# Hidden charm decays of spin-2 partner of X(3872)

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Y.-X. Zheng, Z.-X. Cai, G. Li, S.-D. Liu, J.-J. Wu, and Q. Wu, Phys. Rev. D 109, 014027(2024).

# Catalogue

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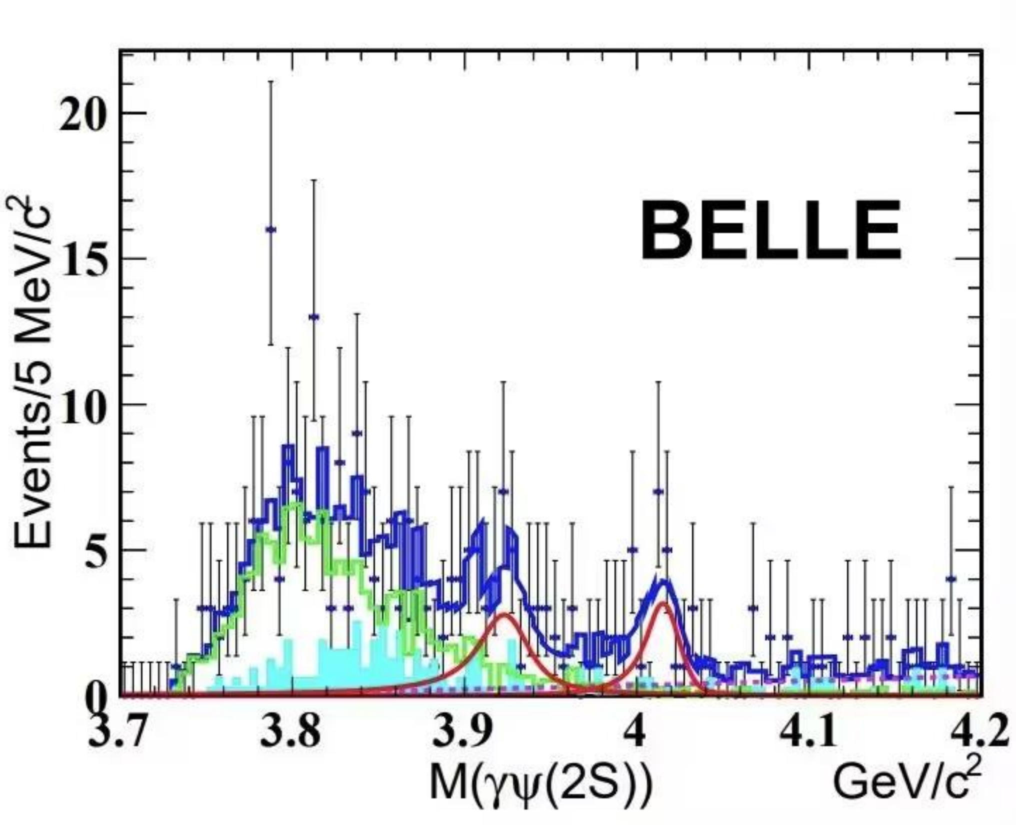
# Background




Belle Collaboration, X. L. Wang, and B. S. Gao et al, Physical Review D 105, 112011 (2022).

The Belle collaboration observed a structure in the invariant mass distribution of the $\gamma\psi(2S)$.

$$M = (4014.3 \pm 4.0 \pm 1.5)\ \text{MeV}$$

$$\Gamma = (4 \pm 11 \pm 6)\ \text{MeV}$$

It is a good candidate for the $\boldsymbol{X_2(2^{++})}$.

# Background

| Particle | Mass(MeV) |
|---|---|
| $D^0$ | 1864.84±0.05 |
| $D^{*0}$ | 2006.85±0.05 |
| $X(3872)$ | 3871.65±0.06 |
| $X_2$ | 4014.3±4.0±1.5 |

**Table 1** The masses of $D^0$, $D^{*0}$, $X(3872)$ and $X_2$.

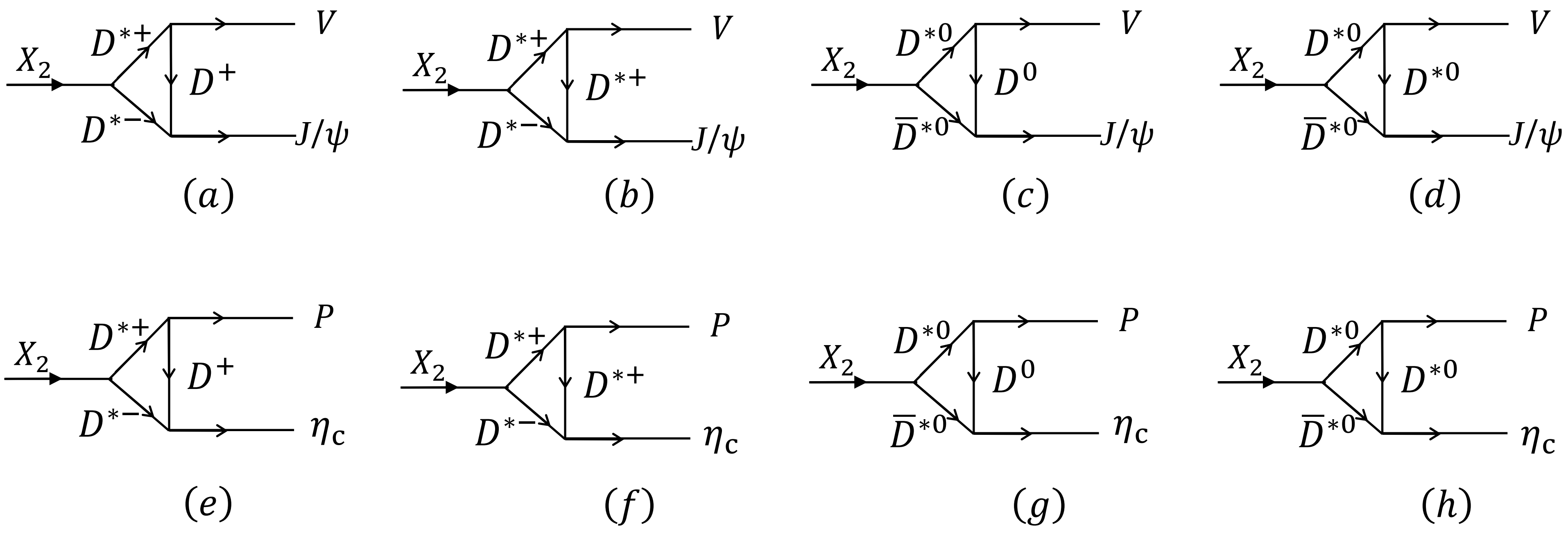
This new structure is a perfect candidate of an isoscalar $D^*\bar{D}^*$molecule, labeled as $X_2$, with quantum numbers $\mathrm{J}^{\mathrm{PC}} = 2^{++}$.

- The mass of $X_2$ is near the threshold of $D^*\bar{D}^*$.
- The width of $X_2$ has the same order of magnitude as predicted.

M. Albaladejo and et al, Eur. Phys. J. C 75 (2015) 547.

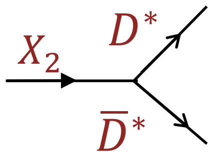
V. Baru and et al, Phys. Lett. B 763 (2016) 20-28.

# Feynman diagrams




$\mathrm{V} = \rho^0, \omega$ $\quad\quad$ $P = \pi^0, \eta, \eta'$

# Effective Lagrangian




$$|X_2\rangle = \frac{1}{\sqrt{2}}\left(\left|D^{*0}\bar{D}^{*0}\right\rangle + |D^{*+}D^{*-}\rangle\right)$$

The effective Lagrangian for the $X_2$ coupling to $D^*\bar{D}^*$can be written as

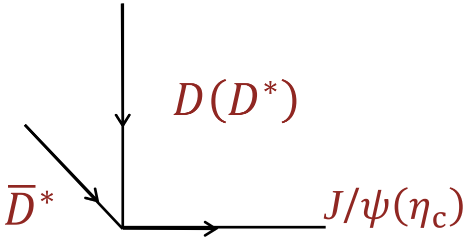
$$\mathcal{L}_{X_2} = \frac{1}{\sqrt{2}}\left(\chi_{\rm nr}^{0} X_{2\mu\nu} D^{*0\dagger\mu}\bar{D}^{*0\dagger\nu} + \chi_{\rm nr}^{\rm c} X_{2\mu\nu} D^{*+\dagger\mu} D^{*-\dagger\nu}\right) + \mathrm{H.c.}$$

$$\chi_{\rm nr}^{0(c)} = \left(\frac{16\pi}{\mu}\sqrt{\frac{2E_B}{\mu}}\right)^{1/2},$$

$E_B$ and $\mu$ are the binding energy of the $X_2$ relative to the $D^*\bar{D}^*$ threshold and the $D^*\bar{D}^*$ reduced mass. For the case of the $D^{*0}\bar{D}^{*0}$ , $\chi_{\rm nr}^{0}$ is 1.32 $\mathrm{GeV}^{-1/2}$, whereas it is 2.36 $\mathrm{GeV}^{-1/2}$ for the $D^{*+}D^{*-}$.

# Effective Lagrangian

In the heavy quark limit, the interactions of the S-wave charmonia $J/\psi$ and $\eta_c$ with the $D$ and $D^*$ mesons are described by the Lagrangian




$$\begin{aligned}\mathcal{L}_S &= ig_{\psi DD}\psi_\mu^\dagger \bar{D}\overleftrightarrow{\partial}^\mu D + g_{\psi D^* D}\epsilon_{\mu\nu\alpha\beta}\partial^\mu\psi^{\dagger\nu}\left(D^{*\alpha}\overleftrightarrow{\partial}^\beta\bar{D} - D\overleftrightarrow{\partial}^\beta\bar{D}^{*\alpha}\right)\\ &- ig_{\psi D^* D^*}\psi_\mu^\dagger\left(D_\nu^*\overleftrightarrow{\partial}^\nu\bar{D}^{*\mu} + D^{*\mu}\overleftrightarrow{\partial}^\nu\bar{D}_\nu^* - D_\nu^*\overleftrightarrow{\partial}^\mu\bar{D}^{*\nu}\right)\\ &- g_{\eta_c D^* D^*}\epsilon_{\mu\nu\alpha\beta}\partial^\mu\eta_c^\dagger D^{*\nu}\overleftrightarrow{\partial}^\alpha\bar{D}^{*\beta} + ig_{\eta_c D^* D}\eta_c\left(D\overleftrightarrow{\partial}^\mu\bar{D}_\mu^* + D_\mu^*\overleftrightarrow{\partial}^\mu\bar{D}\right) + \mathrm{H.c.}\end{aligned}$$

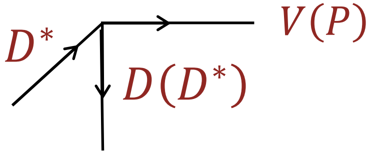
where $D=(D^0, D^+, D_S^+)$, $D^*=(D^{*0}, D^{*+}, D_S^{*+})$, the coupling constants are related to the gauge coupling $g_1=\sqrt{m_\psi}/(2m_D f_\psi)$ with the $J/\psi$ decay constant $f_\psi$= 426 MeV

$$g_{\psi D^* D} = 2g_1\sqrt{m_\psi m_{D^*}/m_D}\,,\quad g_{\psi D^* D^*} = 2g_1 m_{D^*}\sqrt{m_\psi},$$

$$g_{\eta_c D^* D} = 2g_1\sqrt{m_{\eta_c} m_{D^*} m_D}\,,\quad g_{\eta_c D^* D^*} = 2g_1\sqrt{m_{\eta_c}}.$$

# Effective Lagrangian

The Lagrangian relevant to the light vector and pseudoscalar mesons can be written as




$$
\begin{aligned}
\mathcal{L} = & -ig_{D^*D\mathcal{P}}\left(D_i^{\dagger}\partial_\mu \mathcal{P}_{ij} D_j^{*\mu} - D_i^{*\mu\dagger}\partial_\mu \mathcal{P}_{ij} D_j\right) \\
& + \frac{1}{2} g_{D^*D^*\mathcal{P}} \varepsilon_{\mu\nu\alpha\beta} D_i^{*\mu\dagger} \partial^\nu \mathcal{P}_{ij} \overleftrightarrow{\partial}^{\alpha} D_j^{*\beta} - i g_{DD\mathcal{V}} D_i^{\dagger} \overleftrightarrow{\partial}_{\mu} D^j (\mathcal{V}^\mu)_j^i \\
& -2 f_{D^*D\mathcal{V}} \epsilon_{\mu\nu\alpha\beta} (\partial^\mu \mathcal{V}^\nu)_j^i \left(D_i^{\dagger} \overleftrightarrow{\partial}^{\alpha} D^{*\beta j} - D_i^{*\beta\dagger} \overleftrightarrow{\partial}^{\alpha} D^j\right) \\
& + i g_{D^*D^*\mathcal{V}} D_i^{*\nu\dagger} \overleftrightarrow{\partial}_{\mu} D_\nu^{*j} (\mathcal{V}^\mu)_j^i \\
& + 4 i f_{D^*D^*\mathcal{V}} D_{i\mu}^{*\dagger} (\partial^\mu \mathcal{V}^\nu - \partial^\nu \mathcal{V}^\mu)_j^i D_\nu^{*j} + H.c.
\end{aligned}
$$

$$
\mathcal{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix},
$$

Here $\delta = \cos\left(\theta_P + \arctan\sqrt{2}\right)$ and $\gamma = \sin\left(\theta_P + \arctan\sqrt{2}\right)$ with the $\eta$-$\eta'$ mixing angle $\theta_P$ ranging from $-24.6°$ to $-11.5°$.

$$
\mathcal{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\delta\eta + \gamma\eta'}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\delta\eta + \gamma\eta'}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \gamma\eta + \delta\eta' \end{pmatrix}
$$

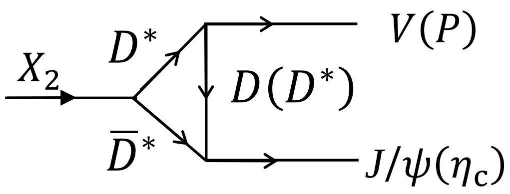
# Effective Lagrangian

The coupling constants can be written as

$$g_{DDV} = g_{D^*D^*V} = \frac{\beta g_V}{\sqrt{2}},$$
$$f_{D^*DV} = \frac{f_{D^*D^*V}}{m_{D^*}} = \frac{\lambda g_V}{\sqrt{2}},$$
$$g_{D^*D^*P} = \frac{g_{D^*DP}}{\sqrt{m_D m_{D^*}}} = \frac{2g}{f_\pi}$$

Here $\beta = 0.9$ and $g_V = m_\rho/f_\pi$ with the pion decay constant $f_\pi = 132$ MeV, $\lambda = 0.56\ \mathrm{GeV}^{-1}$ and $g = 0.59$ based on the matching of the form factors obtained from the light cone sum rule and from the lattice QCD calculations.

## Form factor




$$F(q^2, m^2) = \frac{m^2 - \Lambda^2}{q^2 - \Lambda^2}$$

Here $q$ and $m$ are the momentum and mass of the exchanged meson, $\Lambda = m + \alpha\Lambda_{\mathrm{QCD}}$ with $\Lambda_{\mathrm{QCD}} = 0.22$ GeV. The model parameter α could not be determined from the first principle, but its value was found to be of order of unity. We vary α from 0.7 to 1.4.

# Effective Lagrangian

## Amplitudes

$$\mathcal{M}_P = \frac{x}{2}\chi_{\rm nr}^{c(0)}\sqrt{m_{X_2}}m_{D^*}\varepsilon^{\mu\nu}(X_2)I_{\mu\nu}$$

$$\mathcal{M}_V = \frac{1}{2}\chi_{\rm nr}^{c(0)}\sqrt{m_{X_2}}m_{D^*}\varepsilon^{\mu\nu}(X_2)\varepsilon^{*\alpha}(V)\varepsilon^{*\beta}(J/\psi)I_{\mu\nu\alpha\beta}$$

$$\begin{aligned} I_{\mu\nu\alpha\beta}^{b(d)} = \int \frac{\mathrm{d}^4 q}{(2\pi)^4} g_{\mu\rho}g_{\nu\sigma}\big[4f_{D^*D^*V}\big(p_{3,\eta}g_{\alpha\xi} - p_{3,\xi}g_{\eta\alpha}\big) \\ -g_{D^*D^*V}\big(p_1+q\big)_\alpha g_{\xi\eta}\big]g_{\psi D^*D^*}[(p_2+q)_\delta g_{\beta\gamma} \\ +\big(p_2+q\big)_\gamma g_{\beta\delta} - \big(p_2+q\big)_\beta g_{\gamma\delta}\big]S^{\rho\xi}(p_1,m_{D^*}) \\ \times S^{\sigma\gamma}(p_2,m_{D^*})S^{\delta\eta}(q,m_{D^*})F(q^2,m_{D^*}^2) \end{aligned}$$

$$S(q,m_D) = \frac{1}{q^2 - m_D^2 + \mathrm{i}\epsilon}$$

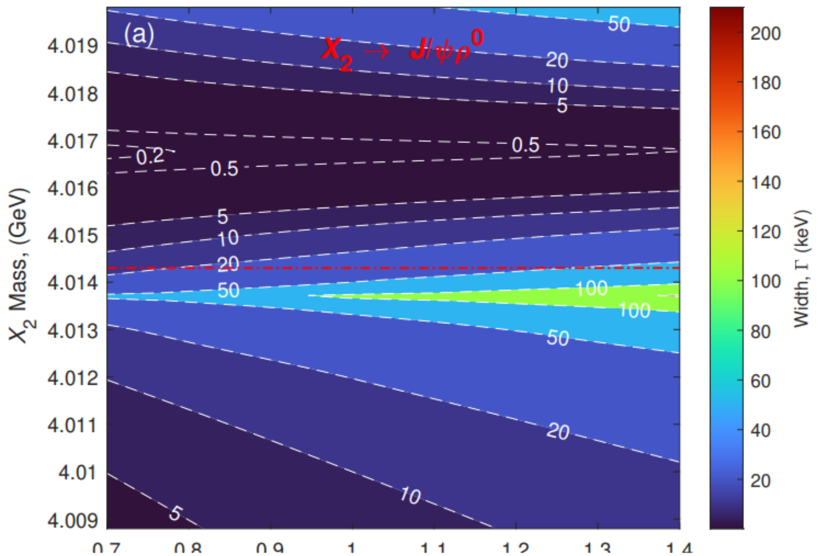
$$S^{\mu\nu}(q,m_{D^*}) = \frac{-g_{\mu\nu} + q^\mu q^\nu/m_{D^*}^2}{q^2 - m_{D^*}^2 + \mathrm{i}\epsilon}$$

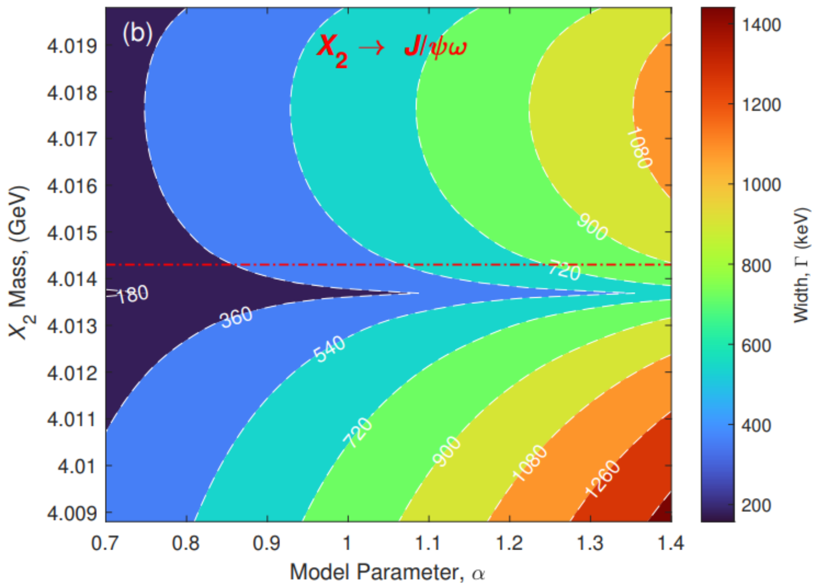
$X_2 \to J/\psi\rho^0$ and $X_2 \to \eta_c\pi^0$: $\mathcal{M}^{(a/e)} + \mathcal{M}^{(b/f)} - \mathcal{M}^{(c/g)} - \mathcal{M}^{(d/h)}$

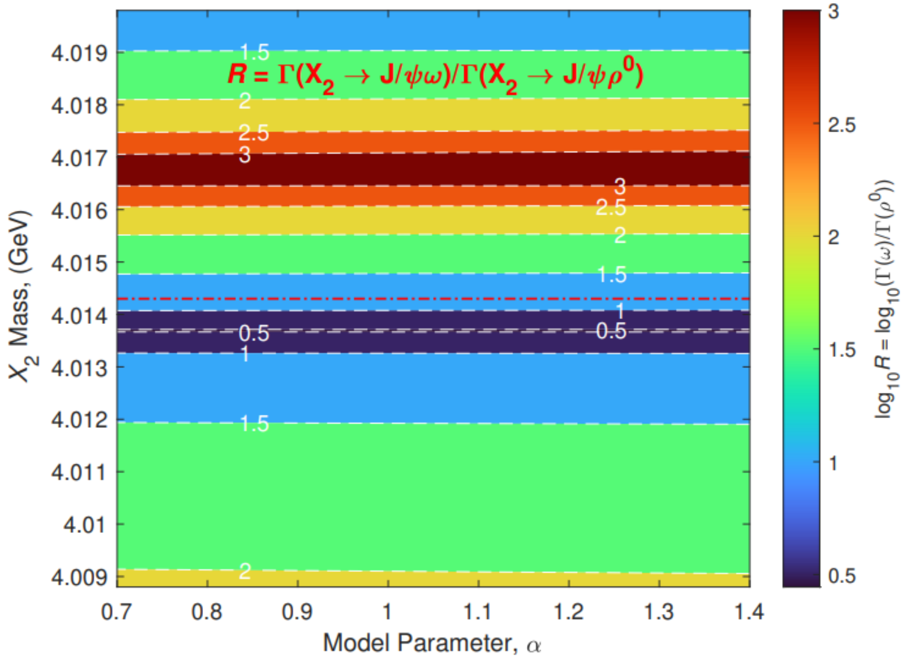
$X_2 \to J/\psi\omega$ and $X_2 \to \eta_c\eta(\eta')$: $\mathcal{M}^{(a/e)} + \mathcal{M}^{(b/f)} + \mathcal{M}^{(c/g)} + \mathcal{M}^{(d/h)}$

$$\Gamma_{X_2} = \frac{1}{2J+1} \times \frac{|\vec{p}|}{8\pi {m_{X_2}}^2}\overline{|\mathcal{M}_{tot}|^2}$$

# Results: $X_2 \to J/\psi V$

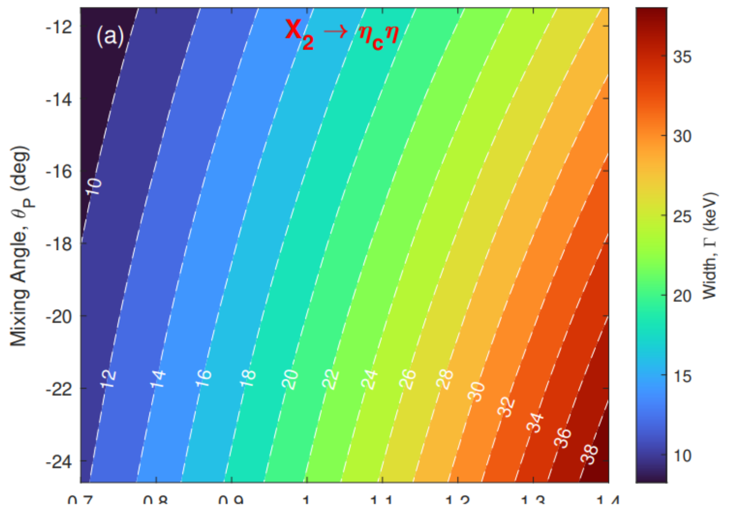


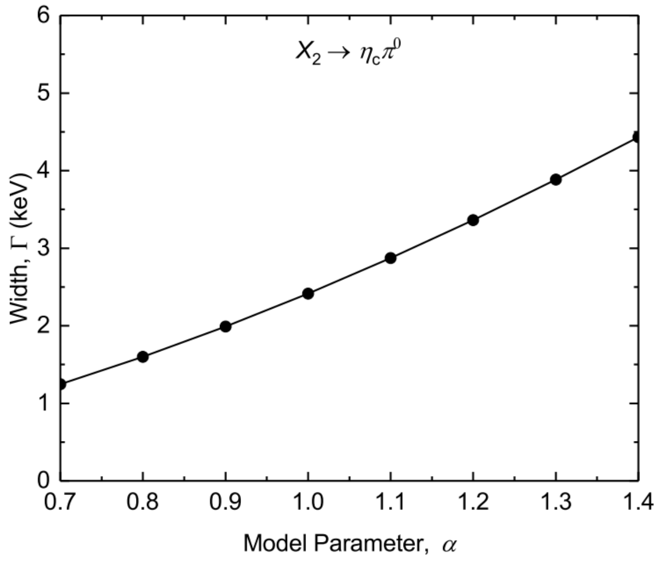



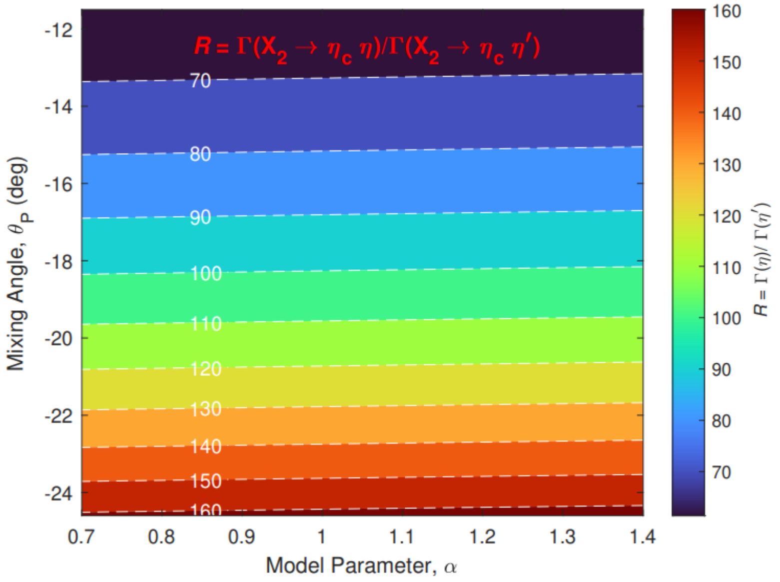



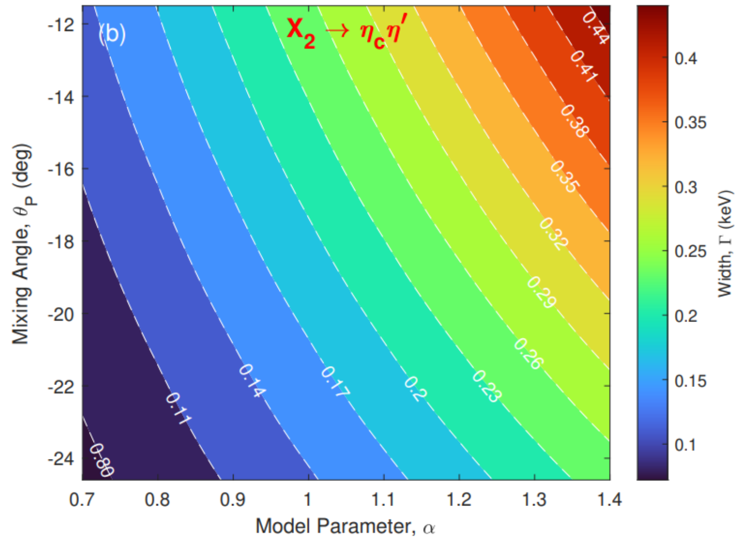

- The process $X_2 \to J/\psi\rho^0$ breaks isospin symmetry, while the process $X_2 \to J/\psi\omega$ is of isospin conservation.
- The interferences between the charged and neutral meson loops provide an important source of the isospin violation.
- At $m_{X_2} = 4.0137$ GeV, $\chi^0_{\rm nr} = 0$. At $m_{X_2} \cong 4.017$ GeV, $\chi^0_{\rm nr} = \chi^c_{\rm nr}$.

# Results: $X_2 \to P\eta_c$



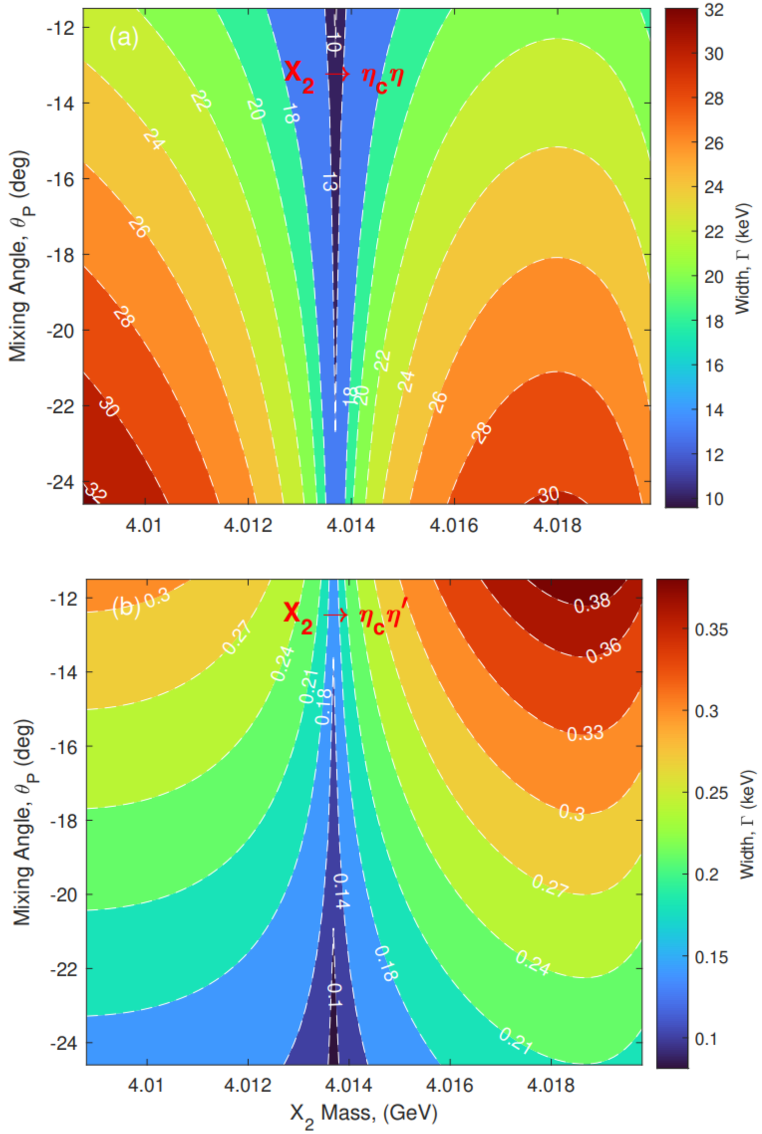



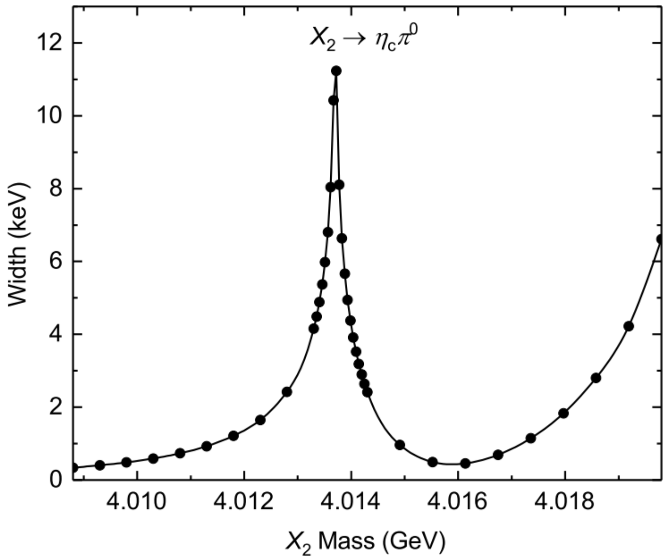

The $X_2$ mass is taken to be 4.0143 GeV.

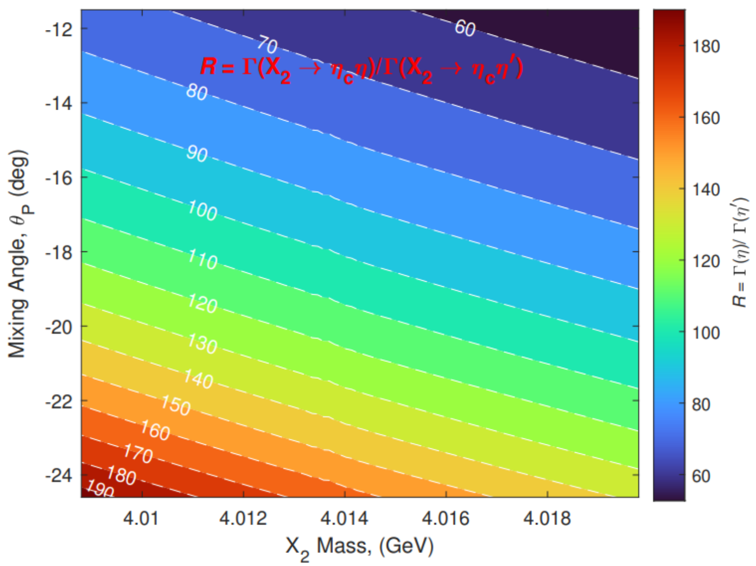
- For a given mixing angle, the widths increase while the model parameter $\alpha$ increases.
- At a given model parameter, the widths for $X_2 \to \eta_c\eta$ and $X_2 \to \eta_c\eta'$ show the opposite variation.
- The processes $X_2 \to \eta_c\eta$ and $X_2 \to \eta_c\eta'$ depend on the $\delta$ and $\gamma$. The $\gamma$ is increased with the increase of $\theta_P$ while $\delta$ is decreased.

# Results: $X_2 \to P\eta_c$




$\alpha = 1$




- At $m_{X_2} = 4.0137$ GeV, the width for $X_2 \to \eta_c\pi^0$ exhibits a maximum value, whereas the two decays of the $X_2 \to \eta_c\eta$ and $X_2 \to \eta_c\eta'$ show minimum widths.
- The decay process $X_2 \to \eta_c\pi^0$ violates isospin symmetry, while the other decays of the $X_2 \to \eta_c\eta$ and $X_2 \to \eta_c\eta'$ follow isospin conservation.
- At $m_{X_2} = 4.0137$ GeV, $\chi^0_{\rm nr} = 0$. At $m_{X_2} \cong 4.017$ GeV, $\chi^0_{\rm nr} = \chi^c_{\rm nr}$, being the same with the case of $X_2 \to J/\psi\rho^0(\omega)$.

# Summary

- In calculations, we assume the $X_2$ as a molecular state of the $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ with equal proportion.
- The calculated results indicate that the widths are all model-$\alpha$ dependent. The relative ratios between the widths of different processes are nearly model-$\alpha$ independent and quite sensitive to the $X_2$ mass.
- At $m_{X_2} = 4.0137$ GeV, the decays that violate the isospin symmetry exhibit widths of peak values, whereas those remaining the isospin symmetry show minimum widths. Near $m_{X_2} = 4.017$ GeV, the opposite happens.

# Thanks for your attention!