularly around the central void and the edges of the sheets.

Part 3

Results

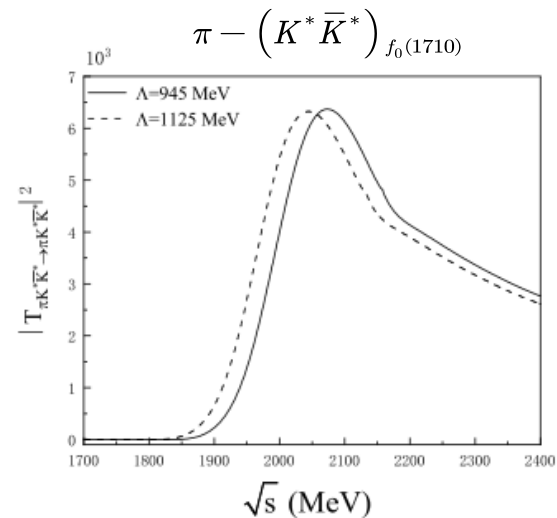


Our work

$\Lambda = 945 \text{ MeV}$	2092 MeV
$\Lambda = 1125 \text{ MeV}$	2054 MeV

PDG
further state: $\eta(2100)$

$2050^{+36}_{-30} {}^{+181}_{-164} \text{ MeV}$	2016	BES3	[1]
$2103 \pm 50 \text{ MeV}$	1989	DM2	[2]



Our work

$\Lambda = 945 \text{ MeV}$	2073 MeV
$\Lambda = 1125 \text{ MeV}$	2031 MeV

PDG
further state: $\pi(2070)$

$2075 \pm 35 \text{ MeV}$	2001	Anisovich et al	[3]
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[1] *BESIII Collaboration*. [Phys. Rev. D 93 \(2016\) 11, 112011](#).

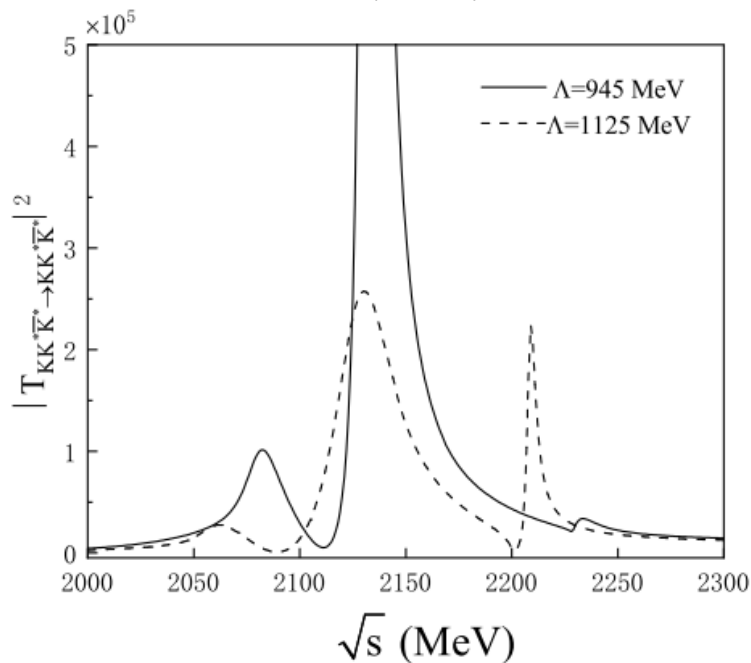
[2] *DM2 Collaboration*. [Phys. Rev. D 39 \(1989\), 701](#).

[3] *Anisovich et al*. [Phys. Lett. B 517 \(2001\), 261-272](#).



Results

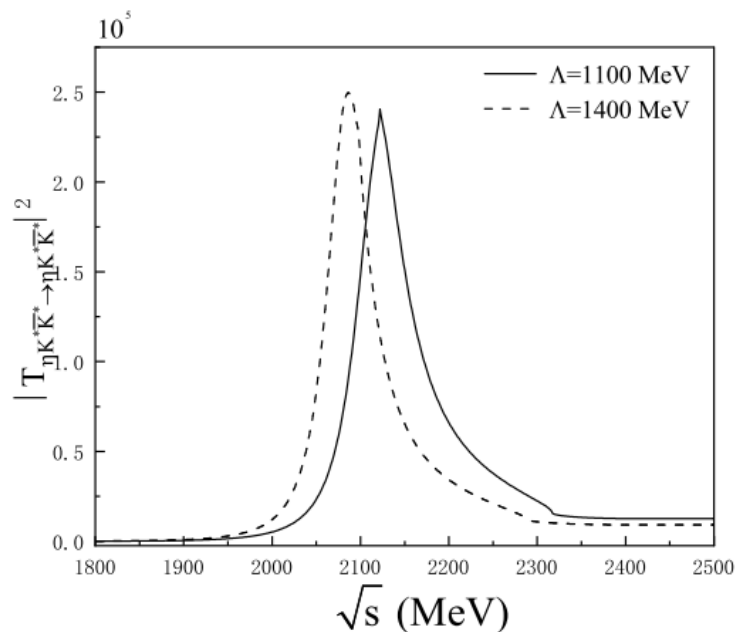
$$K - (K^* \bar{K}^*)_{f_0(1710)}$$



Our work

	State* 1	State 2	State*3
$\Lambda = 945$ MeV	2083 MeV	2134 MeV	2233 MeV
$\Lambda = 1125$ MeV	2062 MeV	2130 MeV	2209 MeV

$$\eta - (K^* \bar{K}^*)_{a_0(1710)}$$



Our work

$\Lambda = 1100$ MeV	2122 MeV
$\Lambda = 1400$ MeV	2086 MeV

PDG
further state: $\pi(2070)$

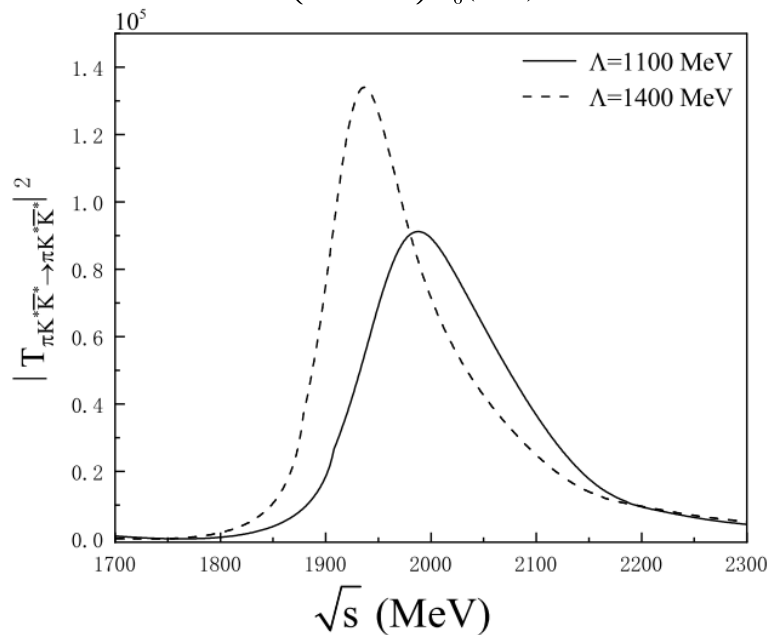
2075 ± 35 MeV	2001	Anisovich et al	[1]
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[1] Anisovich et al. [Phys. Lett. B 517 \(2001\), 261-272.](#)

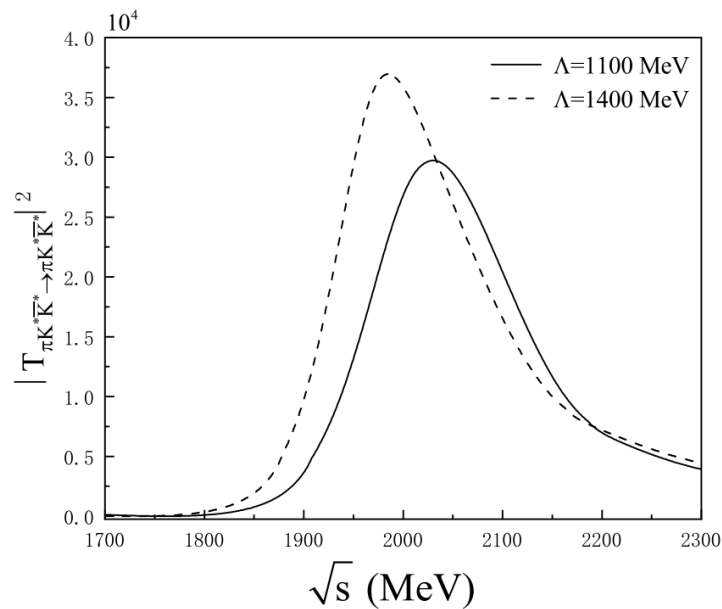


Results

$$\pi - (K^* \bar{K}^*)_{a_0(1710)} \quad I=0$$



$$\pi - (K^* \bar{K}^*)_{a_0(1710)} \quad I=1$$



Our work

	$I=0$	$I=1$
$\Lambda = 1100 \text{ MeV}$	1988 MeV	2030 MeV
$\Lambda = 1400 \text{ MeV}$	1937 MeV	1997 MeV

PDG
further state: $\eta(2010)$

$2010^{+35}_{-60} \text{ MeV}$	2000	Anisovich et al	[1]
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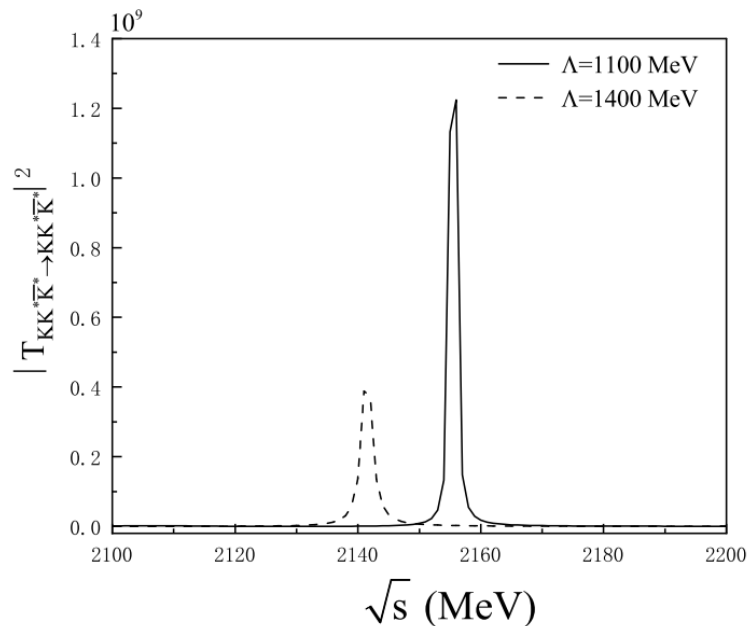
PDG

further state: $\pi(2070)$

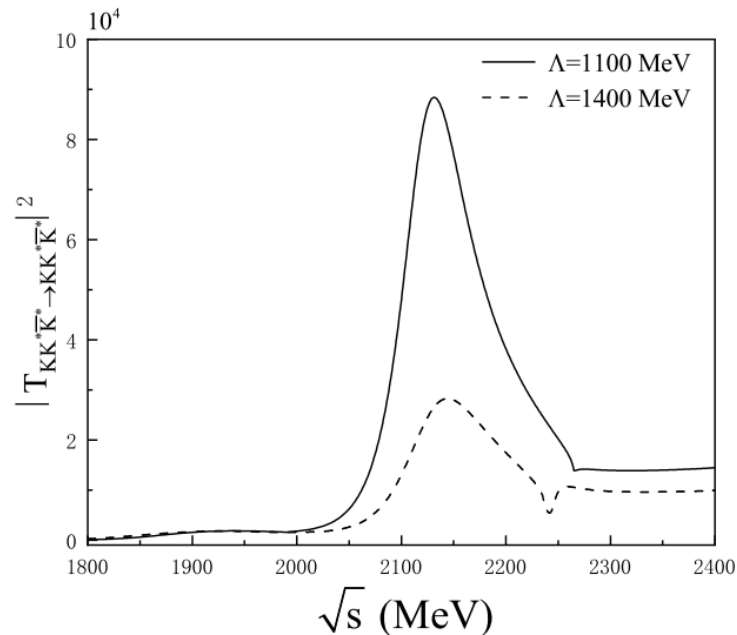
$2075 \pm 35 \text{ MeV}$	2001	Anisovich et al	[2]
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[1] Anisovich et al. [Phys. Lett. B 491 \(2000\), 47-58.](#)

[2] [Anisovich et al. Phys. Lett. B 517 \(2001\), 261-272](#)



$$K - (K^* \bar{K}^*)_{a_0(1710)} \quad I = \frac{1}{2}$$



$$K - (K^* \bar{K}^*)_{a_0(1710)} \quad I = \frac{3}{2}$$

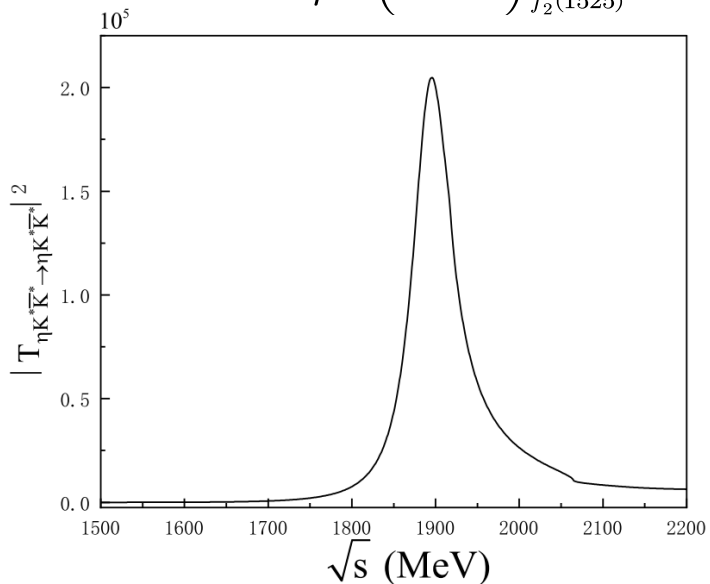
Our work

	$I = \frac{1}{2}$	$I = \frac{3}{2}$
$\Lambda = 1100$ MeV	2156 MeV	2145 MeV
$\Lambda = 1400$ MeV	2132 MeV	2138 MeV



Results

$$\eta - (K^* \bar{K}^*)_{f_2'(1525)}$$

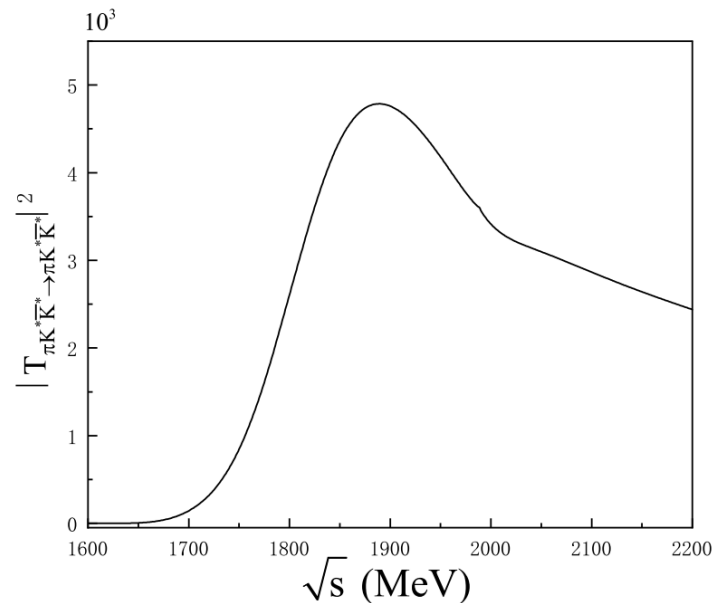


$$\eta_2(1870)$$

Unit in MeV.

Model	Our work	Hybrid State[1]	$s\bar{s} ({}^1D_2)$		$n\bar{n} (2^1D_2)$		PDG (ave.)
			GI[2]	VFV [3]	GI[2]	VFV [3]	
Mass	1896	1900	1890	1853	2130	1863	1842 ± 8

$$\pi - (K^* \bar{K}^*)_{f_2'(1525)}$$



$$\pi_2(1880)$$

Unit in MeV.

Model	Our work	Hybrid state [4]	$q\bar{q} (2^1D_2)$ [2]	PDG
Mass	1889	1800-1900	2130	1879^{+26}_{-5}

[1] N. Isgur and J. E. Paton. [Phys. Rev. D 31 \(1985\), 2910](#)

[2] S. Godfrey and N. Isgur. [Phys. Rev. D 32 \(1985\), 189.](#)

[3] J. Vijande, F. Fernandez, and A. Valcarce. [J. Phys. G 31 \(2005\), 481.](#)

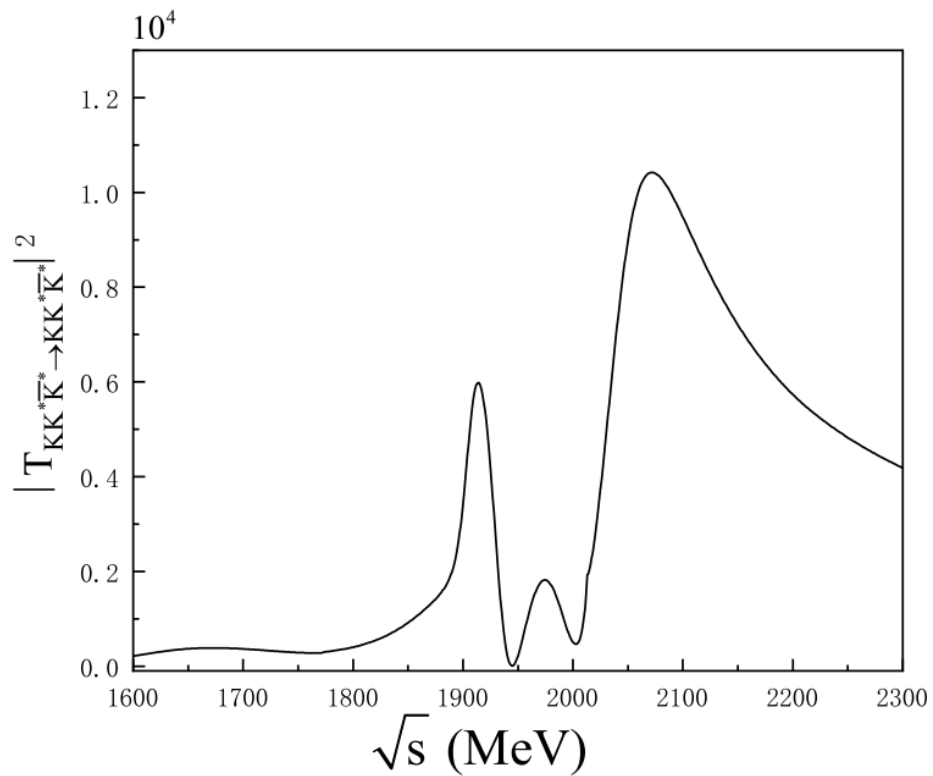
[4] T. Barnes, F. E. Close, and E. S. Swanson. [Phys. Rev. D 52 \(1995\), 5242](#)



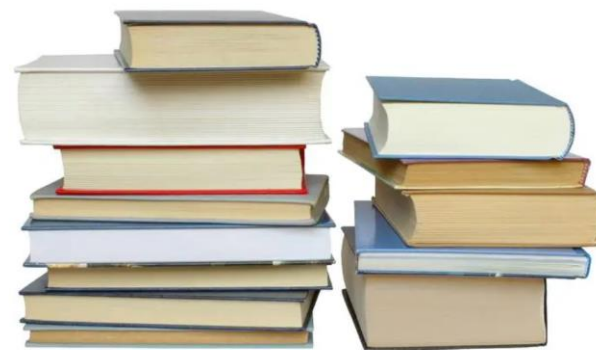
Results

$$K - (K^* \bar{K}^*)_{f_2'(1525)}$$

Unit in MeV.



	State* 1	State* 2	State* 3
Mass	1914	1975	2072



Part 4

Summary

The background of the slide features a complex geometric pattern. It consists of a hyperboloid of two sheets, rendered with a grid of thin, light gray lines that create a sense of depth and curvature. Scattered across the surface of these sheets are numerous small, colored dots in various colors including red, yellow, green, blue, and purple. The dots are more densely clustered on the outer edges of the sheets and more sparsely distributed in the central regions. The overall effect is a sophisticated, mathematical aesthetic.



Summary

$\eta K^* \bar{K}^*$	$0^+(0^-+)$	$\eta(2100)$
	$1^-(0^-+)$	$\pi(2070)$
	$0^+(2^-+)$	$\eta_2(1870)$

$\pi K^* \bar{K}^*$	$0^+(0^-+)$	$\eta(2010)$
	$1^-(0^-+)$	$\pi(2070)$
	$1^-(2^-+)$	$\pi_2(1880)$

$KK^* \bar{K}^*$	$\frac{1}{2}(0^-)$	2080? 2150? 2230?
	$\frac{3}{2}(0^-)$	2140 ?
	$\frac{1}{2}(2^-)$	1910? 1980? 2070?

THANKS

